Anomalous near-field heat transport in many-body systems

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LABORATOIRE CHARLES FABRY

cnrs advancing the frontiers

KITP-Lecture1-Emerging Regimes and Implications of Quantum and Thermal Fluctuational Electrodynamics

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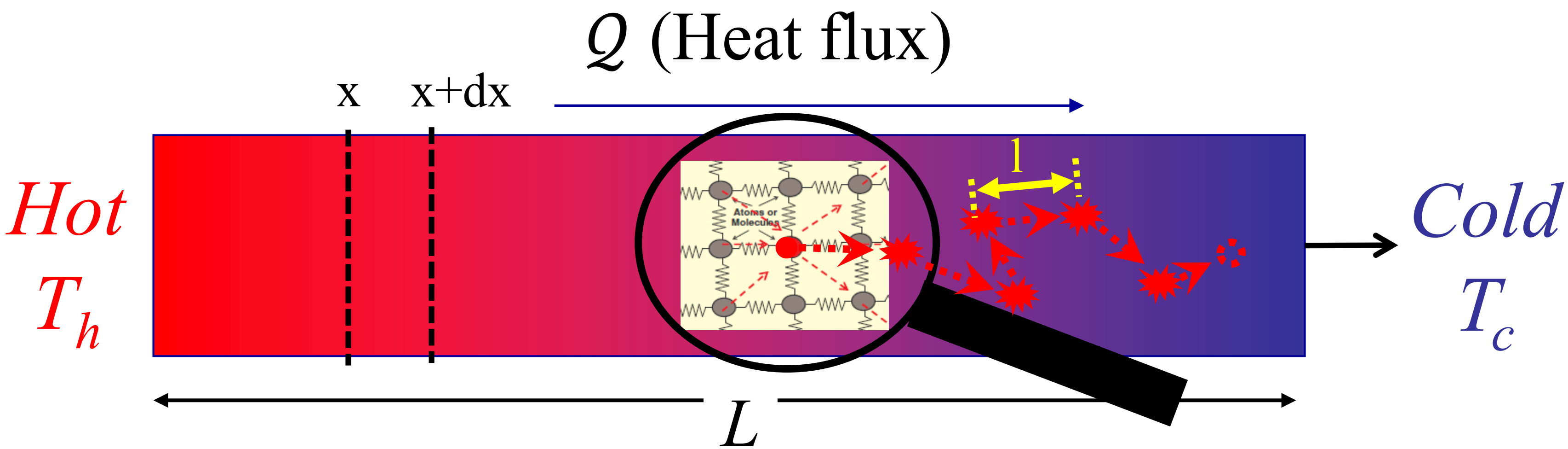
Heat transport in bulk materials : phenomenological approach



L>>l (~200 nm silicone at 300K)

Energy conservation (without source/sink)

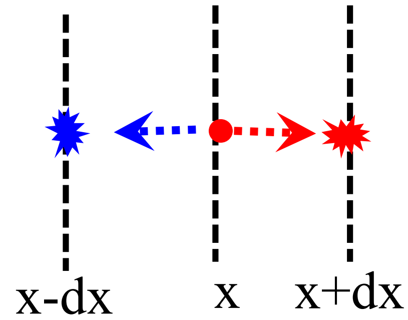
$$\frac{\partial e(x,t)}{\partial t} S dx = S[Q(x) - Q(x+dx)] \approx -S dx \frac{dQ}{dx}$$

$$Q = -\lambda \frac{dT}{dx}$$

(Fourier's law)

$$\frac{\partial T(x,t)}{\partial t} = D \frac{d^2 T}{dx^2}$$

Heat transport in bulk materials : random walk model



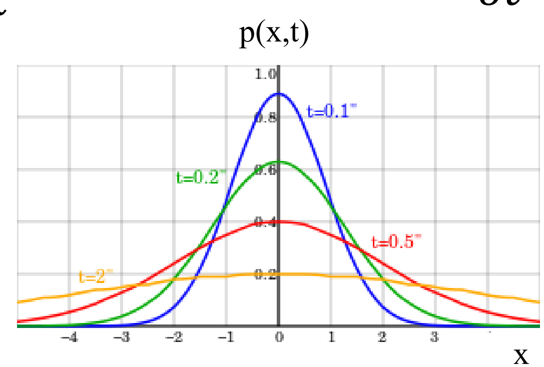
Probability to move to the right (resp. left): α (resp. β)

$$p(x,t+\delta t) = \alpha\, p(x+dx,t) + \beta\, p(x-dx,t) + (1-\alpha-\beta)\, p(x)$$

