



Prescribing the elasticity of knits, stitch-by-stitch

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former Postdoc
(currently @UMass)
Computation

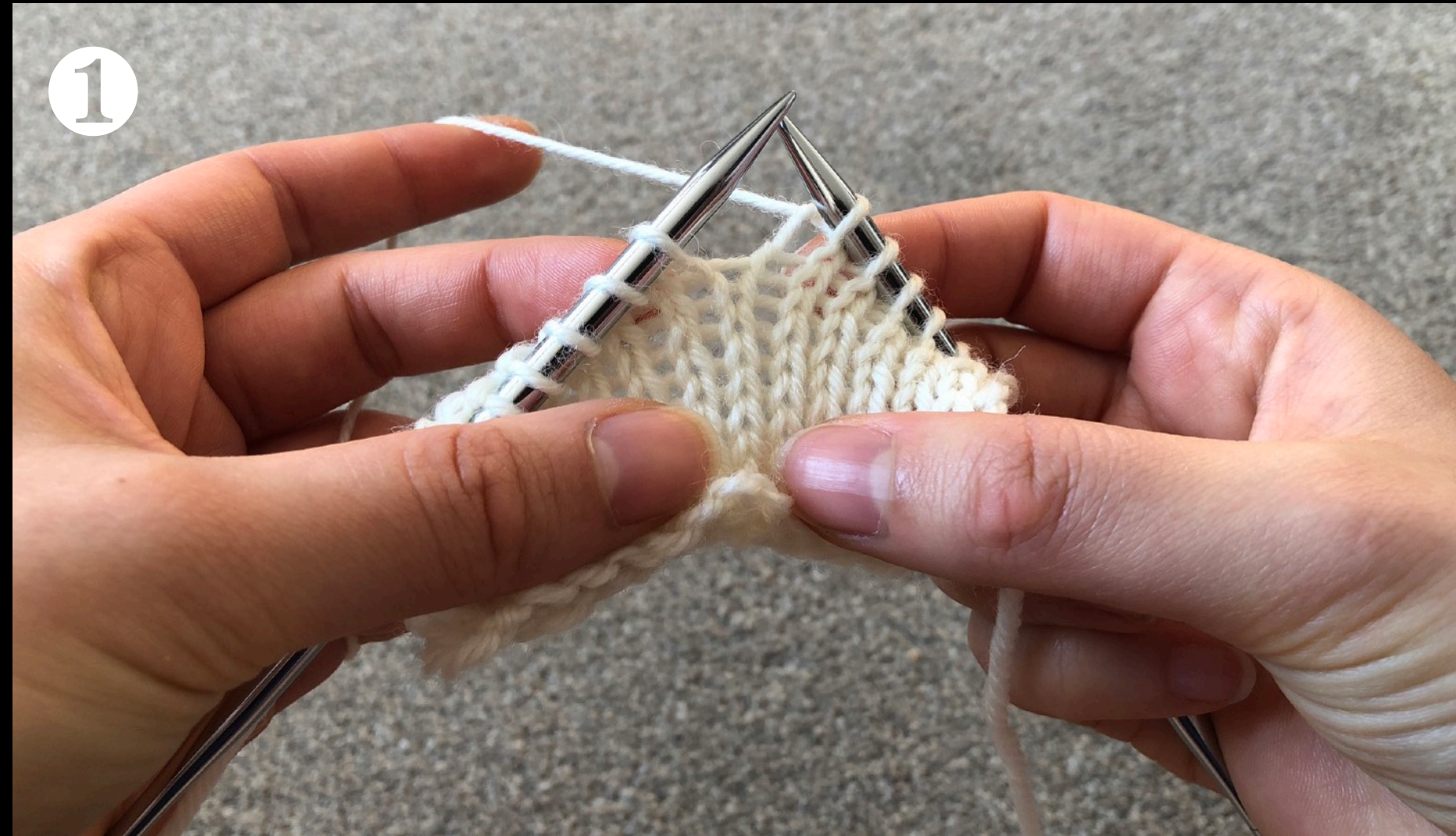


Krishma Singal
PhD student
Imaging &
Experiments

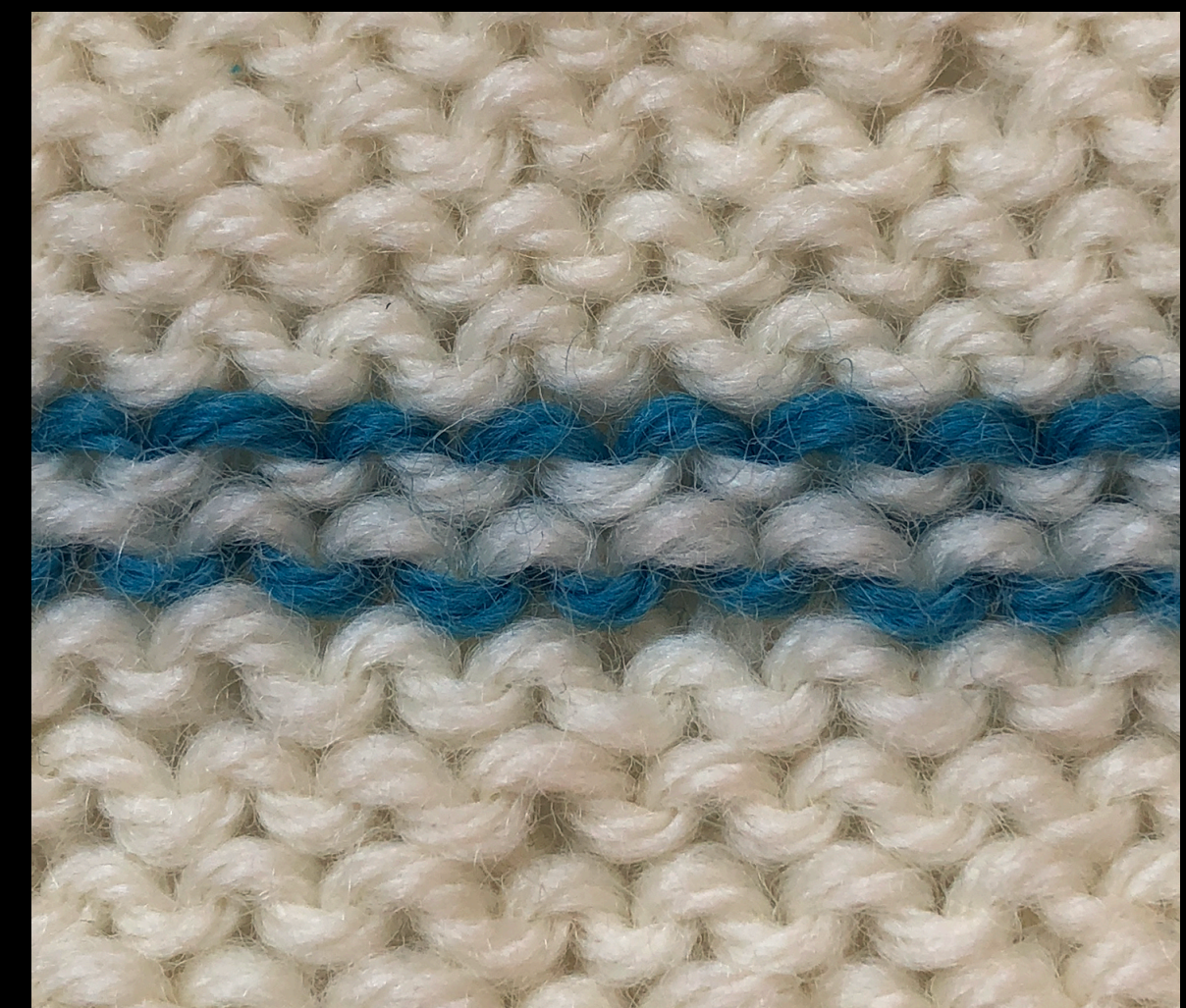
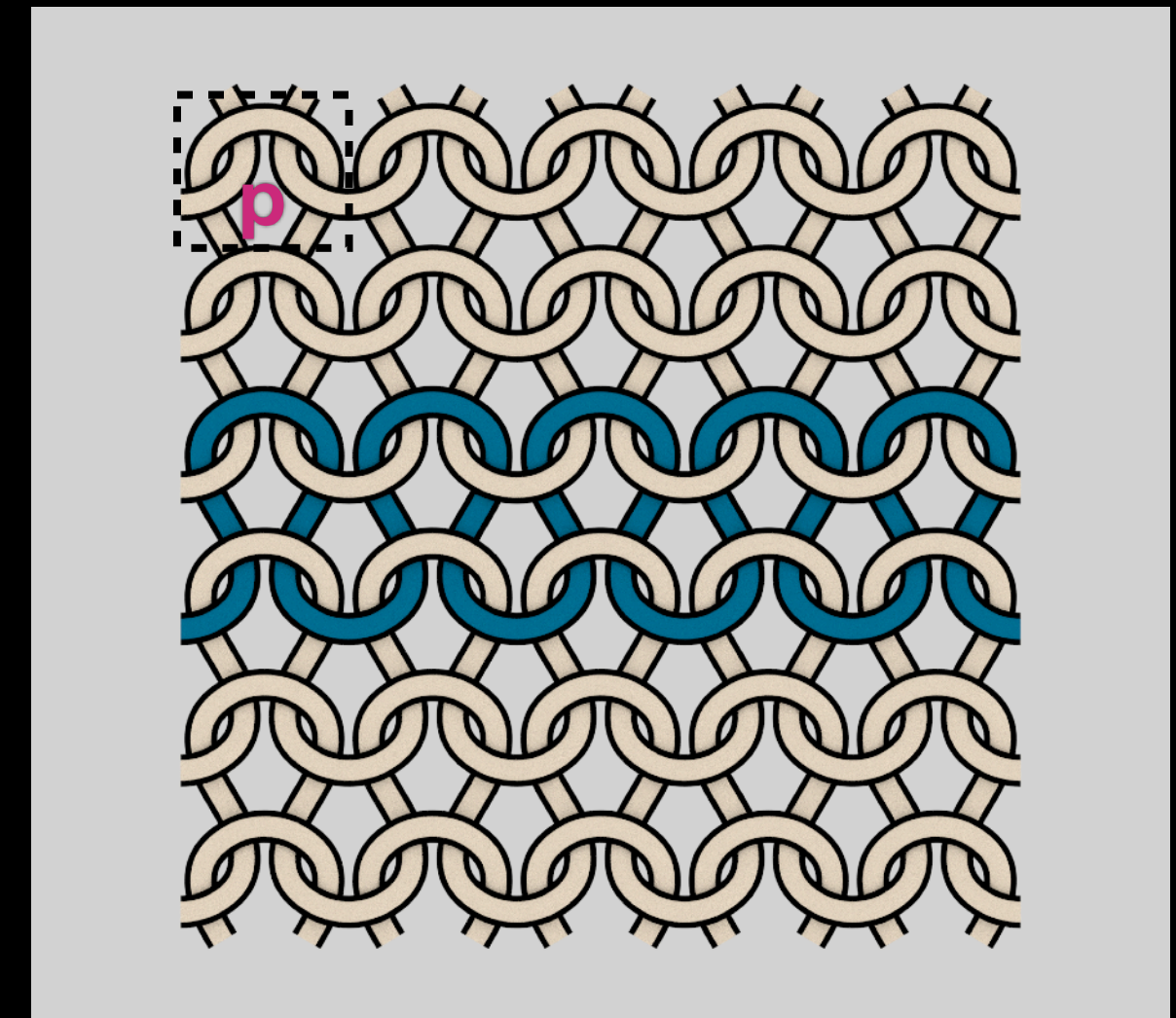
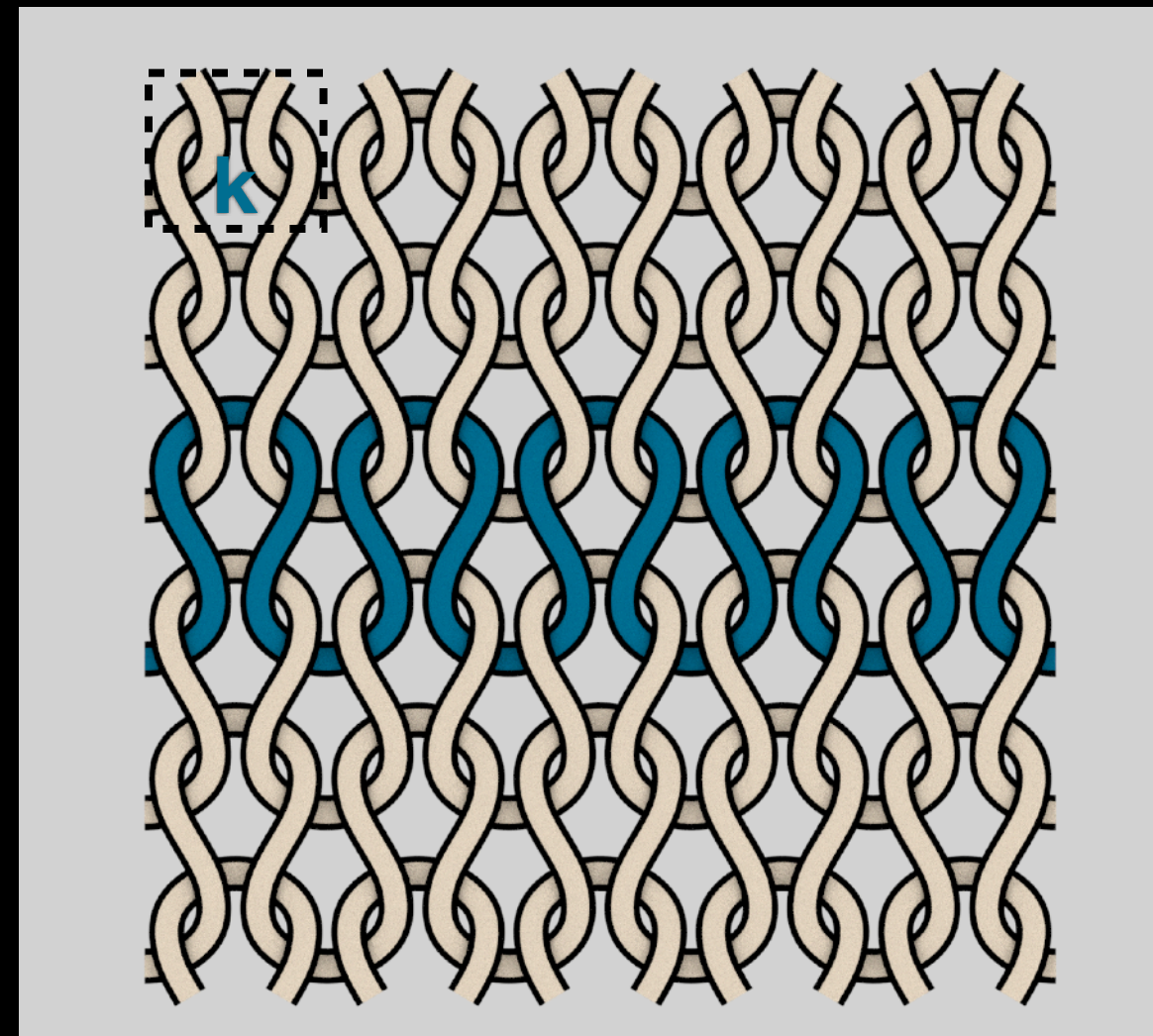
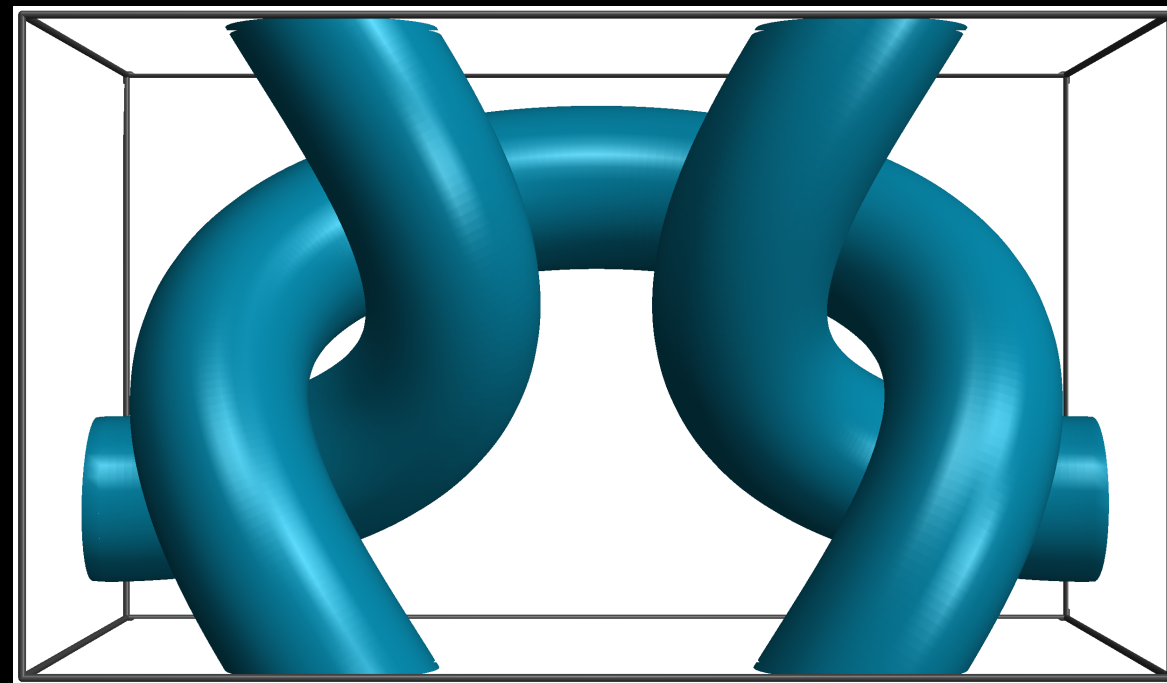


