

# Few- and many-body physics of mass-imbalanced two- components systems

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Saitama, Japan

“Living Near Unitarity”

Kavli Institute for Theoretical Physics

UC Santa Barbara

May 12, 2022

## Pascal Naidon, RIKEN

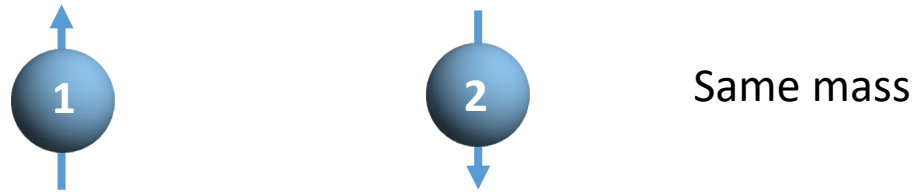
[ribf.riken.jp/~pascal/](http://ribf.riken.jp/~pascal/)



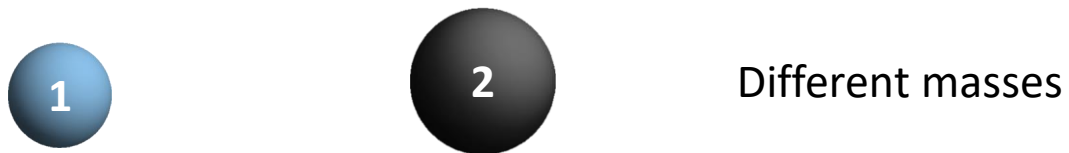
- 2005 PhD in France (Laboratoire Aimé Cotton, Orsay) in ultracold atom theory.  
**Formation of molecules in Bose-Einstein condensates**
- 2005-2008 Postdoctoral researcher at NIST (National Institute of Standards and Technology)  
**Properties of Alkaline-earth atoms for atomic clocks**
- 2008-2012 Postdoctoral researcher at the University of Tokyo.  
**Efimov states in ultracold-atom experiments**
- 2012 Research Scientist at RIKEN  
**Universal few-body and many-body physics**

# Two-component systems

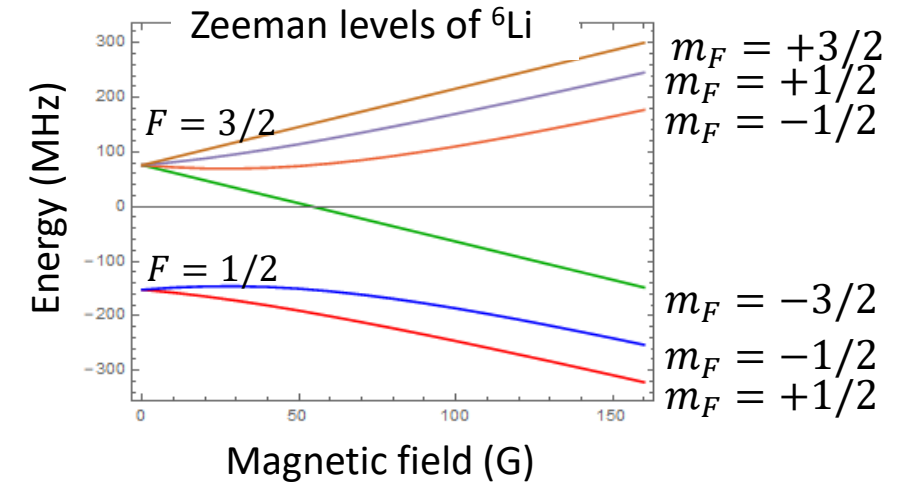
- Identical particles in two different states



- Two different kinds of particles



Spin  $\frac{1}{2}$  particles (e.g. Neutrons)  
Atoms in different hyperfine states



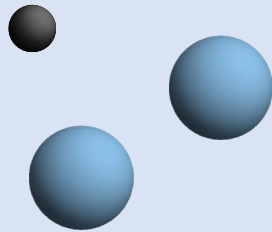
Different species of atoms in a polarised hyperfine state

# Two-component systems

## Boson mixtures

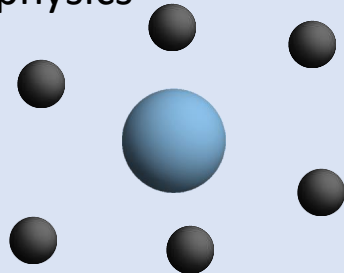
Few-body

① Efimov trimers of bosons

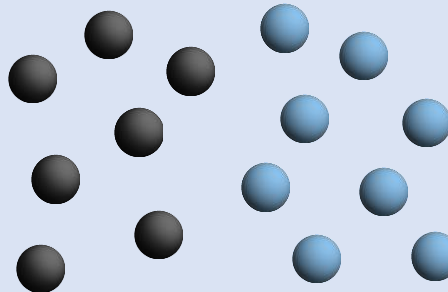


Many-body

② Polaron physics

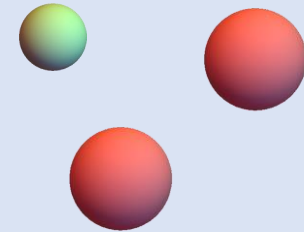


③ Miscibility physics

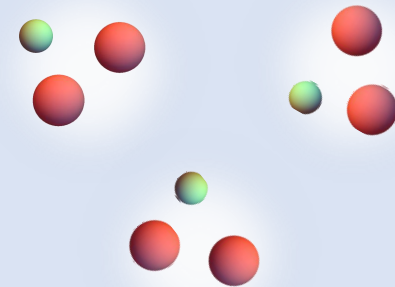


## Fermion mixtures

④ Universal trimers of fermions

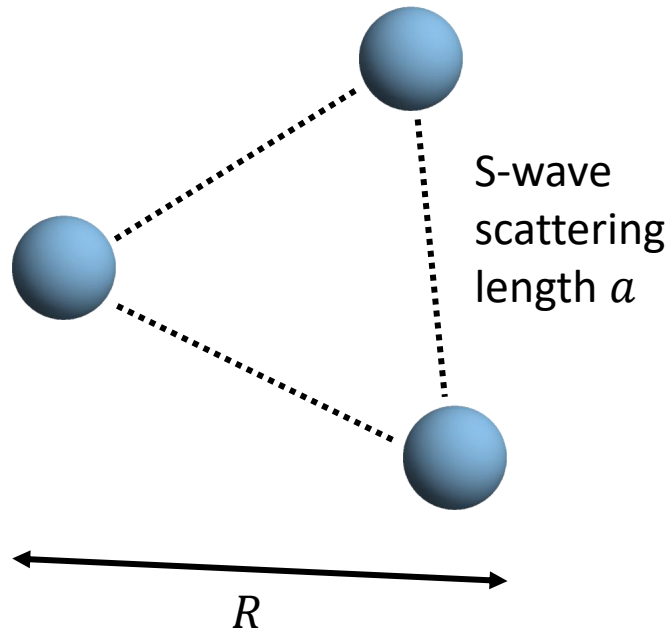


⑤ Trimer phase

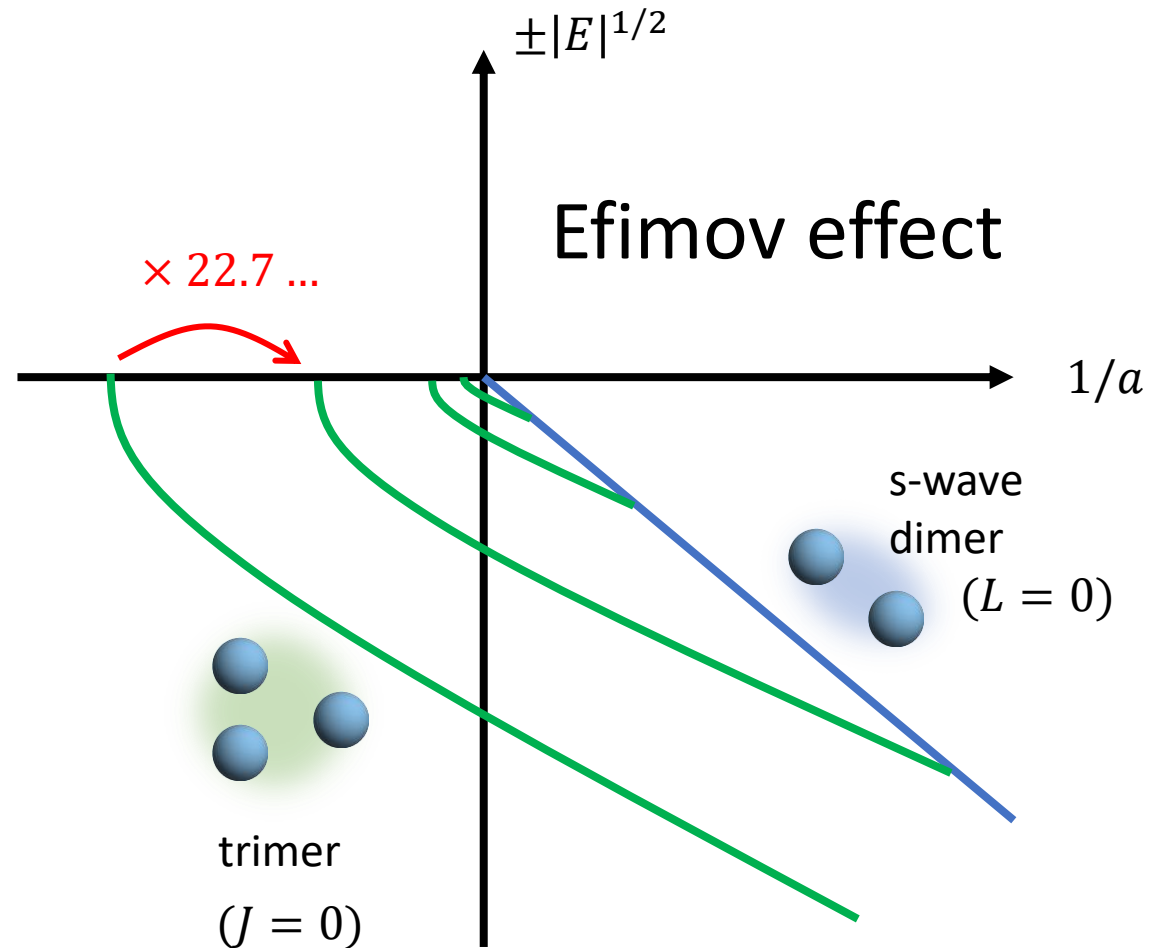


① Efimov trimers of bosons

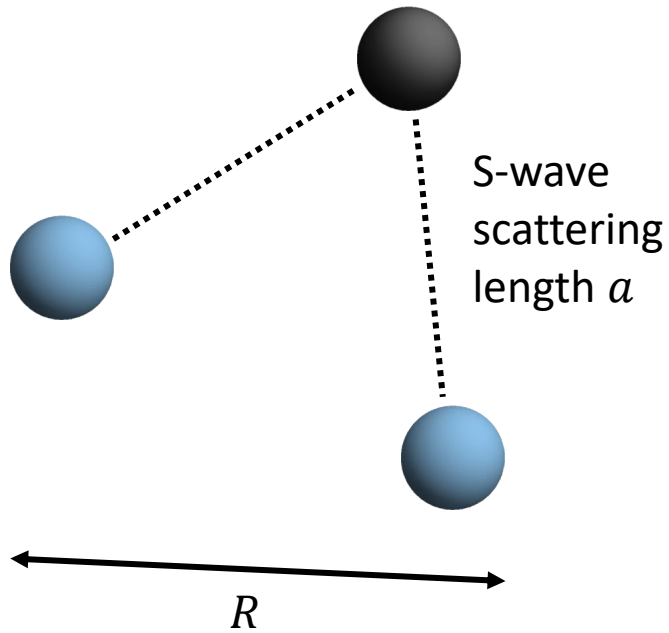
# 3 identical bosons



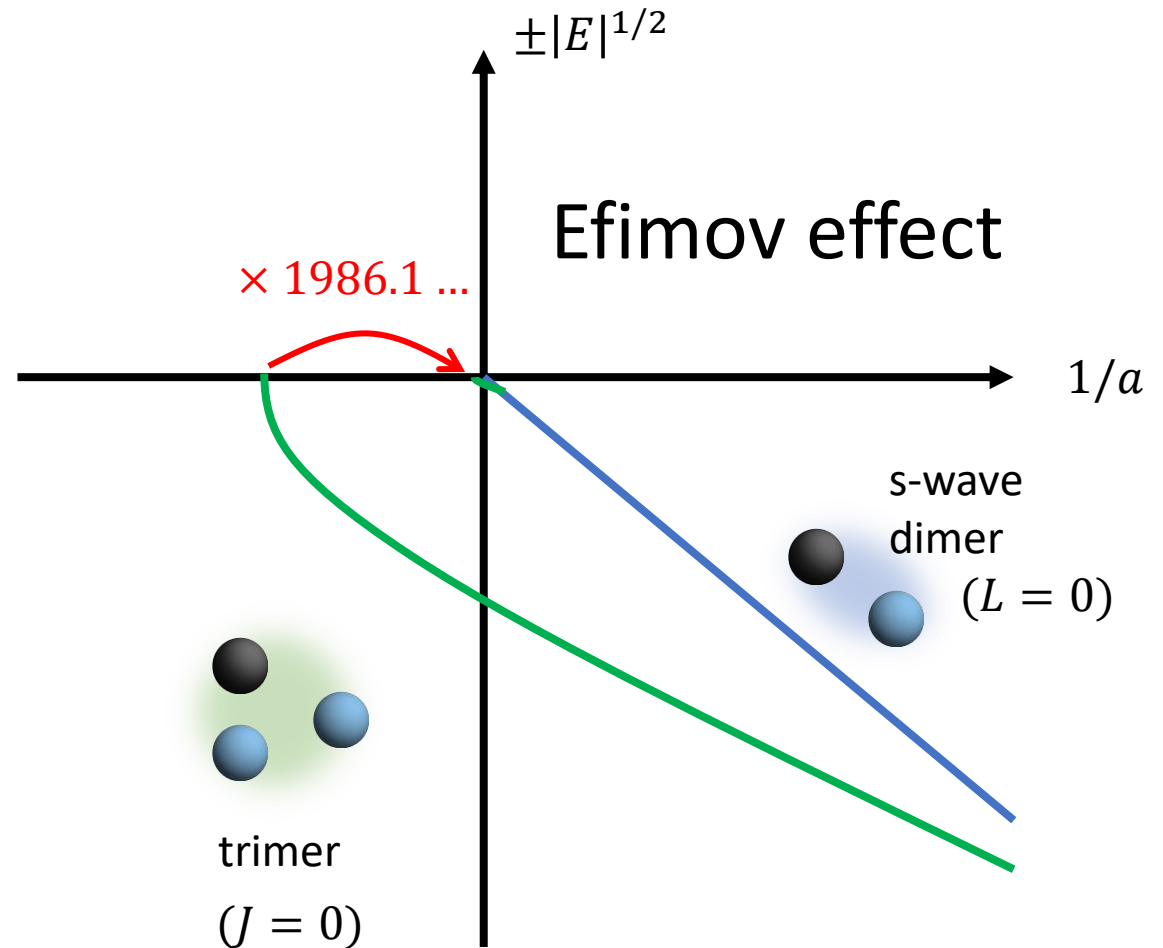
Efimov attraction:  $V_{3B}(R) \sim -\frac{1}{R^2}$



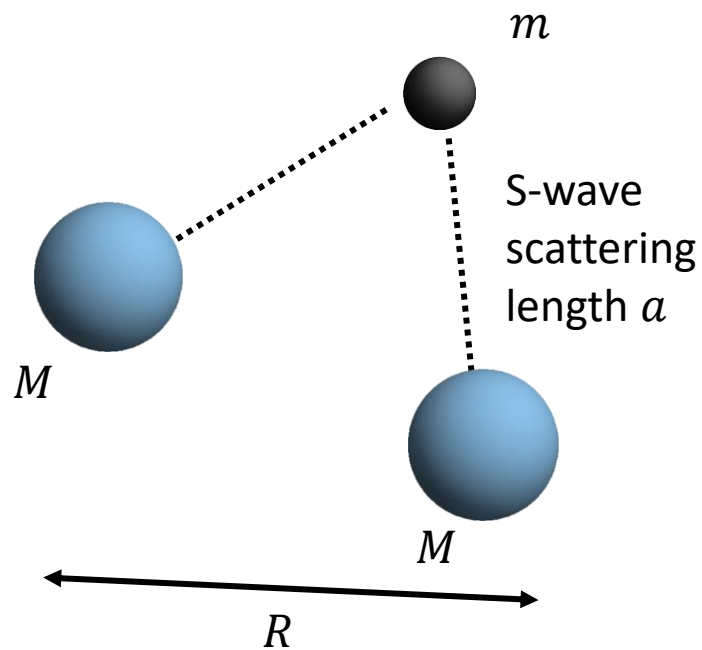
# 2 identical bosons + 1 particle



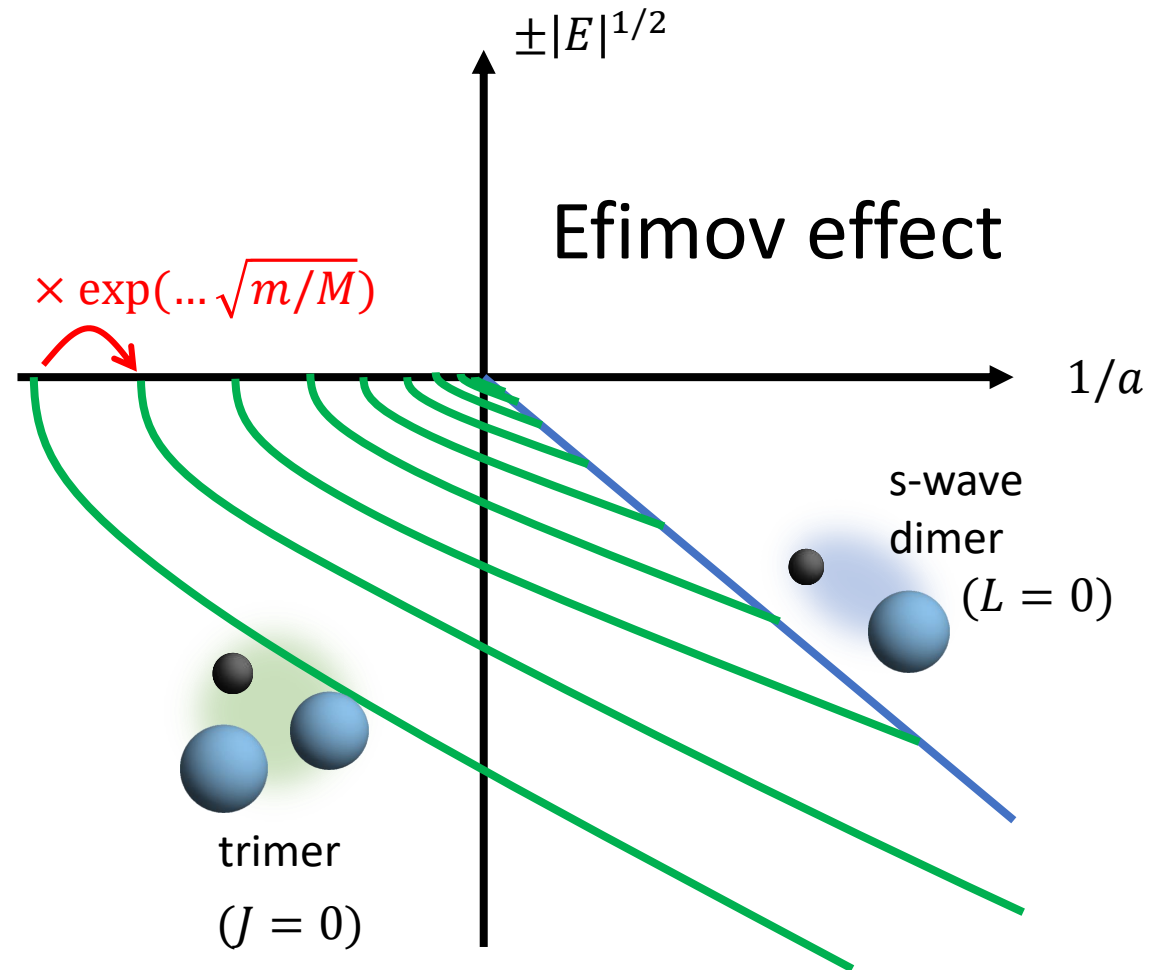
Efimov attraction:  $V_{3B}(R) \sim -\frac{0.17}{R^2}$



# 2 identical bosons + 1 particle



Efimov attraction:  $V_{3B}(R) \sim -\frac{M}{m} \frac{0.16}{R^2}$





## ② Polaron physics

**Two impurities in a Bose-Einstein condensate: from Yukawa to Efimov attracted polarons**

J. Phys. Soc. Jpn. 87, 043002 (2018)

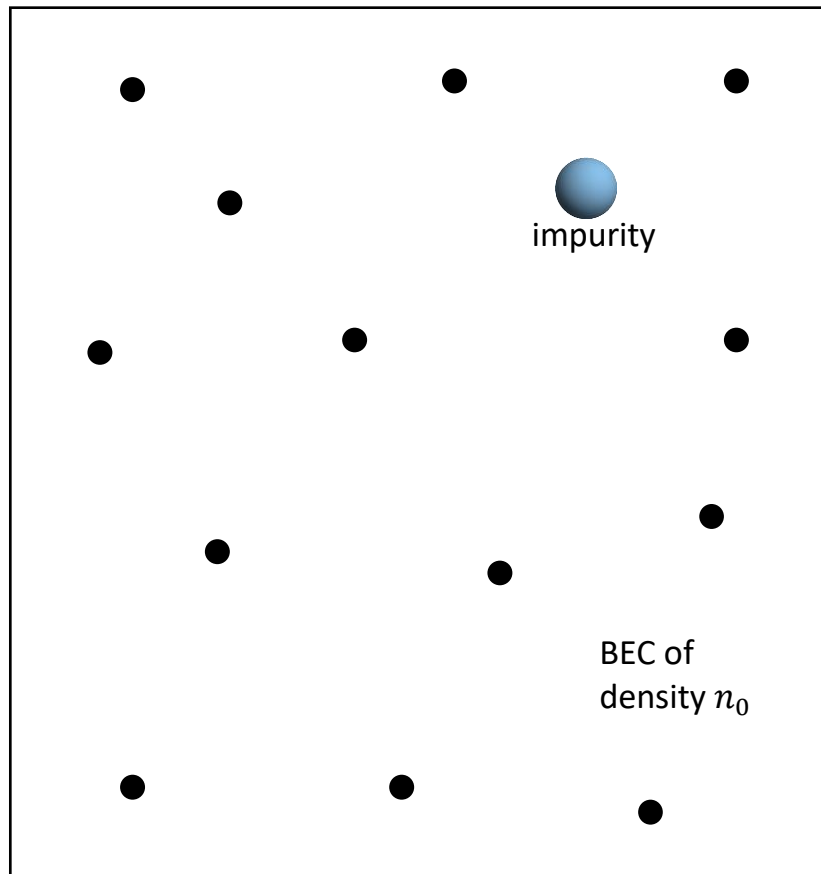
[arxiv:1607.04507]

**Tetramers of two heavy and two light bosons**

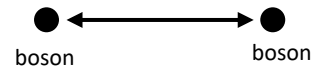
Few-Body Syst 59, 64 (2018)

[arxiv:1802.06237]

# Polaron: particle interacting with a medium



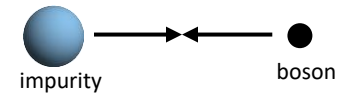
Short-range repulsion



Small scattering  
length  $a_B > 0$

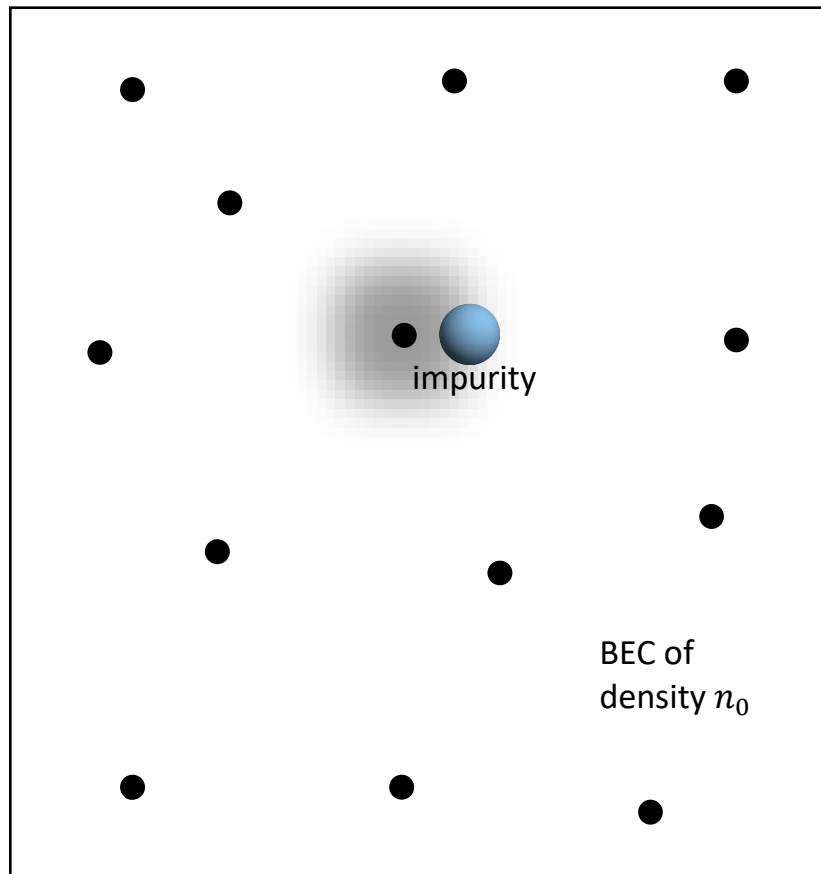
$$n_0 a_B^3 \ll 1$$

Short-range attraction

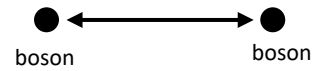


Small or large  
scattering length  $a$

# Polaron: particle interacting with a medium



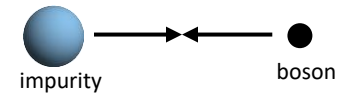
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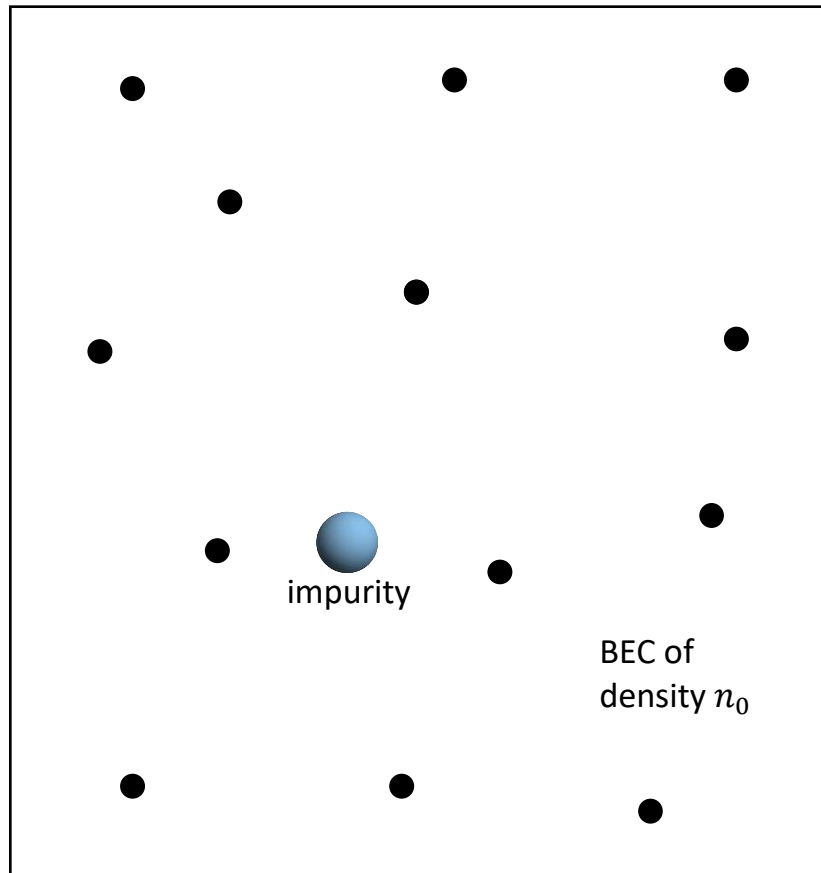
$$n_0 a_B^3 \ll 1$$

Short-range attraction

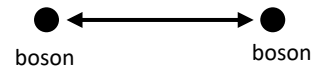


Small or large  
scattering length  $a$

# Polaron: particle interacting with a medium



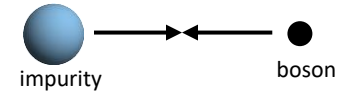
Short-range repulsion



Small scattering  
length  $a_B > 0$

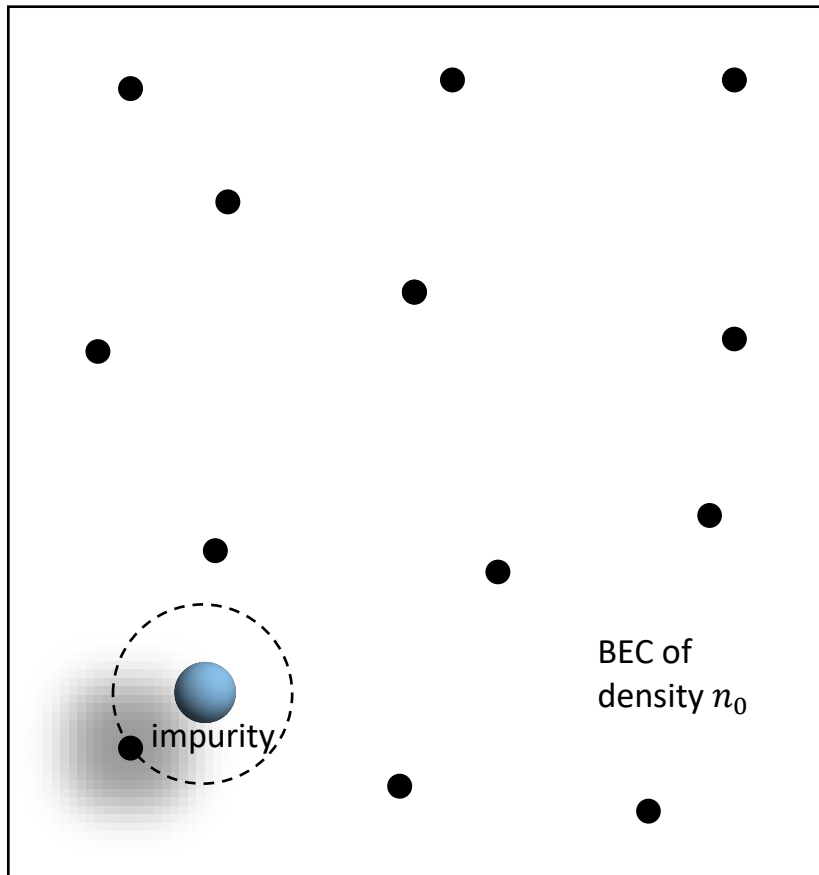
$$n_0 a_B^3 \ll 1$$

Short-range attraction

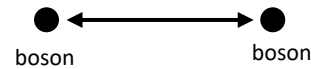


Small or large  
scattering length  $a$

# Polaron: particle interacting with a medium



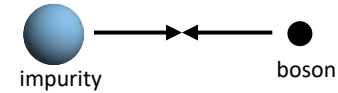
Short-range repulsion



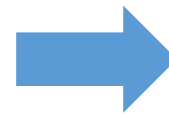
Small scattering  
length  $a_B > 0$

$$n_0 a_B^3 \ll 1$$

Short-range attraction



Small or large  
scattering length  $a$



“Bose polaron”

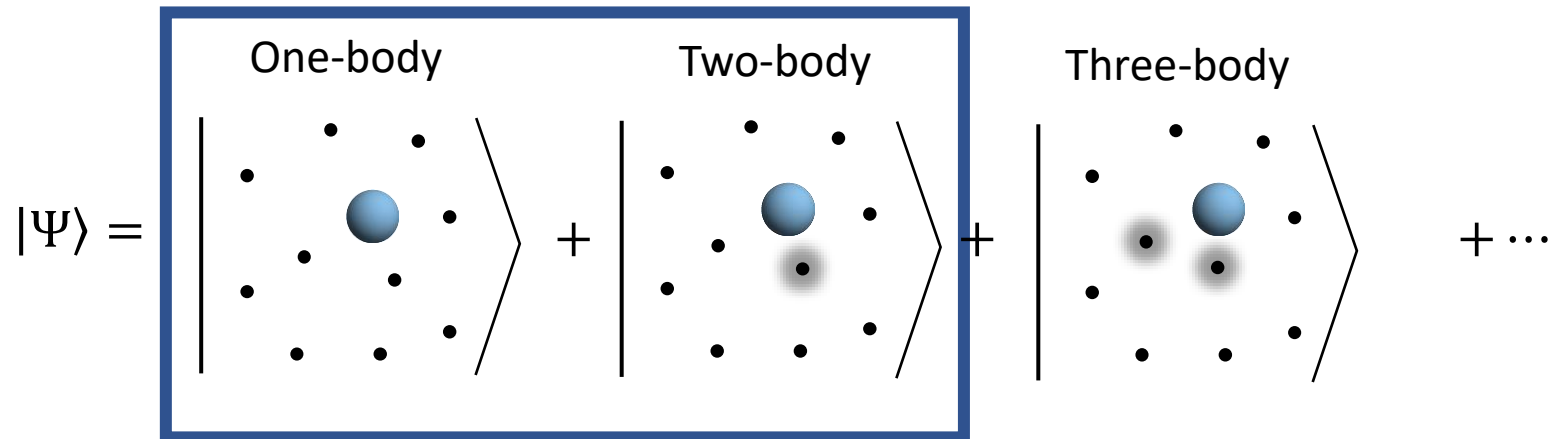
“Artificial nucleon”

How do they interact?

