Casimir force between Weyl semimetals in a chiral medium

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Quantum and Thermal Electrodynamic Fluctuations in the Presence of Matter: Progress and Challenges

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Goals of this work

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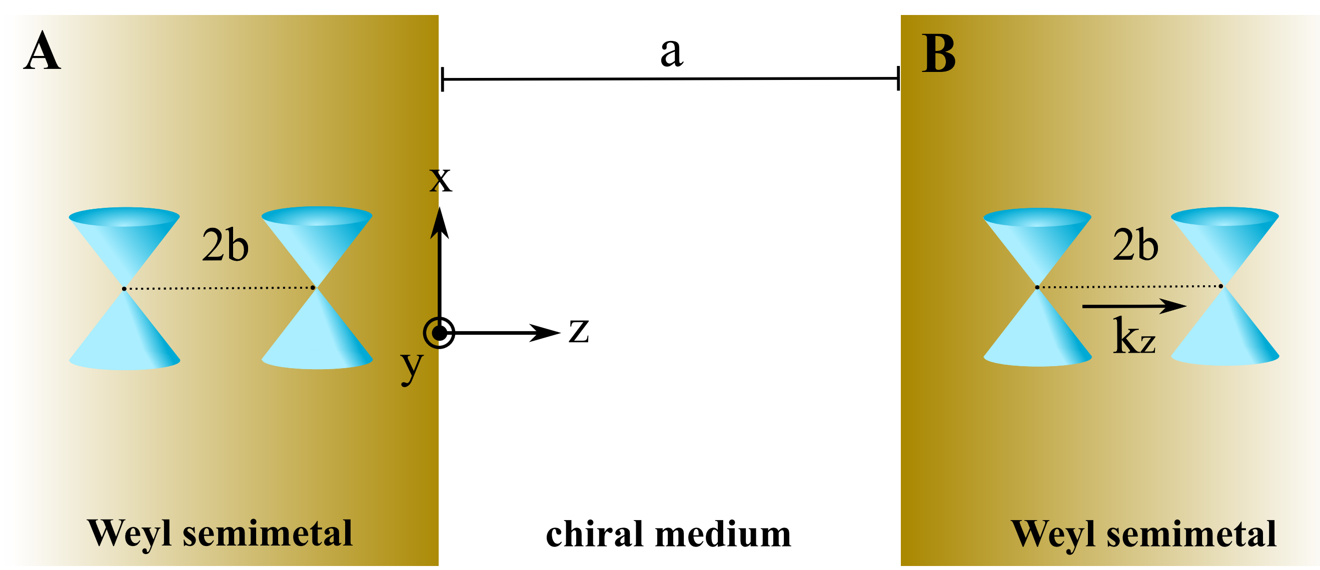


Both **Weyl semimetals** (WSM) and **chiral media** have been shown to give rise to Casimir **repulsion** (or suppression of the attractive force). We consider a combination of both to study potential ways to enhance the repulsion.

Outline

- Small introduction on Casimir Effect and Casimir Repulsion
- Elecrodynamics of Weyl Semimetals (WSM)
- Reflection matrices in the chiral basis
- Chiral media
- Casimir force
- Conclusions and outlook

