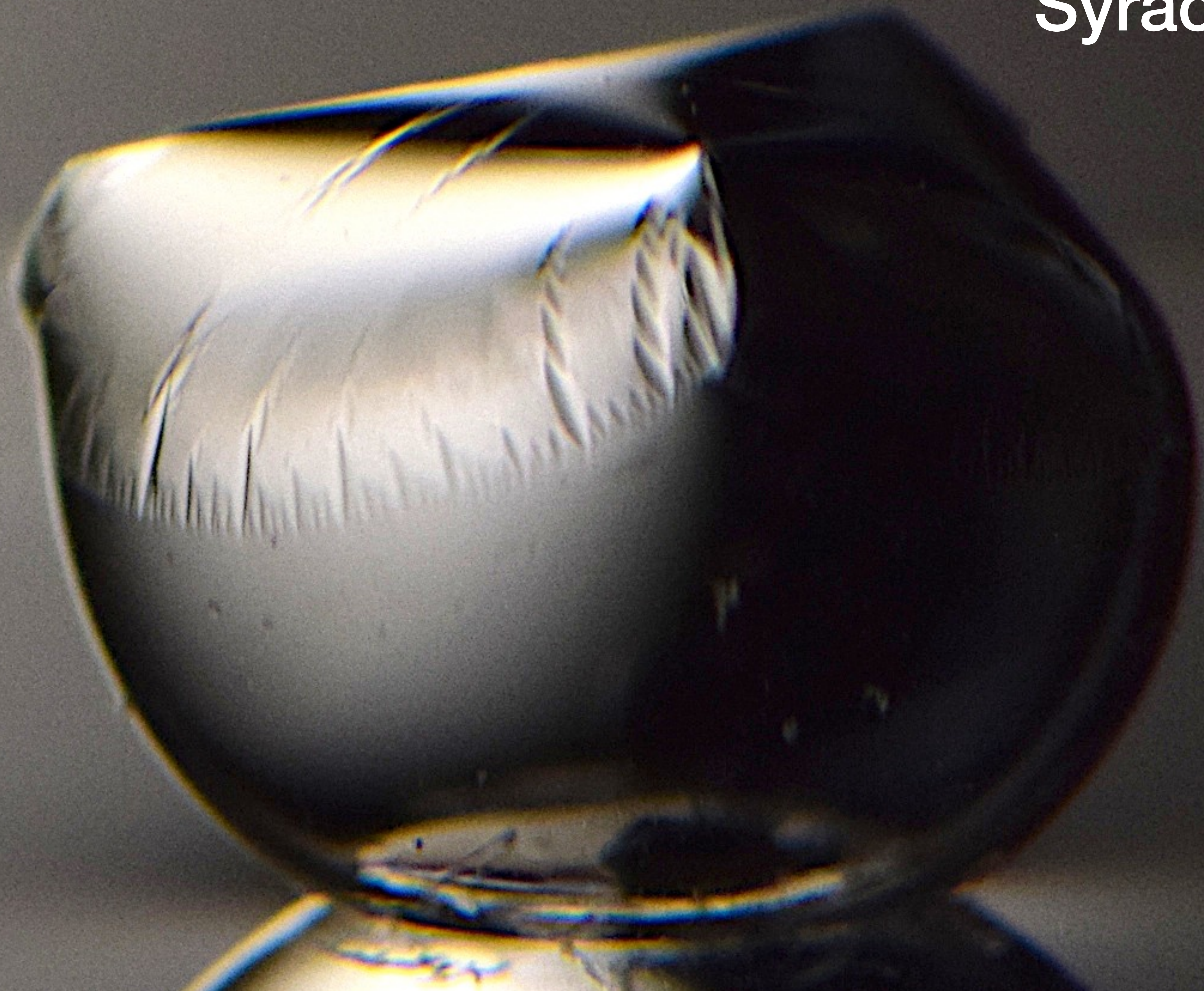
Sheets shaping liquids and liquids shaping sheets

Joseph D Paulsen
Syracuse University



It takes a village...

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simulations

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Chris Santangelo

Dominic Vella

experiments

theory

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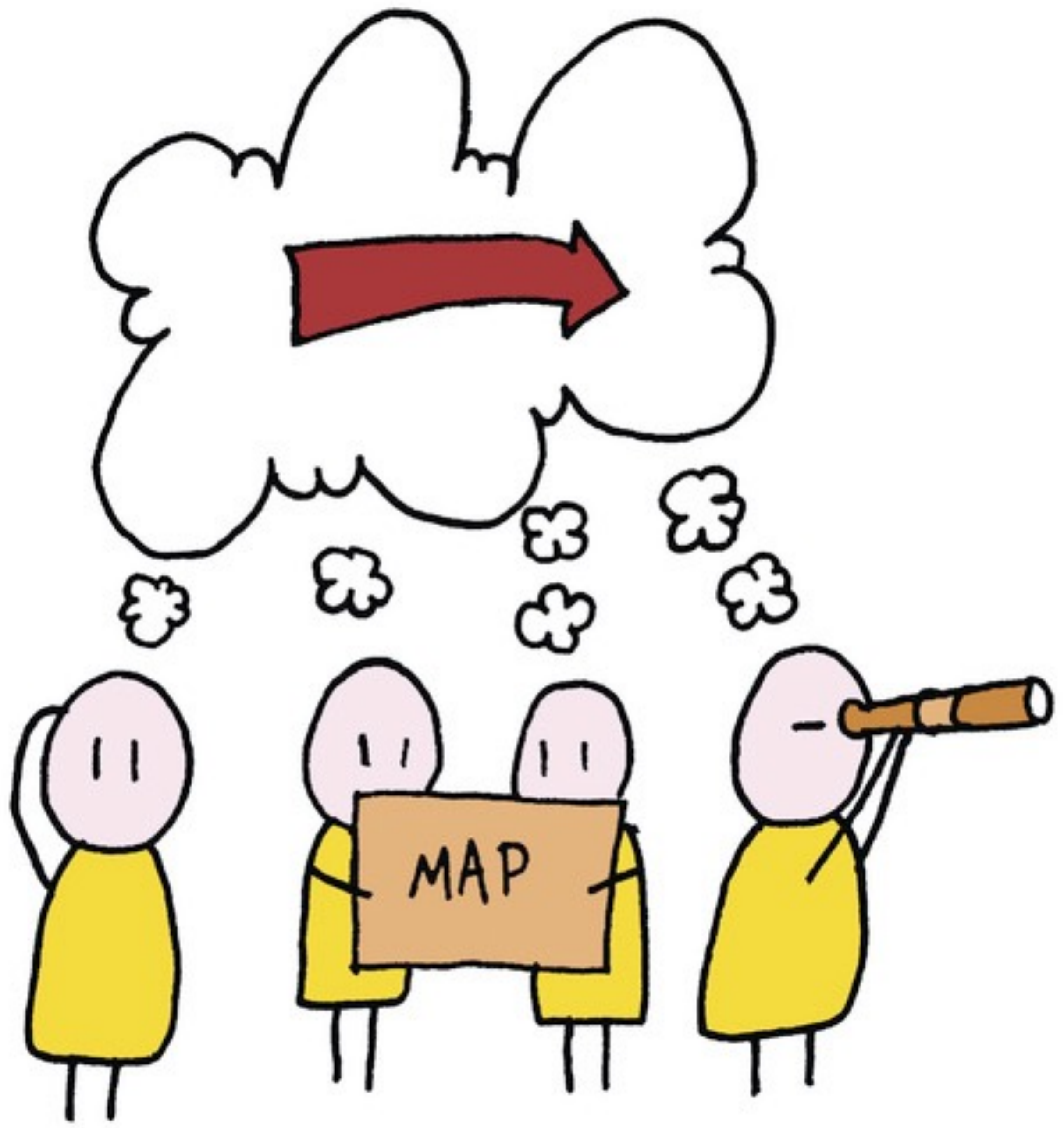
NSF REU DMR-1460784, NSF REU DMR-1757749

*(now Syracuse)



0. Wrapping a droplet with a thin solid

1. Indenting a floating film

2. Beyond the “gross shape”

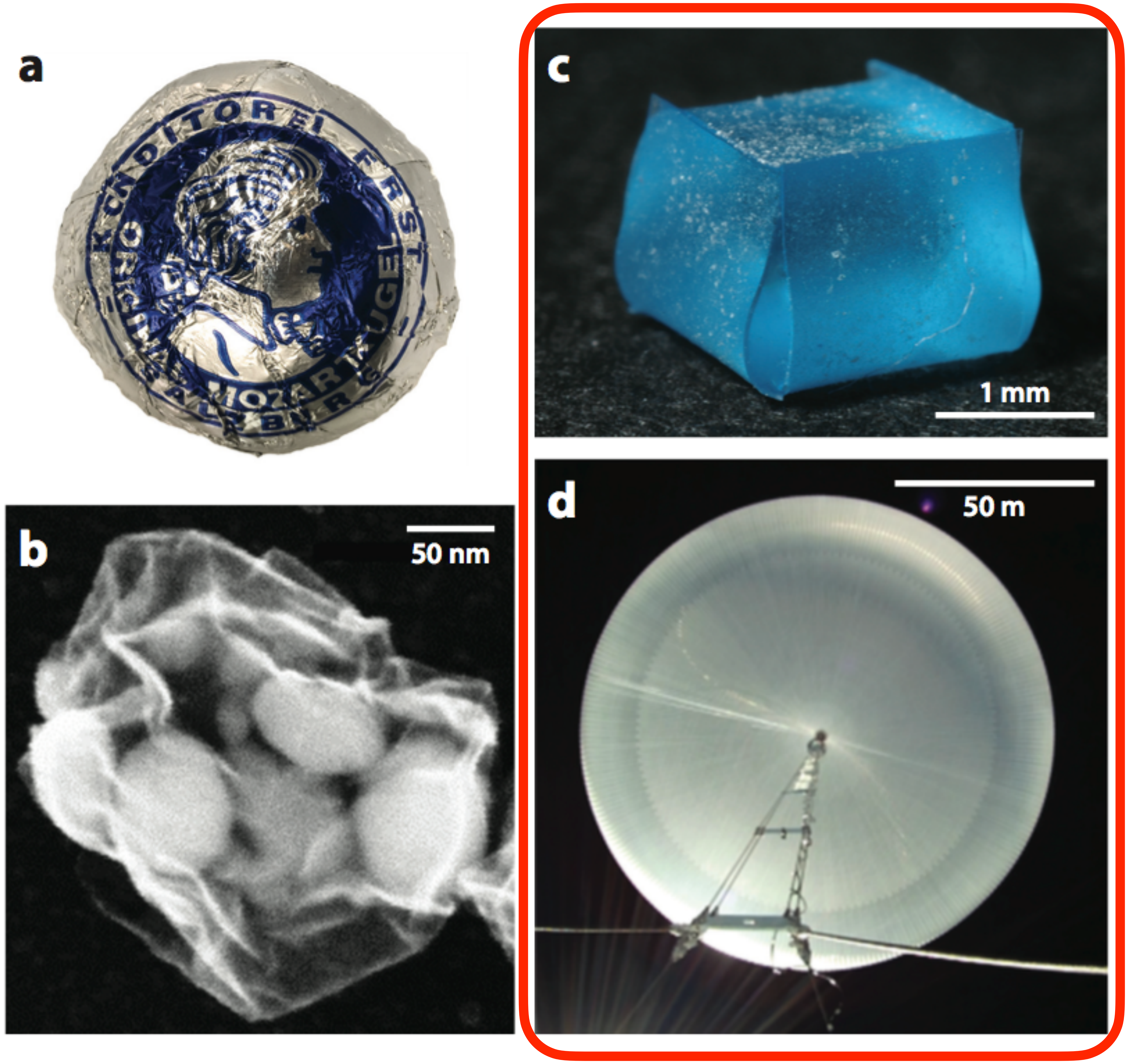
3. Towards solid surfactants

we think we know what’s going on

bleeding edge

Image credit: Rene Sorensen

We wrap objects in thin sheets to conceal, protect, or enhance them



Wrapper & contents

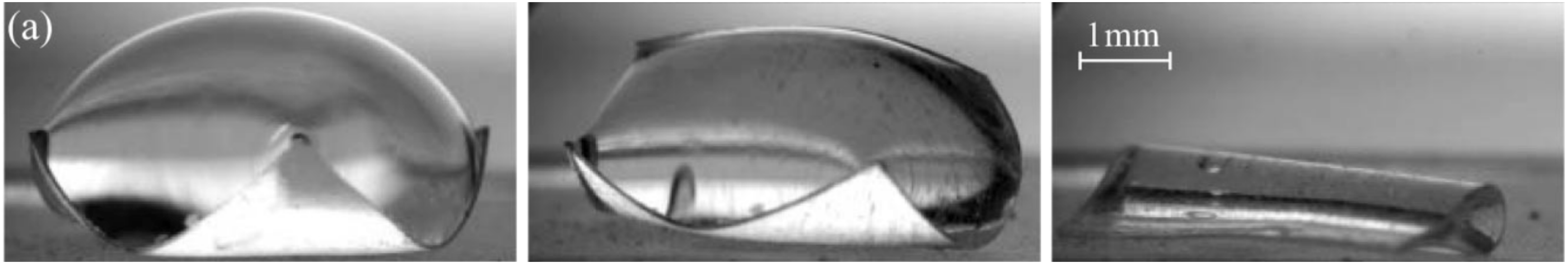
both easily deformed

(a) Wikipedia, cf. Demaine et al. (2009)
(b) Chen et al., Nano Lett. 12 (2012)
(c) Py, Bico, Roman, et al. (2007)
(d) Image courtesy of NASA

