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On the mechanics of kirigami structures

MEchanics and **G**eometry of **A**dvanced **S**tructures **Lab**oratory
(MEGA SLab)

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with

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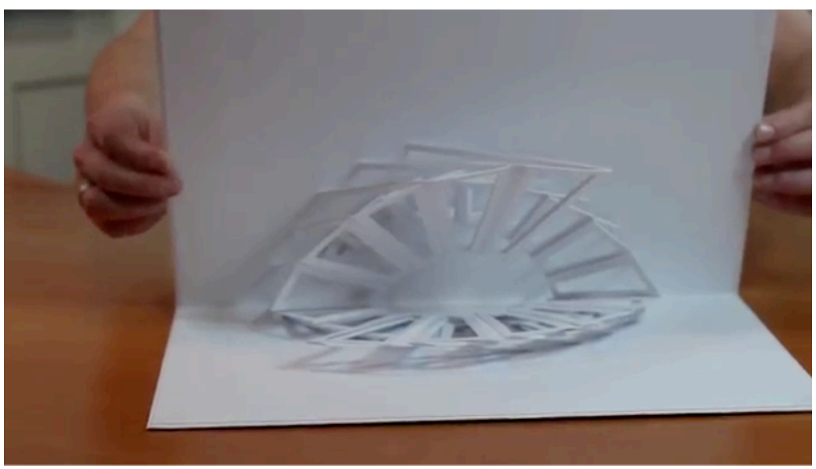
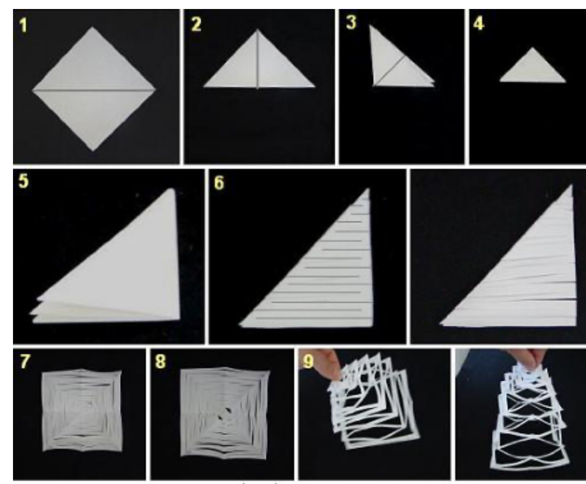
切り紙 (*Kirigami*)
= *Kiru (to cut)* + *Kami (Paper)*

İlhan Koman; Source: sevilokay.files.wordpress.com





www.origami-resource-center.com

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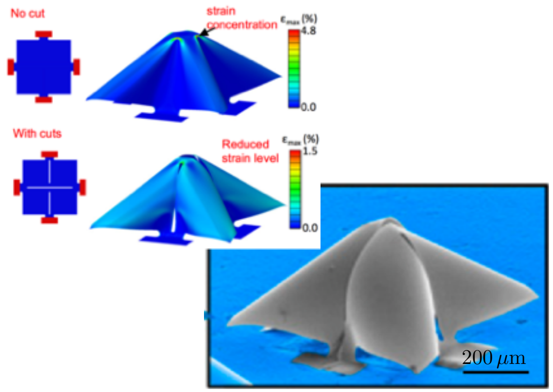
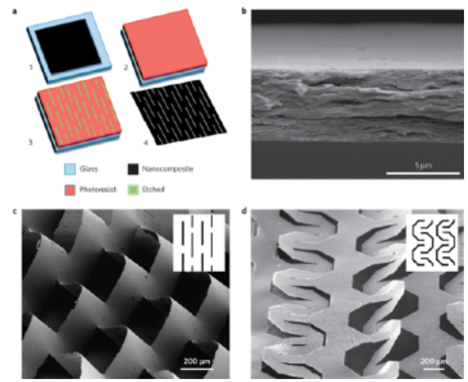
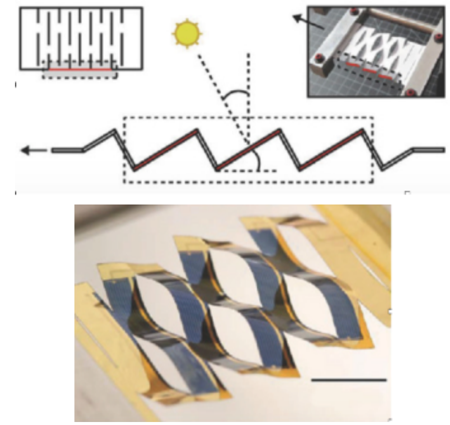
Kirigami in multiple scale



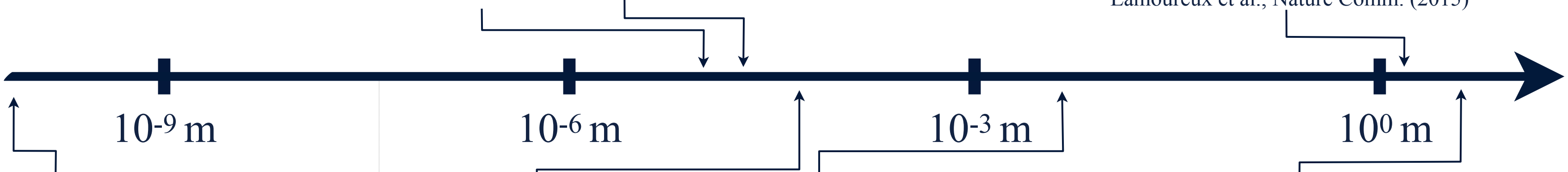
Zhang et al., PNAS (2015)

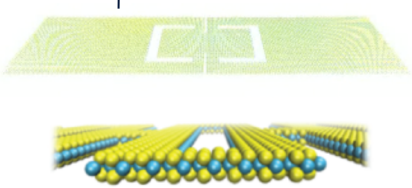


Shyu et al., Nature Materials (2015)

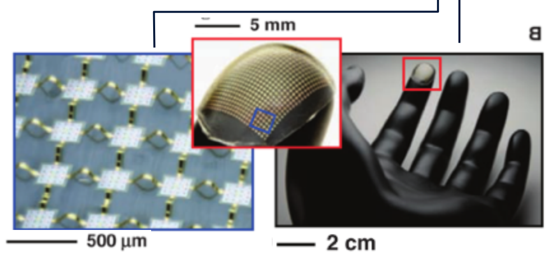


Lamoureux et al., Nature Comm. (2015)





Dias, *et al.*, Soft Matter (2017)



Rogers et al., Science (2010)

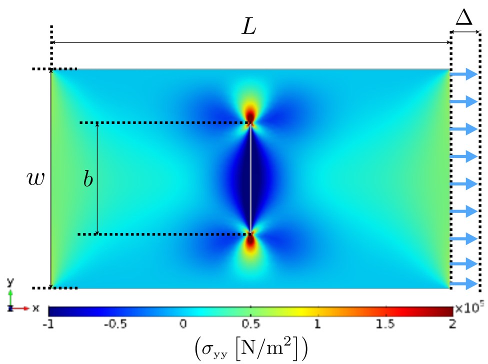
W. Wang, et al., Advanced Functional Materials (2017)

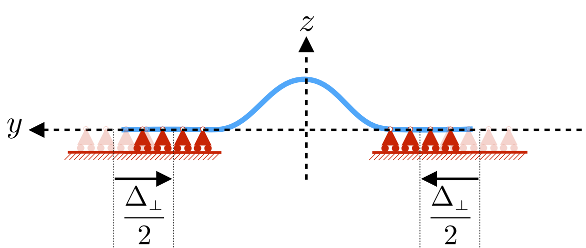
A few challenges we have attempted to address

Programming shapes, motion, and mechanics

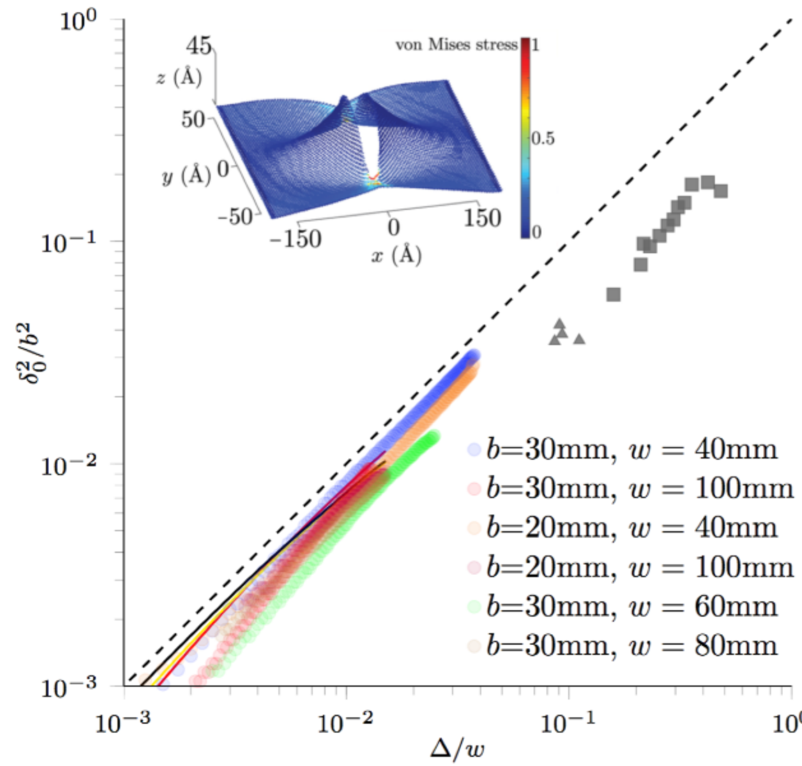
Mechanics of a single cut

Dias, et al., Soft Matter (2017)





$$\left(\frac{\delta_{\max}}{b}\right)^2 \sim \frac{\Delta}{w} + \mathcal{O}\left(\frac{t}{b}\right)^2$$