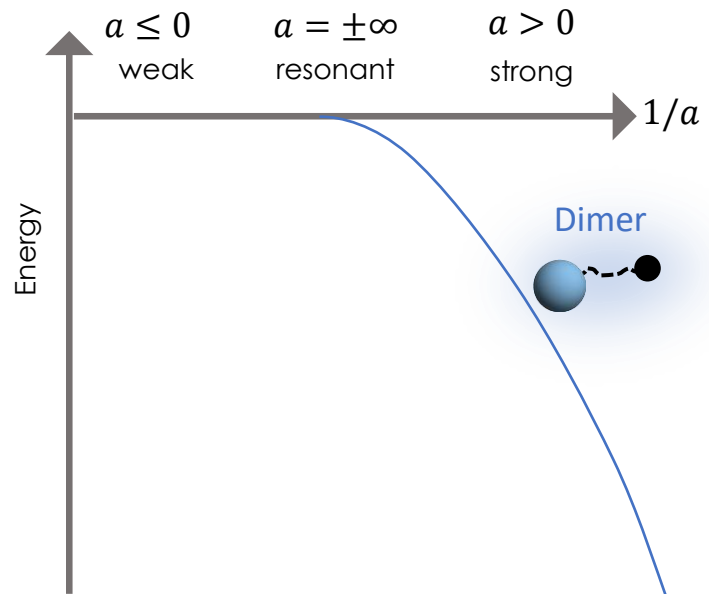
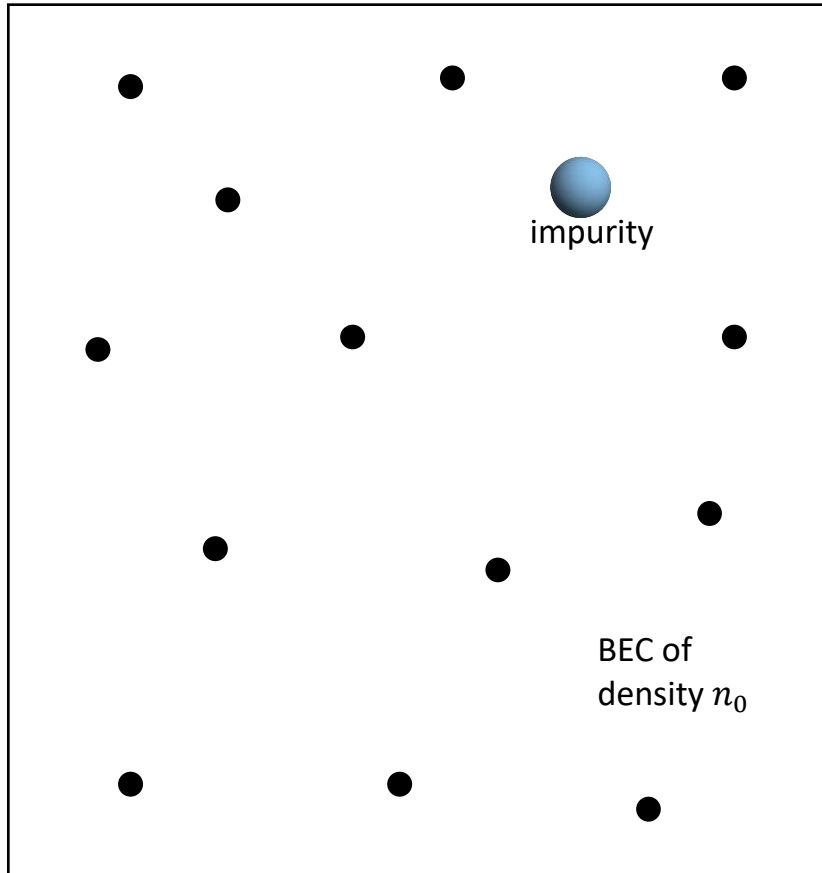
How can they cluster?

# One Bose polaron

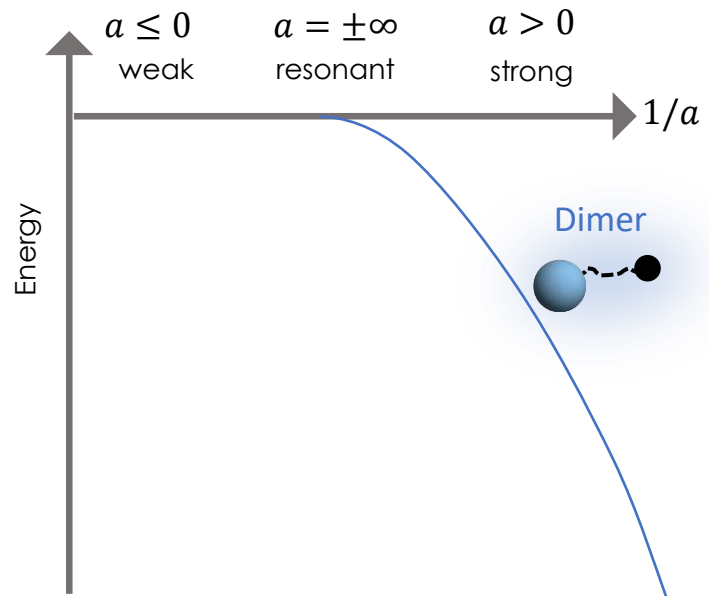
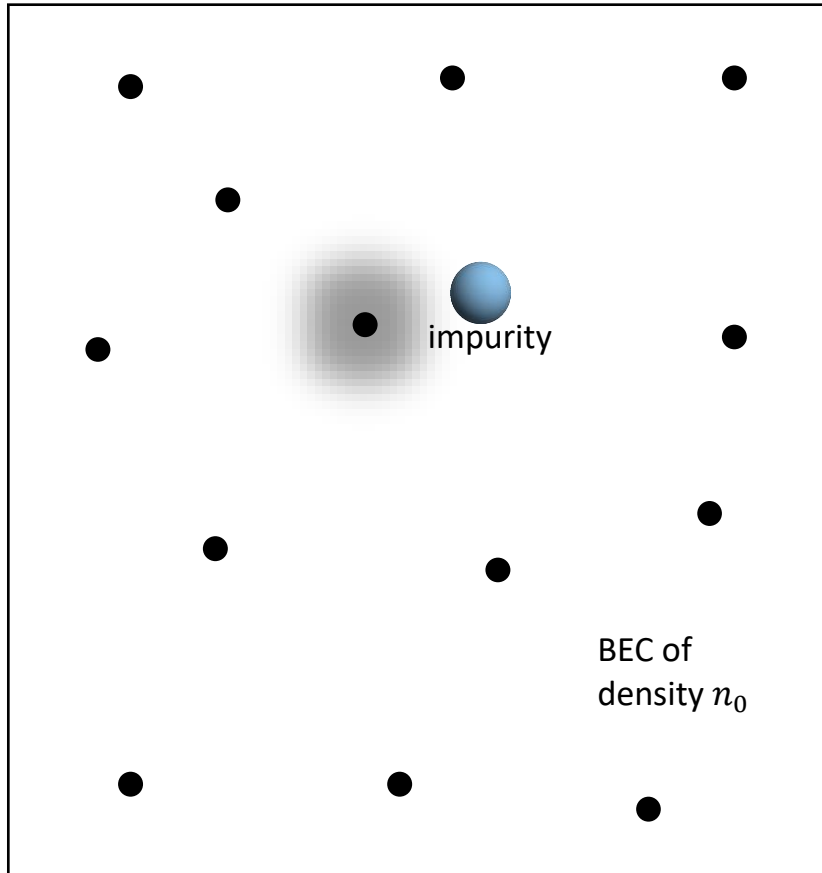
The polaron problem can be treated as a set of coupled few-body problems  
(Truncated basis method)



# One Bose polaron

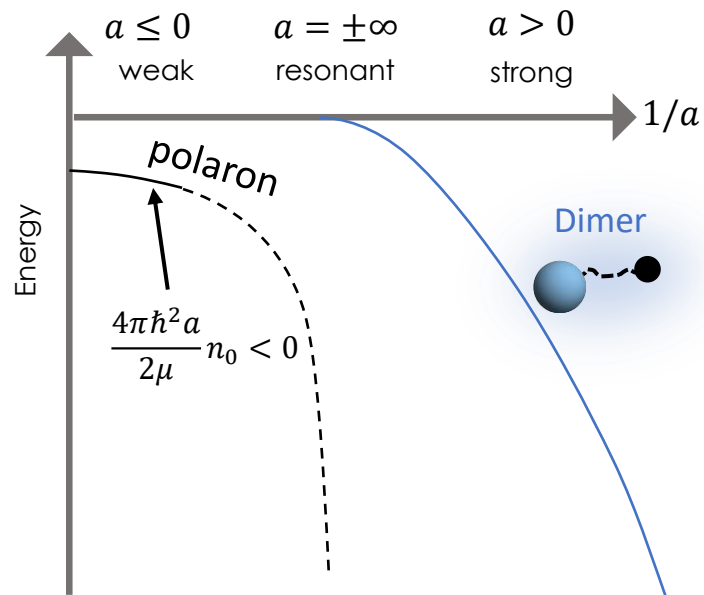
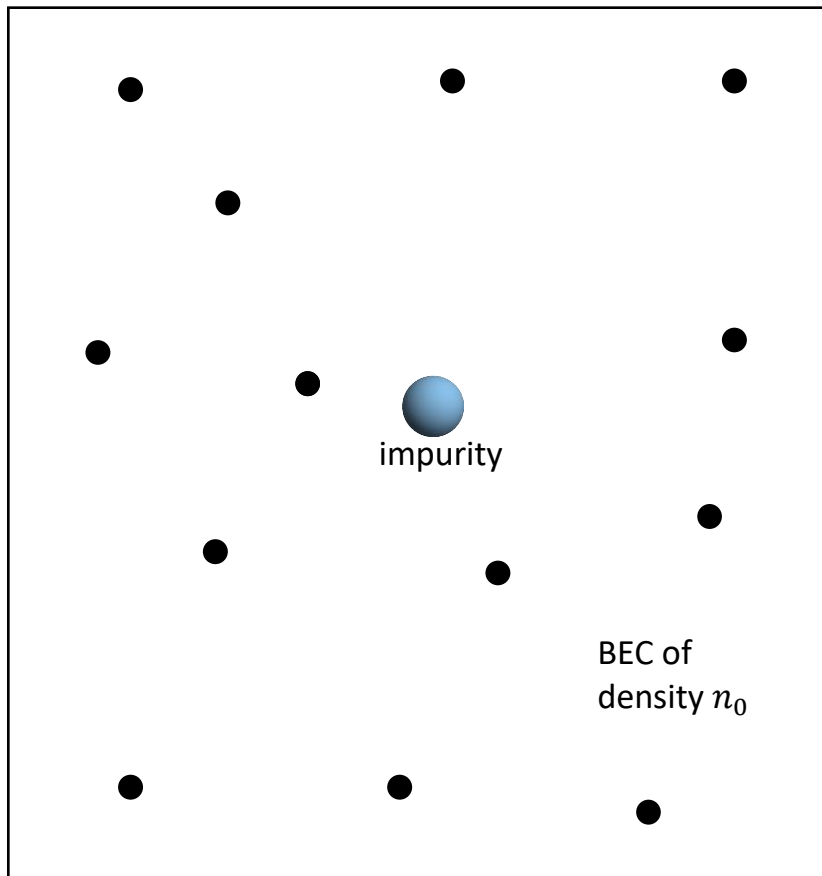


# One Bose polaron

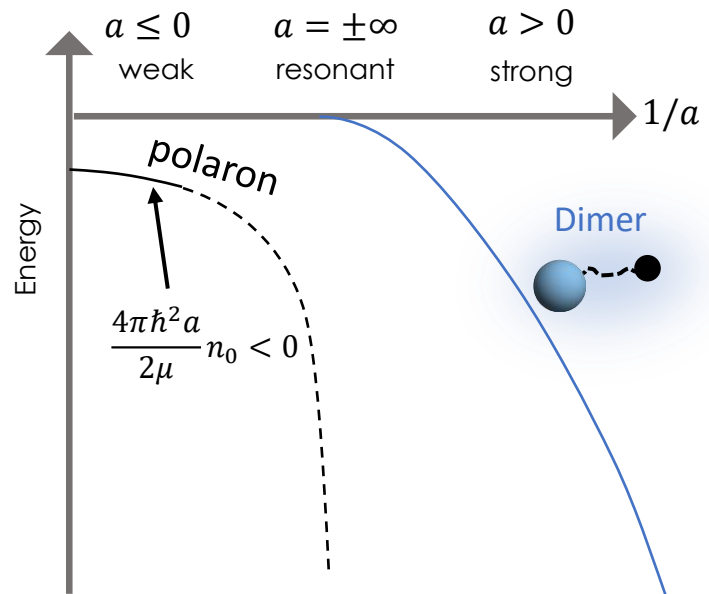
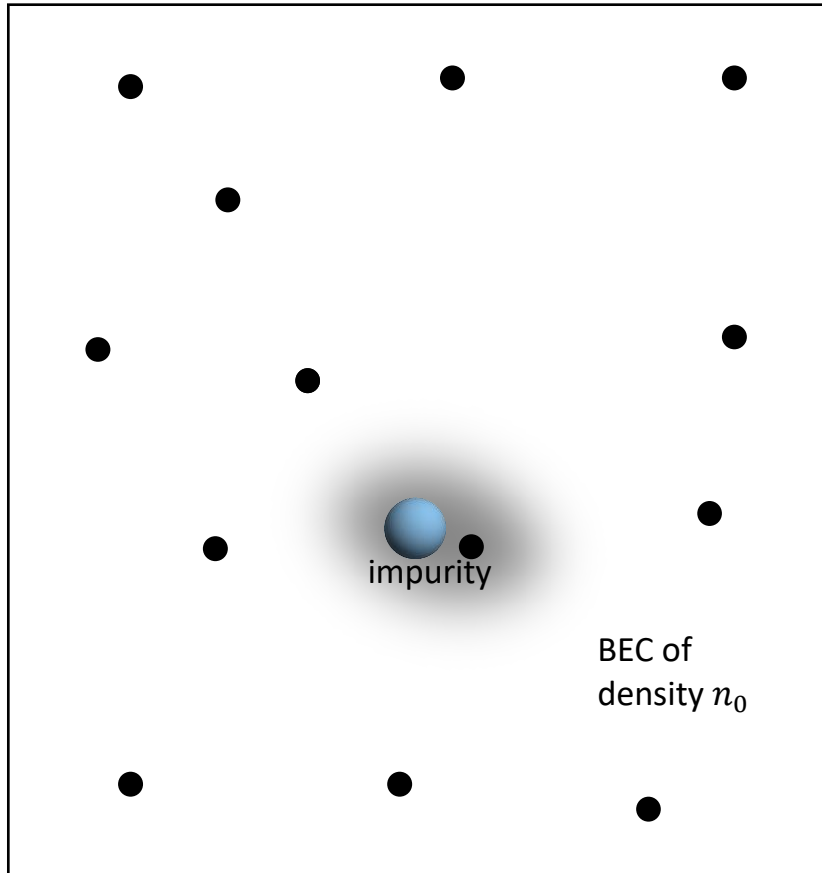




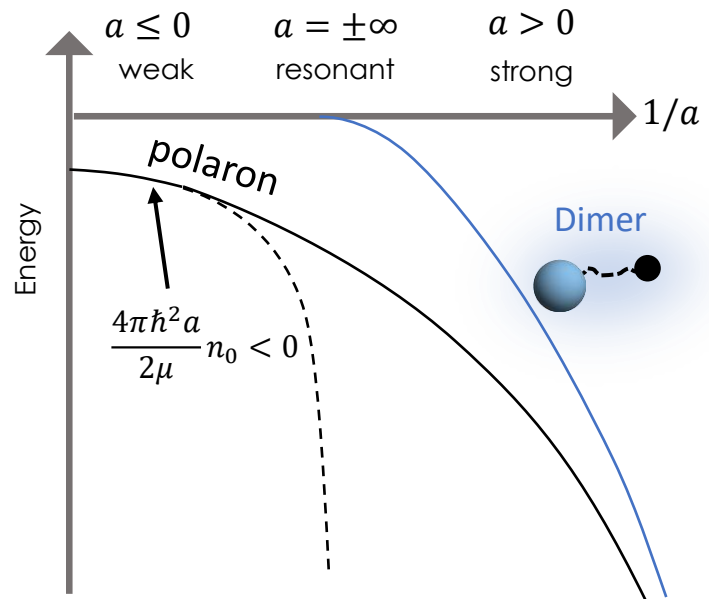
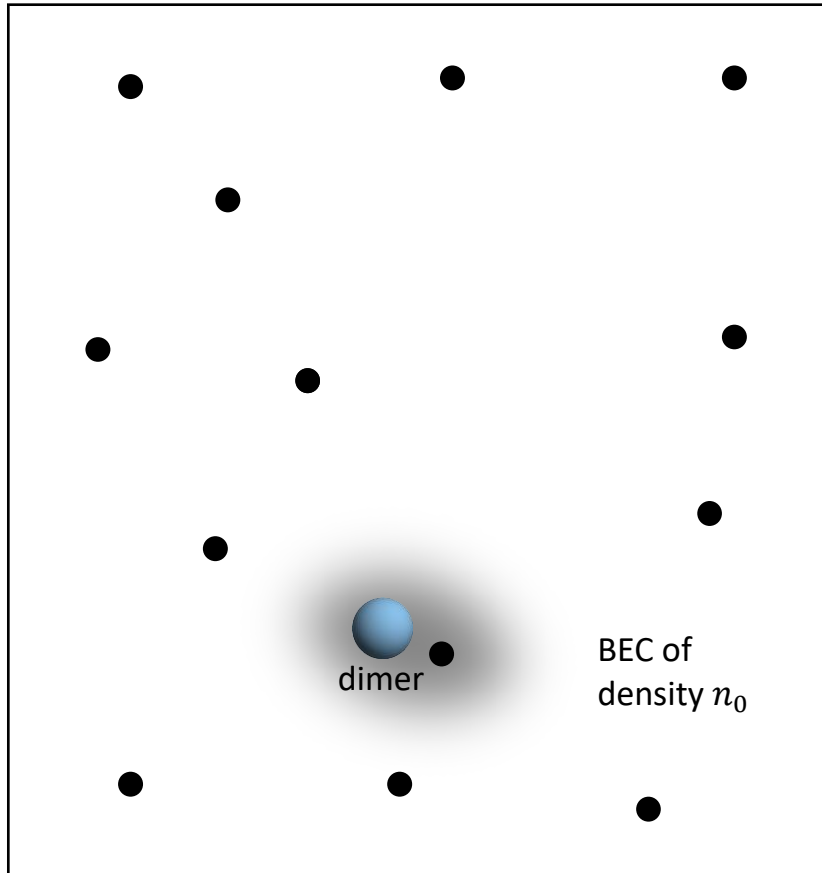
# One Bose polaron



# One Bose polaron

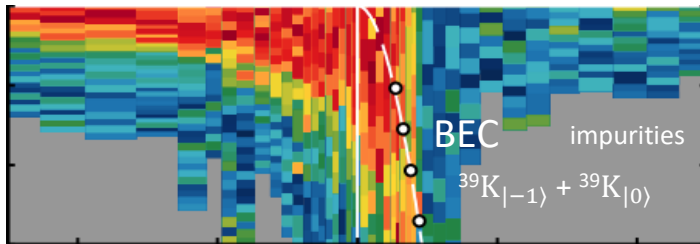


# One Bose polaron

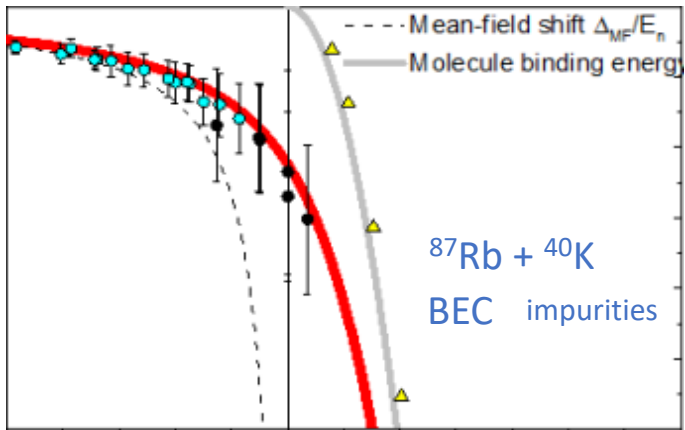


# One Bose polaron

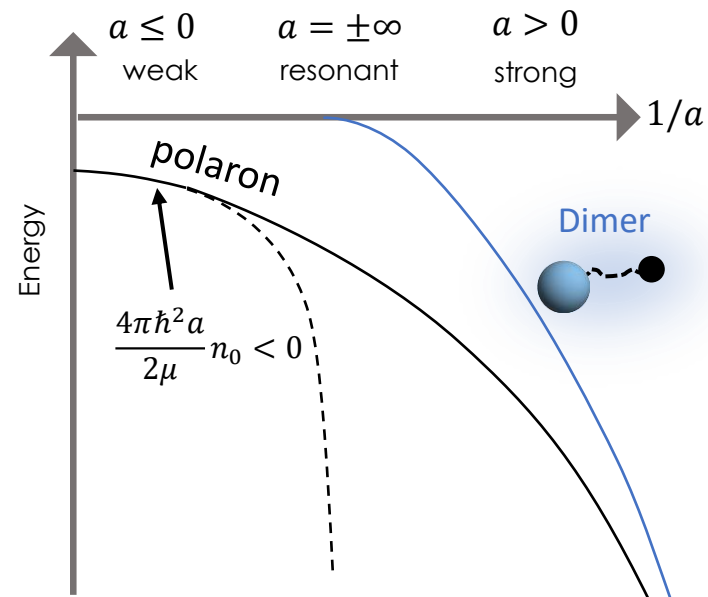
## Observation of the Bose polaron



Jørgensen et al, PRL 117, 055302 (2016)

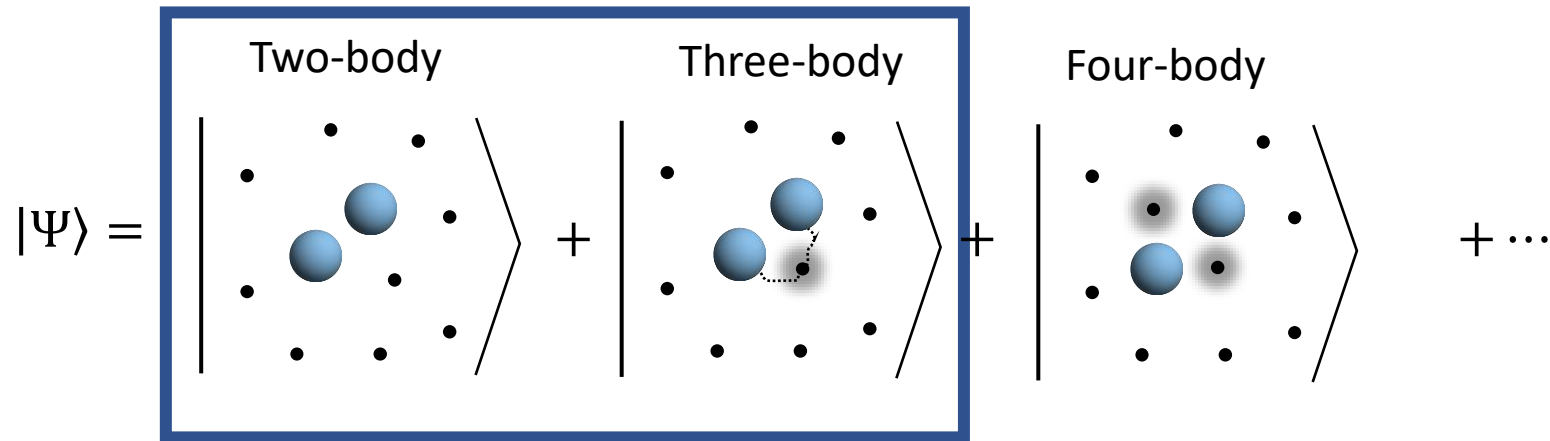


Ming-Guang Hu et al, PRL 117, 055301 (2016)



# Two Bose polarons

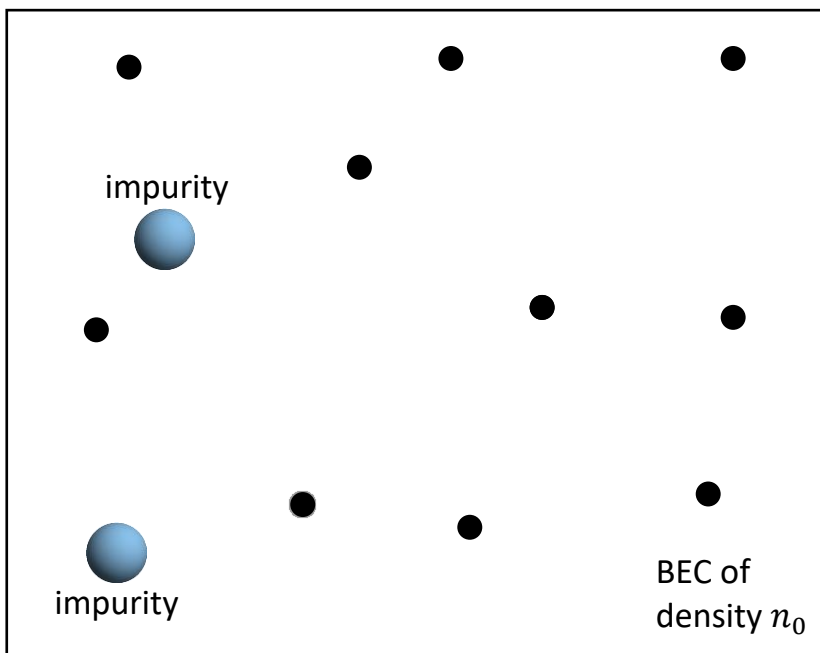
The two-polaron problem can be treated as a set of coupled few-body problems (Truncated basis method)



Excitations can mediate an interaction between the two impurities

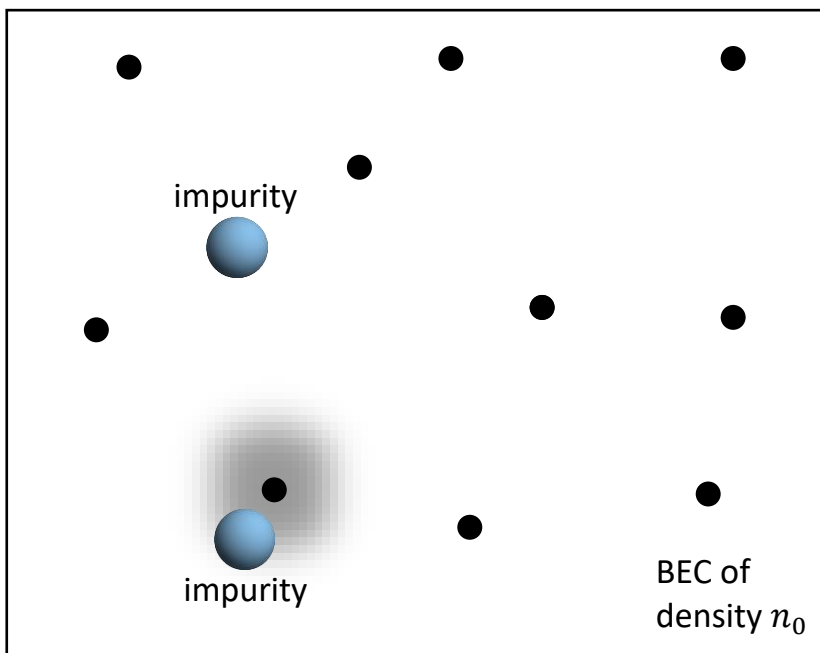
# Two Bose polarons

Weak interaction

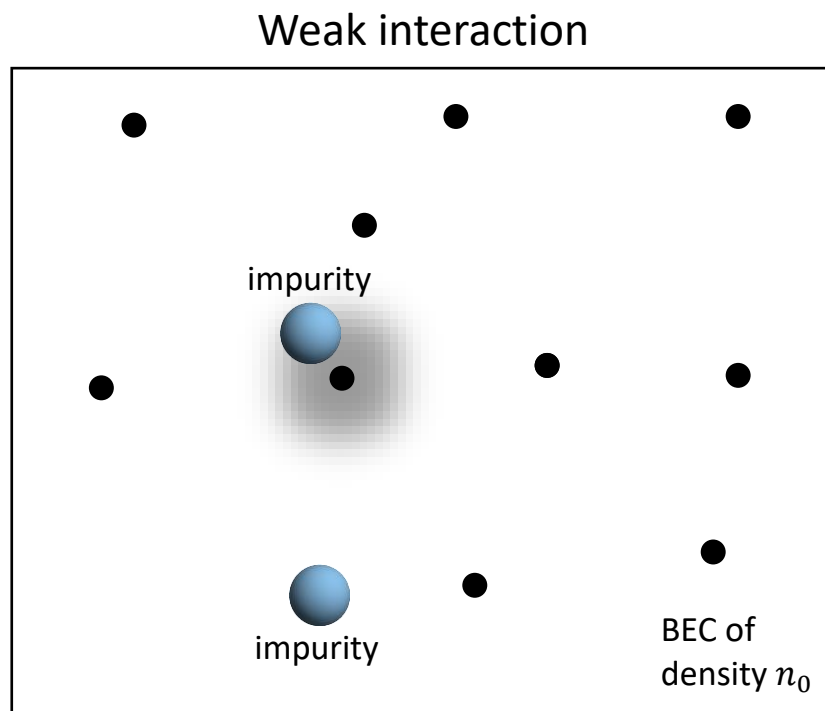


# Two Bose polarons

Weak interaction



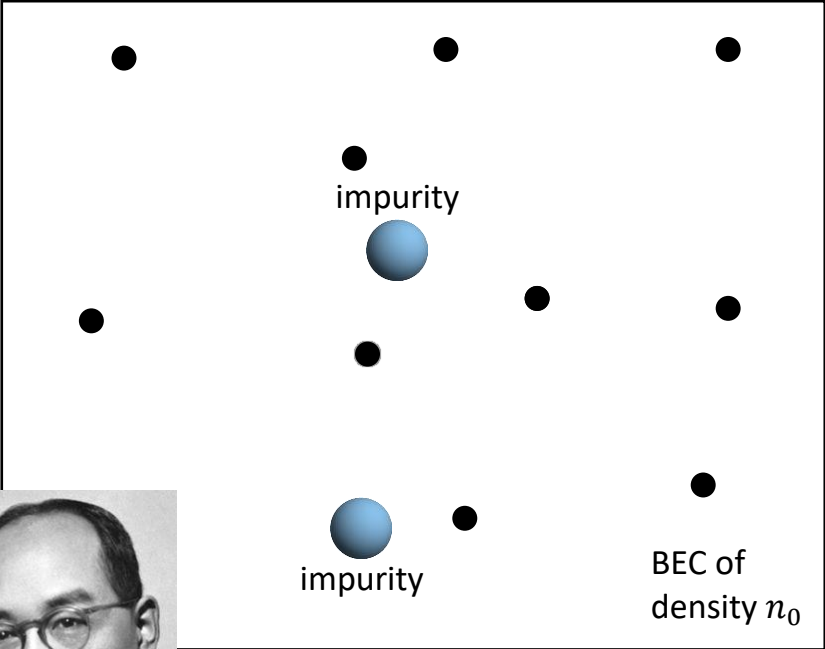
# Two Bose polarons





# Two Bose polarons

Weak interaction



Yukawa potential

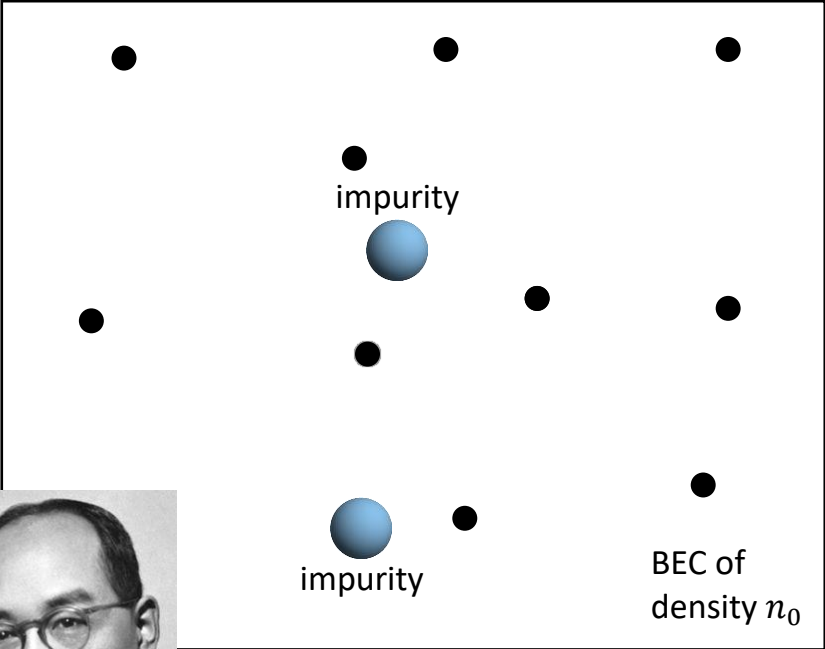
$$V(r) \propto -a^2 n_0 \frac{e^{-\sqrt{2}r/\xi}}{r}$$

A diagram showing two blue spheres representing impurities. A dashed line connects them, with a small black dot at the midpoint. Two blue arrows originate from the spheres, pointing outwards. The distance between the spheres is labeled 'a'.

