

# Frustration and fluctuations in tissue growth

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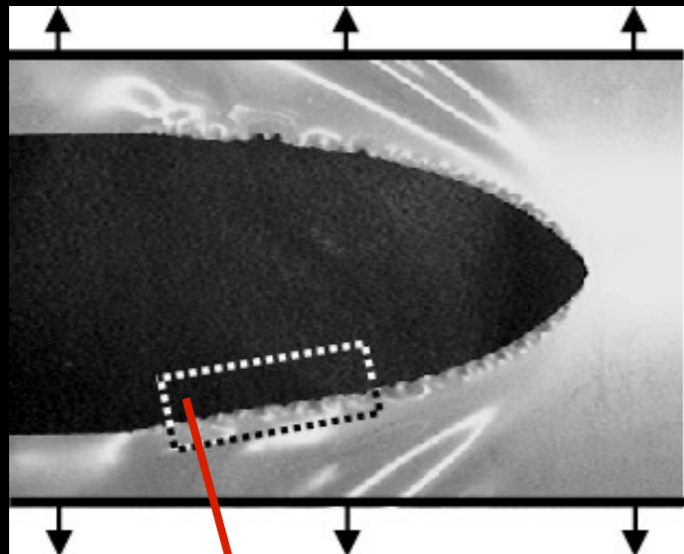




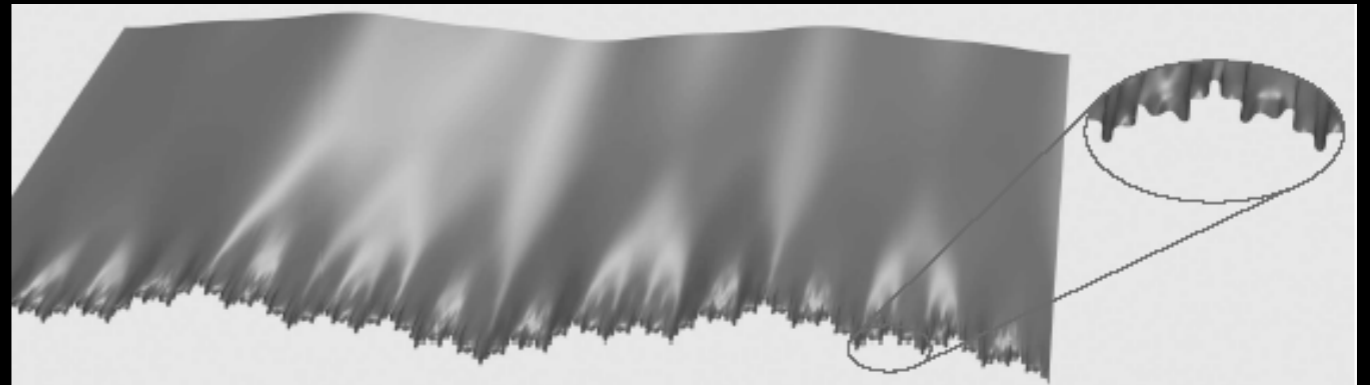
How do organs and organisms reach well-defined size and shape?

*Focus on thin organs*

# Expanding thin elastic sheets: experiments and theory



Sharon et al. Nature 2002



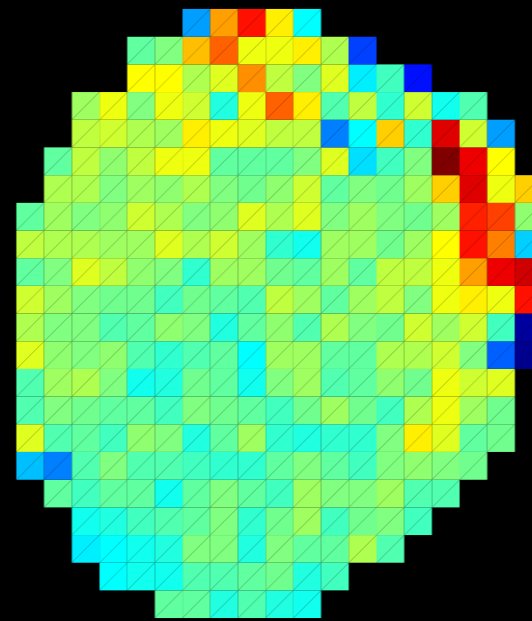
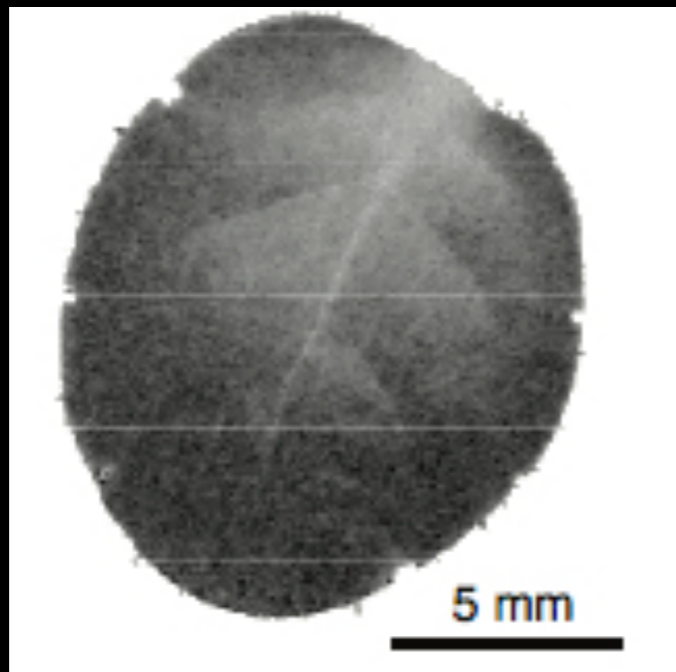
Audoly and Boudaoud PRL 2003