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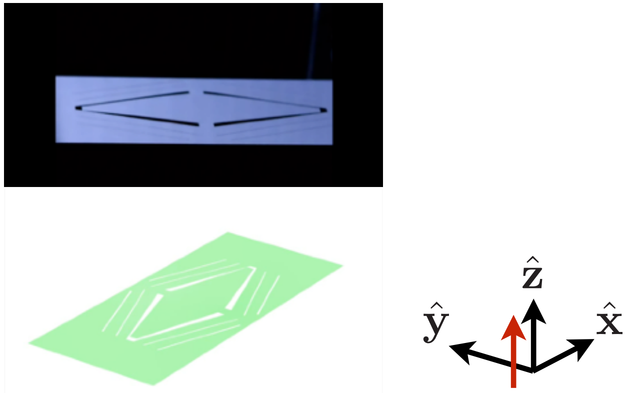
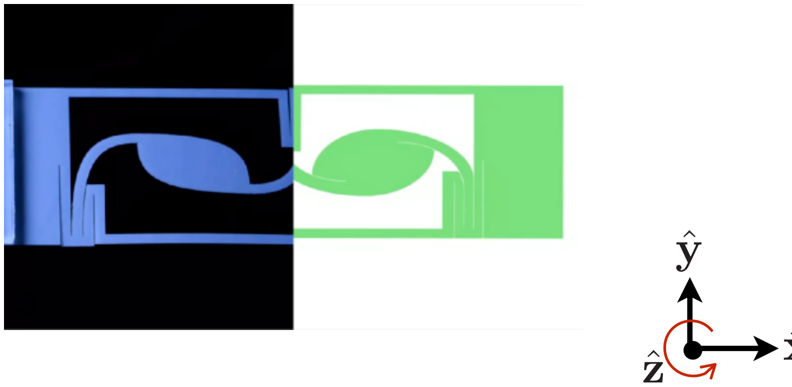
Programming shapes & motion
Kirigami actuators

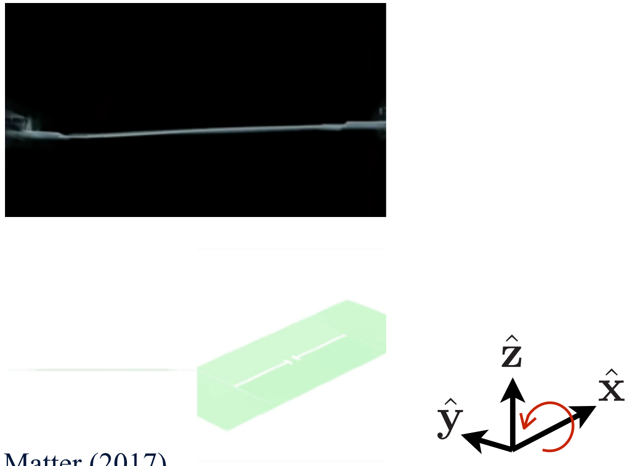
Lift—Displacement along *z*-axis



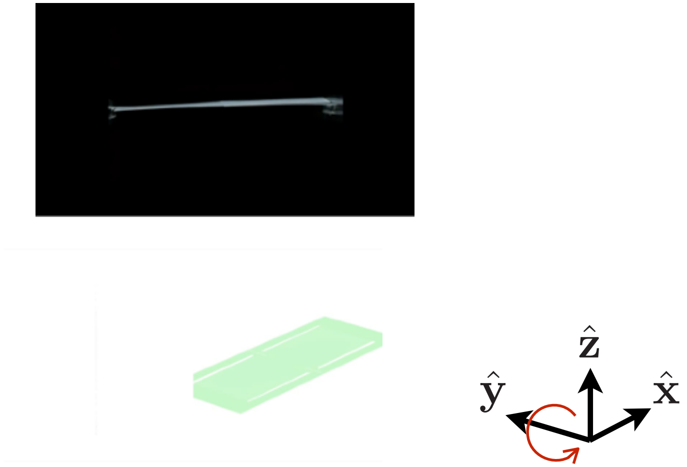
Yaw—Rotation about *z*-axis



Roll—Rotation about *x*-axis



Pitch—Rotation about *y*-axis



Dias, et al., Soft Matter (2017)

Programming mechanics

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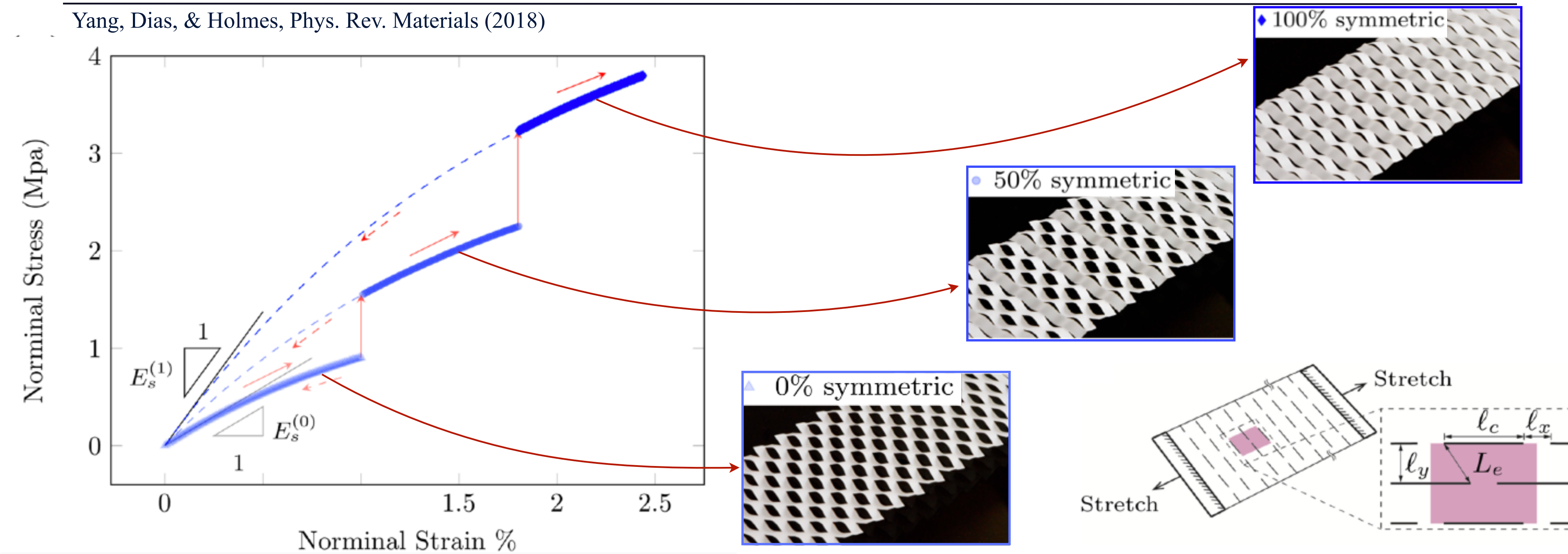
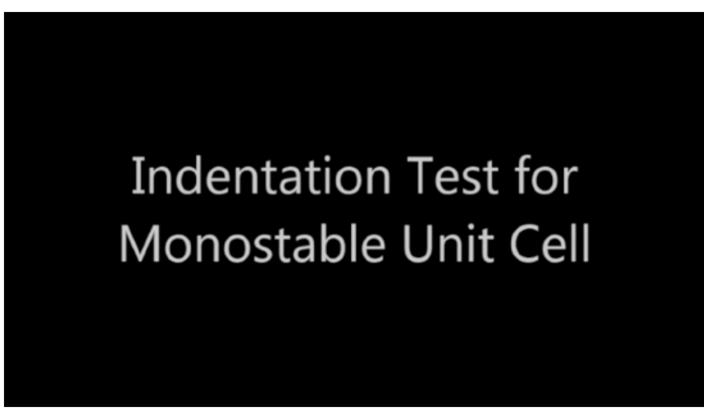
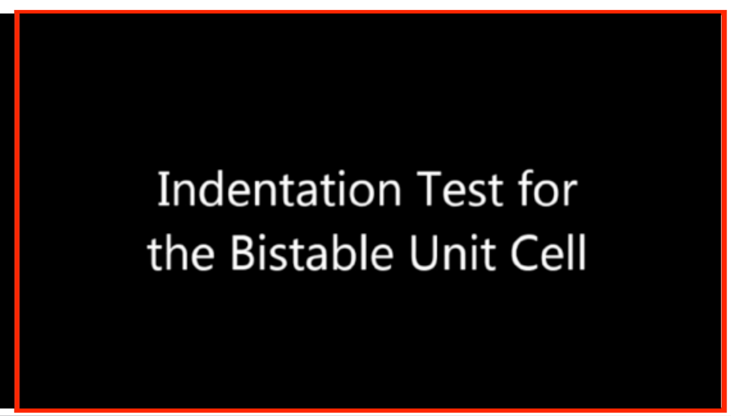
Yang, Dias, & Holmes, Phys. Rev. Materials (2018)







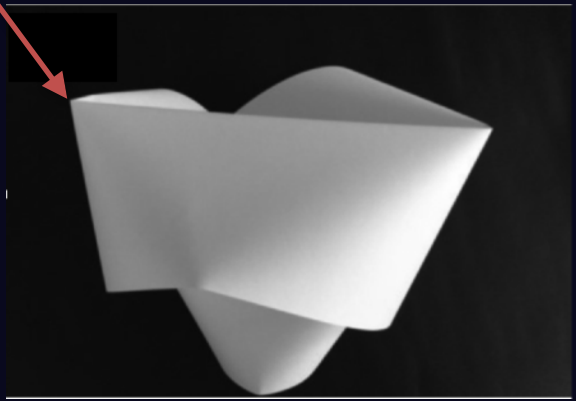
Taking a step back

Building blocks

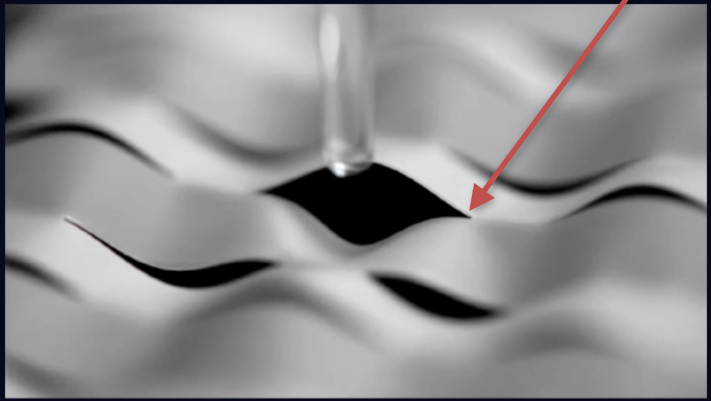
(all about e-cones)

!

?



