

Remodeling Shape and Topology of Fluid Membranes by Curvature and Tension

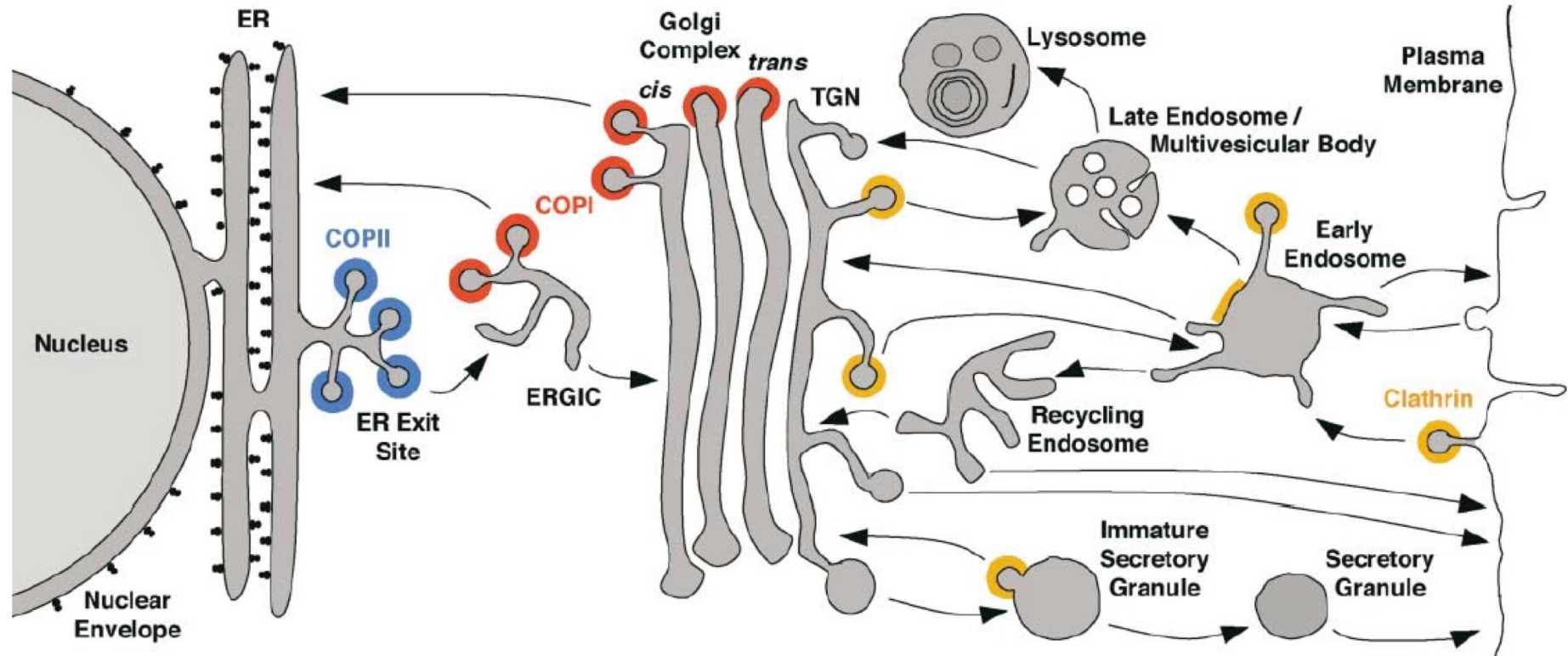
Reinhard Lipowsky

MPI of Colloids and Interfaces, Potsdam, Germany

- Membrane Compartments
- Remodeling of Membrane Shape
- Membrane Necks and Multispheres
- Remodeling of Membrane Topology
- Fission and fusion processes

Intracellular Vesicle Trafficking

Bonifacino, Glick, *Cell* (2004)

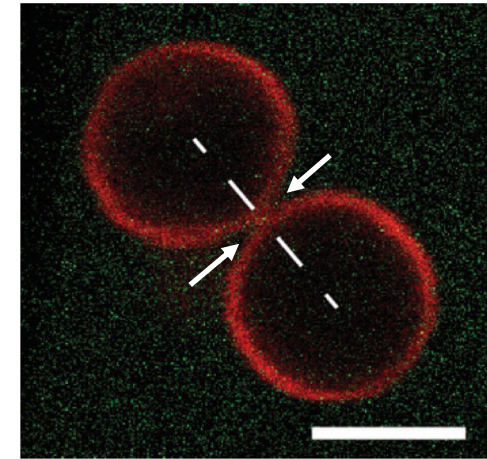


- Colored dots: vesicle formation by budding and fission
- Arrowheads: vesicle uptake by adhesion and fusion

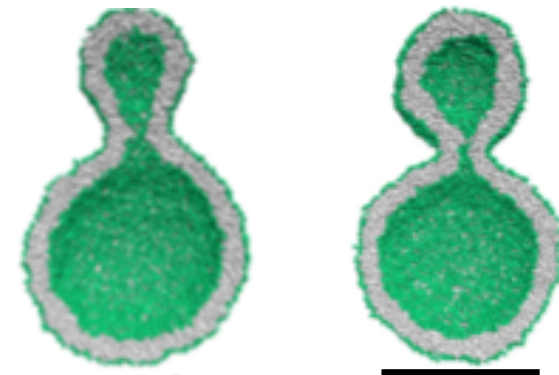
Synthetic Membrane Compartments

Steinkühler et al, *Nature Comm* (2020)

- Giant vesicles or GUVs
- Remodeling observed by optical microscopy
- Nanovesicles or SUVs
- Electron microscopy: limited to a single snapshot for each individual nanovesicle
- Remodeling studied via Molecular Dynamics simulations



5 μm



Ghosh, Satarifard et al,
Nano Letters (2019)

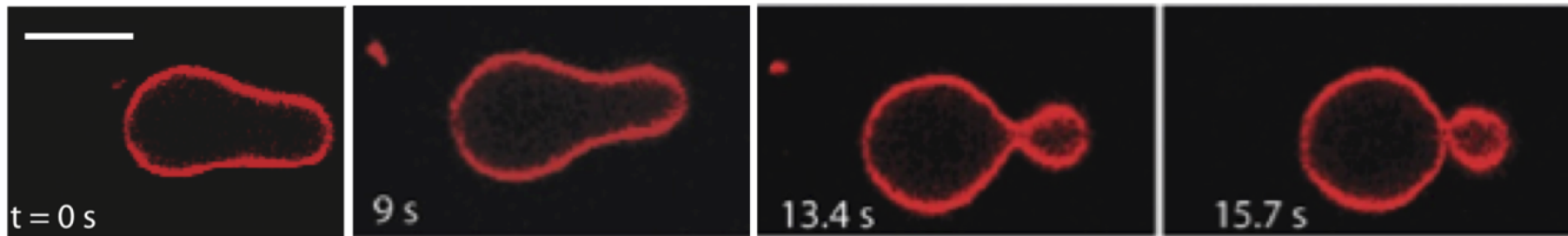
20 nm

- Remodeling of membrane shape
 - Examples for GUVs and nanovesicles
 - Membrane elasticity
 - Membrane necks and multispheres
- Remodeling of membrane topology
 - Relation to fission and fusion processes
 - Fission of membrane necks

Budding of Giant Vesicles

- Pear-like vesicle transformed into two-sphere vesicle
- Snapshots from time lapse over 16 s:

Bhatia et al, *Soft Matter* (2020)