Figure from: “Wrapping liquids, solids, and gases in thin sheets”
JDP, Annu. Rev. Condens. Matter Phys. Vol. 10 (2019)

"Capillary Origami"

Py, Bico, Roman, Baroud, et. al., 2007



"Thick" films (t~60 μm): **long wavelength bending**

What happens for a film that is **1000 times thinner?**

Bending energies (~t^3) are then **×10^9 smaller!**


t = 80 nm circular polystyrene sheet
1 mm
water
silicone oil
fluorinated oil
needle


t = 80 nm circular sheet

How do thin sheets respond to confinement?

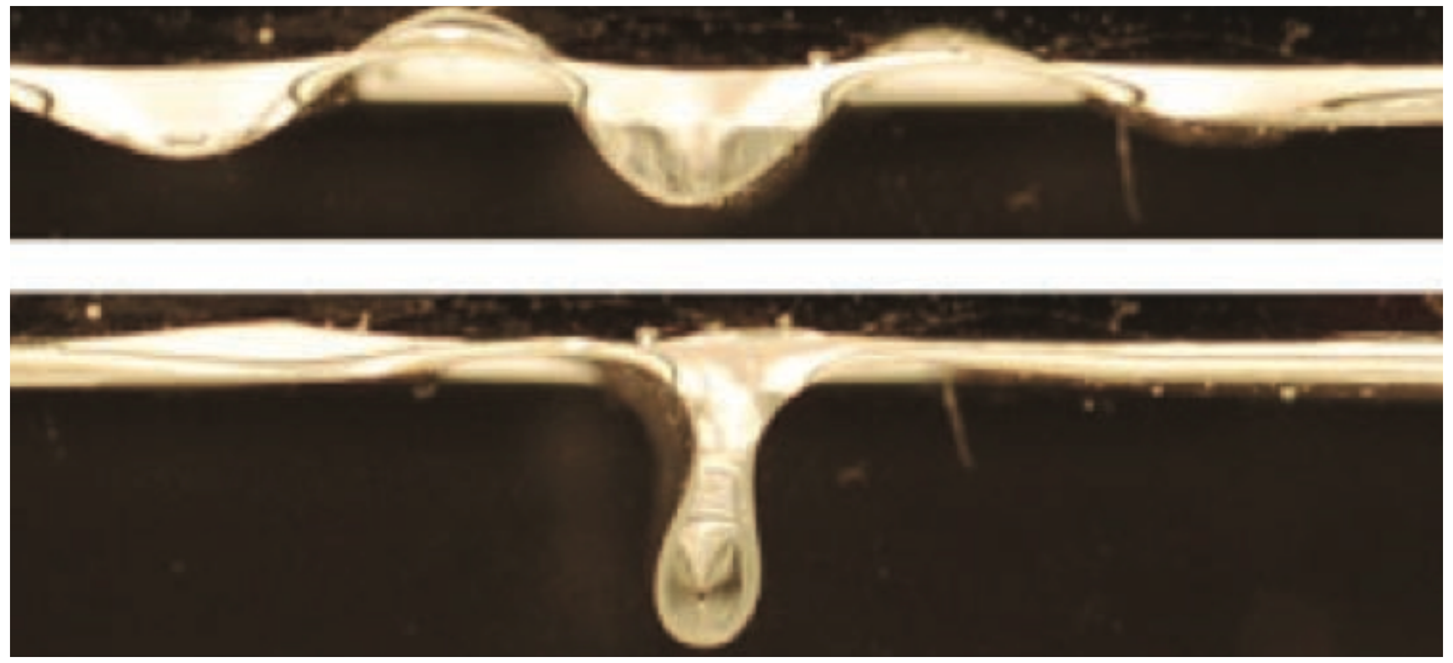
...they wrinkle

Smooth deformation

Huang et al., *PRL* 2010

...they fold

Localized deformation



Pocivavsek et al., *Science* 2008

...they crumple

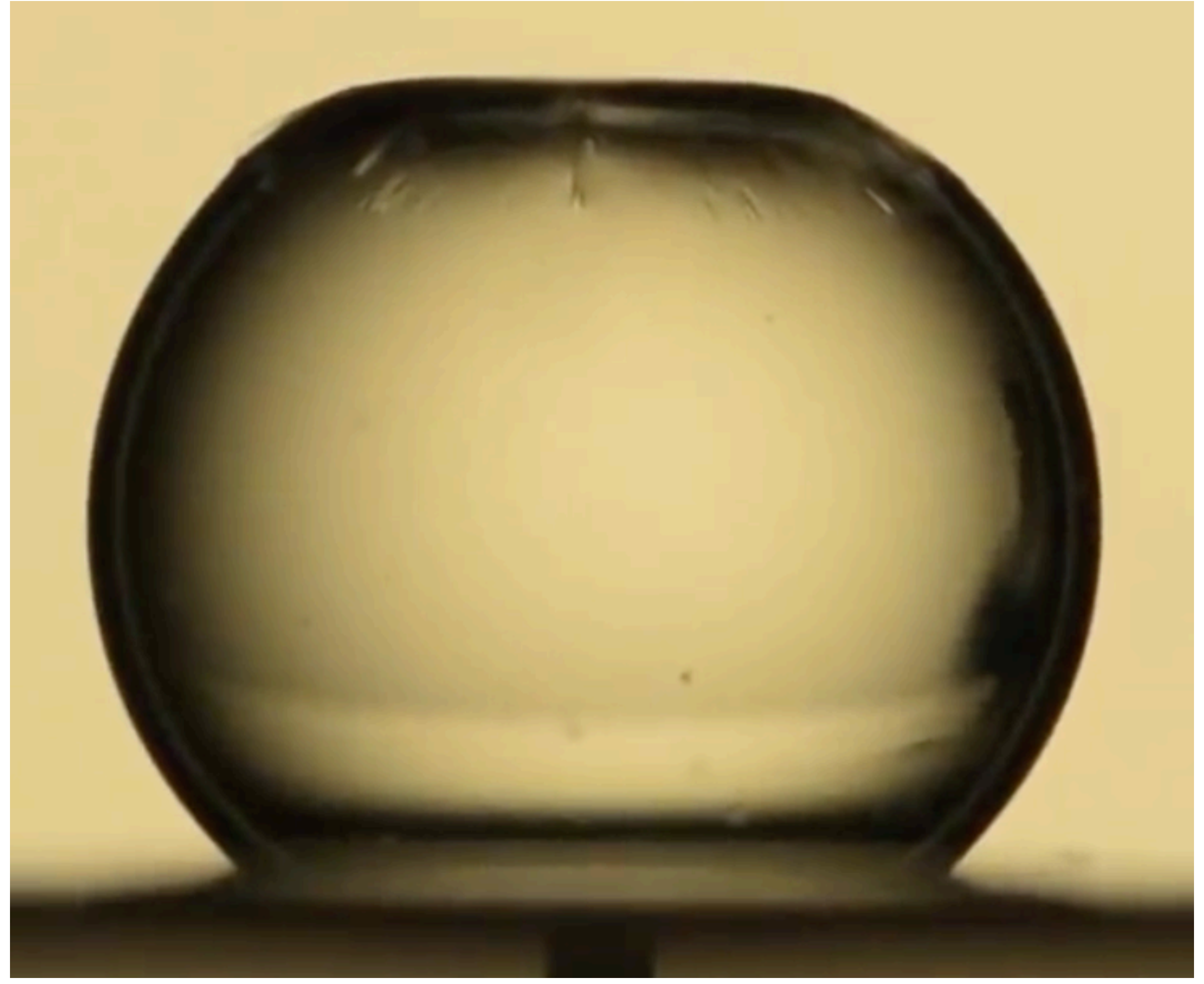
Localized deformation and stress

Subtle physics, nonlinear sheet equations

*Wrapping: **all of these** in **highly curved** geometry!*

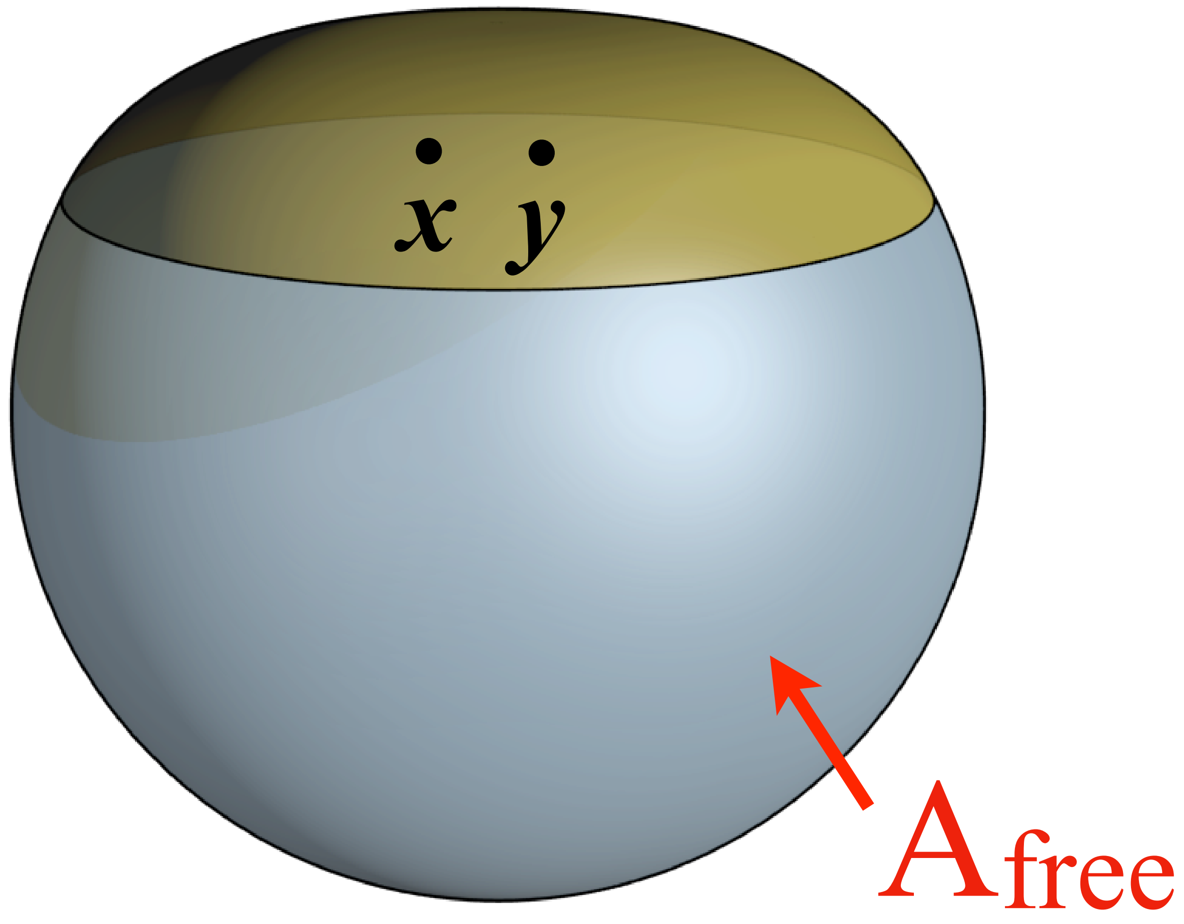
The Plan:

actual shape



=

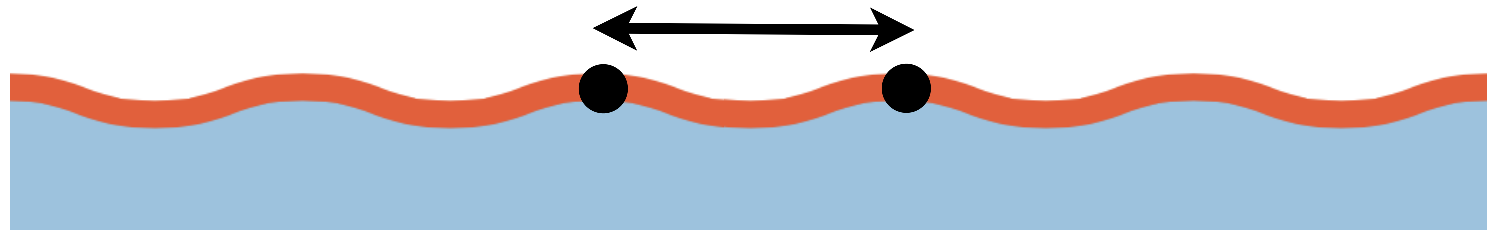
“gross shape”