Fuentealba *et al.*, PRE **91**, 032407 (2015)



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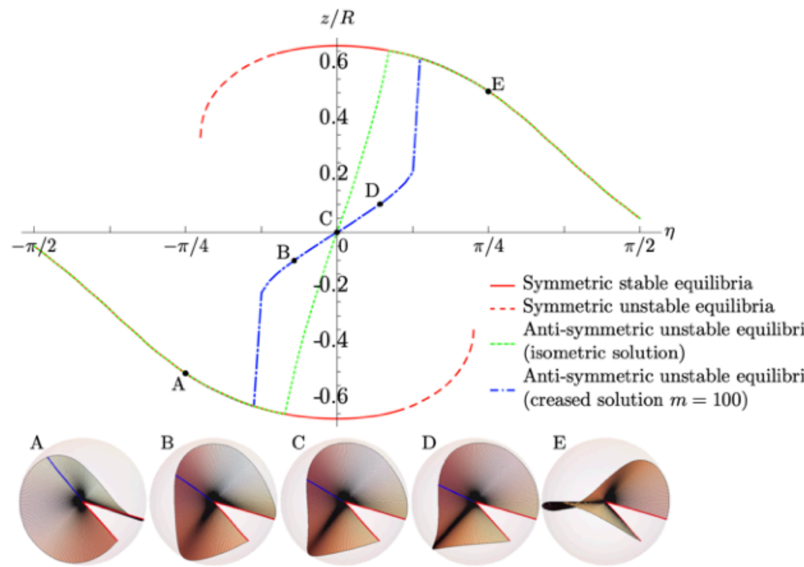
On kirigami mechanics

Souhayl Sadik
Postdoc
Aarhus University

Martin Walker
Collaborator
University of Surrey



Sadik & Dias, JMPS (2021)
Sadik, Walker, & Dias, in preparation (2021)
Sadik & Dias, in preparation (2021)

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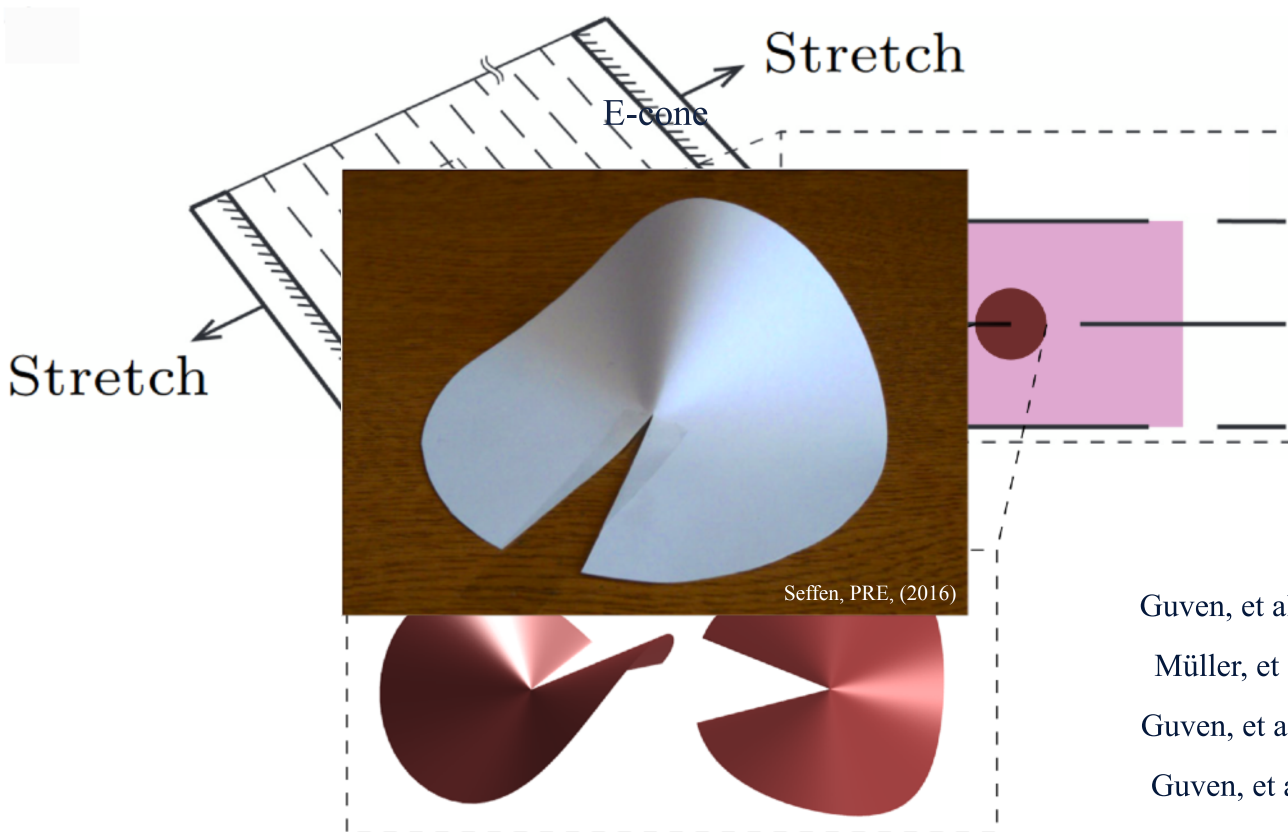
Back to fundamentals: building blocks

Sadik & Dias, JMPS (2021)



Guven, et al., JoP A (2008)

Müller, et al., PRL (2008)

Guven, et al., JoP A (2011)

Guven, et al. EPJE (2013)

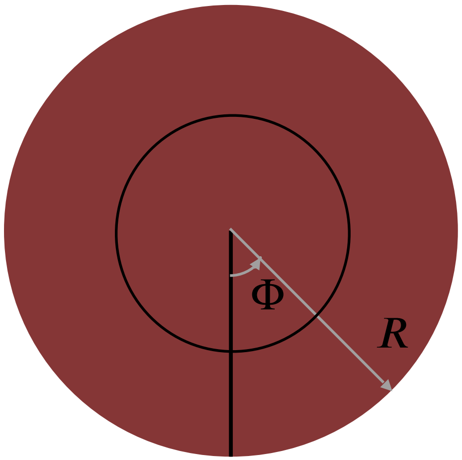
Efrati, et al. PRE (2015)

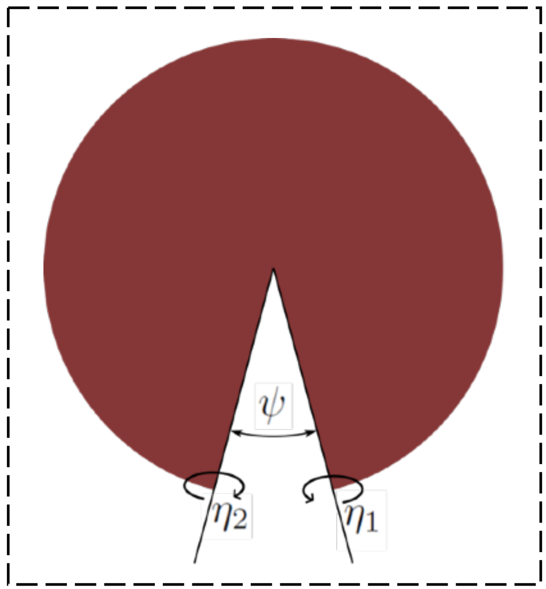
Seffen, PRE, (2016)

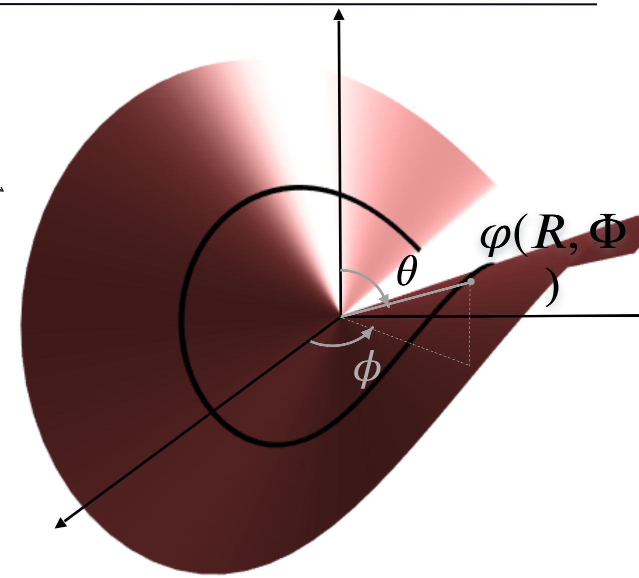
e-cones – Kinematics

Sadik & Dias, JMPS (2021)

$$\varphi(R,\Phi) = (R, \theta(\Phi), \phi(\Phi))$$







$$\phi(0) = \frac{\psi}{2}\,, \quad \phi(2\pi) = 2\pi - \frac{\psi}{2}\,, \quad \theta(0) = \theta(2\pi) = \frac{\pi}{2}\,,$$

$$\sin\left[\theta(0)\right]\theta'(0) = \sin(\eta_1)\,, \quad \sin\left[\theta(2\pi)\right]\theta'(2\pi) = \sin(\eta_2)$$

e-cones – Mechanics
Isometric solutions

Sadik & Dias, JMPS (2021)

$$\mathcal{W} = \frac{h}{4}\Big\{\underbrace{\mu \mathrm{tr}\left[(\boldsymbol{C}-\boldsymbol{G})^2\right] + \frac{\mu\lambda}{2\mu+\lambda}\left[\mathrm{tr}\,(\boldsymbol{C}-\boldsymbol{G})\right]^2}_{\text{Stretching } = 0}\Big\} \underbrace{-\boldsymbol{p}:(\boldsymbol{C}-\boldsymbol{G})}_{\text{Inextensibility Condition}} + \frac{h^3}{12}\Big\{\underbrace{\mu \mathrm{tr}\left[(\boldsymbol{\Theta}-\boldsymbol{B})^2\right] + \frac{\mu\lambda}{2\mu+\lambda}\left[\mathrm{tr}\,(\boldsymbol{\Theta}-\boldsymbol{B})\right]^2}_{\text{Bending}}\Big\}$$

$$\text{Inextensibility } \boldsymbol{C} = \boldsymbol{G} \implies \phi(\Phi) = \frac{\psi}{2} + \int_0^{\Phi} \frac{\sqrt{1-\theta'^2(\eta)}}{\sin[\theta(\eta)]}\, d\eta\,,$$

$$\kappa(\Phi) := \frac{\cot[\theta(\Phi)]\left(1-\theta'^2(\Phi)\right) - \theta''(\Phi)}{\sqrt{1-\theta'^2(\Phi)}}$$

$$\kappa''(\Phi) + \frac{\kappa^3(\Phi)}{2} + (1+\alpha)\,\kappa(\Phi) = 0$$

$$\kappa(\Phi) = \kappa_o\, \mathrm{cn}\left(\frac{\kappa_o}{2k}(\Phi-\Phi_o)\,|k^2\right)\,, \quad k^2 = \frac{\kappa_o^2}{2\kappa_o^2 + 4(1+\alpha)}$$

e-cones – Stress fields

Sadik & Dias, JMPS (2021)