Scale bar: 5 μm

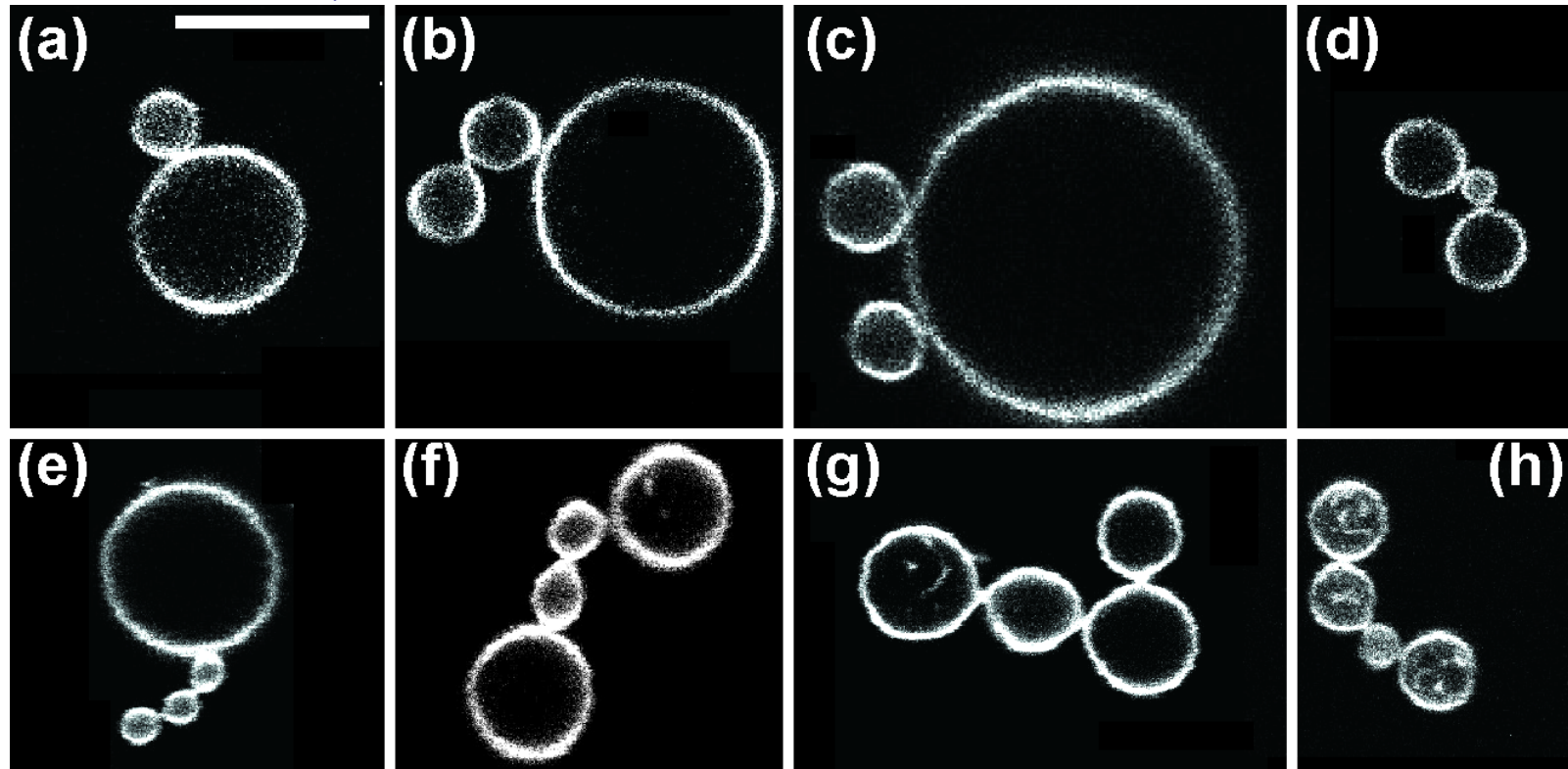
‘Fluid worm hole in three dimensions’

- Membrane exposed to asymmetric sucrose/glucose solutions
- Membrane forms two spheres connected by a single neck
- Same membrane system leads to proliferation of necks !

Multispheres with Many Necks

Scale bar: 10 μm

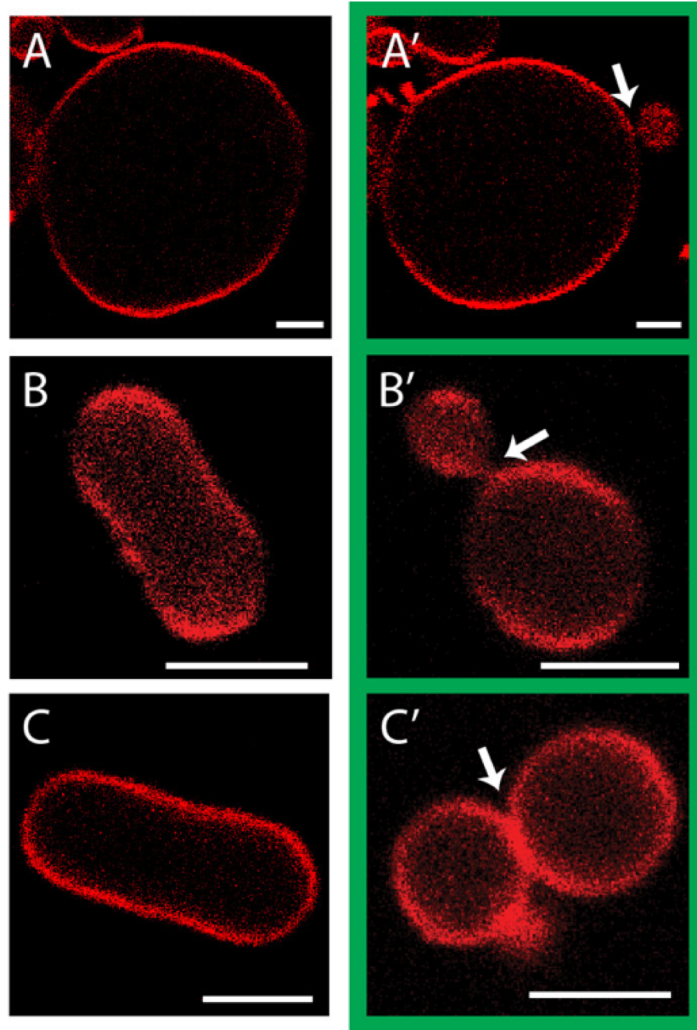
Bhatia et al, *Soft Matter* (2020)



- One membrane forms several spheres connected by necks
- Each shape involves only two different sphere radii

Controlled Budding of GUVs

Steinkühler et al, *Nature Comm* (2020)



No GFP

+ GFP

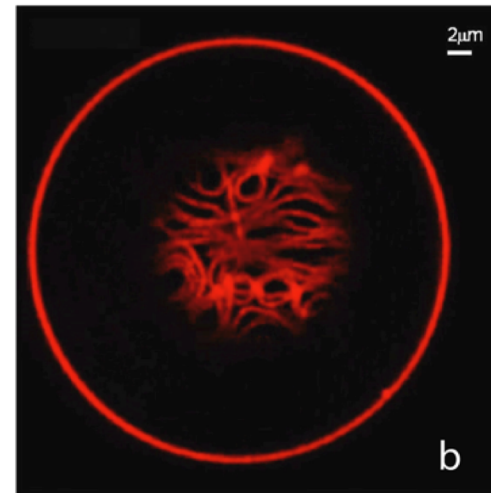
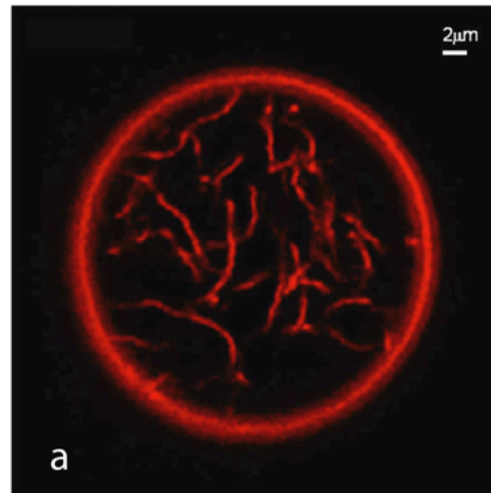
- Vesicles exposed to His-tagged GFP in the exterior solution
- GFP binds to anchor lipids in the vesicle membrane
- Membrane curvature fine-tuned by **nanomolar** concentration of GFP
- Low densities of membrane-bound GFP generate strongly curved membranes

GUVs and Aqueous Phase Separation

Li et al, *PNAS* (2011) Liu et al, *ACS Nano* (2016)

- Aqueous phase separation within giant vesicles
- Example: PEG and dextran in water
- Formation of many stable nanotubes, no pulling forces
- Wetting properties determine patterns of nanotubes:

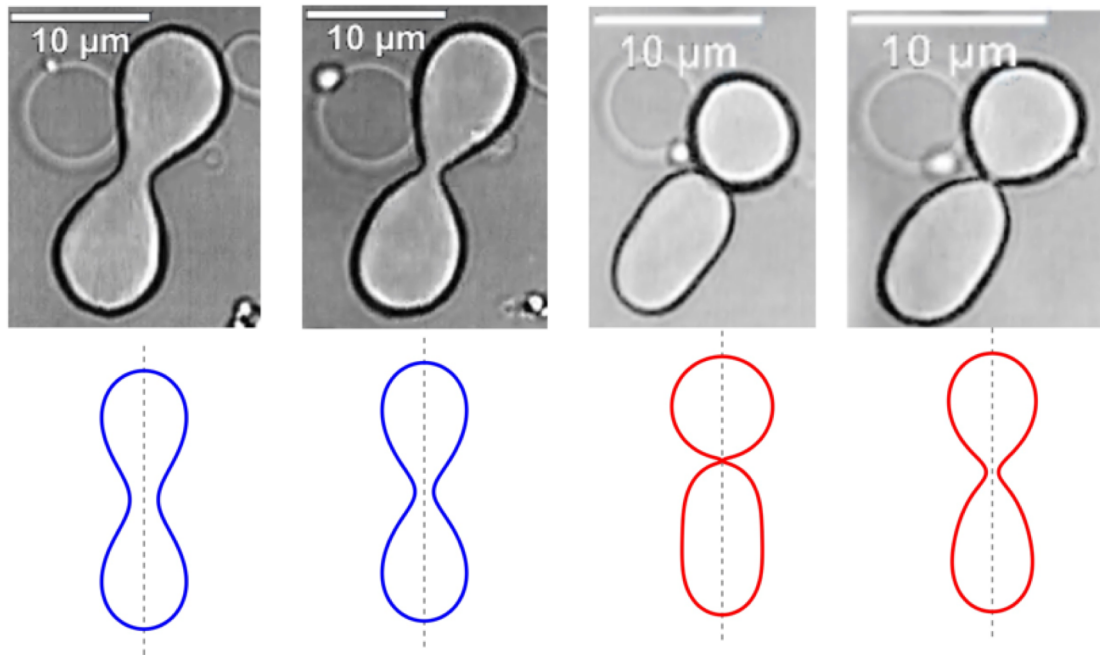
Complete wetting:
tubes stay in one
liquid phase