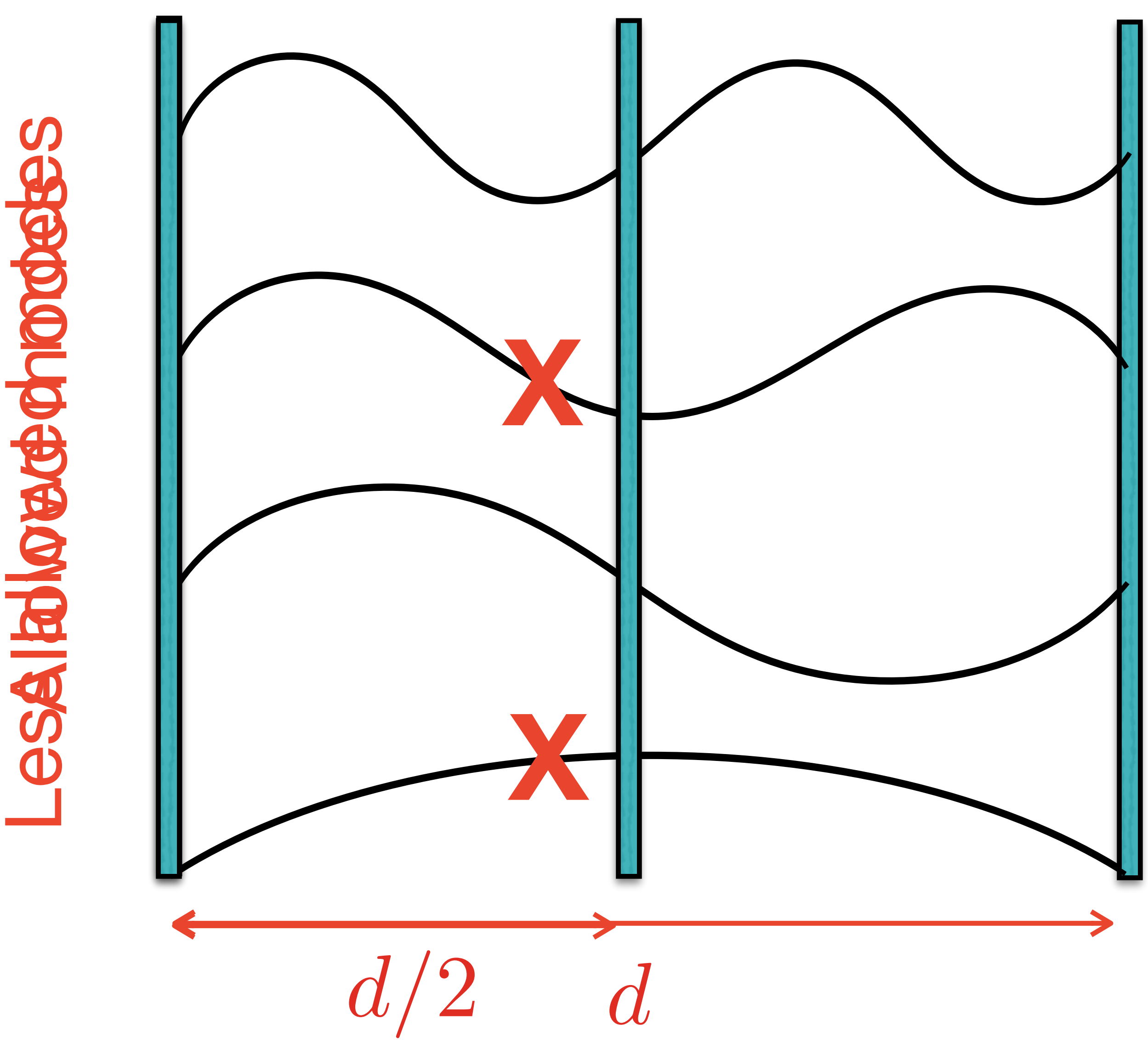
Casimir Effect and Casimir Repulsion

Casimir force

Electromagnetic field

Fourier mode of the EM field with frequency ω ⟷ Cuantization ⟷ **Quantum harmonic oscillator with frequency ω**

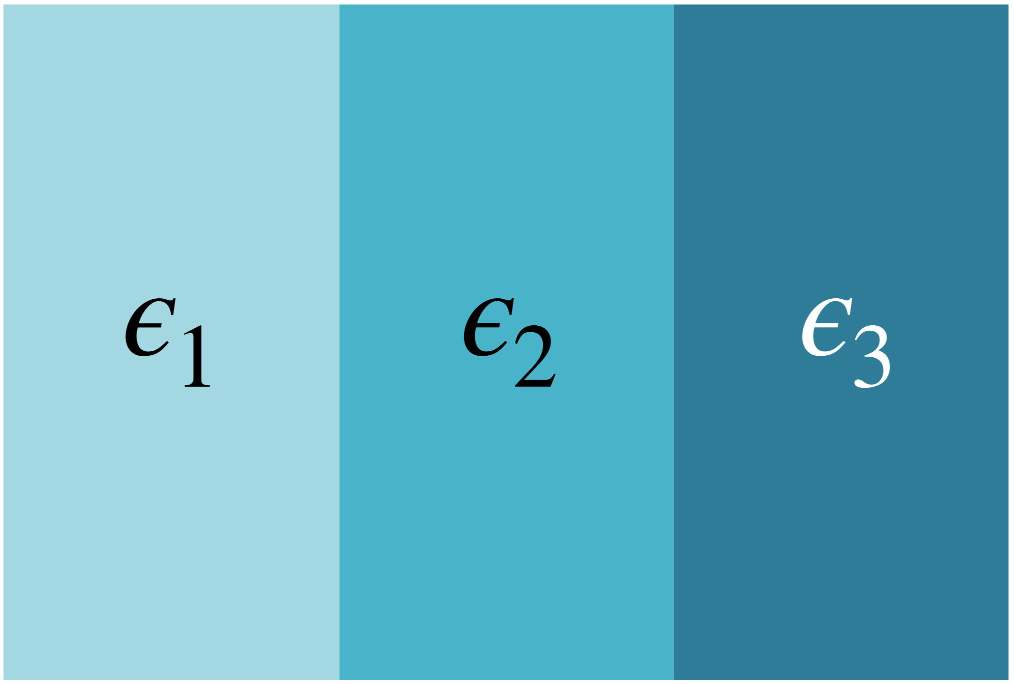
$$E_0 = \frac{1}{2}\hbar\omega$$



$$E(d/2) < E(d)$$

Attractive force!

Casimir Repulsion



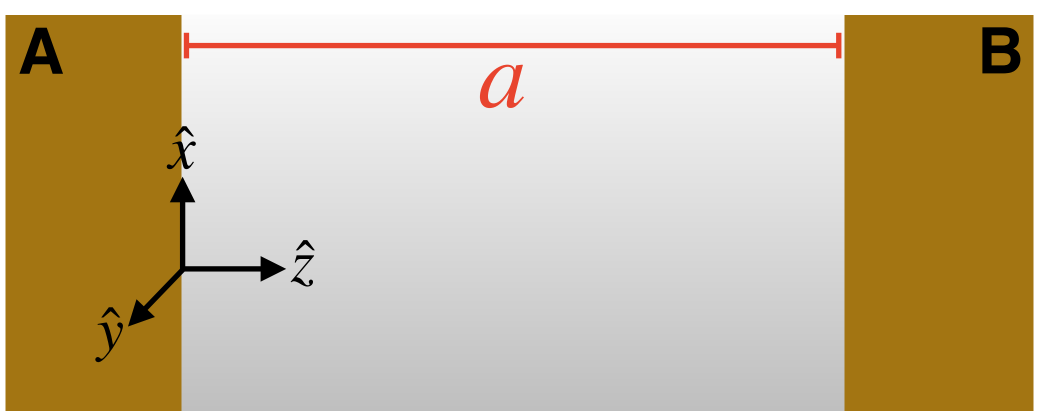
- Was achieved experimentally with $\epsilon_1 < \epsilon_2 < \epsilon_3$ [*Munday et al. Nature* ***457****, 170 (2009)]*
- Proposals on anisotropic and chiral metamaterials
- Topological materials: many proposals, but one needs to be careful to use the 'correct' Lifshitz formula **with a prime on one of the reflection matrices** *[Fialkovsky et al. PRB* ***97****, 165432 (2018)].* Particular proposal with WSM *[Wilson et al. PRB* ***91****, 235115 (2015)].*
- With a Chiral medium between two perfectly conducting plates [*Jiang & Wilczek, PRB* ***99*** *125403 (2019)*], Lifshitz formula calculated with a non-reciprocal Green's function method.