Inhomogeneously growing thin sheets buckle by default

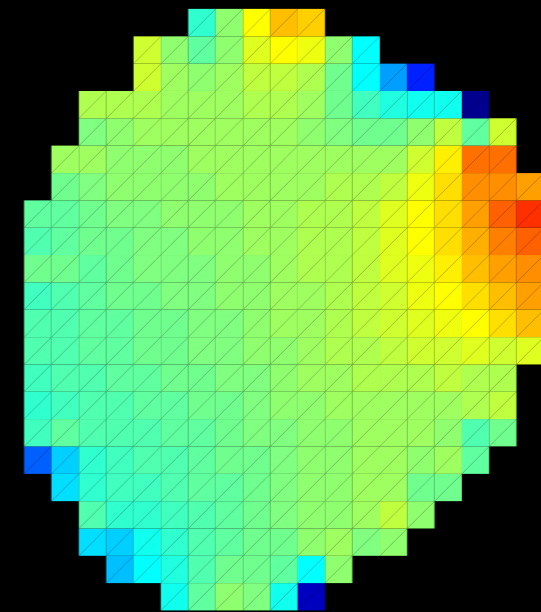


How do leaves achieve flatness?

# Does flatness constrain growth?



Growth  
 $1/A \cdot dA/dt$



Growth from contour  
and conformal map

Alim et al. *Phys Biol* 2016

Similar results: G. Mitchison, *JTB* 2016

Conformal maps retrieve  
2D isotropic growth

But local fluctuations

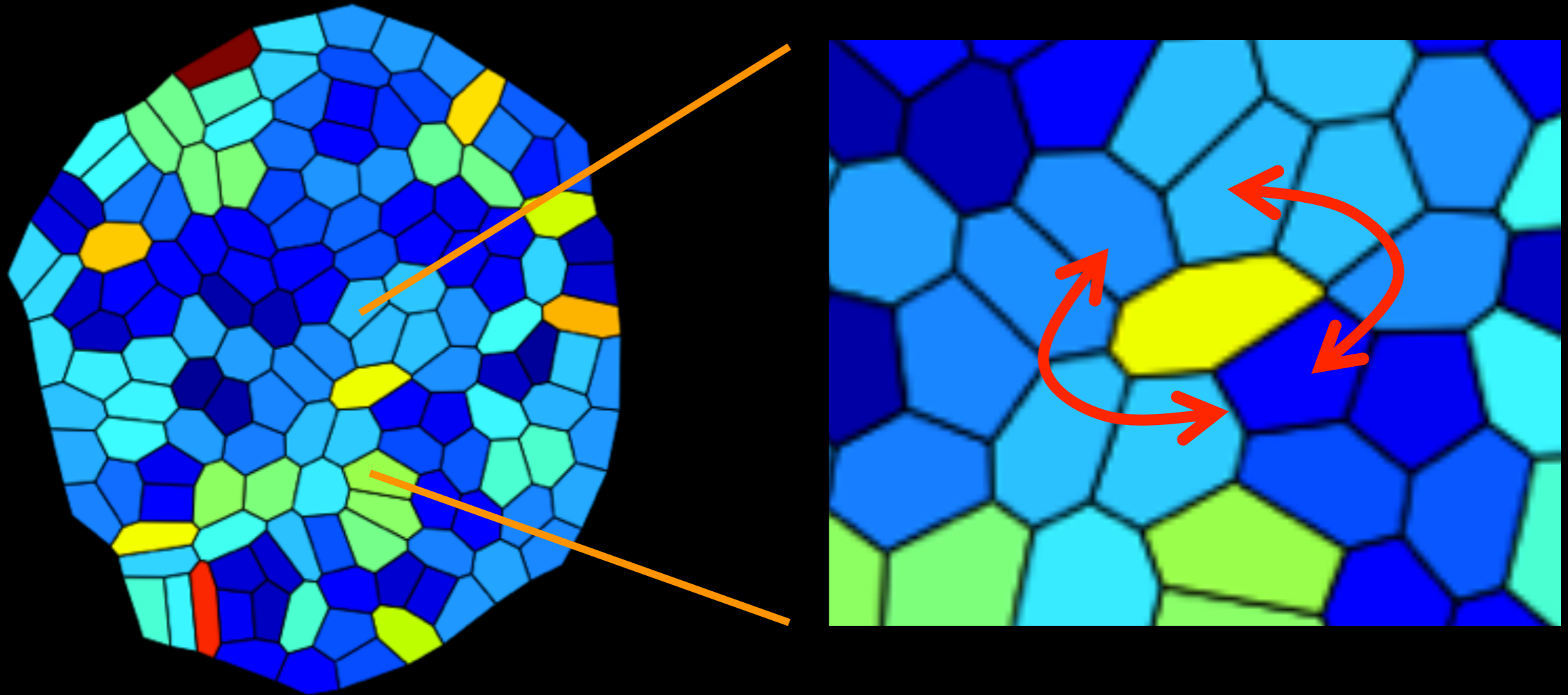


What is the impact of local fluctuations on overall shape of a flat organ?