DMR 1847172

The knitted stitch



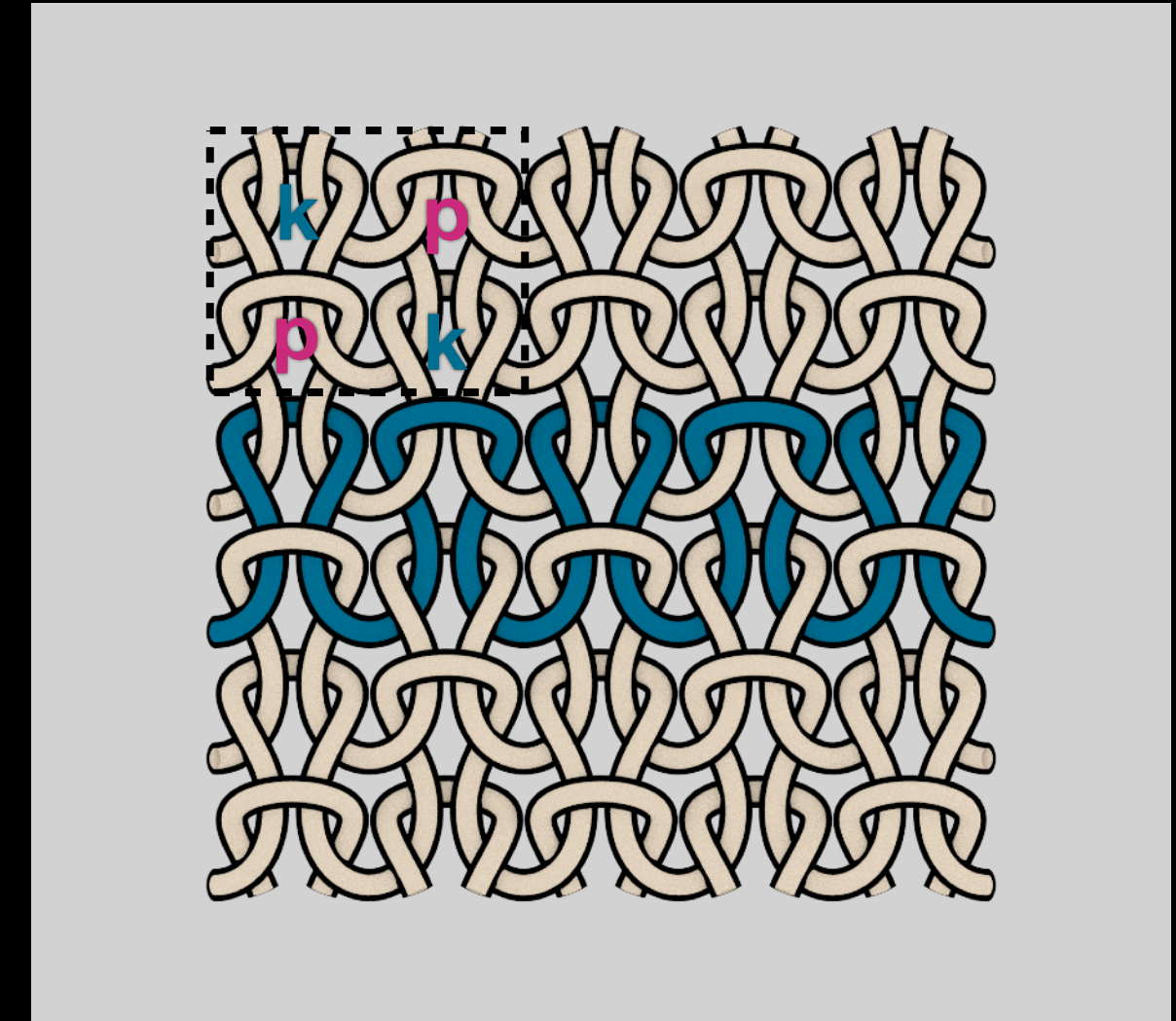
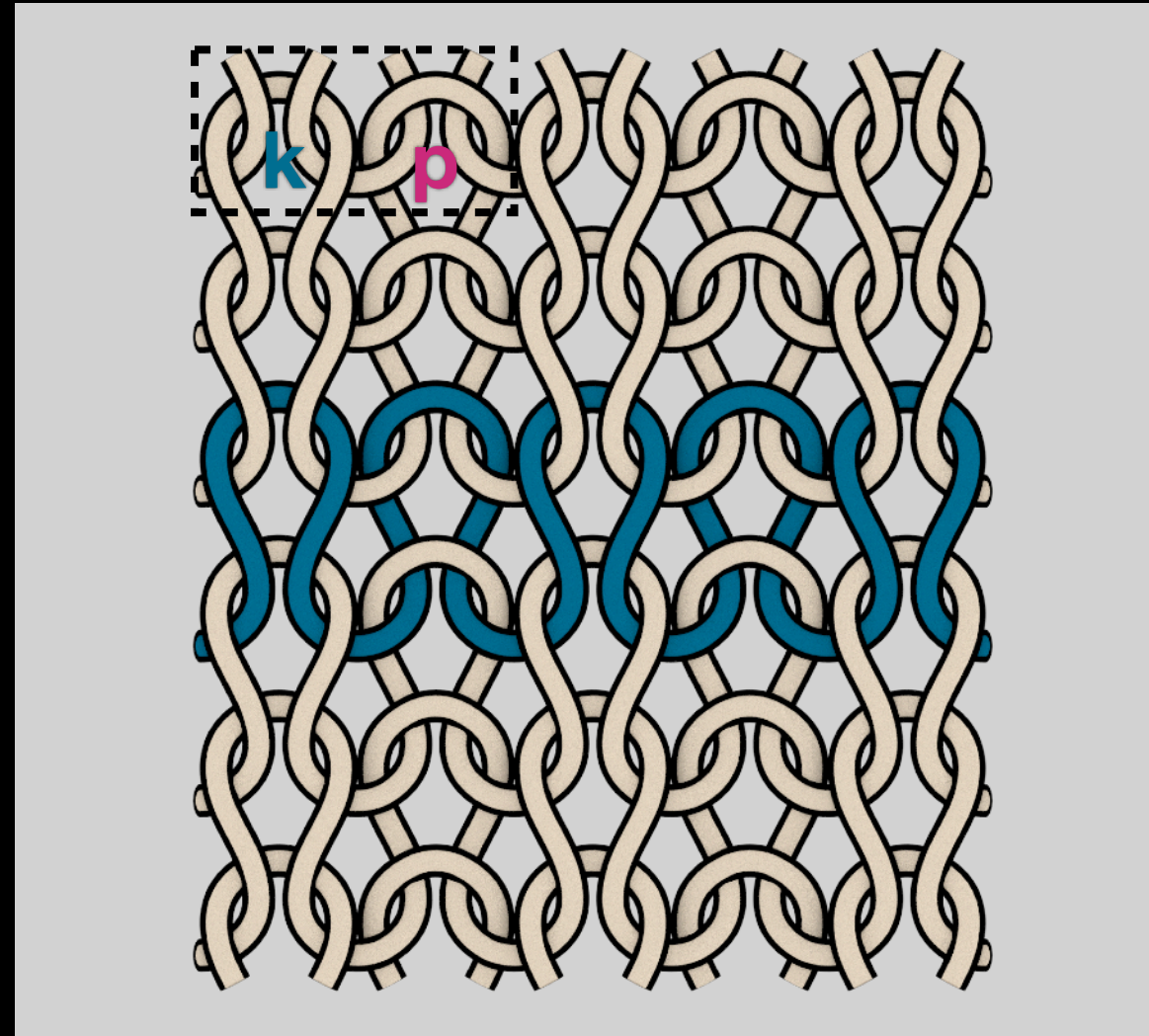
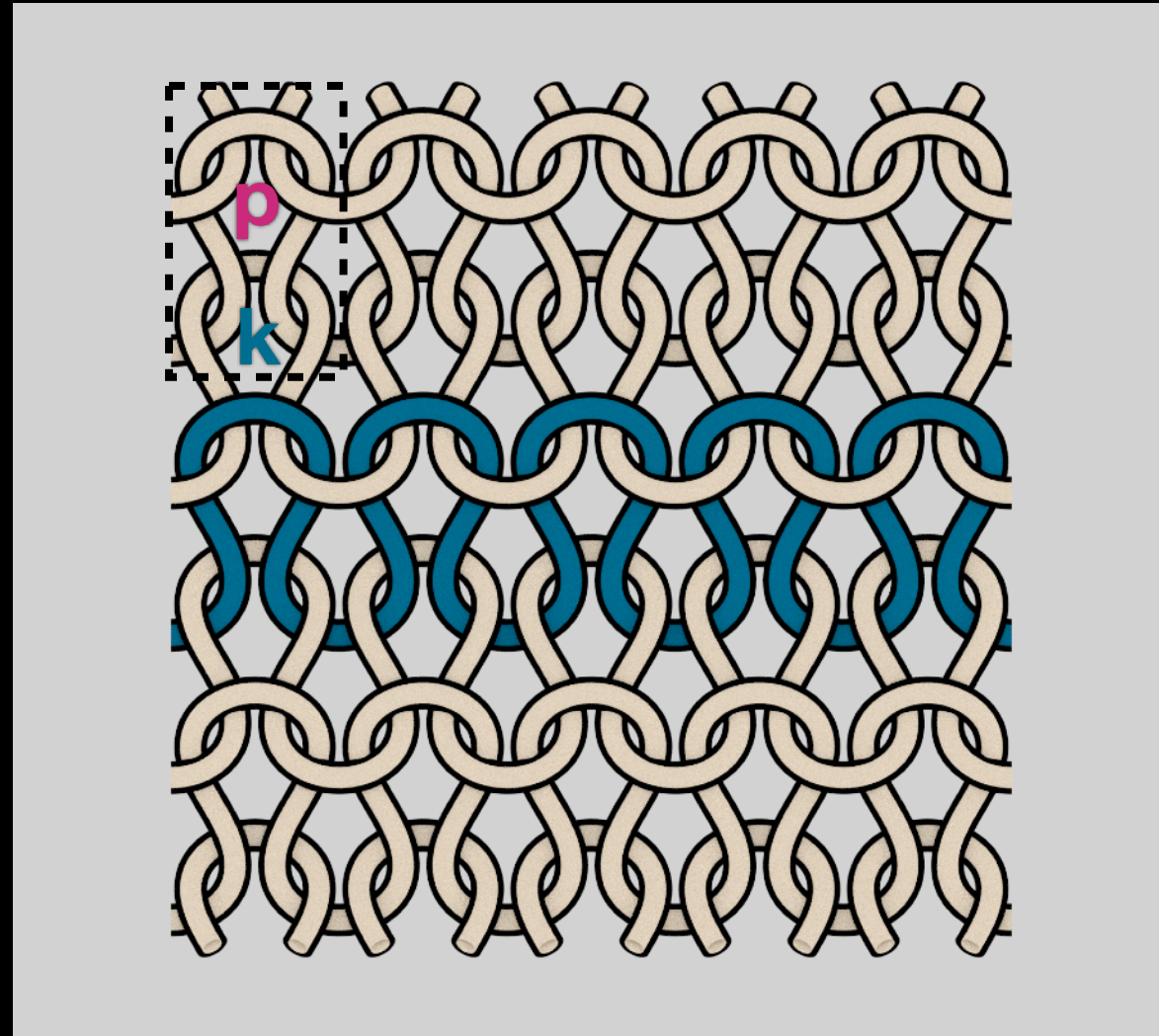
The knitted stitch