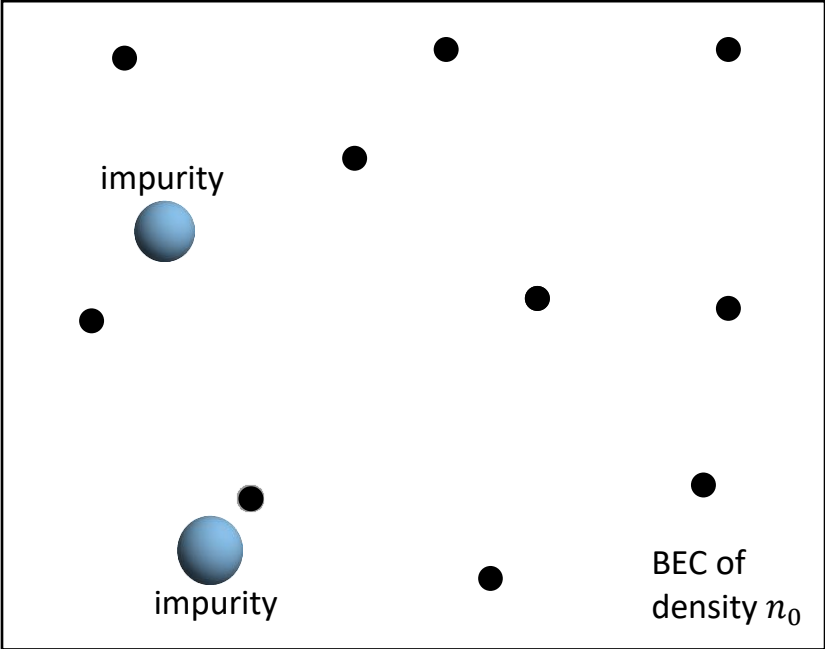
$$\xi = (8\pi n_0 a_B)^{-1/2}$$

# Two Bose polarons

Weak interaction



Resonant interaction



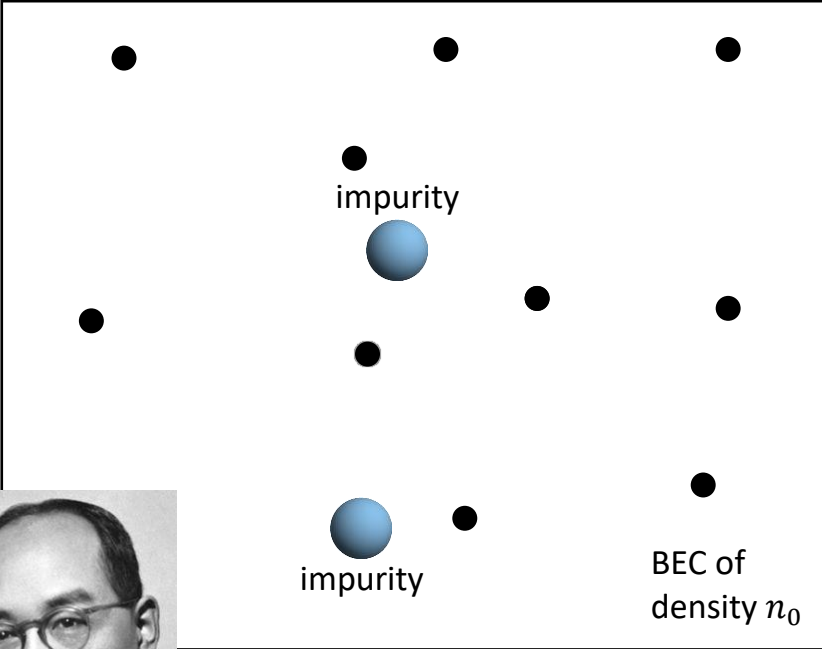
Yukawa potential

$$V(r) \propto -a^2 n_0 \frac{e^{-\sqrt{2}r/\xi}}{r}$$

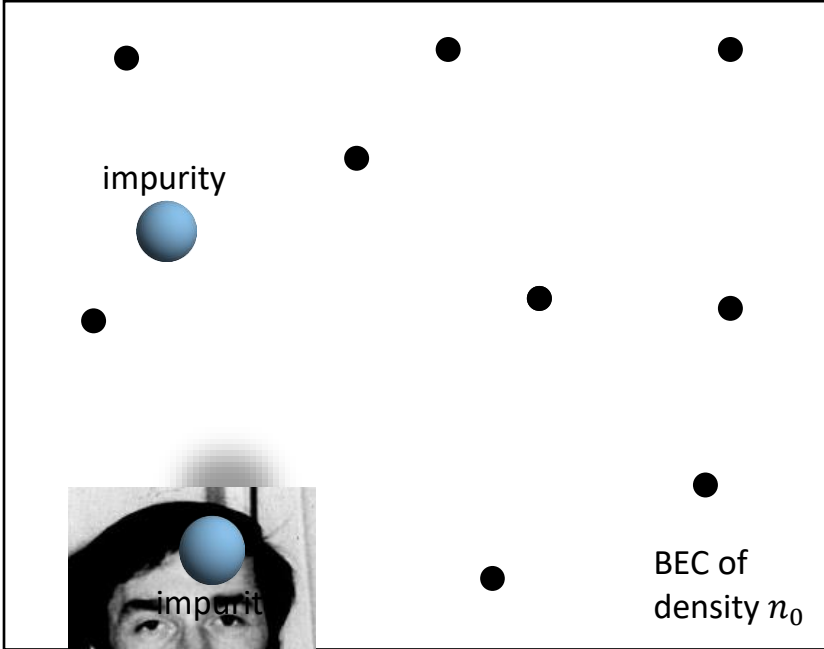
A diagram illustrating the Yukawa potential between two particles (blue spheres). The distance between them is labeled  $r$ . The interaction range is indicated by a dashed line of length  $a$ .

# Two Bose polarons

Weak interaction



Resonant interaction



Yukawa potential

$$V(r) \propto -a^2 n_0 \frac{e^{-\sqrt{2}r/\xi}}{r}$$

Efimov potential

$$V(r) = -\frac{\hbar^2 0.16}{m r^2}$$

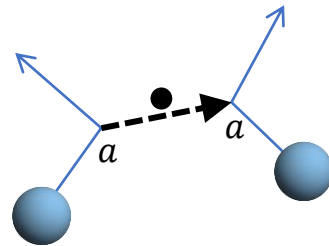
# Two Bose polarons

Weak interaction



Yukawa potential

$$V(r) \propto -a^2 n_0 \frac{e^{-\sqrt{2}r/\xi}}{r}$$

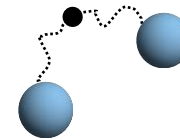


Resonant interaction



Efimov potential

$$V(r) = -\frac{\hbar^2 0.16}{m r^2}$$



“exchange of virtual bosons”



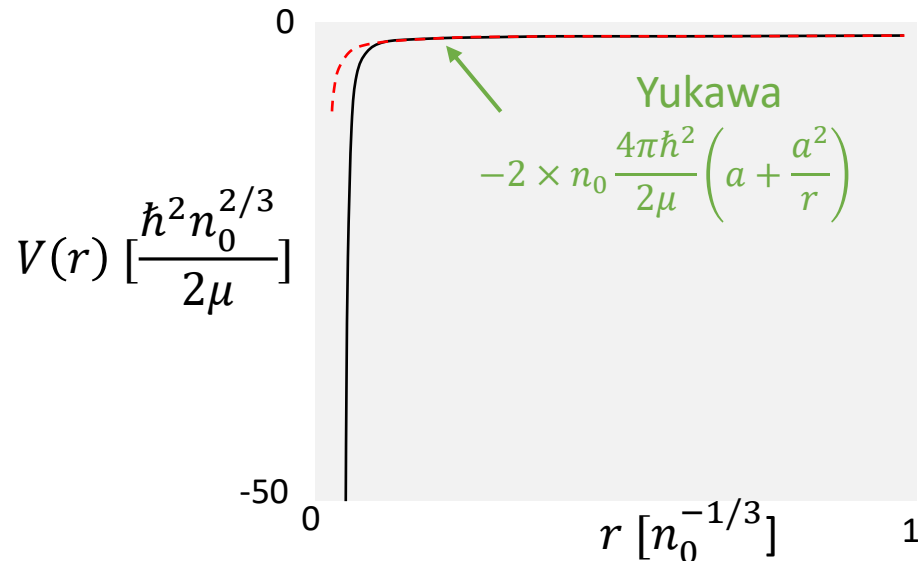
“exchange of real bosons”

# Two Bose polarons

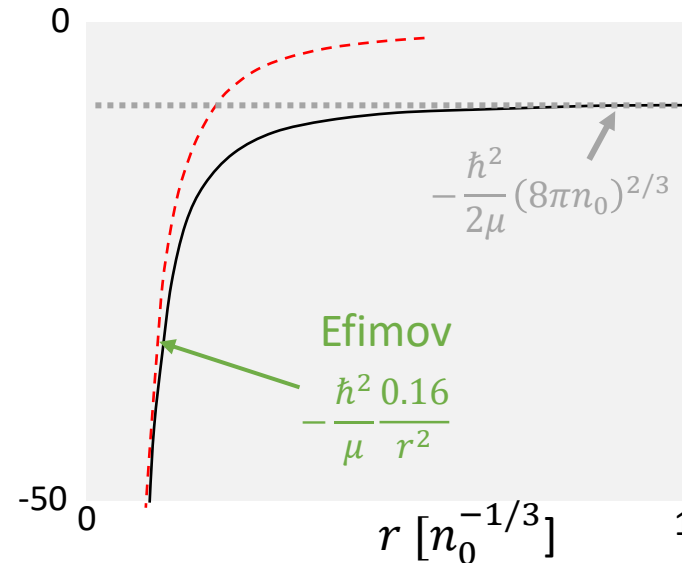
Effective potential (Born-Oppenheimer) between polarons:

$$V(r) = -\frac{\hbar^2 \kappa^2}{2\mu} \quad \frac{1}{a} - \kappa + \frac{1}{r} e^{-\kappa r} + \frac{8\pi n_0}{\kappa^2} = 0 \quad (a_B \rightarrow 0)$$

For small  $a$  ( $a = -0.05 n_0^{-1/3}$ )



At resonance  $a = \pm\infty$



# Two Bose polarons

Calculation of the spectrum at a given mass ratio

Mass of  
impurities



$$\frac{M}{m} = 19$$

Mass of  
bosons



Ex:

impurities: Cesium-133

Bosons: Lithium-7

$$n_0 a_B^3 = 0.00001$$

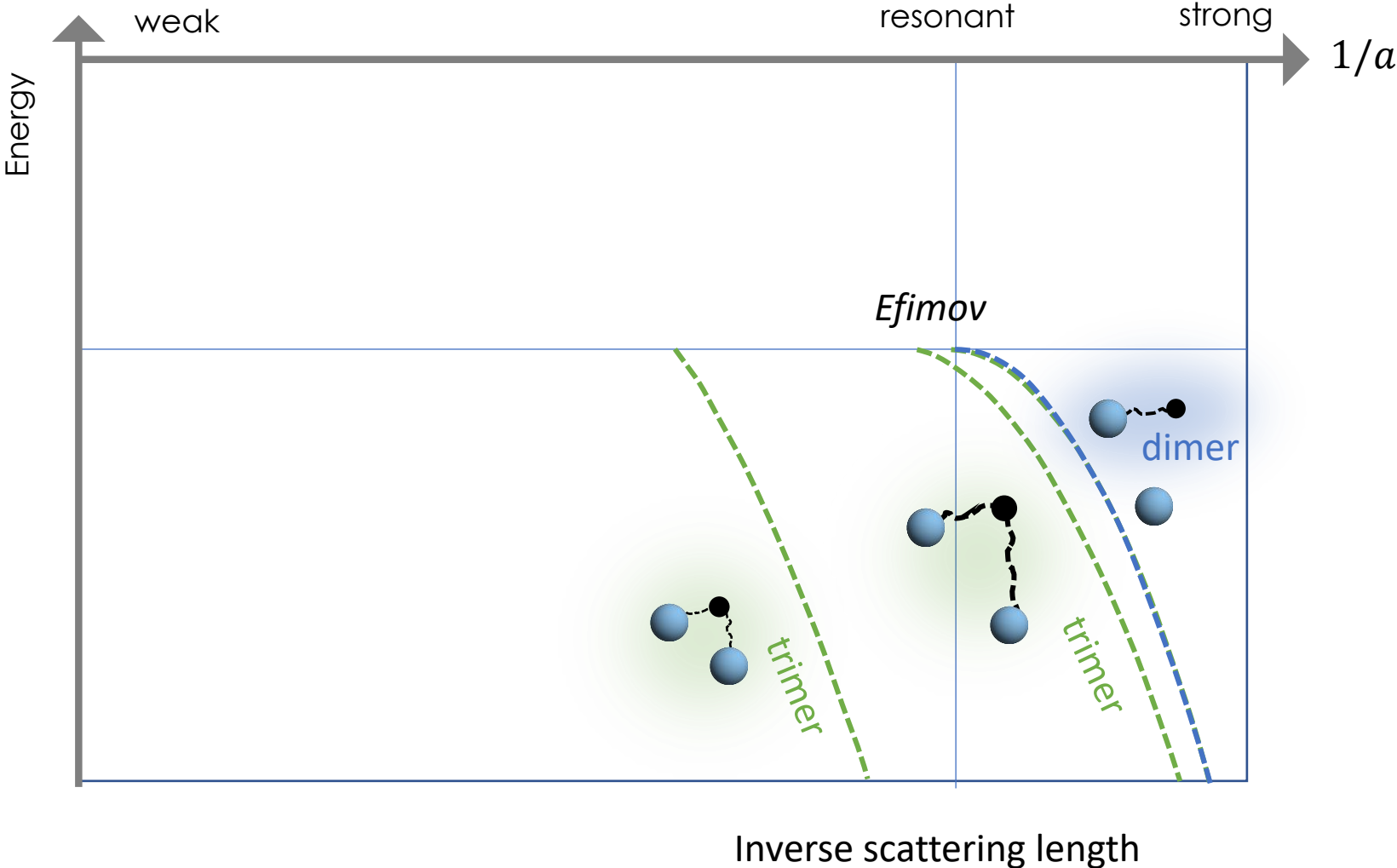


Density of  
bosons

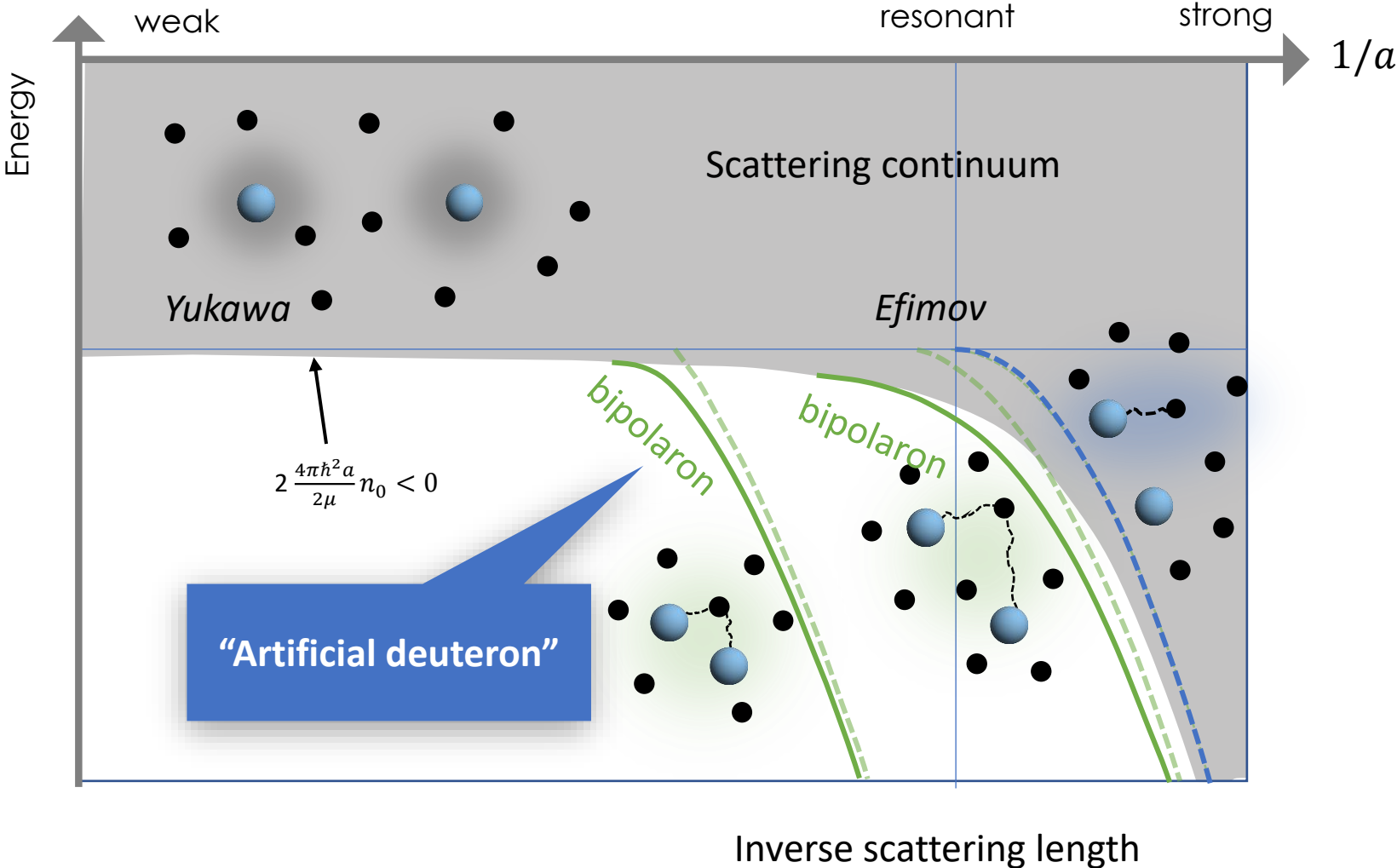


Scattering length  
of bosons

# Two Bose polarons



# Two Bose polarons





# Two Bose polarons

The two-polaron problem can be treated as a set of coupled few-body problems

$$|\Psi\rangle = \left[ \begin{array}{c} \text{Two-body} \\ \left| \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right\rangle \\ \text{Two-body} \end{array} \right] + \left[ \begin{array}{c} \text{Three-body} \\ \left| \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right\rangle \\ \text{Three-body} \end{array} \right] + \left[ \begin{array}{c} \text{Four-body} \\ \left| \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right\rangle \\ \text{Four-body} \end{array} \right] + \dots$$

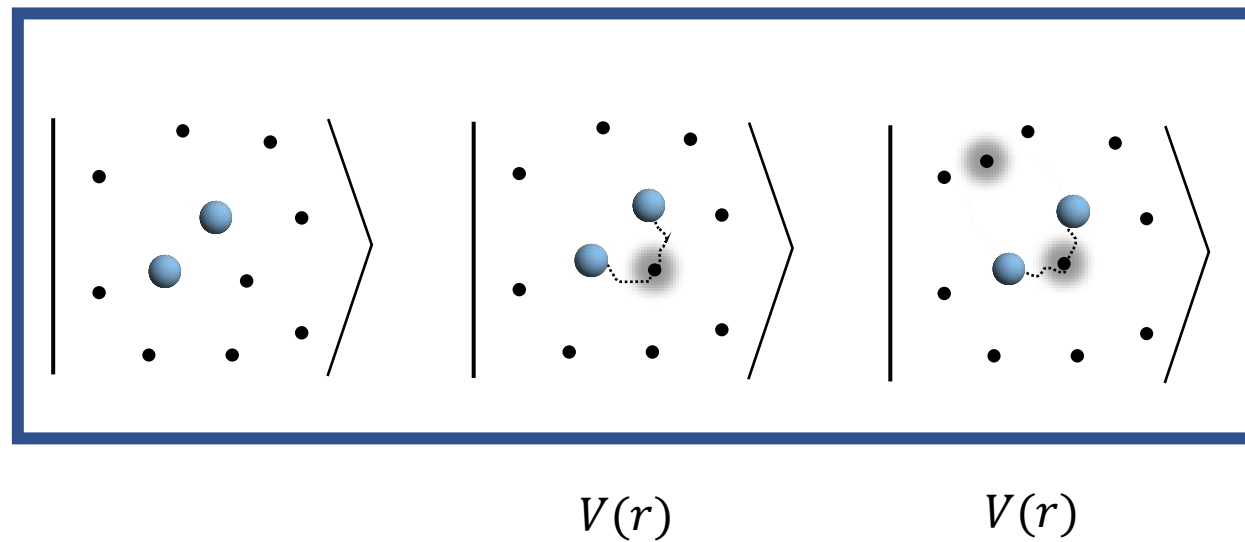
$V(r)$

# Two Bose polarons

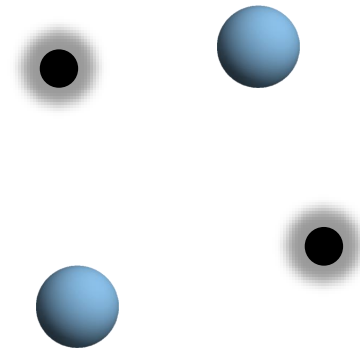
The two-polaron problem can be treated as a set of coupled few-body problems

$$|\Psi\rangle = \left[ \begin{array}{c} \text{Two-body} \\ \left| \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right\rangle \\ \text{Two-body} \end{array} \right] + \left[ \begin{array}{c} \text{Three-body} \\ \left| \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right\rangle \\ \text{Three-body} \\ V(r) \end{array} \right] + \left[ \begin{array}{c} \text{Four-body} \\ \left| \begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right\rangle \\ \text{Four-body} \\ 2V(r) \end{array} \right] + \dots$$

# Two Bose polarons



FOUR-BODY PROBLEM: 2+2 bosons



# FOUR-BODY PROBLEM: 2+2 bosons

Separable interaction model:

$$V = \frac{4\pi\hbar^2}{2\mu} g |\phi\rangle\langle\phi| \quad g < 0 \quad \phi(p) = \begin{cases} 1 & \text{for } p \leq \Lambda \\ 0 & \text{for } p > \Lambda \end{cases}$$

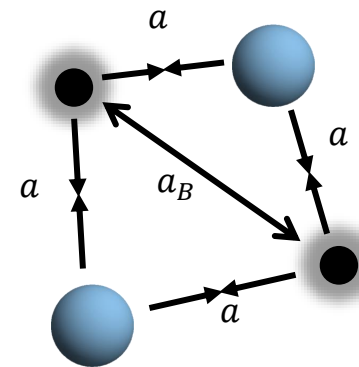
$$\text{Scattering length } a = \left( g^{-1} + \frac{2}{\pi} \Lambda \right)^{-1}$$

Contact interaction limit:  $\Lambda \rightarrow \infty$

$$V_B = \frac{4\pi\hbar^2}{m} g_B |\phi_B\rangle\langle\phi_B| \quad g_B > 0 \quad \phi_B(p) = \begin{cases} 1 & \text{for } p \leq \Lambda_B \\ 0 & \text{for } p > \Lambda_B \end{cases}$$

$$\text{Scattering length } a_B = \left( g_B^{-1} + \frac{2}{\pi} \Lambda_B \right)^{-1}$$

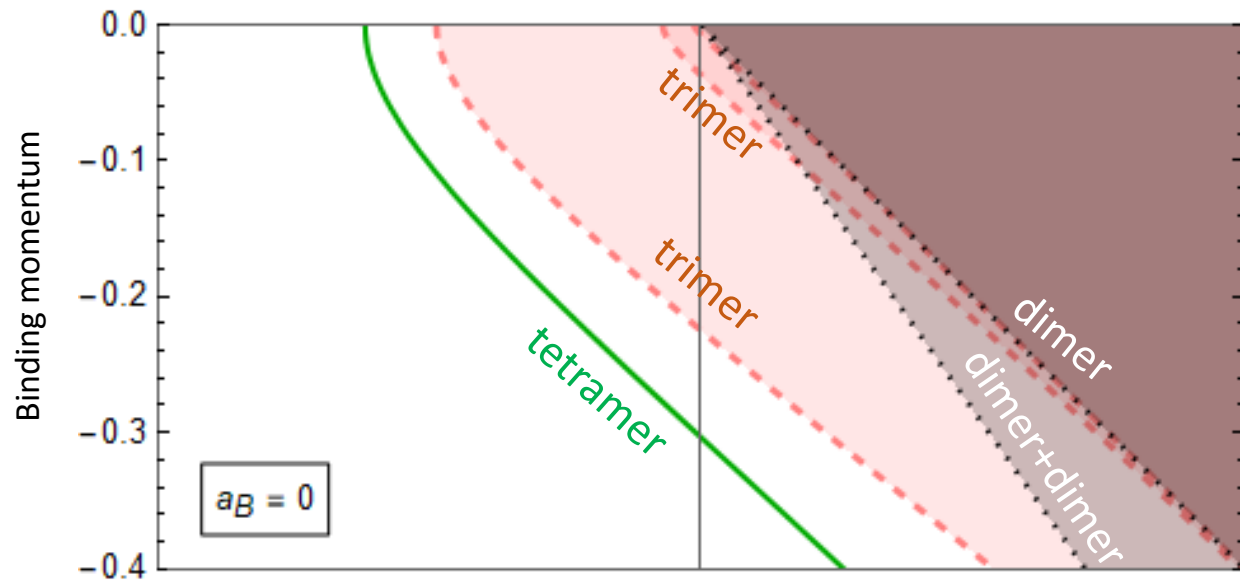
Strong repulsion limit:  $g_B \rightarrow \infty$



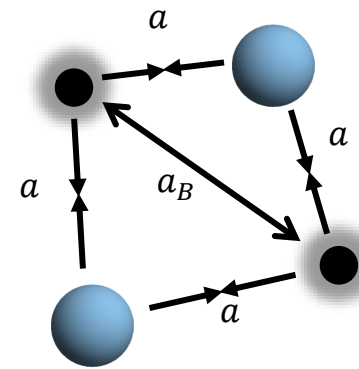
3-body cutoff  $\Lambda_3 \sim l_{vdW}^{-1}$

# FOUR-BODY PROBLEM: 2+2 bosons

Energy spectrum  $\frac{M}{m} = 19$  ( $^{133}\text{Cs}$  in  $^7\text{Li}$ )



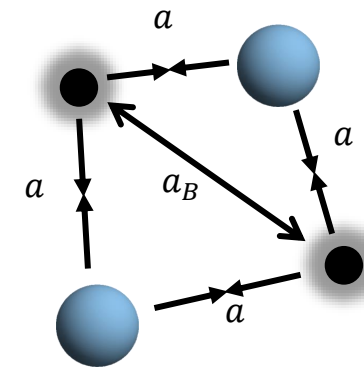
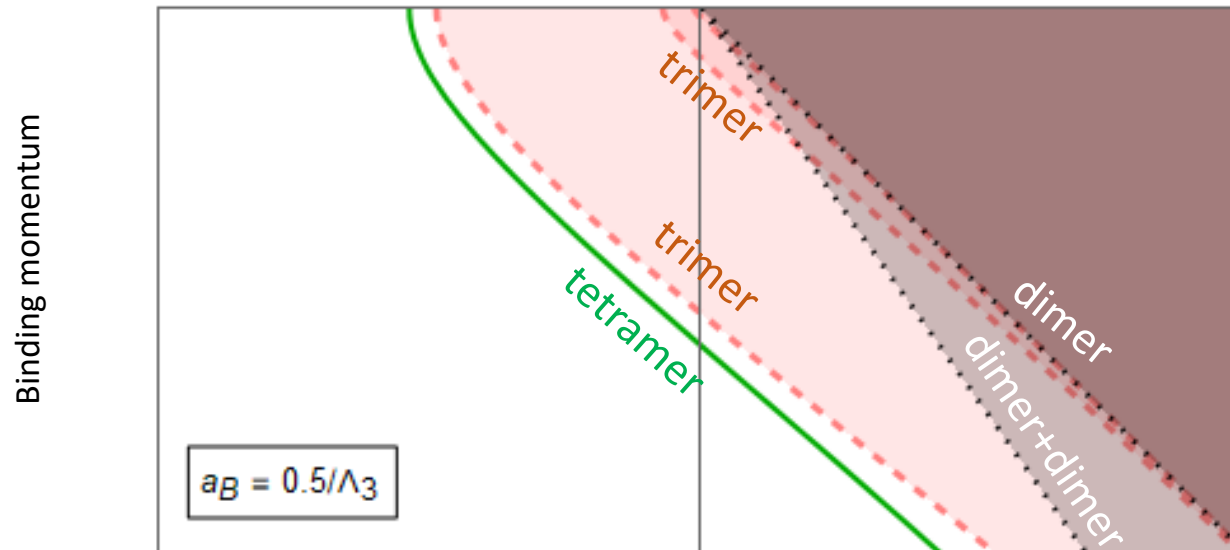
Inverse scattering length  $1/a$



3-body cutoff  $\Lambda_3 \sim l_{vdW}^{-1}$

# FOUR-BODY PROBLEM: 2+2 bosons

Energy spectrum  $\frac{M}{m} = 19$  ( $^{133}\text{Cs}$  in  $^7\text{Li}$ )

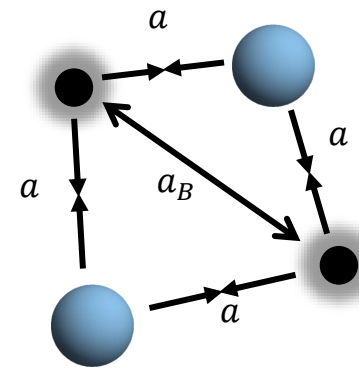
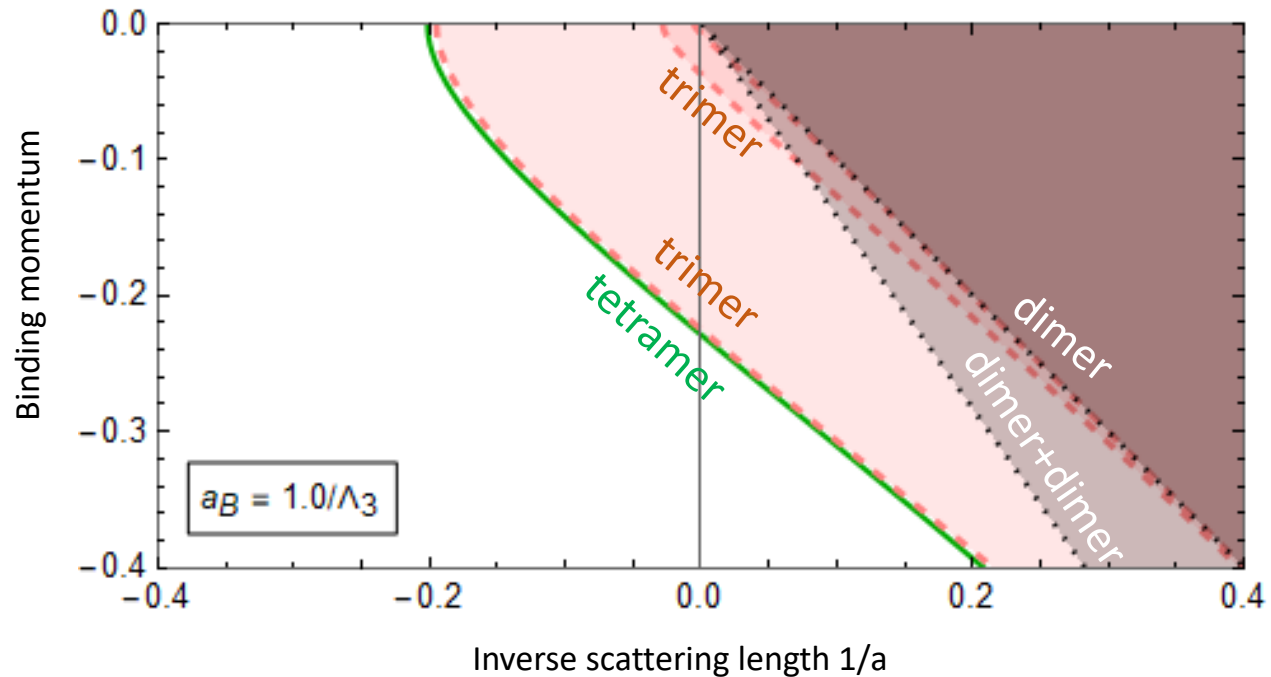


$$\text{3-body cutoff } \Lambda_3 \sim l_{vdW}^{-1}$$

Inverse scattering length  $1/a$

# FOUR-BODY PROBLEM: 2+2 bosons

Energy spectrum  $\frac{M}{m} = 19$  ( $^{133}\text{Cs}$  in  $^7\text{Li}$ )

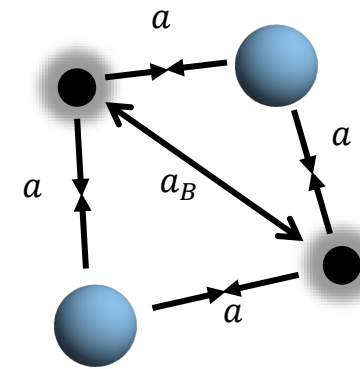
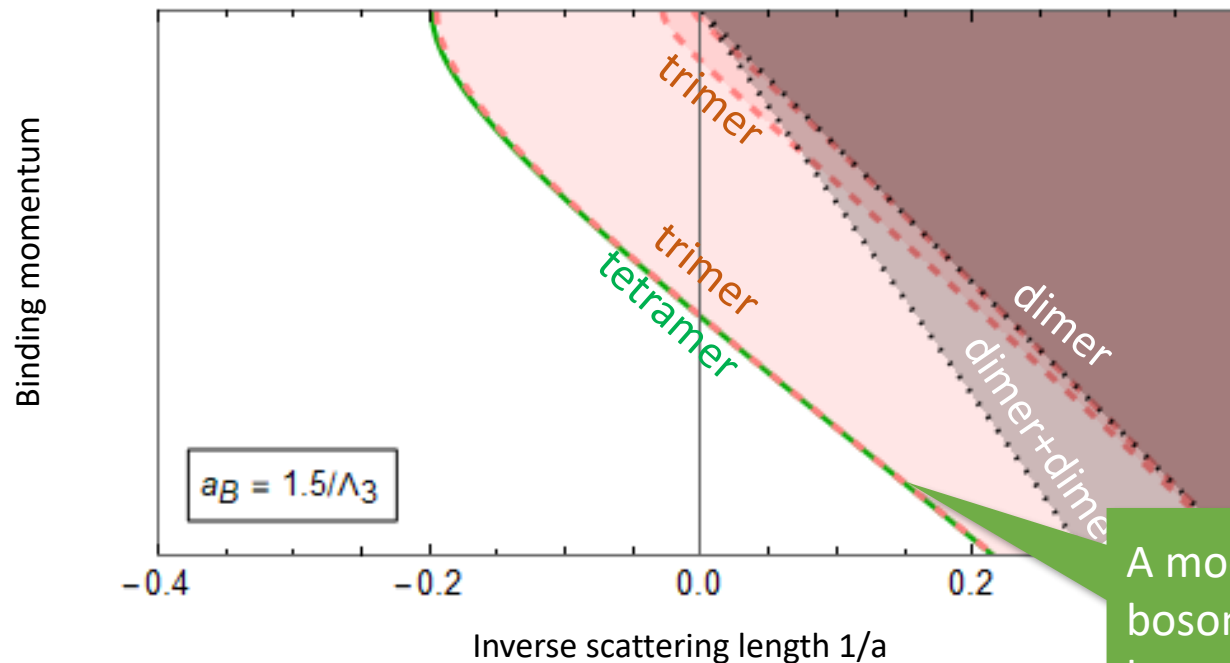


3-body cutoff  $\Lambda_3 \sim l_{vdW}^{-1}$



# FOUR-BODY PROBLEM: 2+2 bosons

Energy spectrum  $\frac{M}{m} = 19$  ( $^{133}\text{Cs}$  in  $^7\text{Li}$ )



3-body cutoff  $\Lambda_3 \sim l_{vdW}^{-1}$

A moderate repulsion between bosons can suppress the four-body bound state

## ② Polaron physics: summary

- The 3-body Efimov effect in mass imbalanced systems close to unitarity affects the polaron physics. The mediated interaction changes from Yukawa to Efimov type, and leads to bipolarons.
- A simple single-excitation picture captures this physics.