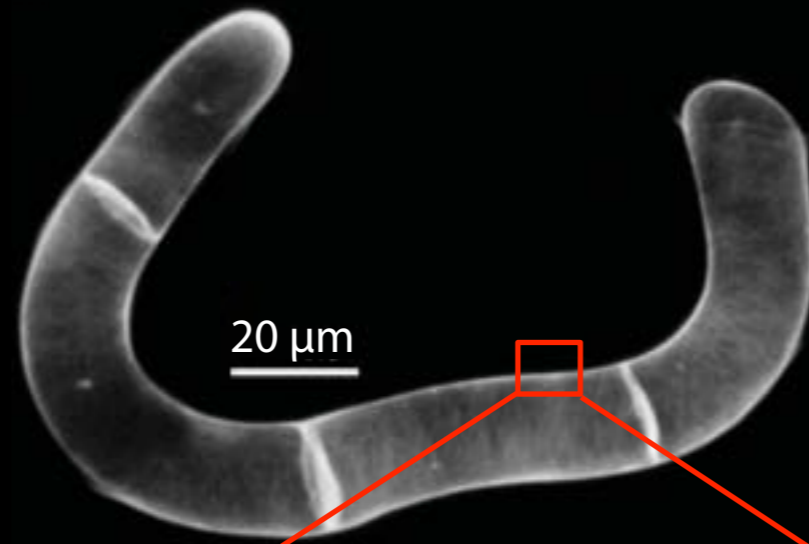
Antoine Fruleux

# Heterogeneous growth and frustration

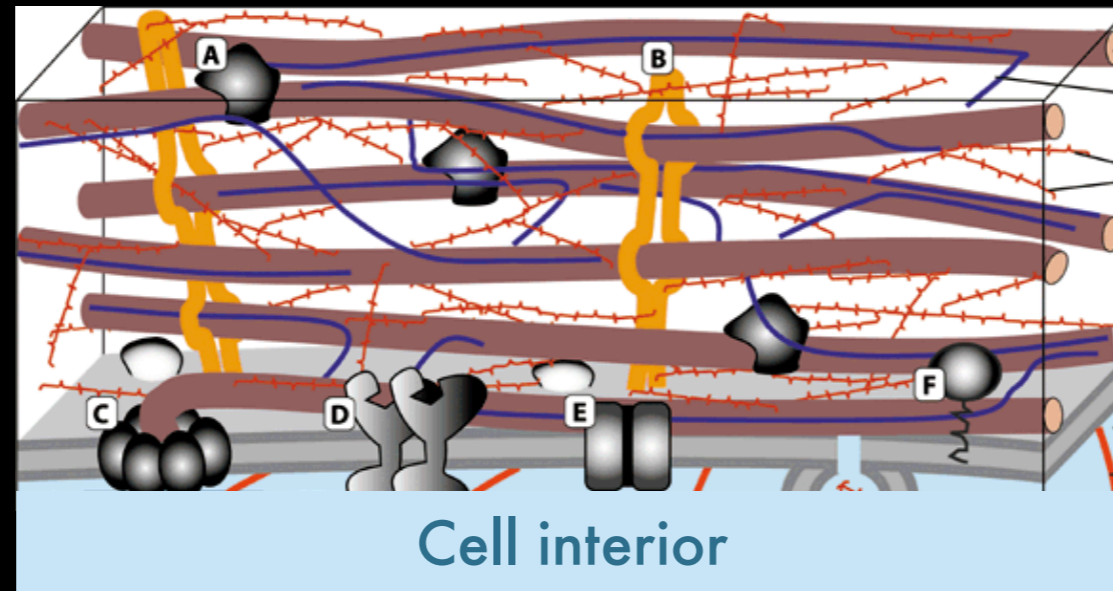


Circumferential mechanical stress  
around fast growing cells

Do plant cells react?



Plant cell



Cell wall  
0.1 to  
10μm  
Thick

Cell  
membrane

Plant cells react: Cellulose is deposited in the direction of maximal mechanical stress

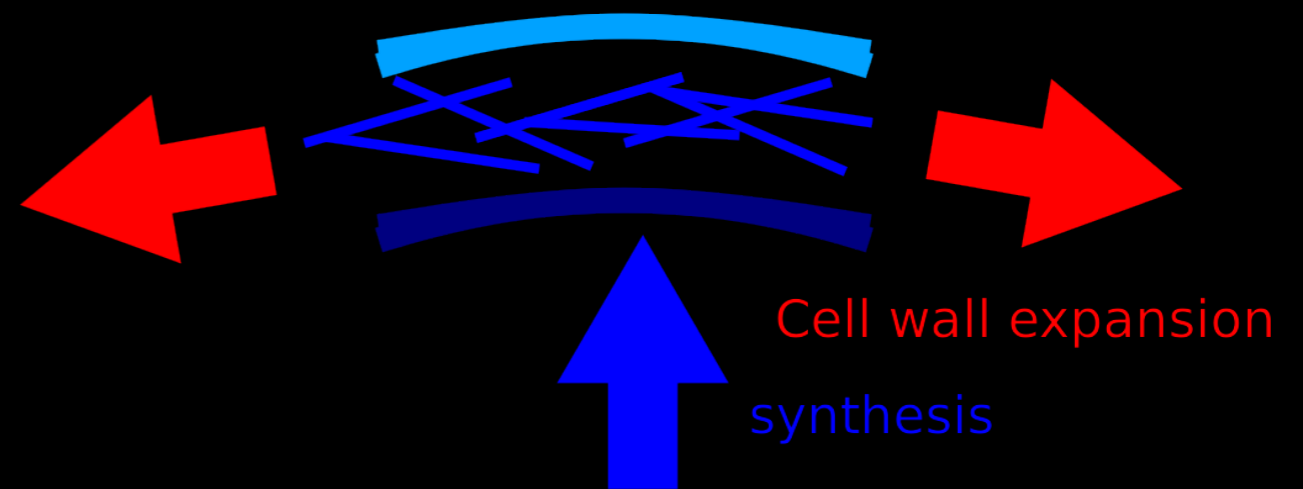
What is the effect of such mechanical feedback on organ shape?



