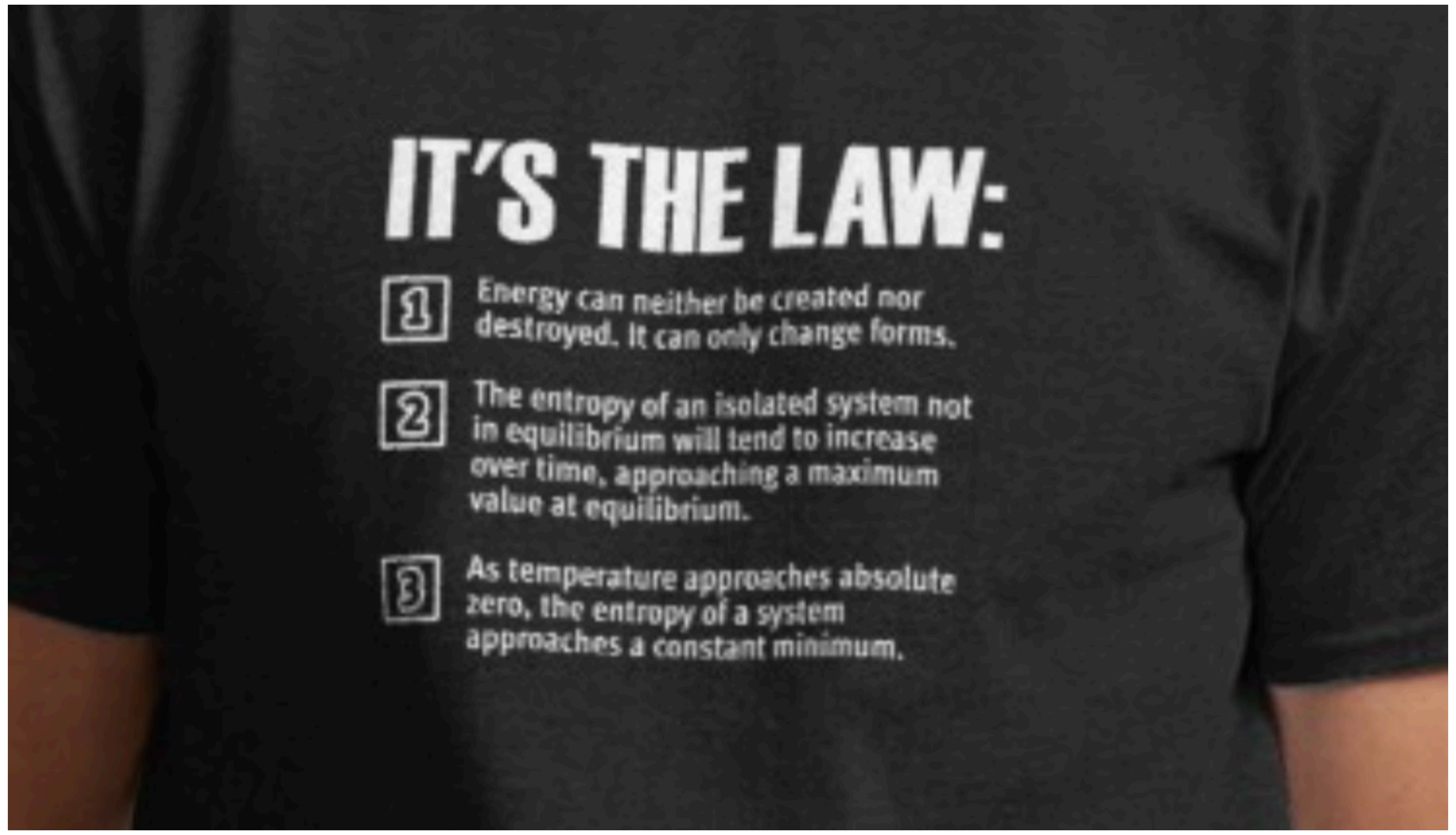
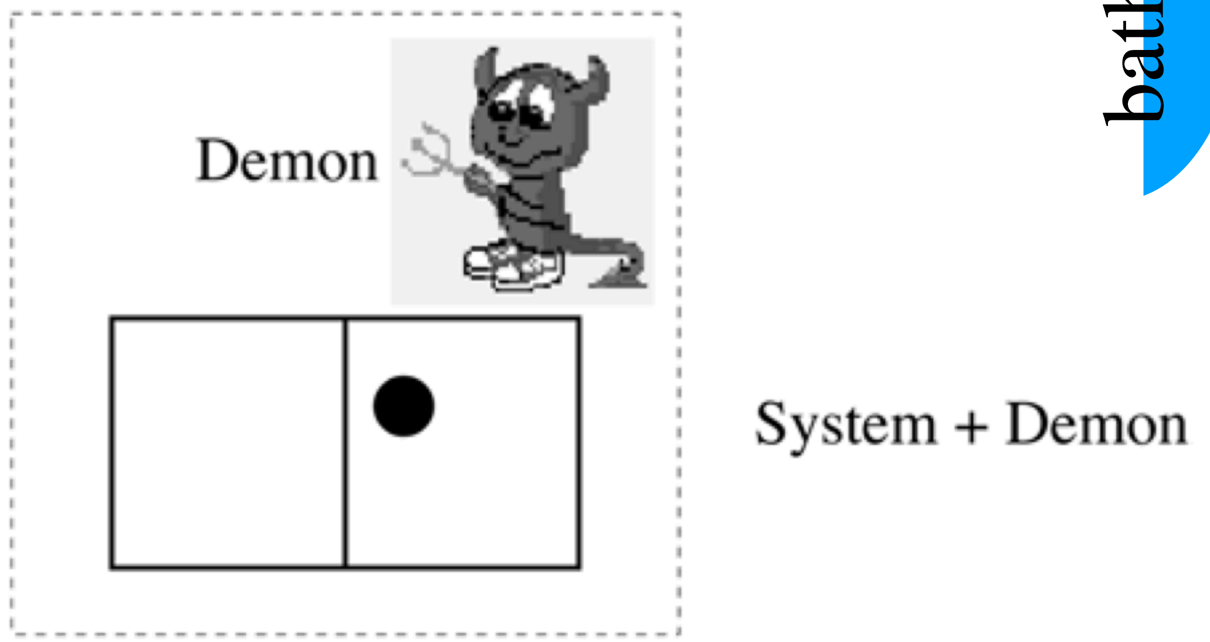
Master Equations on Trial for violating the 2nd Law