Assuming $\alpha = \beta$ (non asymmetry) and making Taylor expansions

$$\frac{\partial p(x,t)}{\partial t} = D\,\frac{\partial^2 p}{\partial x^2} \quad \text{with} \quad D = \alpha\,\frac{dx^2}{\delta t}$$

$$p(x,t) = \frac{1}{\sqrt{4\pi D t}}\, e^{-\frac{x^2}{4Dt}}$$

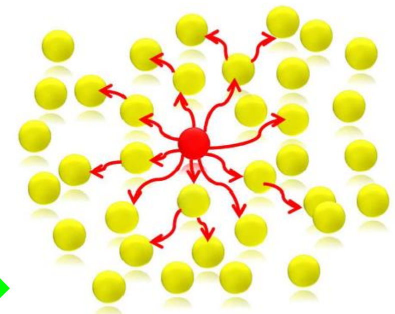


$$M_n \equiv \int p(x,t)\, x^n\, dx < C$$

(all momentum are bounded)

$$M_2 \equiv var = 2Dt$$

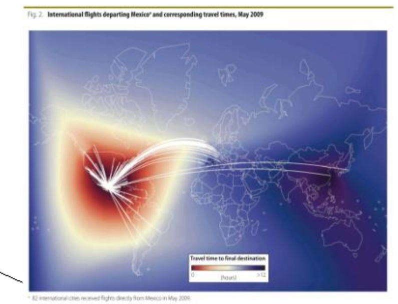
How heat spreads in many body systems?

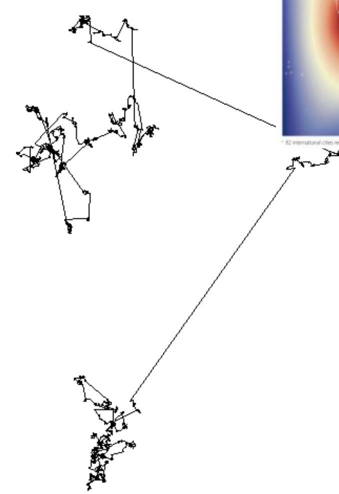


Nanopartilcle network

Gaussian
(diffusive)

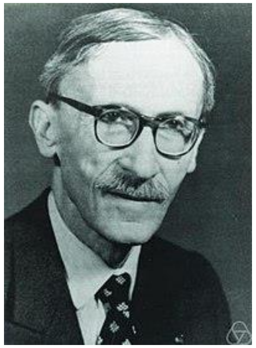
Anomalous
(sub/superdiffusive)





(Fourier)

Diffusive or not diffusive?



(Levy)

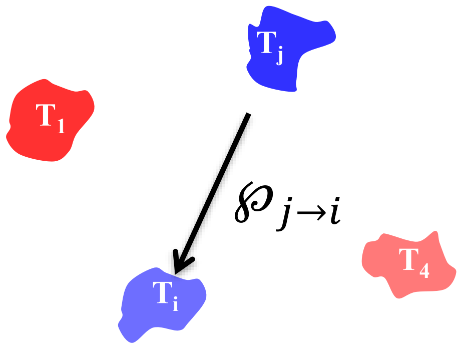
Outline

- Near-field heat transport in many body systems
  - Landaue formalism, summation rules and supercurrent
  - Heat transport as a generalized random walk
- Identifying the regimes of transport by analyzing the thermal conductance scaling
  - Heat transport in diluted systems
  - Heat transport in dense systems
- Open problems and concluding remarks

Heat transfer in many-body systems

Energy balance (neglecting the background contribution)

$$\frac{dT_i}{dt} = \sum_{j \neq i} \wp_{j \to i}\,(T_1, \ldots, T_N)$$



Using the Landauer formalism (small objects)

$$\wp_{j \to i} = \int_0^{\infty} [\theta(T_j, \omega)\Im_{ji}(\omega) - \theta(T_i, \omega)\Im_{ij}(\omega)]\,\frac{d\omega}{2\pi}$$

with the transmission coefficient (arbitrary non-reciprocal materials)