Non-reciprocal media

- Casimir energy between two plane plates separated by a distance *a* in non-reciprocal media [*Jiang 2019*]

$$E_C = \hbar \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2 k_\parallel}{(2\pi)^2} \ln \det(\mathbb{I} - R_B U_{BA} R_A U_{AB})$$

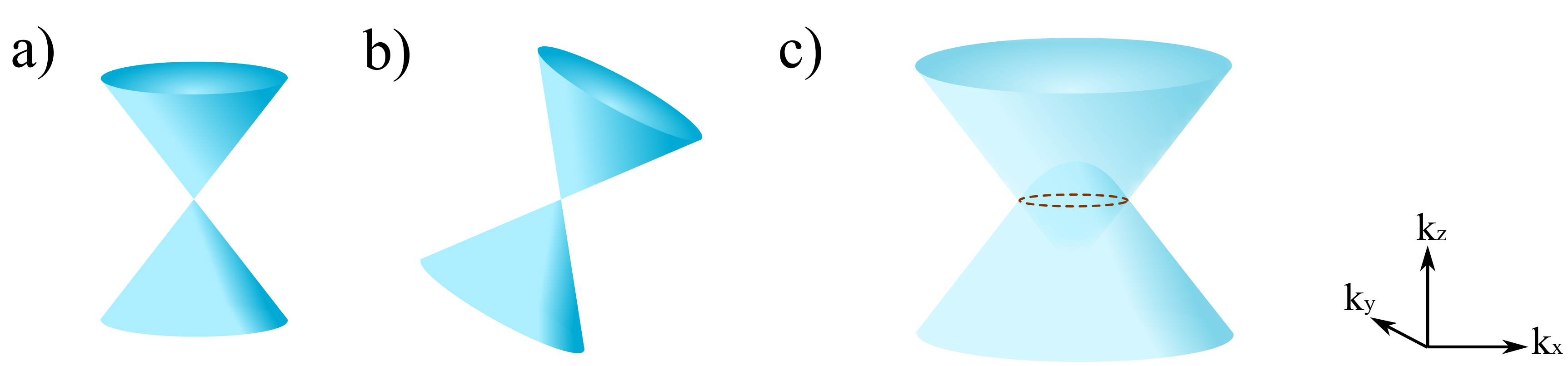
- R_B is the reflection matrix for the plate filling the space $z > a$ and R_A for the plate filling the space $z < 0$
- $U_{AB}\,(U_B A)$ represents the translation matrix from B to A (A to B)



Weyl Semimetals

Weyl Semimetals (WSM)

- 3D analogues of graphene
- Gapless band structure protected by topology and symmetry
- Linearly dispersing low-energy excitations that behave as massless **Weyl fermions** with ≠ chirality
- Weyl nodes where the conduction and valence band touch, come in pairs and require some **symmetry to be broken**
- Break of time-reversal produces a **bulk Hall effect** → non-vanishing Hall conductivity
- Particular optical and electrodynamical properties



(a) untilted and (b) tilted WSM. (c) Nodal Line Semimetal

Electrodynamics of WSM

- Action $S = S_0 + S_A$, the ordinary Maxwell action S_0:

$$S_0 = -\frac{1}{4}\int d^3r\,dt\,F_{\mu\nu}F^{\mu\nu} - \frac{1}{c}\int d^3r\,dt\,A_\nu j^\nu,$$

- Plus an axionic term S_A

fully antisymmetric tensor

$$S_A = \frac{e^2}{32\pi^2\hbar c}\int d^3r\,dt\,\theta(\mathbf{r},t)\,\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta},$$

$\theta(\overrightarrow{b}\cdot\vec{r} - 2b_0 t)$

$2\overrightarrow{b}$ separation between Weyl cones

[*Grushin PRD **86** (2012), Zyuzin & Burkov PRB **86** (2012)*]

Electrodynamics of WSM

- By solving the Euler-Lagrange equations, one finds the modified Maxwell equations inside the material

$$\nabla \cdot \mathbf{E} = \rho - \frac{e^2 b}{2\pi^2 \hbar c} \alpha\, \hat{z} \cdot \mathbf{B},$$

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{1}{c}\mathbf{j} + \frac{e^2 b}{2\pi^2 \hbar c}\hat{z} \times \mathbf{E},$$

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$

Correction to the charge density

we consider only TR symmetry to be broken, $b_0 = 0$, and **b** = b $\hat{z}$

Hall conductivity $\sigma_{xy} = -\sigma_{yx} = \dfrac{e^2 b}{2\pi^2 \hbar}$

Dispersion relation and solutions

- Solve the modified ME in Fourier space with planes wave ansatz
- We can introduce an effective dielectric function by defining the displacement field **D**(**k**,ω)=$\epsilon(\omega)$**E**(**k**,ω) with

$$\epsilon(\omega) = \begin{pmatrix} \epsilon_0 & i\sigma_{xy}/\omega & 0 \\ -i\sigma_{xy}/\omega & \epsilon_0 & 0 \\ 0 & 0 & \epsilon_0 \end{pmatrix},$$

the ferromagnetic WSM is a gyrotropic medium with the gyrotropic parameter proportional to the separation *b* of the Weyl cones

In the frequencies relevant for our work we can take $\epsilon_0 \approx 1$

- Dispersion relation: $$\omega_\pm^2(\mathbf{k}) = k^2 + \frac{\sigma_{xy}^2}{2} \pm \sqrt{k_z^2\sigma_{xy}^2 + \frac{\sigma_{xy}^4}{4}}.$$