+

“small-scale features”

(wrinkles, crumples, folds)



…which allow in-plane compression at negligible cost

Gross shape **can compress** but not stretch:

$$|\boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{y})| \leq |\boldsymbol{x} - \boldsymbol{y}|$$

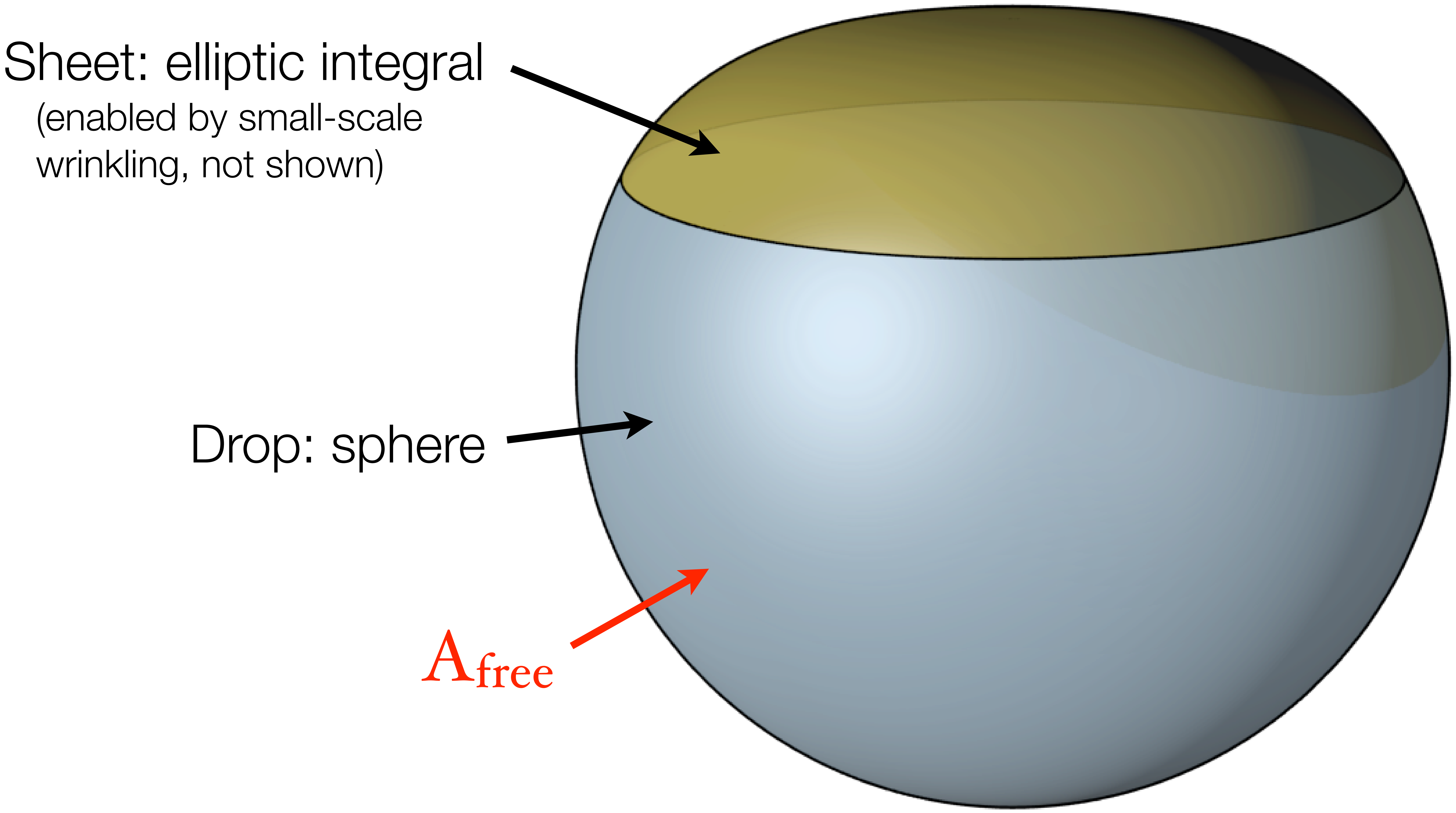
where $\boldsymbol{f}$: planar sheet → gross shape

Geometric model: $U = \gamma A_{free}$ (System seeks **gross shape** that minimizes A_{free})

JDP, Démery, et al., Nat. Mater. 2015

Test case 1: Large droplet (gross shape is axisymmetric)