Antoine Fruleux  
ENS Lyon

# A stochastic viscous model for cell wall expansion

- Viscous cell wall on long time scales
- Tension due to cell pressure
- Increase in synthesis when tension is increased (density and orientation of fibres)
- Fluctuations in synthesis



wall tension

$$\bar{\sigma}(\bar{\dot{\gamma}}, \rho, \bar{s})$$

strain rate tensor

$$\bar{\dot{\gamma}} = (\partial_{\vec{r}} \vec{v}) + (\partial_{\vec{r}} \vec{v})^T$$

→  $\vec{v}$  flow velocity

$$\bar{\omega} = (\partial_{\vec{r}} \vec{v}) - (\partial_{\vec{r}} \vec{v})^T$$

$$\partial_{\vec{r}} \cdot [\bar{\sigma} - \bar{1}P] = \vec{0}$$

wall components:

→  $\rho$  density

$$\partial_t \rho + \partial_{\vec{r}} \cdot \{\rho \vec{v}\} = \kappa_\rho$$

convection + synthesis

→  $\bar{s}$  anisotropy

$$\partial_t \bar{s} + (\vec{v} \cdot \partial_{\vec{r}}) \bar{s} + \bar{\omega} \cdot \bar{s} - \bar{s} \cdot \bar{\omega} = \bar{\kappa}_s$$

advection + corotation + synthesis

$$\bar{\kappa}_\rho = \langle \kappa_\rho \rangle - \frac{1}{\tau_\rho} (\Delta \rho - \beta_\rho \Delta \sigma)$$

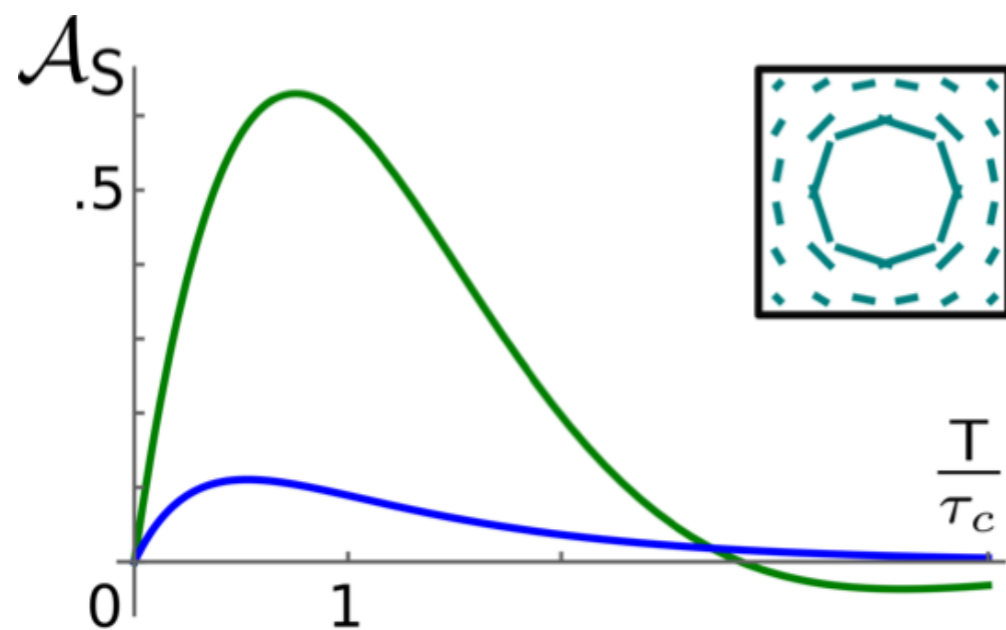
$$\bar{\kappa}_s = -\frac{1}{\tau_s} (\bar{s} - \beta_s \bar{\sigma}_a)$$

*Generic model* Similar to models for animal tissues, with feedback in addition

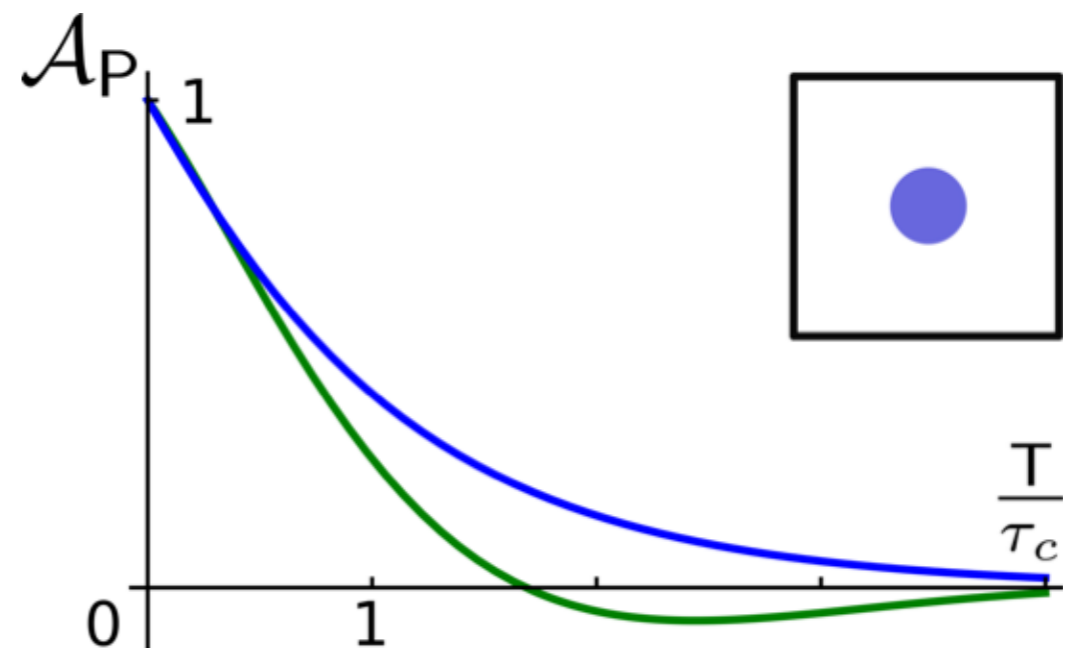
# Response to local reduction in synthesis

*Low anisotropic mechanical feedback*

*High anisotropic mechanical feedback*



Tissue anisotropy



Tissue density

wall tension

$$\bar{\sigma}(\bar{\dot{\gamma}}, \rho, \bar{s})$$

strain rate tensor

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$$\partial_t \bar{s} + (\vec{v} \cdot \partial_{\vec{r}}) \bar{s} + \bar{\omega} \cdot \bar{s} - \bar{s} \cdot \bar{\omega} = \bar{\kappa}_s$$

advection + corotation + synthesis

$$\bar{\kappa}_\rho = \langle \kappa_\rho \rangle - \frac{1}{\tau_\rho} (\Delta \rho - \beta_\rho \Delta \sigma) + \Delta \bar{\kappa}_\rho$$