Partial wetting:
tubes bind to
liquid-liquid
interface

Active Shape Oscillations of GUVs

- Min proteins D and E in interior solution
- MinD/E binds to membrane and unbinds via ATP hydrolysis



Litschel et al,
Angewandte Chemie (2018)

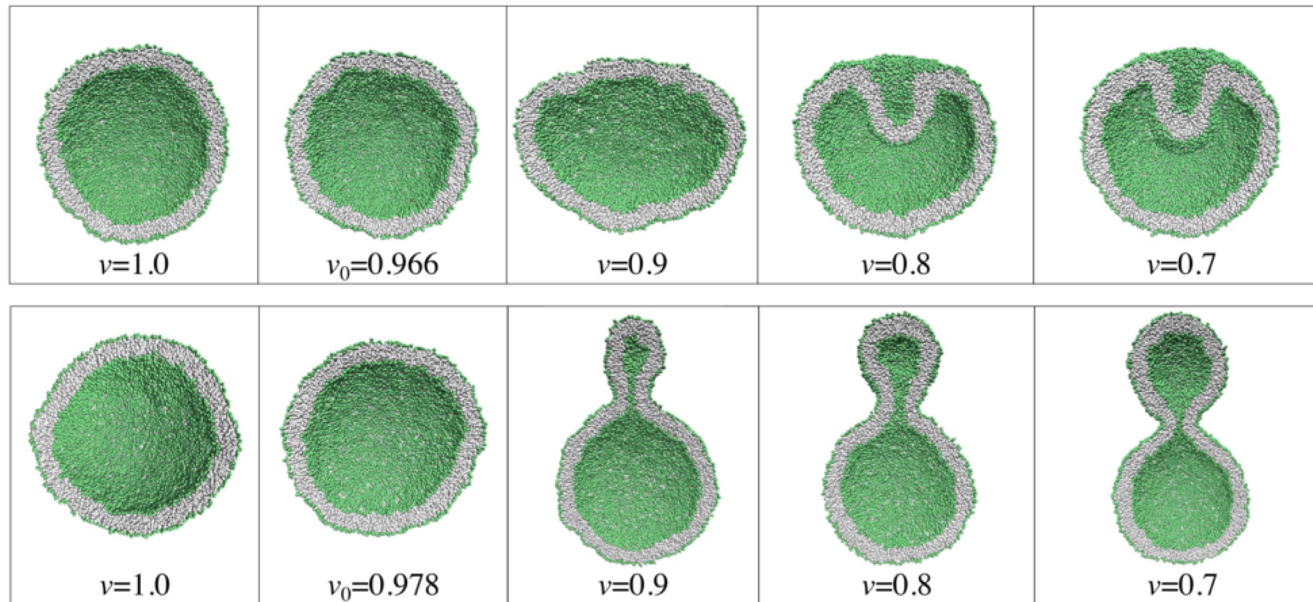
Christ et al, *Soft Matter* (2021)

- Cyclic closure and opening of membrane neck
- Time-dependent spontaneous curvature

Remodeling of Nanovesicle Shape

- Spherical nanovesicles with diameter of 36 nm
- Shape transformations by volume reduction

Ghosh, Satarifard et al,
Nano Letters (2019)



- Inner leaflet compressed

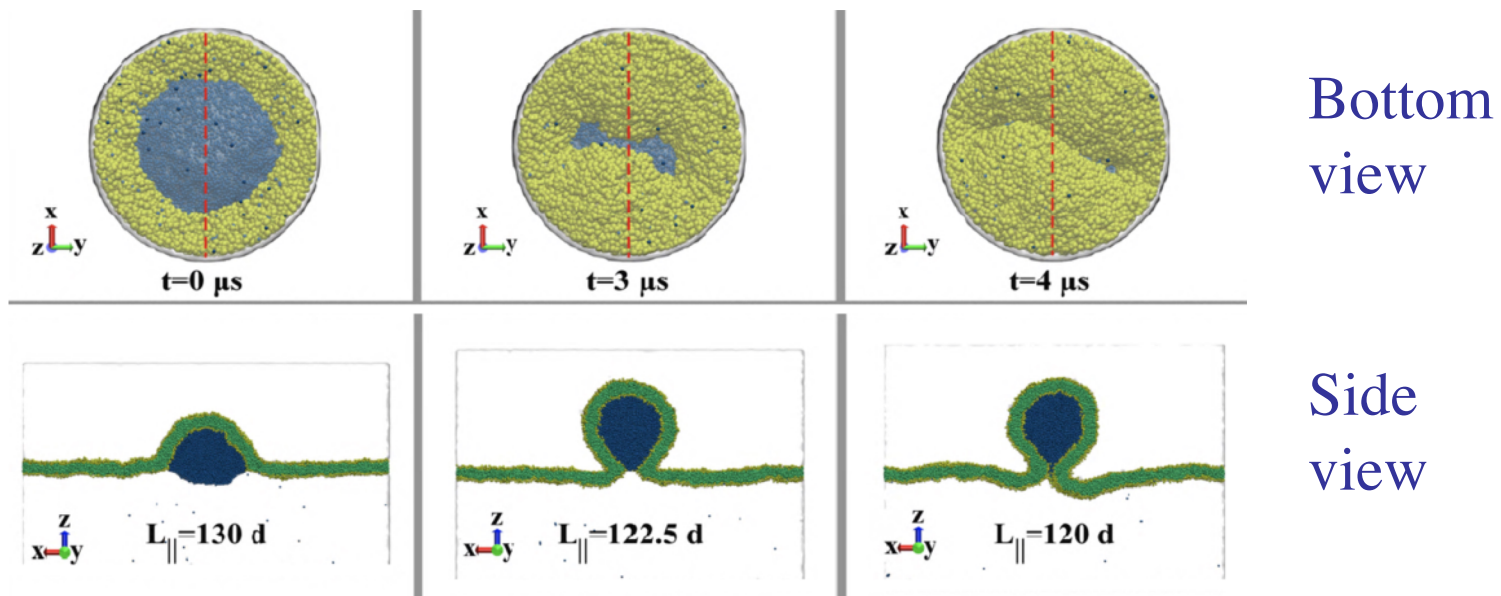
- Outer leaflet compressed

- Controlled by mechanical tensions in the two leaflets

Non-axisymmetric Membrane Necks

Satarifard et al, ACS Nano (2018)

- Engulfment of nanodroplet (blue) by lipid bilayer
- Neck shape controlled by mechanical bilayer tension:



- Formation of tight-lipped neck reveals **negative line tension of contact line**

Elasticity of Fluid Membranes

- Biomembrane as thin elastic sheet
- Elastic Deformations
- Fluid Membranes

Stretching



Membrane tension

Shearing



Shear -> Flow

Bending



Curvature elasticity

Theory of Membrane Elasticity

- Elastic stretching: Area A and tension Σ

$$\text{Mechanical tension } \Sigma = K_A (A - A_0)/A_0$$

area compressibility modulus K_A , optimal area A_0

$$\text{Stretch energy } E_{\text{st}} = \frac{1}{2} K_A (A - A_0)^2/A_0$$

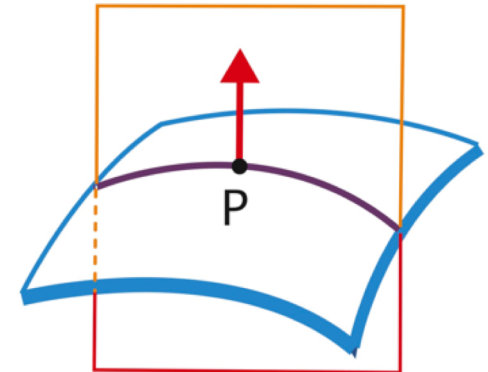
- Elastic bending: mean curvature M

$$\text{Bending energy } E_{\text{be}} = \int dA 2 \kappa (M - m)^2$$

bending rigidity κ , spontaneous curvature m

- Total elastic energy: Stretch energy + bending energy

$$E_{\text{el}} = E_{\text{st}} + E_{\text{be}} = \frac{1}{2} K_A (A - A_0)^2/A_0 + E_{\text{be}}$$