Global minimum of $U = \gamma A_{free}$ among all axisymmetric configurations



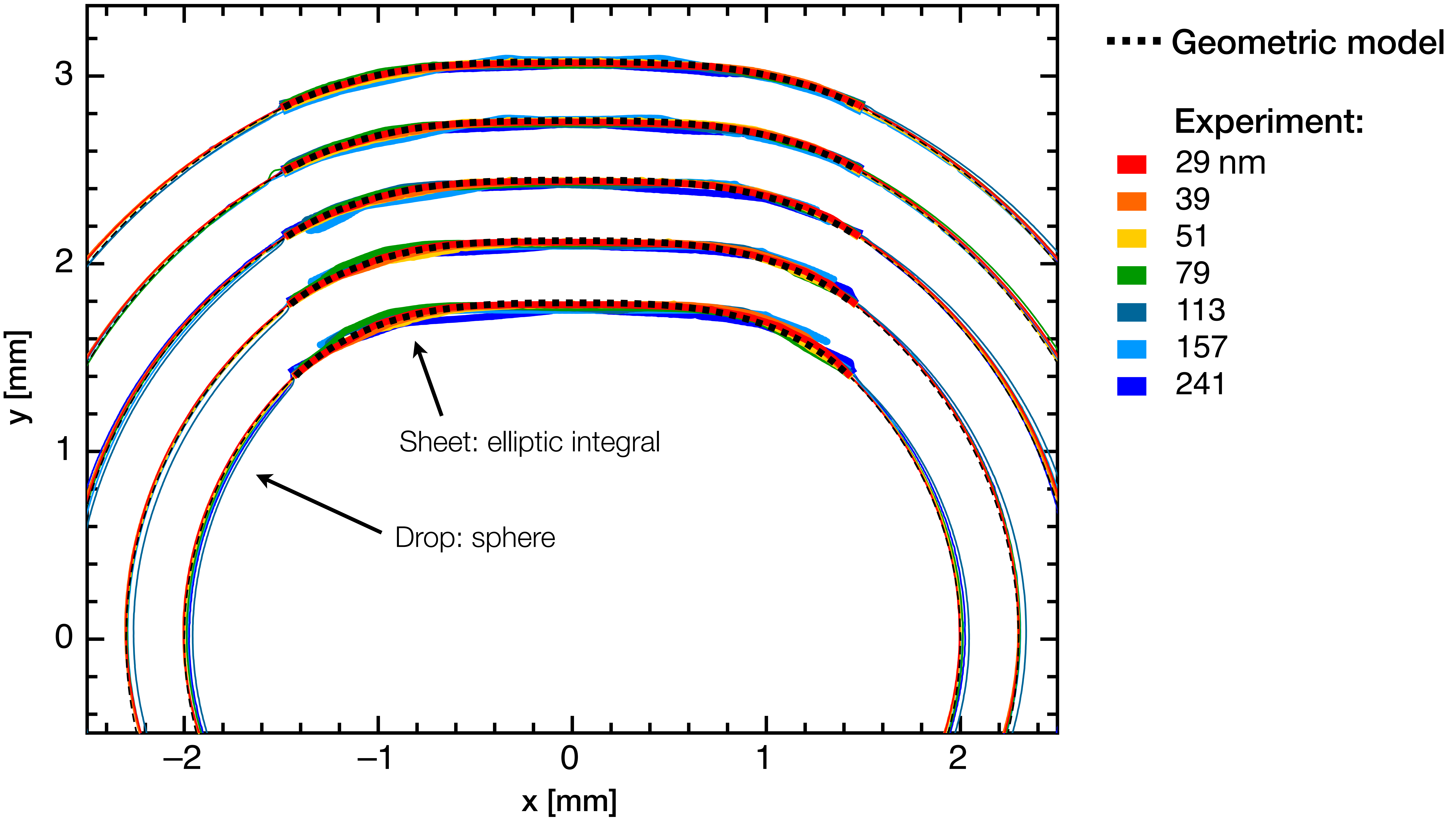
Single parameter: W/R

W: sheet radius

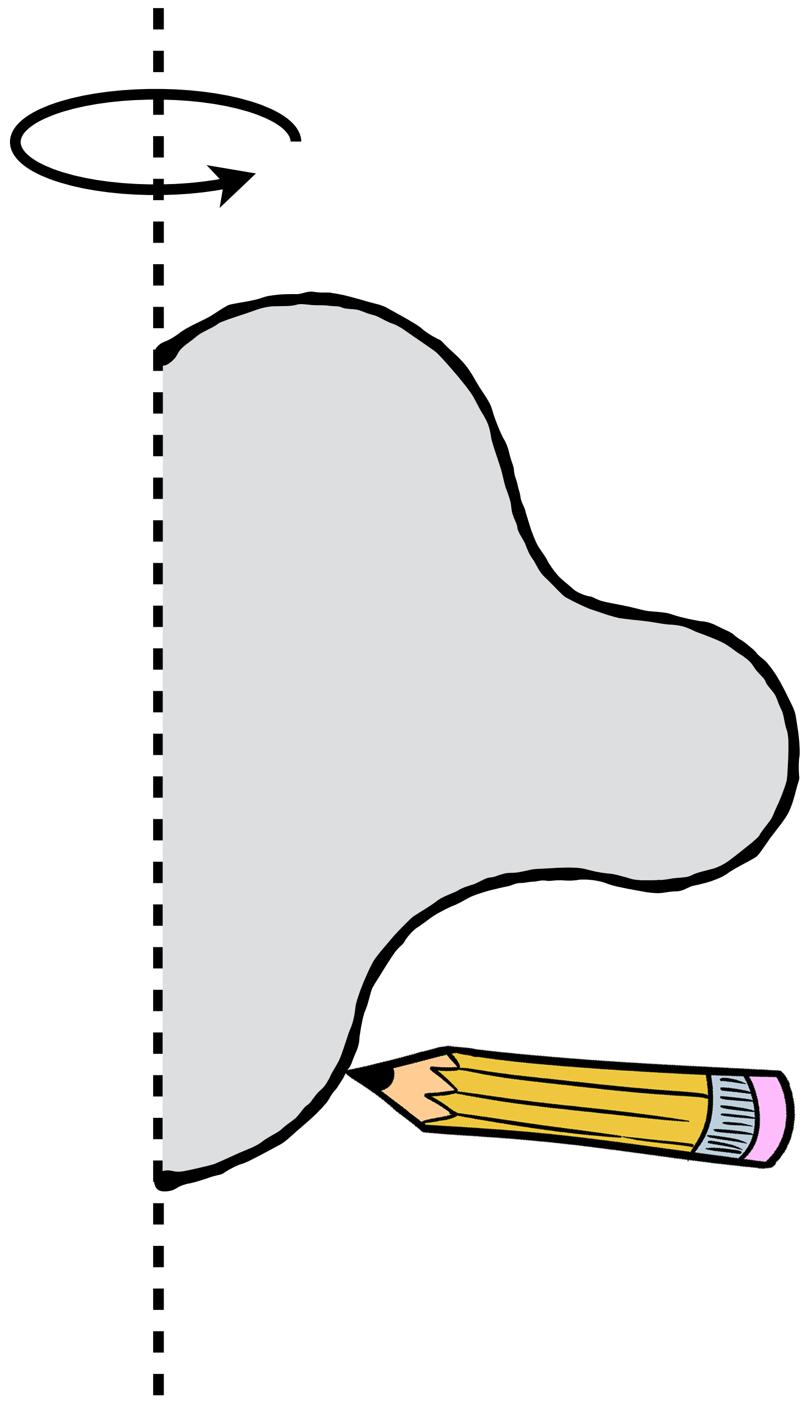
$(4\pi/3)$ $\mathbf{R}^3$: drop volume

JDP, Démery, et al., *Nature Materials* 2015

Comparing geometric model ($U = \gamma A_{free}$) with experiment



JDP, Démery, et al., *Nature Materials* 2015



1. Draw a line starting and ending on z-axis
2. Consider its surface of revolution

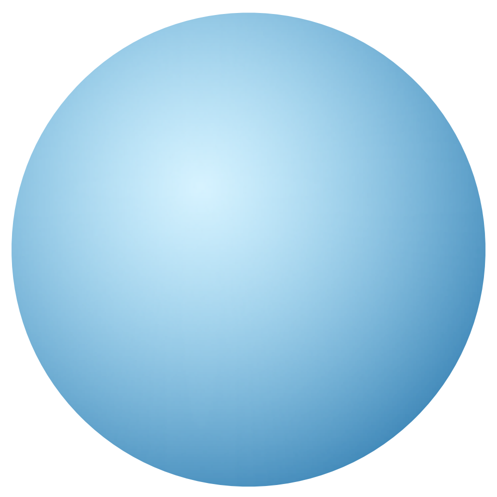
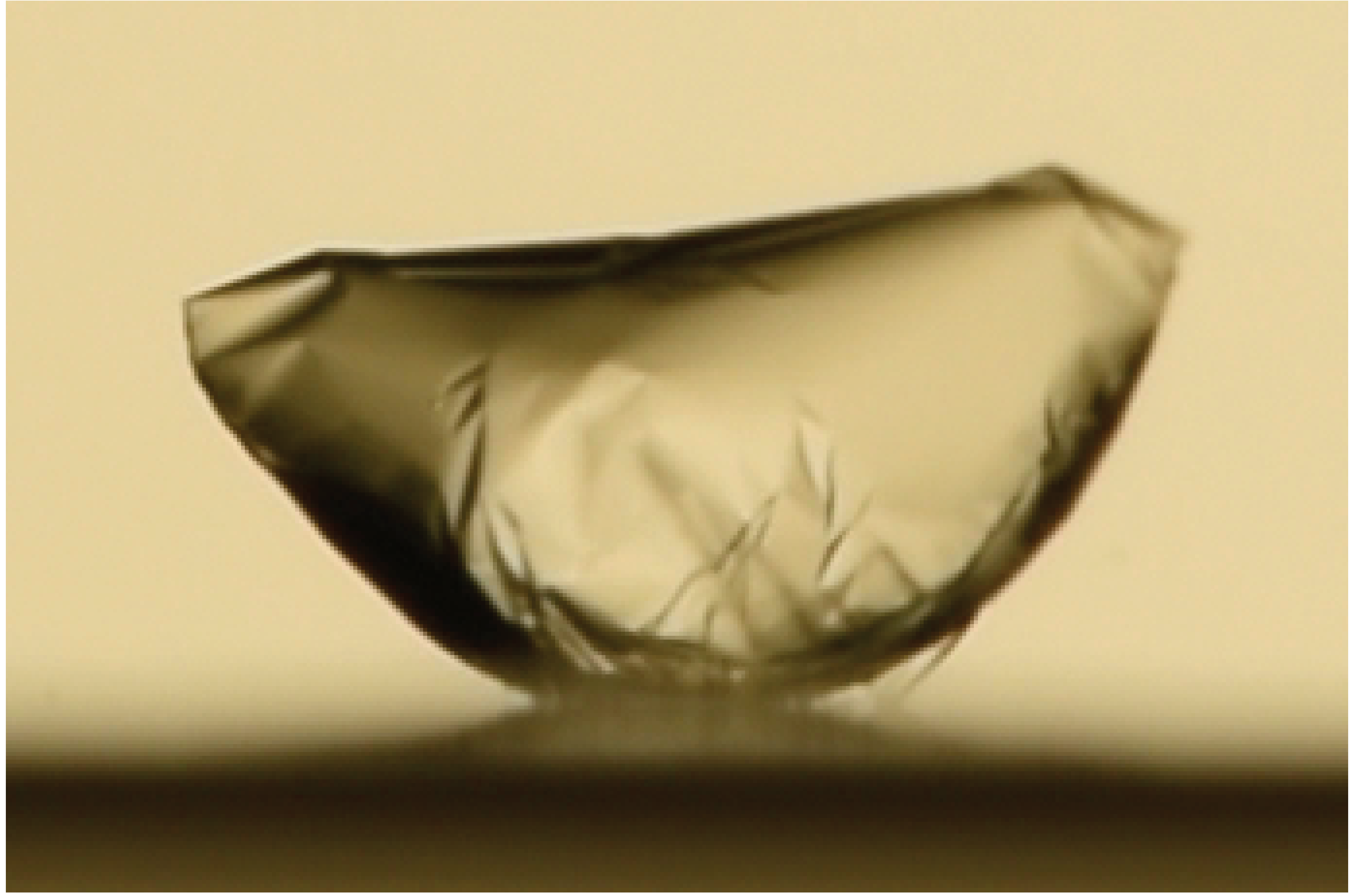
Optimization:	**Solution:**	**Example:**
Maximize volume for given surface area	Sphere	 liquid droplet, soap bubble
Maximize volume for given arclength	Elliptic integral: $$f(x) = \int_x^a \frac{t^2}{\sqrt{a^4 - t^4}}\,dt$$ where $$a = \frac{4r\sqrt{2\pi}}{\Gamma(1/4)^2}$$	 G.I. Taylor 1919: Parachute Paulsen (not me) 1994: Mylar balloon

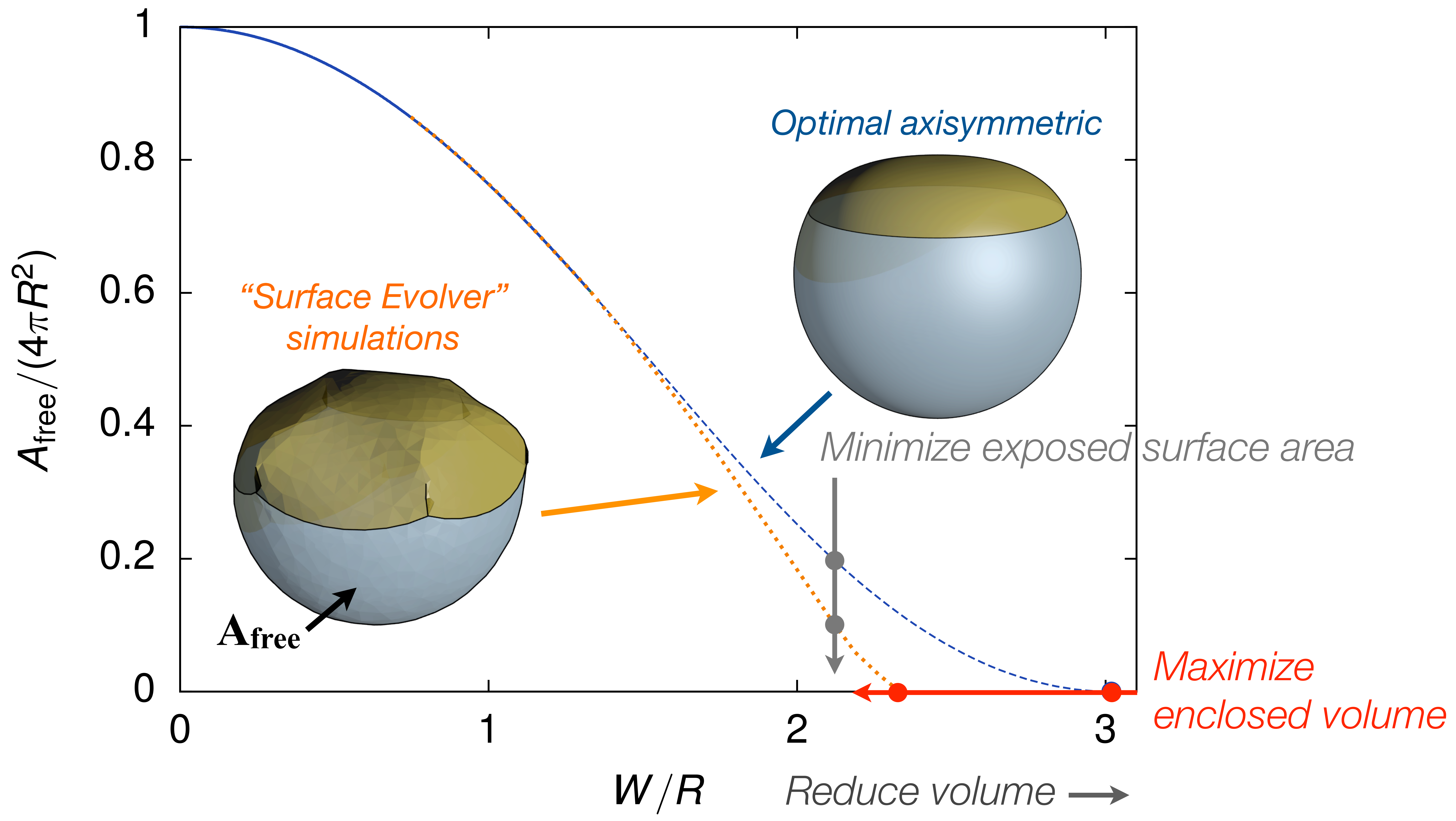
photo credit: rctoys.com

What about small drops?