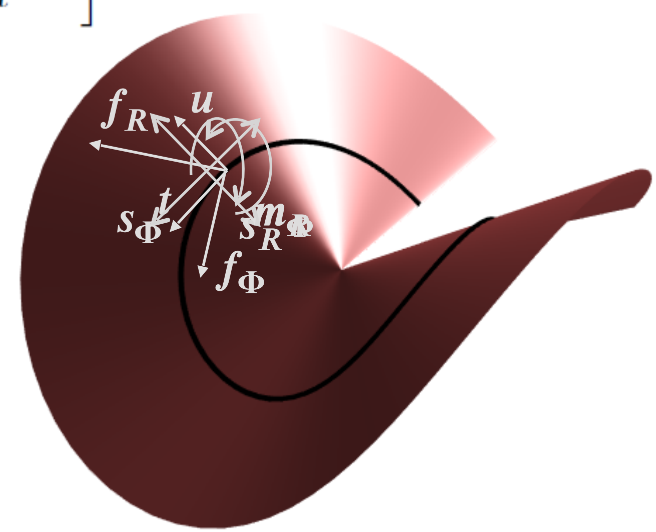
The surface traction, shear, and moment fields on the surface

$$\bar{\boldsymbol{f}}_R = \left[\frac{h^3\mu(\lambda+\mu)}{3(\lambda+2\mu)}\frac{\frac{1}{2}\kappa^2(\Phi)+\alpha}{R^2} - \frac{a'(\Phi)+(\ln(R)+1)b'(\Phi)+\int_0^\Phi b(\eta)d\eta}{R^2} - \frac{c(\Phi)}{R}\right]\boldsymbol{u} - \left[\frac{a(\Phi)+\ln(R)b(\Phi)}{R^2}\right]\boldsymbol{t}$$

$$\bar{\boldsymbol{f}}_\Phi = -\left[\frac{a(\Phi)+\ln(R)b(\Phi)}{R^2}\right]\boldsymbol{u} + \left[-\frac{h^3\mu(\lambda+\mu)}{3(\lambda+2\mu)}\frac{\frac{1}{2}\kappa^2(\Phi)+\alpha}{R^2} + \frac{\int_0^\Phi b(\eta)d\eta}{R^2}\right]\boldsymbol{t}$$

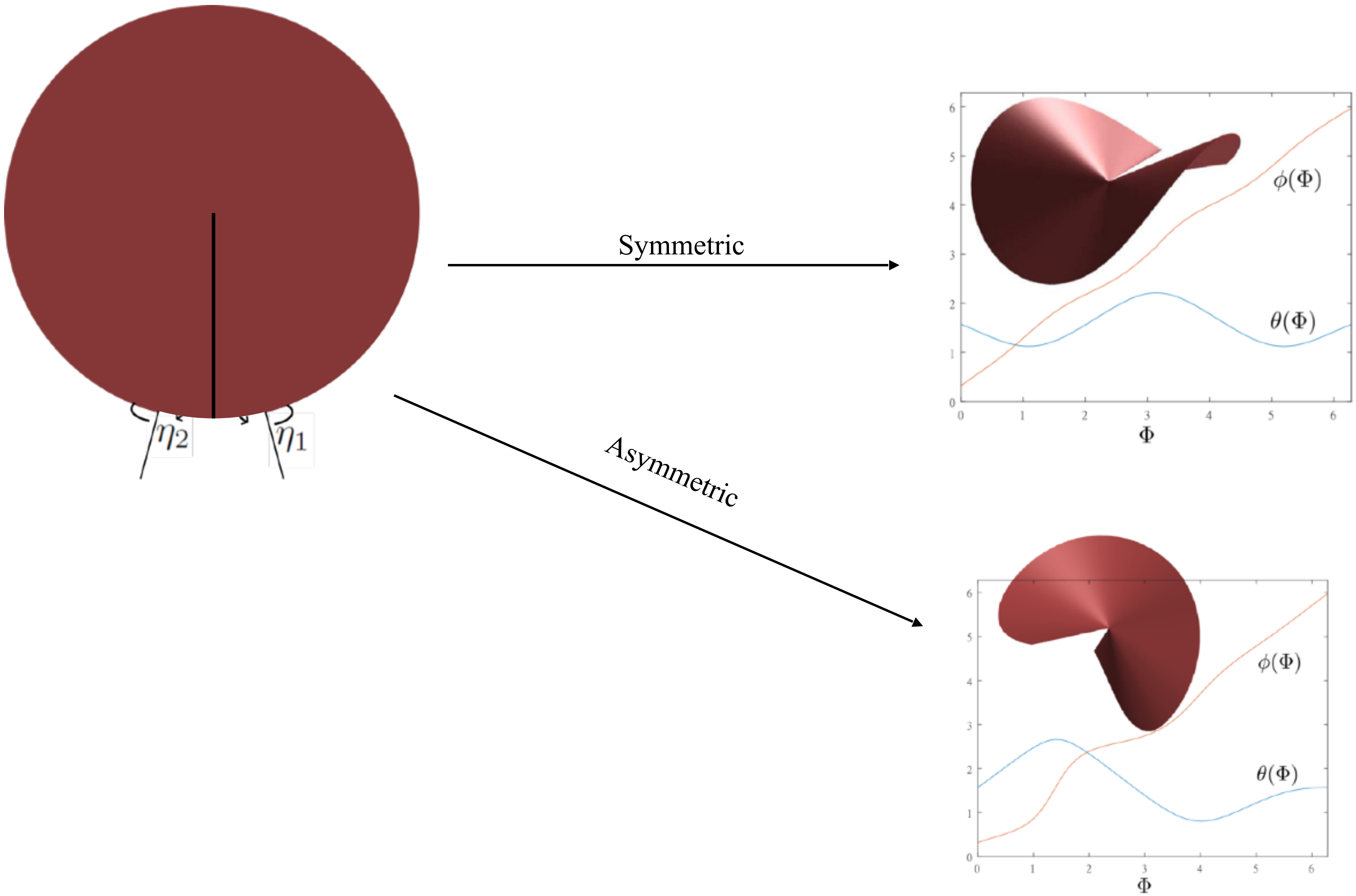
$$\bar{s}_R = \frac{h^3\mu(\lambda+\mu)}{3(\lambda+2\mu)}\frac{\kappa(\Phi)}{R^2} \qquad \bar{s}_\Phi = \frac{h^3\mu(\lambda+\mu)}{3(\lambda+2\mu)}\frac{\kappa'(\Phi)}{R^2}$$

$$\bar{\boldsymbol{m}}_R = \frac{h^3\mu\lambda}{6(\lambda+2\mu)}\frac{\kappa(\Phi)}{R}\boldsymbol{u} \qquad \bar{\boldsymbol{m}}_\Phi = \frac{h^3\mu(\lambda+\mu)}{3(\lambda+2\mu)}\frac{\kappa(\Phi)}{R}\boldsymbol{t}$$



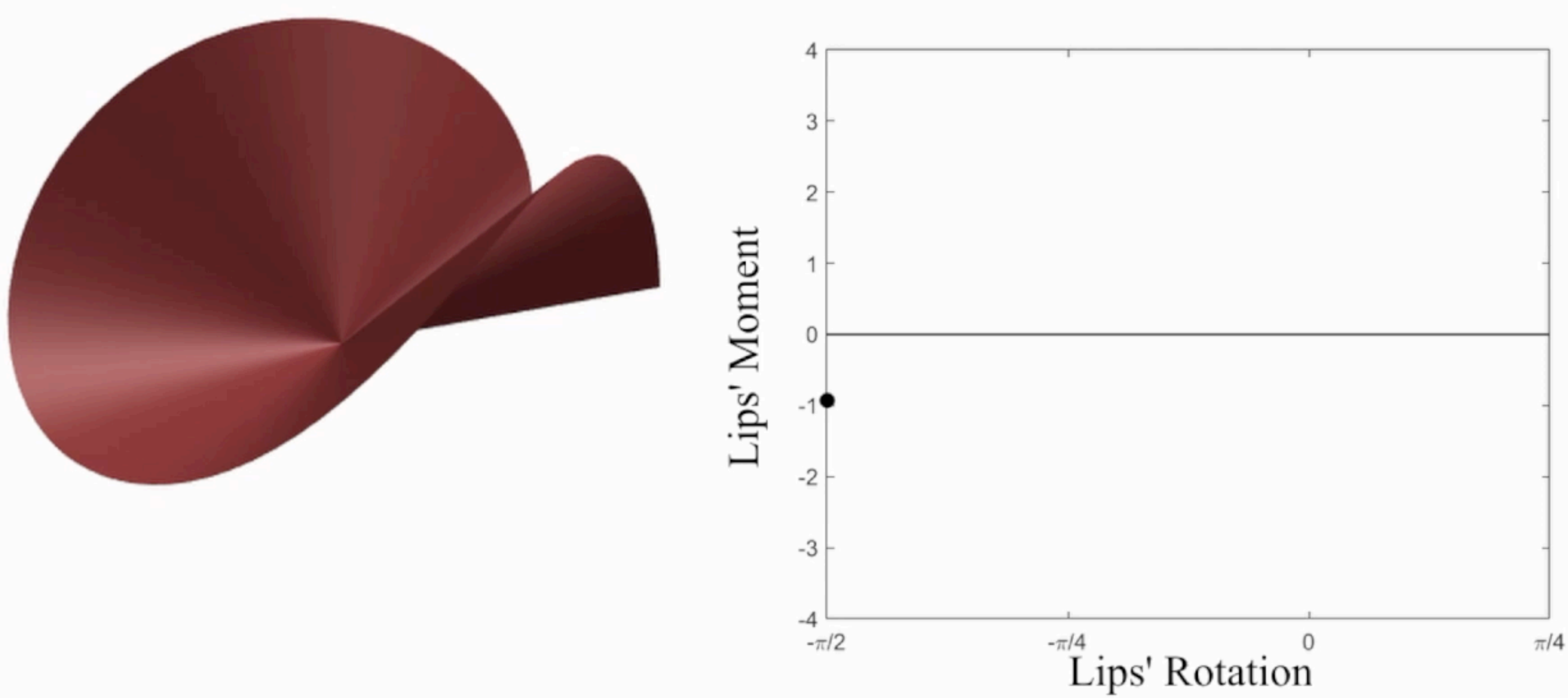
e-cones – Shape Reconstruction

Sadik & Dias, JMPS (2021)