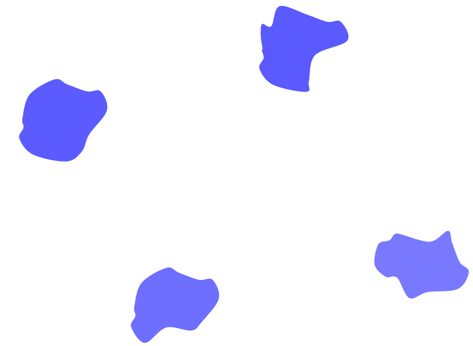
$$\Im_{ji}(\omega) = \frac{4}{3}\left(\frac{\omega}{c}\right)^4 Im\,Tr[\boldsymbol{\alpha}_i \mathcal{G}_{\boldsymbol{ij}} \frac{\boldsymbol{\alpha}_j - \boldsymbol{\alpha}_j^{\dagger}}{\boldsymbol{2i}} \mathcal{G}_{\boldsymbol{ij}}^{\dagger}]$$

Polarizability Full Green

PRL **107**, 114301(2011), RMP **93**, 025009 (2021)

Summation rules in many-body systems

At equilibrium (T_i=T_j=T=Cte)



$$\sum_{j\neq i} \wp_{j\rightarrow i}=0$$

Hence

$$\int_0^\infty \theta(T,\omega)\left[\sum_{j\neq i}\Im_{ji}(\omega)-\sum_{j\neq i}\Im_{ij}(\omega)\right]\frac{d\omega}{2\pi}=0$$

This relation holds for any temperature

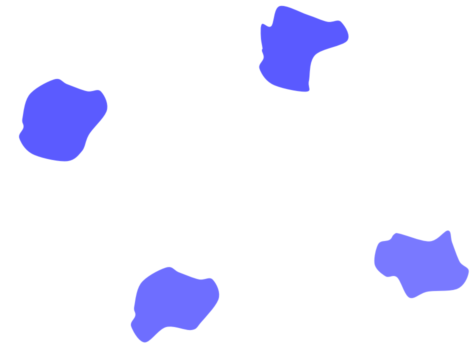
$$\sum_{j\neq i}\Im_{ji}(\omega)=\sum_{j\neq i}\Im_{ij}(\omega) \qquad \forall\, i=\overline{1,N}$$

PRL 118, 173902 (2017)

In 2 body systems

$$\Im_{12}(\omega)=\Im_{21}(\omega)$$

Supercurrent in non-reciprocal systems



At equilibrium

$$\wp^{eq}{}_{j\to i} = \int_0^\infty \theta(T_i,\omega)[\mathfrak{I}_{ji}(\omega) - \mathfrak{I}_{ij}(\omega)]\,\frac{d\omega}{2\pi}$$

If

$$\mathfrak{I}_{ji}(\omega) \neq \mathfrak{I}_{ij}(\omega)$$

a supercurrent can exist

In 2 body systems $(\mathfrak{I}_{12}=\mathfrak{I}_{21})$

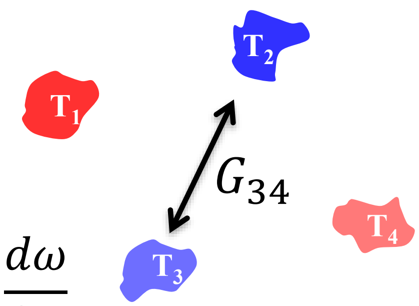
$$\wp^{eq}{}_{1\to 2} = \wp^{eq}{}_{2\to 1} = 0$$ No supercurrent

According to summation rules, the net power received by each body

$$\sum_j \wp^{eq}{}_{j\to i} = 0$$ No heating/cooling

Heat transport as a generalized random walk

Close to the thermal equilibrium



$$\frac{dT_i}{dt} = \sum_{j \neq i} G(r_i, r_j)(T_j - T_i)$$

$$G(r_i, r_j) = \lim_{|T_i - T_j| \to 0} \frac{1}{|T_i - T_j|} \int_0^\infty [\theta(T_i, \omega) - \theta(T_j, \omega)] \Im_{ij}(\omega) \frac{d\omega}{2\pi}$$

In the continuous limit

$$\frac{dT(r,t)}{dt} = \int_V G(r,r')T(r',t)dr' - T(r,t) \int_V G(r,r')dr'$$

which is analog to the Chapman-Kolmogorov master equation

$$\frac{dT(r,t)}{dt} = \int_V p(r,r') \frac{T(r',t)}{\tau(r)} dr' - \frac{T(r,t)}{\tau(r)}$$