(no longer axisymmetric)



Breaking axial symmetry ***improves coverage***



JDP, Démery, et al., *Nature Materials* 2015

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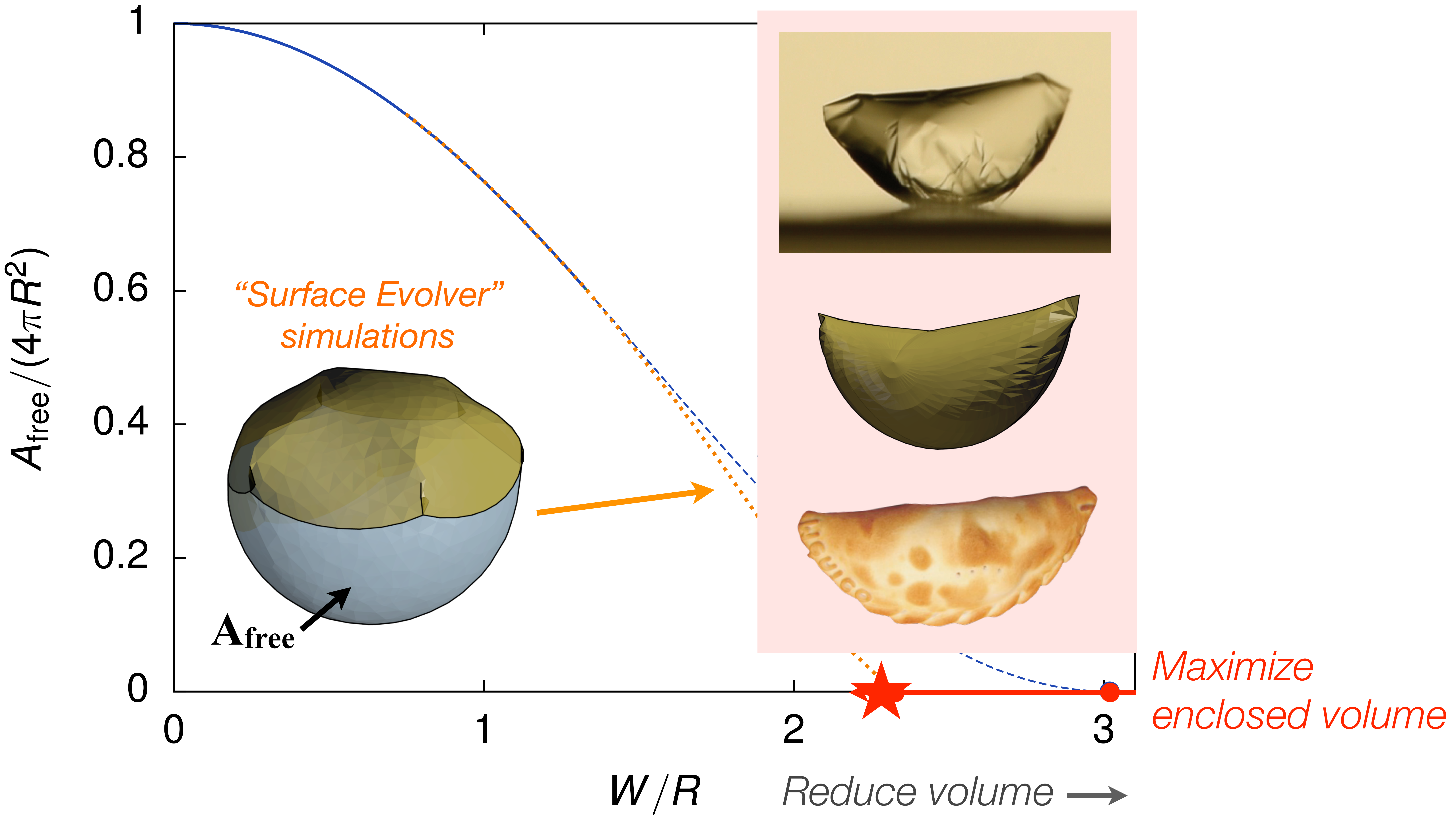

JDP, Démery, et al., *Nature Materials* 2015

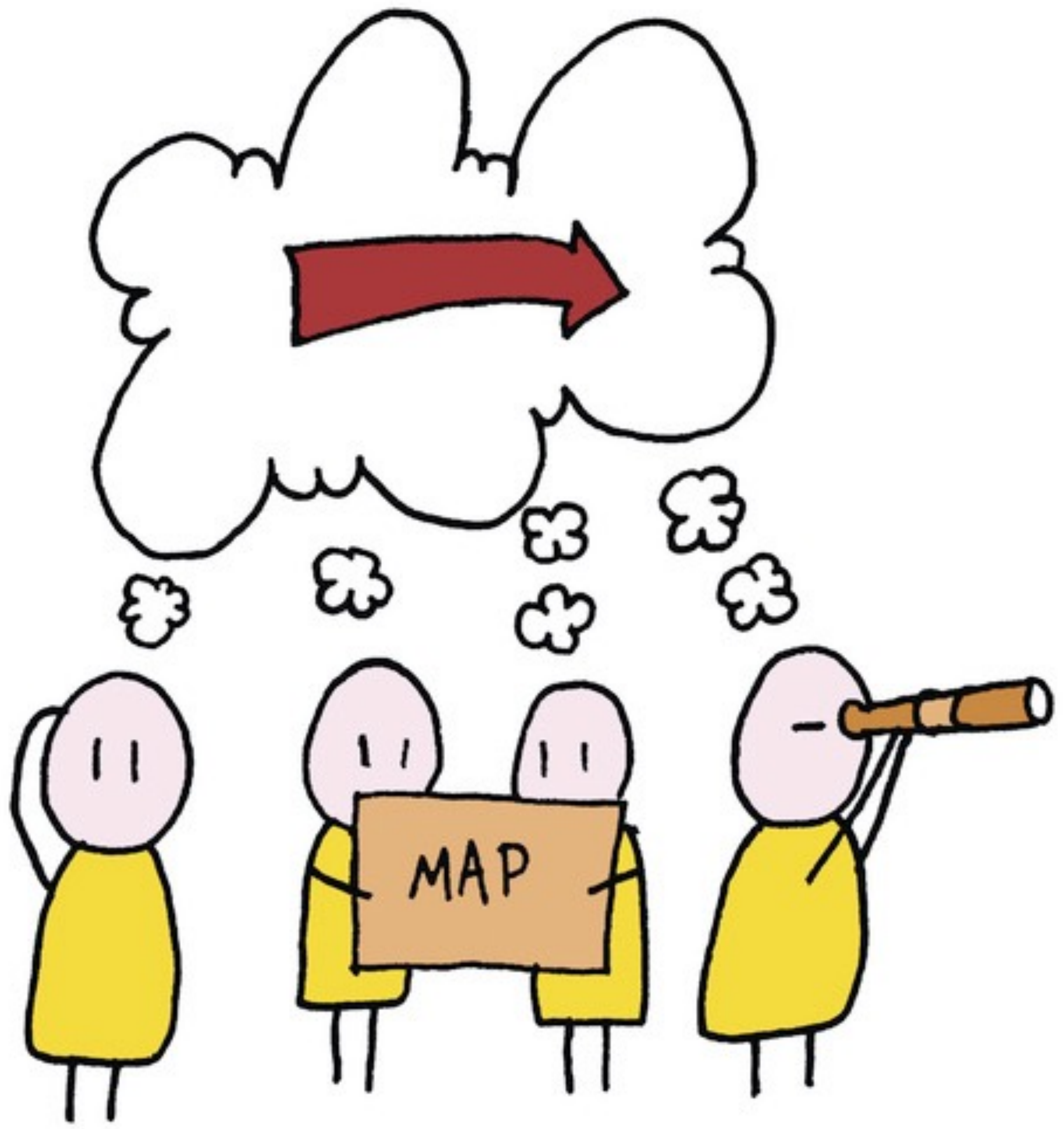

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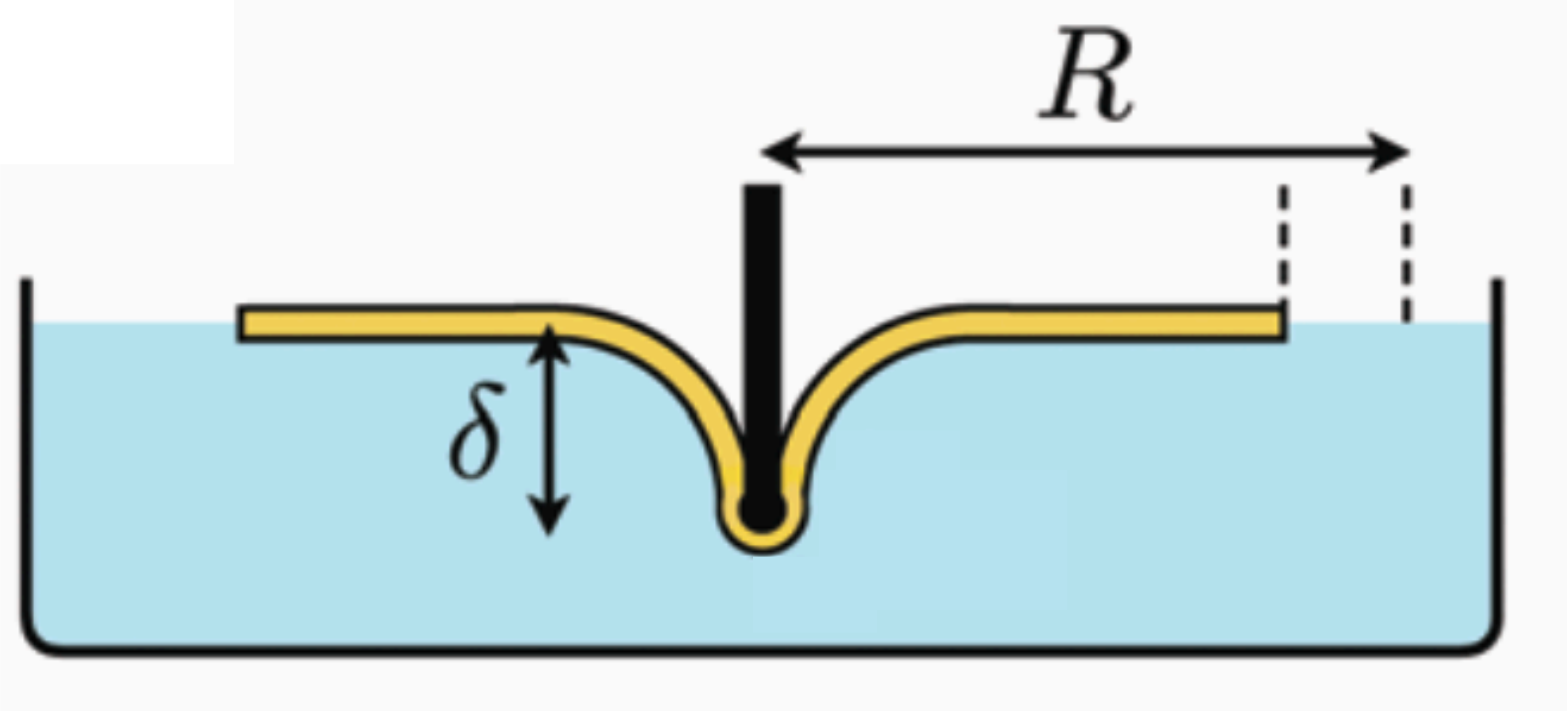
Image credit: Rene Sorensen

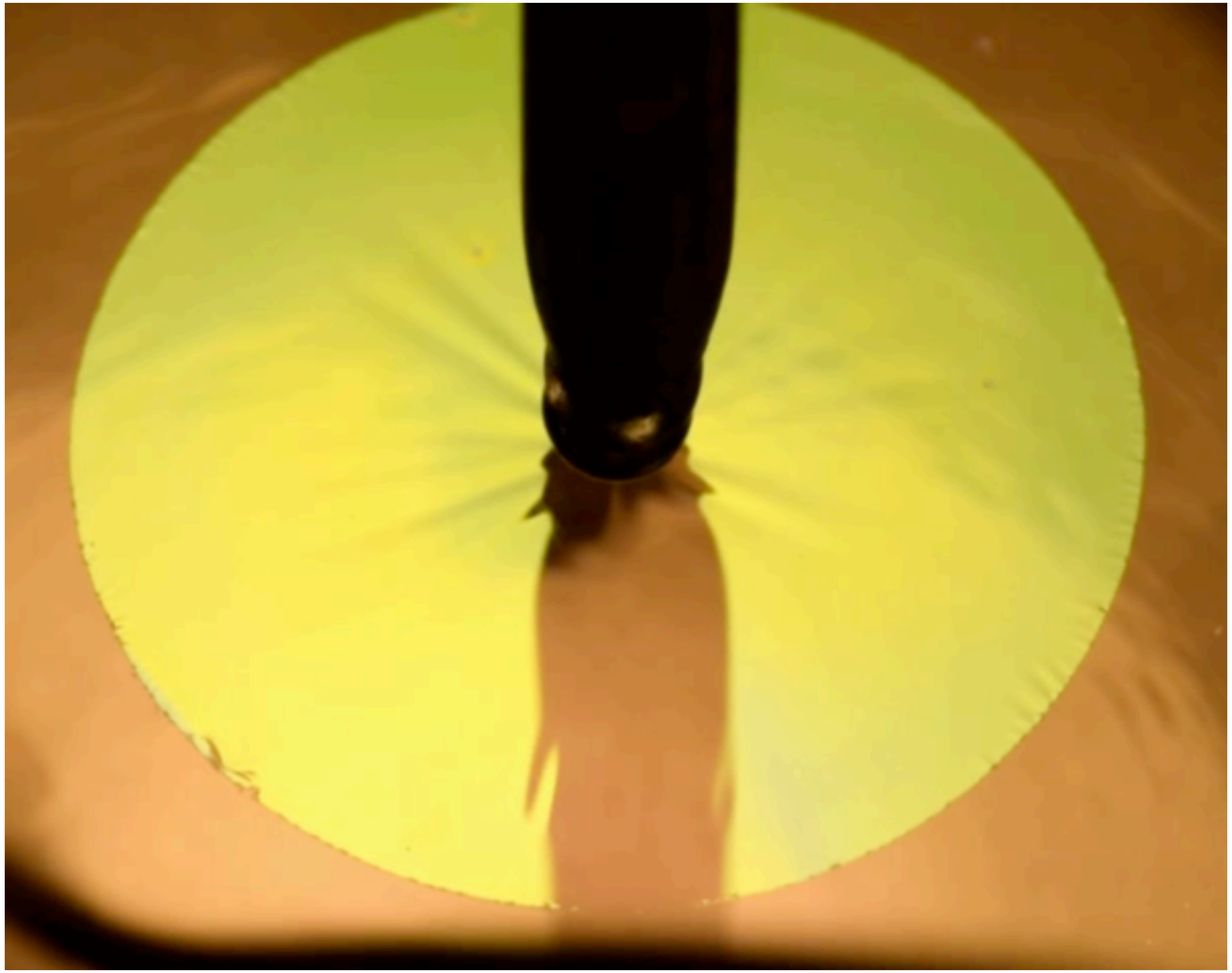
Indentation:

Holmes & Crosby, PRL 2010
Vella, Davidovitch, et al., PRL 2015, PRE 2018
Paulsen, Hohlfeld, et al., PNAS 2016
Ripp, Démery, Zhang, JDP, Soft Matter 2020

Monica Ripp



side view



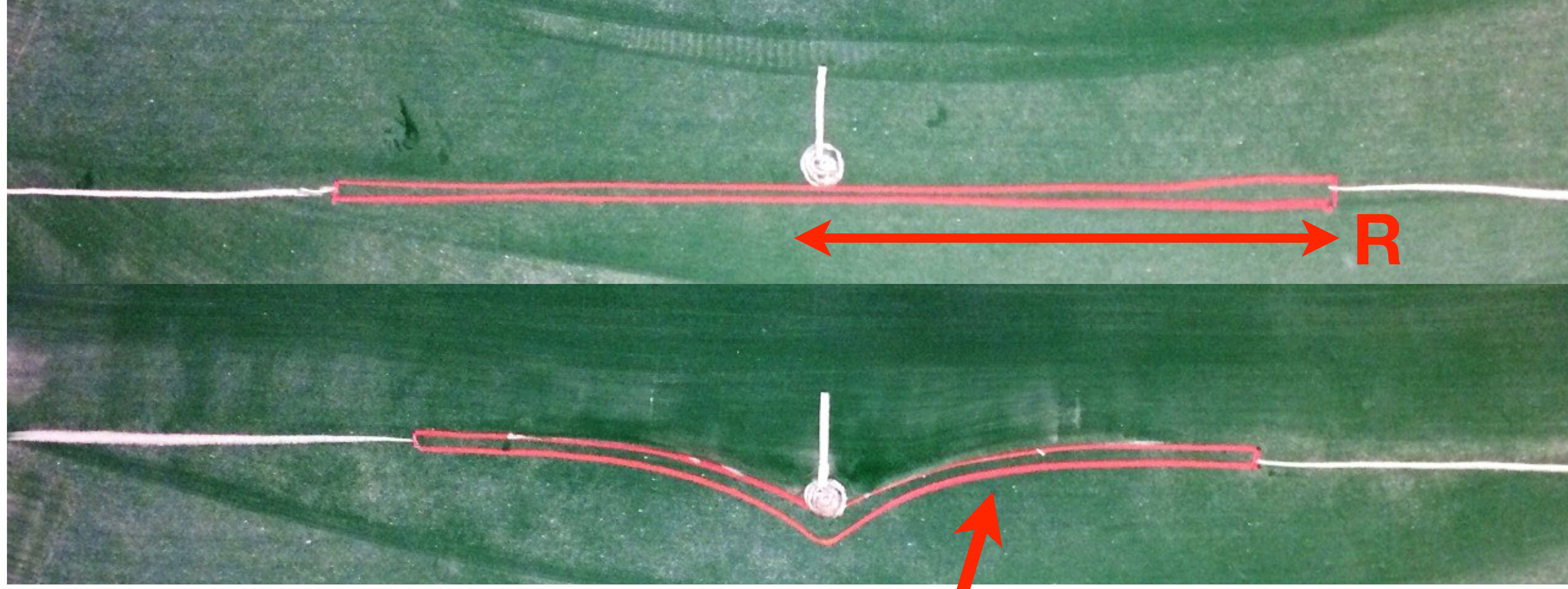
top view

Geometric model for poking:

- **Energy functional,** now with *gravity*

$$U = U_{\text{gravity}} + \gamma(\Delta A_{\text{free}})$$ ← fluid energies only

$$= \pi \int_0^\infty \left[\rho g r \zeta(r)^2 + 2\gamma R\left(\sqrt{1+\zeta'(r)^2} - 1\right)\right] \mathrm{d}r$$



Problem boils down to finding axisymmetric height function: *ζ(r)*

Vincent Démery

- Optimal *ζ(r)* satisfies **Euler-Lagrange equation:**

$$\zeta''(r) = \frac{r\zeta(r)}{\ell_{\text{curv}}{}^3}\left[1+\zeta'(r)^2\right]^{3/2} \quad \text{where} \quad \ell_{\text{curv}} = \ell_c{}^{2/3} R^{1/3}$$

- **Small slopes:** Reduces to $\zeta''(r) = r\zeta(r)/\ell_{\text{curv}}{}^3$ (with same Airy function solution from Dominic's talk)

- **BONUS:** Geometric model gives access to ***large slopes behavior***:

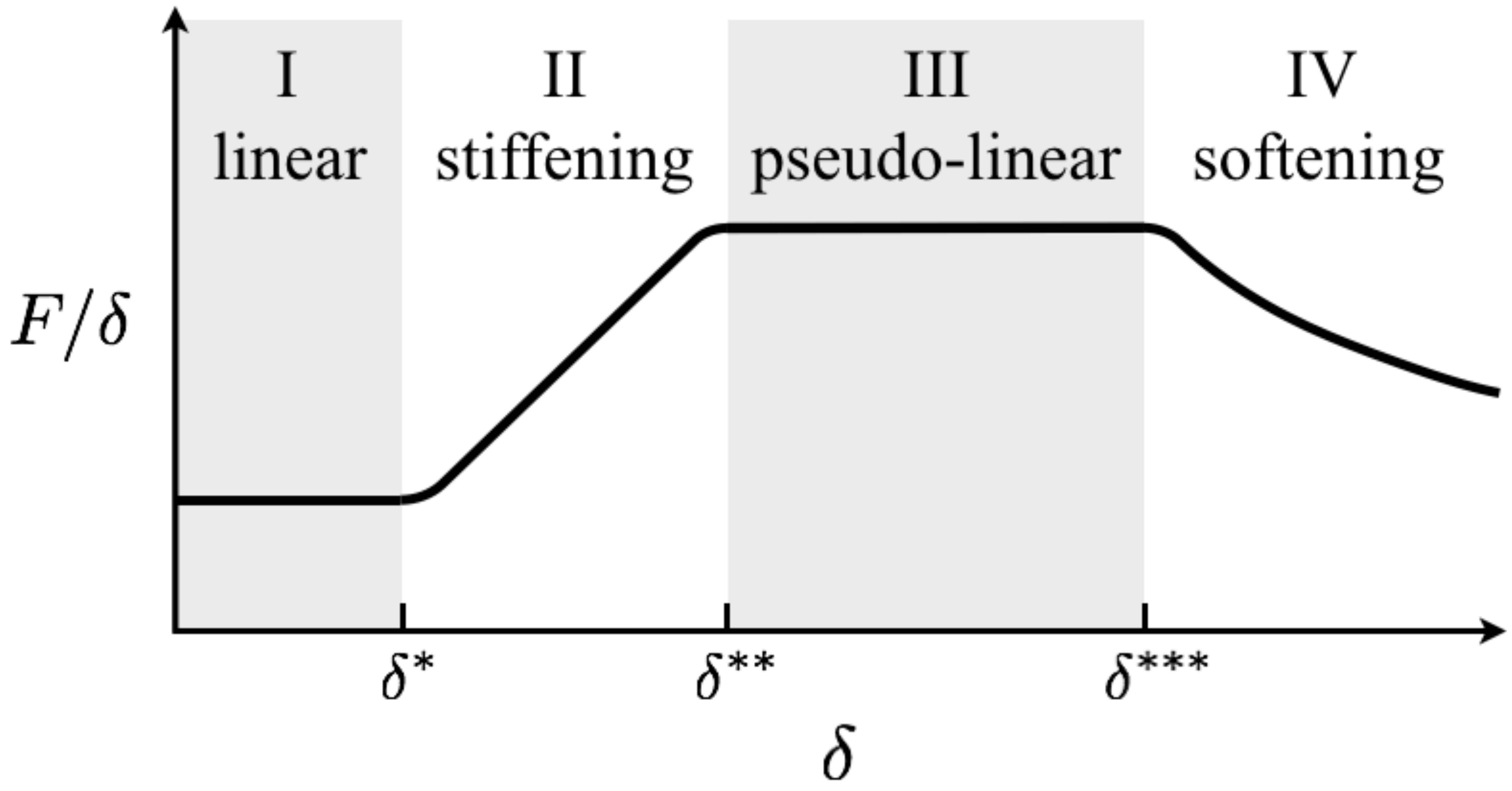
Profile: $\zeta(r) \sim -[3\log(1/r)/2]^{2/3}$ as $r \to 0$

Energy: $U \sim 2\pi\gamma R\delta$

Force: $F \sim 2\pi\gamma R$

Ripp, Démery, Zhang, JDP, Soft Matter 2020

“Spring constant” F/δ, felt by indenter



Force:

I: $F \sim \gamma\delta$

II: $F \sim \sqrt{Y\rho g}\delta^2$

III: $F \approx 4.581(\gamma R_{\text{film}})^{2/3}(\rho g)^{1/3}\delta$

IV: $F \sim 2\pi\gamma R_{\text{film}}$

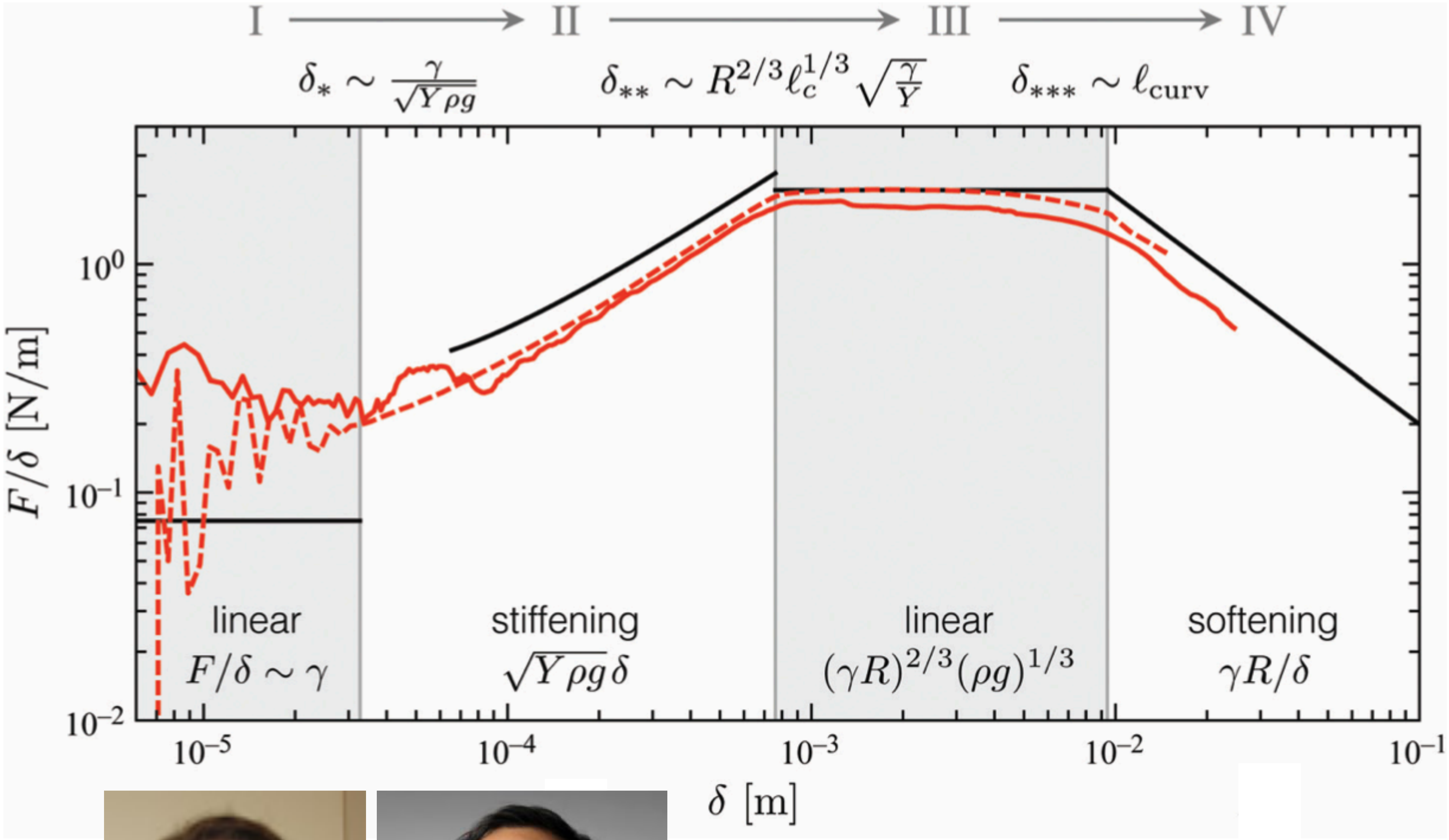
■ Föppl–von Kármán (cf. Benny, Dominic’s talks)