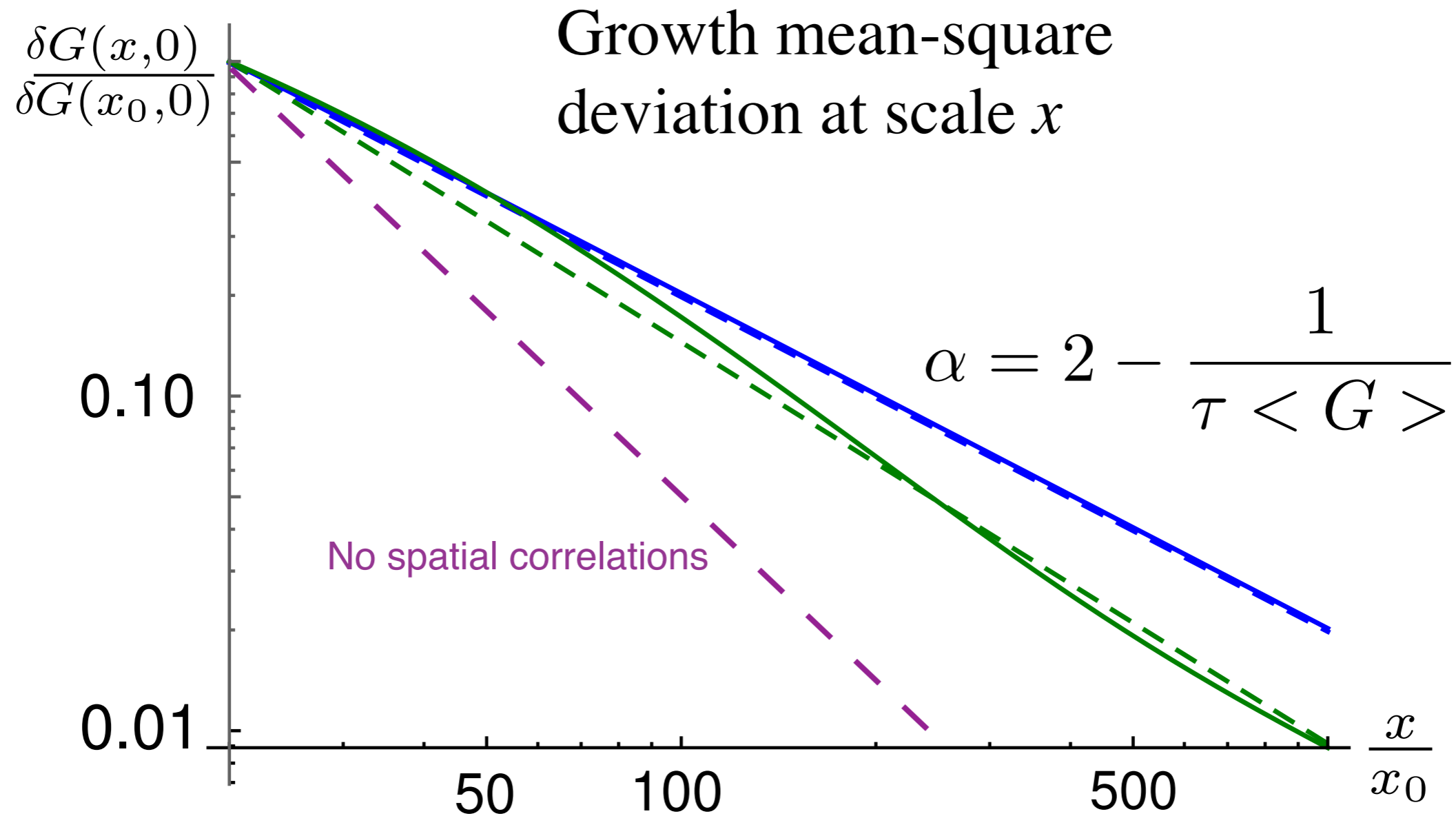
$$\bar{\kappa}_s = -\frac{1}{\tau_s} (\bar{s} - \beta_s \bar{\sigma}_a) + \Delta \bar{\kappa}_s$$

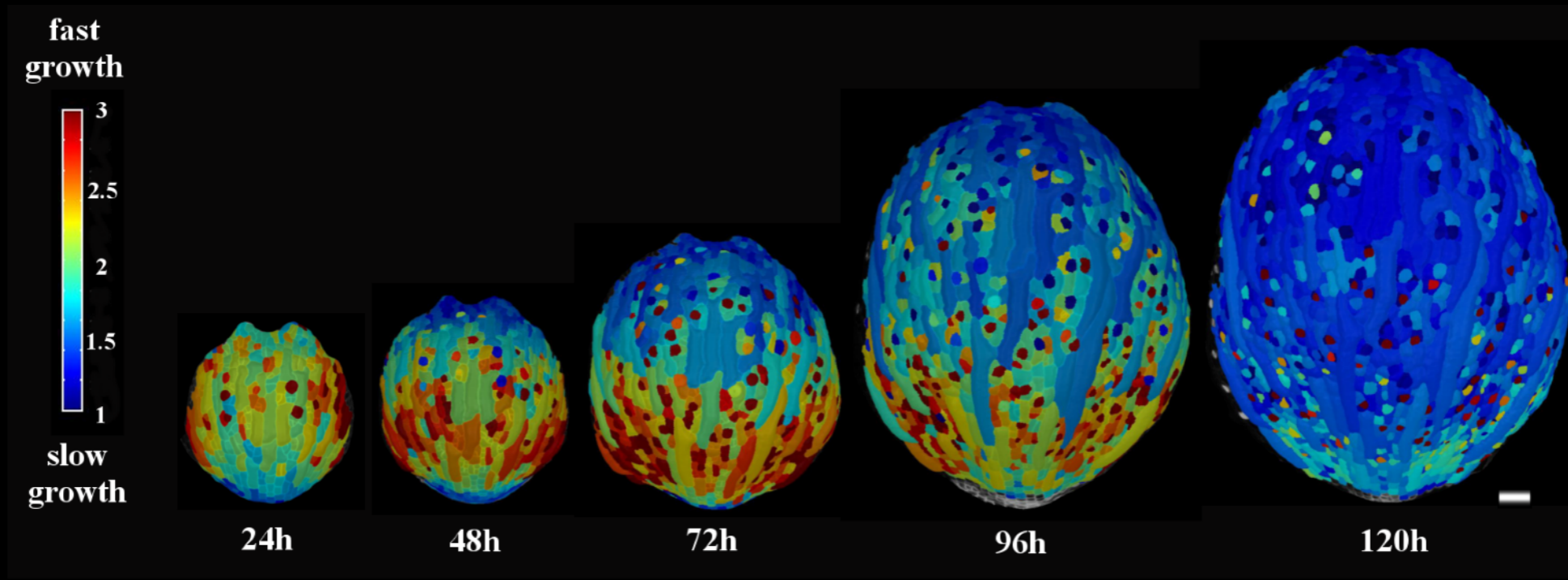
$$\langle \Delta \kappa_\rho(\vec{r}, t) \Delta \kappa_\rho(\vec{r}', t') \rangle = K g \left( \frac{|\vec{r} - \vec{r}'|}{\xi_0} \right) \delta(t - t')$$