$$p(r,r') = G(r,r')\left(\int_V G(r,r')dr'\right)^{-1}$$

Probablility distribution

$$\tau(r) = \left(\int_V G(r,r')dr'\right)^{-1}$$

Rate of jump

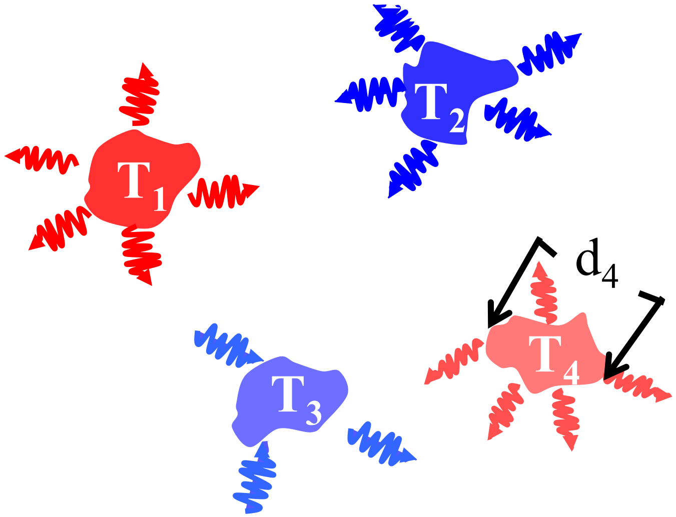
Heat transport regimes vs conductance scaling

Assuming $G \sim \frac{1}{|r_i - r_j|^\gamma}$ ➡ $\wp_i = -\sum_j \frac{\Gamma_{d;\alpha}}{|r_i - r_j|^\gamma}(T_i - T_j) \approx (-\Delta)^{\alpha/2} T\big|_i$ with $\alpha = \gamma\text{-d}$

where $(-\Delta)^{\alpha/2}$ is the fractional Laplacian

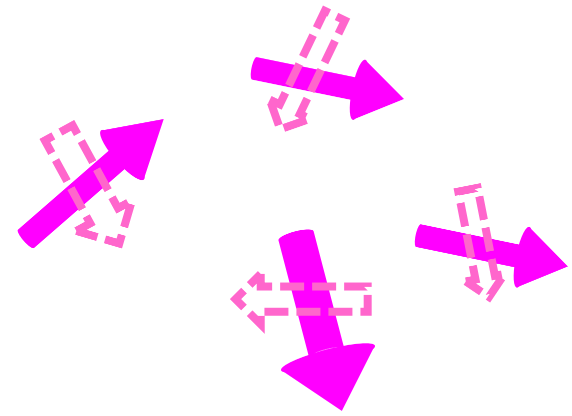
$\gamma = 2 + d \Rightarrow (-\Delta)^{\alpha/2} = (-\Delta)$		diffusion
$d < \gamma < 2 + d \Rightarrow$		superdiffusion
$\gamma = d$	$\Rightarrow (-\Delta)^{\alpha/2} = I$	ballistic

Many body heat transfer in the dipolar approximation



$$d_i < \min(\lambda_{T_j})$$

$$\lambda_{T_j} = c\hbar/(k_B T_j)$$



Small objects in interaction

N fluctuating dipoles in mutual interaction

Energy balance:

$$\rho_i C_i V_i \frac{dT_i}{dt} = \underbrace{-\int_{S_i} \langle \Pi(r,t) \rangle . dS_i = \wp_i \equiv \int_{V_i} \langle j.E \rangle dV_i}_{\text{Poynting theorem}}$$

Many-body heat transfer in the dipolar approximation

Absorbed power : $\wp_i = \int_{V_i} \langle j.E \rangle dV_i = \left\langle \frac{d\tilde{p}_i}{dt}, E \right\rangle$ $\quad j = \frac{d\tilde{p}}{dt}\delta(r)$

Using the decomposition $\tilde{p}_i(t) = 2\,\mathrm{Re}\left(\int_0^\infty p_i(\omega) \frac{e^{-i\omega t}}{2\pi} d\omega \right)$

$$\wp_i = 2\int_0^\infty \frac{d\omega}{2\pi}\omega \int_0^\infty \frac{d\omega'}{2\pi} \mathrm{Im}[\langle p_i(\omega) E_i^\dagger(\omega') \rangle e^{-i(\omega-\omega')t}]$$

Local field

$$E_i = \omega^2 \mu_0 \sum_j \mathcal{g}_0(r_i, r_j) p_j$$

Dipole moments

$$p_i = p_i^{fluc} + p_i^{ind} \quad \text{with} \quad p_i^{ind} = \varepsilon_0 \alpha_i \sum_j \mathcal{g}_0(r_i, r_j) p_j$$

$$\langle p_i(\omega) E^\dagger_i(\omega) \rangle = \omega^2 \mu_0 \sum_{j\alpha\beta k\gamma} M_{ij,\alpha\beta} \langle p_{j,\beta}^{fluc}(\omega) p_{k,\gamma}^{fluc\,\dagger}(\omega) \rangle N_{ki,\gamma\alpha}$$