### **Outlook**

- More consistent picture with more excitations?
- More polarons.

## ③ Miscibility physics



Dmitry Petrov

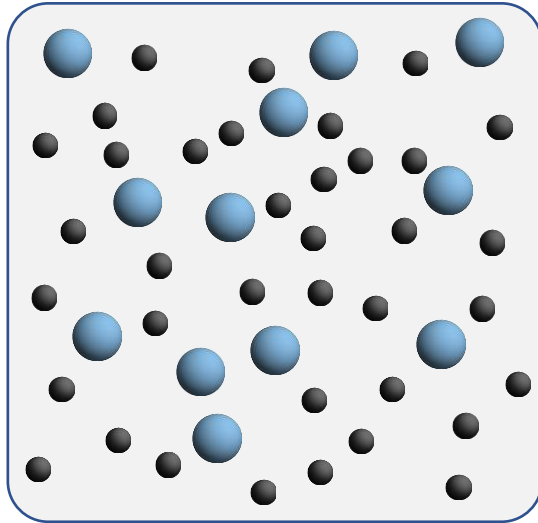
LPTMS, Orsay

**Mixed bubbles in Bose-Bose mixtures**

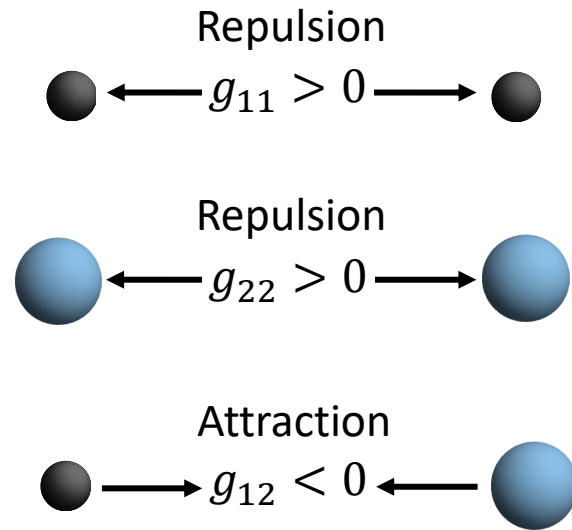
Phys. Rev. Lett. 126, 115301 (2021)

[arXiv:2008.05870]

# Mixture of cold atoms: many-body



**Mixture**



The coupling constants are given by the scattering lengths

$$g_{ij} = \frac{4\pi\hbar^2 a_{ij}}{m}$$

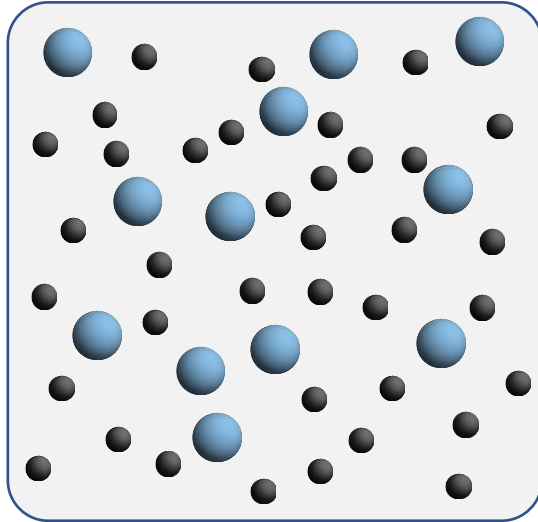
**Weak interactions, far from unitarity!  
(no Efimov effect)**

Mean-field theory:

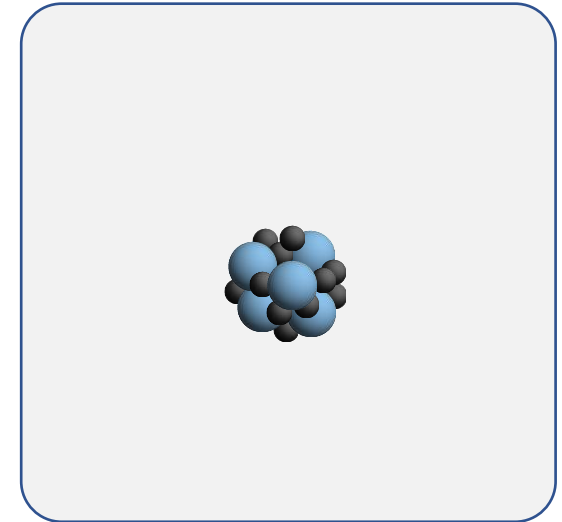
$$E/V = \frac{1}{2} g_{11} n_1^2 + \frac{1}{2} g_{22} n_2^2 + g_{12} n_1 n_2$$

# Mixture of cold atoms: many-body

$$\downarrow \sqrt{g_{11}g_{22}}$$



**Mixture**



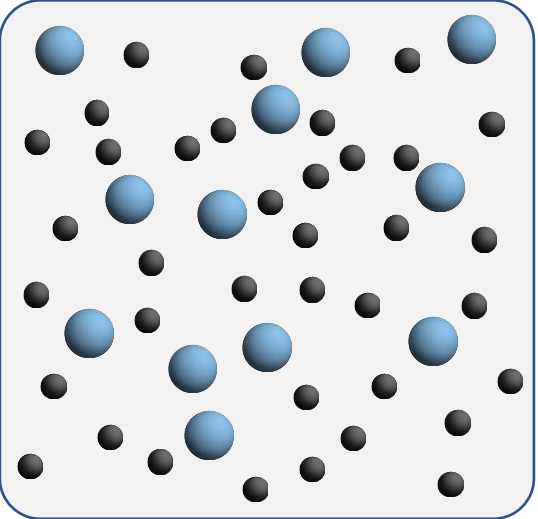
**Collapsed (solid)**

Mean-field theory:

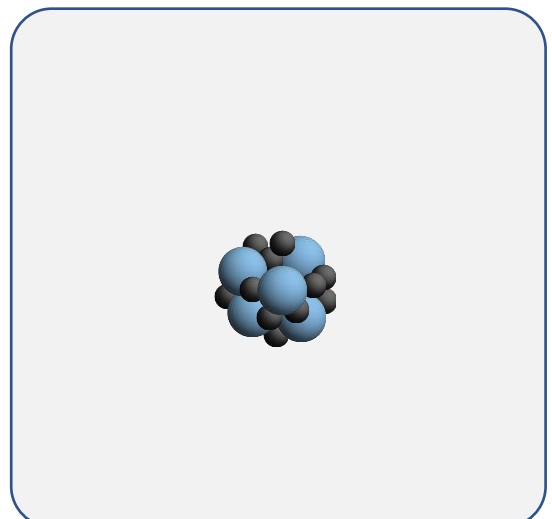
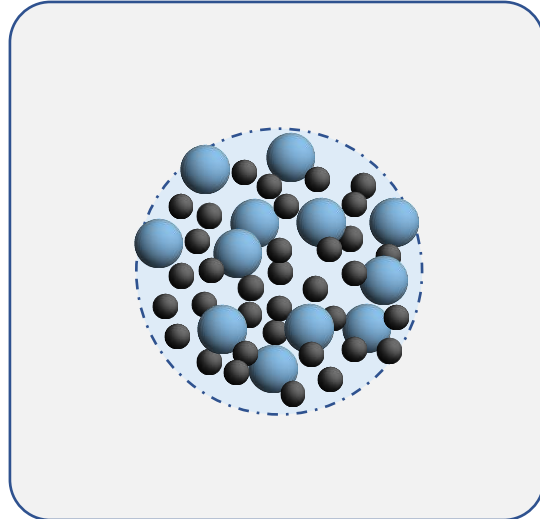
$$E/V = \frac{1}{2}g_{11}n_1^2 + \frac{1}{2}g_{22}n_2^2 + g_{12}n_1n_2$$

# Mixture of cold atoms: many-body

$\sqrt{g_{11}g_{22}}$




Mixture



Collapsed (solid)

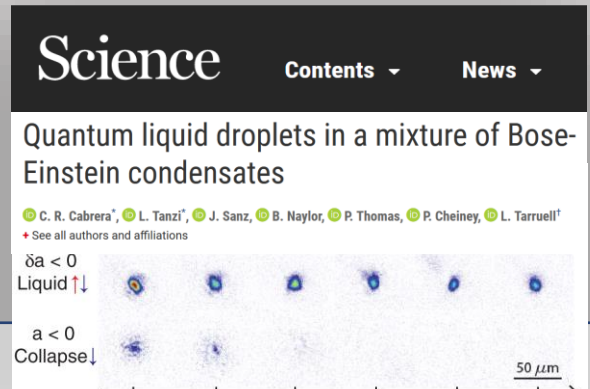
Beyond mean-field

$E/V =$



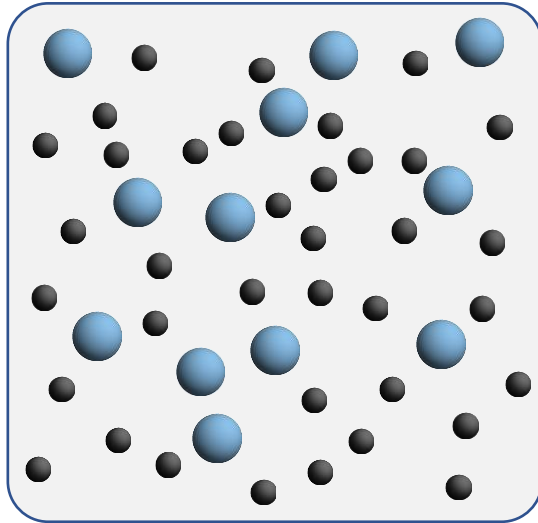
**Dmitry Petrov**  
Prediction (2015)

**Observation (2018)**

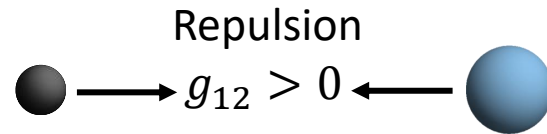
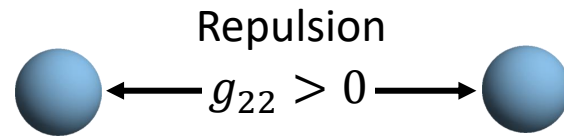
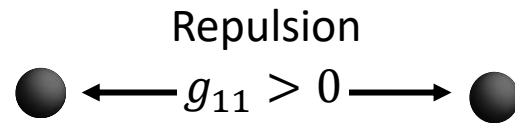


$\propto n^{5/2}$   
 $\Psi(n_1, n_2)$

# Mixture of cold atoms: repulsive



**Mixture**

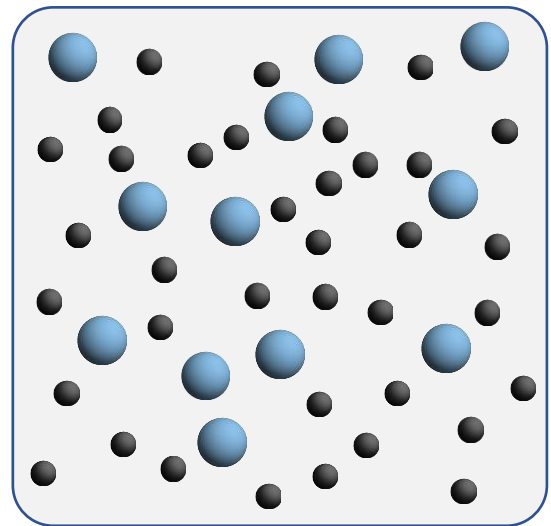


Mean-field theory:

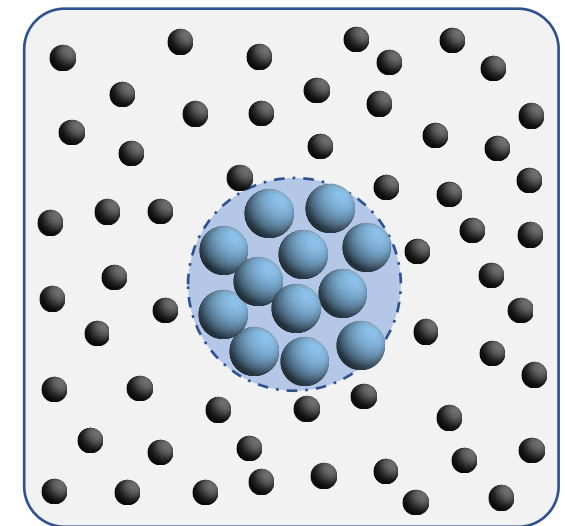
$$E/V = \frac{1}{2} g_{11} n_1^2 + \frac{1}{2} g_{22} n_2^2 + g_{12} n_1 n_2$$

# Mixture of cold atoms: repulsive

$$\downarrow \sqrt{g_{11}g_{22}}$$



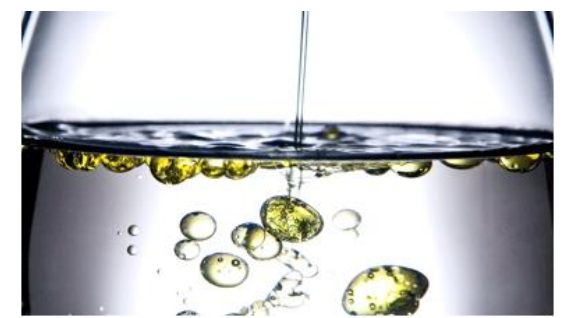
Mixture



Separated

Mean-field theory:

$$E/V = \frac{1}{2} g_{11} n_1^2 + \frac{1}{2} g_{22} n_2^2 + g_{12} n_1 n_2$$

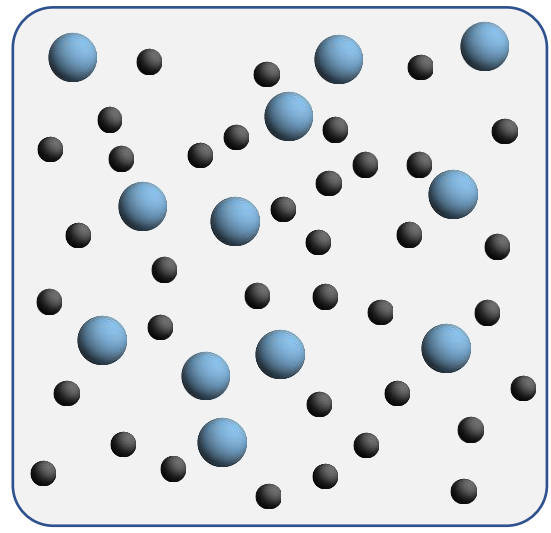


Like oil and water (but gases!)

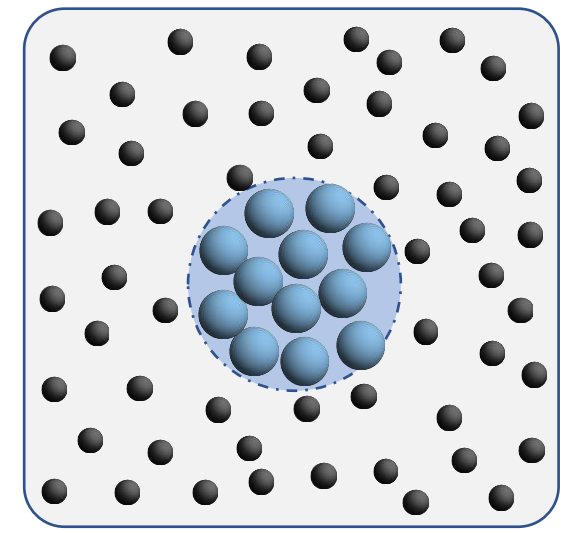


# Mixture of cold atoms: repulsive

$$\downarrow \sqrt{g_{11}g_{22}}$$

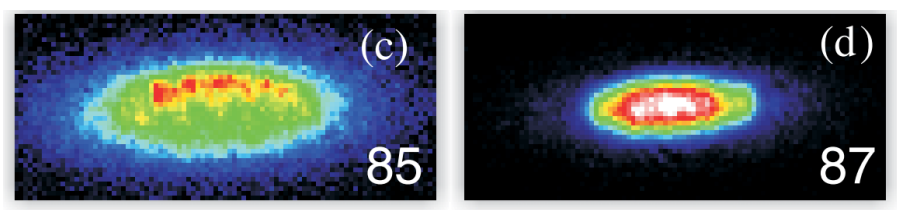


**Mixture**

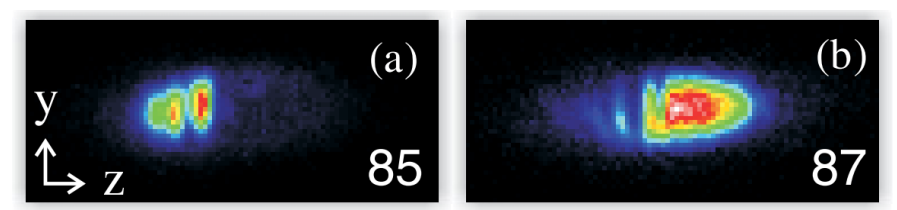


**Separated**

Rubidium atom mixture ( $^{85}\text{Rb} + ^{87}\text{Rb}$ )

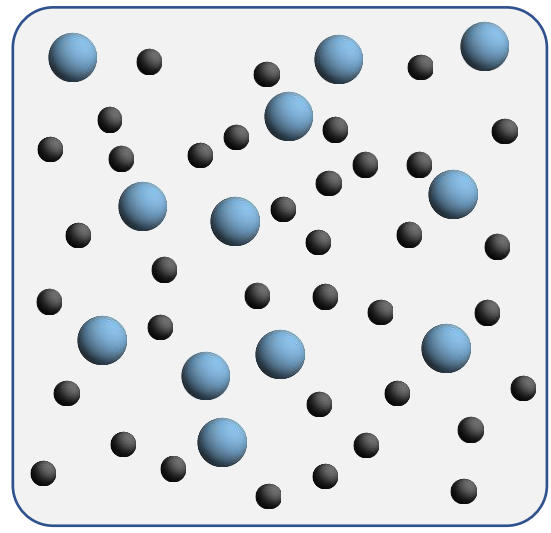
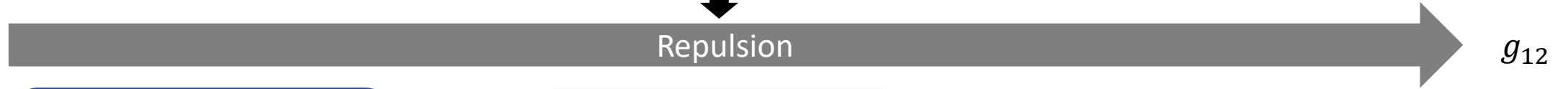


Rubidium atom mixture ( $^{85}\text{Rb} + ^{87}\text{Rb}$ )



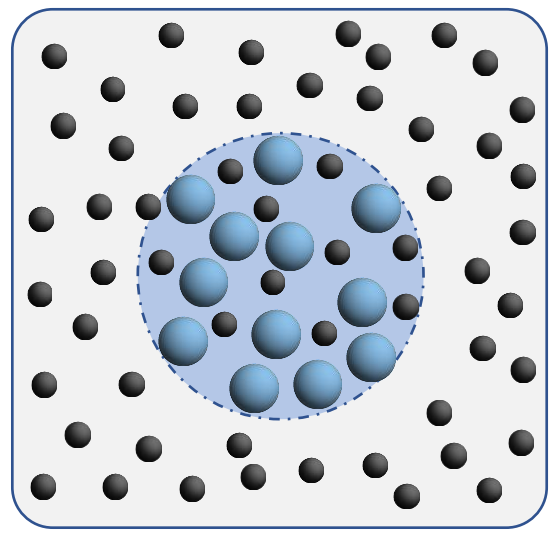
# Mixture of cold atoms: repulsive

$$\downarrow \sqrt{g_{11}g_{22}}$$

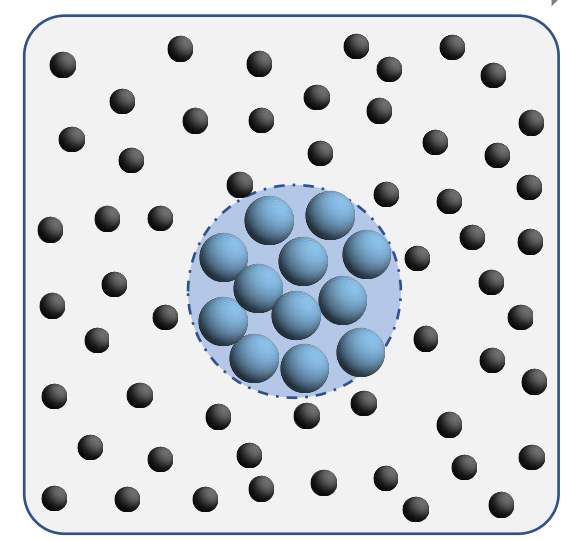


**Mixed**

$$g_{12} < \sqrt{g_{11}g_{22}} - \epsilon$$



**Partially mixed**  
"mixed bubble"



**Separated**

$$g_{12} > \sqrt{g_{11}g_{22}}$$

Beyond mean-field theory:

$$E/V = \frac{1}{2} g_{11} n_1^2 + \frac{1}{2} g_{22} n_2^2 + g_{12} n_1 n_2 + E_{LHY}(n_1, n_2)$$

### ③ Miscibility physics: Summary

- Close to miscibility/immiscibility threshold of a Bose mixture, quantum fluctuations allow the existence of partially mixed bubbles, bearing some similarity to quantum liquid droplets.
- Compared to the liquid droplet, the mixed bubble may be difficult to observe experimentally.

# ④ Universal trimers of fermions



Ludovic Pricoupenko  
Sorbonne Université

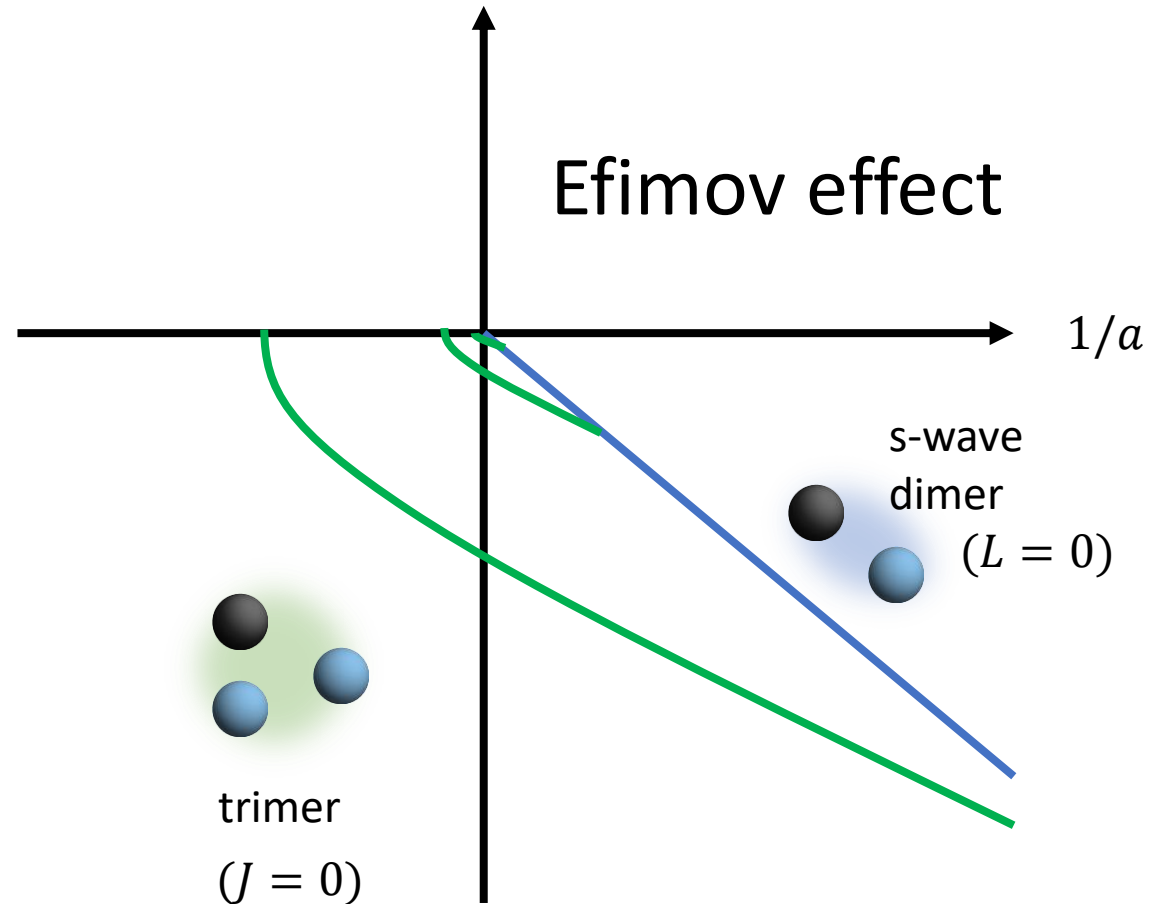
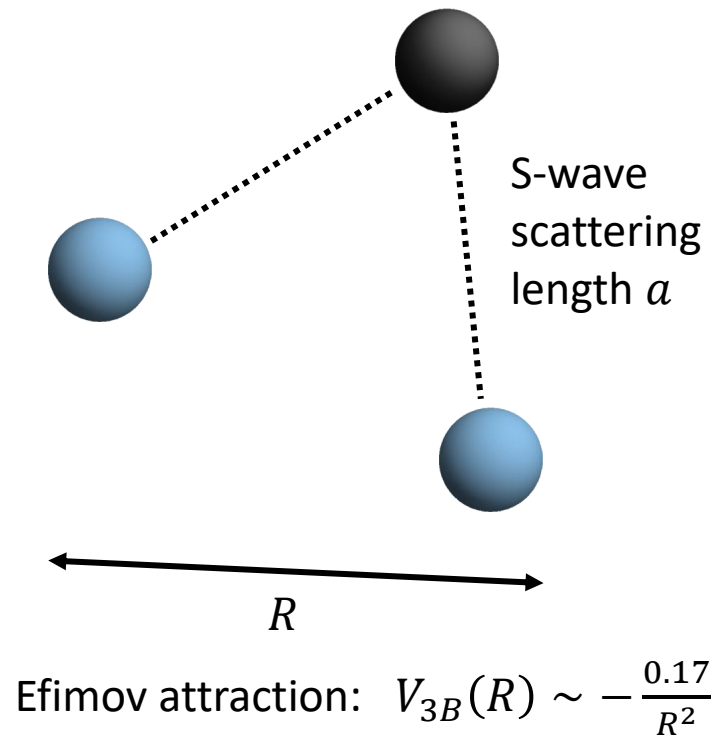


Christiane Schmickler  
RIKEN

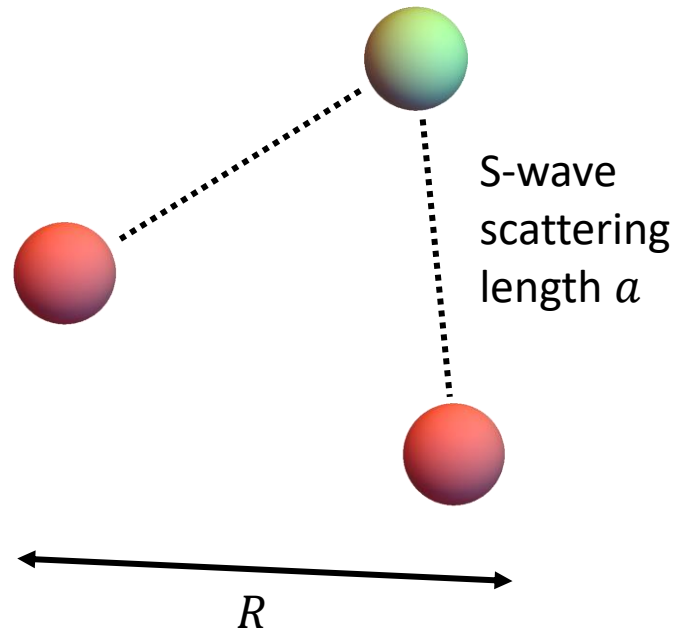
**Shallow Trimers of Two Identical Fermions  
and One Particle in Resonant Regimes**

[arxiv:2112.02983]