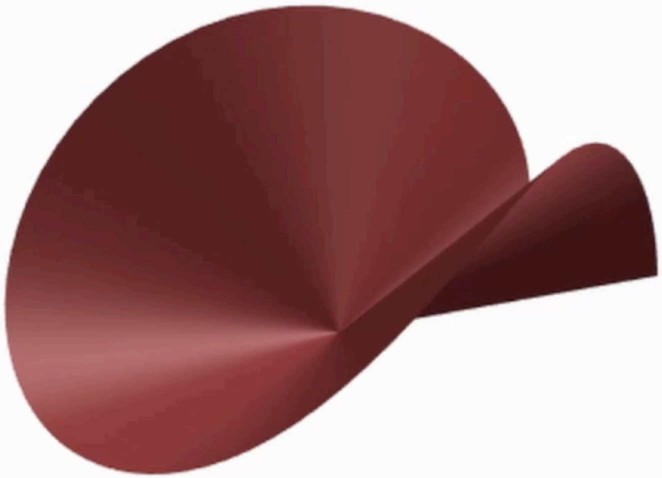
Symmetric solutions

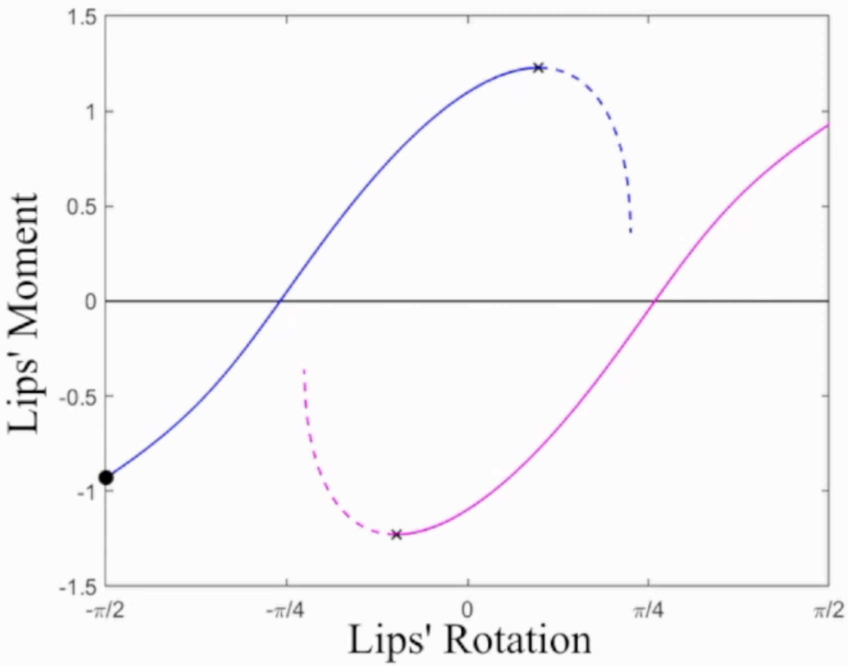
Sadik & Dias, JMPS (2021)



Stability of e-cones

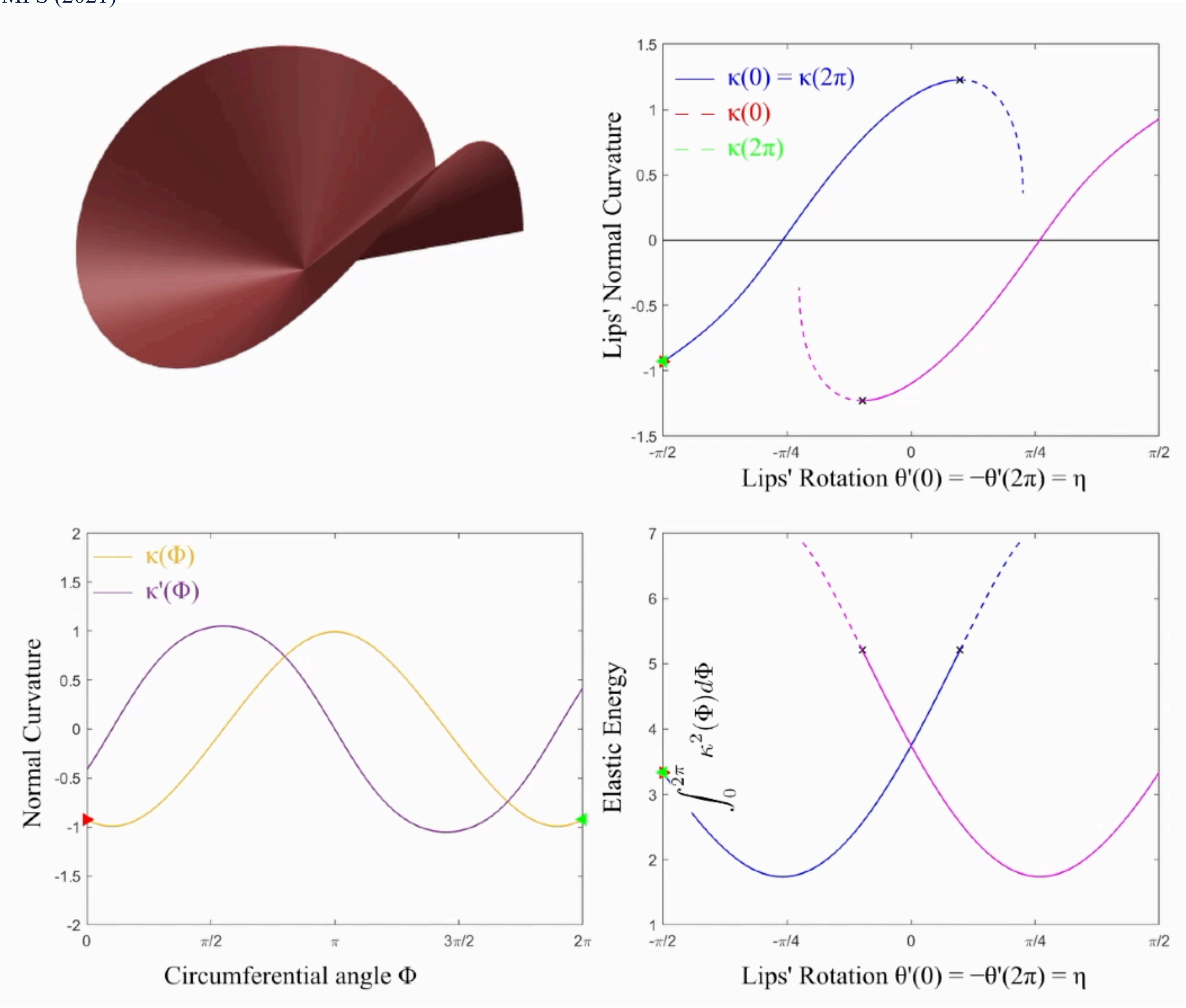
Sadik & Dias, JMPS (2021)





Stability of e-cones

Sadik & Dias, JMPS (2021)



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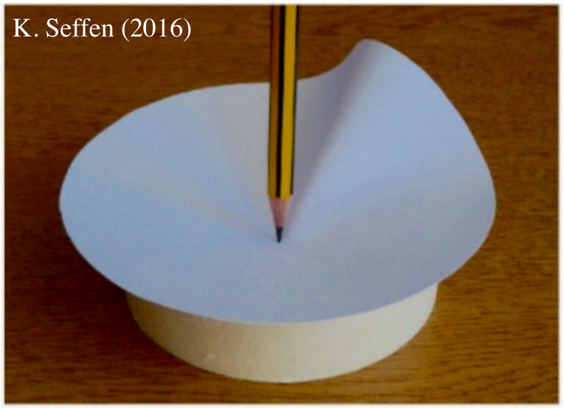
More on the deformation motifs

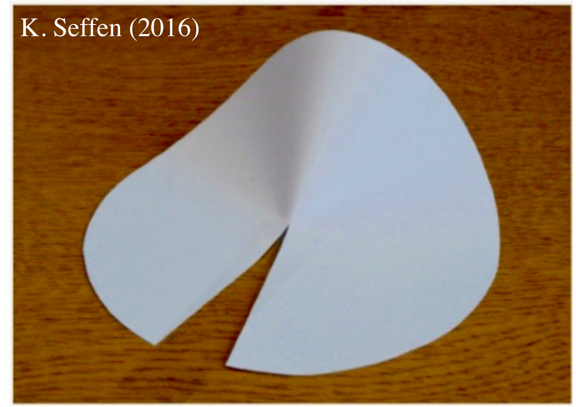
Chaieb, *et al*., PRL (1998)
Cerda & Mahadevan, PRL (1998)

E. Efrati, *et al*., PRE (2015)
Chopin & Kudrolli, Soft Matter (2016)

d-cones



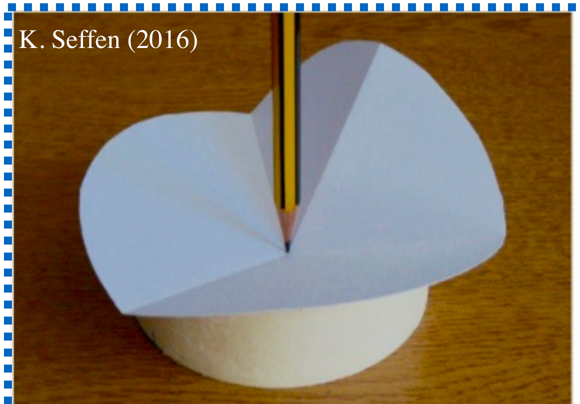
e-cones

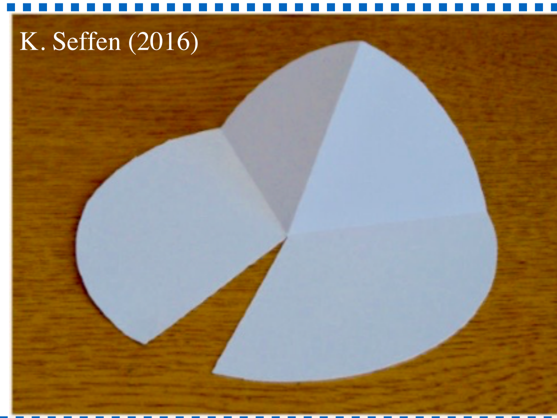


f-cones:
Farmer & Calladine, IJMS (2005)

Seffen, PRE (2016)

Andrade, Adda-Bedia, Dias. PRE (2019)





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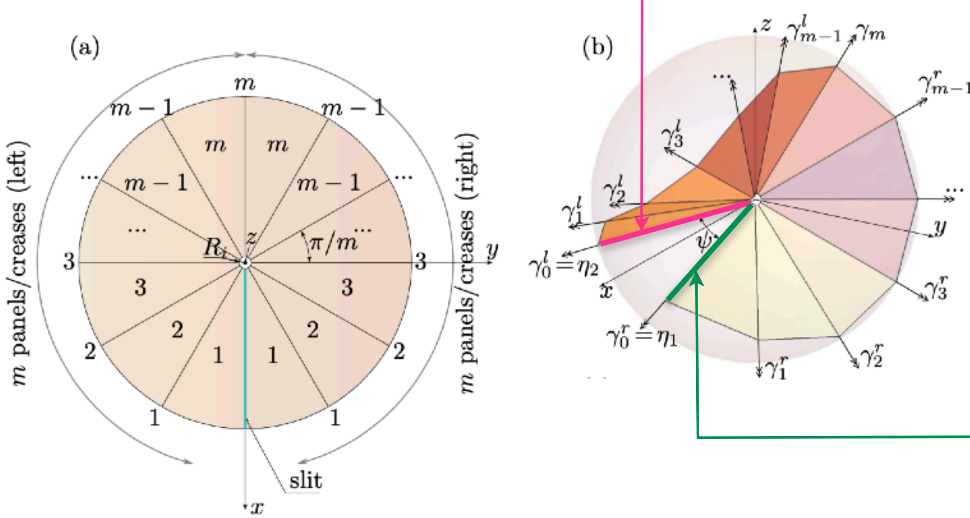
Origamising e-cones