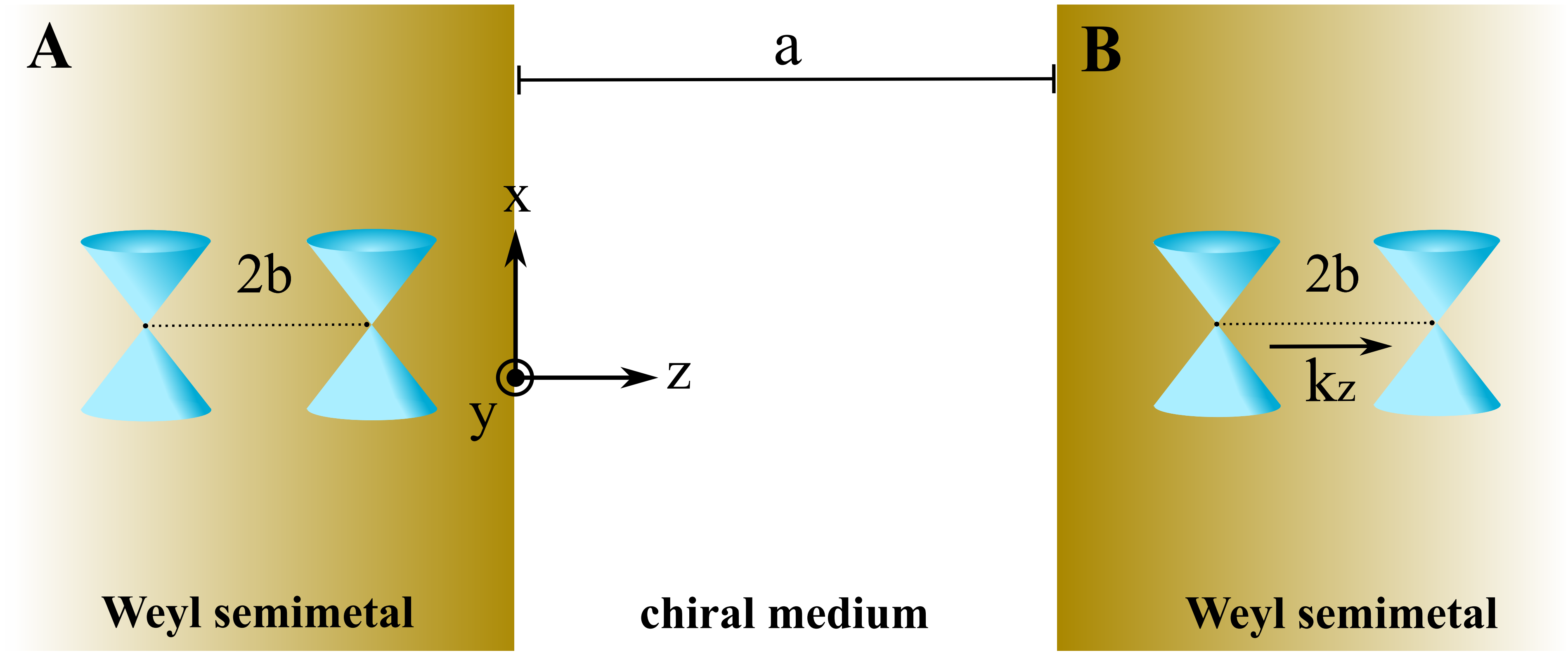
- Unnormalized polarization vectors (TE/TM basis $\hat{e}_1 = \hat{y} \times \hat{k}$, $\hat{e}_2 = \hat{y}$):

$$\mathbf{D}_\pm = \omega_\pm k_z (k^2 - \omega_\pm^2)\hat{e}_1 - ik\sigma_{xy}\left(\omega_\pm^2 - k_x^2\right)\hat{e}_2,$$

Reflection matrices in the chiral basis

Our system

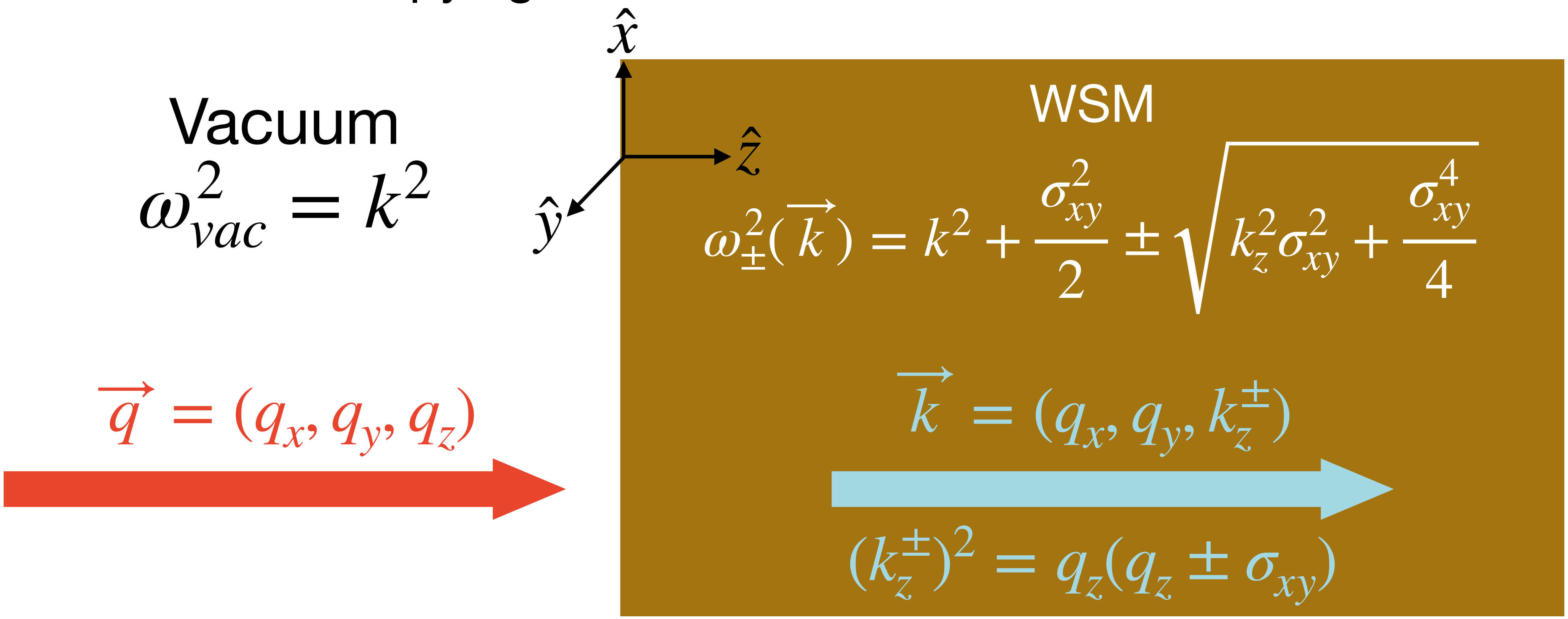
- Our system will consist on two semi-infinite WSM separated by a distance *a*, and the gap between them filled by a chiral medium (more on this later)



- So to calculate the Casimir force using the Lifshitz formula, we need the **reflection matrices**

Reflection matrix

- For a WSM occupying $z > 0$:



- For $q_z < \sigma_{xy}$ $\rightarrow$ only one polarization propagating inside the material, (the other is an **evanescent wave)**
- The two polarizations for the transmitted field are

$$\mathbf{e}^{\pm}(\mathbf{q}) = q_z^2\hat{x} \mp i\omega q_z\hat{y} - q_x k_z^{\pm}\hat{z}.$$

The chiral basis

- Inside the WSM there is only one polarization for each $\hat{k}_{\pm}$
- Outside, due to the rotational symmetry of our system, the **chiral basis** is the most appropiate choice:

waves propagating from left to right (A to B)

$$\hat{e}_{R,L} = \frac{1}{2}(\hat{e}_1 \pm i\hat{e}_2) = \frac{1}{\sqrt{2}}\left(\frac{q_z}{q}\hat{x} \pm i\hat{y} - \frac{q_x}{q}\hat{z}\right),$$

$$\hat{e}'_{R,L} = \frac{1}{2}(\hat{e}'_1 \pm i\hat{e}'_2) = \frac{1}{\sqrt{2}}\left(-\frac{q_z}{q}\hat{x} \pm i\hat{y} - \frac{q_x}{q}\hat{z}\right),$$

waves propagating from right to left (B to A)

- When the direction of propagation is reversed, the correct right-handed set of vectors that form the basis changes

The reflection matrix

- We write the incoming field $\overrightarrow{E}_0$ in the $\hat{e}_{L,R}$ basis
- The reflected field $\overrightarrow{E}_r$ in the $\hat{e}'_{L,R}$ basis
- Transmitted field $\overrightarrow{E}_{\pm}$ will be in the direction determined by its wavevector $\hat{k}_{\pm}$, $\hat{e}_{\pm}$
- We find the reflection matrix which in the chiral basis has an off-diagonal form:

$$R(q_z) = \frac{1}{\sigma_{xy}} \begin{pmatrix} 0 & \sigma_{xy} + 2k_z^- - 2q_z \\ \sigma_{xy} - 2k_z^+ + 2q_z & 0 \end{pmatrix}$$

- $R(q_z)$ **acts on a vector in the basis** $\hat{e}_{L,R}$ and **returns a vector in the** $\hat{e}'_{L,R}$ basis

The reflection matrix

- For the **mirrored system**, that is, a WSM occupying the $z < 0$ region, we need to again write incoming, reflected and transmitted waves

- We get

$$R'(q_z) = \frac{1}{\sigma_{xy}} \begin{pmatrix} 0 & \sigma_{xy} - 2k_z^+ + 2q_z \\ \sigma_{xy} + 2k_z^- - 2q_z & 0 \end{pmatrix}$$

- which we need to calculate the Casimir energy through the Lifshitz formula

Wick rotation

- In order to use these matrices in the Lifshitz formula, we need to rotate them to imaginary frequencies
- It was done but assuming $\sigma_{xy} > 0$
- We will contemplate as well the case $\sigma_{xy} < 0$ because the sign of the Hall conductivity changes if the sample is mirrored in space
- We get:

$$R(ip_z) = \begin{pmatrix} 0 & \mathcal{R}(p_z) \\ \mathcal{R}^*(p_z) & 0 \end{pmatrix}, \qquad R'(ip_z) = \begin{pmatrix} 0 & \mathcal{R}^*(p_z) \\ \mathcal{R}(p_z) & 0 \end{pmatrix}.$$

- With:

$$\mathcal{R}(p_z) = 1 - g(p_z) + ih(p_z),$$

$$g(p_z) = \frac{1}{|\sigma_{xy}|}\sqrt{2p_z\left(\sqrt{p_z^2+\sigma_{xy}^2} - p_z\right)},$$

$$h(p_z) = \frac{1}{\sigma_{xy}}\sqrt{2p_z\left(\sqrt{p_z^2+\sigma_{xy}^2} + p_z\right)} - 2\frac{p_z}{\sigma_{xy}}.$$

Chiral materials

Chiral materials

- Eigenmodes are not TE-TM waves, but chiral states
- Photons with $\neq$ chiralities propagate at $\neq$ velocities
- Broken inversion symmetry
- Translation matrices for a photon going from A to B and B to A:

$$U_{BA} = \begin{pmatrix} e^{ik_L^+ a} & 0 \\ 0 & e^{ik_R^+ a} \end{pmatrix}, \quad U_{AB} = \begin{pmatrix} e^{ik_L^- a} & 0 \\ 0 & e^{ik_R^- a} \end{pmatrix},$$

- $k_R^\pm$ ($k_L^\pm$) are the z component of the wave vectors of **right** (**left**) circularly polarized photons travelling in the **+** (A → B, left to right) direction or **-** (B → A, right to left) direction
- In vacuum $k_R^\pm$ and $k_L^\pm$ are all the same
- Chiral basis: Natural choice to work both with chiral materials and WSMs

Chiral materials

Chiral media

Optically active materials

- Photons with $\neq$ chirality propagate with $\neq$ velocity
- **Independently of their direction of propagation**
- $k_R^{\pm} = ip_z + \delta k_z$
- $k_L^{\pm} = ip_z - \delta k_z$
- $\delta k_z = \alpha_0 \rho$
- α_0 specific rotation
- δ mass concentration of optically active molectules

Faraday materials

- Faraday effect
- Photon propagation velocity depends on their **chirality** and their **direction of propagation.**
- $k_R^{\pm} = k_L^{\mp} = ip_z \pm \delta k_z$
- $\delta k_z = \mathscr{V} B$
- $\mathscr{V}$ Verdet constant
- B magnetic field in the direction of propagation

Chiral materials

Chiral media

Optically active materials

No relevant for our work (all dependence on the material vanishes for the Casimir energy)

- $k_L^{+} = ip_z - \delta k_z$
- $\delta k_z = \alpha_0 \rho$
- α_0 specific rotation
- δ mass concentration of optically active molectules

Faraday materials

- Faraday effect
- Photon propagation velocity depends on their **chirality** and their **direction of propagation.**
- $k_R^{\pm} = k_L^{\mp} = ip_z \pm \delta k_z$
- $\delta k_z = \mathscr{V} B$
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- B magnetic field in the direction of propagation