PRL, **107**, 114301 (2011), PRB, **88**, 104307 (2013)

Many body heat transfer in the dipolar approximation

Using the fluctuation dissipation theorem:

$$\langle p_{j,\beta}^{fluc}(\omega) p_{k,\gamma}^{fluc\,\dagger}(\omega')\rangle = \hbar\varepsilon_0 \delta_{jk}\delta_{\beta\gamma}\delta(\omega-\omega') Im(\alpha_j)[1+2n(\omega,T_j)]$$

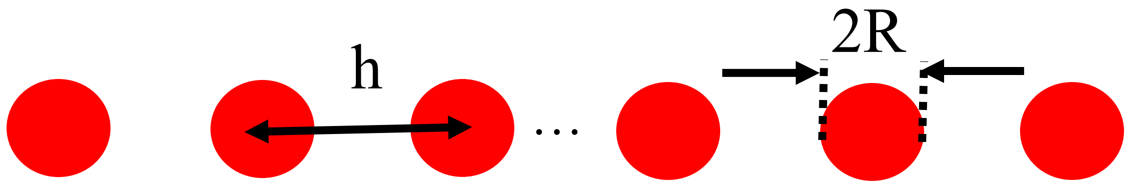
with

$$n(\omega,T) = [e^{\frac{\hbar\omega}{k_B T}} - 1]^{-1}$$

$$\wp_i = 3\int_0^\infty \frac{d\omega}{2\pi}\sum_j \underbrace{\frac{4}{3}\frac{\omega^4}{c^4} Im(\alpha_i) Im(\alpha_j) Im\left(tr\left[\mathcal{G}_{ij}\mathcal{G}_{ij}^{\dagger}\right]\right)}_{\mathfrak{I}_{ij}(\omega)}\left[\theta(T_i,\omega)-\theta(T_j,\omega)\right]$$

PRL, **107**, 114301 (2011)