$$\langle \Delta \kappa_s(\vec{r}, t) \Delta \kappa_s(\vec{r}', t') \rangle = S g \left( \frac{|\vec{r} - \vec{r}'|}{\xi_0} \right) \delta(t - t')$$

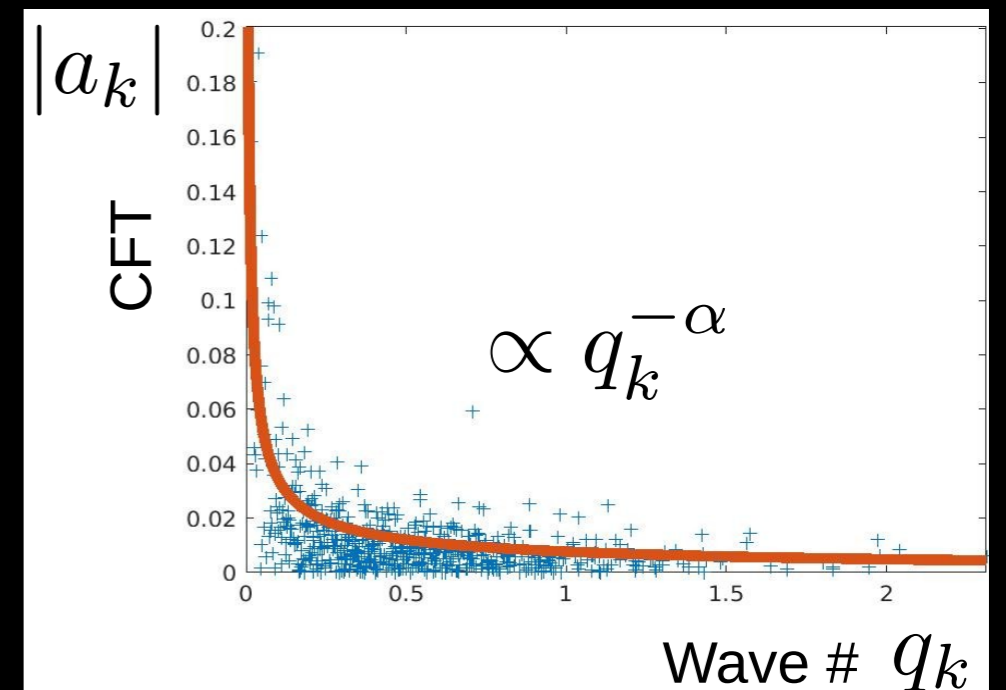