Carsten Henkel

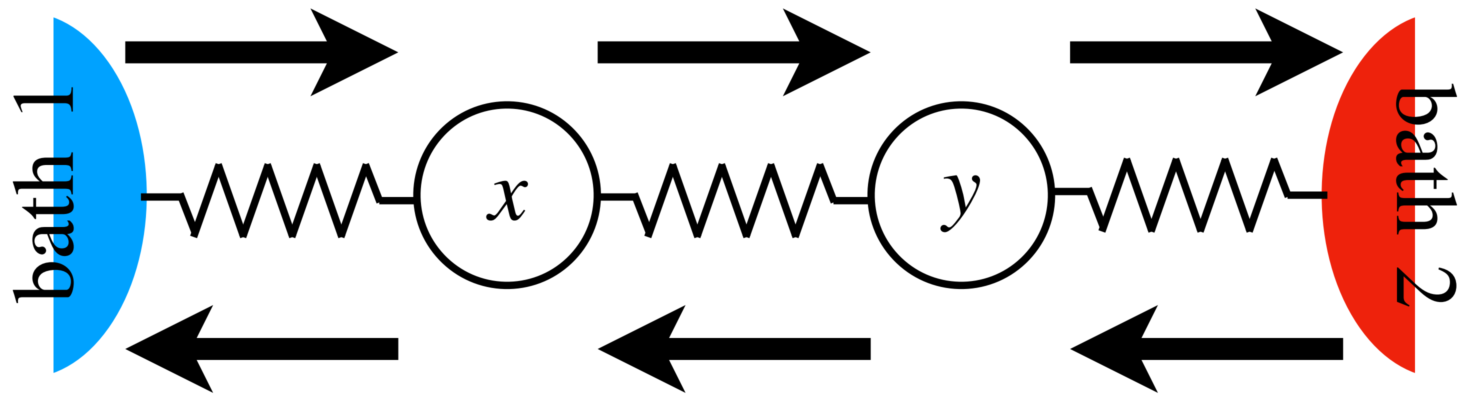
Physics & Astronomy, University of Potsdam, Germany