Heat transport in diluted 1D systems

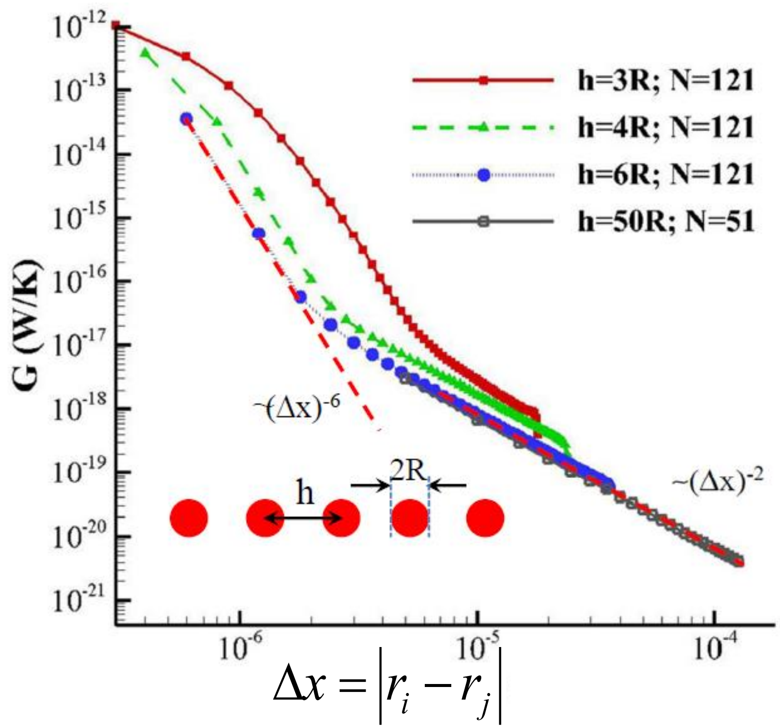


SiC

R=100 nm

T=300 K

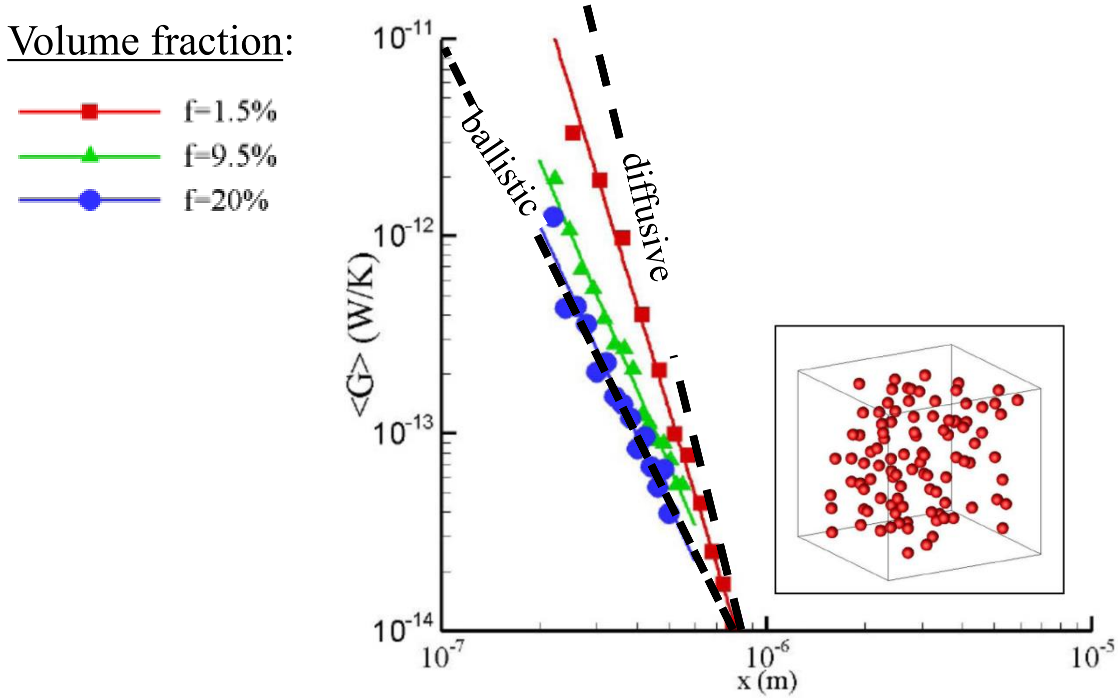
PRL, **107**, 114301 (2011)
PRL, **111**, 174301 (2013)



$G \sim \frac{1}{\Delta x^2}$ → $1 < \gamma = 2 < 3$ → superdiffusion

Long range correlations due to phonon-polariton (i.e. collective modes)

Heat transport in diluted random 3D systems



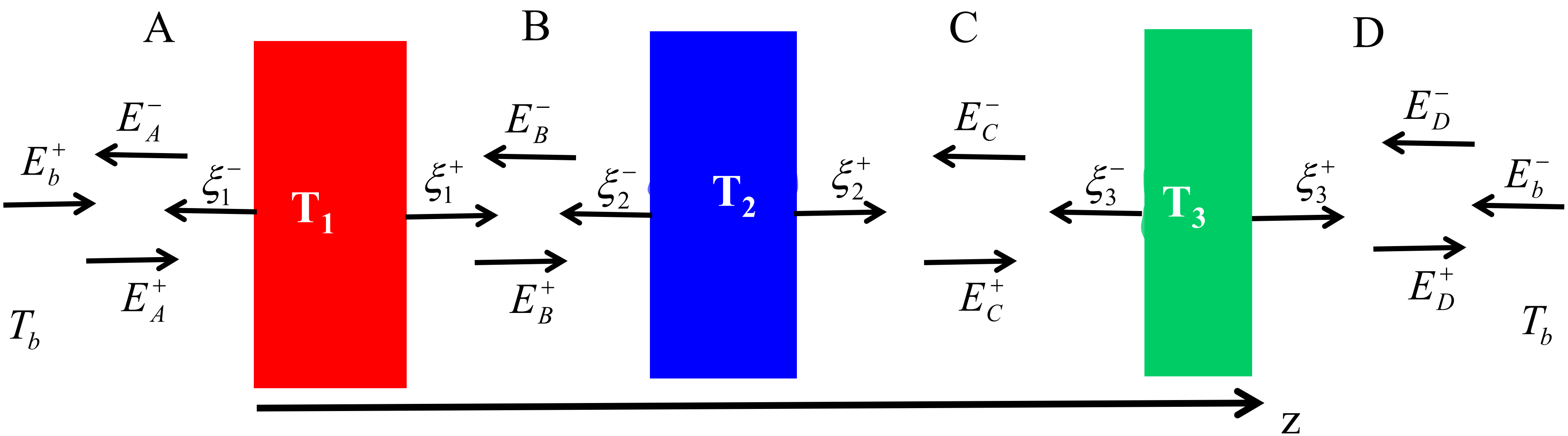
Clusters of SiC nanoparticles

Averaging
over 250 realizations

$3 < \gamma < 5$ ➡ superdiffusion

PRL, **111**, 174301, 2013

Heat transport in dense systems



Normal component of Poynting vector in each cavity:

$$\langle S_z \rangle = \langle E \times H \rangle . e_z = \sum_p \int \frac{d^2 k}{(2\pi)^2} \sum_{\phi,\phi'} \int_0^\infty \frac{d\omega}{2\pi} F_p^{\phi\phi'}(k,\omega) \langle E_p^{\phi}(k,\omega), E_p^{\phi'}(k,\omega) \rangle$$