Stockinette

Reverse stockinette

Patterns made from knits and purls



Garter stitch



1x1 ribbing



Seed stitch

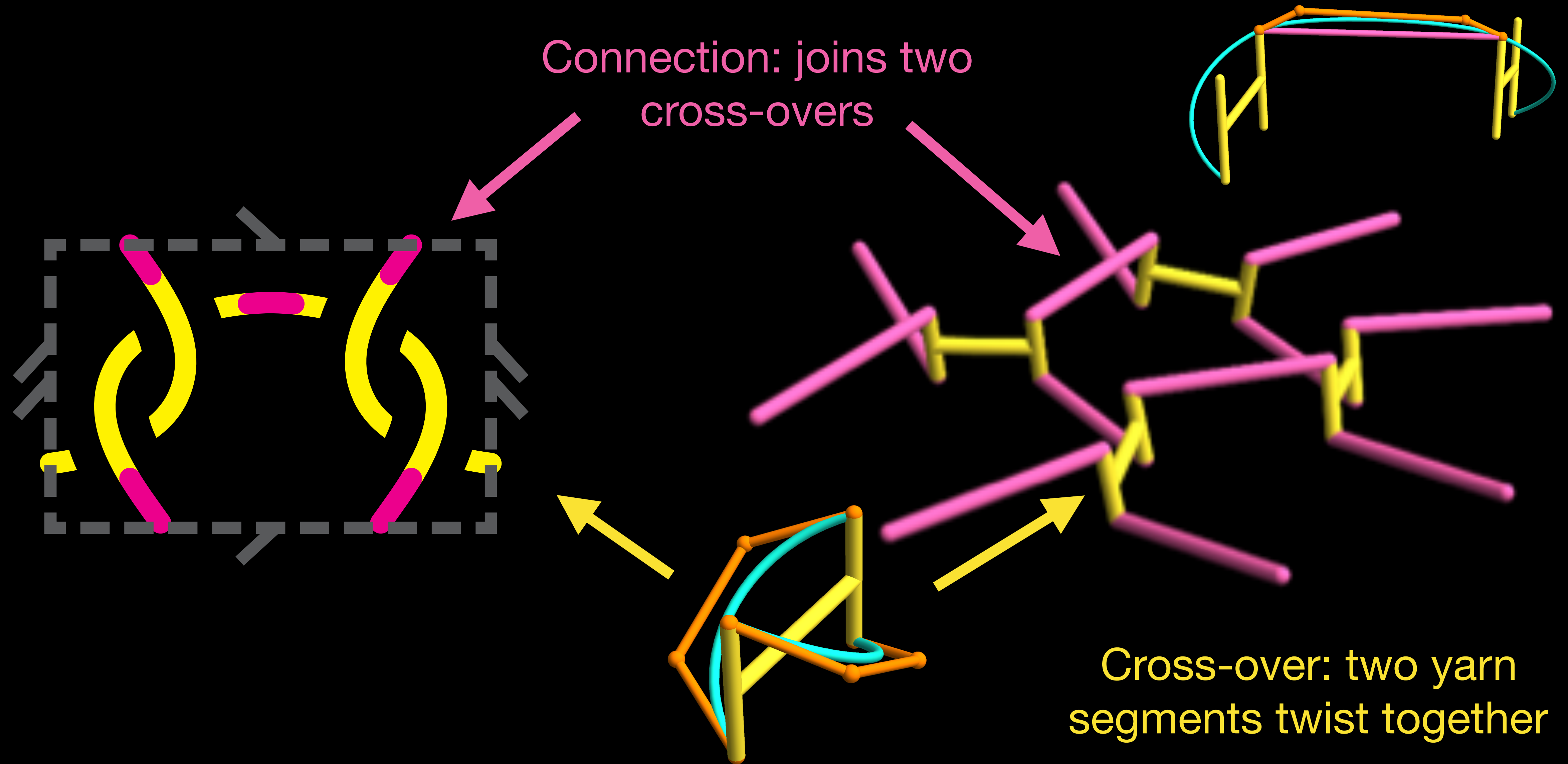
Knitting machine



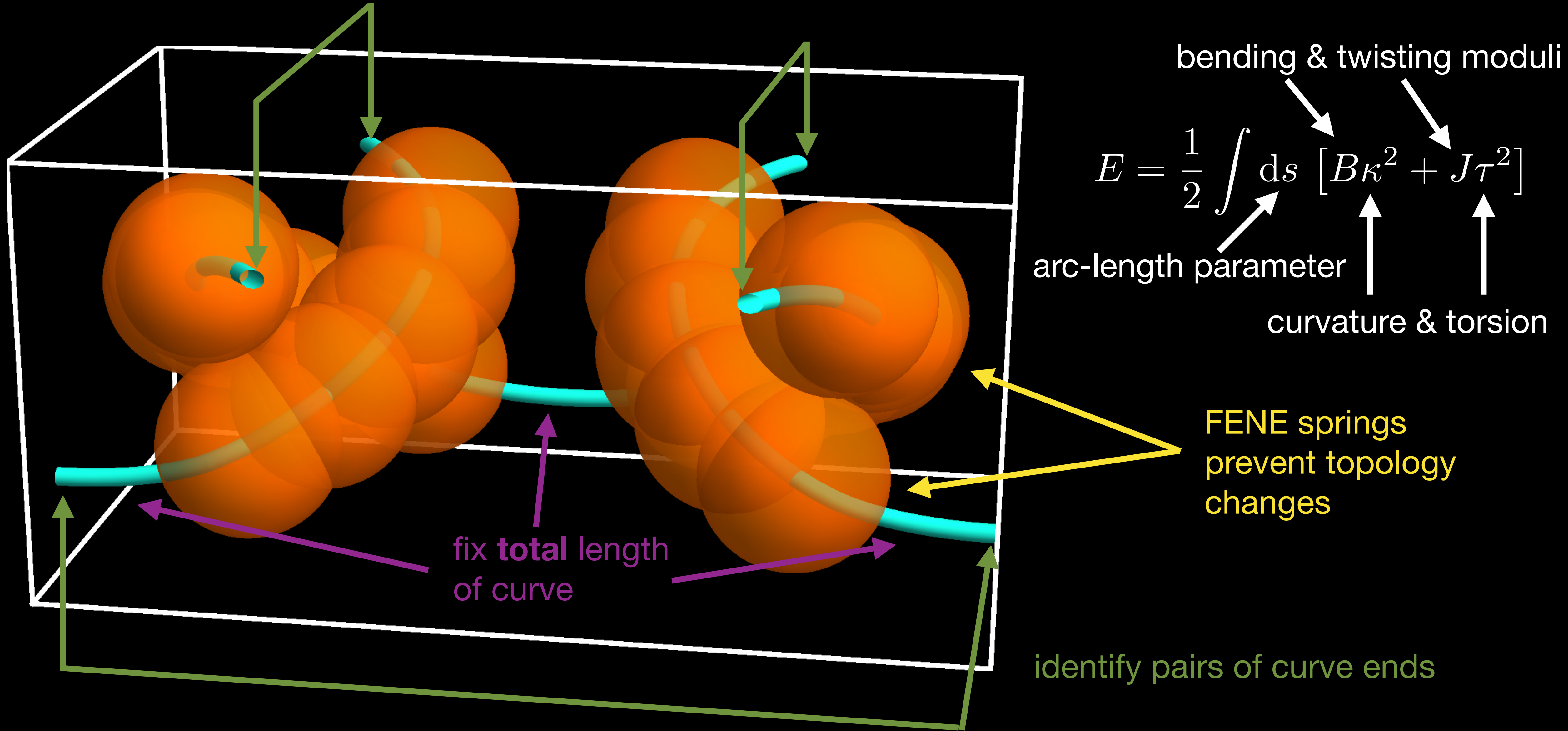
Experimental setup



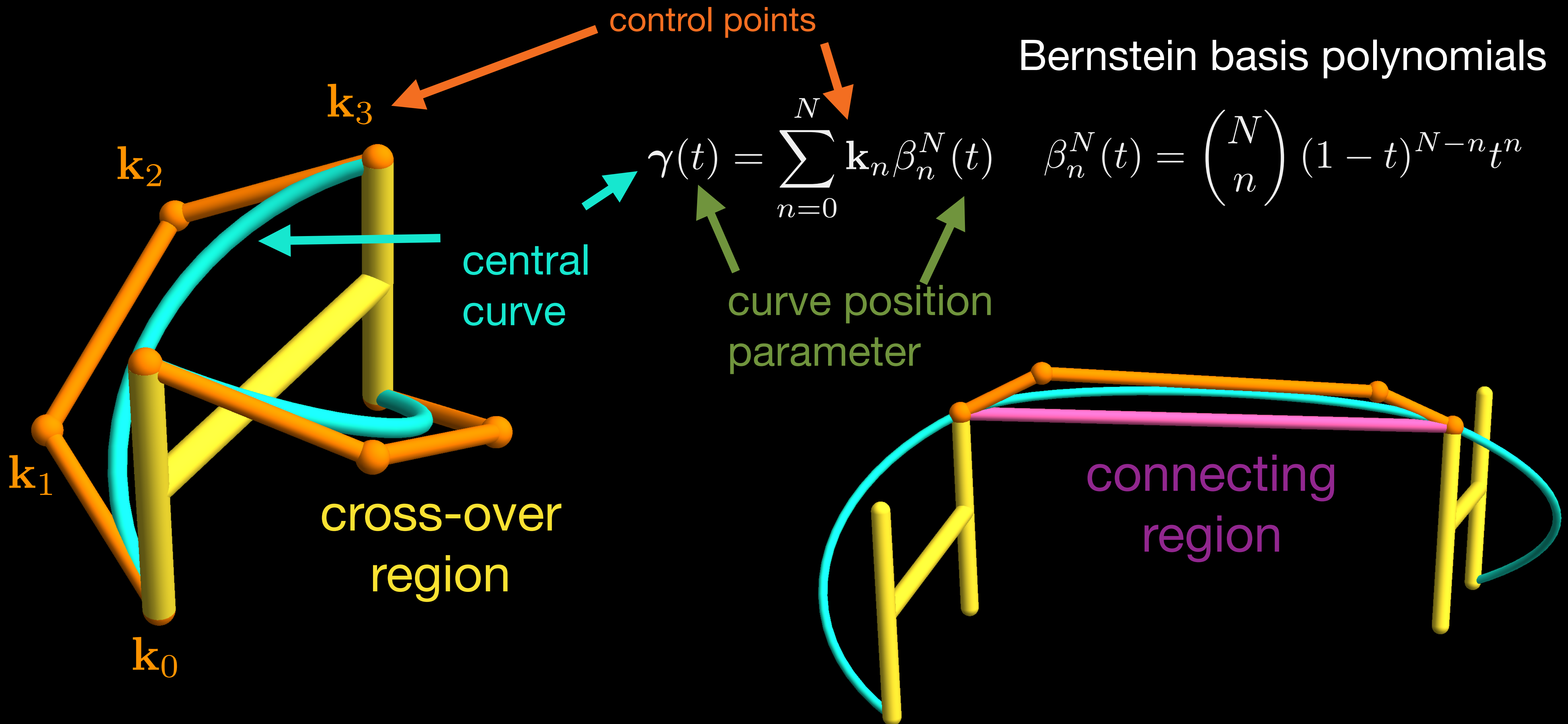
Lattice model geometry



Simple yarn model

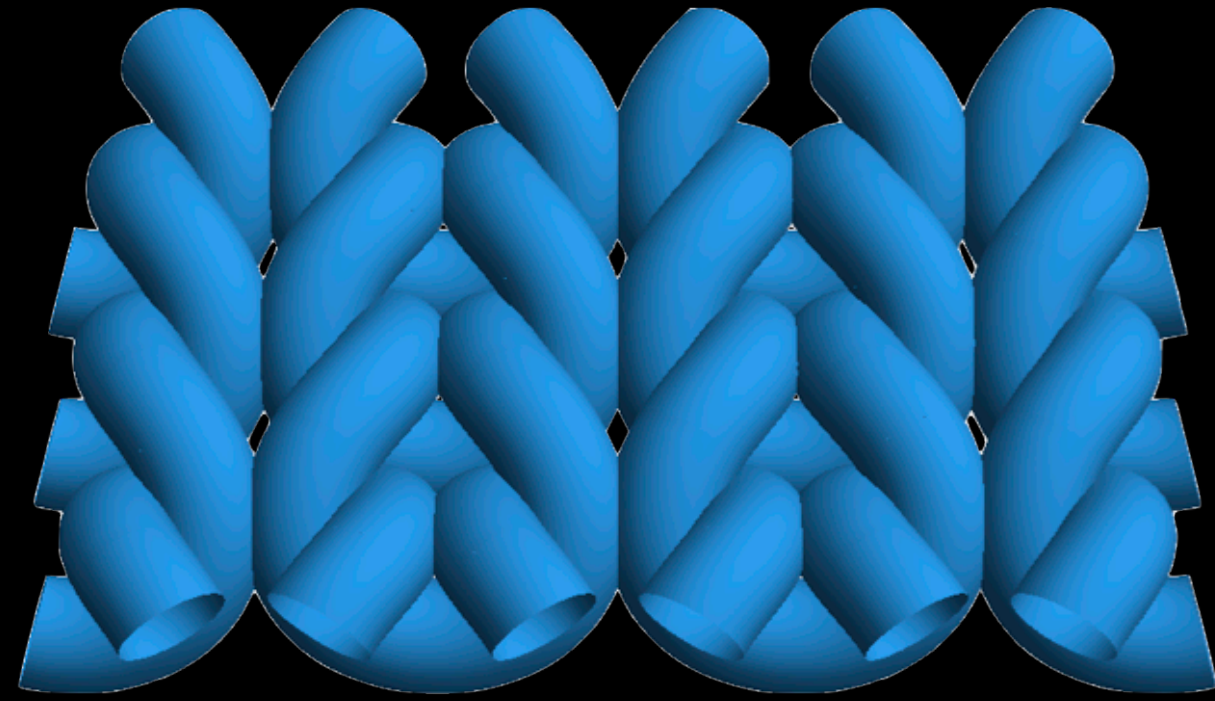


Bézier curves: expanding in a polynomial basis

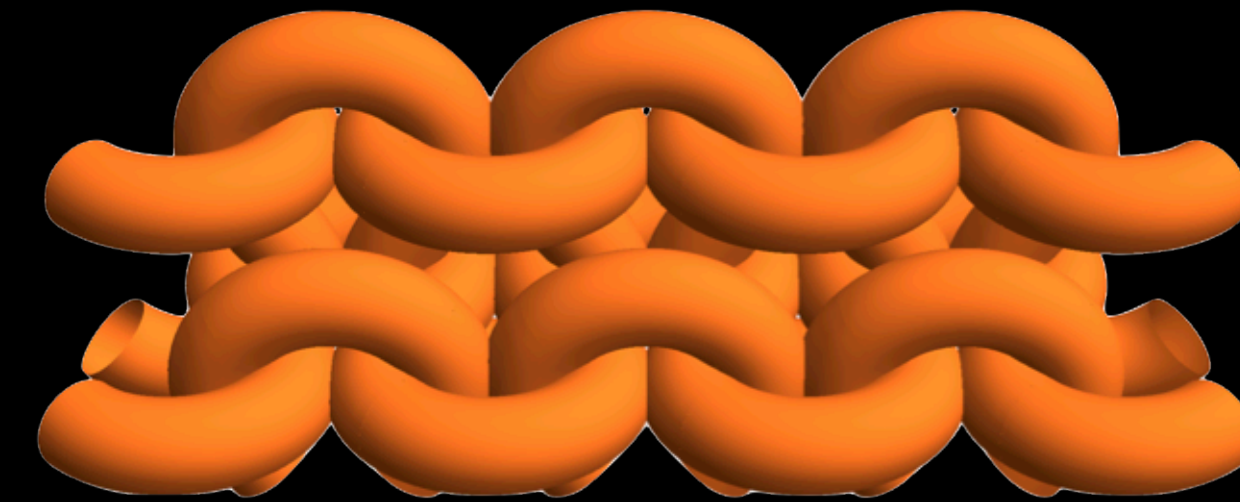


Minimal energy configurations

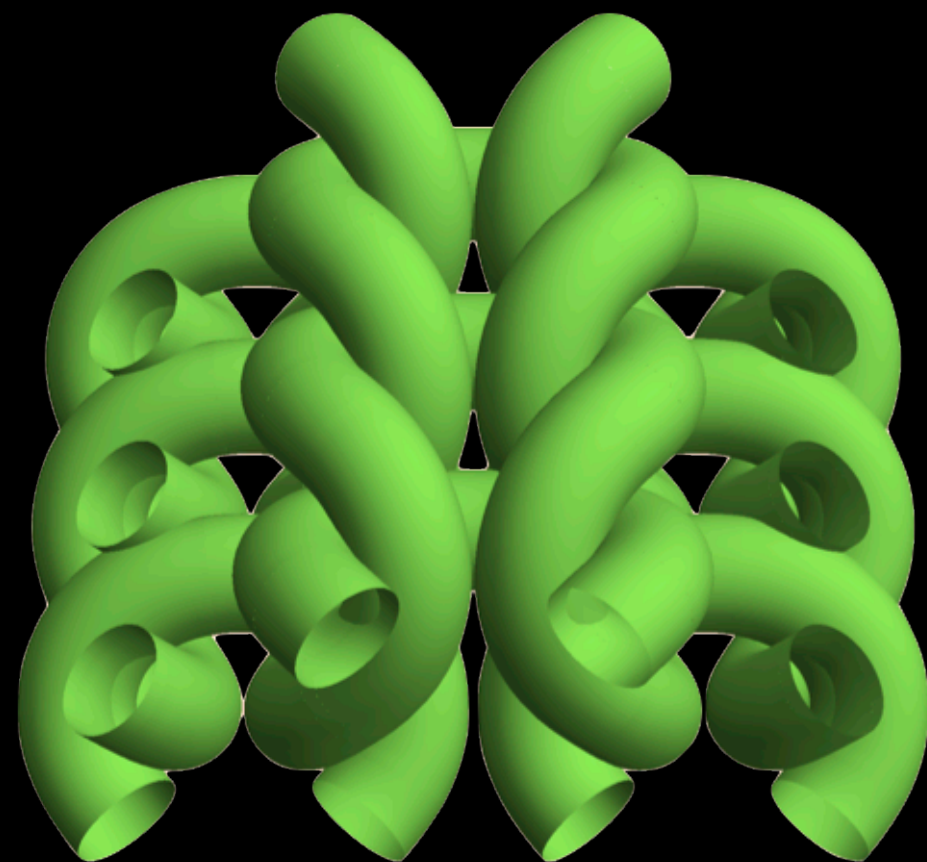
Stockinette



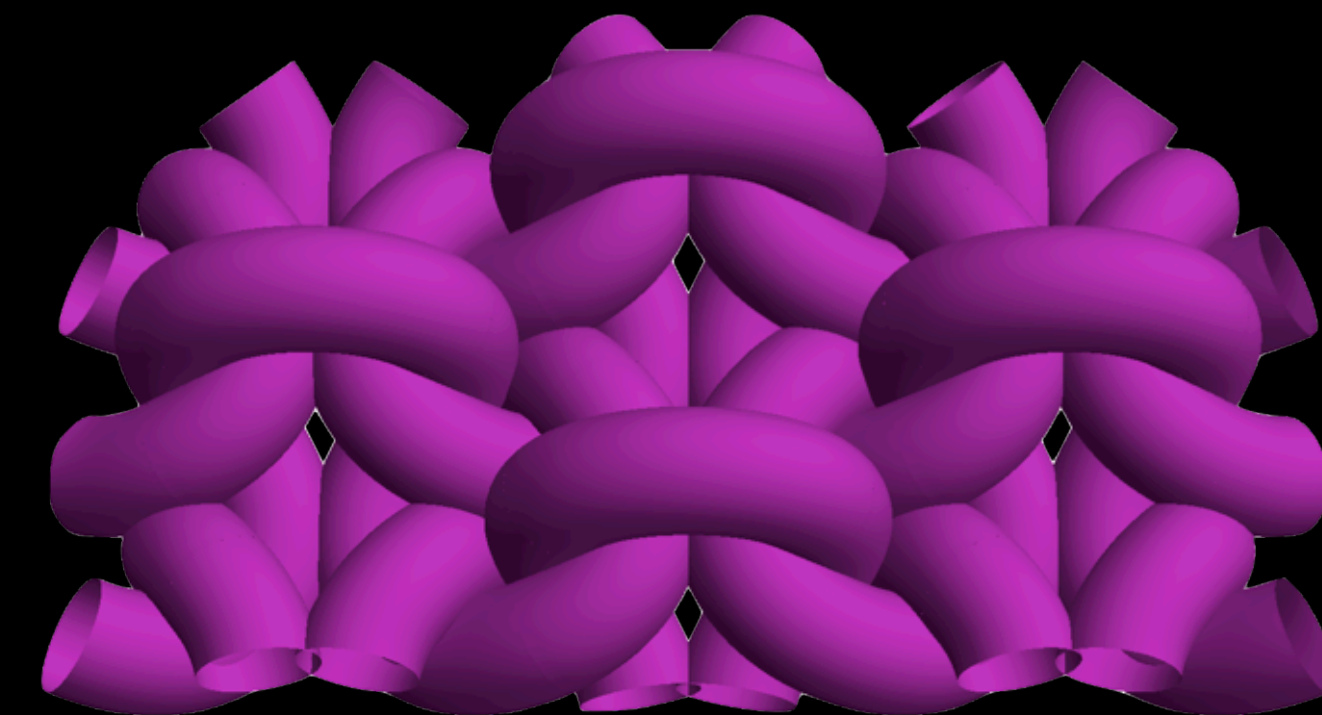
Garter



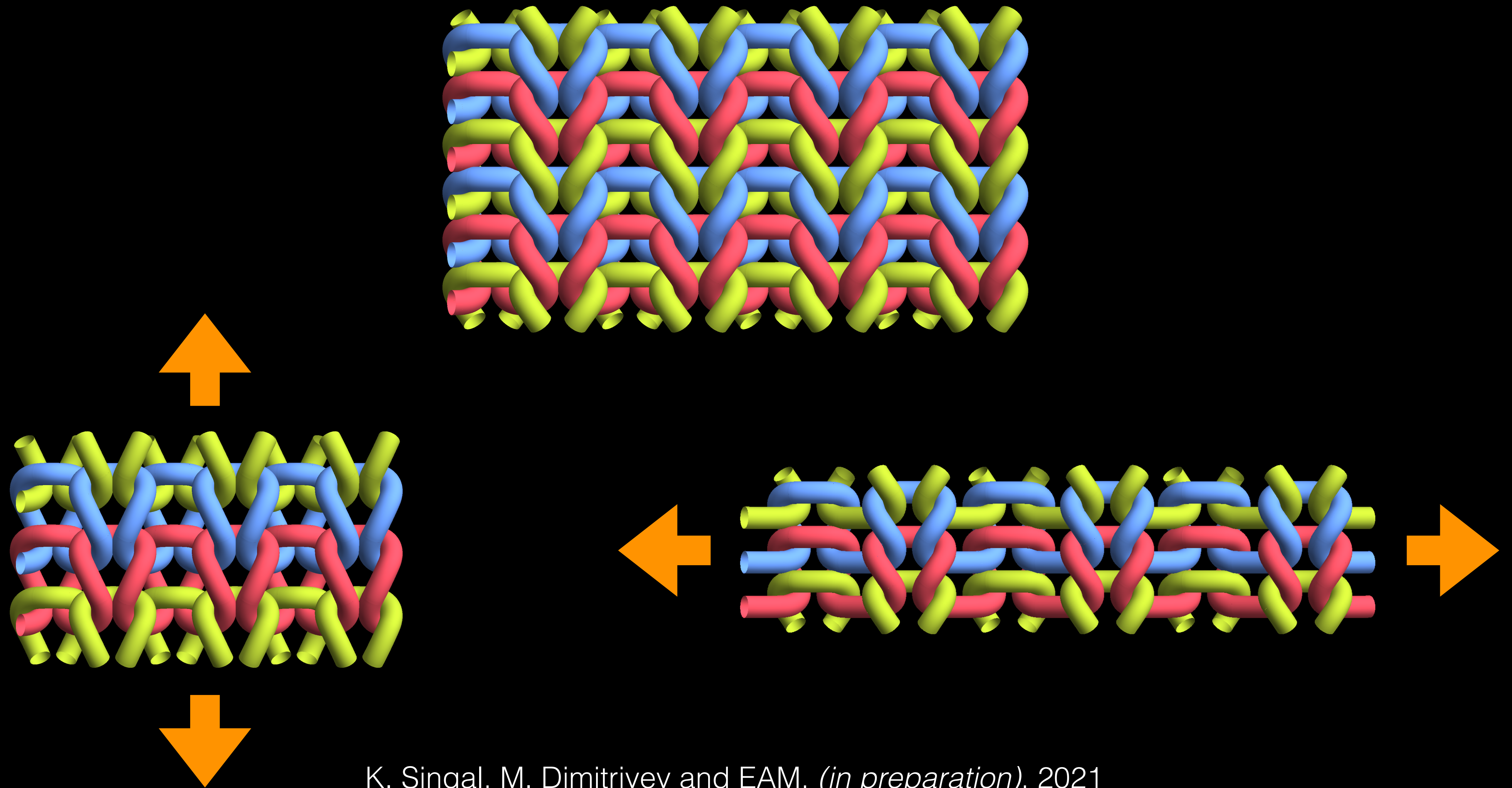
Rib



Seed