redbubble.com

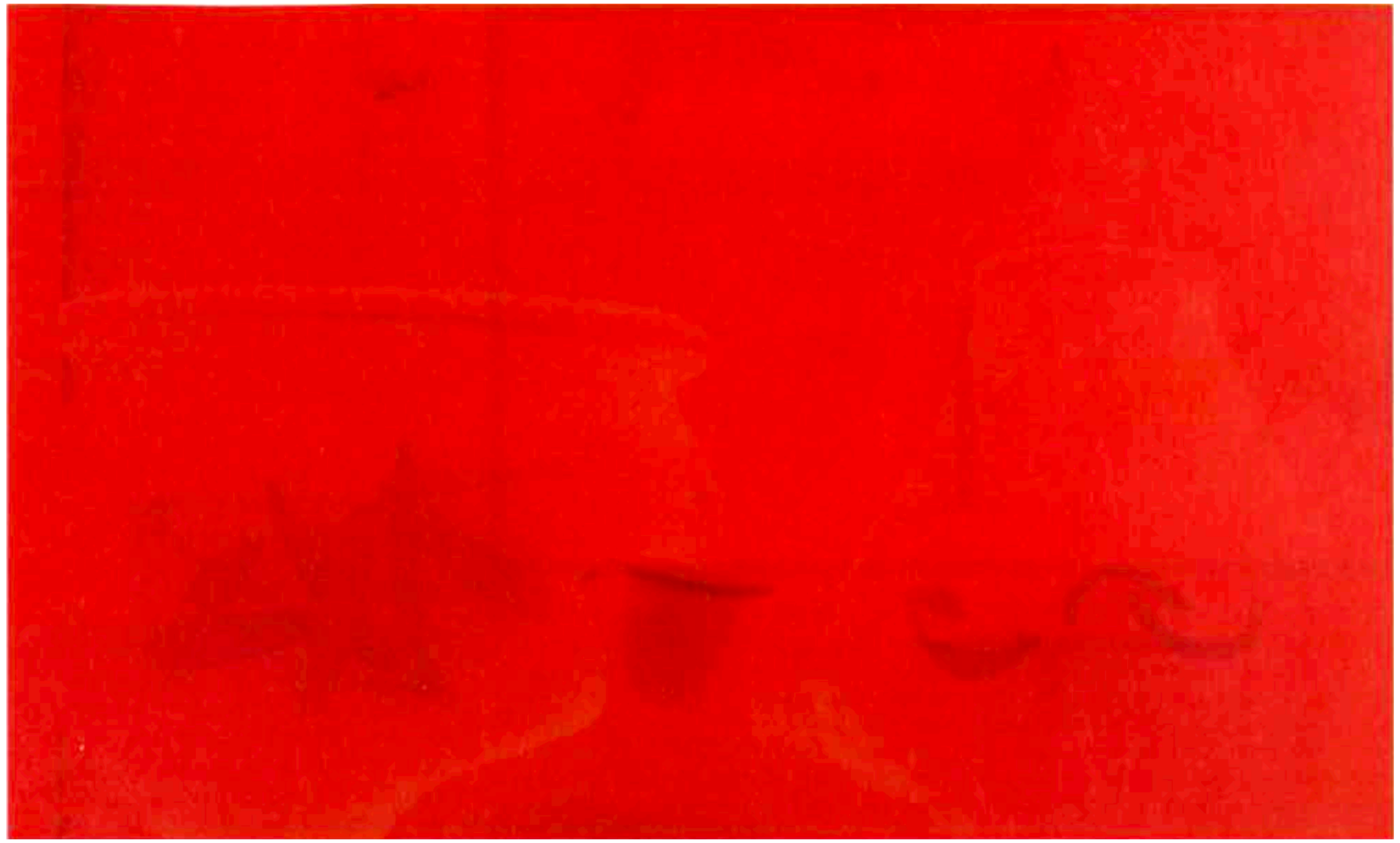


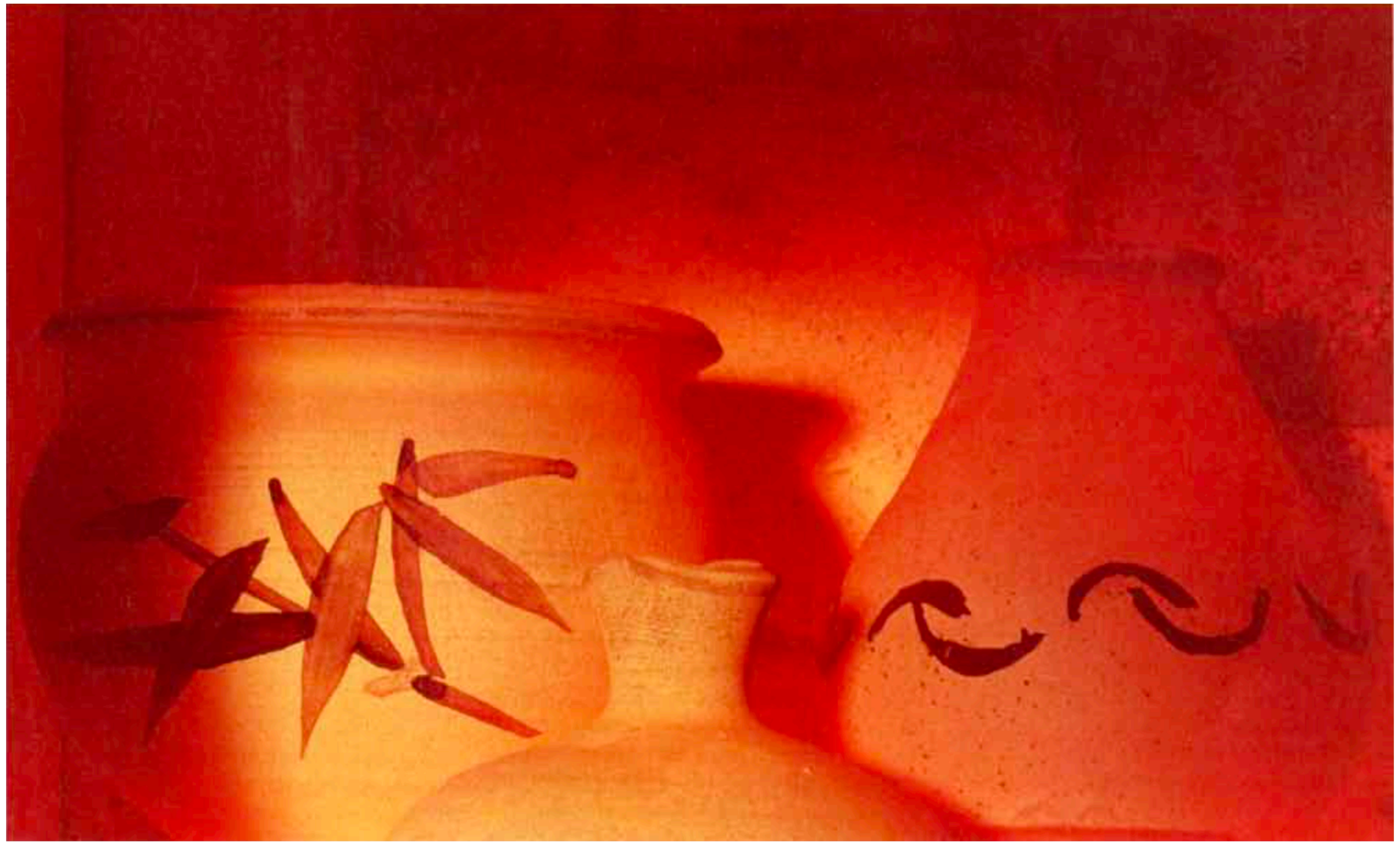
Plenio & Vitelli (*Contemp Phys* 2001)



Flectro'22 Workshop – KITP St. Barbara 20 June–05 Aug 2022

Equilibrium (Radiation) is useless





Charles H. Bennett
Demons, Engines and the Second Law
Sci. Am. **257** (Nov 1987) 108