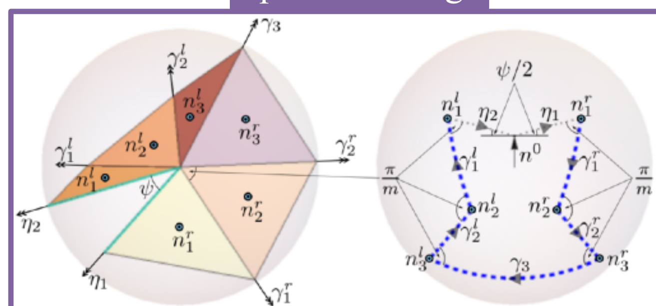
Sadik, Walker, & Dias, in preparation (2021)

$$\boldsymbol{u}^l_{1,0} = \boldsymbol{u}^l_{2,0} = (\cos(\psi/2) \ \ -\sin(\psi/2) \ \ 0)^\mathsf{T}$$



$$\boldsymbol{u}^r_{1,0} = \boldsymbol{u}^r_{2,0} = (\cos(\psi/2) \ \ \sin(\psi/2) \ \ 0)^\mathsf{T}$$

Spherical image

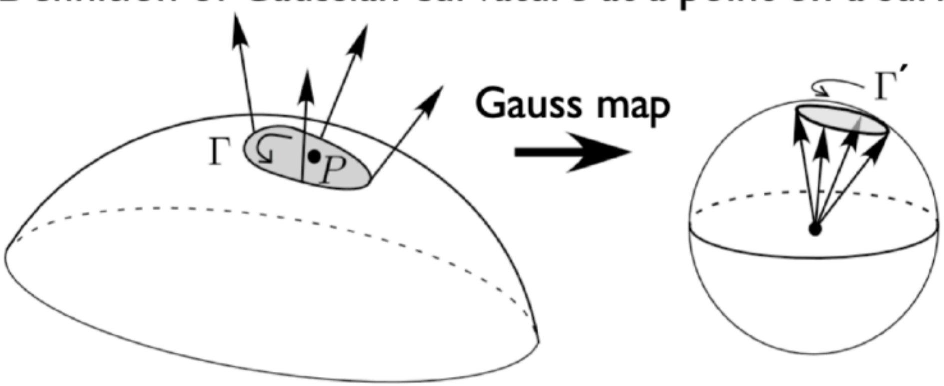


$$\boldsymbol{u}^{\{r,l\}}_{1,j} = \sin(\pi/m)\left[\sin(\gamma^{\{r,l\}}_{j-1})\boldsymbol{n}^{\{r,l\}}_{j-1} \pm \cos(\gamma^{\{r,l\}}_{j-1})\boldsymbol{n}^{\{r,l\}}_{j-1} \times \boldsymbol{u}^{\{r,l\}}_{2,j-1}\right] + \cos(\pi/m)\,\boldsymbol{u}^{\{r,l\}}_{2,j-1}$$

The Gauss map

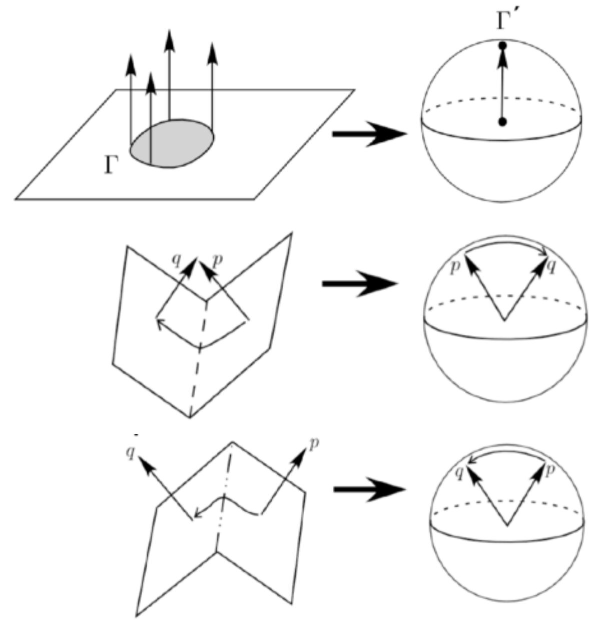
K. F. Gauss, Disquisitiones generales circa superficies curvas (1828)

Definition of Gaussian curvature at a point on a surface:



Then the curvature κ of the surface at P is

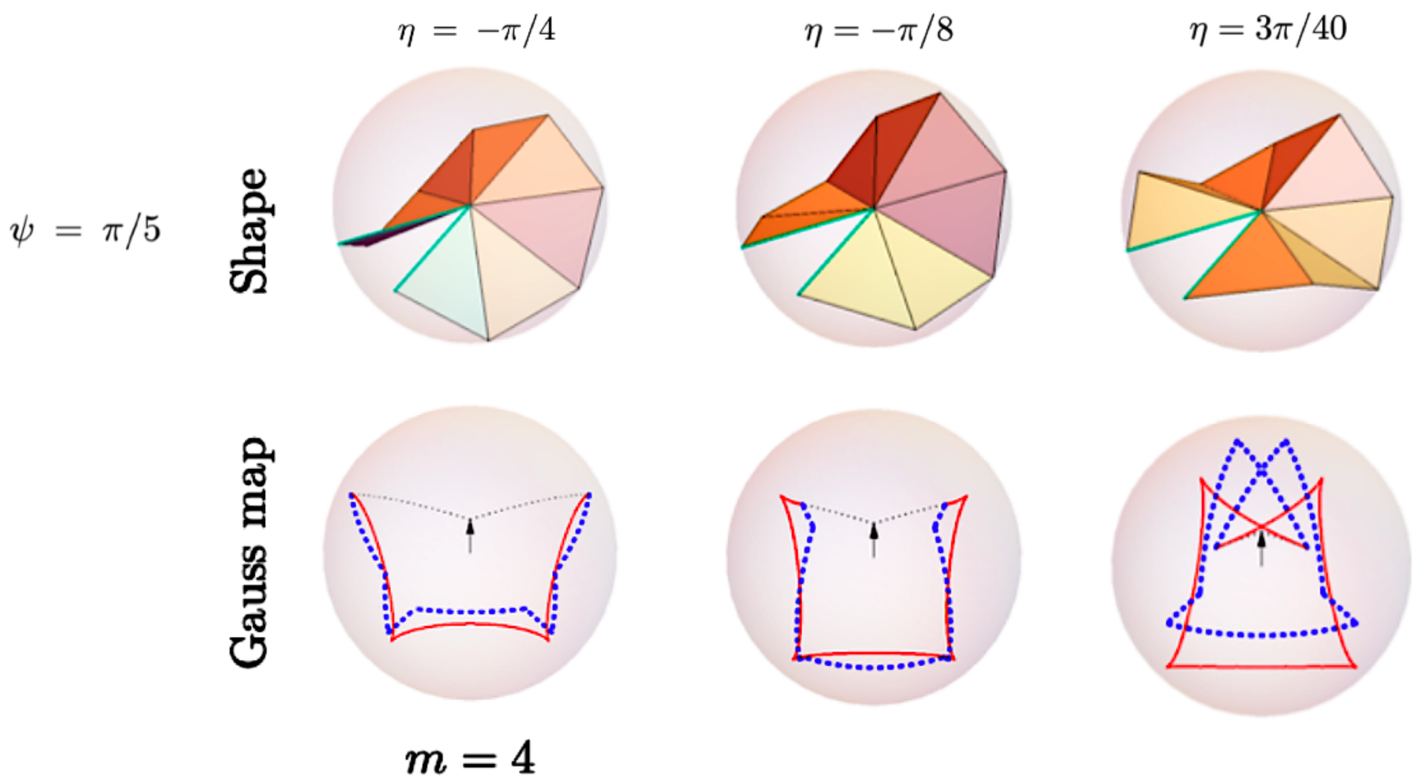
$$\kappa = \lim_{\Gamma \to P} \frac{\text{Area in } \Gamma'}{\text{Area in } \Gamma}$$



T. Hull (2016)

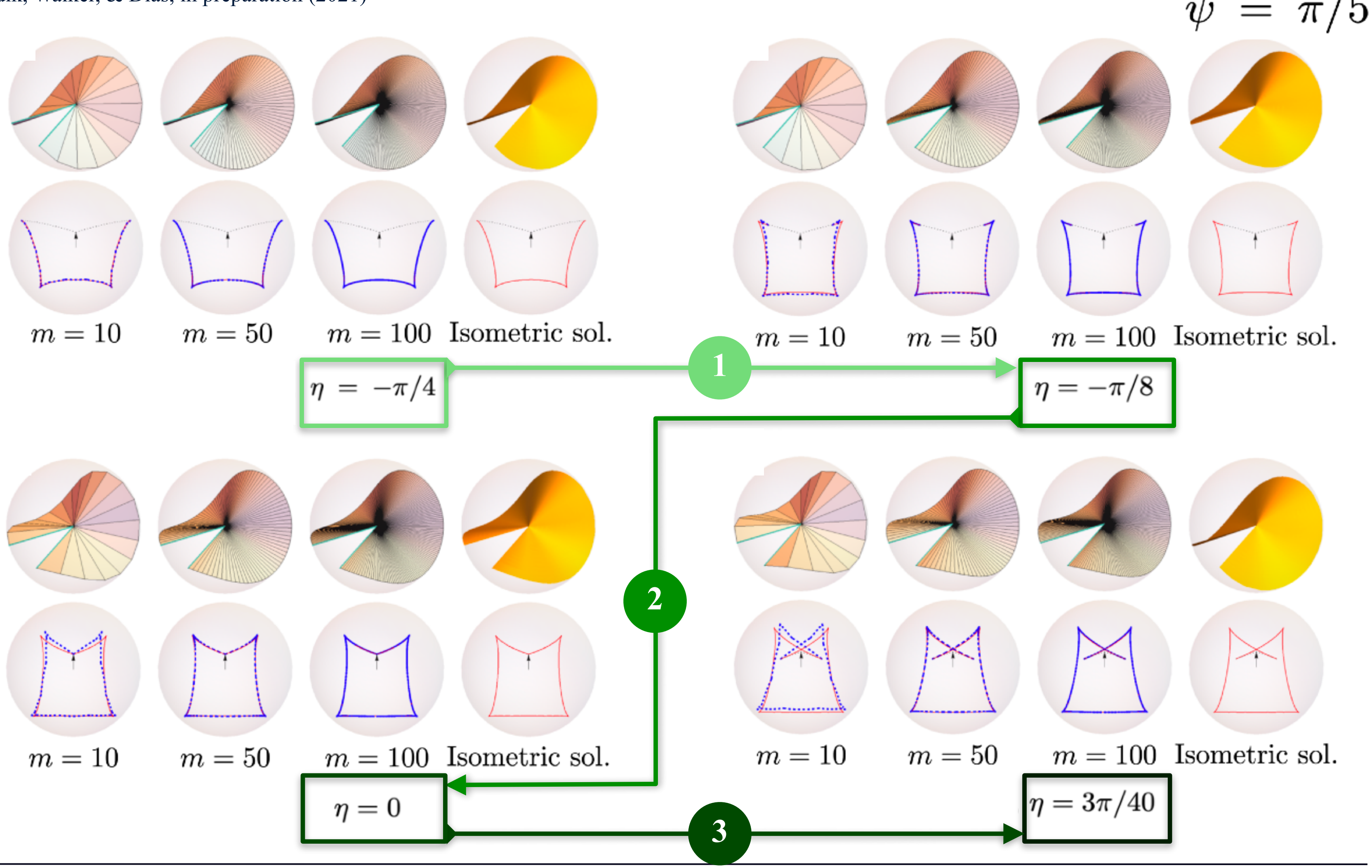
Origamising e-cones

Sadik, Walker, & Dias, in preparation (2021)