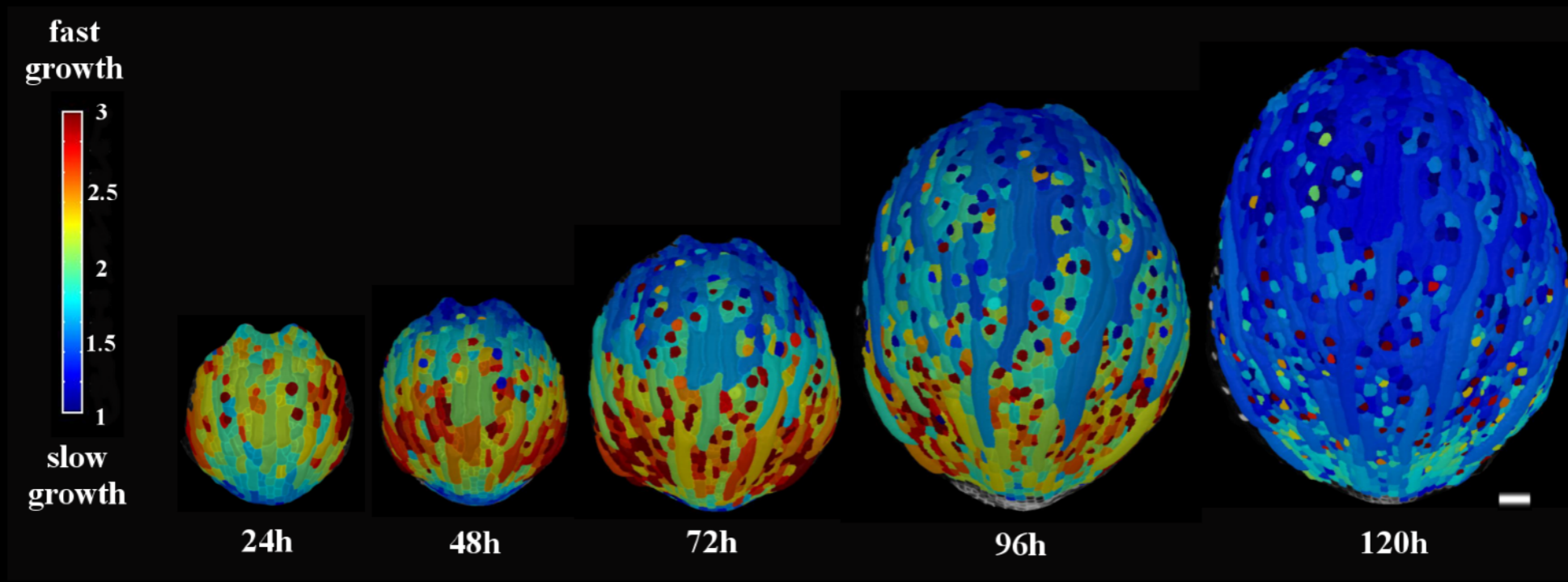
# Growth fluctuations



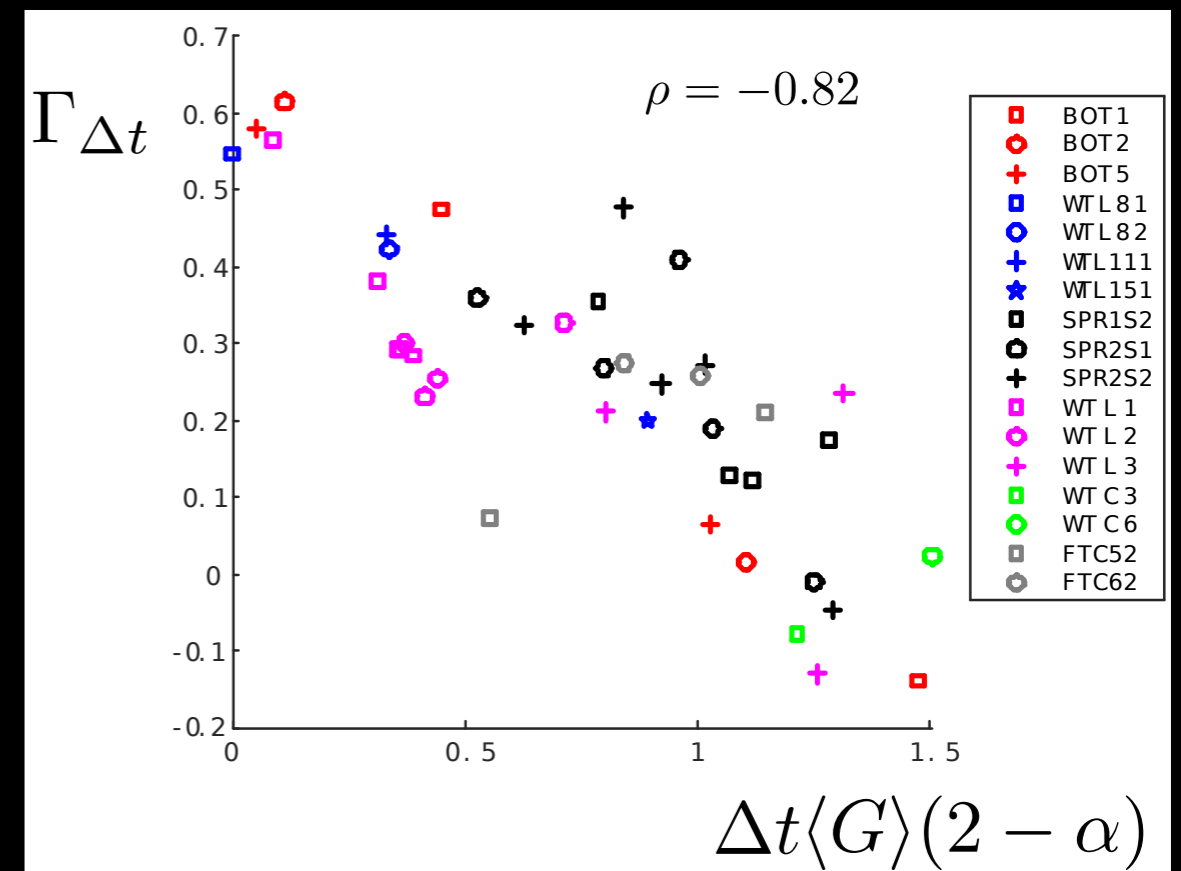


- Live imaging of growing flowers
- Quantification of rate of increase in area
- New spectral decomposition method