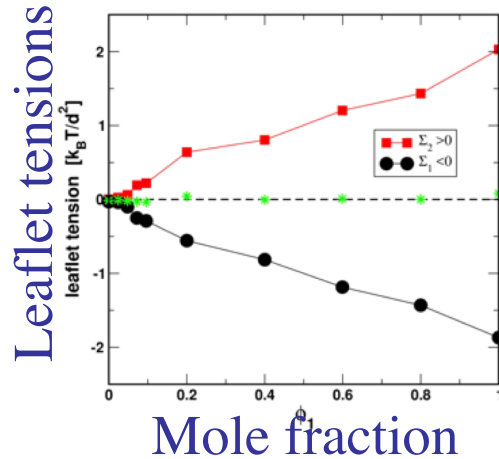
Composite Nature of Membrane Tension

- Mechanical tension Σ plus spontaneous tension $\sigma = 2 \kappa m^2$
- Spontaneous tension leads to spontaneous tubulation
- Total membrane tension $\Sigma_{\text{tot}} = \Sigma + \sigma$
Lipowsky, Faraday Disc (2013)
- Spontaneous tension $\sigma = 2 \kappa m^2$ is material parameter
- Tension σ measured by micropipette aspiration of tubulated GUVs
Bhatia et al, ACS Nano (2018)
- Mechanical tension Σ depends on vesicle size and shape
- Difficult to measure, exact computation for multispheres

Bilayer and Leaflet Tensions

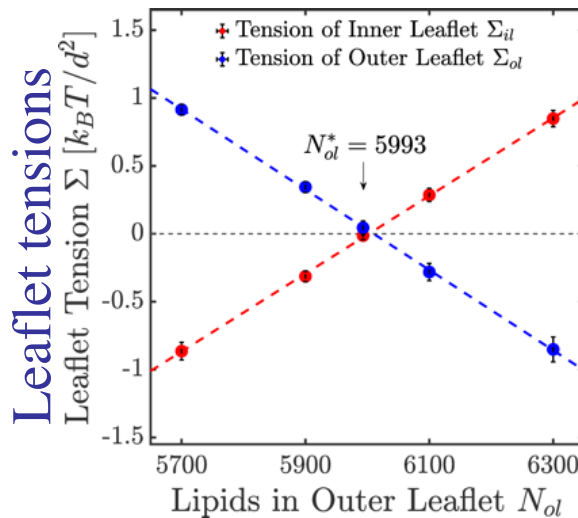
- Bilayer membrane consists of two leaflets, l_1 and l_2
- Mechanical bilayer tension $\Sigma = \Sigma_{l_1} + \Sigma_{l_2}$
- Tensionless bilayers: $\Sigma = 0$ implies $\Sigma_{l_2} = -\Sigma_{l_1}$
=> One leaflet stretched and one leaflet compressed
- Unique reference state with $\Sigma_{l_2} = \Sigma_{l_1} = 0$

Leaflet Tensions without Flip-Flops



Sreekumari, RL, *JCP*(2018)

- Planar bilayer, two lipids A and B
- Lower leaflet contains only lipid B
- Leaflet tensions versus mole fraction of lipid A in upper leaflet



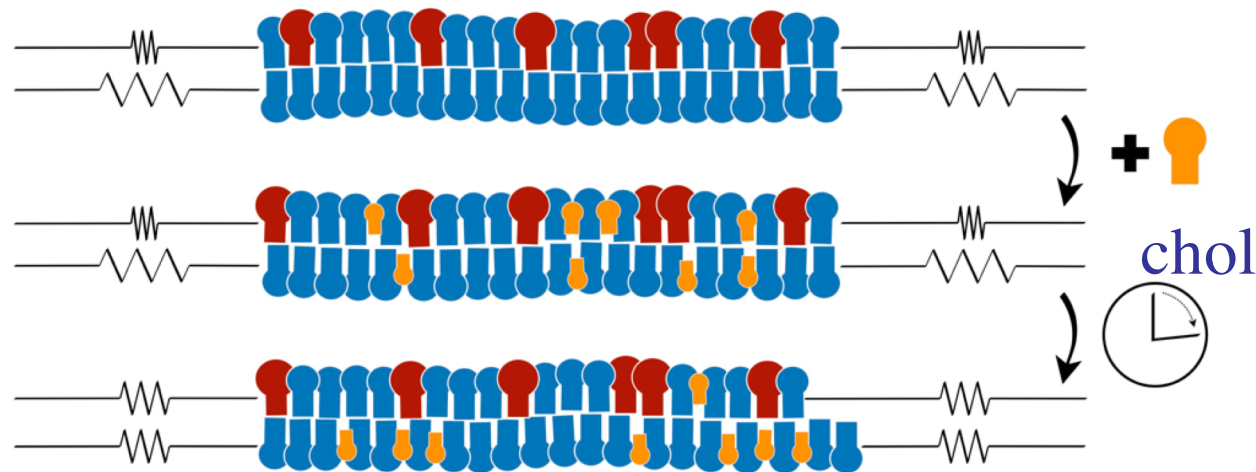
Ghosh, Satarifard et al, *Nano Letters* (2019)

- Spherical nanovesicle, one lipid A
- Fixed total number of lipids $N = N_{ol} + N_{il}$
- Leaflet tensions versus lipid number N_{ol} of outler leaflet

Leaflet Tensions with Flip-Flops

Miettinen, RL, *Nano Letters* (2019)

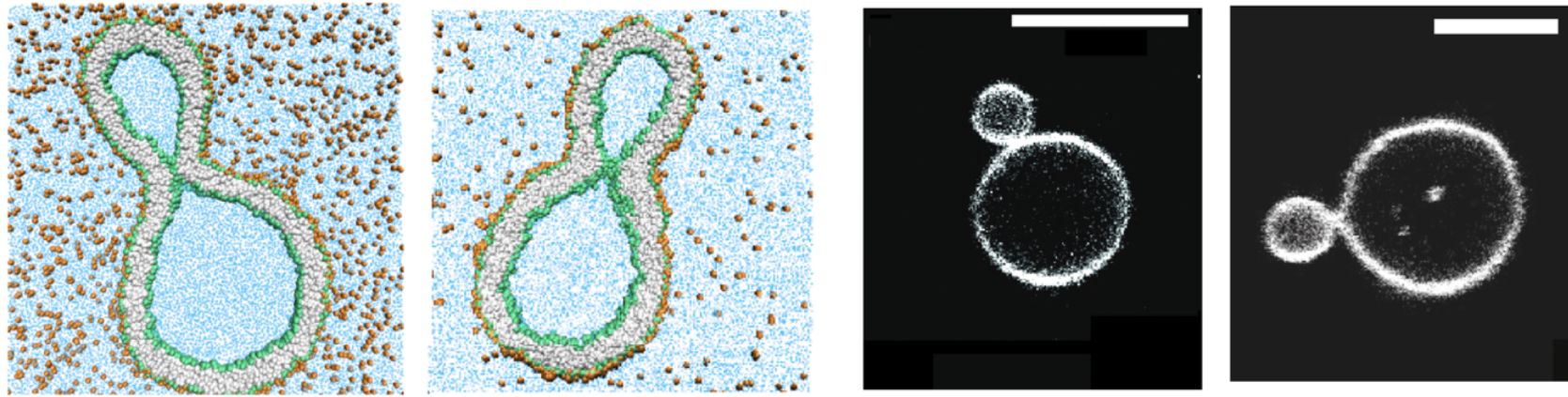
- Tensionless planar bilayer with two lipids (blue and red)
- Addition of cholesterol (orange) to both leaflets:



- Cholesterol undergoes flip-flops between leaflet
- Leaflet tensions decay to zero

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 - Fission of membrane necks

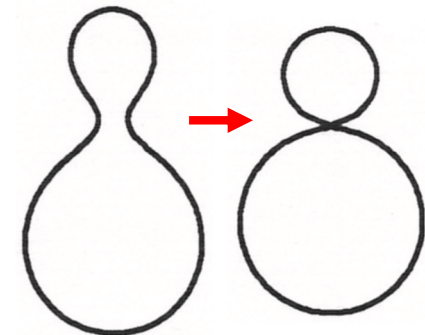
Separation of Length Scales



Nanoscale: Hourglass-shaped neck

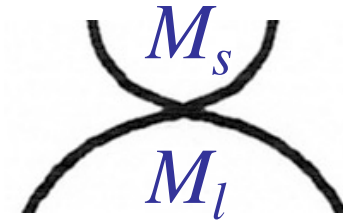
Micron scale: Pointlike neck

- Bilayer thickness \ll vesicle size $R_{ve} = \sqrt{A/(4\pi)}$
- Simple relations for local properties of necks
- Neck closure: principal curvatures of waistline diverge but mean curvature M_{wl} remains finite