Casimir energy from the non-reciprocal Lifshitz formula

Casimir energy

$$E_C = \hbar \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2k_\parallel}{(2\pi)^2} \ln\det(\mathbb{I} - R_B U_{BA} R_A U_{AB})$$

$$R_B(ip_z) = \begin{pmatrix} 0 & \mathcal{R}_B(p_z) \\ \mathcal{R}_B^*(p_z) & 0 \end{pmatrix},$$

$$U_{AB} = \begin{pmatrix} e^{ik_L^- a} & 0 \\ 0 & e^{ik_R^- a} \end{pmatrix},$$

$$U_{BA} = \begin{pmatrix} e^{ik_L^+ a} & 0 \\ 0 & e^{ik_R^+ a} \end{pmatrix},$$

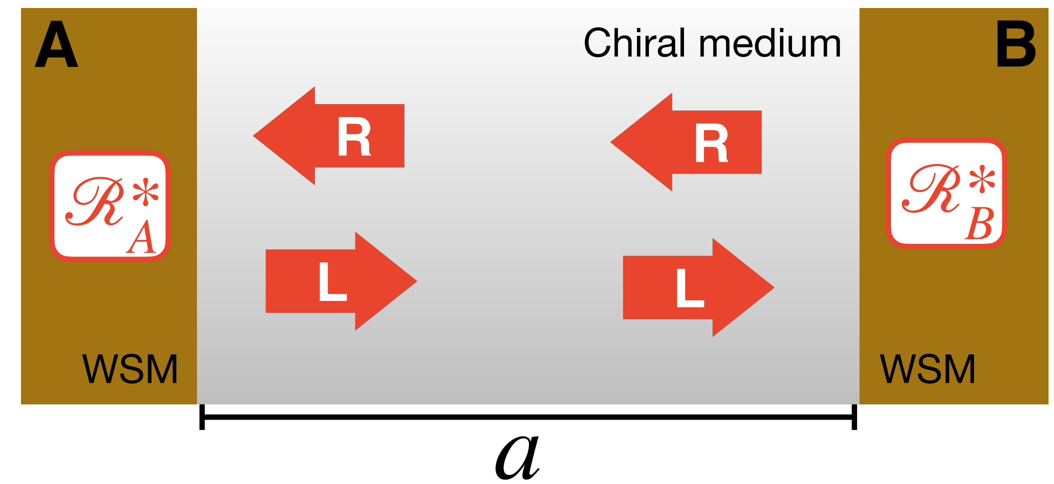
$$R_A(ip_z) = \begin{pmatrix} 0 & \mathcal{R}_A^*(p_z) \\ \mathcal{R}_A(p_z) & 0 \end{pmatrix},$$

$$\mathscr{R}_{A,B}(p_z) = \mathscr{R}(\sigma_{xy} = \sigma_{xy}^{A,B})$$

Casimir energy

$$E_C = \hbar \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2 k_\parallel}{(2\pi)^2} \ln\det(\mathbb{I} - R_B U_{BA} R_A U_{AB})$$

becomes diagonal



$$\begin{pmatrix} 1 - \mathcal{R}_B \mathcal{R}_A\, e^{i(k_R^+ + k_L^-)a} & 0 \\ 0 & 1 - \mathcal{R}_B^* \mathcal{R}_A^*\, e^{i(k_L^+ + k_R^-)a} \end{pmatrix}$$

A WSM acts as a regular **imperfect mirror** for chiral photons, but the reflection coefficient **depends on their chirality.**
(Linear TE/TM photons = mixed polarizations)