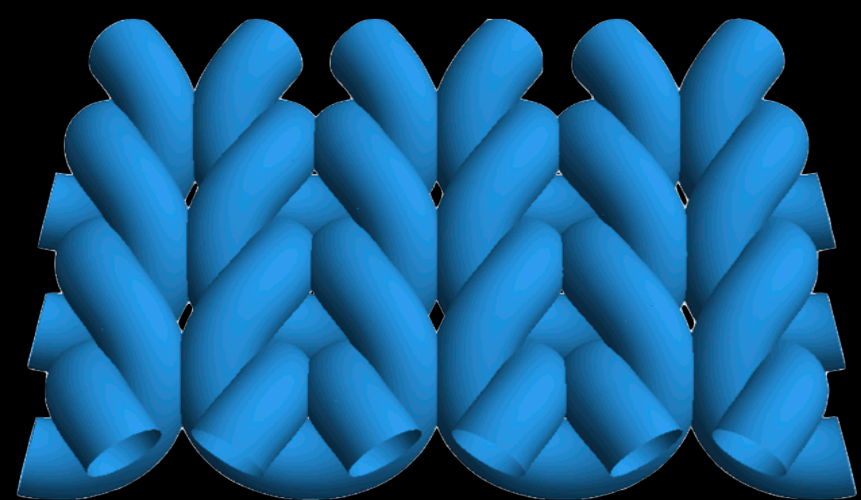


Simulation results: applying uniaxial forces



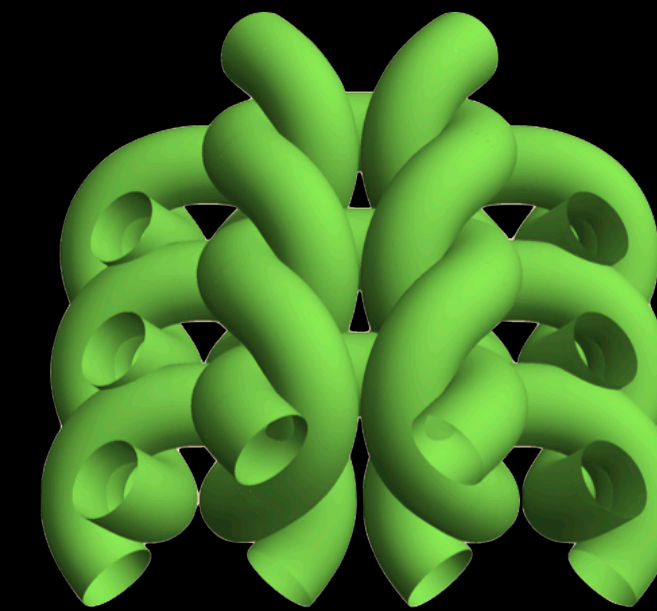
Stress-strain relations

Stockinette

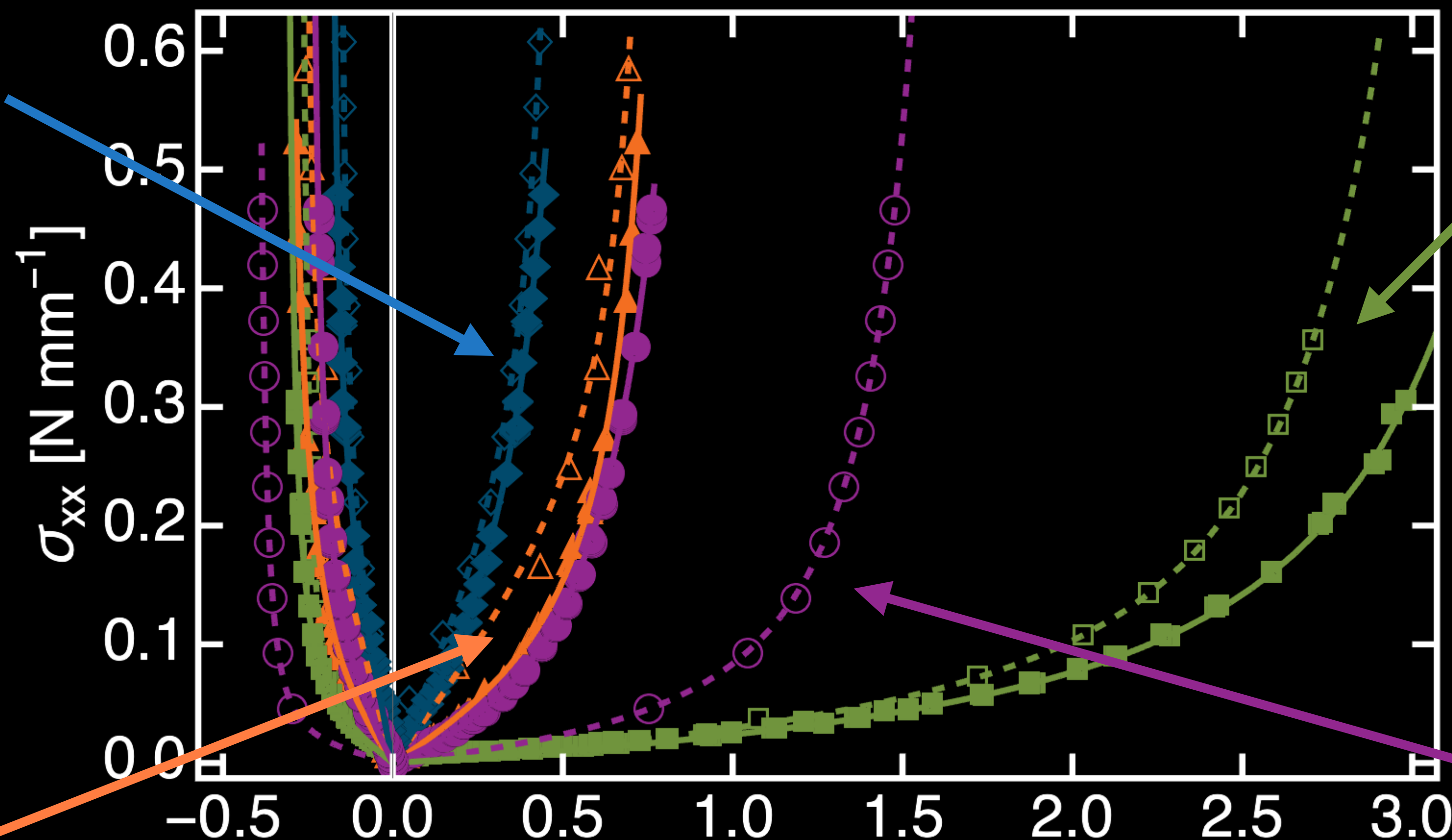
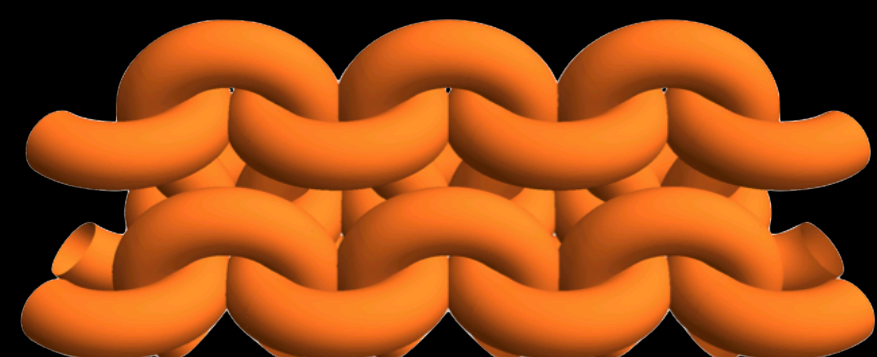


Force \longleftrightarrow

Rib



Garter

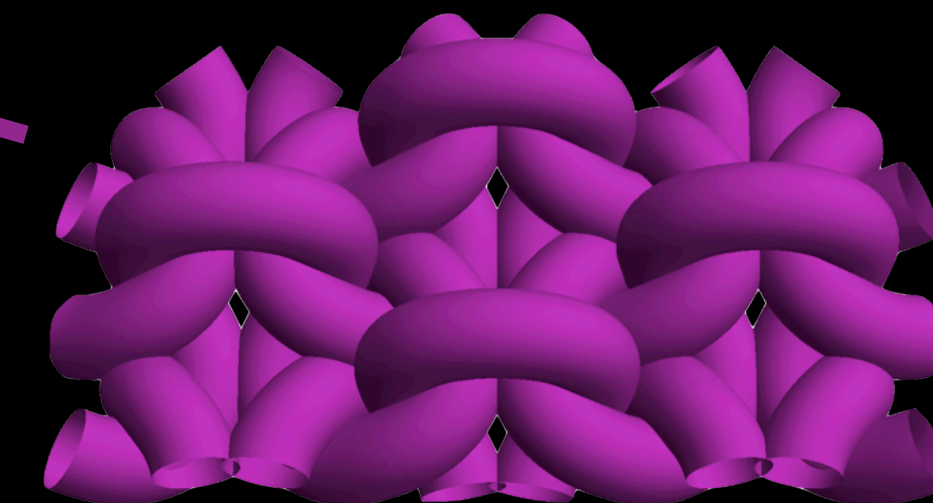


Response \updownarrow

ϵ_{xx}

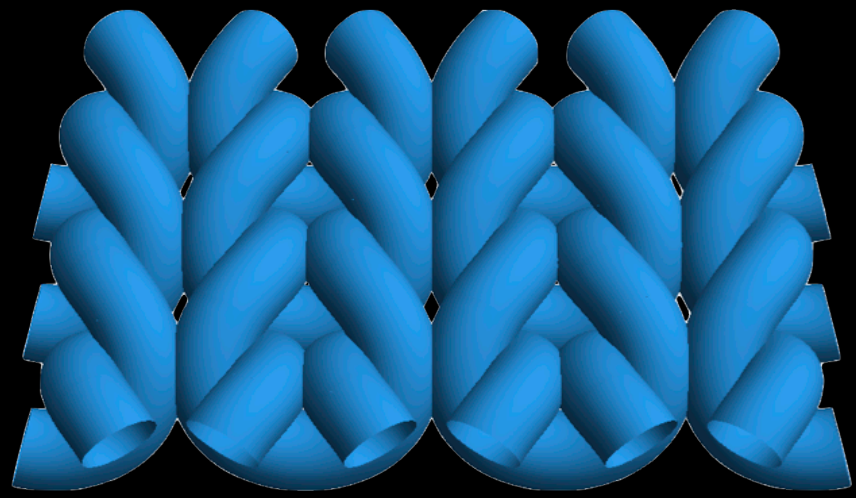
Response \longleftrightarrow

Seed



Stress-strain relations

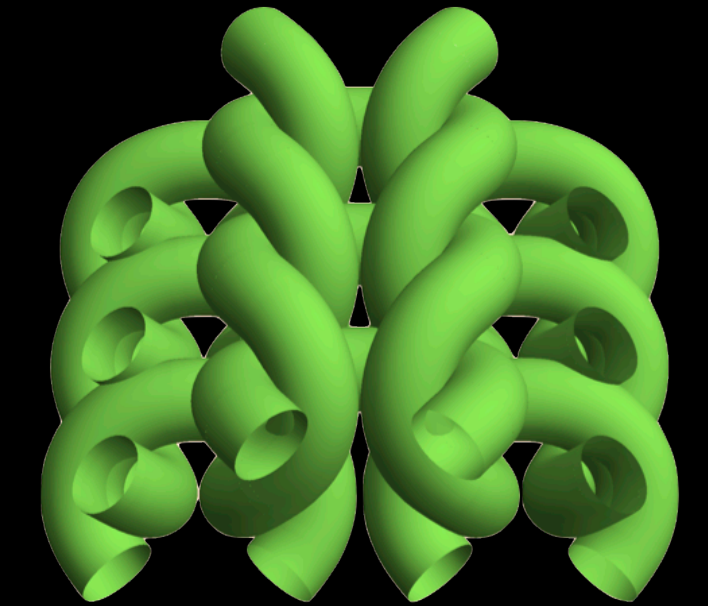
Stockinette



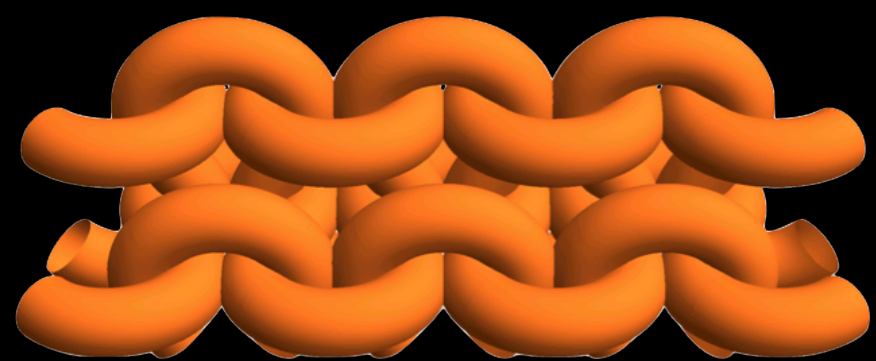
Force



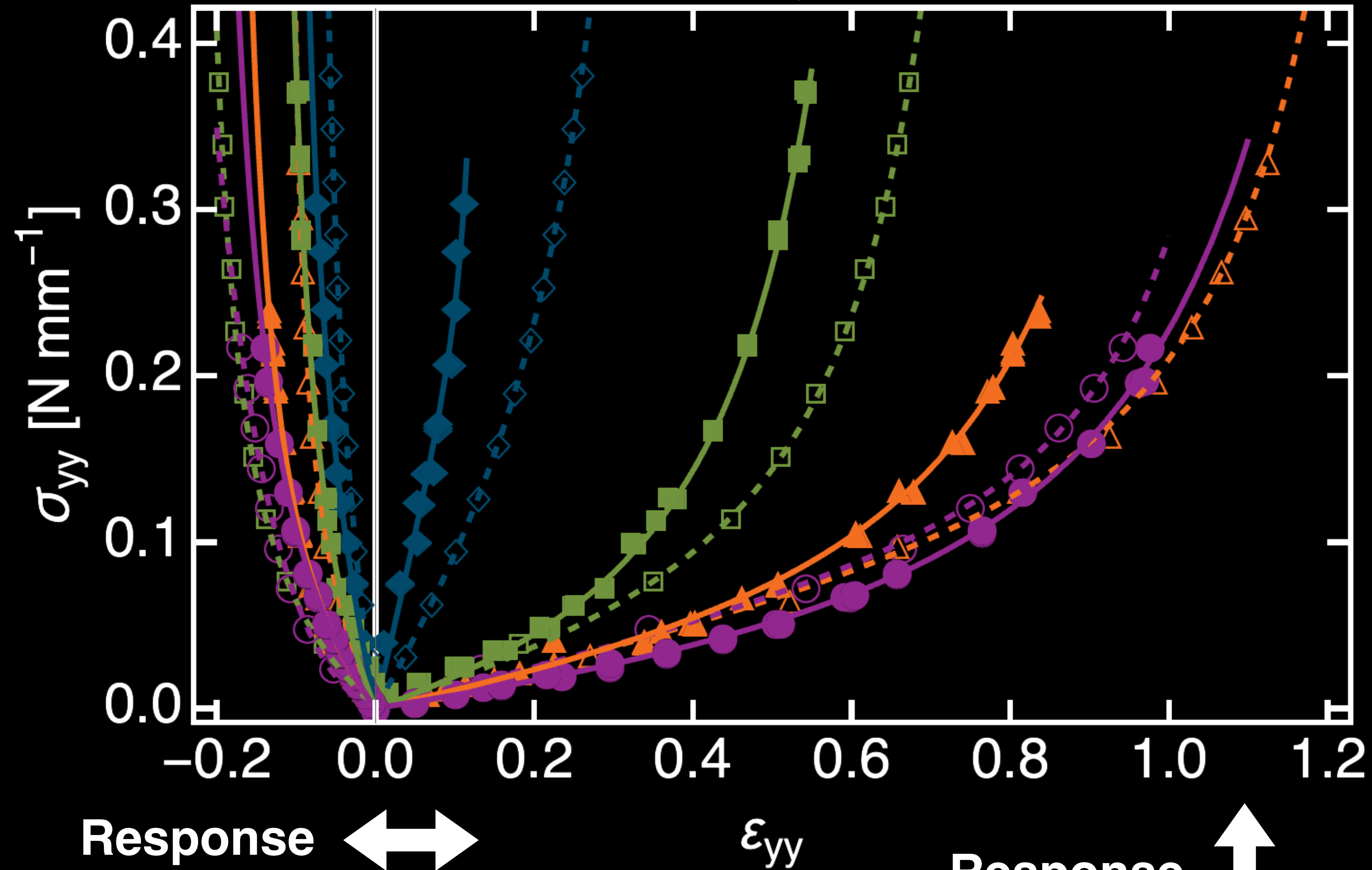
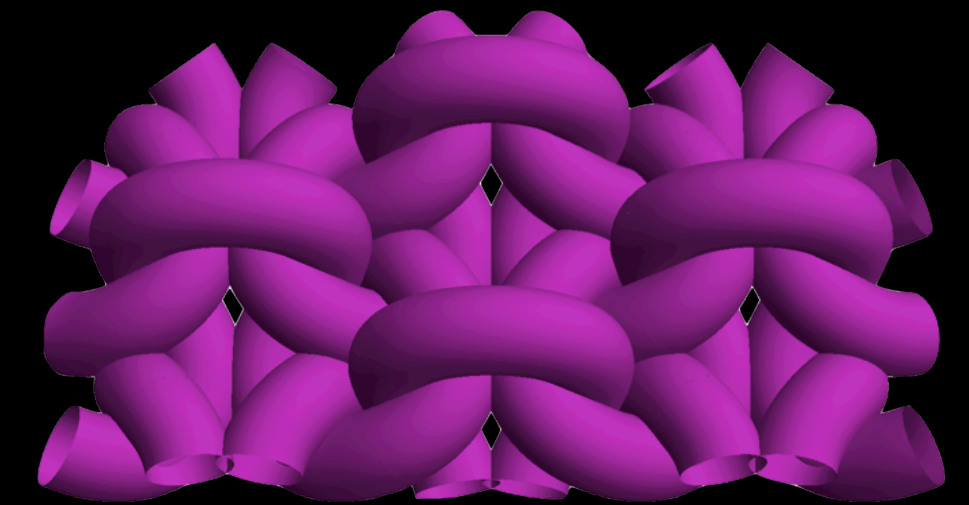
Rib