# 2 identical bosons + 1 particle

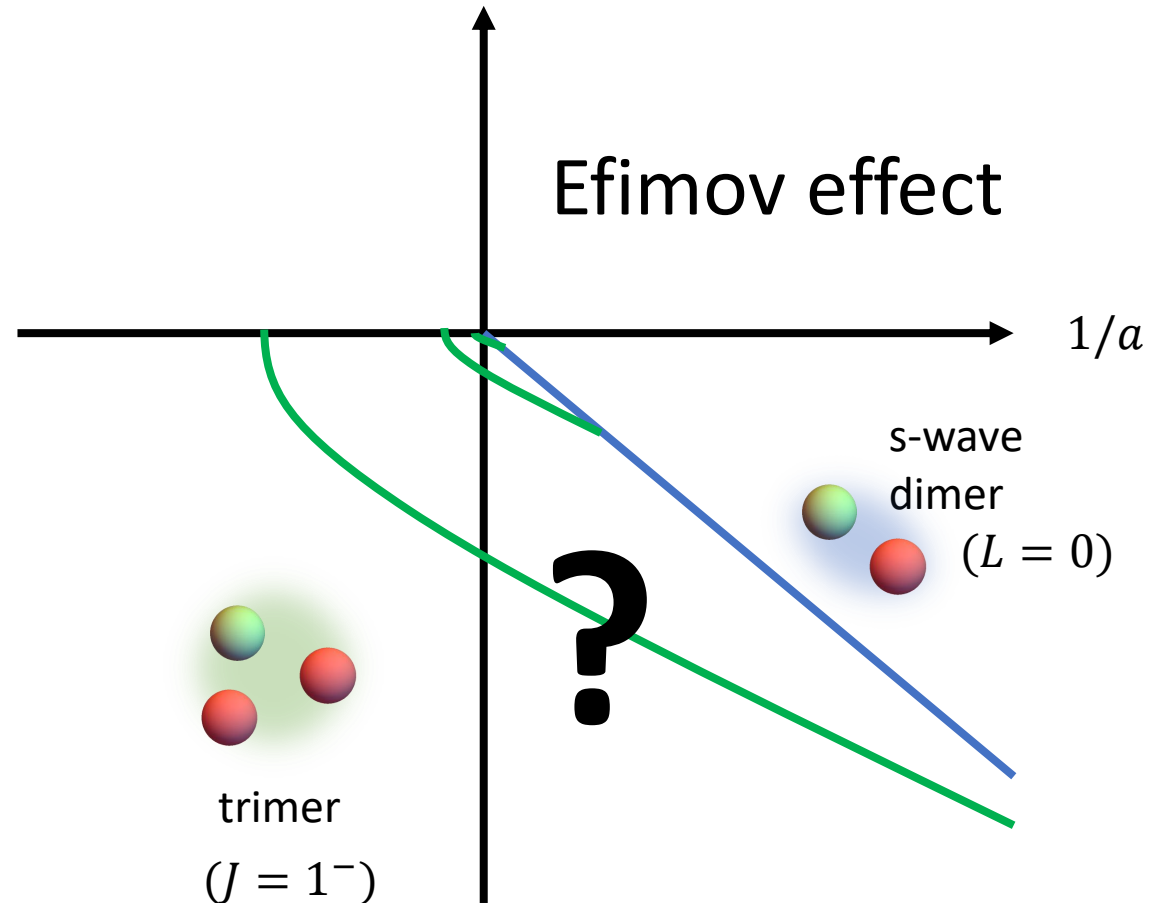


# 2 identical fermions + 1 particle

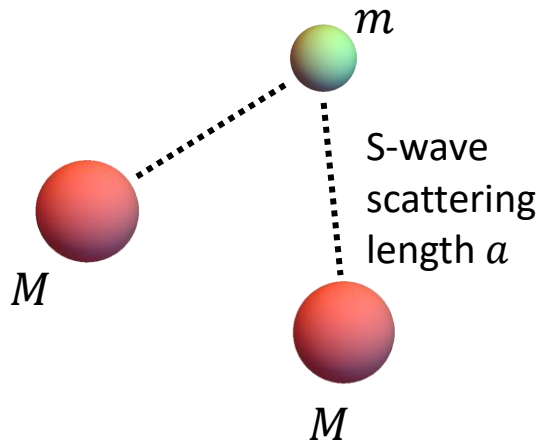


Efimov attraction:  $V_{3B}(R) \sim -\frac{0.17}{R^2}$

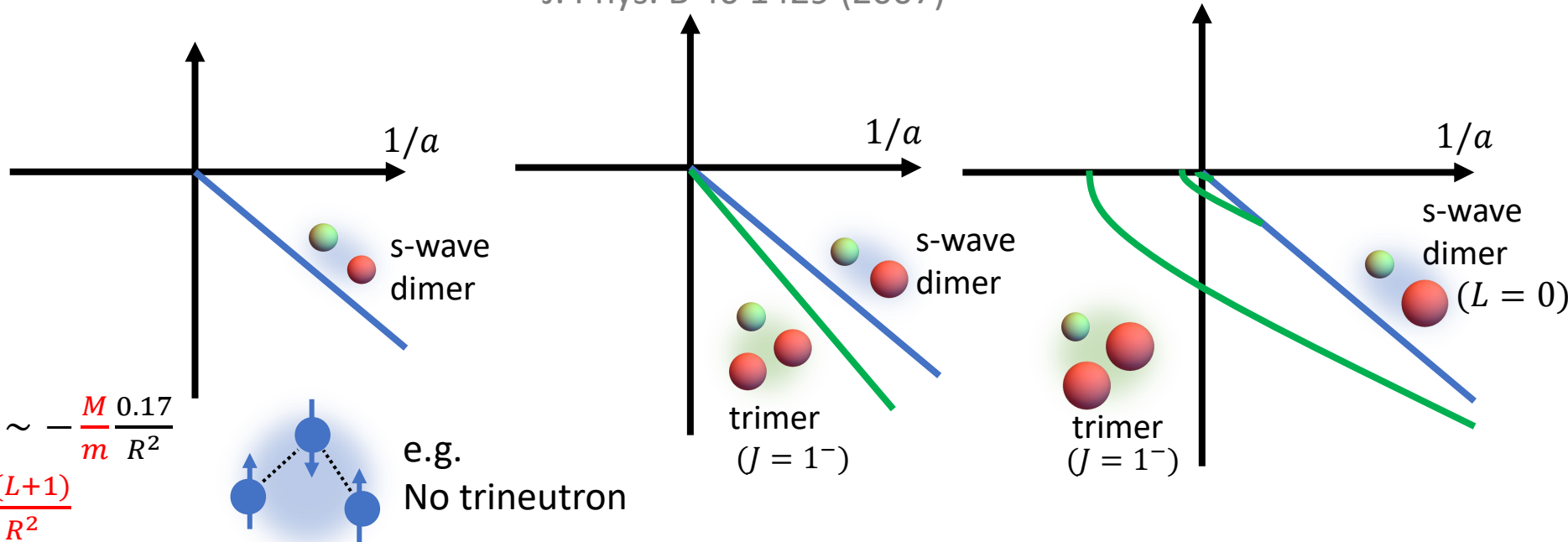
Centrifugal repulsion:  $+\frac{L(L+1)}{R^2}$



# 2 identical fermions + 1 particle



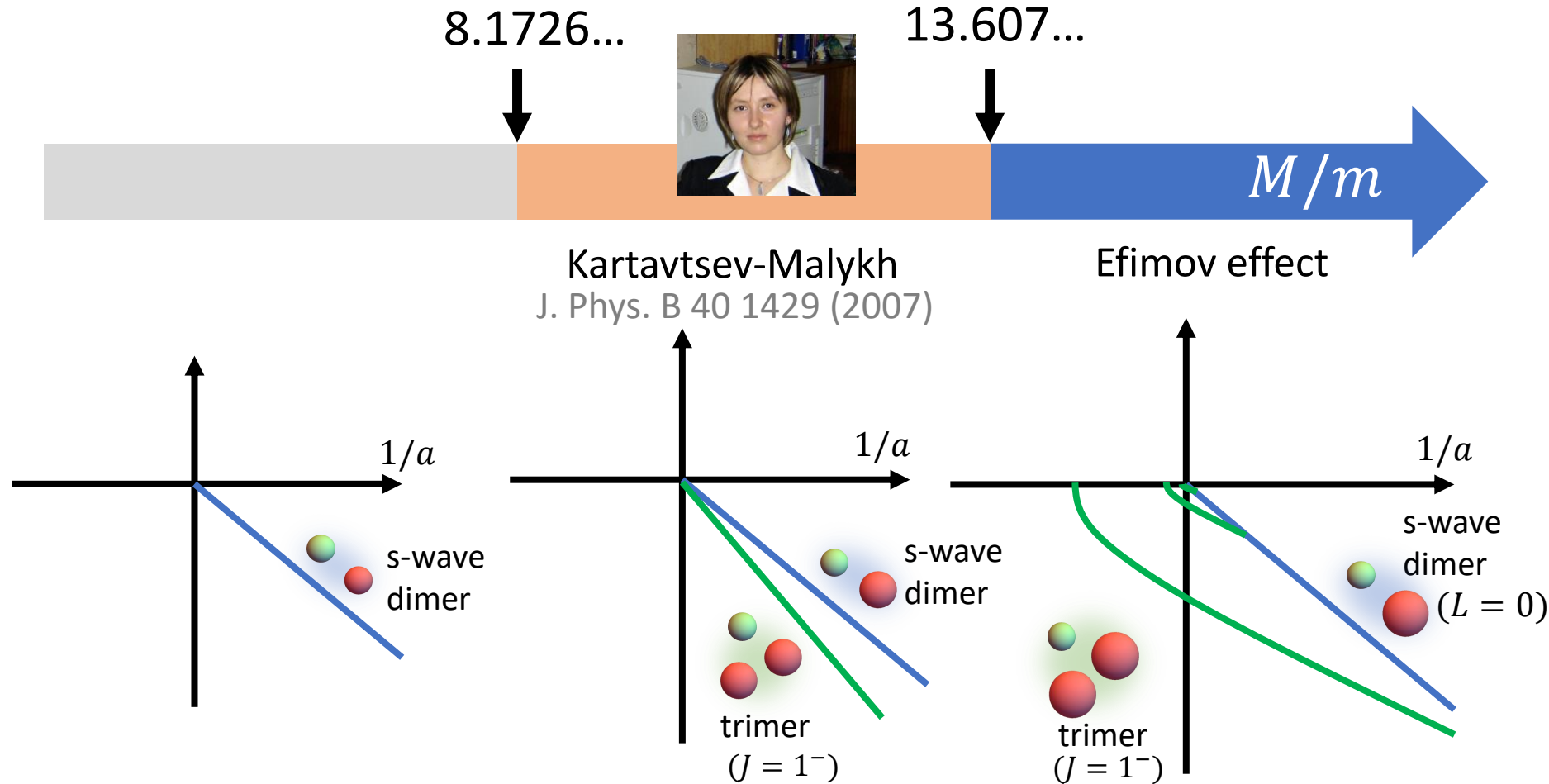
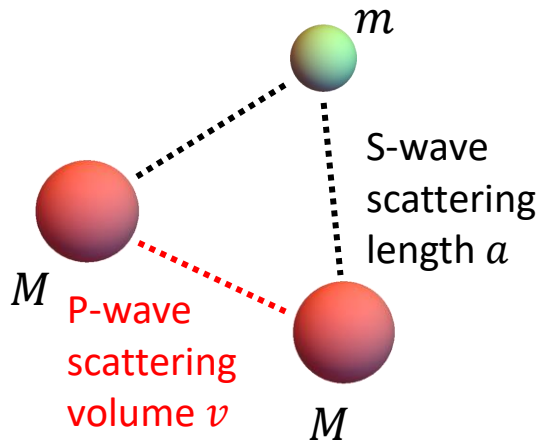
Kartavtsev-Malykh  
J. Phys. B 40 1429 (2007)



Efimov attraction:  $V_{3B}(R) \sim -\frac{M}{m} \frac{0.17}{R^2}$

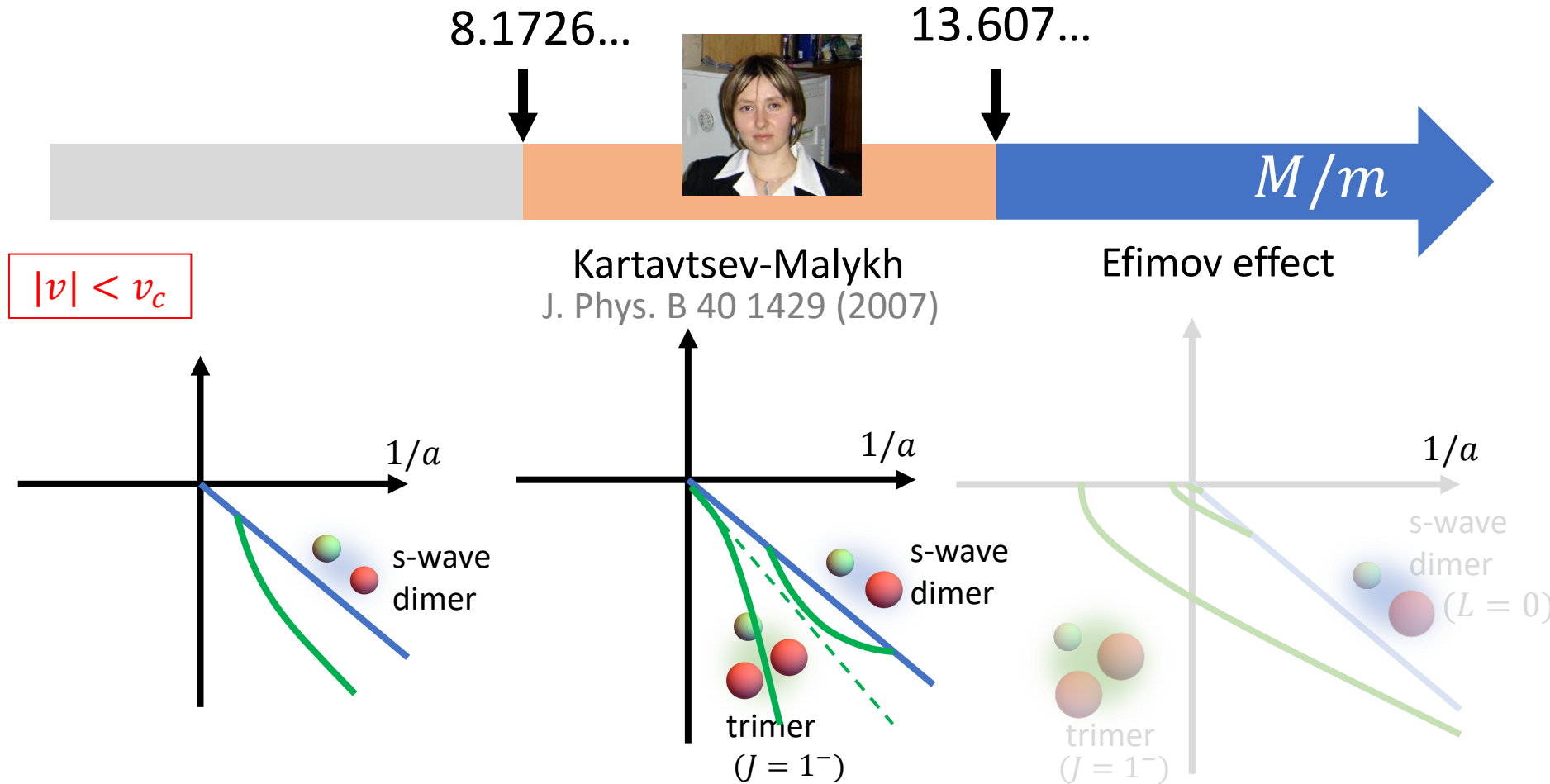
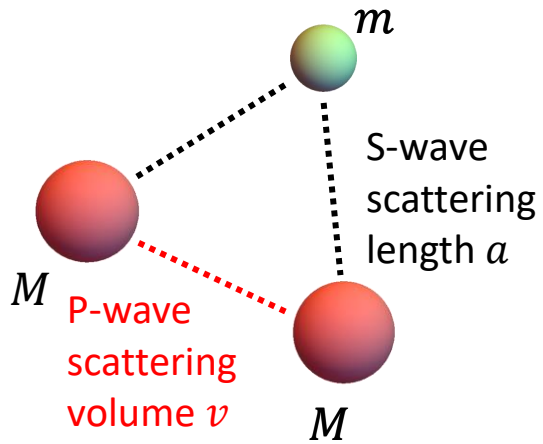
Centrifugal repulsion:  $+\frac{L(L+1)}{R^2}$

# 2 identical fermions + 1 particle

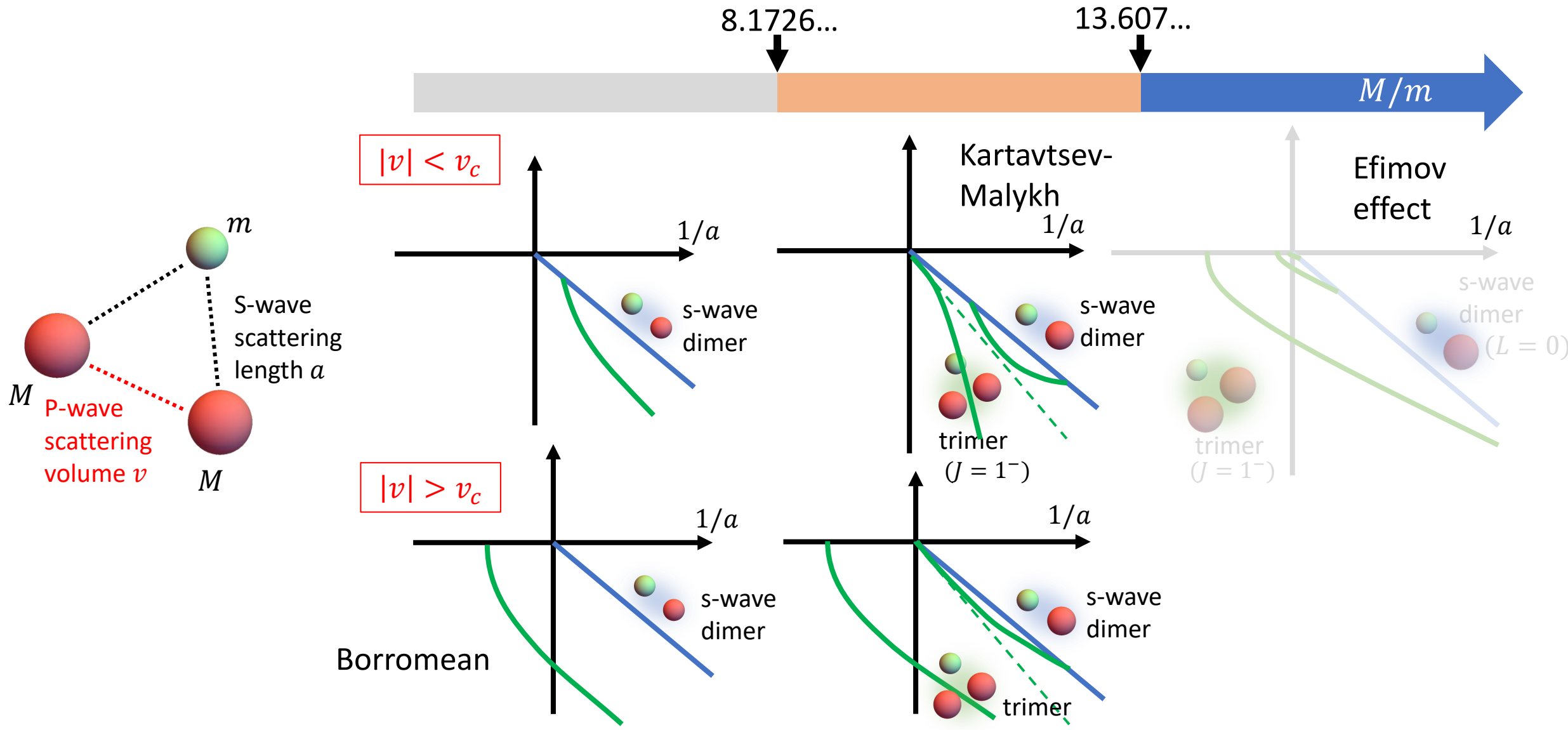




# 2 identical fermions + 1 particle

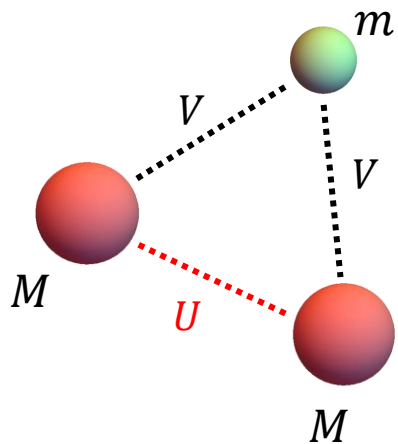


# 2 identical fermions + 1 particle



# 2 identical fermions + 1 particle

Separable interactions:



**S wave:**  $V = \xi |\chi\rangle\langle\chi|$       With  $\langle\mathbf{k}|\chi\rangle = \chi(k)$

**P wave:**  $U = \sum_m g_m |\Phi_m\rangle\langle\Phi_m|$       With  $\langle\mathbf{k}|\Phi_m\rangle = \phi(k)\mathbf{k} \cdot \mathbf{e}_m$

**Form factors:**

Cut-off:  $\begin{cases} 1 & \text{for } k \leq \Lambda \\ 0 & \text{for } k > \Lambda \end{cases}$

Gaussian:  $\exp(-k^2/\Lambda^2)$

Yamaguchi:  $\left(1 + \frac{k^2}{\Lambda^2}\right)^{-1}$

Yamaguchi-squared:  $\left(1 + \frac{k^2}{\Lambda^2}\right)^{-2}$

The model parameters  $\xi, \Lambda_0$  and  $g_m, \Lambda_1$  are adjusted to give **the same low-energy parameters:**

**S wave:** scat. length  $a$ , eff. range  $\bar{r}_e$

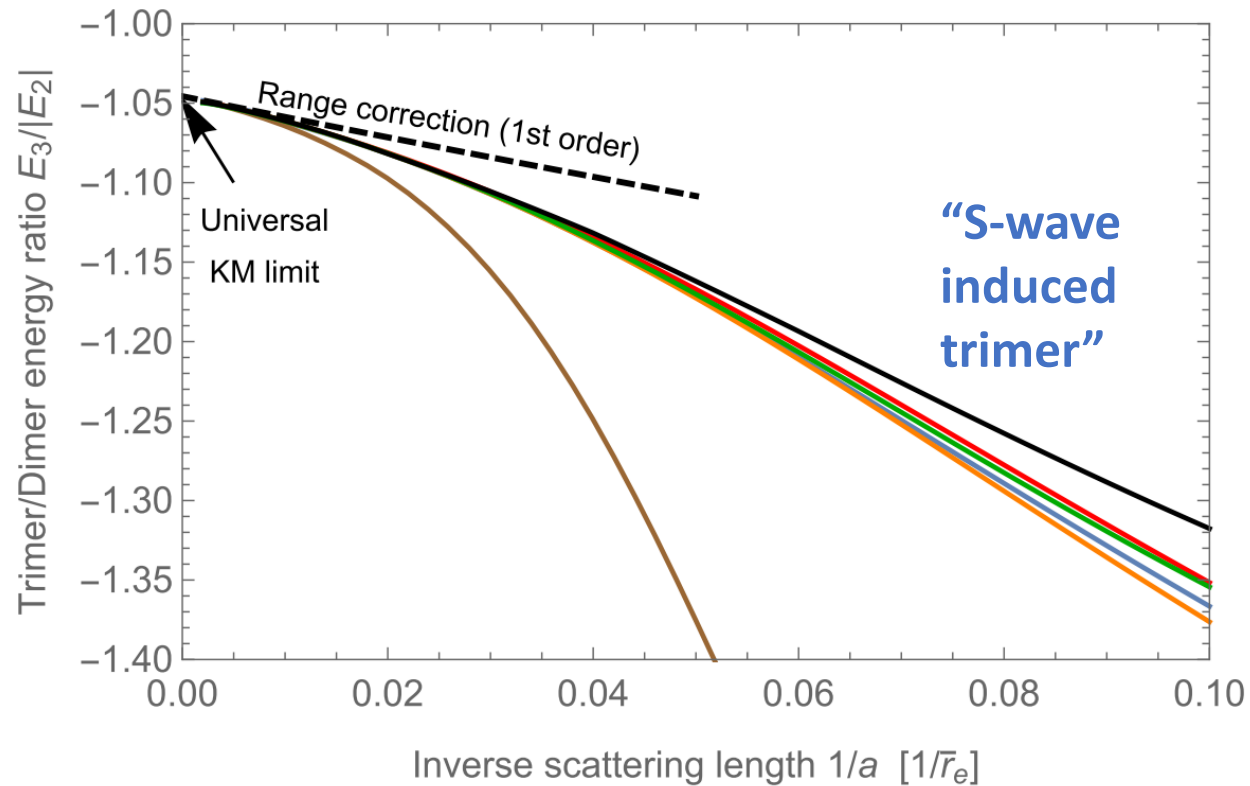
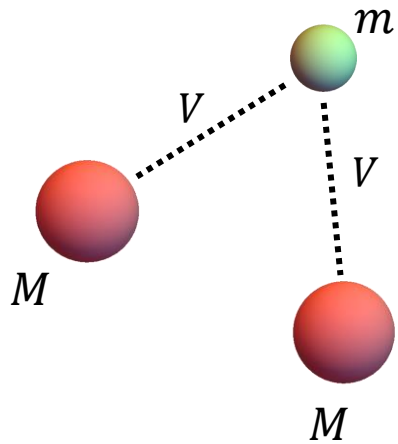
**P wave:** scat. volume  $v$ , width parameter  $\bar{\alpha}$

$$k \cot \delta_S = -\frac{1}{a} + \frac{1}{2} r_e k^2 + \dots \quad \text{and} \quad \bar{r}_e = r_e (a \rightarrow \infty)$$

$$k \cot \delta_P = -\frac{1}{k^2 v} - \alpha + \dots \quad \text{and} \quad \bar{\alpha} = \alpha (v \rightarrow \infty)$$

# 2 identical fermions + 1 particle

Without p-wave interaction:

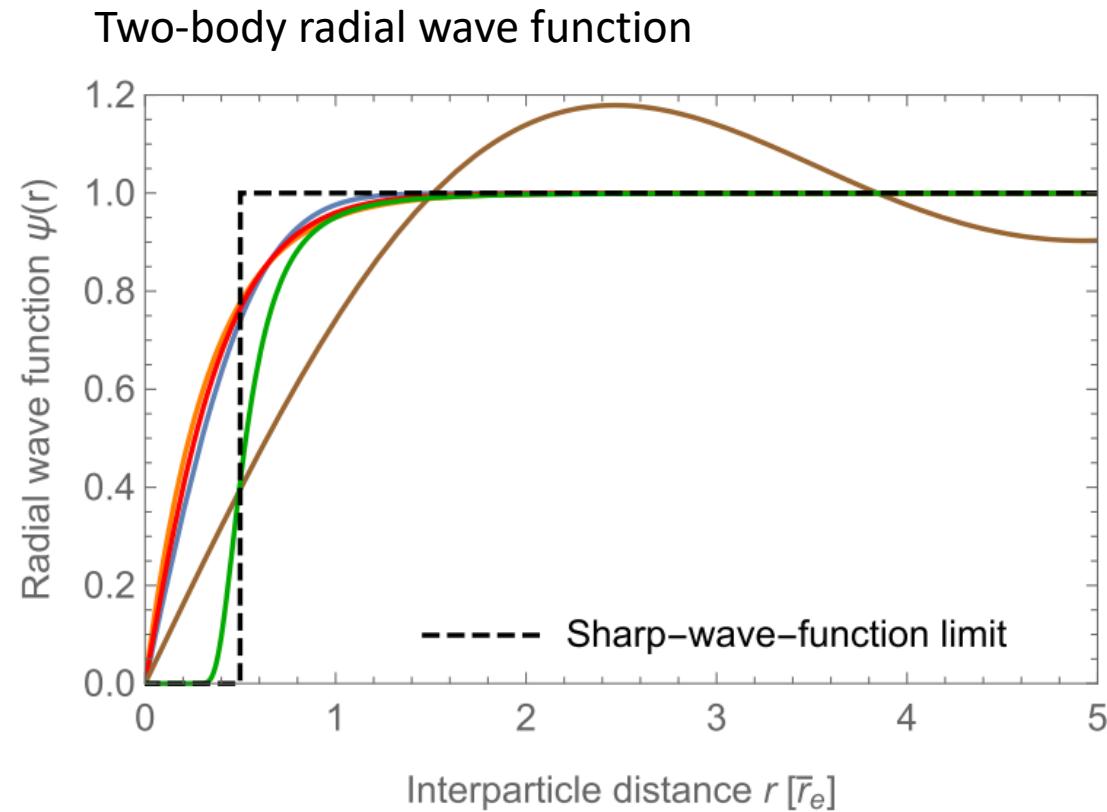
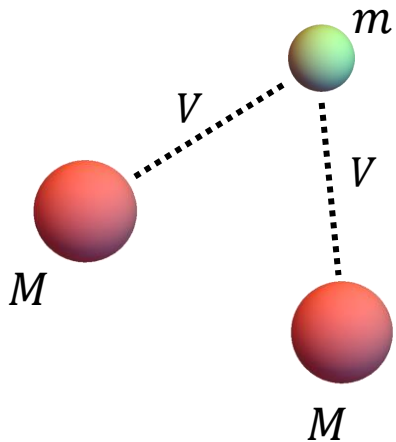


Mass ratio  $\frac{M}{m} = 9$

- Gaussian
- Yamaguchi
- Yamaguchi-squared
- Cutoff
- Lennard-Jones
- Sharp-wave-function limit

# 2 identical fermions + 1 particle

Without p-wave interaction:

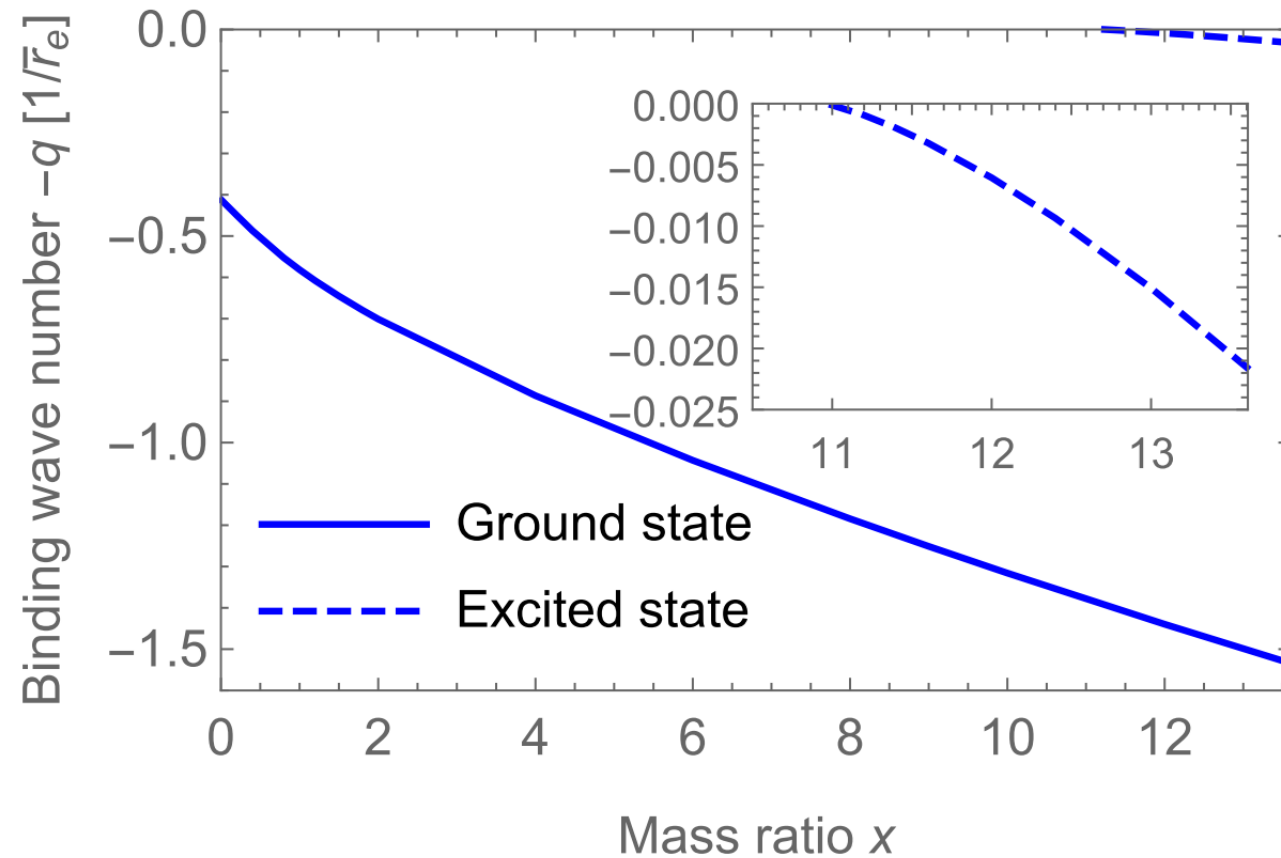
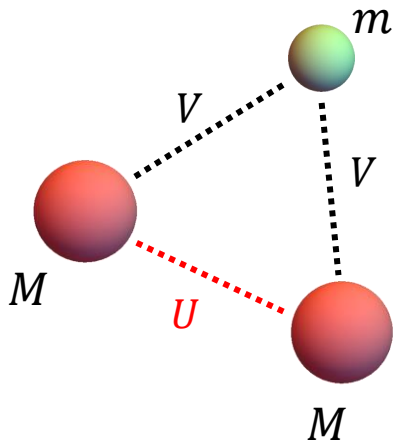


Mass ratio  $\frac{M}{m} = 9$

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# 2 identical fermions + 1 particle