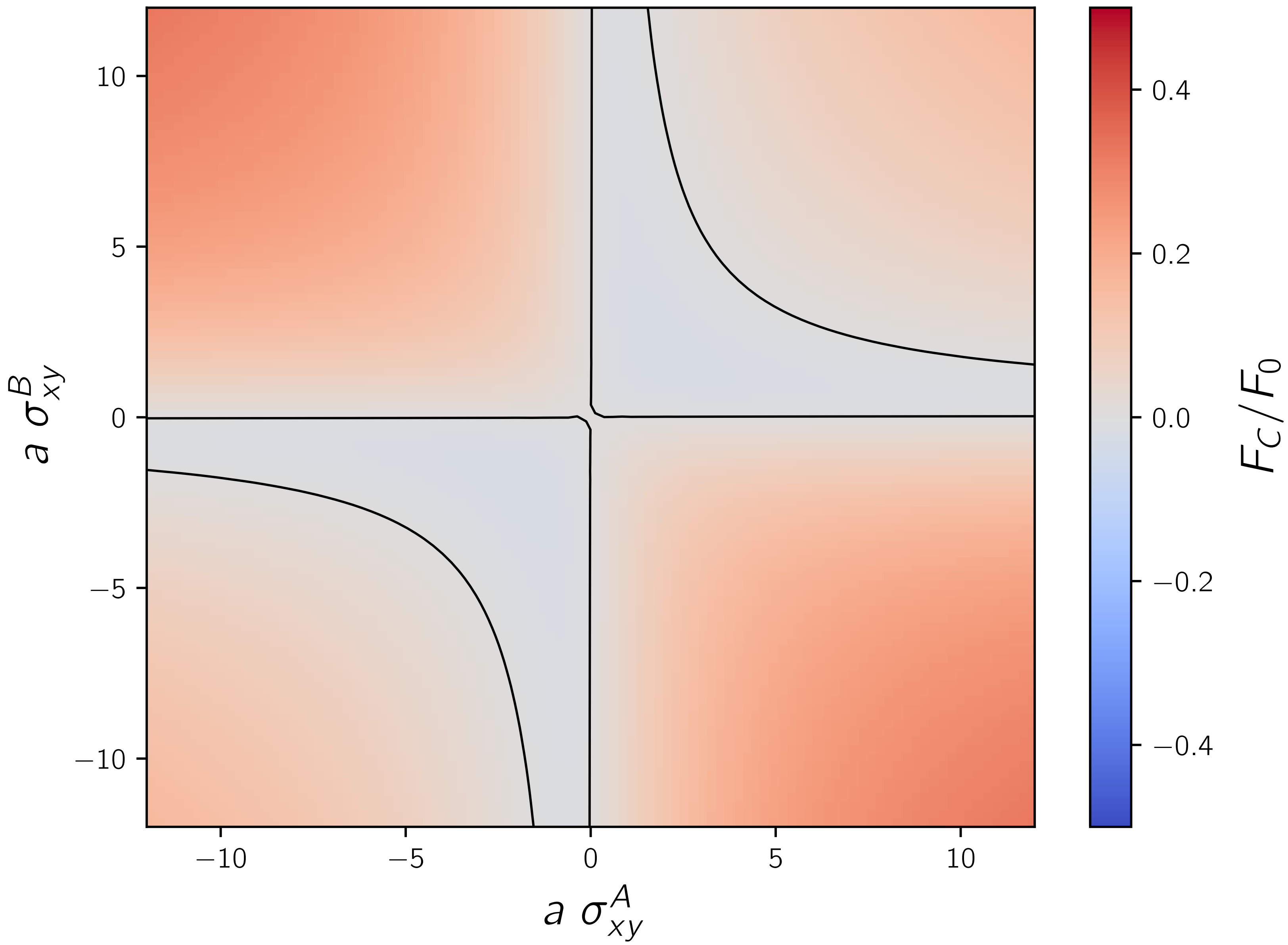
Effect of chiral media

$$\begin{pmatrix} 1 - \mathcal{R}_B \mathcal{R}_A \boxed{e^{i(k_R^+ + k_L^-)a}} & 0 \\ 0 & 1 - \mathcal{R}_B^* \mathcal{R}_A^* \boxed{e^{i(k_L^+ + k_R^-)a}} \end{pmatrix}$$

- **Optical active media:** $k_R^\pm = ip_z + \delta k_z$ and $k_L^\pm = ip_z - \delta k_z$
  ➡ the dependence on the characteristics of the material (given by δk_z) vanishes
- **Faraday materials**: $k_R^\pm = k_L^\mp = ip_z \pm \delta k_z$
  ➡ $e^{i(k_R^+ + k_L^-)a} = e^{-2p_z a} e^{2i\delta k_z a}$ and $e^{i(k_L^+ + k_R^-)a} = e^{-2p_z a} e^{-2i\delta k_z a}$
  ➡ an added phase shift at each reflection

Casimir force

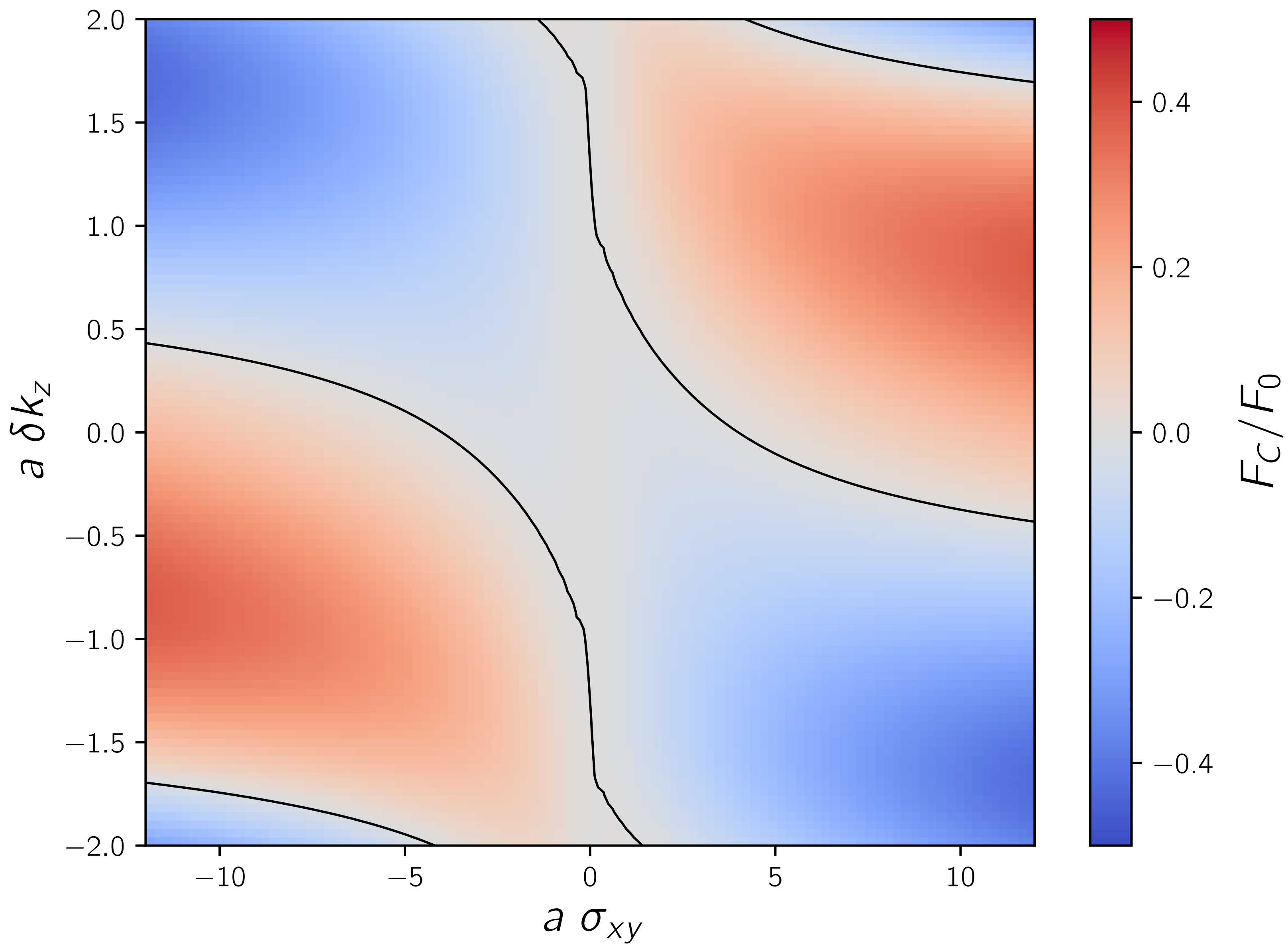
Casimir force in vacuum



- Force between two **different** WSMs separated by a **vacuum** gap of lenght *a*. **Red** areas indicate **attraction,** and **blue** areas indicate **repulsion**.
- **Same sign** of the Hall conductivity is a **necessary condition** for repulsion.
- Flipping the sample changes the separation of Weyl cones and hence the sign of σ_{xy}, and it changes the force from attractive to repulsive

- Even though **repulsion** can be achieved in vacuum, its magnitude is only about **5% of the magnitude** of the Casimir force between two perfectly conducting plates. A strong **suppression of the attractive force** can be achieved though.