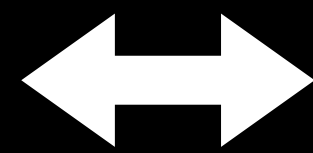
Garter



Seed



Response

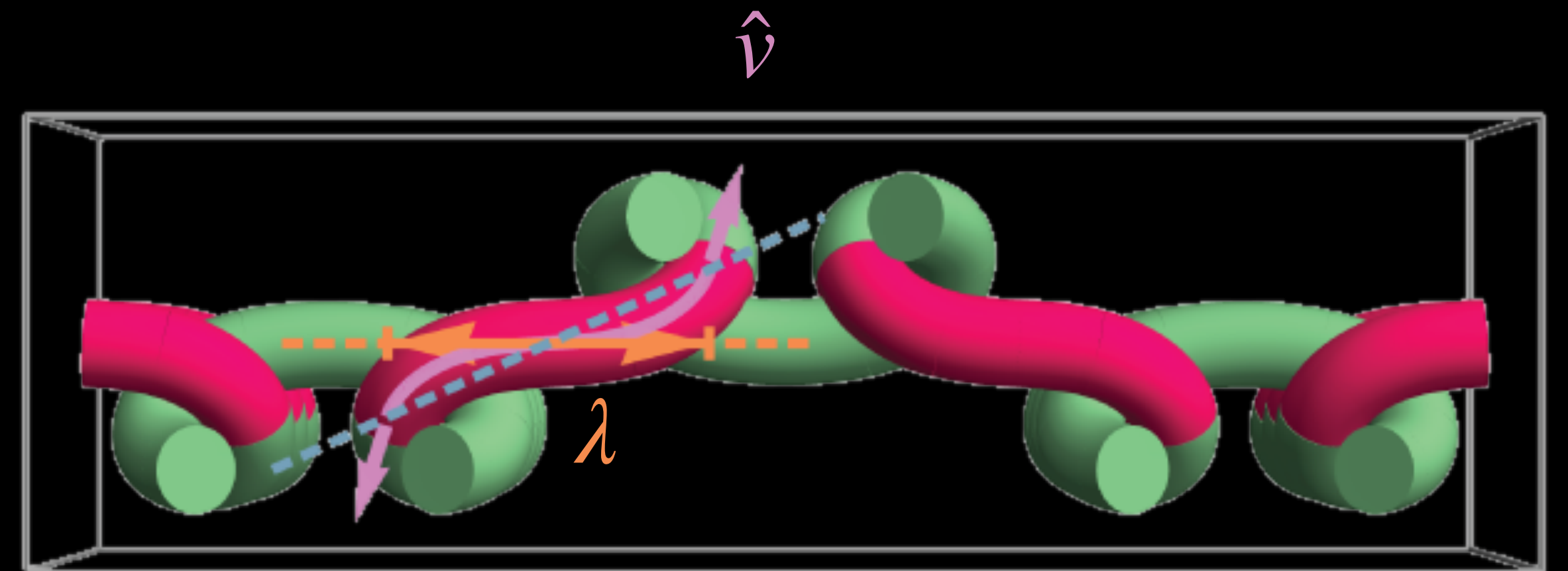
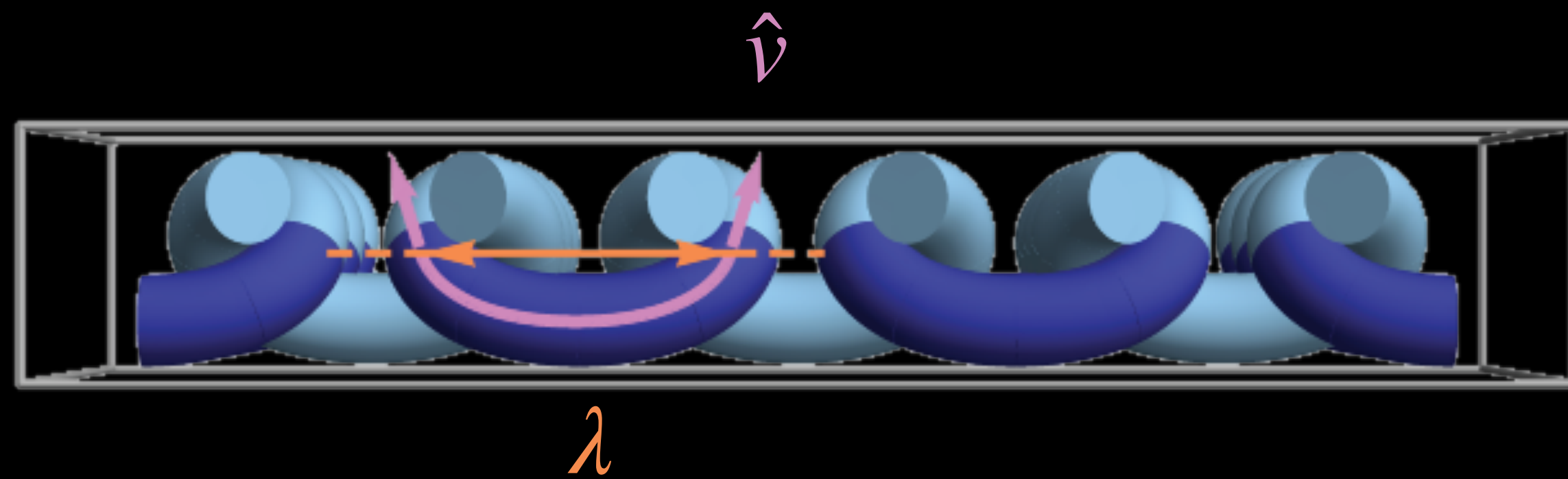
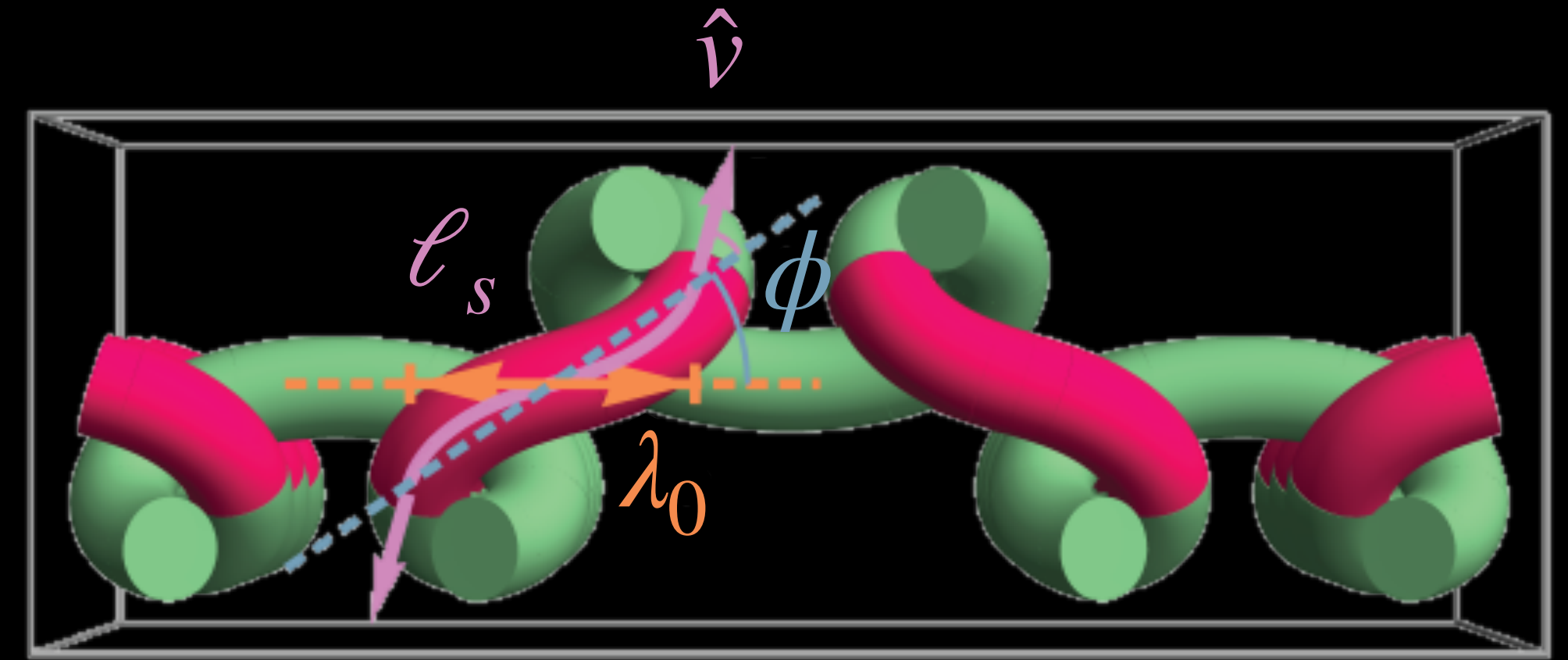
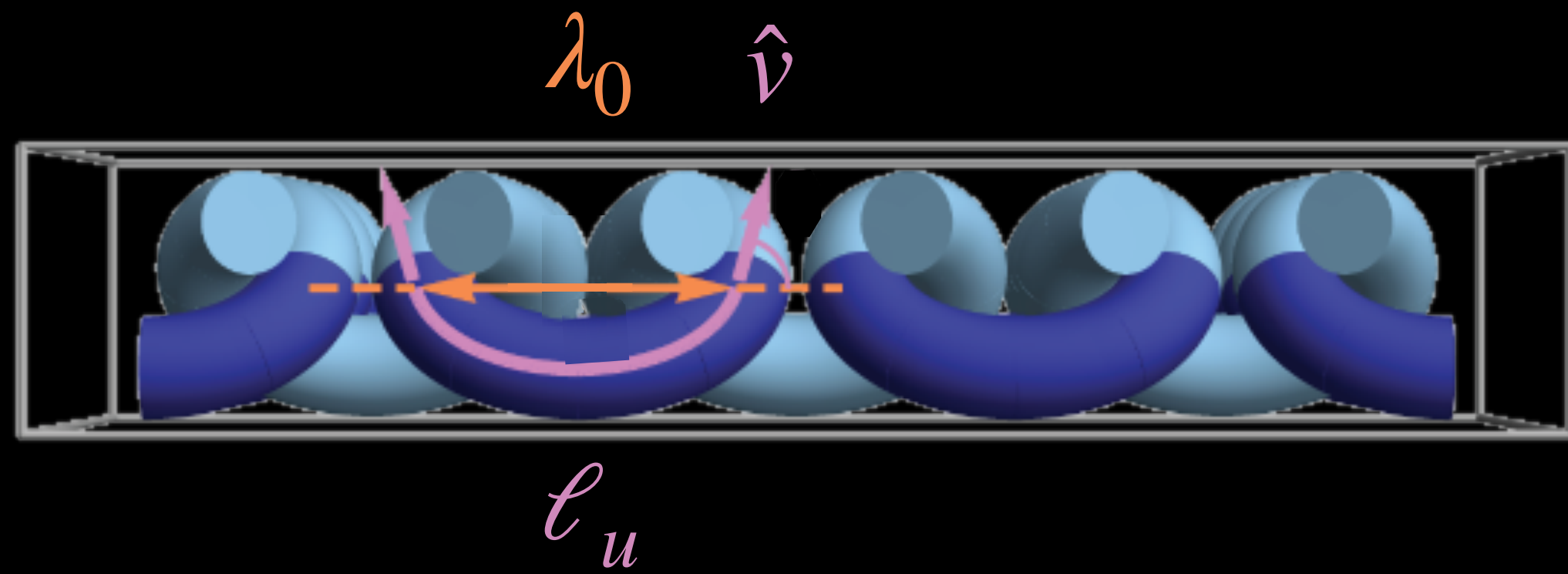


ϵ_{yy}

Response



Topology, symmetry & low strain behavior

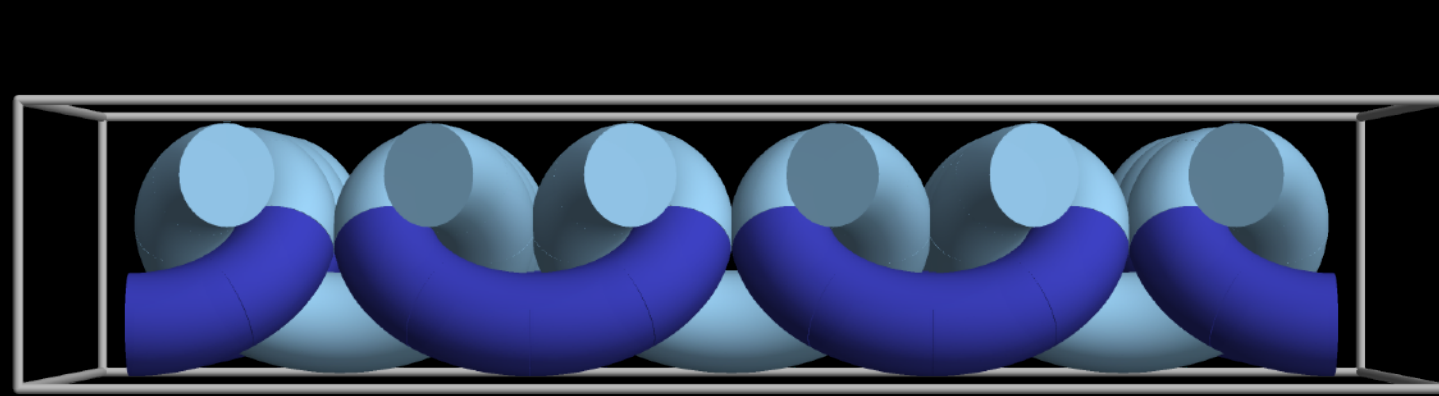


$$k_u \sim \frac{180B}{\lambda_0^3 v_z^2} \left(1 + \mathcal{O}(\Delta\lambda^2) \right)$$

$$k_s \sim \frac{12B}{\lambda_0^3 v_z^2} \left(1 + v_x \cot \phi + \mathcal{O}(\Delta\lambda^2) \right)$$

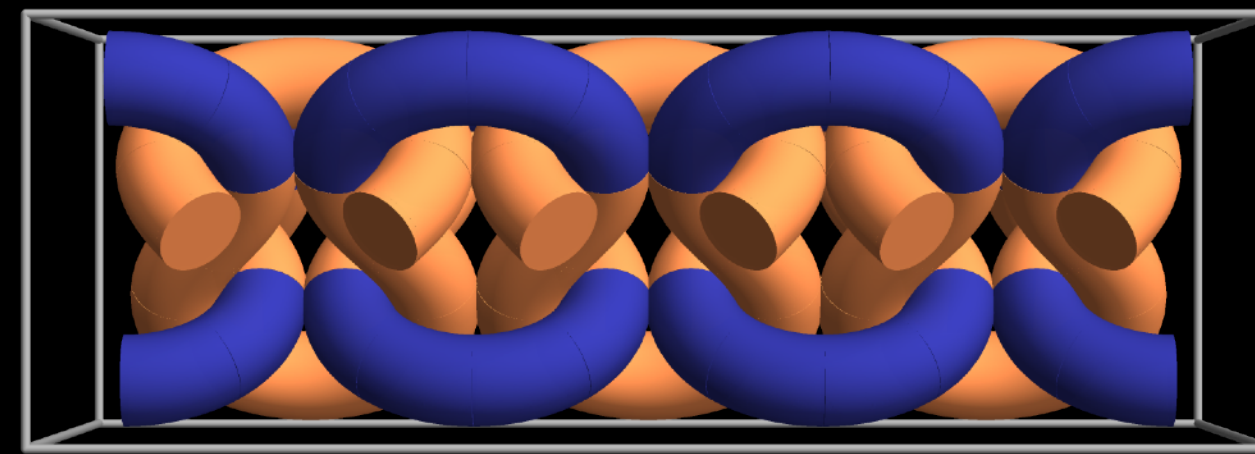
Topology, symmetry & low strain behavior