Origamising e-cones

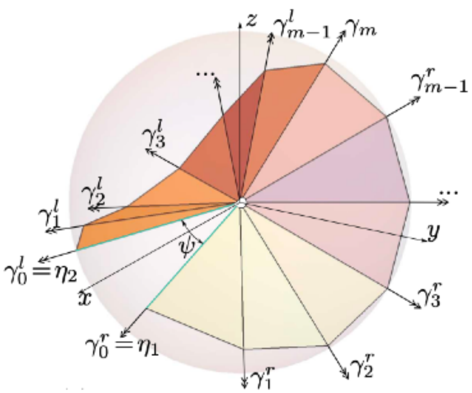
Sadik, Walker, & Dias, in preparation (2021)

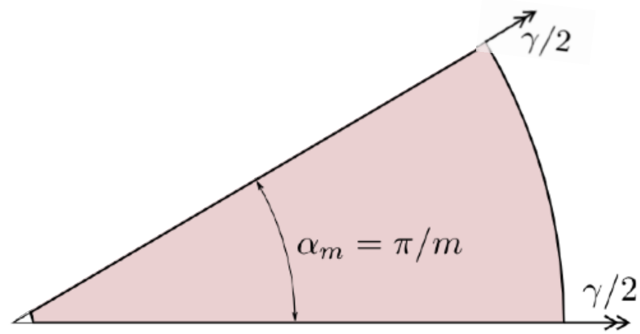


Origamising e-cones

Stretching is back

Sadik, Walker, & Dias, in preparation (2021)





$$\mathcal{W} = \frac{1}{2} k_b \left\{ \sum_{j=1}^{m-1} \left[\left(\gamma_j^r\right)^2 + \left(\gamma_j^l\right)^2 \right] + \left(\gamma_m\right)^2 \right\}$$

$$k_b = \frac{Eh^3}{12(1-\nu^2)} \frac{1 + \cos\left(\frac{\pi}{m}\right)}{\left(\frac{\pi}{m} + \sin\left(\frac{\pi}{m}\right)\right)} \ln\left(\frac{R_o}{R_i}\right)$$

$$k_s = \frac{Eh\, m^2 \left[\left(R_o^2 - R_i^2\right)^2 + 4R_i^2 R_o^2 \left[\log\left(\frac{R_o}{R_i}\right)\right]^2 \right]}{8\pi^2 (R_o^2 - R_i^2)}$$

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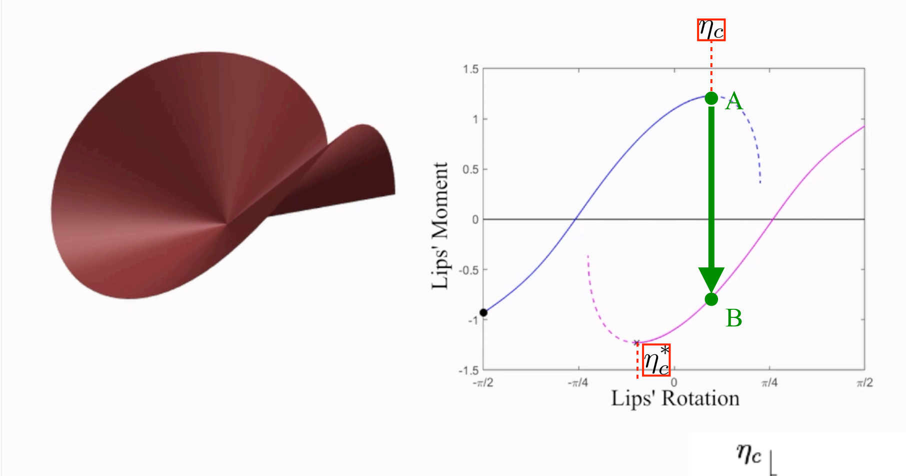
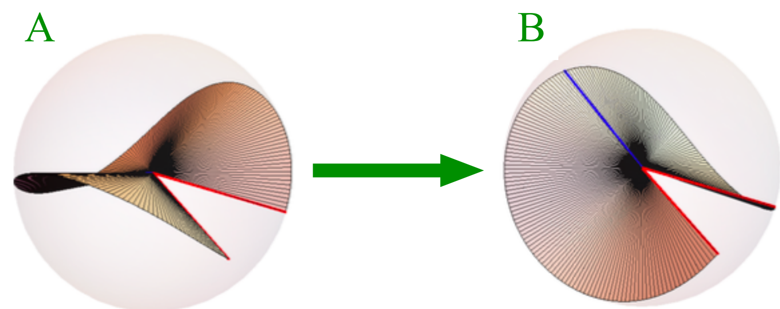
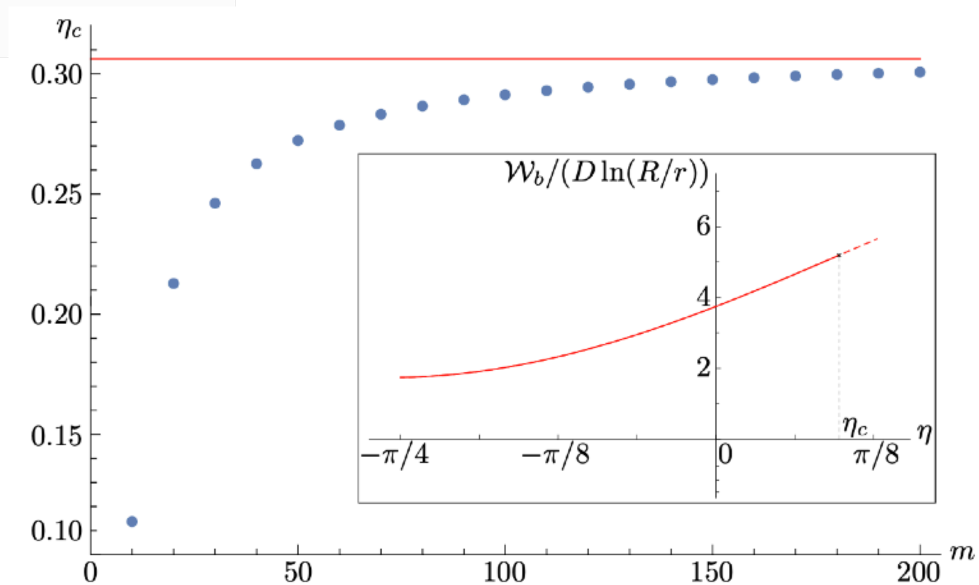
Snapping behaviour
Pure bending

Sadik, Walker, & Dias, in preparation (2021)

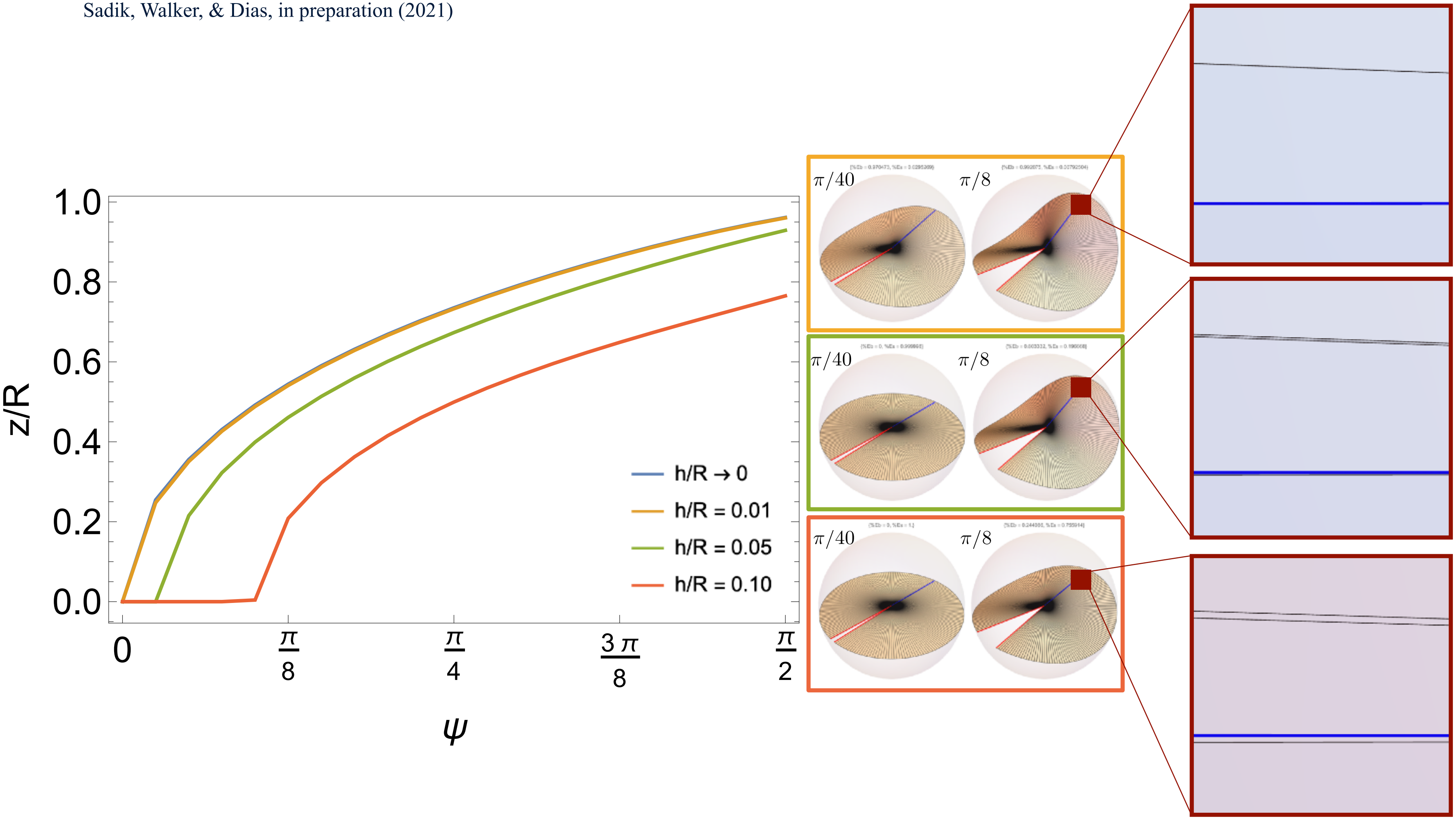






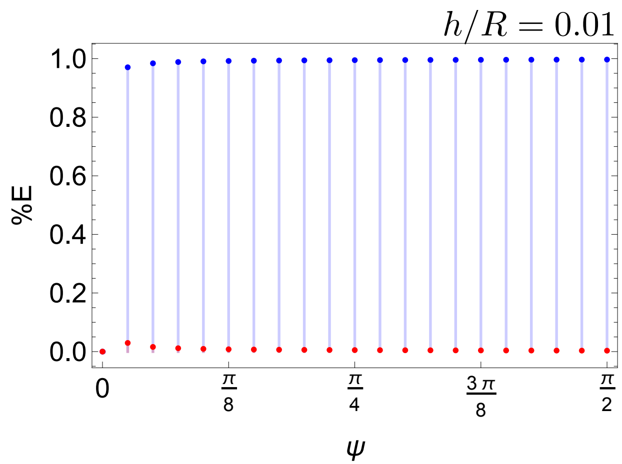
Behaviour close to the instability point