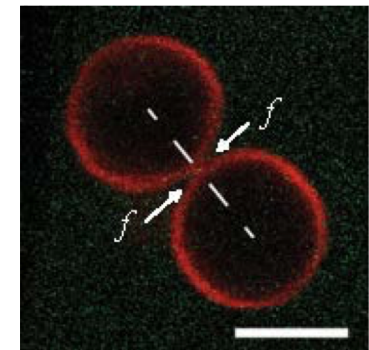
Local Properties of Membrane Necks

- Limiting value: $M_{wl} \approx \frac{1}{2} (M_l + M_s) \equiv M_{ne}$
- Defines neck geometry via neck curvature M_{ne}
- Stability of closed neck depends on spontaneous curvature m :



neck is stably closed for $M_{ne} \leq m$

- Constriction force at neck, $f = 8\pi \kappa (m - M_{ne})$
- Total membrane tension $\Sigma_{tot} = 4 \kappa m M_{ne}$
- Mechanical membrane tension $\Sigma = 2 \kappa m (2M_{ne} - m)$
- Relations between **local** neck geometry and elastic parameters



Multispheres: Geometry

RL, *Advances Biomembranes and Lipid Selfassembly*, Vol. 30 (2019)

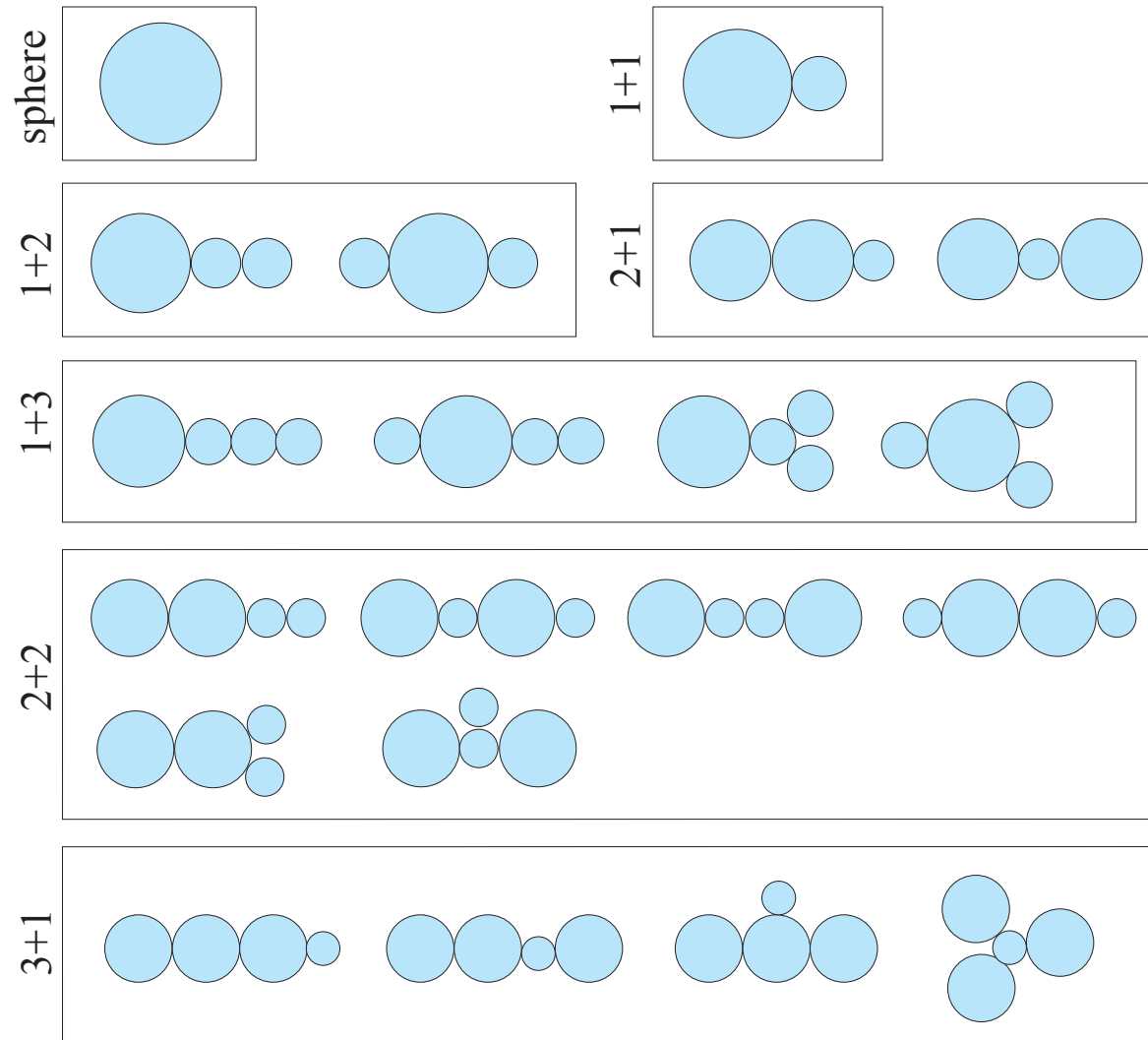
- Multispheres with large and small spheres
- Rescaled large sphere radius r_l and small sphere radius r_s
- Multispheres consisting of N_l large and N_s small spheres
- $(N_l + N_s)$ -geometry determined by two simple equations:

$$N_l r_l^2 + N_s r_s^2 = 1$$

$$N_l r_l^3 + N_s r_s^3 = v$$

- Two nonlinear equations for two unknowns r_l and r_s
- Simple equations generate morphological complexity

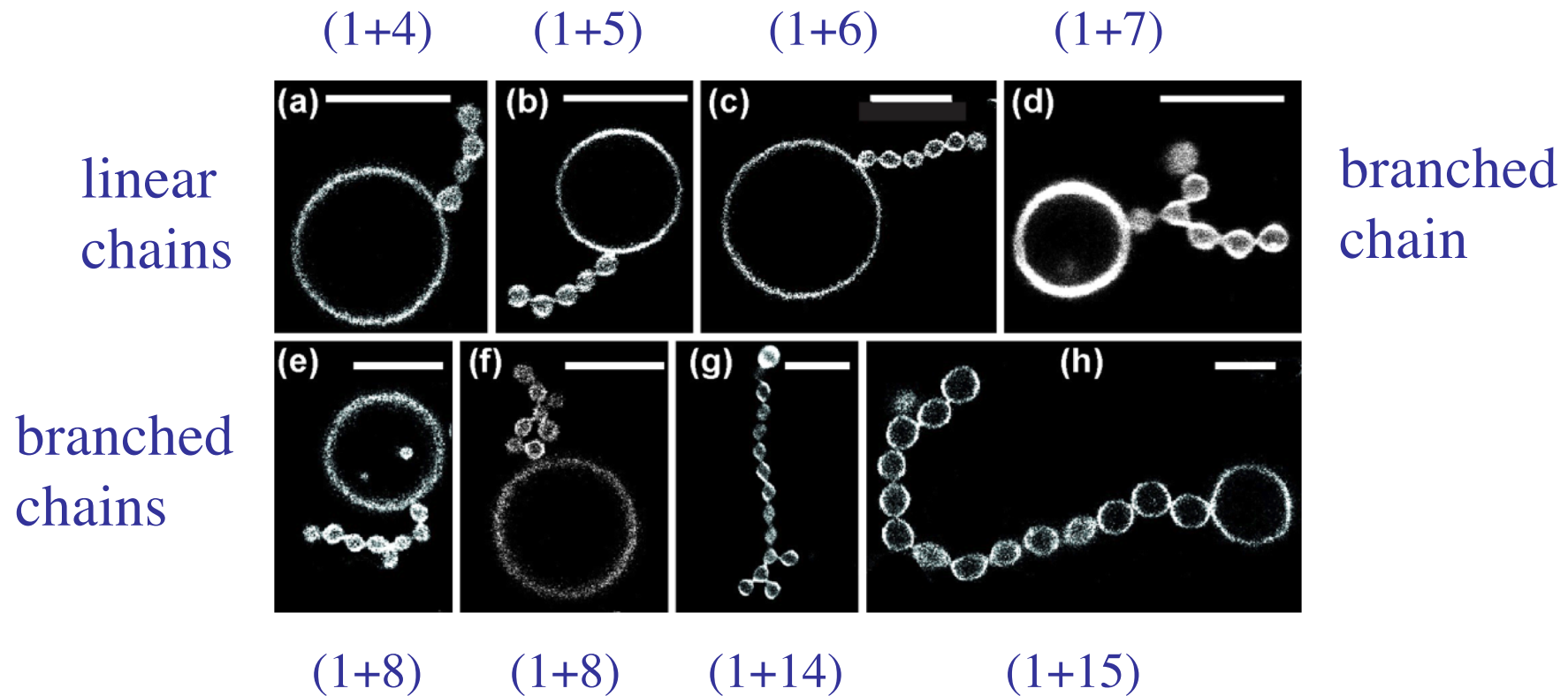
Multispheres up to $N_l + N_s \leq 4$



$(1+N_s)$ -Multispherical Vesicles

Bhatia et al, Soft Matter (2020)