Stockinette

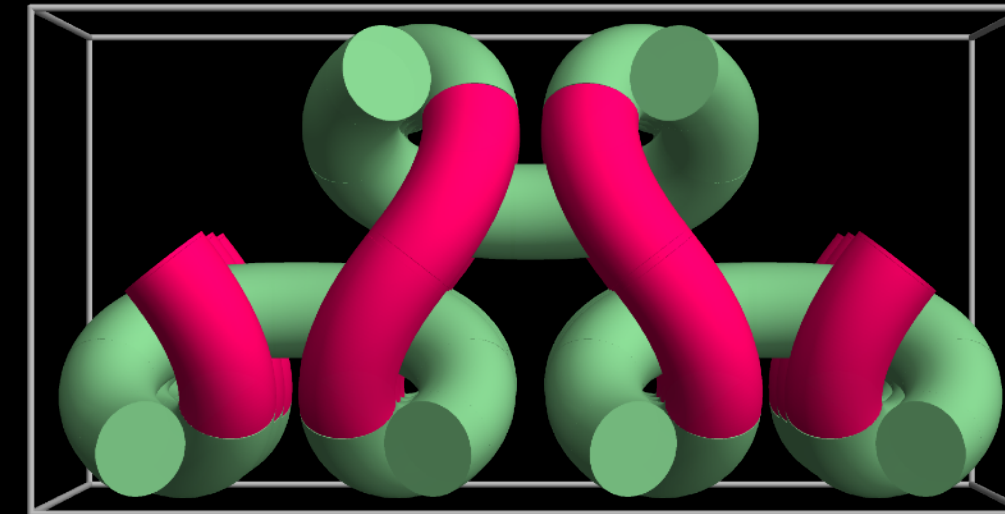


ϵ_{xx}

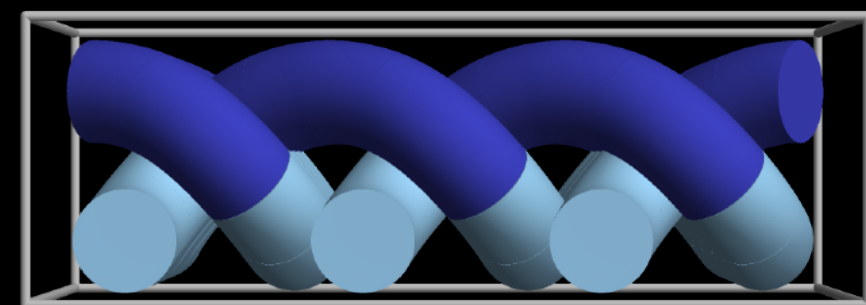
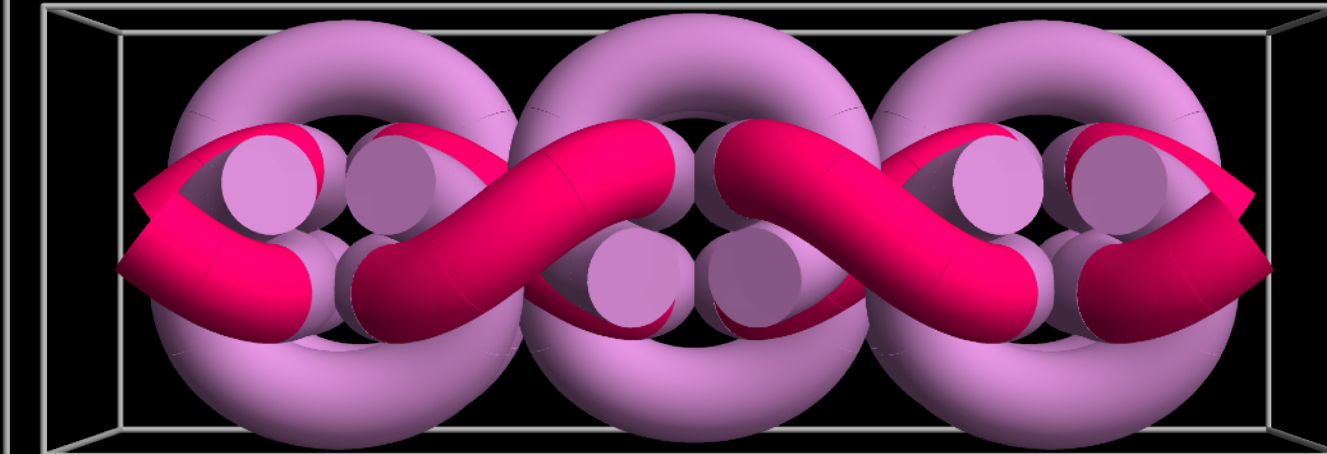
Garter



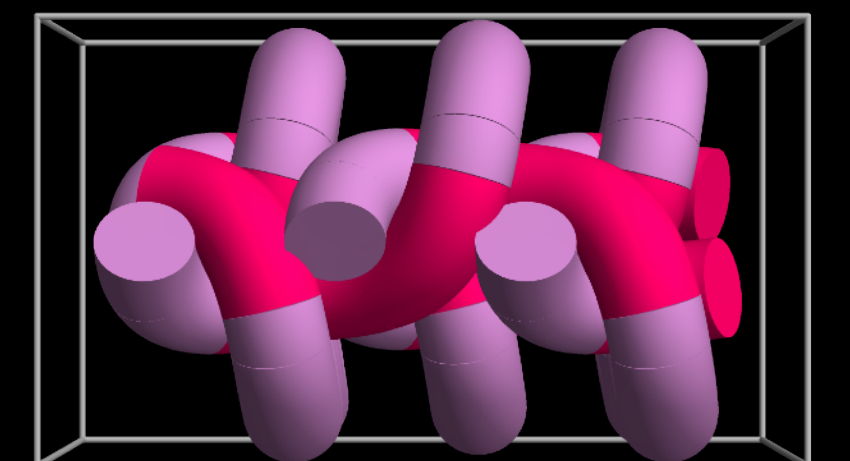
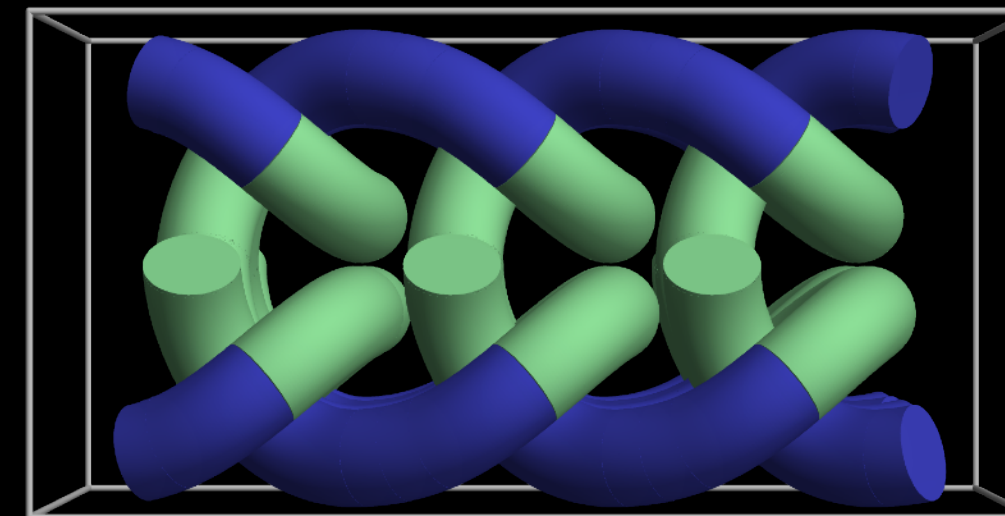
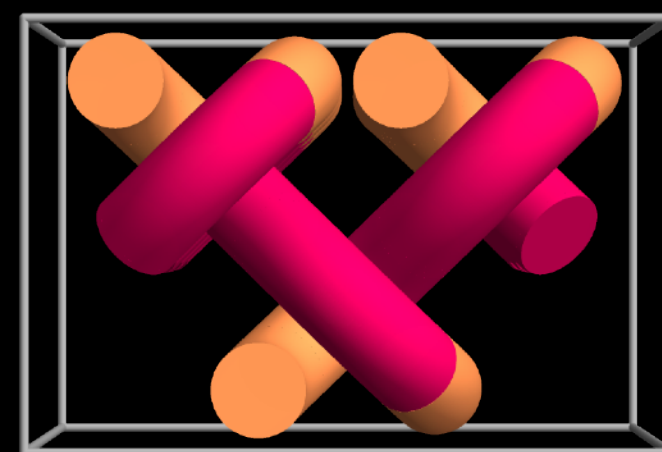
Rib



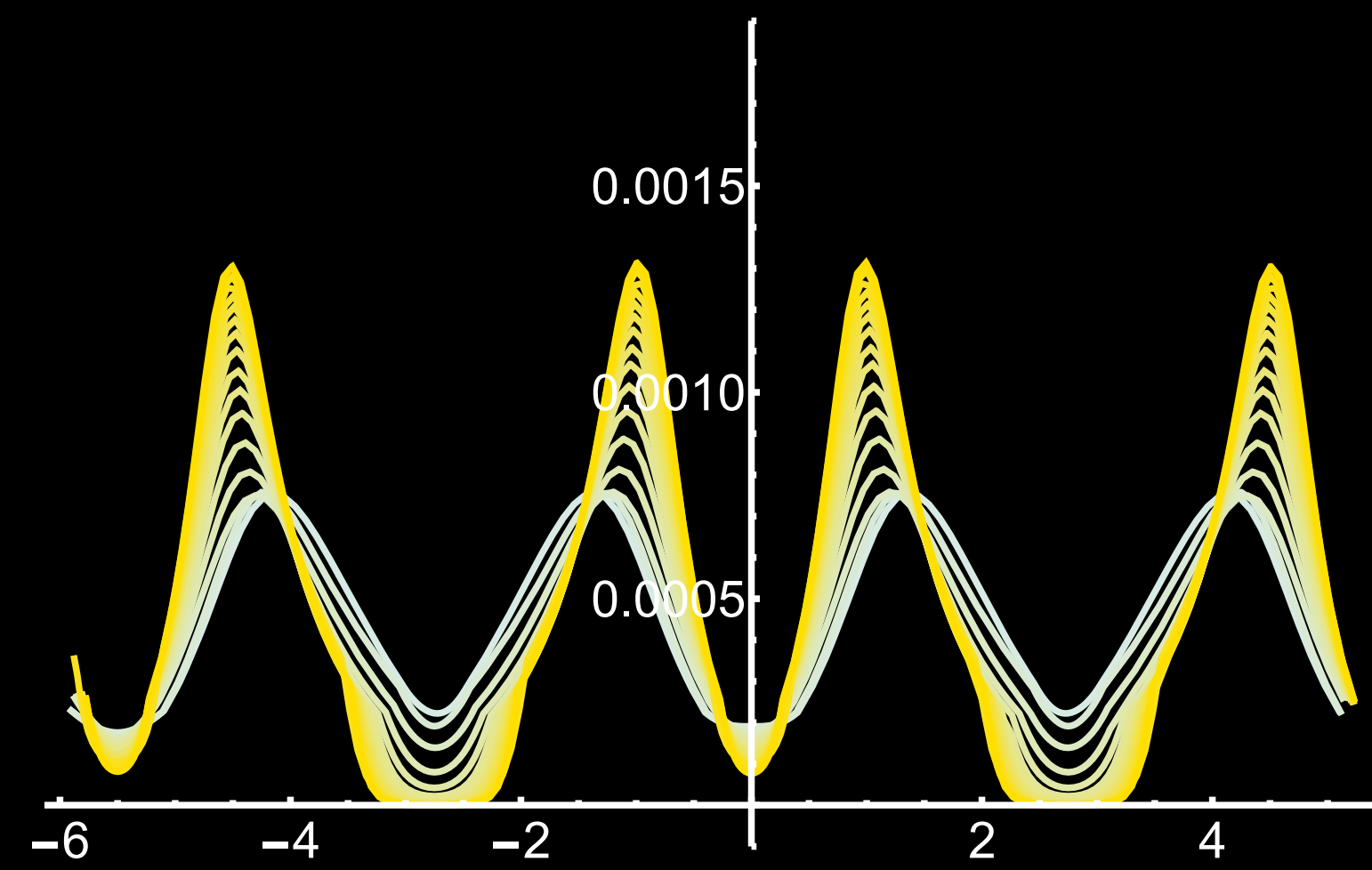
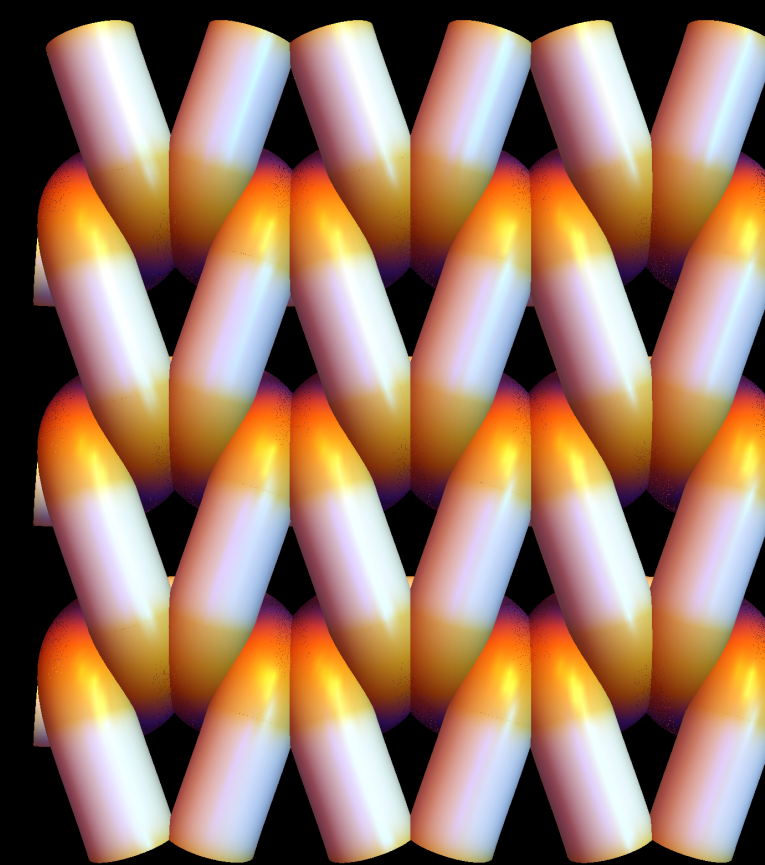
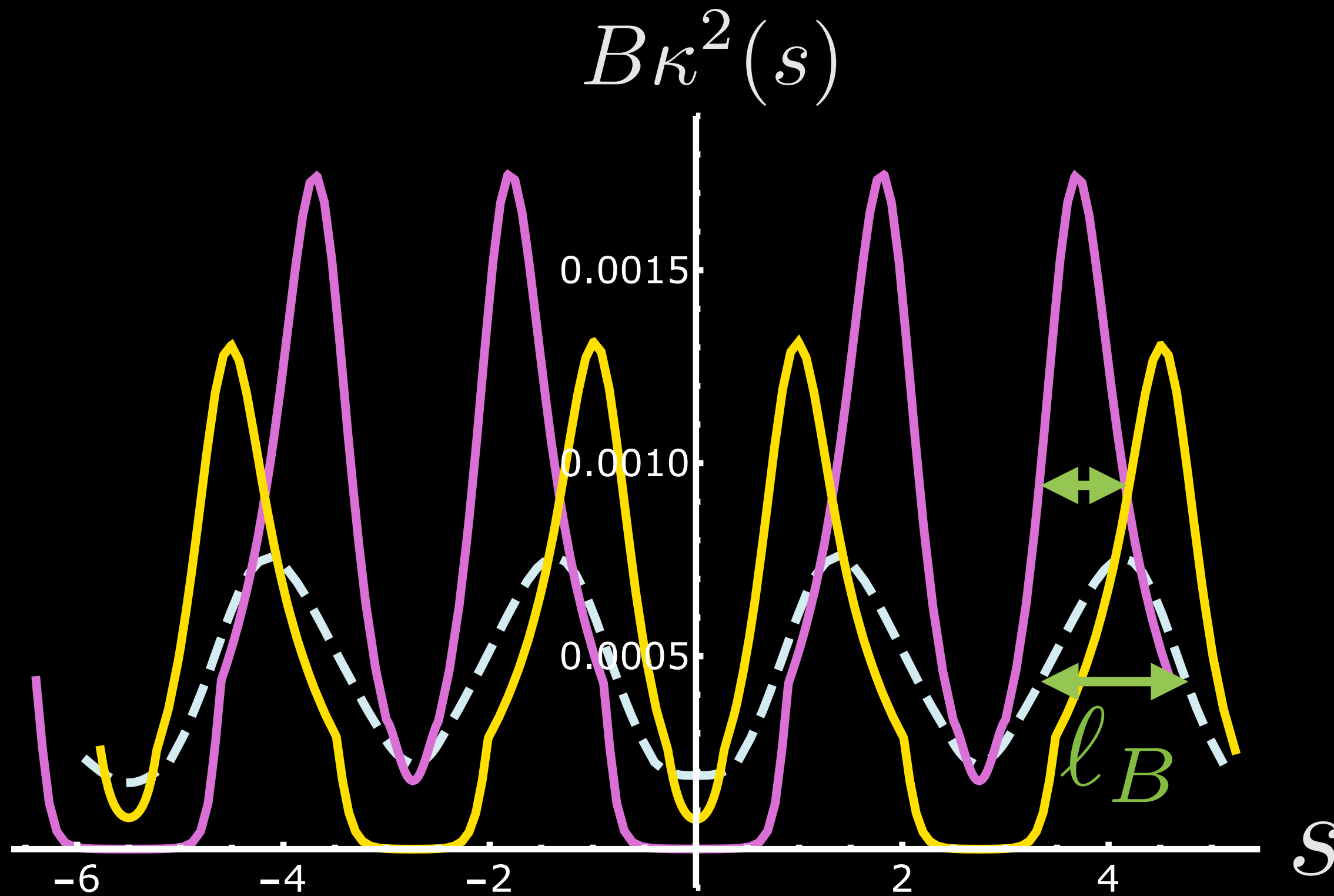
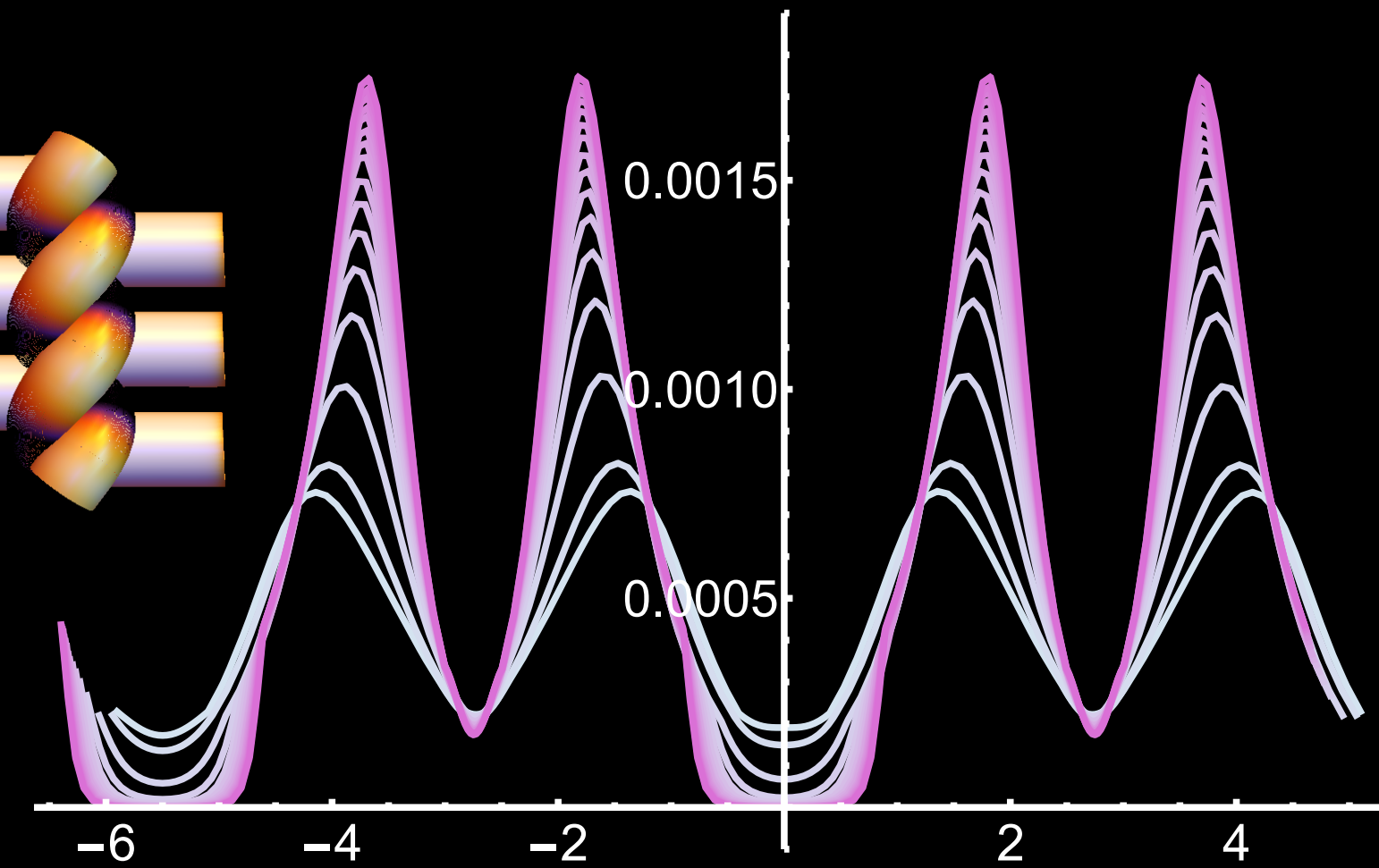
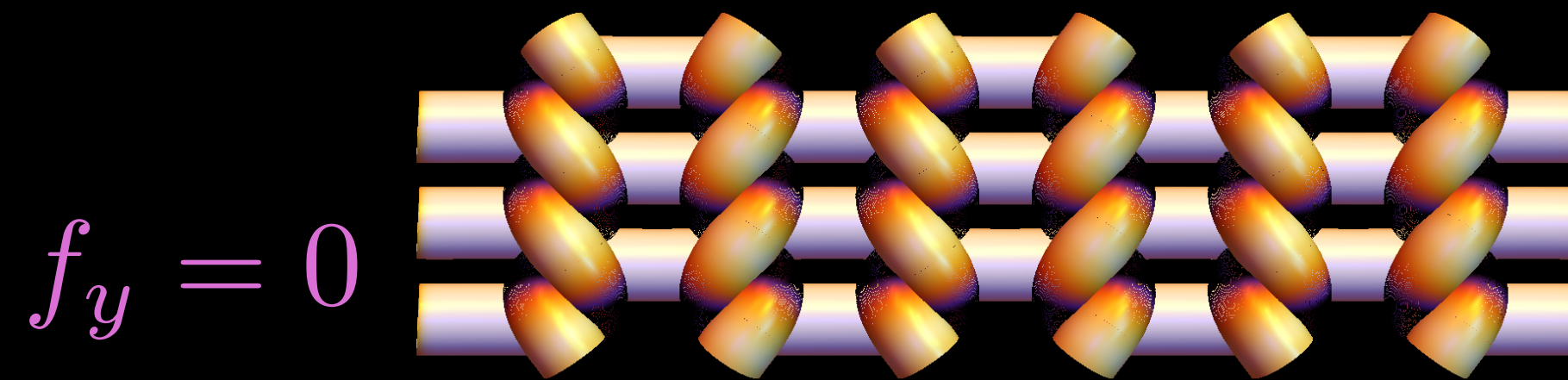
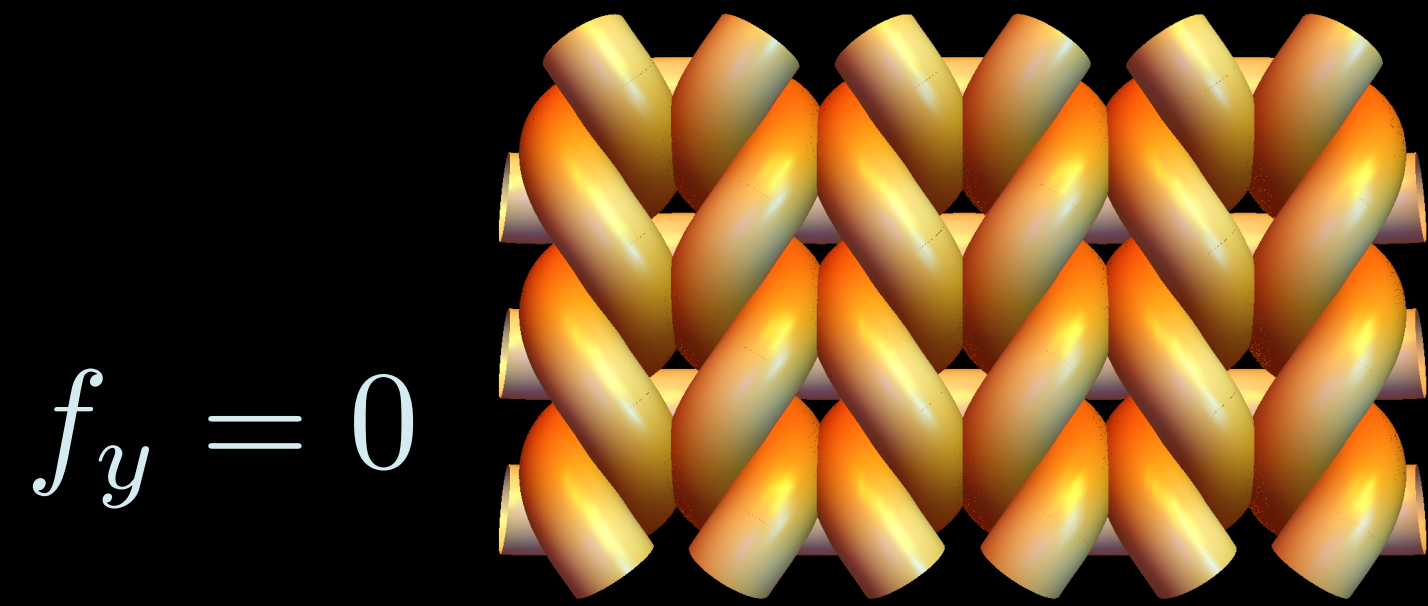
Seed