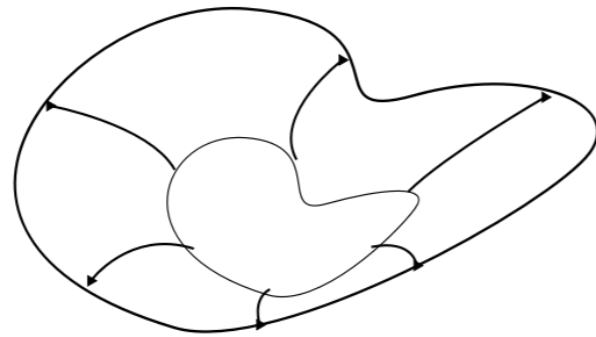




- Temporal correlations vs. time delay / theoretical correlation time



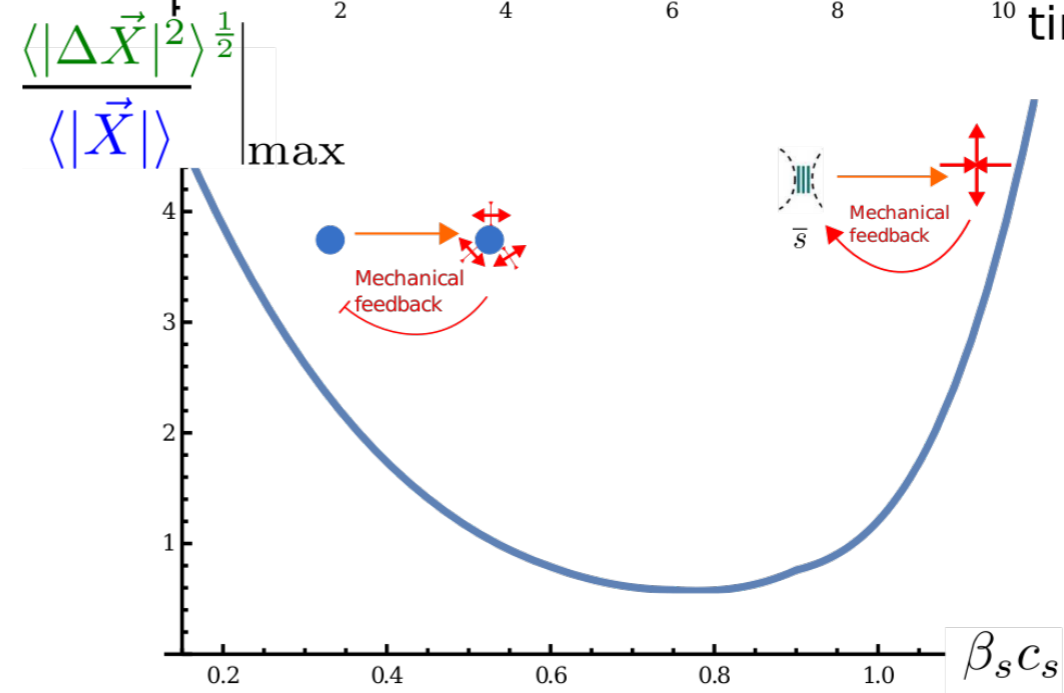
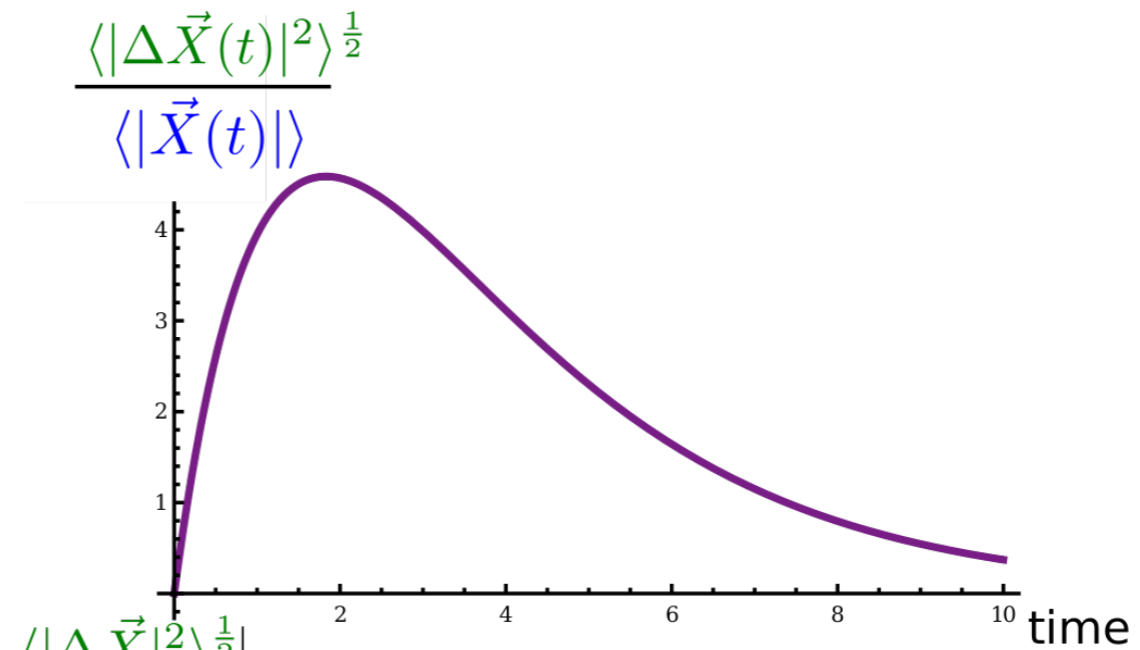
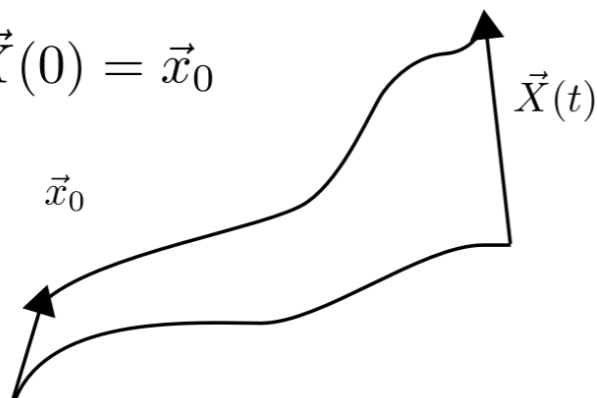
# Fluctuations of organ contours



advected contours

$$\frac{d\vec{X}}{dt}(t) = \vec{v}(\vec{X}(t), t)$$

$$\vec{X}(0) = \vec{x}_0$$



→ Optimal mechanical feedback

To be tested in experiments

# Conclusions

- ▶ Long-range correlations in growing tissue
- ▶ Range modulated by mechanical feedback
- ▶ An optimum of response to mechanical stress may ensure a minimal variability in shape

Fruleux & Boudaoud, PNAS (2019);  
Phys Rev Res (2021);  
in progress (2021).

# Open questions

- ▶ Is mechanical feedback always ‘stabilising’?
- ▶ How does this framework extend to 3D tissues?
- ▶ Do fluctuations enable living systems to sense their state?
- ▶ Are fluctuations important for size determination?