- $(1+N_s)$ -spheres with one large sphere and a chain of N_s small spheres:

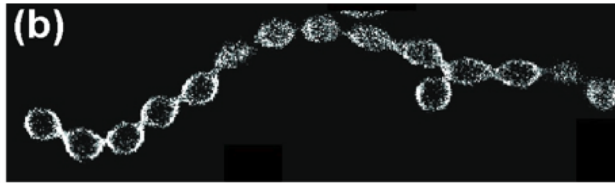
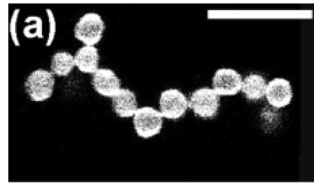


(N_*) -Multispherical Vesicles

Bhatia et al, *Soft Matter* (2020)

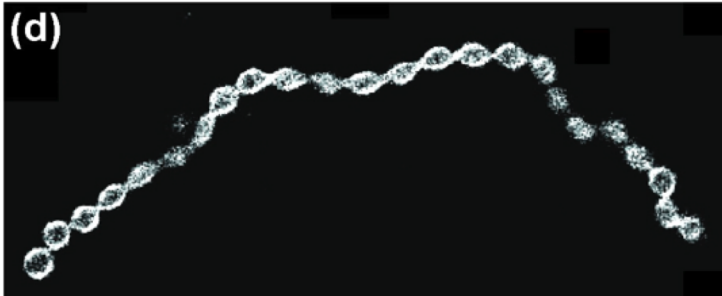
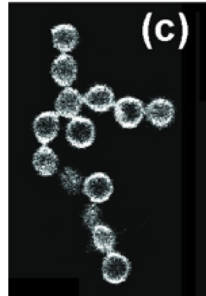
- Multispheres consisting of N_* equally sized spheres:

$N_* = 14$
branched



$N_* = 15$
branched

$N_* = 15$
branched



$N_* = 24$
linear

$N_* = 39$
branched



- Surprising mobility: linear \Leftrightarrow branched chains

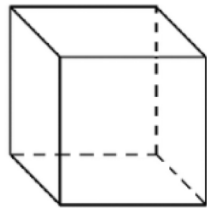
- Remodeling of membrane shape
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Topology of Surfaces

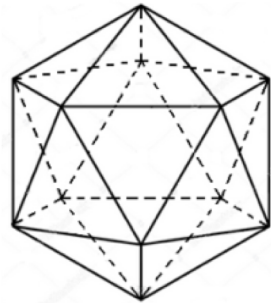
- Closed surface with F faces, E edges, and V vertices



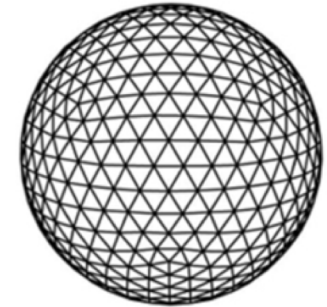
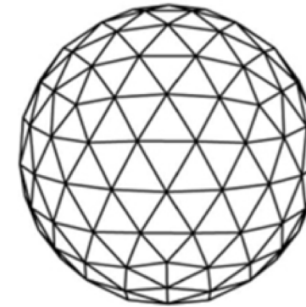
tetrahedron



cube



icosahedron

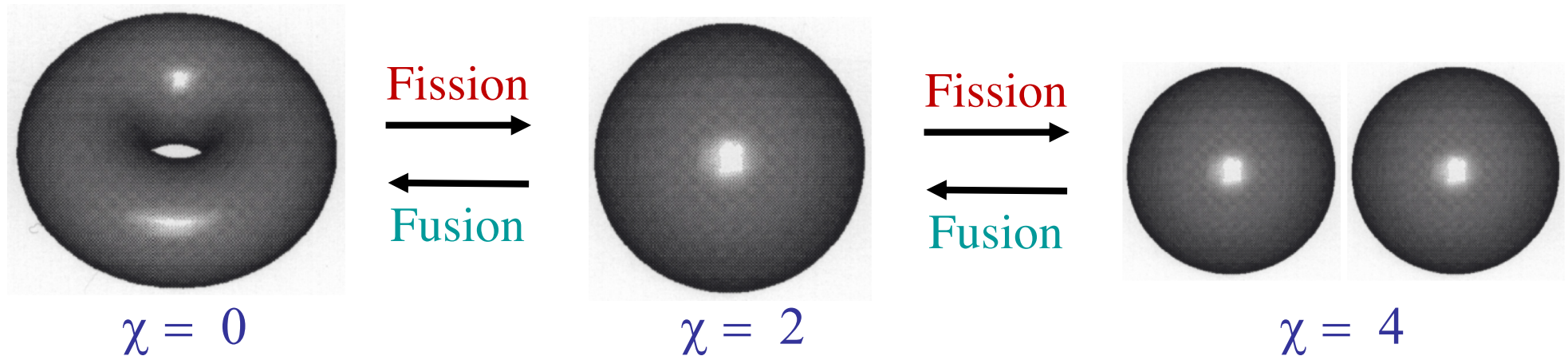


sphere

- Euler characteristic $\chi = F - E + V$
- For tetrahedron, cube, ..., and sphere: $\chi = 2$
- Euler characteristic is topological invariant
- Euler characteristic is additive: $\chi = 2 + 2 = 4$ for two spheres

Remodeling of Membrane Topology

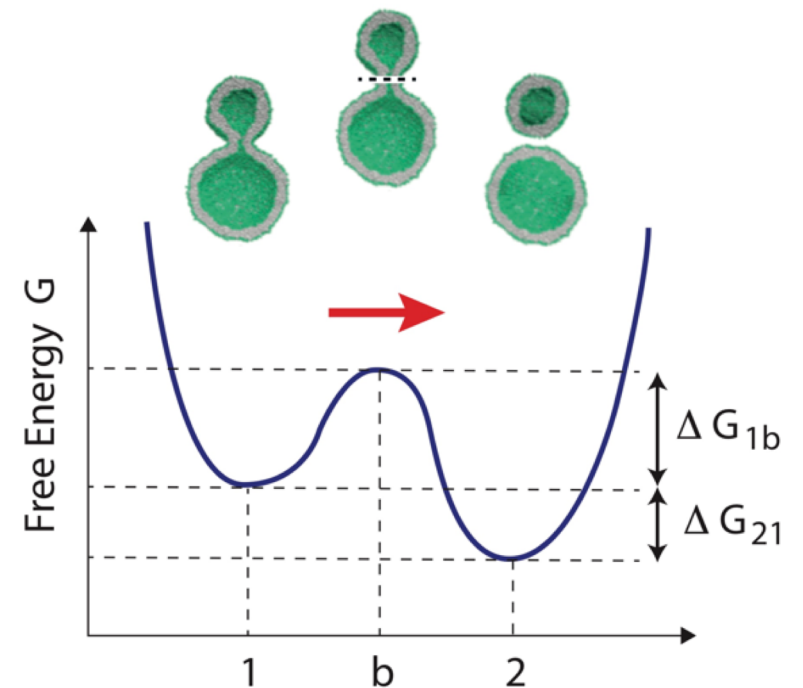
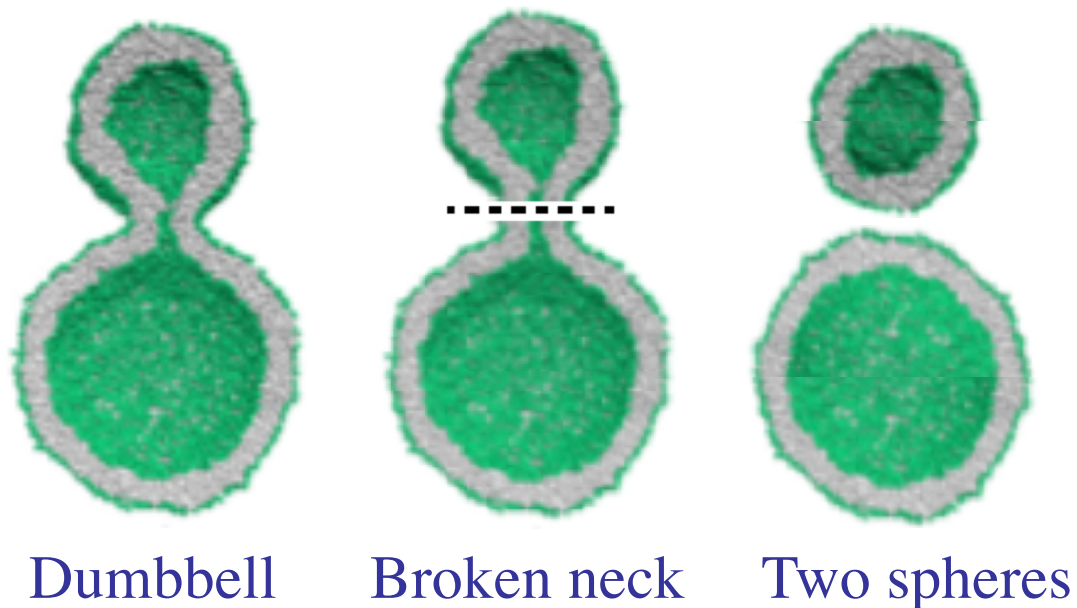
- Topological classification via Euler characteristic χ :