<u>From the scattering theory</u>:

$$E_B^+ = \xi_1^+ + \Im_1^+ E_b^+ + \Re_1^- E_B^-$$

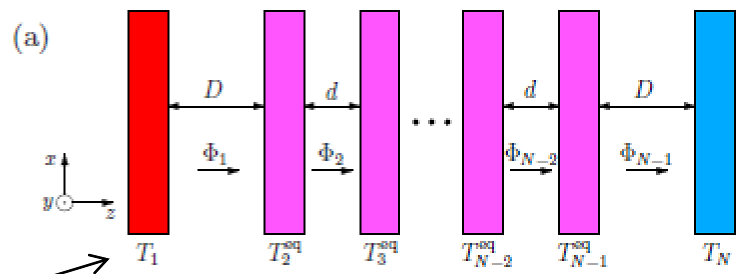
$$E_B^- = \xi_2^- + \Im_2^- E_C^- + \Re_2^+ E_B^+$$

$$\ldots$$

where, $\Im_i^{\pm}$ and $\Re_i^{\pm}$ are the transmission and reflection operators

PRB, 95, 205404, 2017

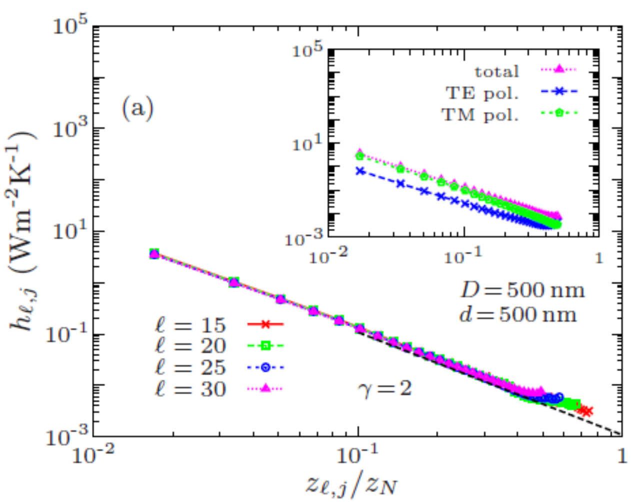
Transition to ballistic regime

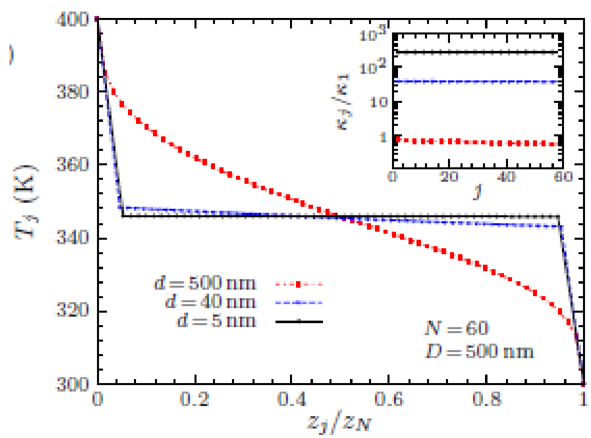


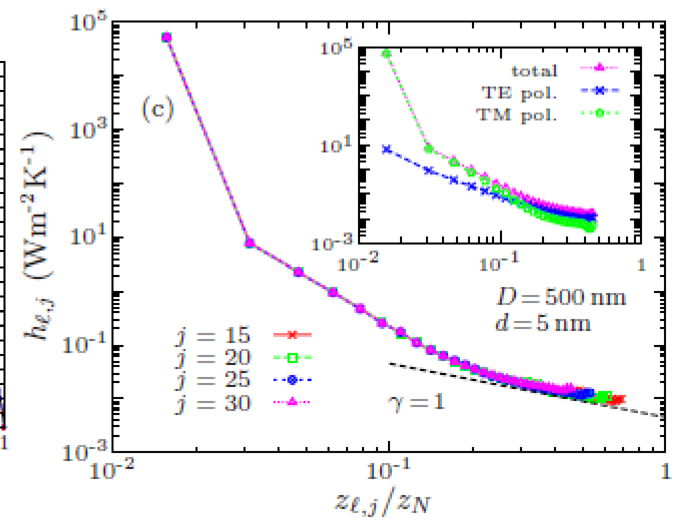
SiC slabs (200 nm thick)