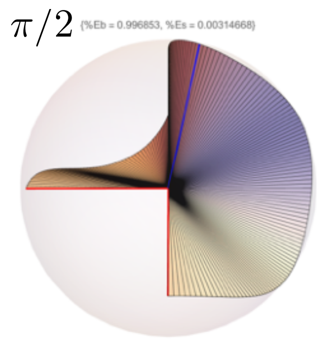
Sadik, Walker, & Dias, in preparation (2021)

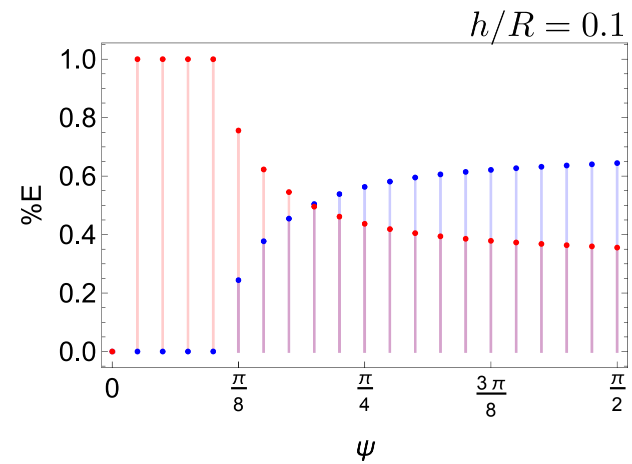


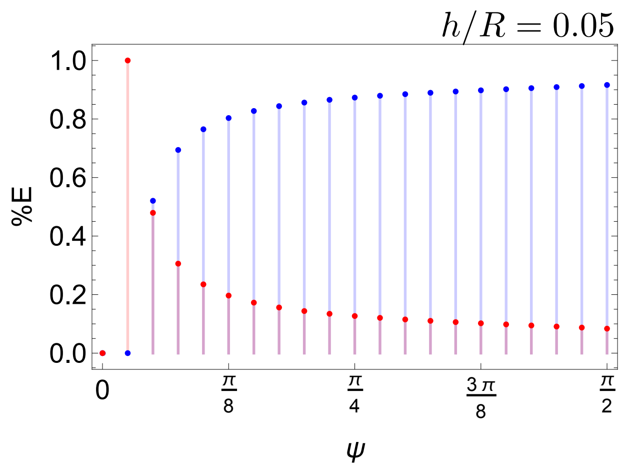
Energy landscape

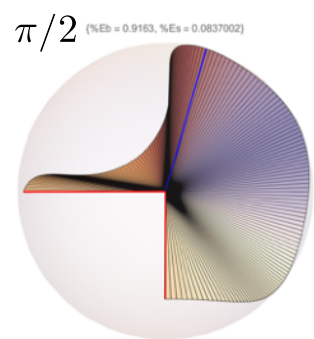
Sadik, Walker, & Dias, in preparation (2021)

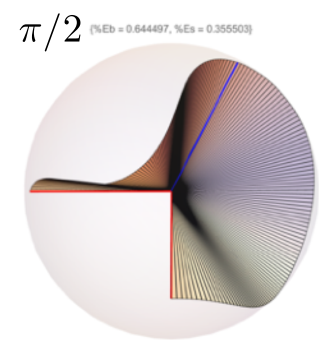












● E_b ● E_s

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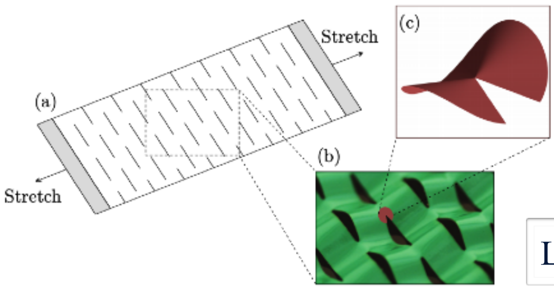
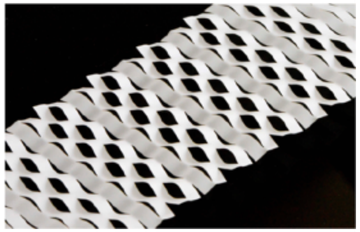
Final remarks on Kirigami



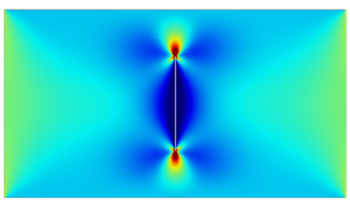
Kirigami

Local behaviour

Global behaviour

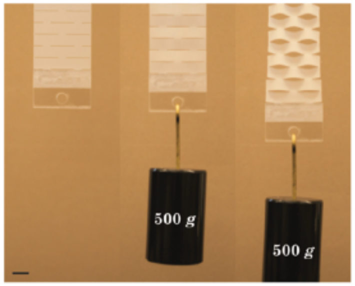


Homogenisation



Stress localisation
crack-crack interactions

Macroscopic response
Mechanics and form

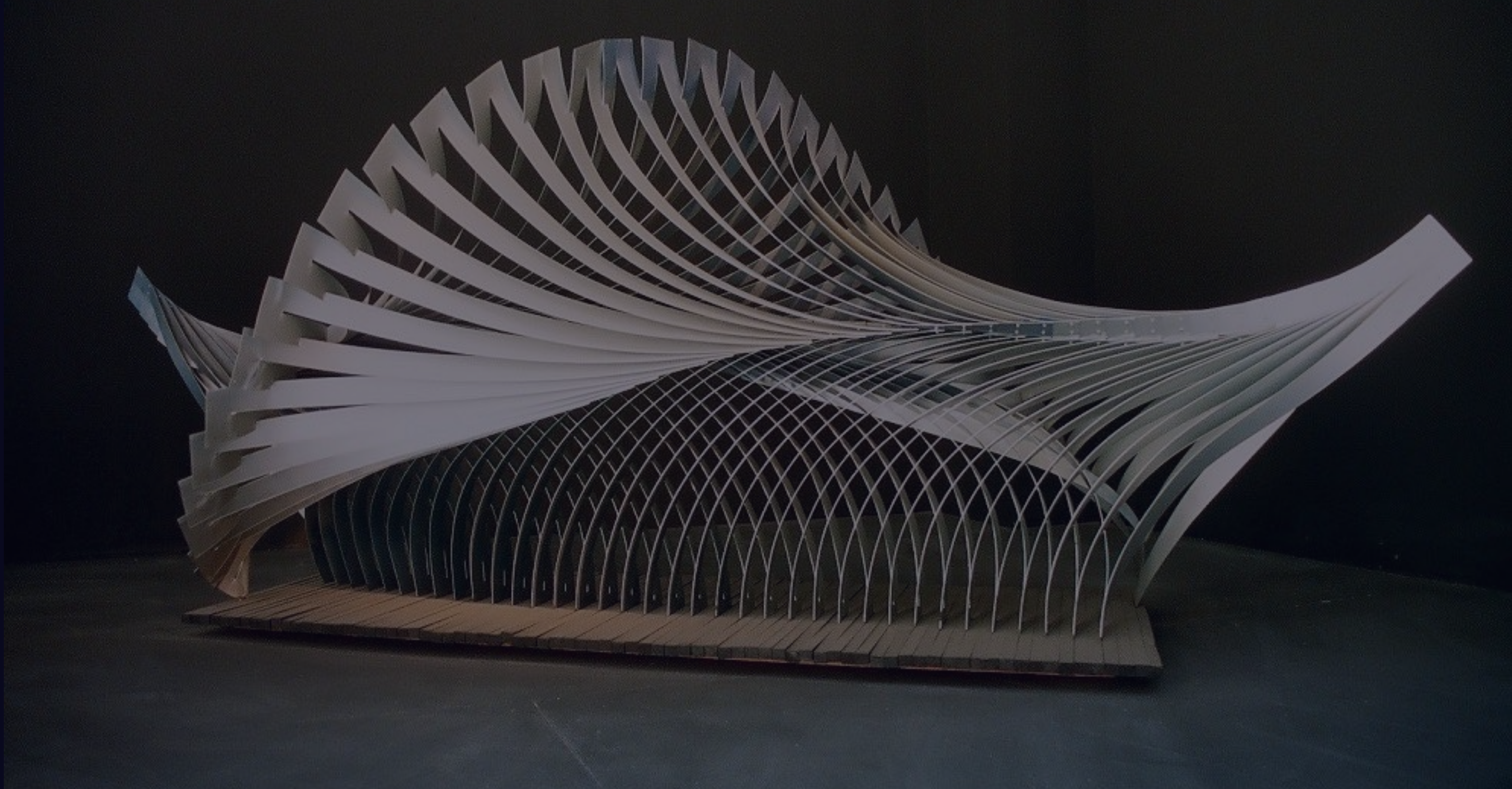


Fundamental contributions to new motifs in mechanics of thin sheets

Constitutive laws for non-simply connected plates and shells

Dias, et al., Soft Matter (2017)
Yang, Dias, & Holmes, Phys. Rev. Materials (2018)
Kaspersen, HInes, Moore, Rasmussen, & Dias, DIS '19 (2019)
Sadik & Dias, JMPS (2021)
Sadik, Walker, & Dias, in preparation (2021)
Sadik & Dias, in preparation (2021)

Thank you for your attention!



İlhan Koman; Source: sevilokay.files.wordpress.com