- Topological transformation \Leftrightarrow change $\Delta\chi = \chi_{\text{fin}} - \chi_{\text{ini}}$
- **Fission**: Euler characteristic $\Delta\chi > 0$
- **Fusion**: Euler characteristic $\Delta\chi < 0$

Fission of Membrane Necks

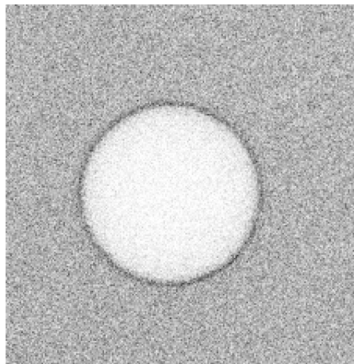
- Membrane fission implies disruption/cut of membrane
- Work of fission proportional to length of cut
- Shortest possible cut for dumbbell across membrane neck:



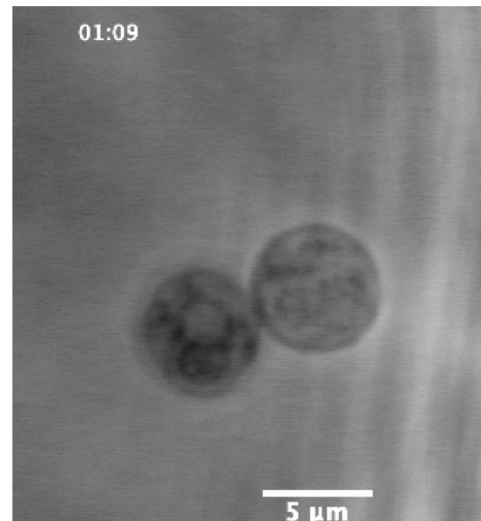
Neck Fission of GUVs

Steinkühler et al: *Nature Comm.* (2020)

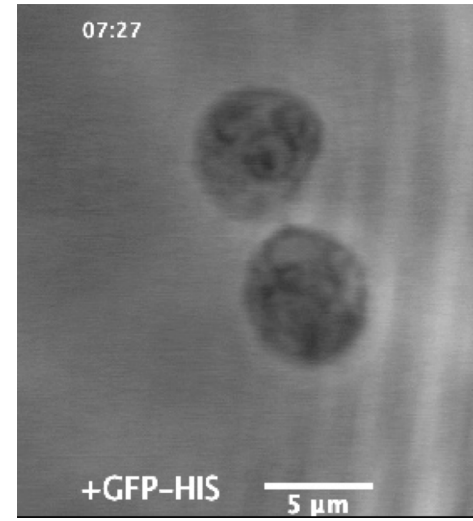
- Osmotic deflation + GFP binding
 - Osmotic deflation: Spherical GUV \rightarrow dumbbell GUV
- Increase in GFP \rightarrow Neck cleavage \rightarrow Two daughter GUVs



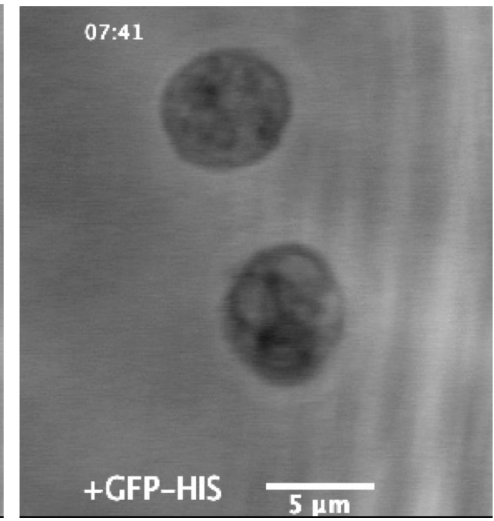
Adsorption of GFP onto GUV membrane



Deflation leads to dumbbell with membrane neck



Directly after neck cleavage



Complete division into two smaller GUVs

Constriction Force from GFP

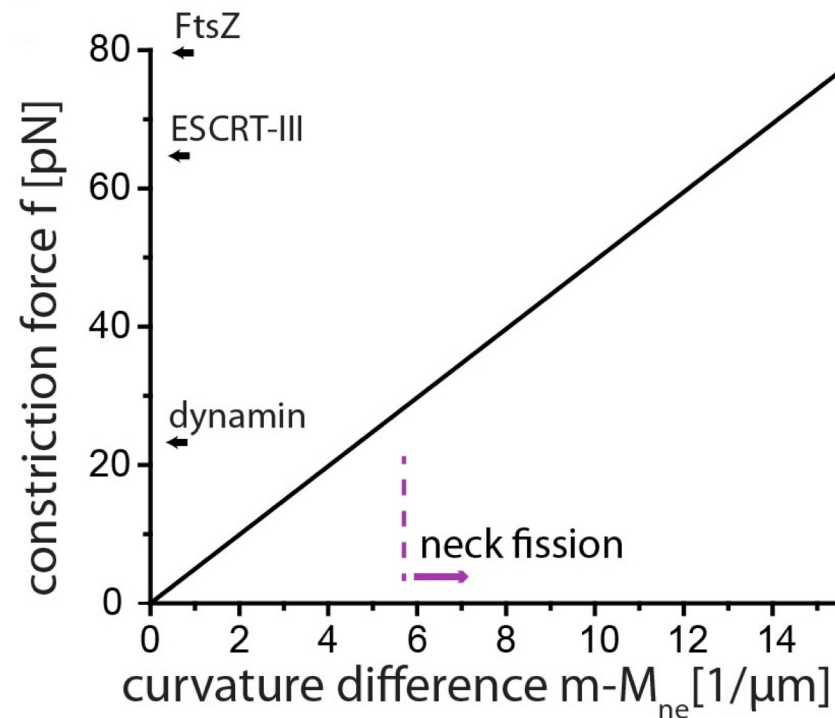
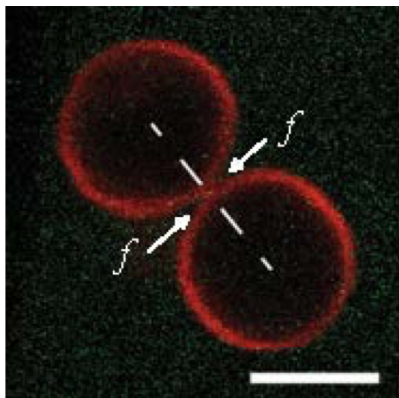
- Small GFP concentrations X in the solution generate large spontaneous curvature

Steinkühler et al:
Nature Comm. (2020)

$$m = \frac{1.86}{\mu\text{m}} \frac{X}{\text{nM}} \quad \text{for } 0 < X < 24 \text{ nM}$$