Vella, Davidovitch, et al., *PRL* 2015, *PRE* 2018

■ Geometric model: $U = U_{\text{gravity}} + \gamma(\Delta A_{\text{free}})$

Ripp, Démery, Zhang, JDP, *Soft Matter* 2020

“Spring constant” F/δ, felt by indenter



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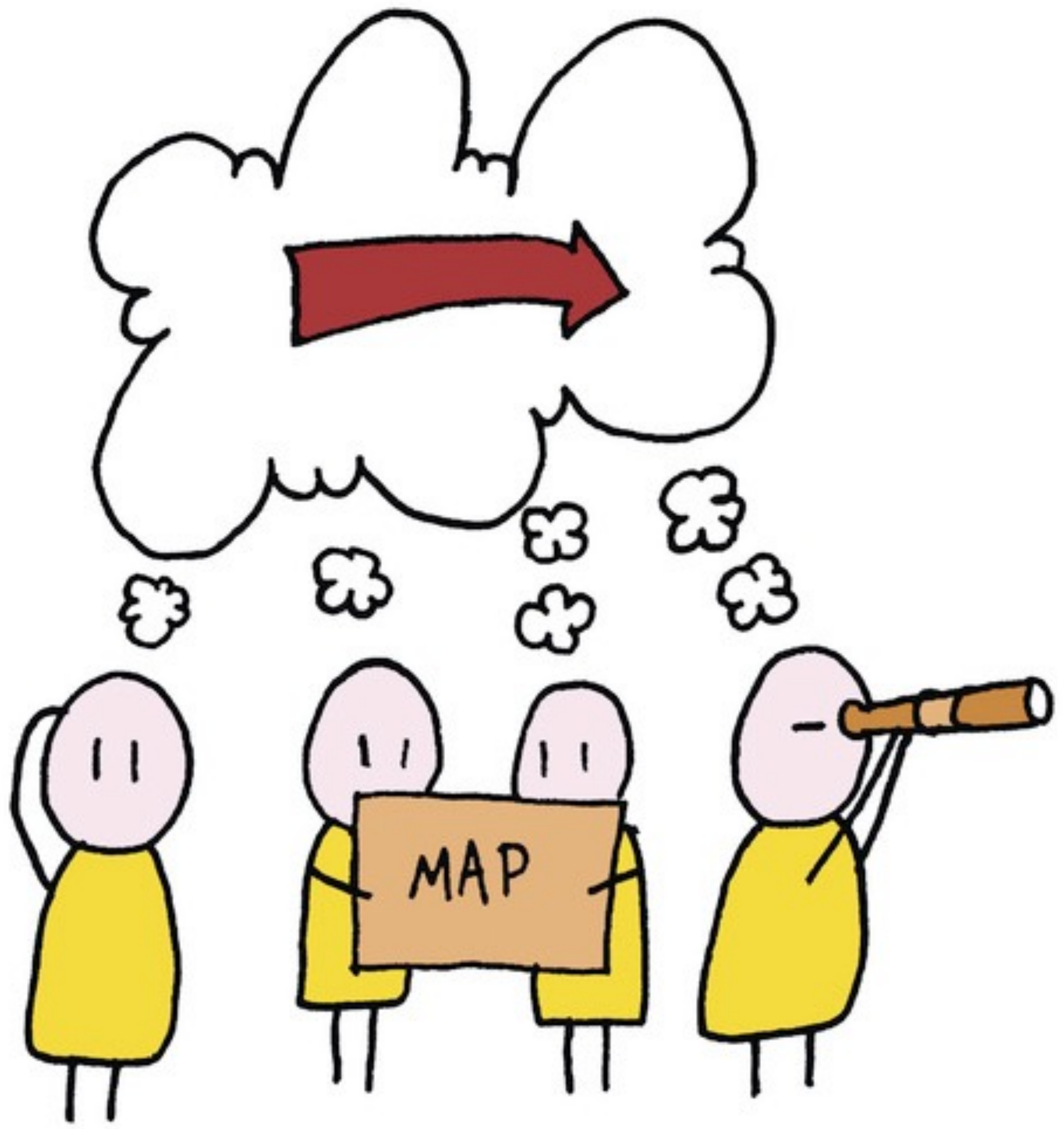

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Image credit: Rene Sorensen

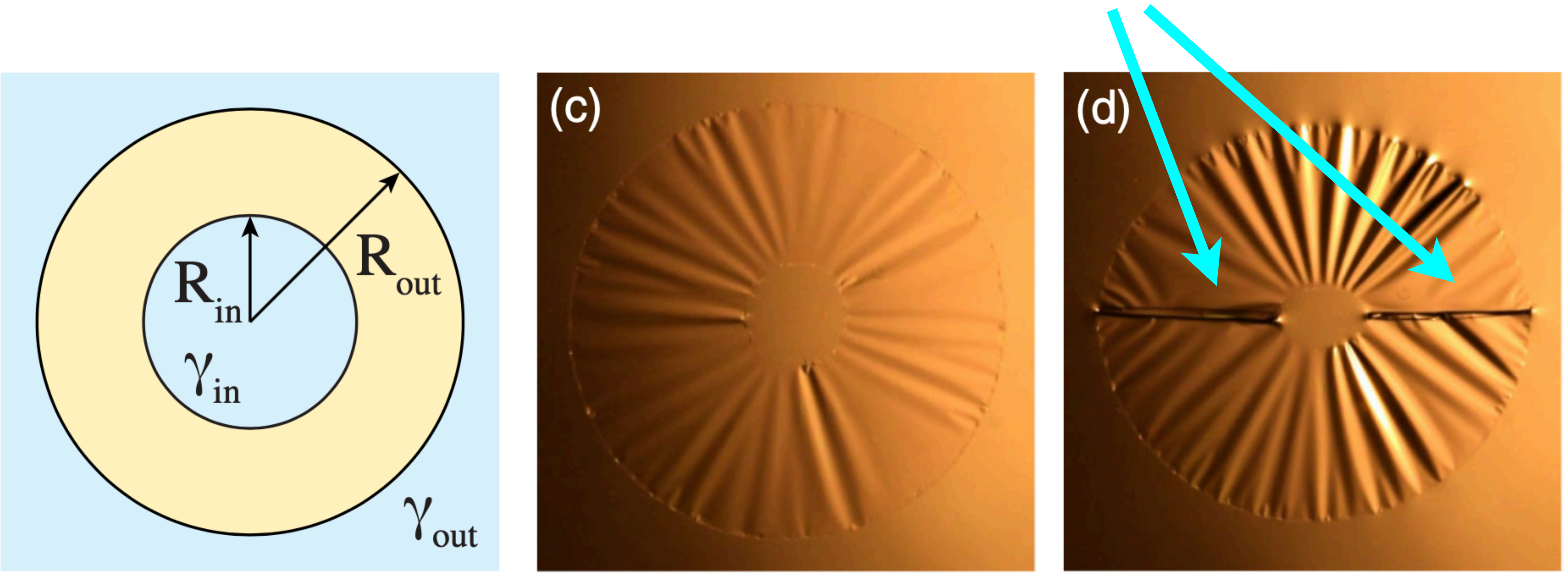
A very reasonable outlook:

The geometric model $U = \gamma A$ is great for explaining gross shapes, forces, **BUT** it doesn't predict small-scale features (wrinkles, crumples, folds), *i.e.*, how excess length is stored

Recent progress:

Annular film on a flat liquid bath:

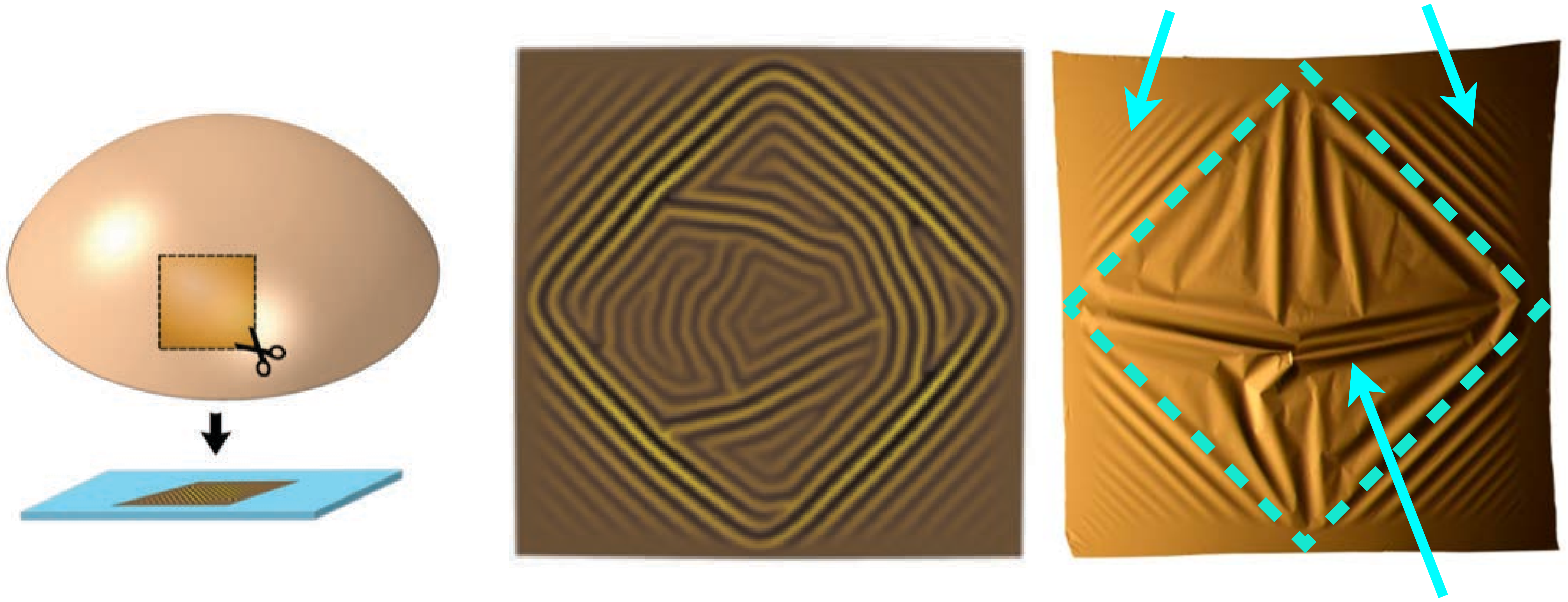
optimal gross shape **implies folds** (vs. wrinkles)



JDP, Démery, Davidovitch, Menon, et al., PRL 118 (2017)