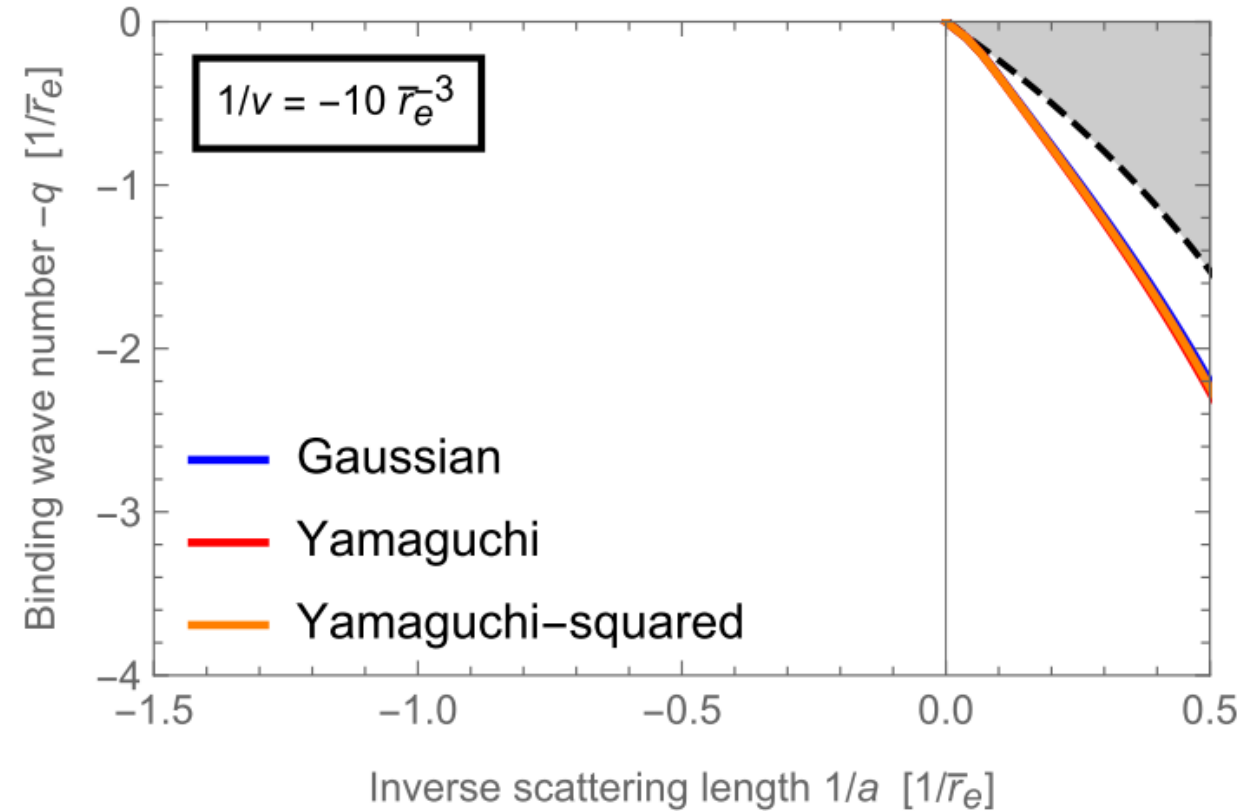
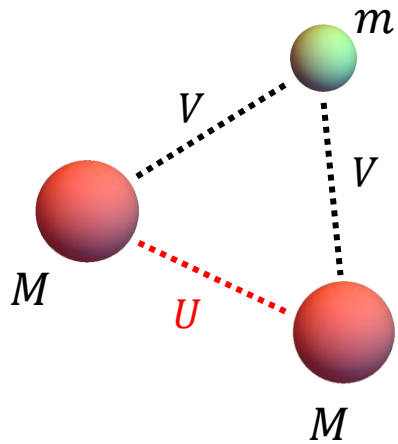
With p-wave interaction:  $a = \infty$  and  $v = \infty$



“P-wave induced trimer”

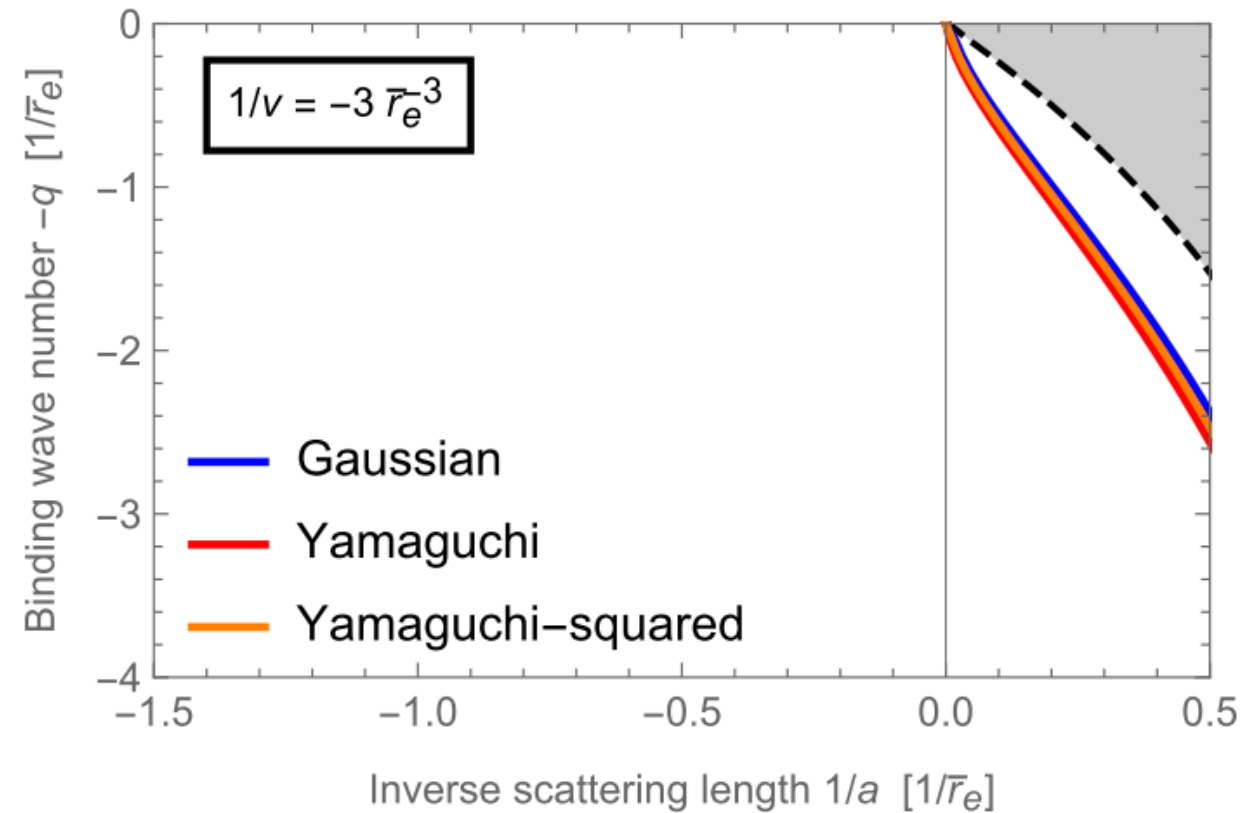
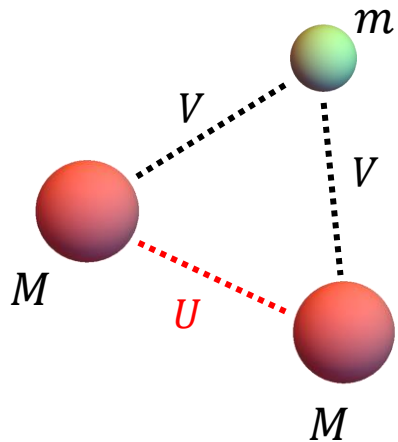
# 2 identical fermions + 1 particle

With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )



# 2 identical fermions + 1 particle

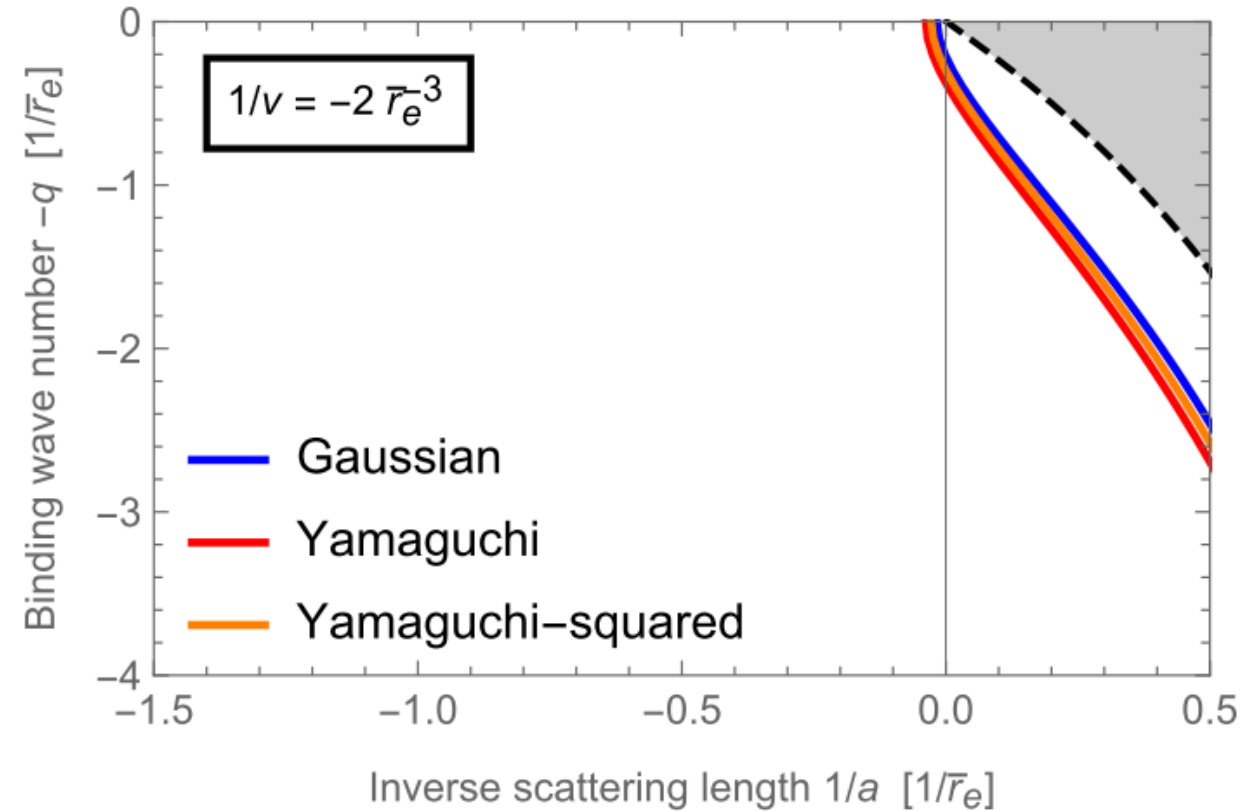
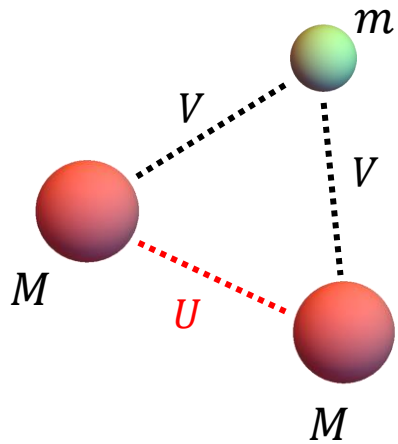
With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )





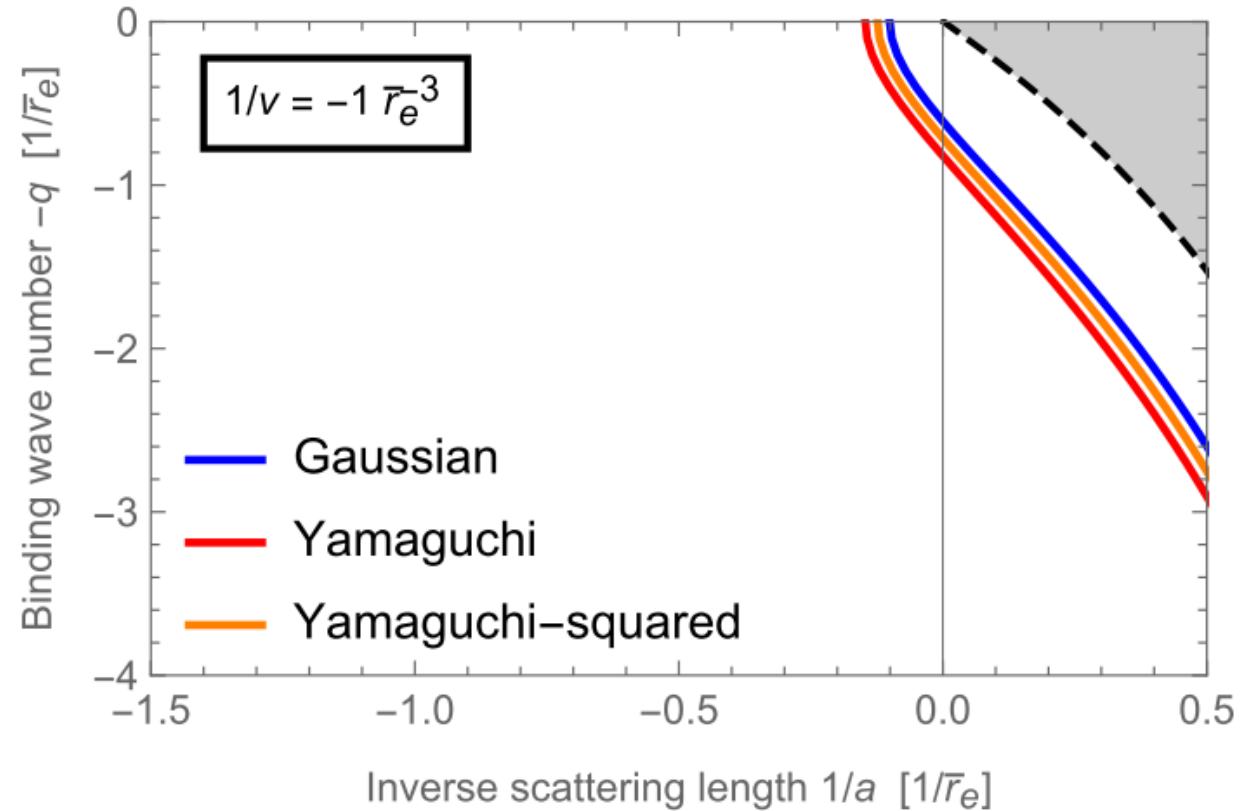
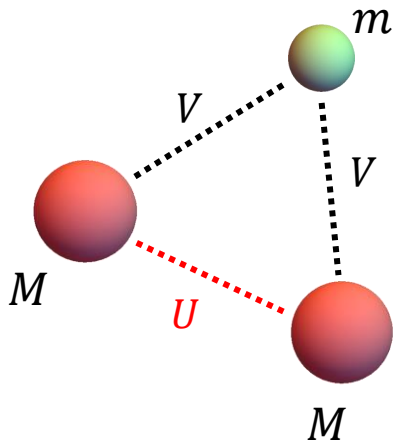
# 2 identical fermions + 1 particle

With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )



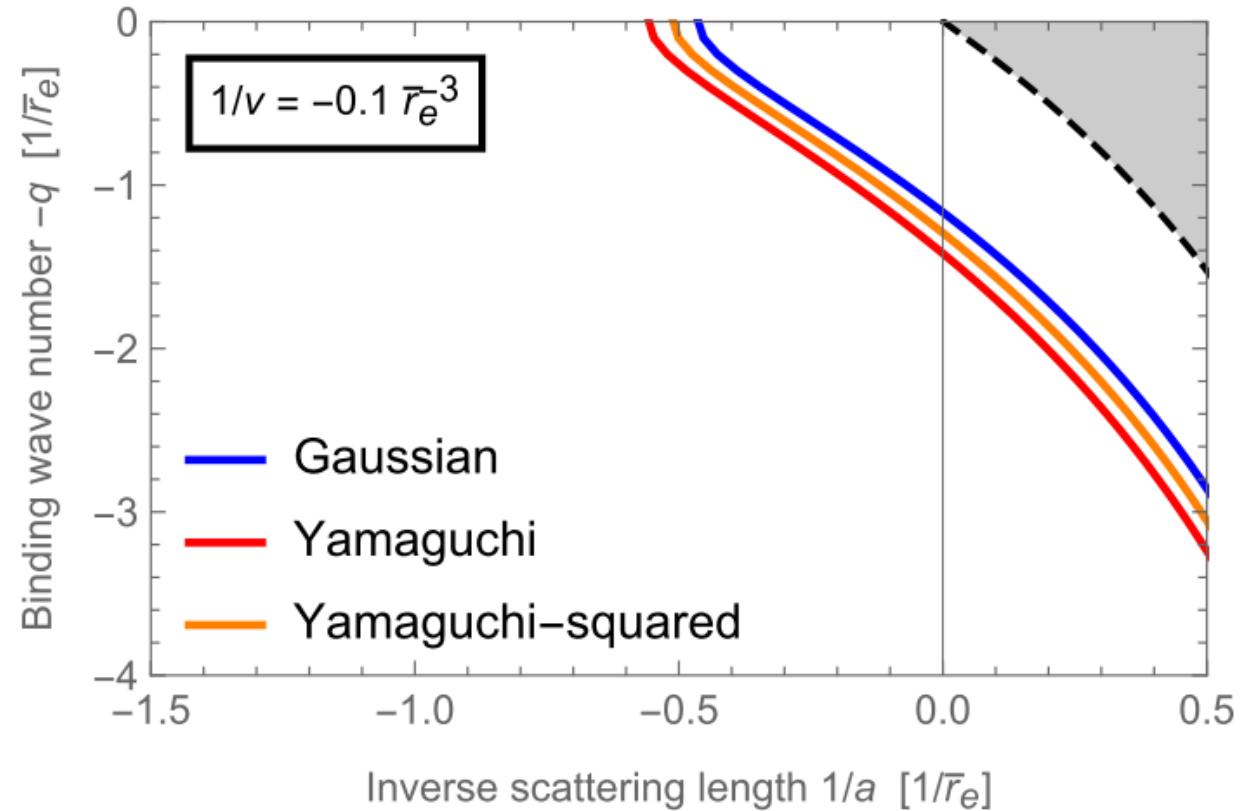
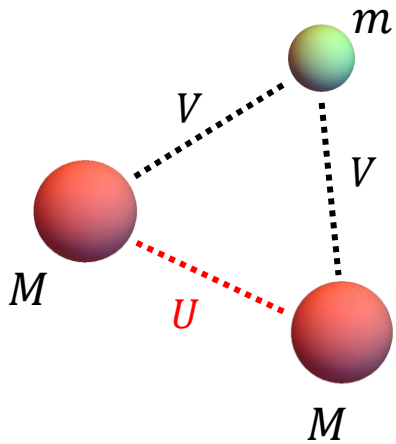
# 2 identical fermions + 1 particle

With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )



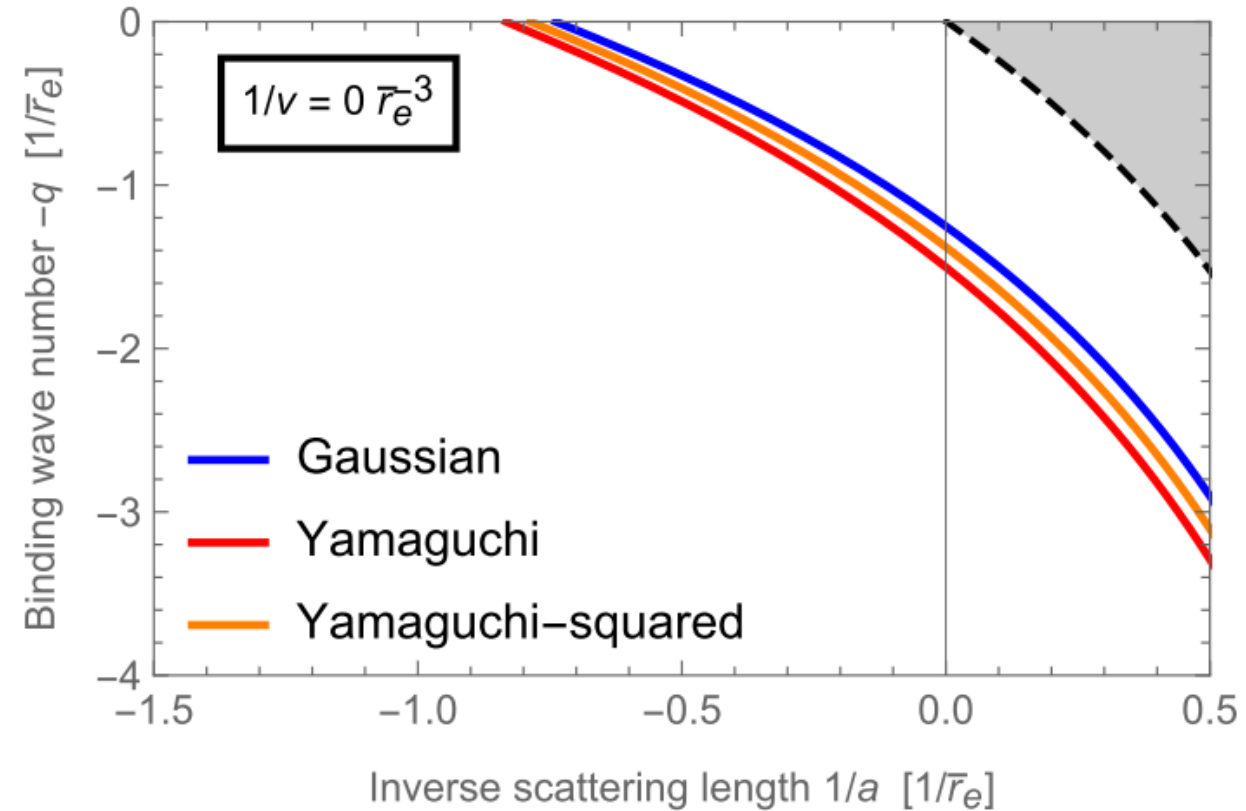
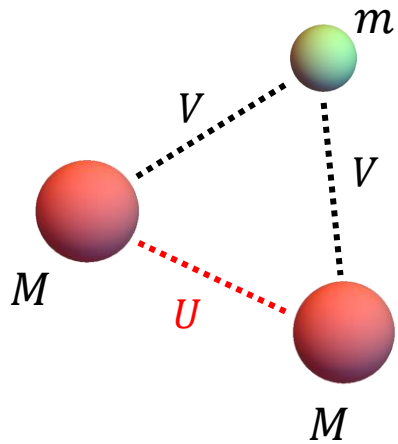
# 2 identical fermions + 1 particle

With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )

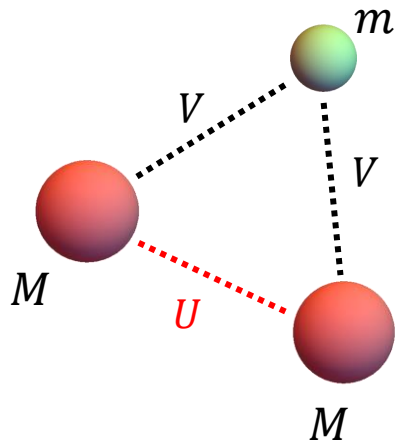


# 2 identical fermions + 1 particle

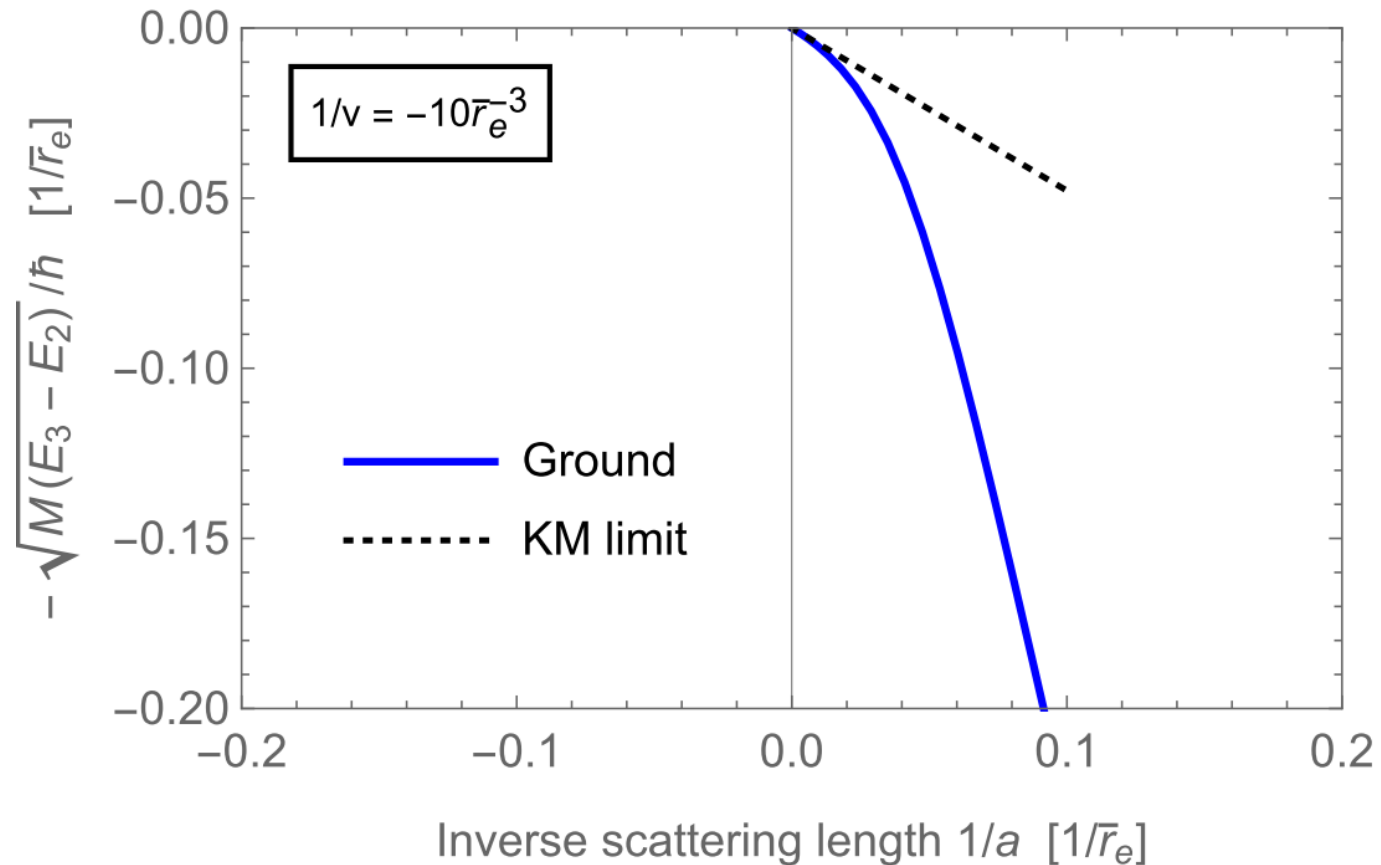
With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )



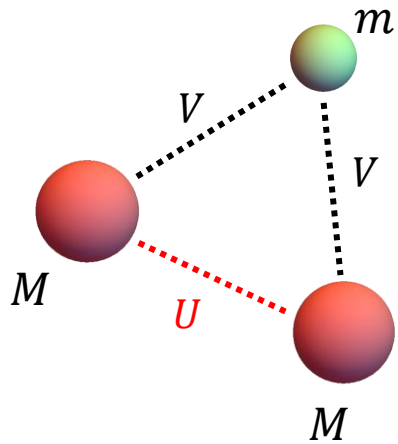
# 2 identical fermions + 1 particle



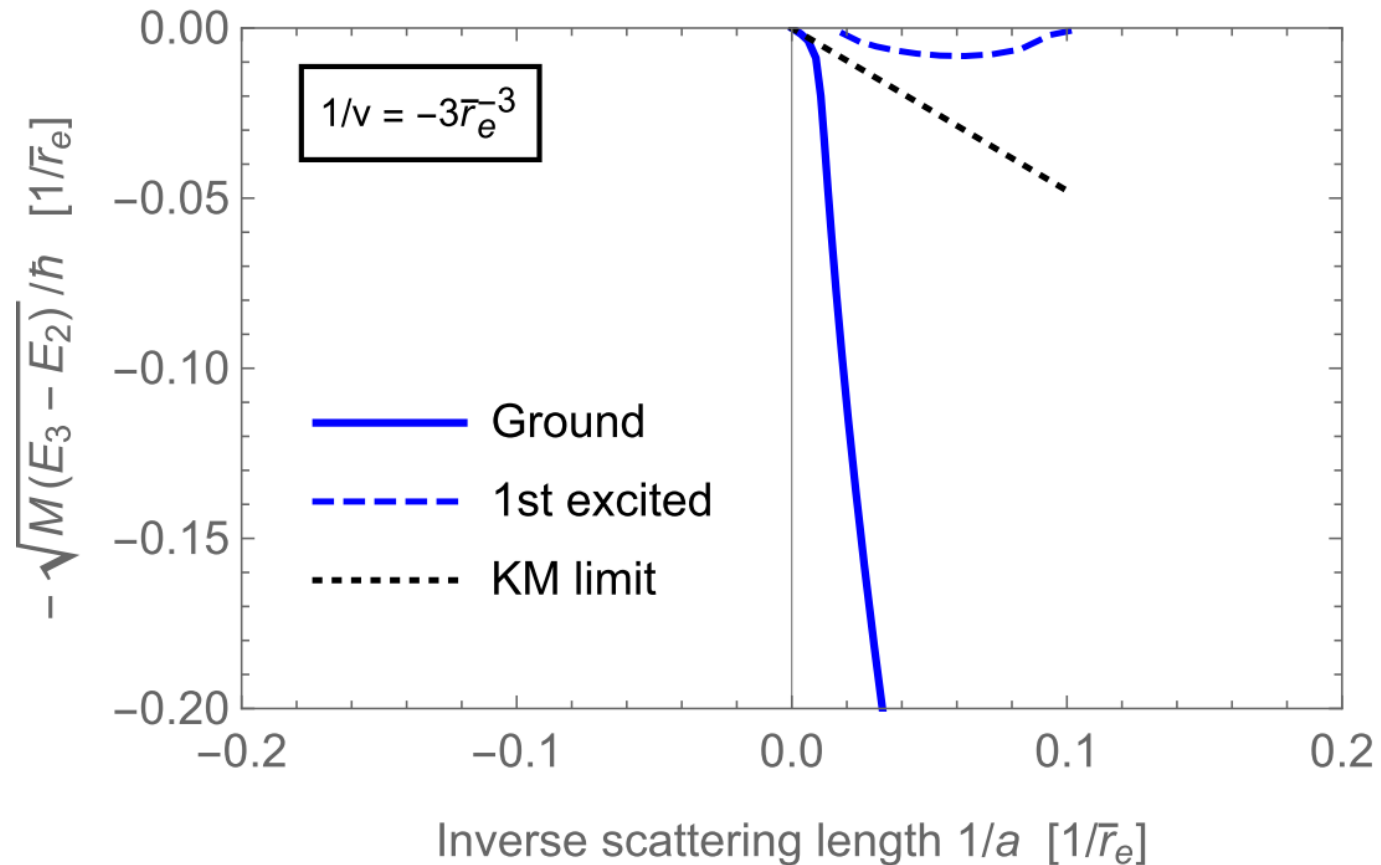
With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )



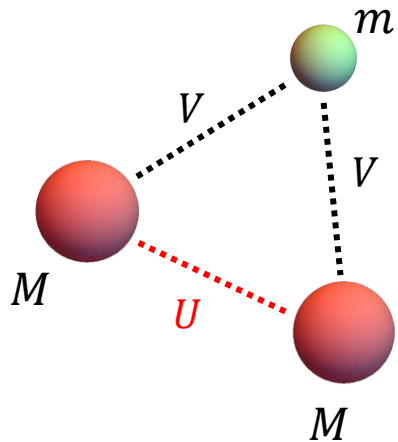
# 2 identical fermions + 1 particle



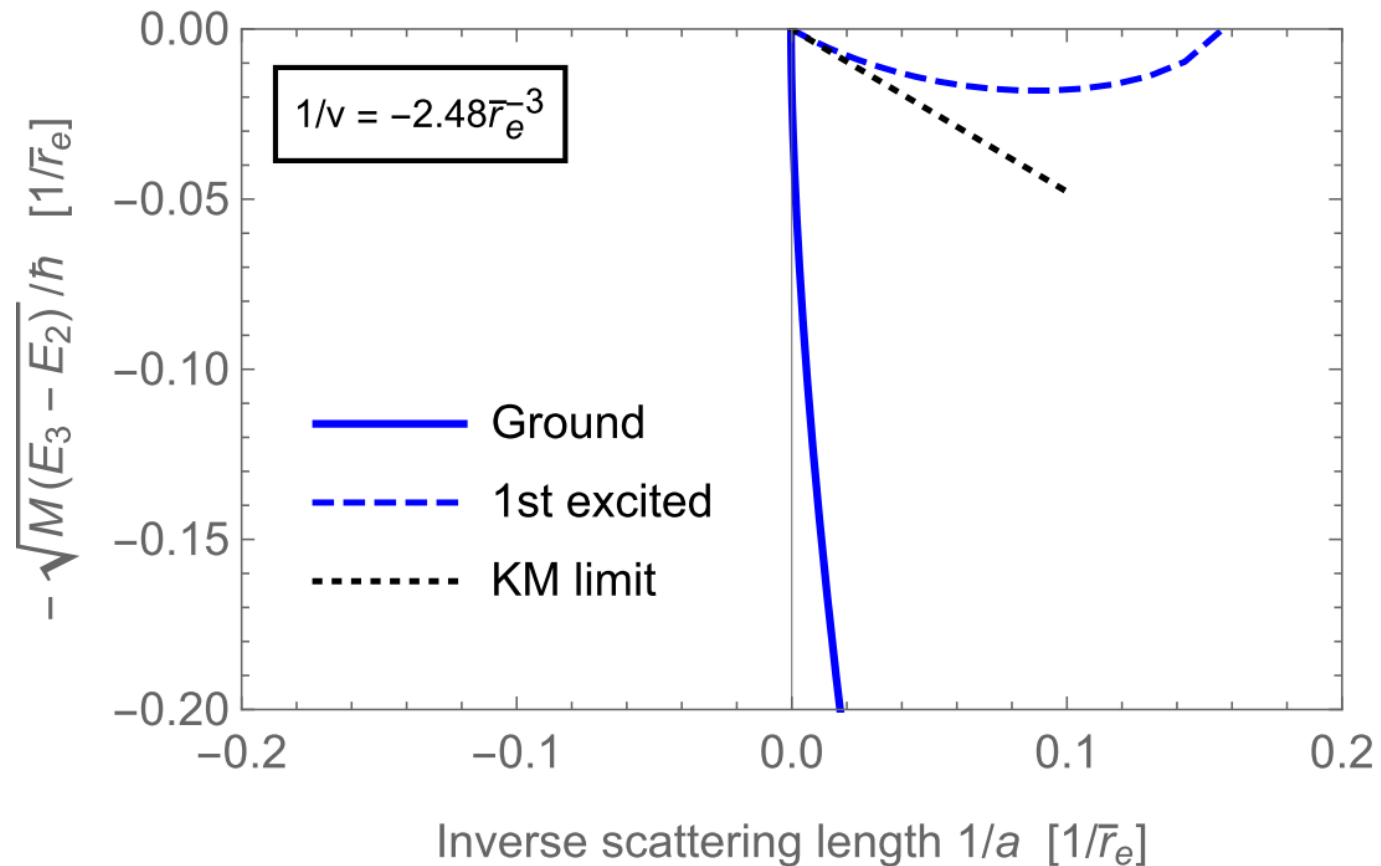
With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )



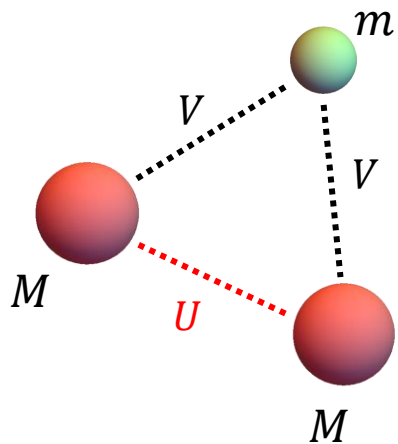
# 2 identical fermions + 1 particle



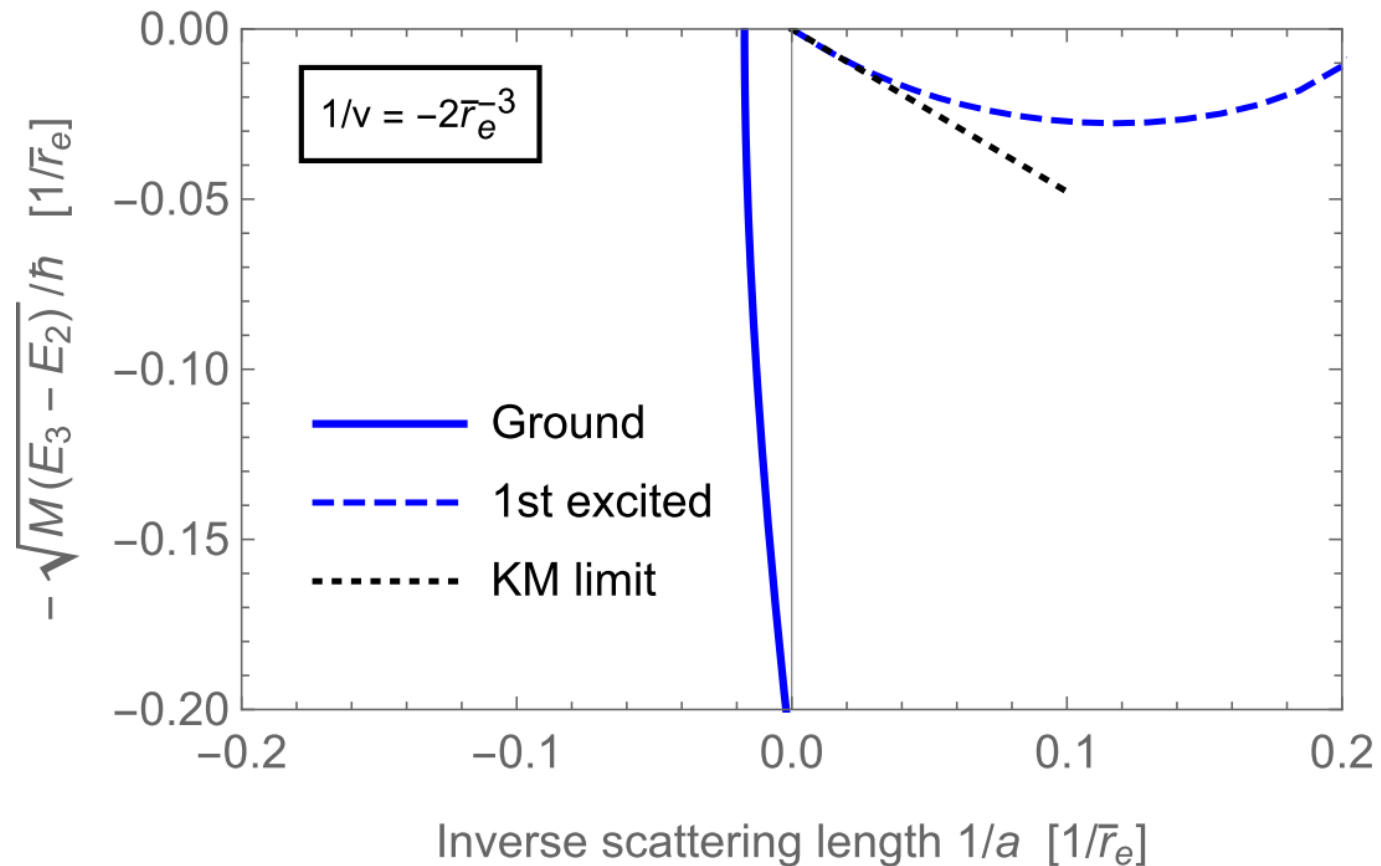
With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )



# 2 identical fermions + 1 particle

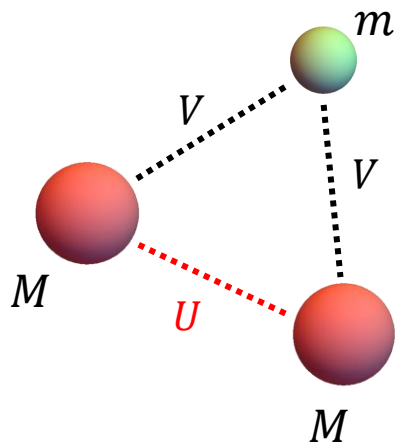


With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )

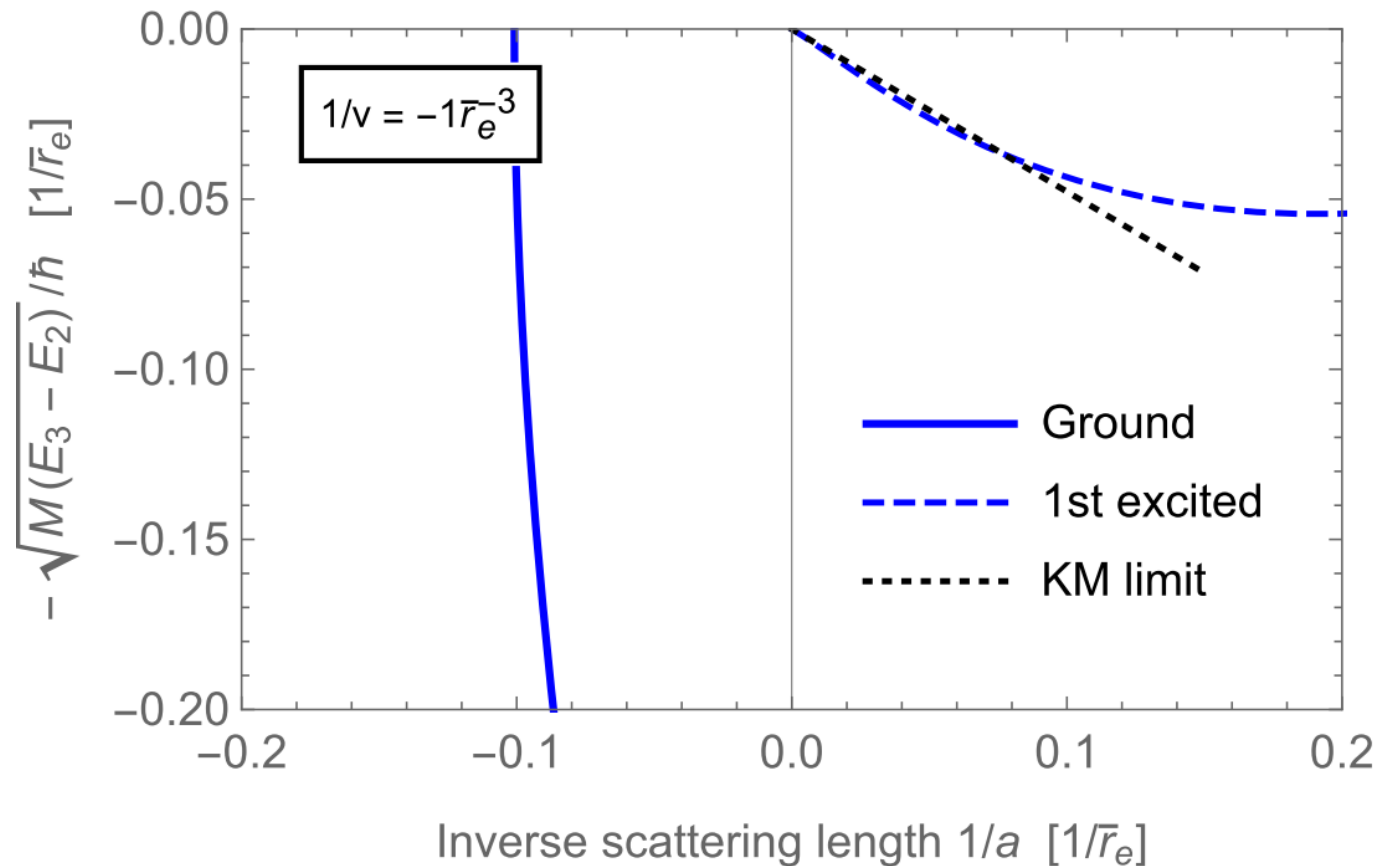




# 2 identical fermions + 1 particle

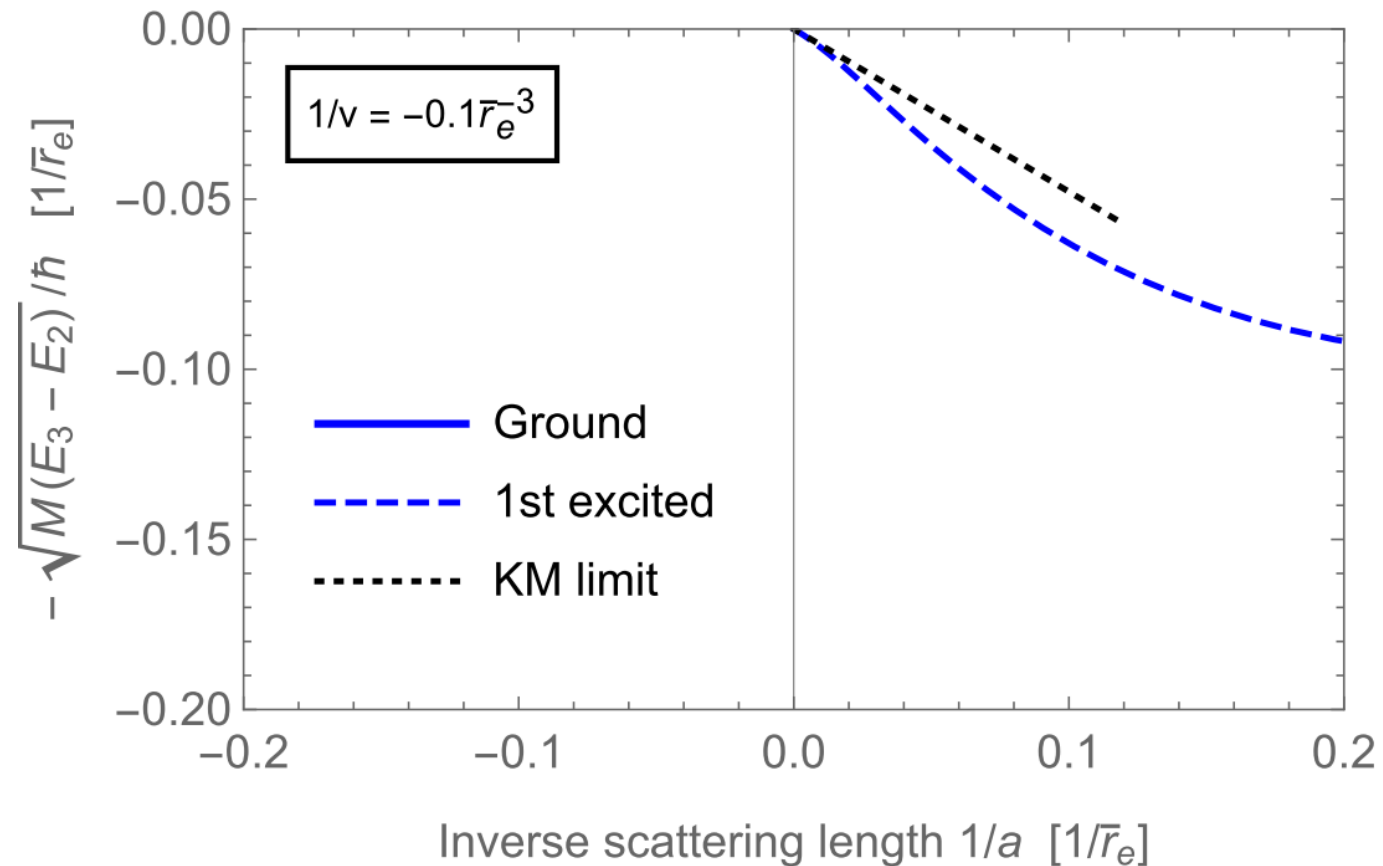
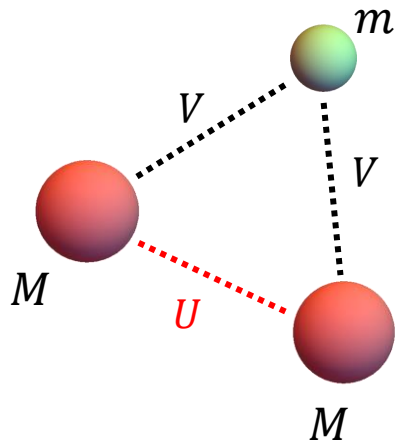


With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )

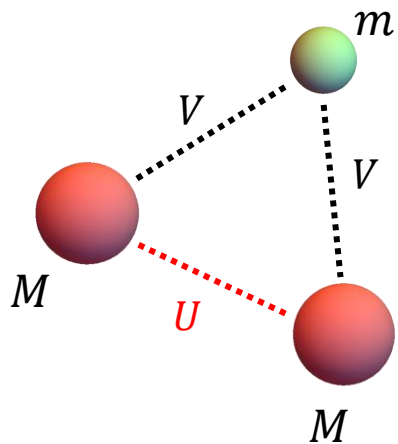


# 2 identical fermions + 1 particle

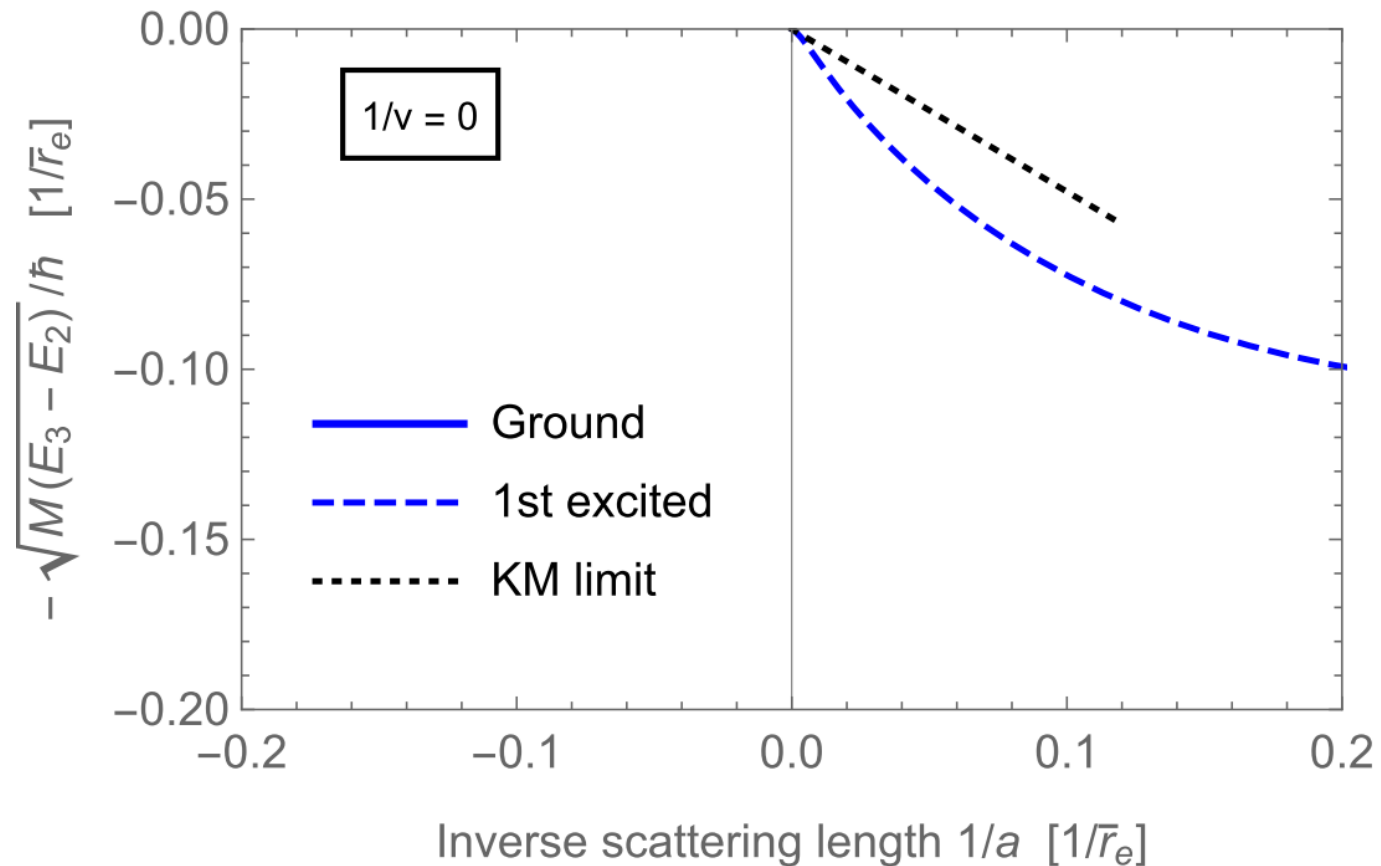
With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )



# 2 identical fermions + 1 particle

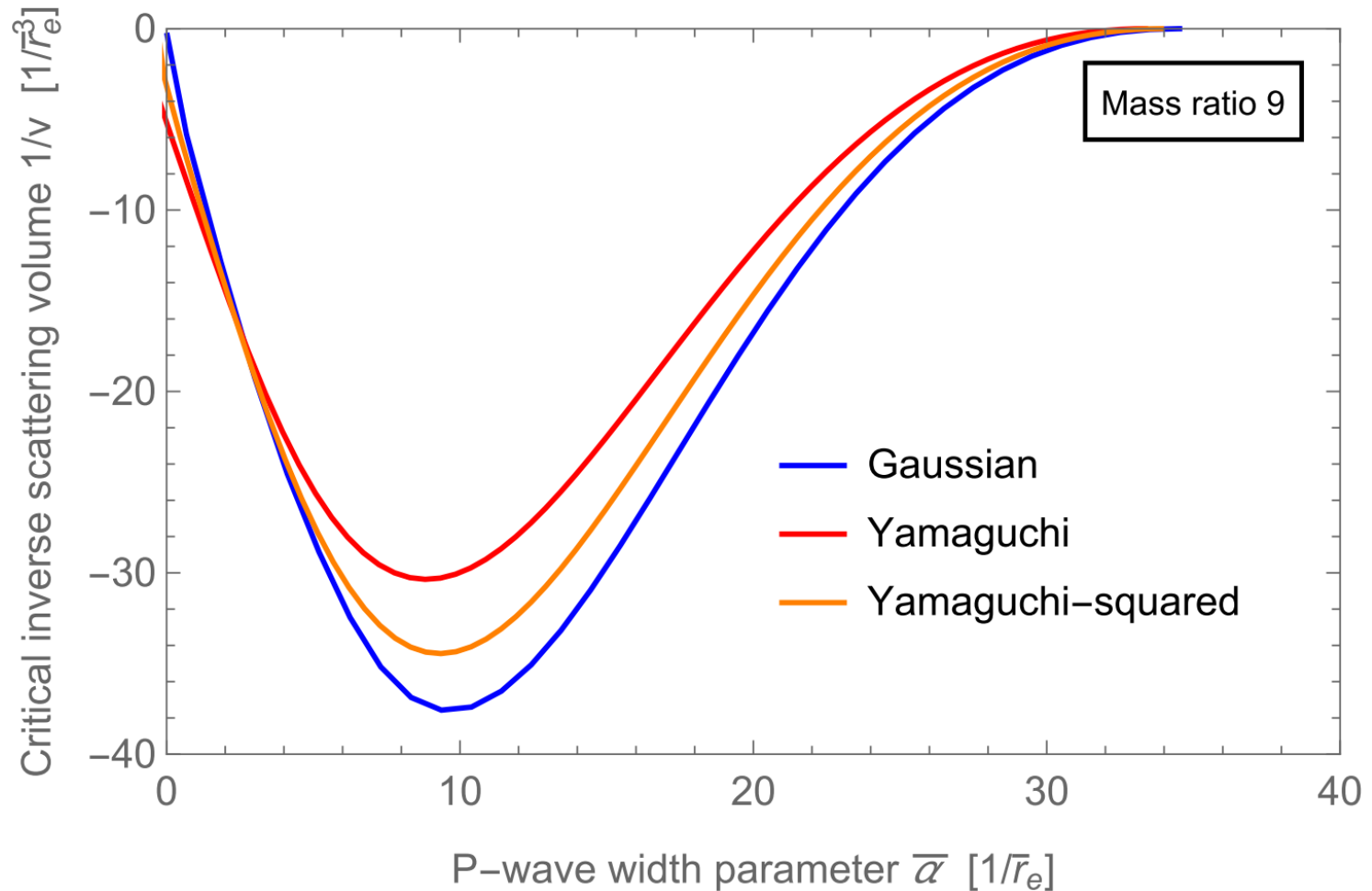
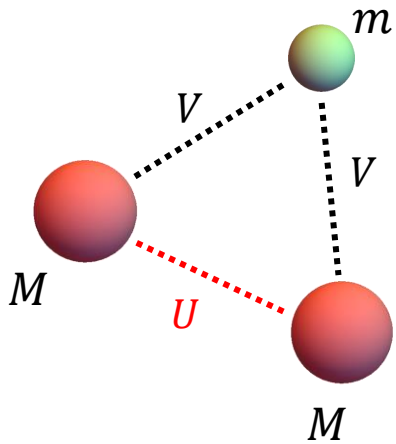


With p-wave interaction: (Mass ratio  $\frac{M}{m} = 9$ )



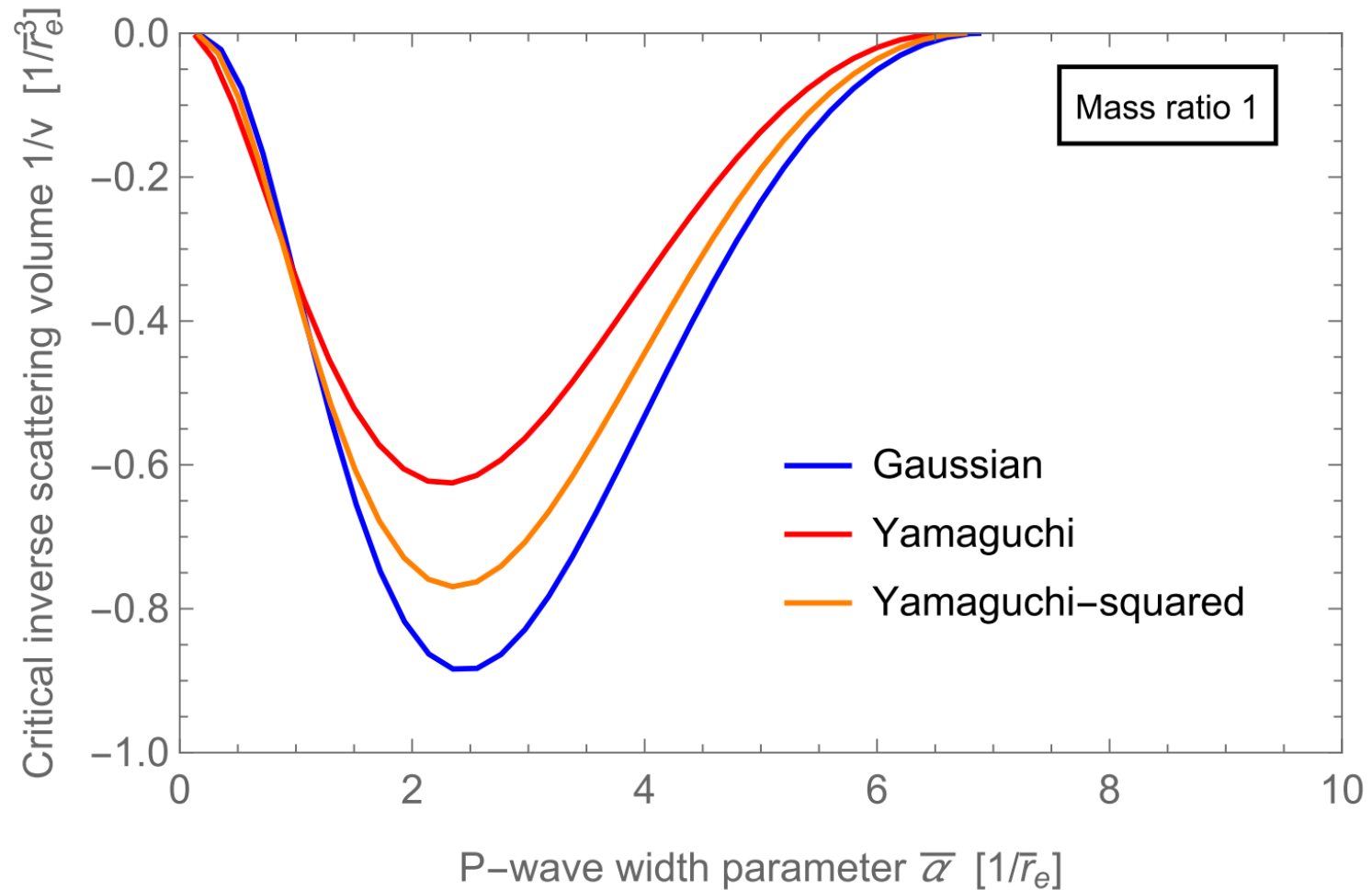
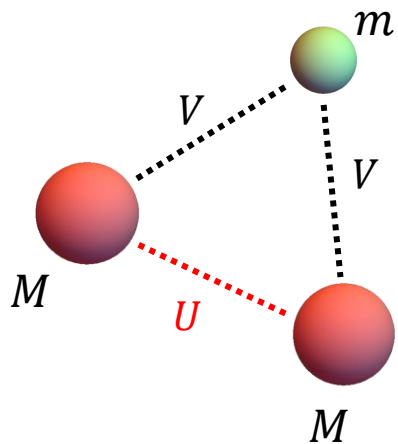
# 2 identical fermions + 1 particle

Critical scattering volume for a Borromean state

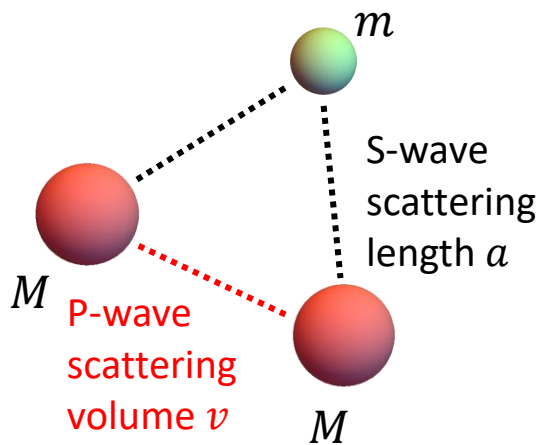


# 2 identical fermions + 1 particle

Critical scattering volume for a Borromean state



## ④ Universal trimers of fermions: summary

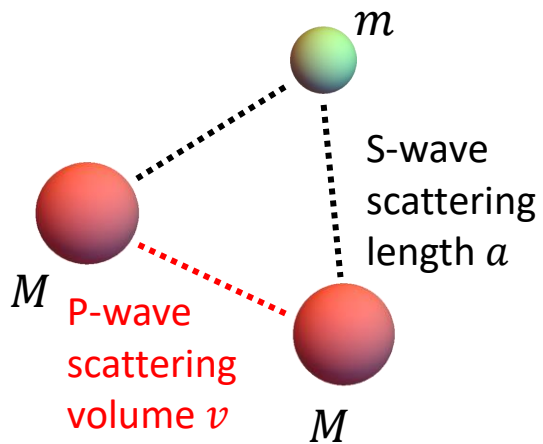


- In 2 fermions + 1 particle systems with p-wave interactions, the universal trimers predicted by Kartavtsev and Malykh for mass ratios between 8.17 and 13.6 always exist for (very!) large scattering lengths.
- However, the p-wave interaction enriches the spectrum by at least one additional state, which can be Borromean and exist *for any mass ratio*.

# ④ Universal trimers of fermions: summary

## Outlook

- Beyond the separable approximation?
- Van der Waals universality for atoms?
- Coupled-channel interactions?
- Stability against recombination?