ϵ_{yy}

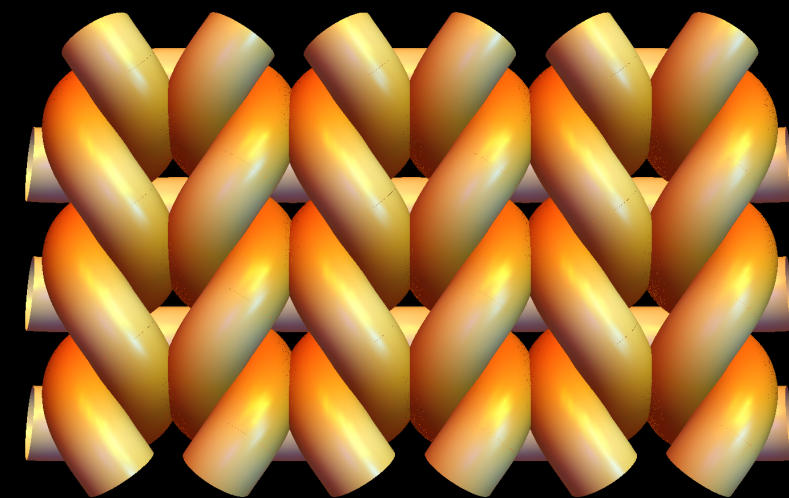


Curvature concentration

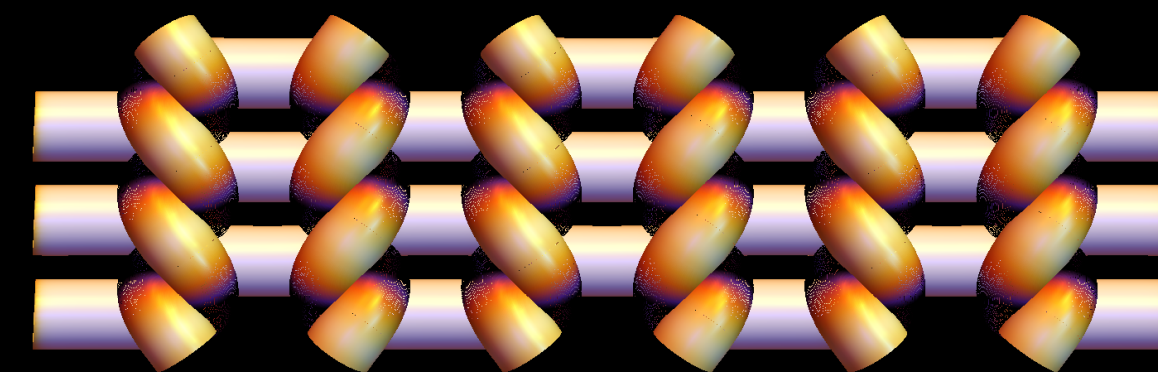
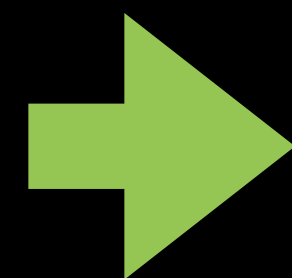


Curvature concentration & nonlinear response

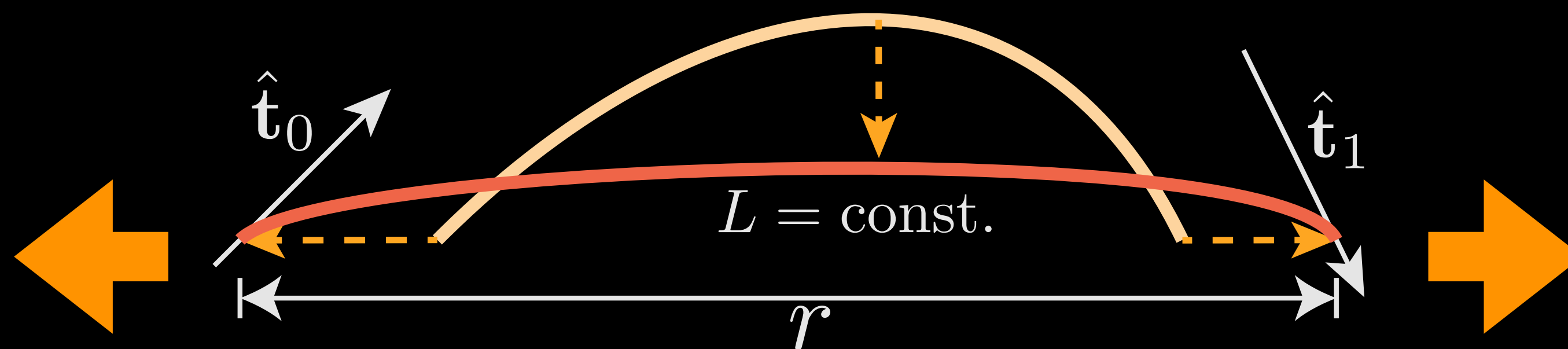
$$E = \frac{1}{2} B \int_0^L ds \kappa^2 + T(L - L_0) \quad \ell_B \equiv \sqrt{\frac{B}{t}}$$



Low tension T
Long bending length ℓ_B



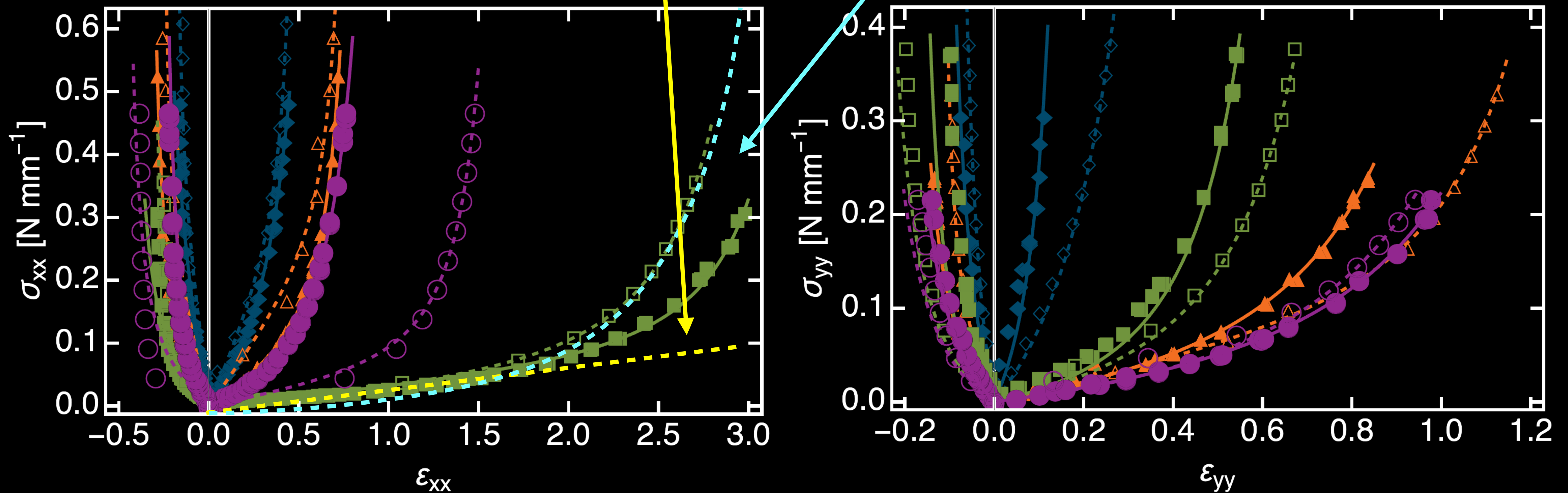
High tension T
Short bending length ℓ_B



$$E(r) \sim \frac{B/L}{1 - r/L}$$

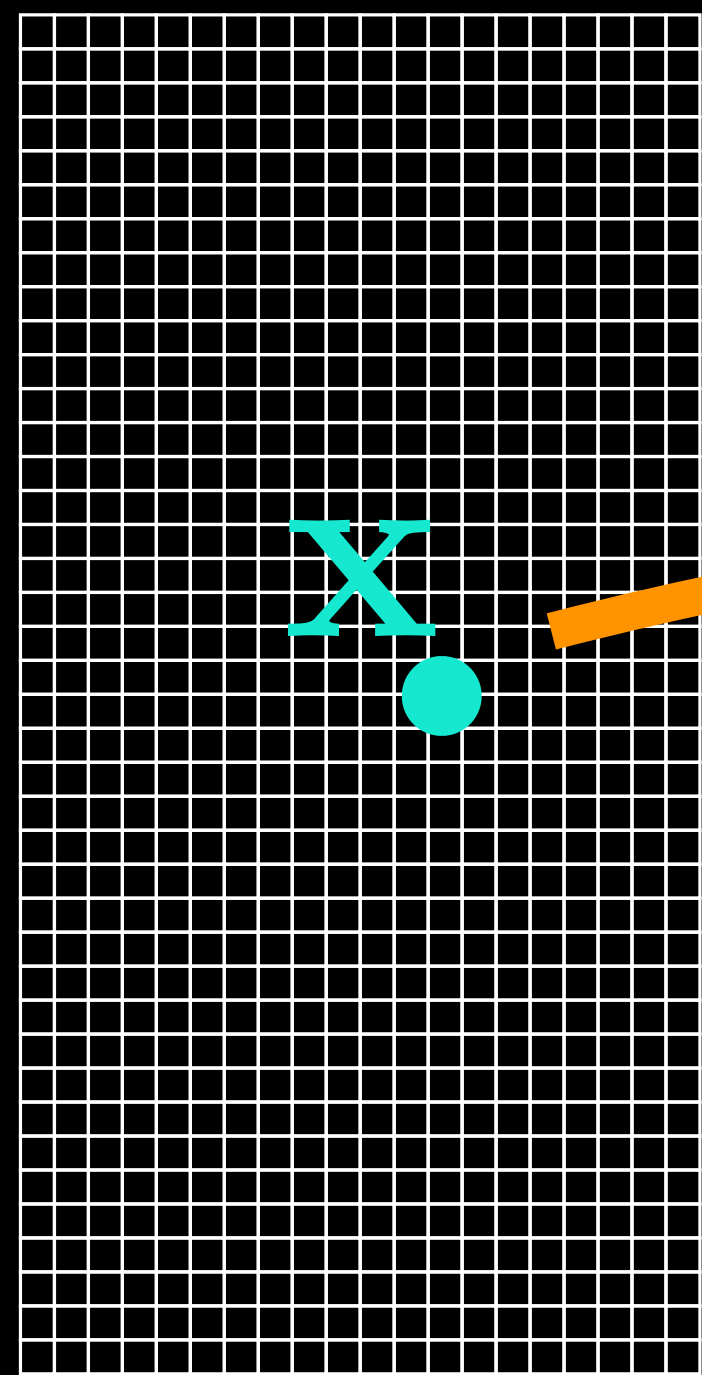
High tension limit & elastic nonlinearity

$$\sigma_i(\varepsilon_i) = k_i \varepsilon_i + \beta_i \left[\frac{1}{(1 - \alpha_i \varepsilon_i)^2} - 1 \right]$$