Starting point when suggesting this talk

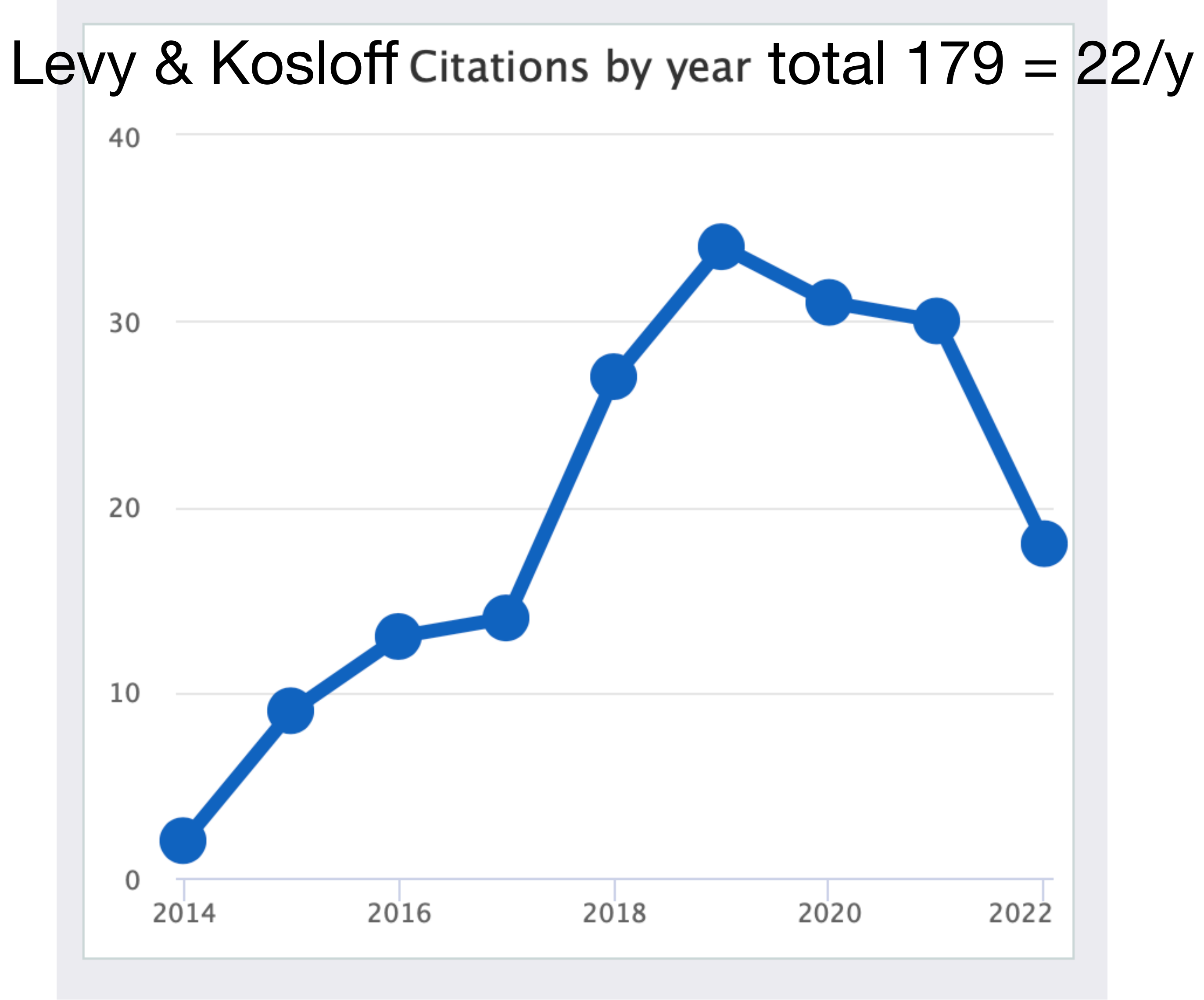
A. Levy and R. Kosloff
The local approach to quantum transport may violate the second law of thermodynamics
Europhys. Lett. **107** (2014) 20004