Casimir force in a Faraday material



- Force between two WSM with the same **Hall conductivity** WSMs separated by a gap filled with a **Faraday material**. **Red** areas indicate **attraction,** and **blue** areas indicate **repulsion**.
- For a fixed distance between the plates, we can see stronger repulsive forces of up to **50% of the magnitude of the static Casimir force** between two perfect mirrors.

- The sign of δk_z determines which chirality propagates faster than the other. It can be changed by flipping the orientation of the external magnetic field. We see that **repulsion in enhanced** when δk_z and σ_{xy} have opposite signs.

Force as a function of the distance

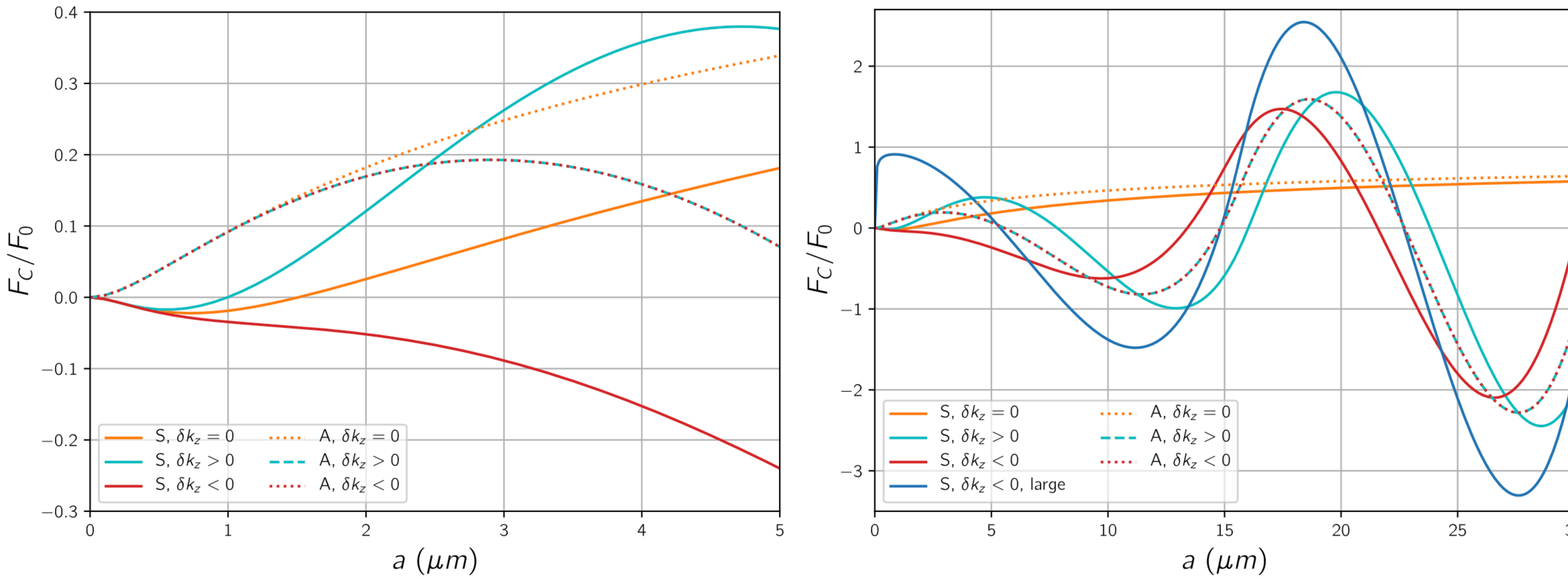


FIG. 4. Casimir force as a function of the distance, for long and short distances. The $\delta k_z = 0$ case corresponds to vacuum filling the gap between the two WSMs. The nonvanishing δk_z are set to be $\delta k_z = \pm 2 \times 10^5\ \mathrm{m}^{-1}$, and the values of the Hall conductivities of the WSMs are such that $\sigma_{xy}^A/c = 2.65 \times 10^6\ \mathrm{m}^{-1}$. The curves marked with a *S* are the ones corresponding to a symmetric configuration where $\sigma_{xy}^A = \sigma_{xy}^B$, while *A* stands for the antisymmetric configuration with $\sigma_{xy}^A = -\sigma_{xy}^B$. The line marked "*large*," shown in the plot for large distances, corresponds to a value $\sigma_{xy}^A/c = 2650 \times 10^6\ \mathrm{m}^{-1}$ and illustrates the perfect conductor limit.

- S: = Hall conductivity, A: ≠ Hall conductivity
- Short distances: = σ_{xy} is needed, Faraday material + external magnetic field opposite to the splitting of the Weyl cones result in an enhancement of friction.
- Large distances: the presence of the chiral medium creates oscillations since it introduces a phase shift $e^{\pm 2i\delta k_z a}$ at each reflection

Conclusions

Conclusions and outlook

- We studied the role of chirality in the Casimir force both on the plates (reflection matrices) and on the gap (translation matrices)
- We found that each photon sees the WSM as an imperfect mirror with a Fresnel coefficient dependent on its chirality
- We found that filling the gap with an optically active media has no effect on the Casimir force between two WSMs
- We studied the Casimir interaction between two WSM allowing them to be different, and found that a flip on a sample might switch from an attractive to a repulsive regime
- Found that a Faraday material in the gap might lead to a substantial enhancement of the repulsive force.
- This system allows for the manipulation of several parameters, some intrinsic to the material employed ($|\sigma_{xy}|$, $\mathscr{V}$) but also some external ones like the orientation of the plates or the direction of an external magnetic field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