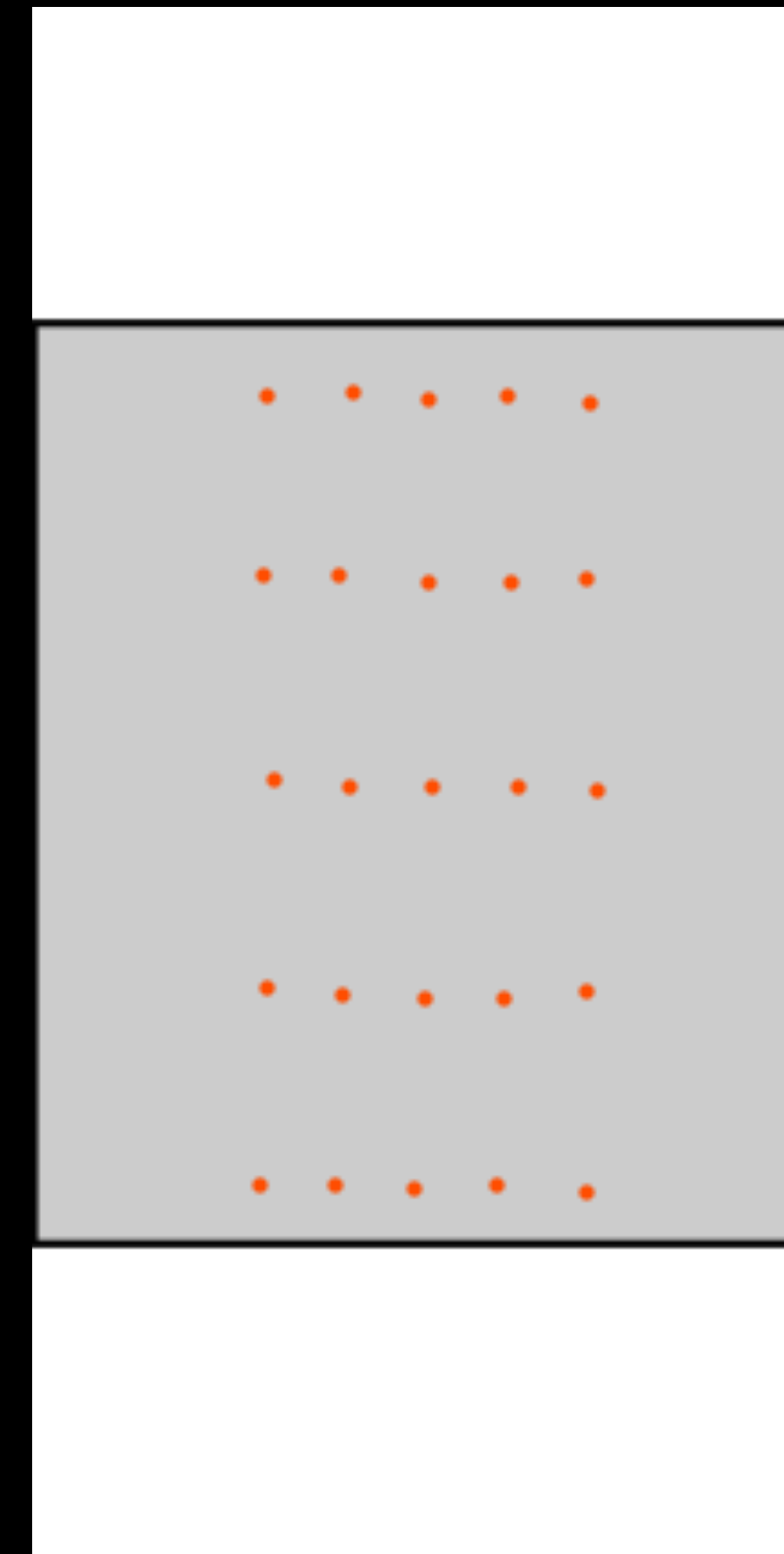
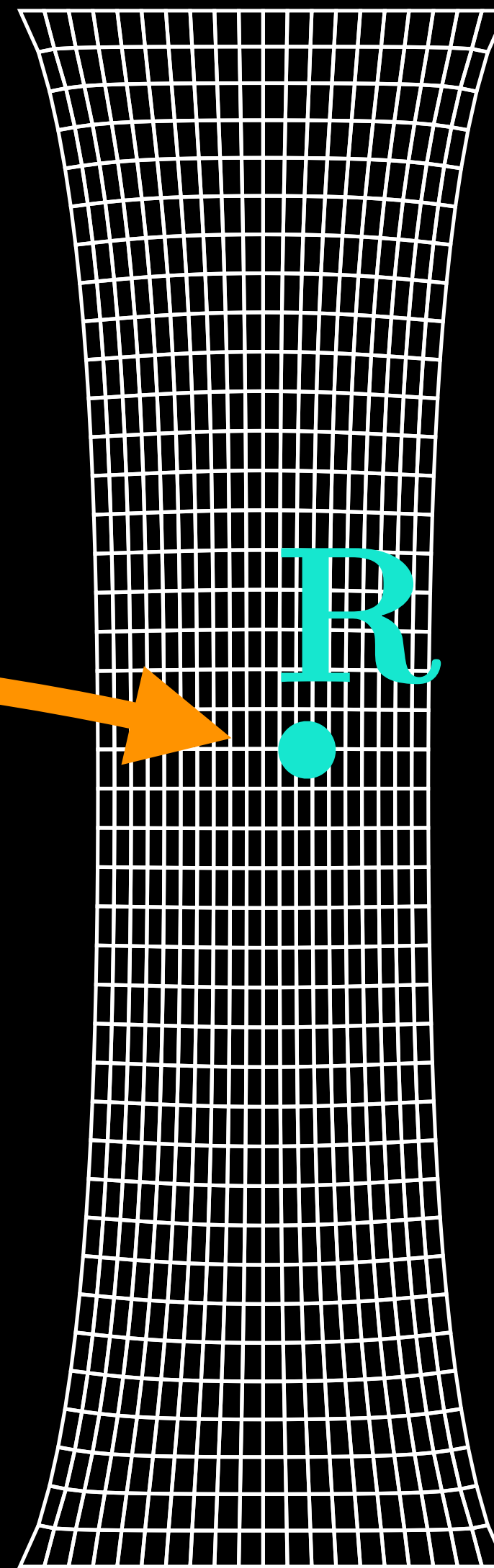


Towards a continuum model

$$\sigma_i(\varepsilon_i) = k_i \varepsilon_i + \beta_i \left[\frac{1}{(1 - \alpha_i \varepsilon_i)^2} - 1 \right]$$



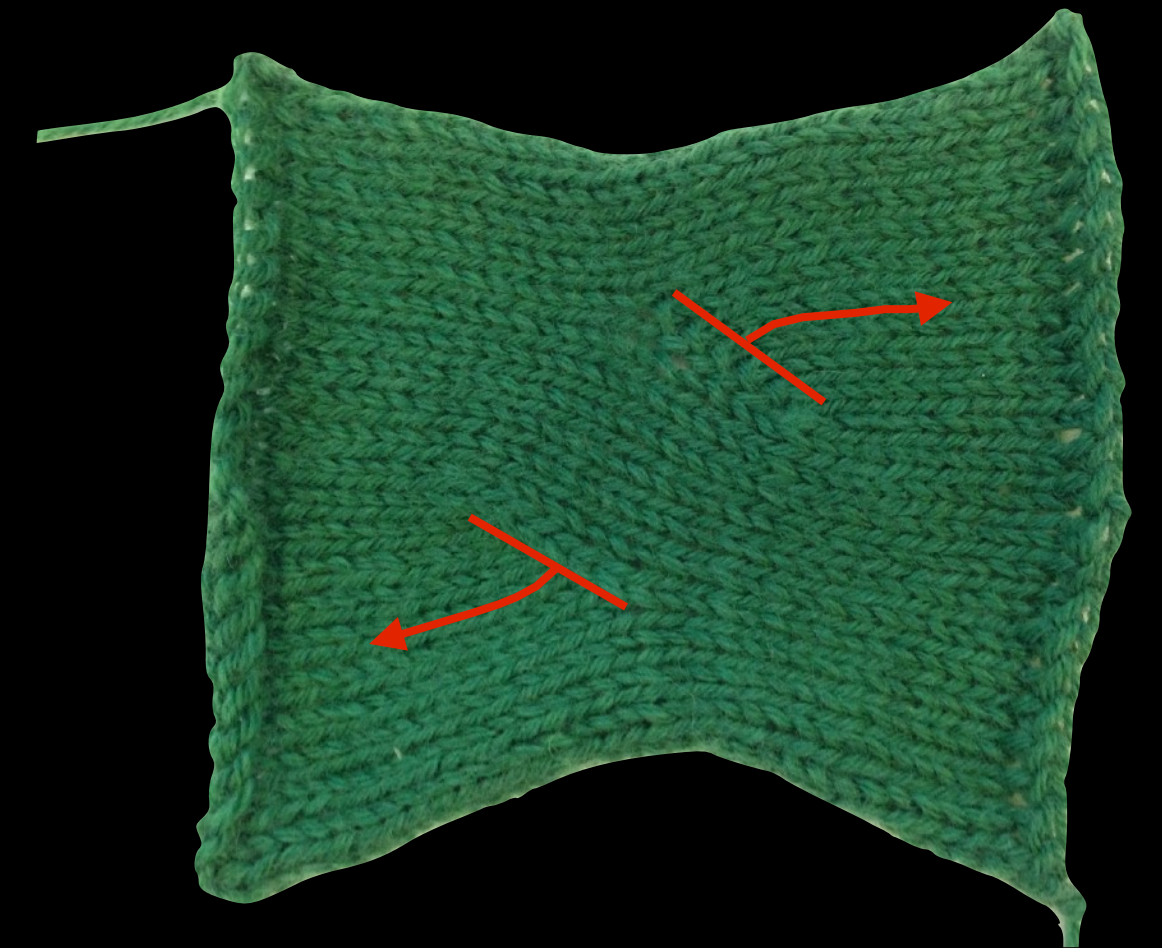
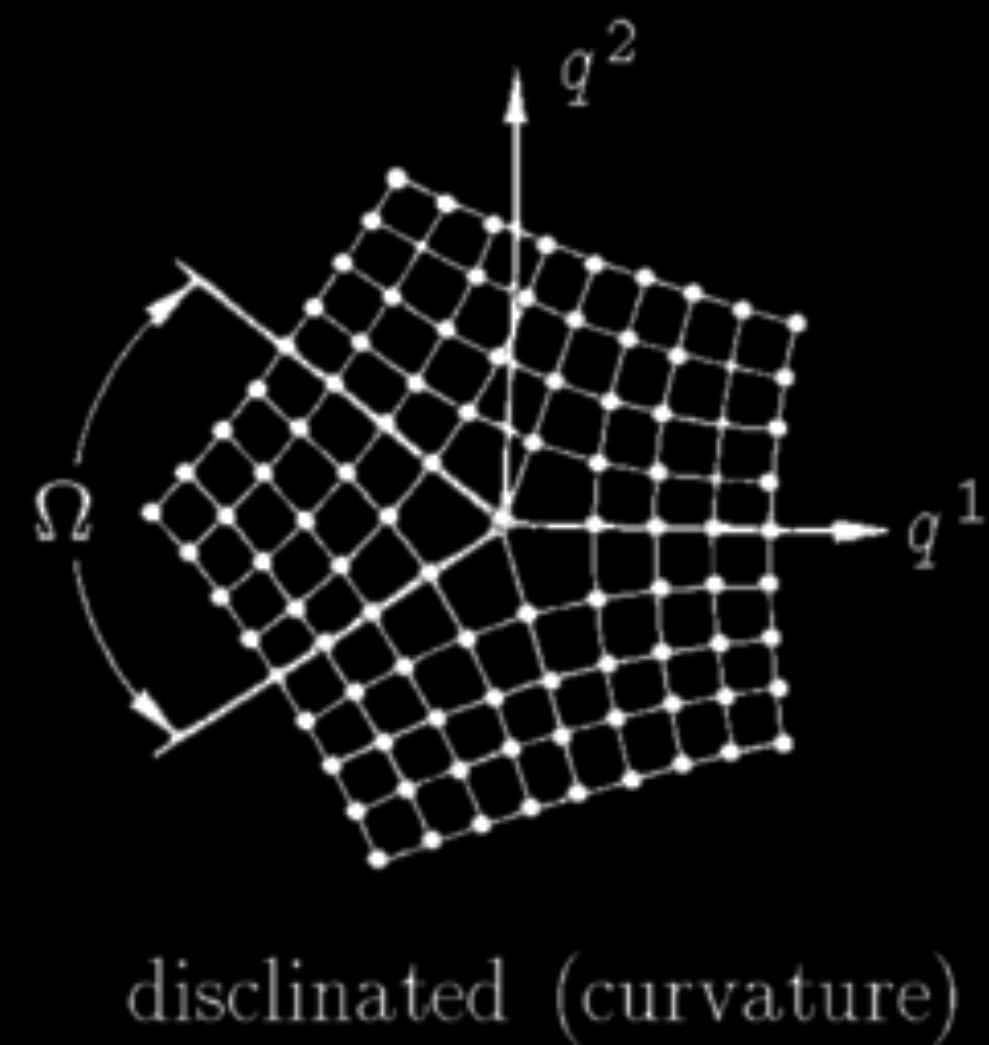
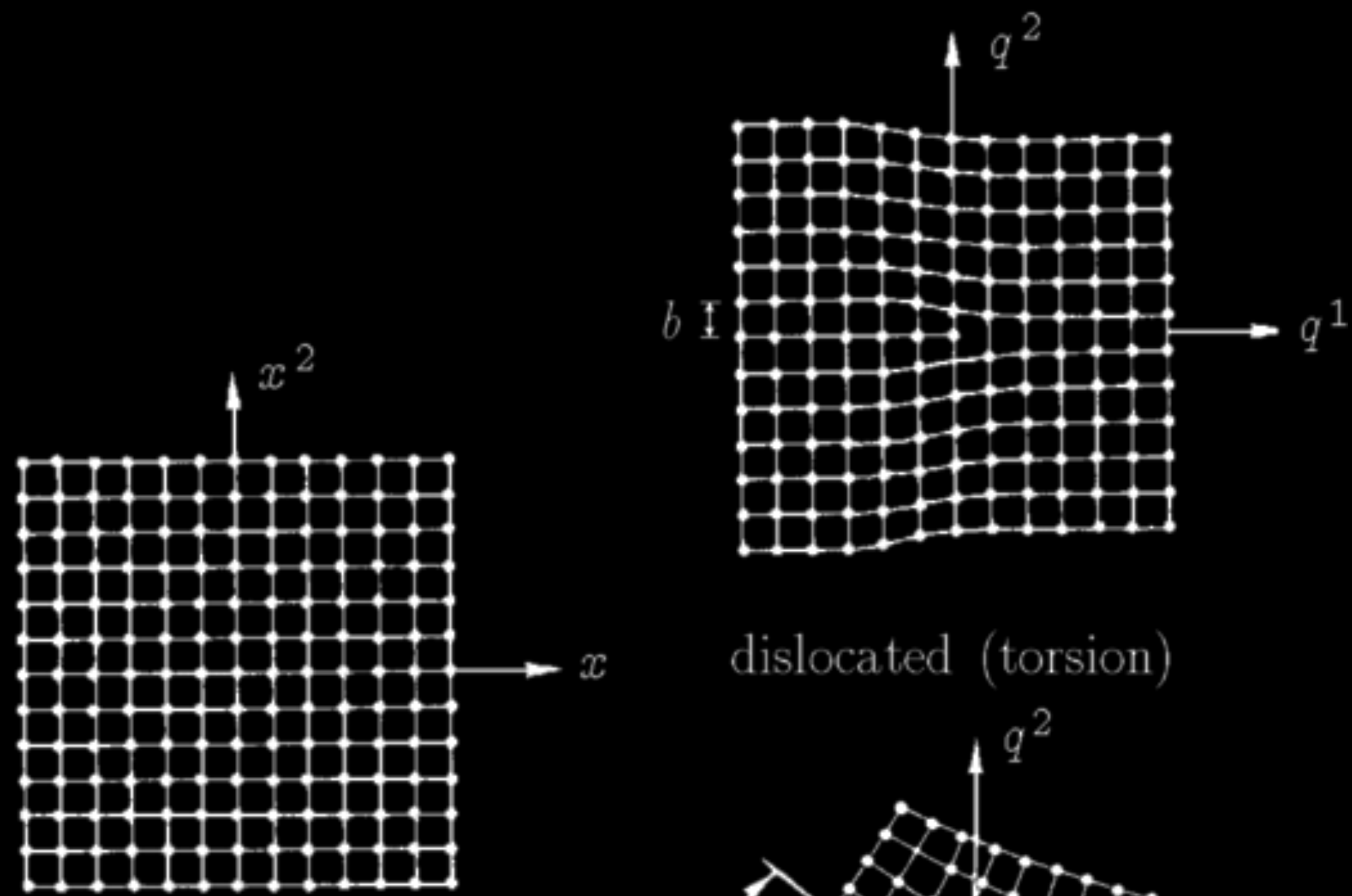
$$\mathbf{x} \mapsto \mathbf{R}(\mathbf{x})$$



Equilibrium equations:

$$\partial_j \sigma_{ij} = 0$$

Knits are a crystalline material... with defects



Future work: Defects in knits



Thank you!