- Constriction force

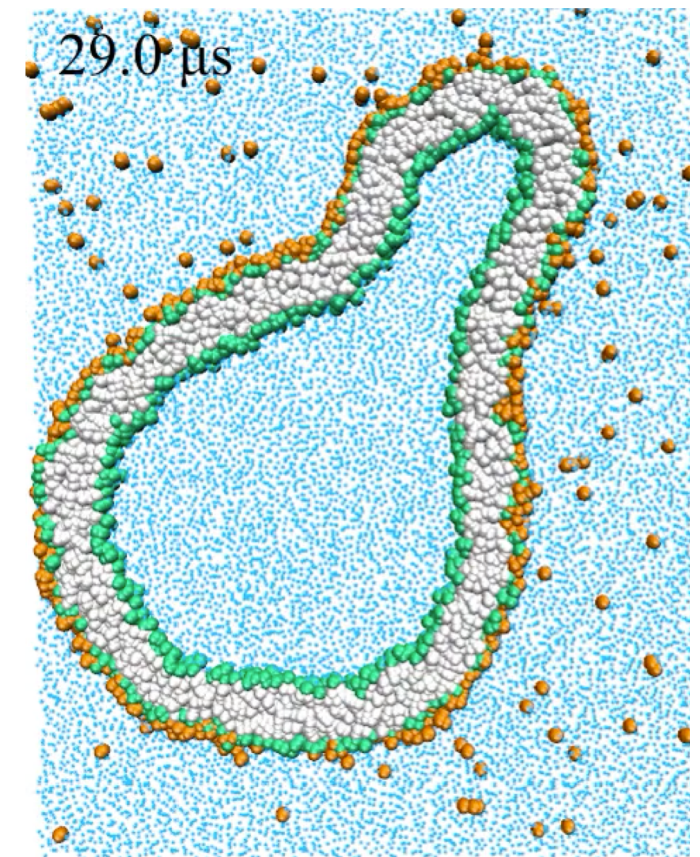
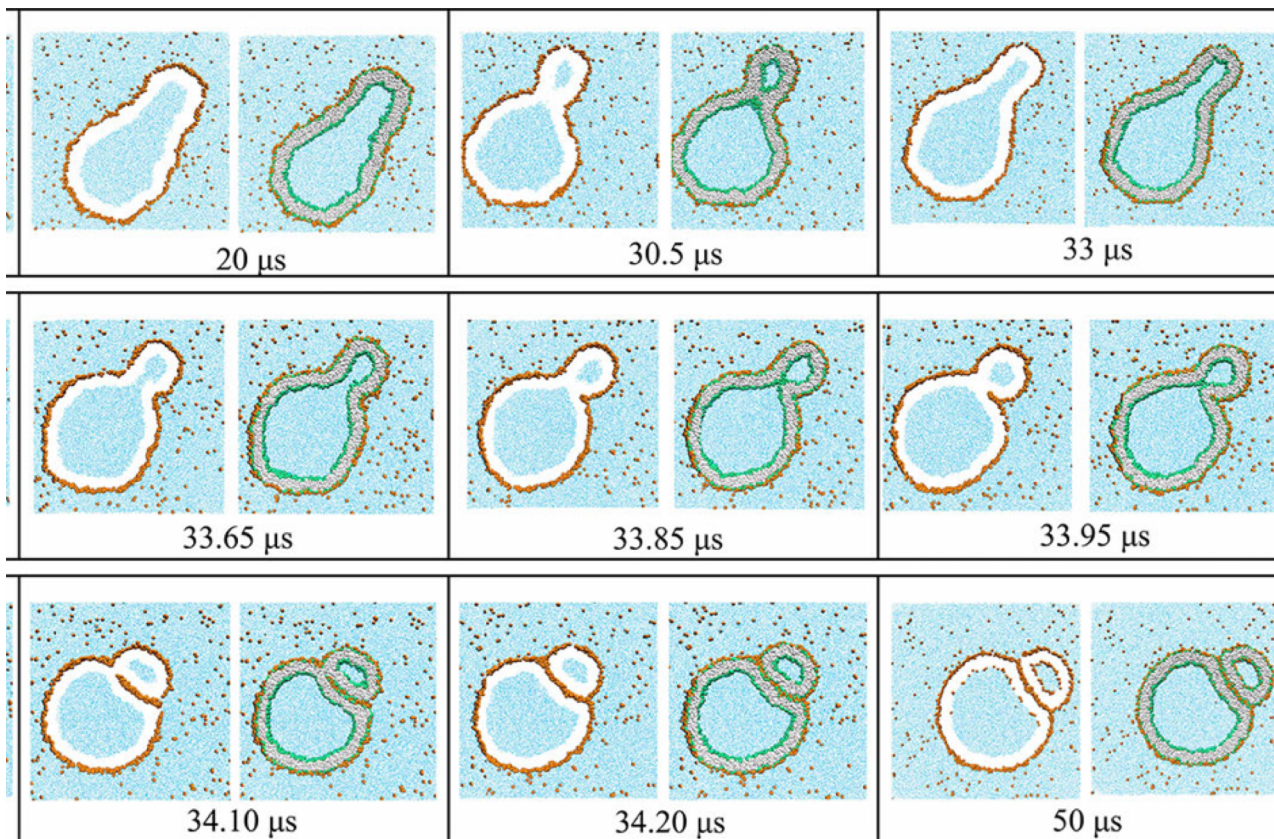
$$f = 8\pi \kappa (m - M_{\text{ne}})$$



Neck Fission of Nanovesicles

Ghosh et al, ACS Nano (2021)

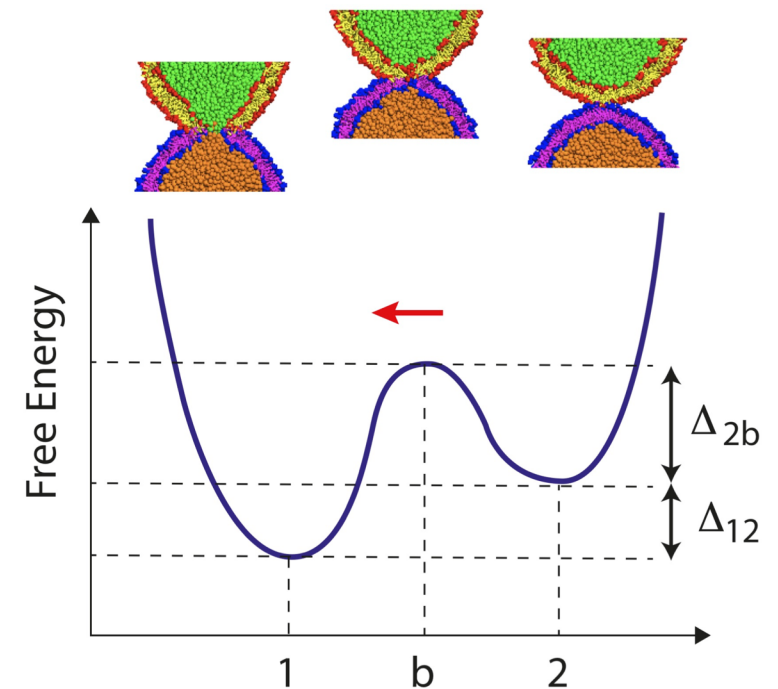
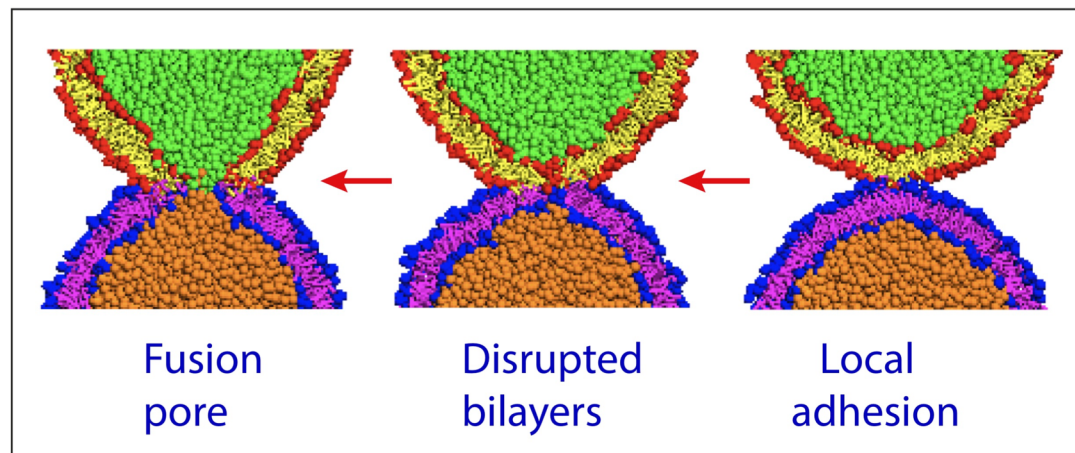
- Nanovesicle exposed to small solutes (orange) that adsorb onto vesicle membrane:



Fusion of Nanovesicles

Shillcock, RL, *Nature Materials* (2005) Grafmüller et al, *Phys. Rev. Lett.* (2007)
Gao et al, *Soft Matter* (2008)

- Two nanovesicles under mechanical tension $\Sigma \sim (A-A_0)/A_0$
- After local contact, fusion pore opens within $2 \mu\text{s}$



- Important role of tension in both fusion and fission processes !

Coworkers

Experiment



Tripta
Bhatia



Rumiana
Dimova



Yonggang
Liu



Jan
Steinkühler



Ziliang
Zhao

Joint projects with:
Petra Schwille
Seraphine Wegner

Simulation



Rikhia
Ghosh



Andrea
Grafmüller



Markus
Miettinen



Vahid
Satarifard



Aparna
Sreekumari

Theory



Simon
Christ

- Ongoing projects: Joachim Spatz, Petra Schwille, Tony Hyman

Recent References

Experiment

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Patterns of flexible nanotubes ... *ACS Nano* (2016) 10, 463-474.
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Simple sugars shape giant vesicles into multispheres ... *Soft Matter* (2020) 16, 1246-1258.
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Active shape oscillations of giant vesicles ... *Soft Matter* (2021) 17, 319-330

Simulation

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Nanodroplets at membranes create tight-lipped ... *ACS Nano* (2018) 12, 12424-12435.
- M. Miettinen, RL.
Bilayer membranes with frequent flip-flops ... *Nano Letters* (2019) 19, 5011-5016
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Advances in Biomembranes and Lipid Selfassembly, Vol. 30 (Academic Press, 2019) Ch. 3