Diluted (d=500 nm)

Dense (d=5 nm)







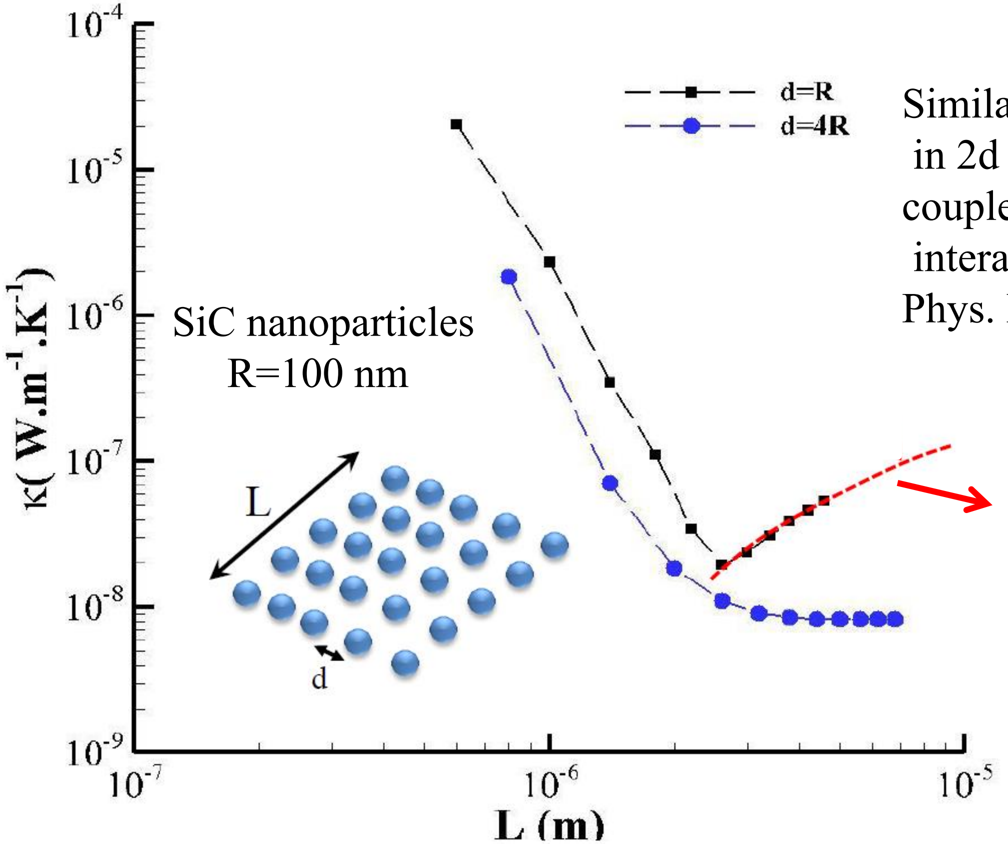
Superdiffusive (TM dominates)

Ballistic (TE dominates)

PRB **97**, 035423 (2018)

Diverging conductivity in 2D systems

$$\phi = \kappa \frac{\Delta T}{L} \quad \text{with} \quad \kappa = G \frac{L}{N\pi R^2}$$



Similar to phononic conduction in 2d systems of oscillators coupled through anharmonic interactions (S. Lepri et al., Phys. Rep. 377, 1, 2003)

$O(\log(L))$

$$G = O\left(\frac{\log(L)}{L}\right)$$

no algebraic decay of the pdf

Summary/prospects

- Anomalous (non-gaussian) heat transport regimes in many body systems :

➡ Superdiffusive regime in diluted 1D and 3D systems due to collective behavior

➡ Transition to a ballistic regime in dense systems due to the TE modes contribution

➡ Role plays by the non-local response on the transition?

➡ Transport regimes in non-reciprocal systems?

➡ Diverging (logarithmic) conductivity in 2D systems

Unlike phononic systems there is not yet rigorous explanation for this behavior…

➡ How to deal with large many-body systems?
Toward a hydrodynamic description of transport (round table 06/30)

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