# ⑤ Trimer phase of fermionic mixtures



Shimpei Endō  
Tohoku University



Antonio M. García-García  
Shanghai Jiao Tong University

**Scattering of universal fermionic clusters in the resonating group method**

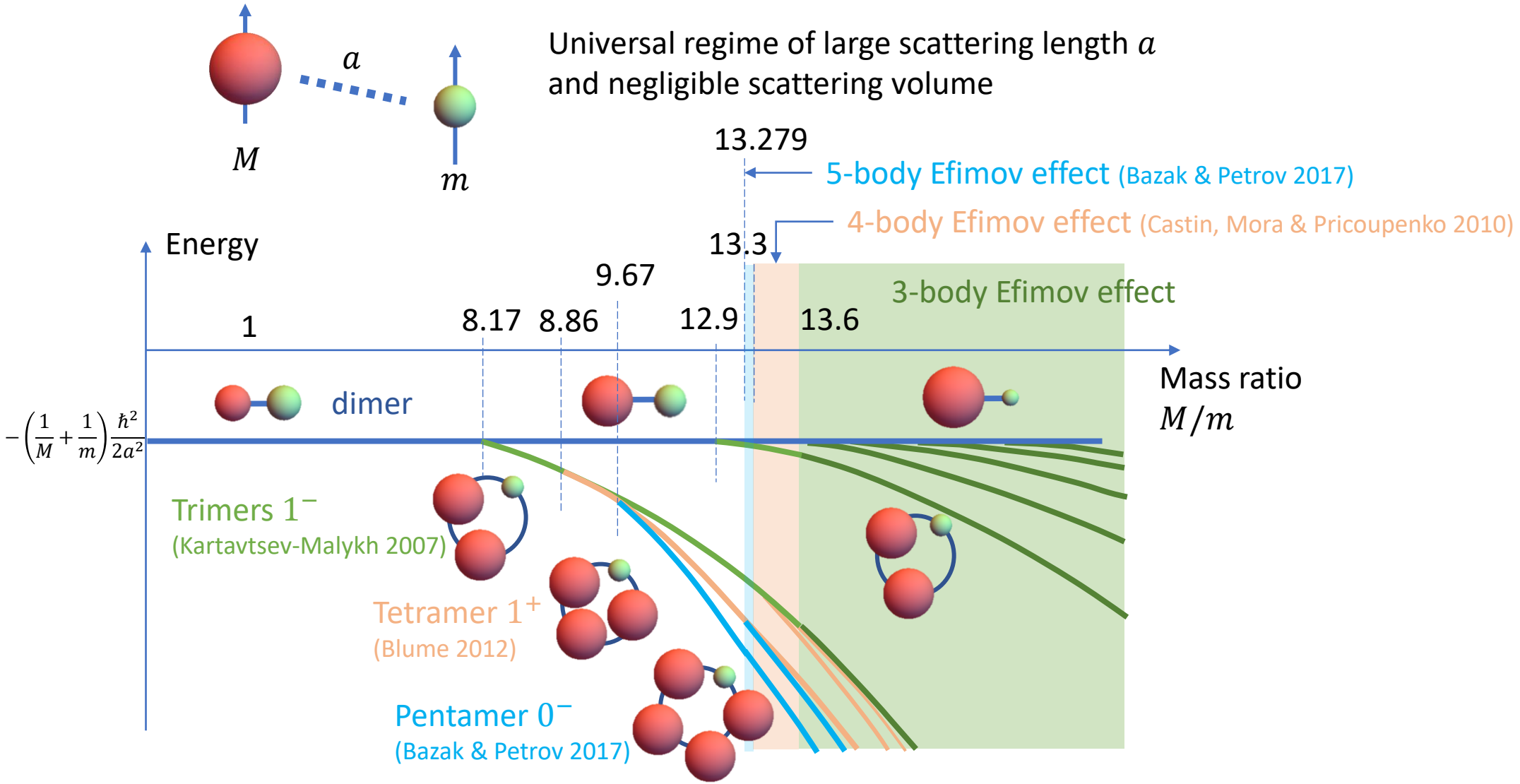
J. Phys. B, 49, 3, 034002 (2016) [[arxiv:1507.06373v1](https://arxiv.org/abs/1507.06373v1)]

**Universal clusters as building blocks of stable quantum matter**

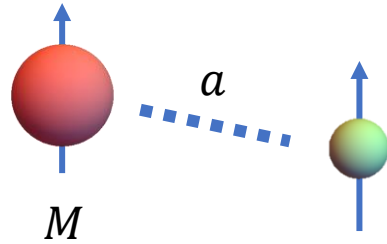
Phys. Rev. A 93, 053611 (2016) [[arxiv:1507.06309v2](https://arxiv.org/abs/1507.06309v2)]



# N+1 fermions

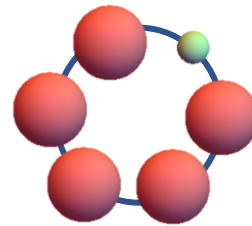


# N+1 fermions

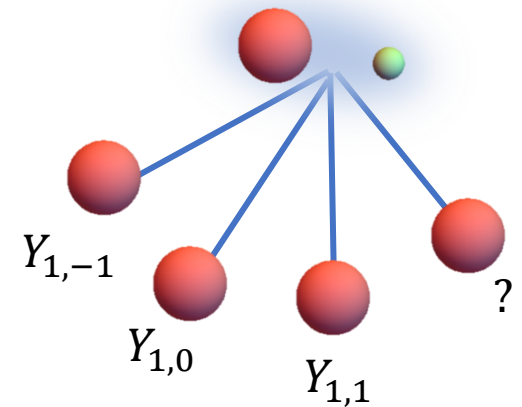


Universal regime of large scattering length  $a$  and negligible scattering volume

- Is there a 6-body Efimov effect?
- Is there a universal hexamer?
- What about the large N limit?



Shell-model argument  
(Bazak & Petrov):



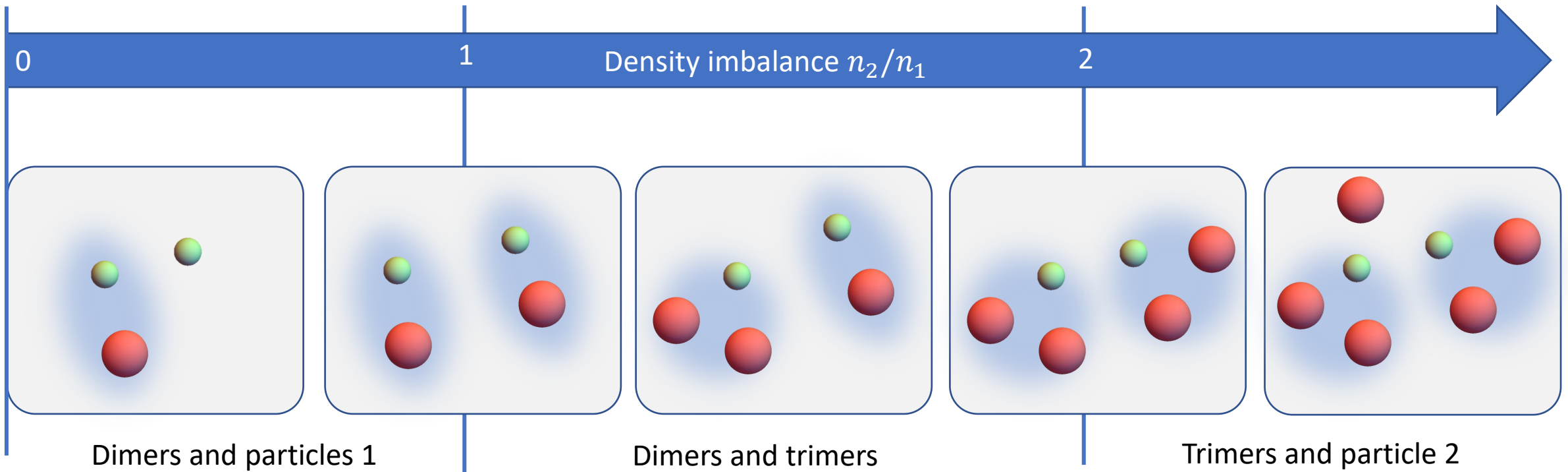
# Two-component fermion mixture

$$8.17 < \frac{M}{m} < 8.86 \dots$$

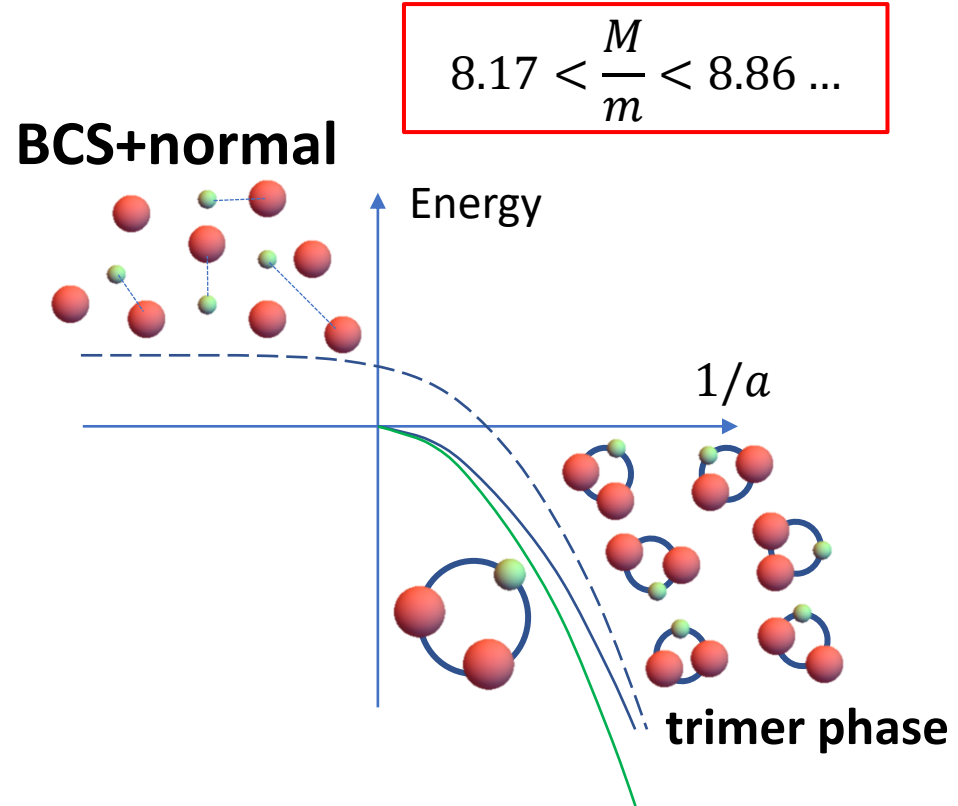
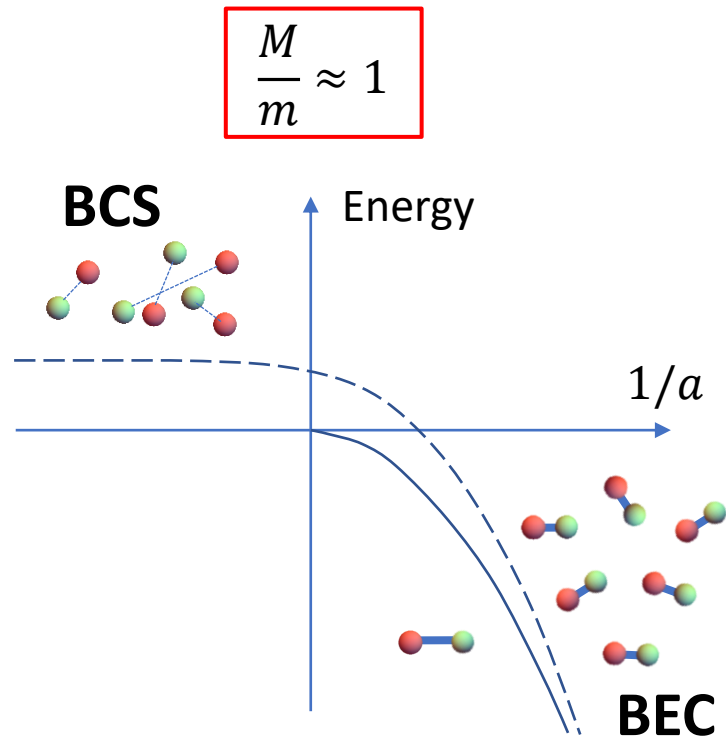
Two dimers are favoured over one trimer

$$|E_2| < |E_3| < 2|E_2|$$

No tetramers, pentamers, etc...



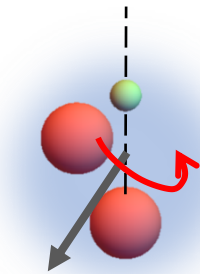
# Two-component fermion mixture



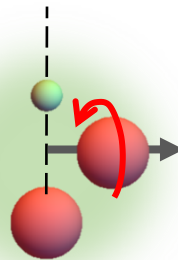
# Two-component fermion mixture

The trimers are

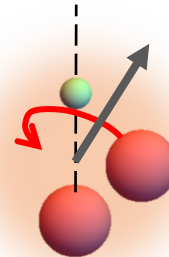
- **fermions**
- with **one unit of angular momentum  $J = 1$**
- So there are three degenerate rotational states :



$m = -1$



$m = 0$

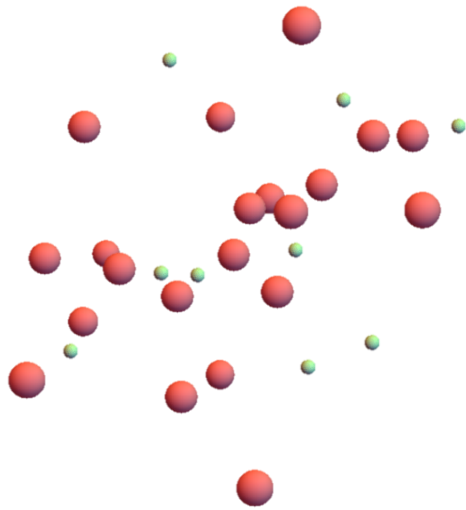


$m = +1$

# Trimer mixture phase

*At low density, the system forms a mixture of trimers*

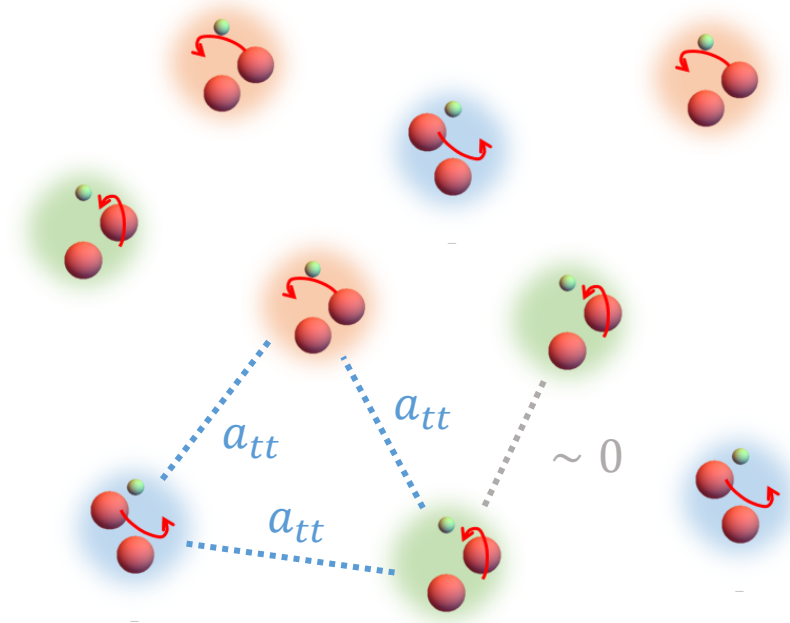
2-component Fermi system



interaction strength



3-component Fermi system



**SU(3) symmetry:** same scattering length  $a_{tt}$  for all pairs

# Trimer mixture phase

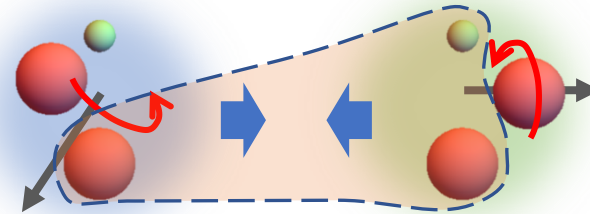
*Is the phase superfluid or not?*

It depends on whether the trimer-trimer interaction is attractive or not.

No 6-body bound state means the interaction is weak, but is it repulsive or attractive?



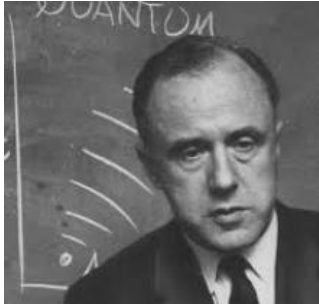
On the one hand, identical fermions tend to repel



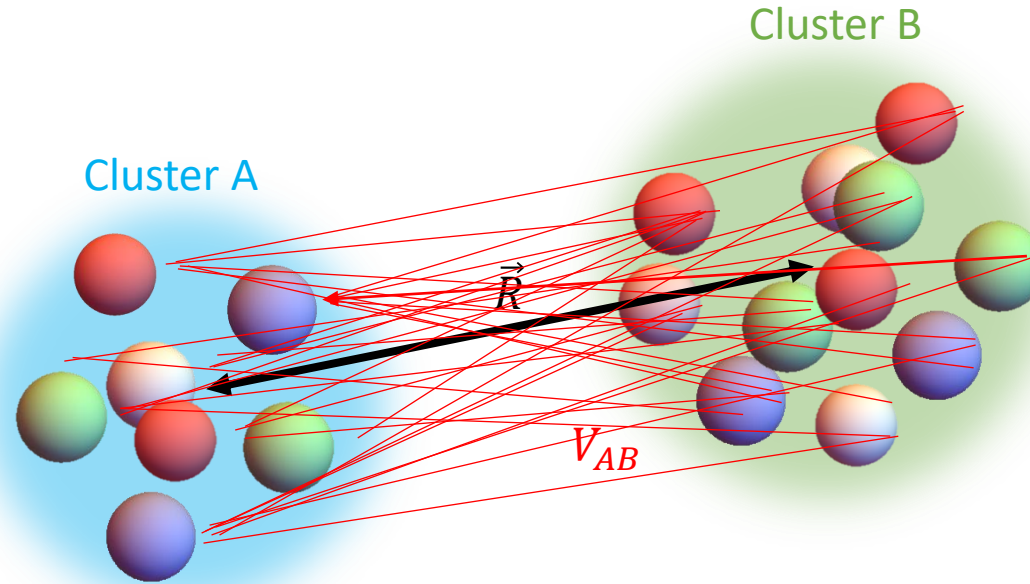
On the one hand, there is 3-body Efimov attraction

**How to solve the six-body problem?**

# The Resonating Group Method (RGM)



**John A. Wheeler**  
 "Molecular Viewpoints in  
 Nuclear Structure" (1937)  
 Physical Review 52, 1083



Anti-symmetrisation

$$\Psi = S \left[ \begin{array}{ccc} \text{cluster A} & \text{cluster B} & \text{relative motion} \\ \phi_A(1,2, \dots, n) & \phi_B(n+1, n+2, \dots, N) & \psi(\vec{R}) \end{array} \right] \quad \text{No excitation (rearrangement) during collision}$$

Variational principle

$$\rightarrow K \cdot \left( -\frac{\hbar^2}{2\mu} \vec{\nabla}^2 \psi \right) + V \cdot \psi = 0$$

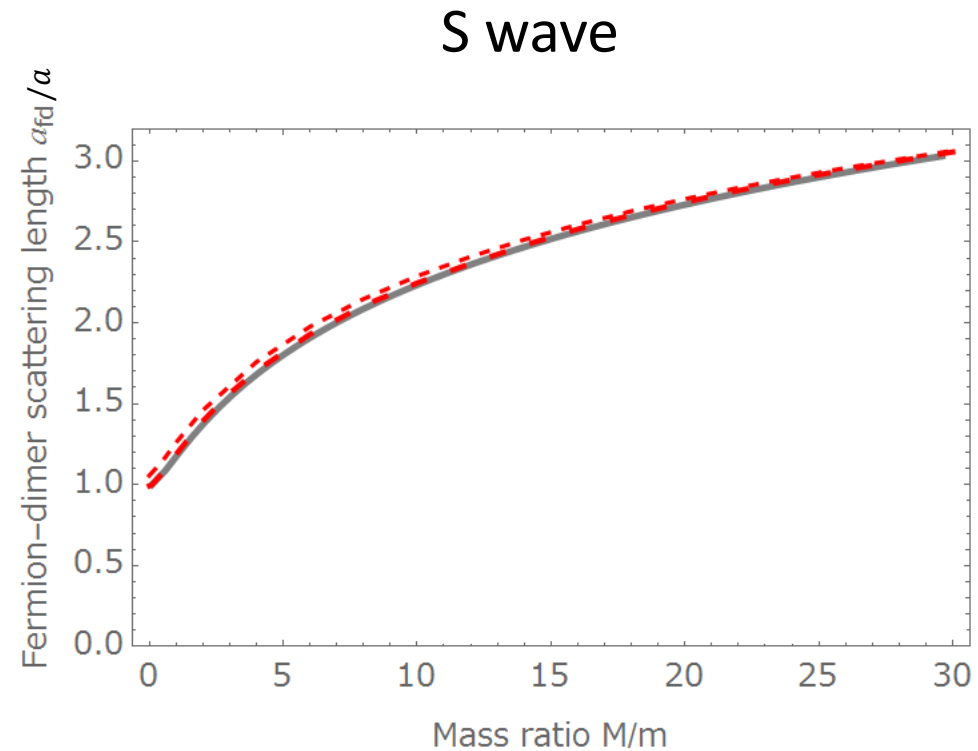
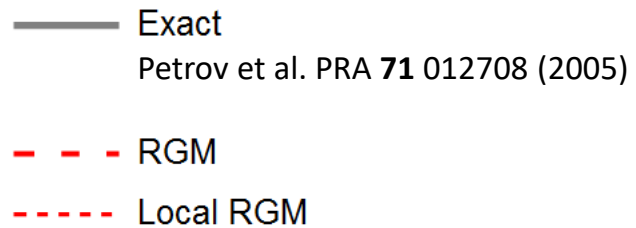
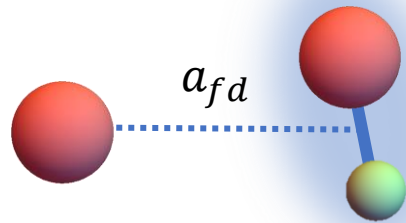
$$\left\{ \begin{array}{l} K \cdot \varphi = \langle \phi_A \phi_B | S(\phi_A \phi_B \varphi) \rangle \approx \varphi \\ \text{Effective potential (non-local)} \longrightarrow \text{Local potential} \\ V \cdot \varphi = \langle \phi_A \phi_B | V_{AB} | S(\phi_A \phi_B \varphi) \rangle \quad V_{\text{loc}}(r) = \int d^3r' V(r, r') \end{array} \right.$$



# The Resonating Group Method

*How well does it work for universal fermionic clusters?*

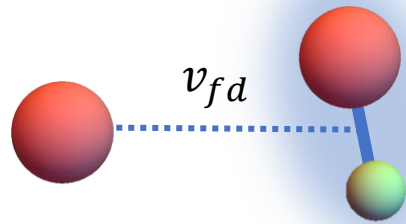
Fermion + dimer scattering



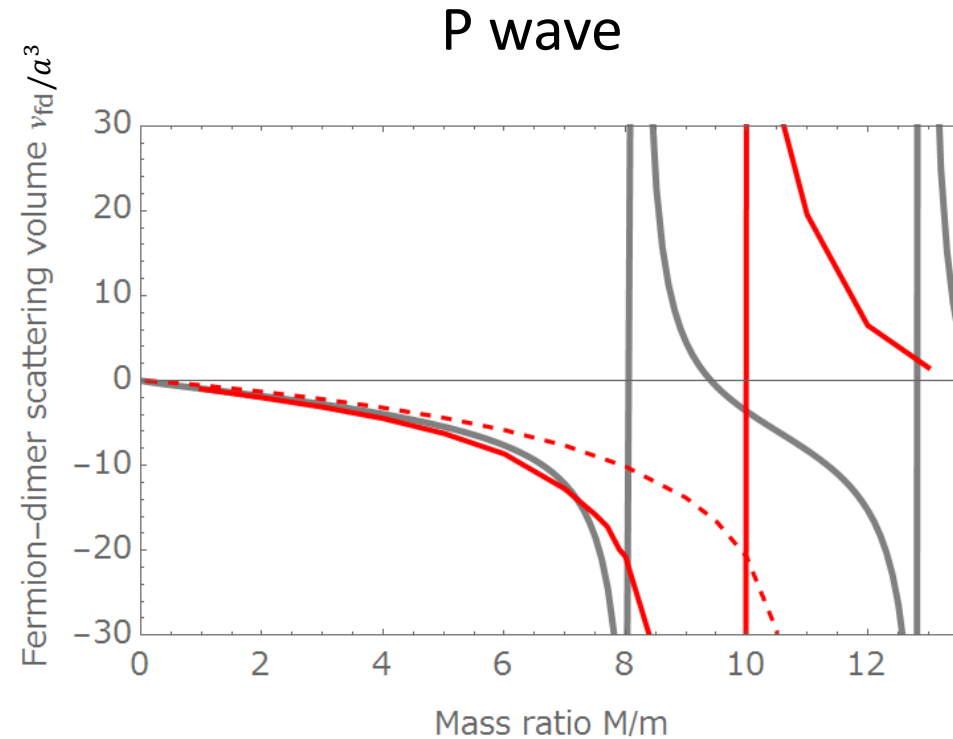
# The Resonating Group Method

*How well does it work for universal fermionic clusters?*

Fermion + dimer scattering



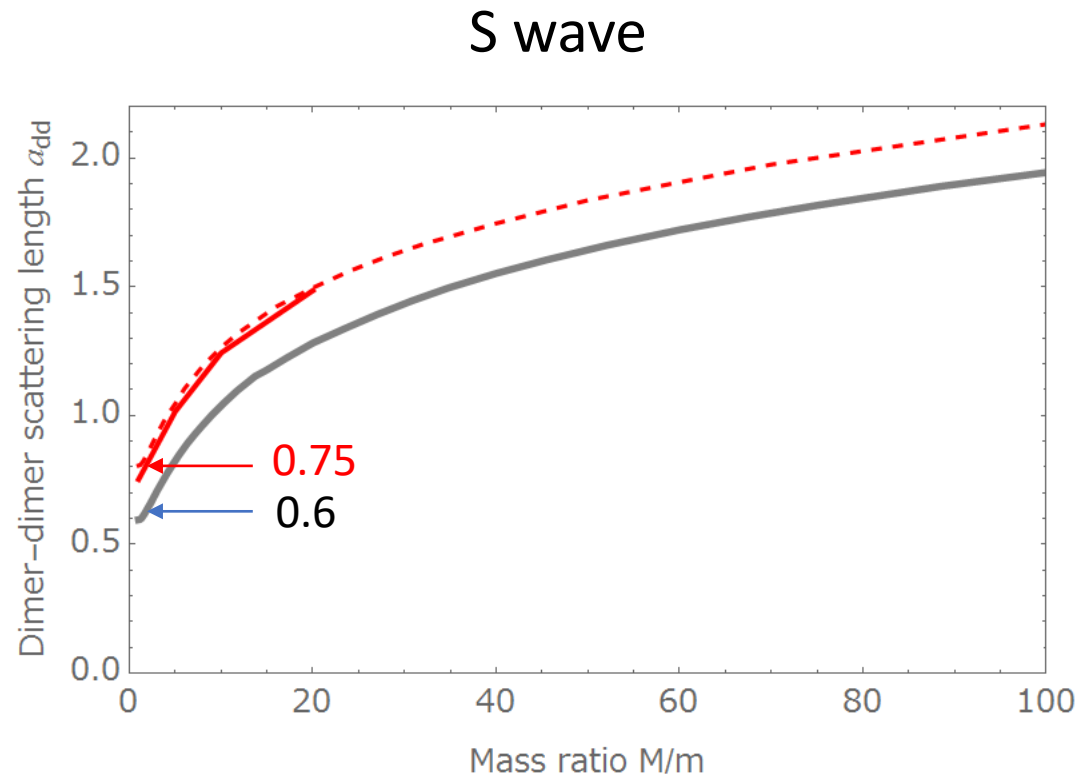
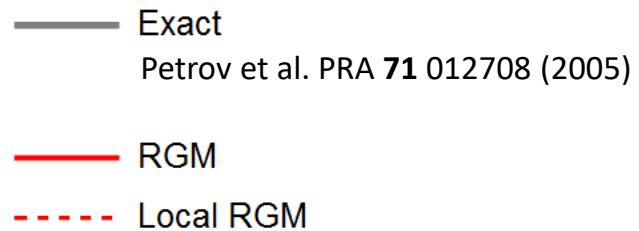
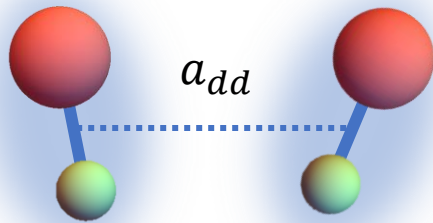
- Exact  
Endo et al, Few-body Syst. (2011)
- RGM
- - - Local RGM



# The Resonating Group Method

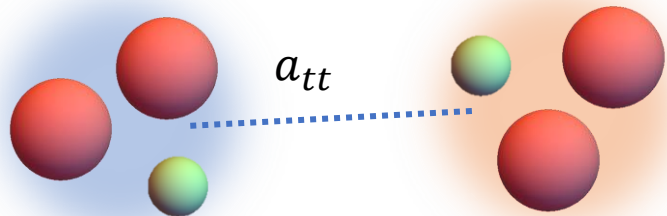
*How well does it work for universal fermionic clusters?*

Dimer + dimer scattering



# The Resonating Group Method

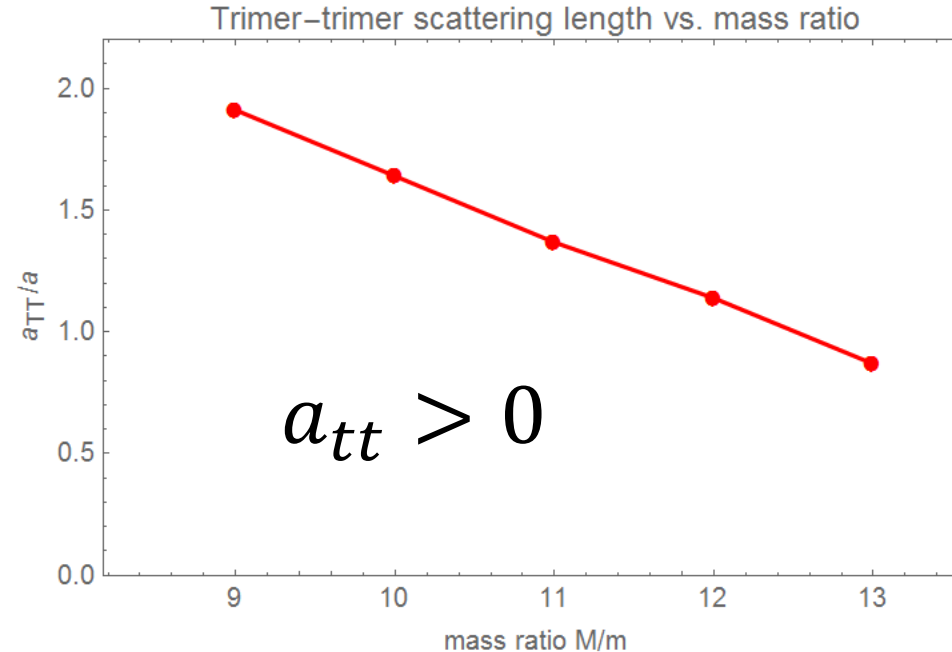
Trimer + Trimer scattering



Local potential  $V$  given by  
9-dimensional integrals,  
calculated by Monte-Carlo

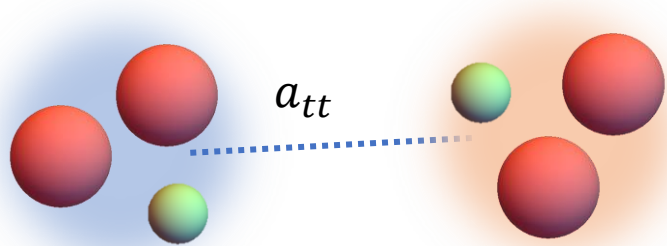
—●— Local RGM

S wave



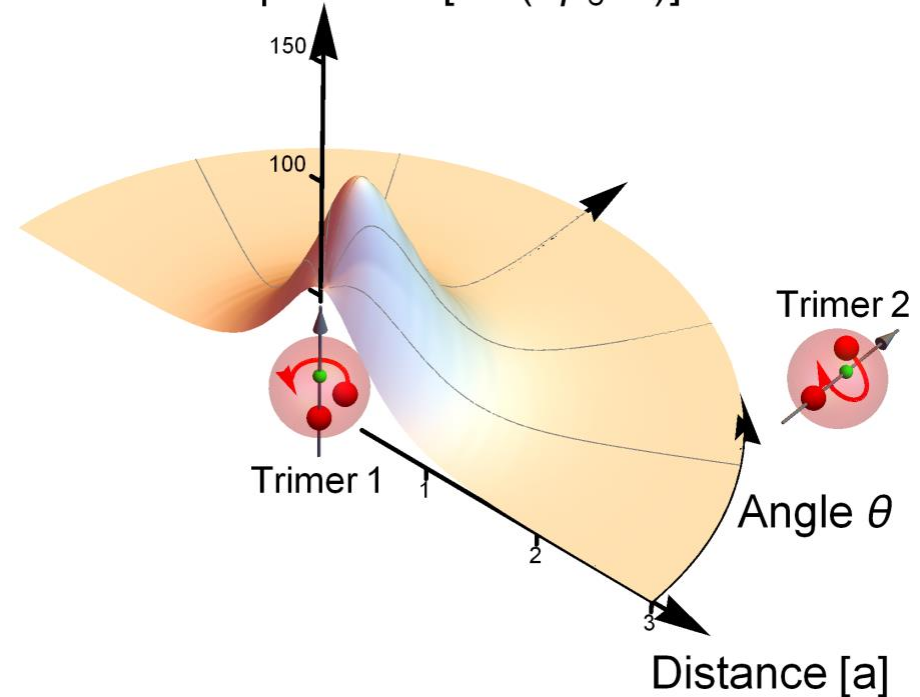
# The Resonating Group Method

Trimer + Trimer scattering



(local approximation)

Trimer-trimer potential  $[\hbar^2/(2\mu_6 a^2)]$

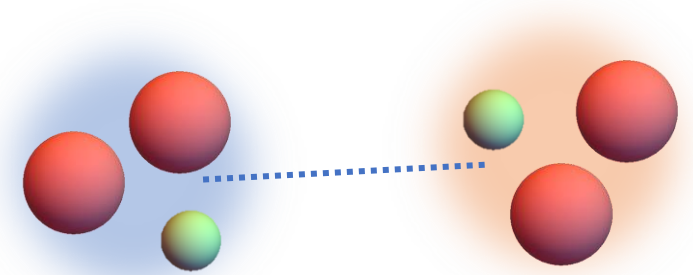


## ⑤ Trimer phase: Summary

- 2-component mass-imbalanced Fermi systems close to unitarity can turn into a 3-component Fermi system of universal trimers
- The resonating group method predicts a repulsion between universal trimers, implying the existence of a many-body phase that is a non-superfluid  $SU(3)$  Fermi gas.

### Outlook

- Can we solve this 6-body problem?
- What if we include the p-wave interaction?



# General summary

- Two-component boson mixtures near unitarity exhibit the 3-body Efimov effect, which plays a role in many-body settings like the polaron problem. Non-resonant interactions also lead to interesting phenomena like liquid droplets and partially-mixed bubbles.
- Two-component fermion mixtures near unitarity can exhibit a rich spectrum of universal trimers below the Efimov critical mass ratio. The universal trimers may form an  $SU(3)$  Fermi gas.