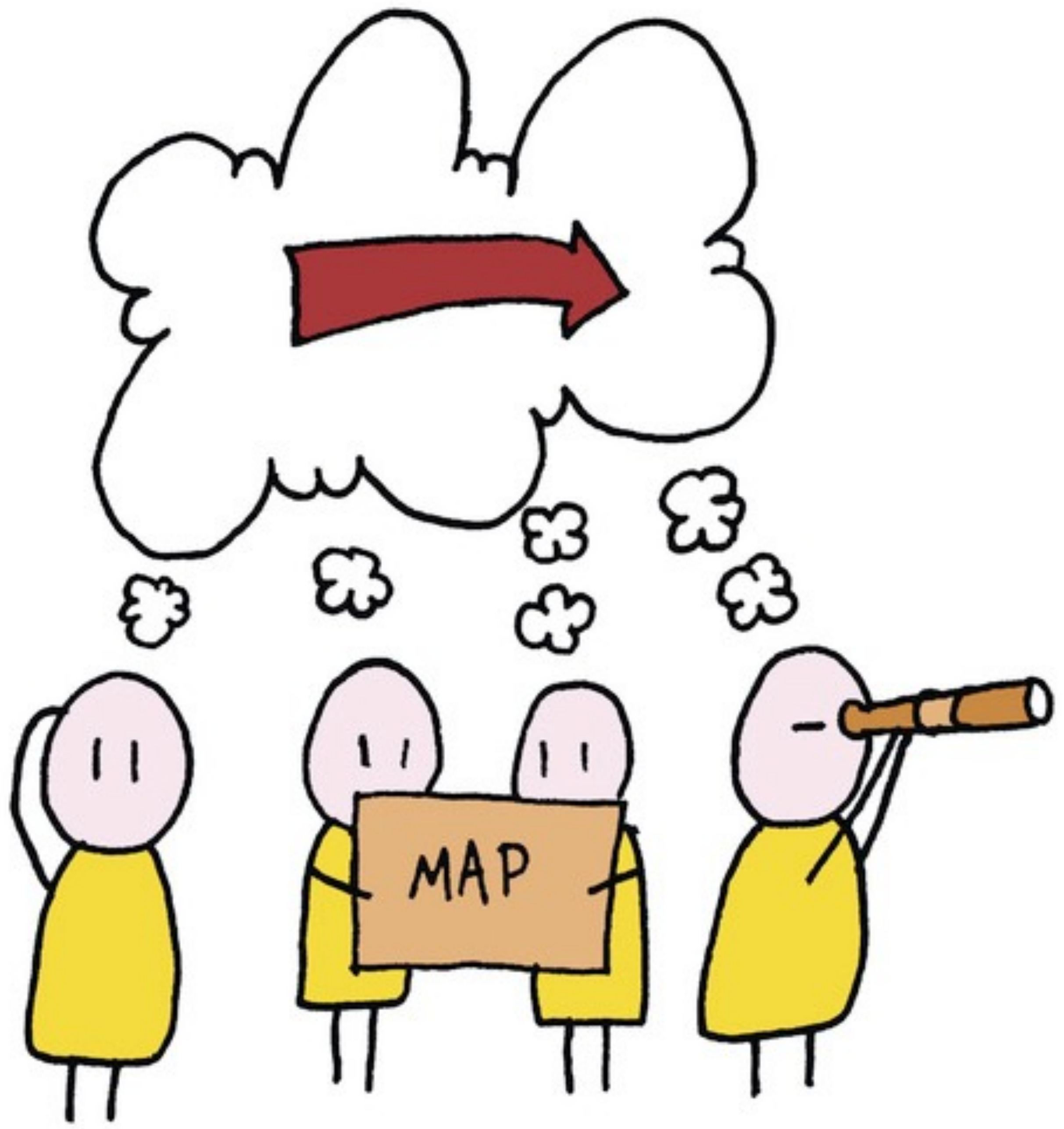
Stamping a curved shell onto a plane:

wrinkle direction **fixed** by optimal gross shape



wrinkles '**free**'

Tobasco, JDP, Katifori, et al. (arXiv 2004.02839)



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Some folding transitions and wrinkle layouts may be understood as a geometric optimization

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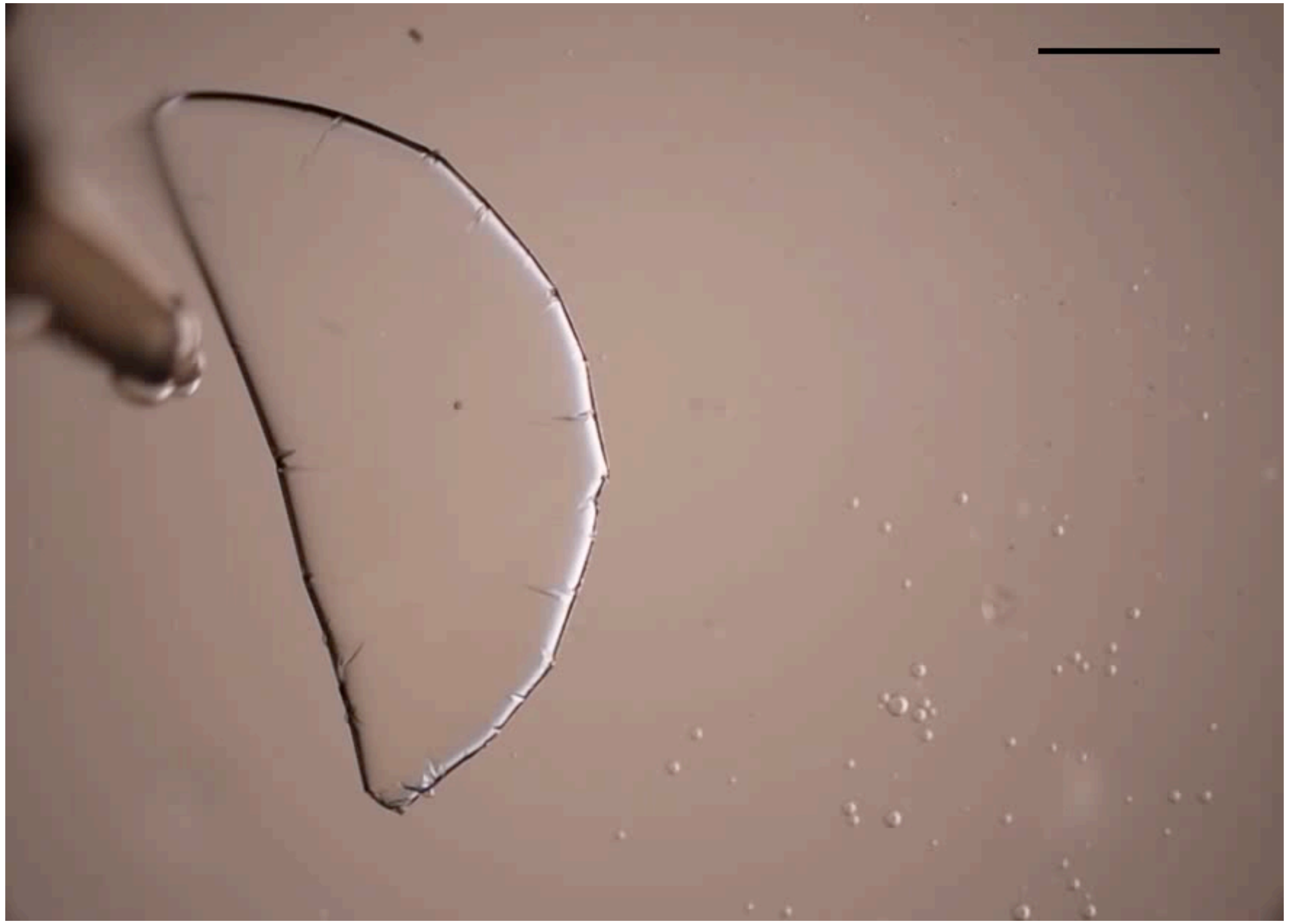
Image credit: Rene Sorensen

Elastic films as **"solid surfactants"**



Complementary to molecular & particle surfactants

Interesting properties for applications:

-Achieve non-spherical shapes

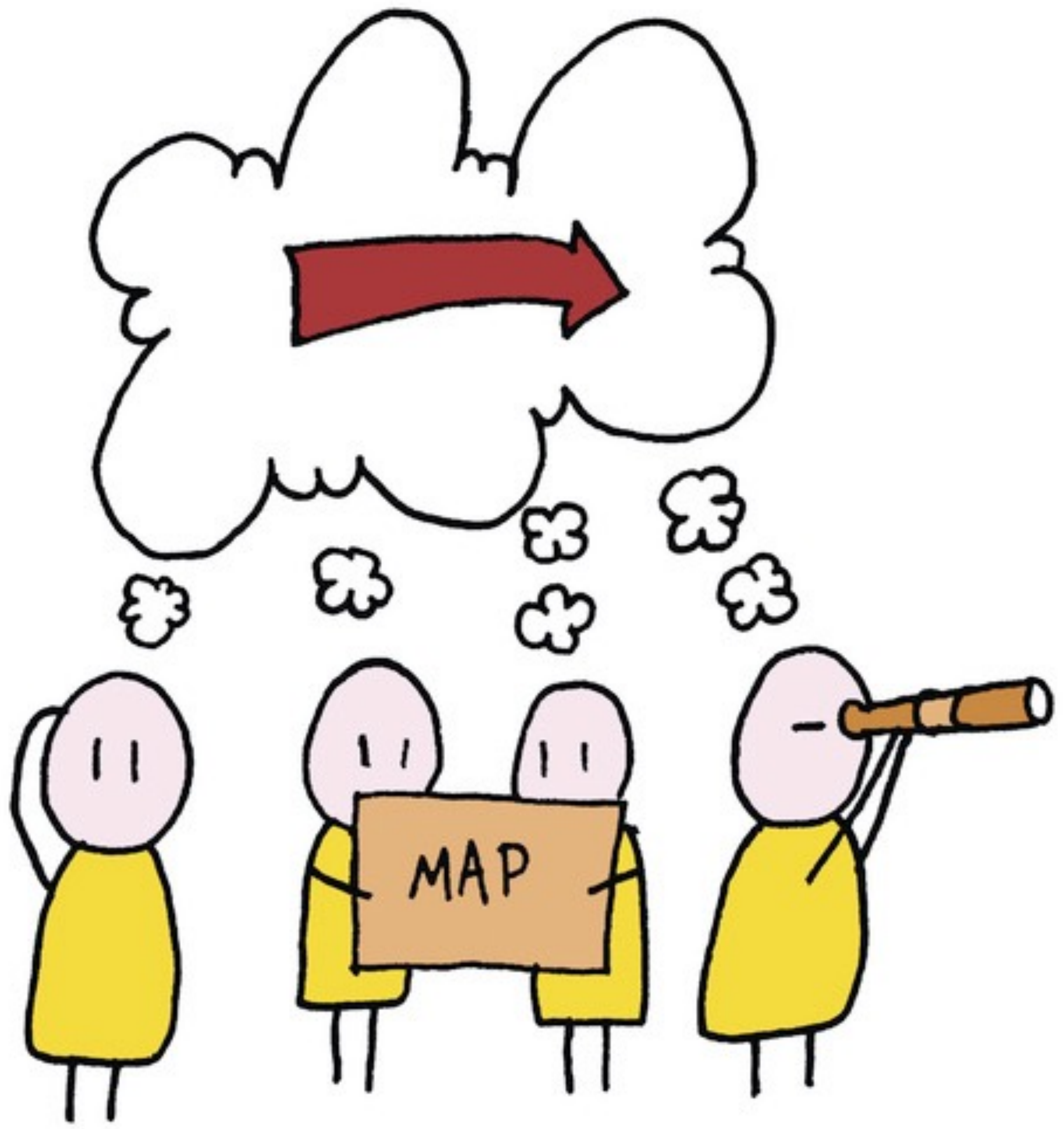
-Tailor mechanical, optical properties

-Platform for surface patterning
(physical, chemical)

-Sequester/protect liquid cargo

Kumar, JDP, Russell, Menon
Science 359 (2018)

Conclusions



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Ultrathin sheets: A tool for tailoring droplets, emulsions, interfaces (shape, rheology, surface chemistry, …)

"Wrapping liquids, solids, and gases in thin sheets"
JDP, Annu. Rev. Condens. Matter Phys. Vol. 10 (2019), **arXiv:1804.07425**