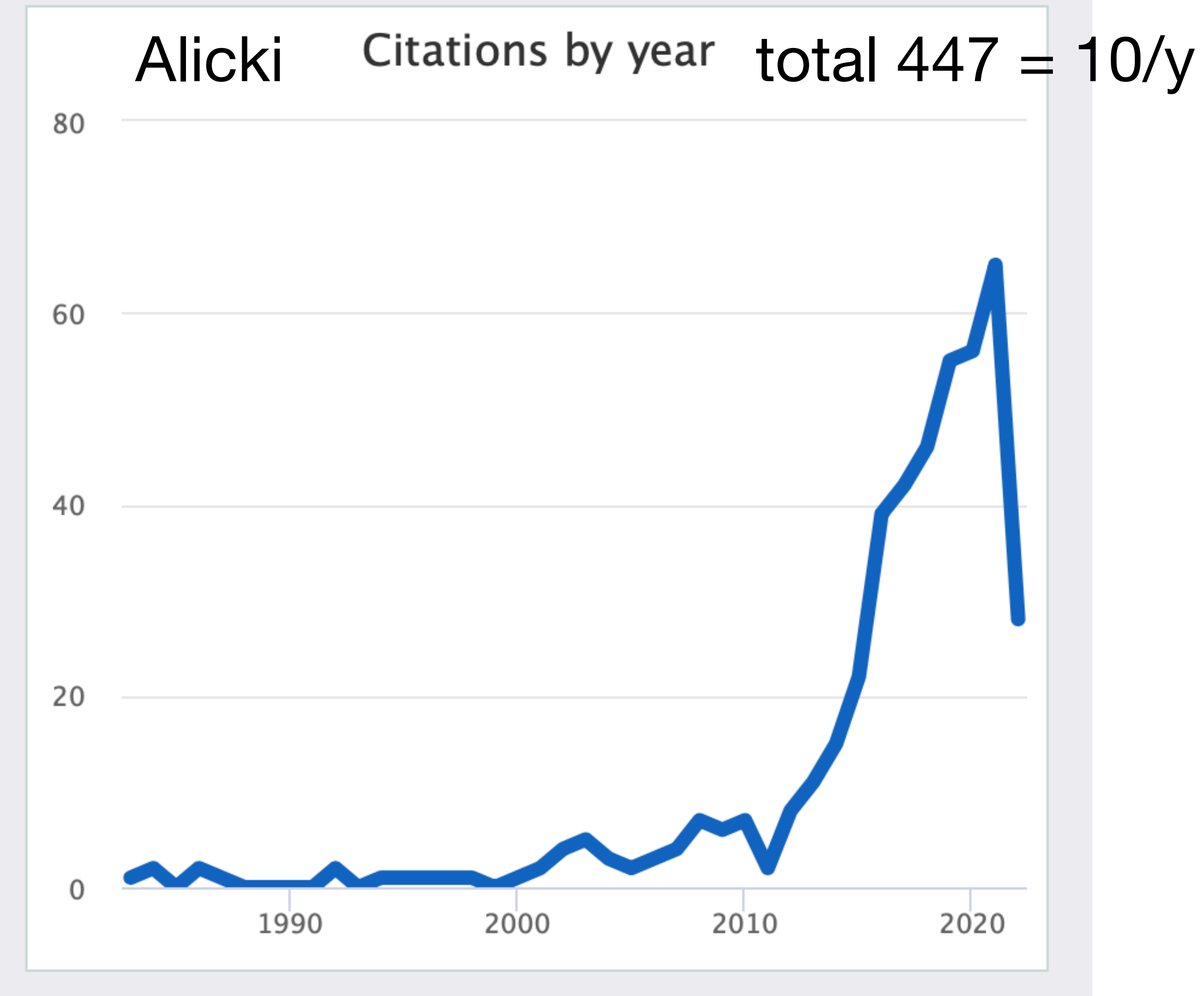
Gabriele De Chiara, Gabriel Landi, Adam Hewgill, Brendan Reid, Alessandro Ferraro, Augusto J Roncaglia, and Mauro Antezza
Reconciliation of quantum local master equations with thermodynamics
New J. Phys. **20** (2018) 113024

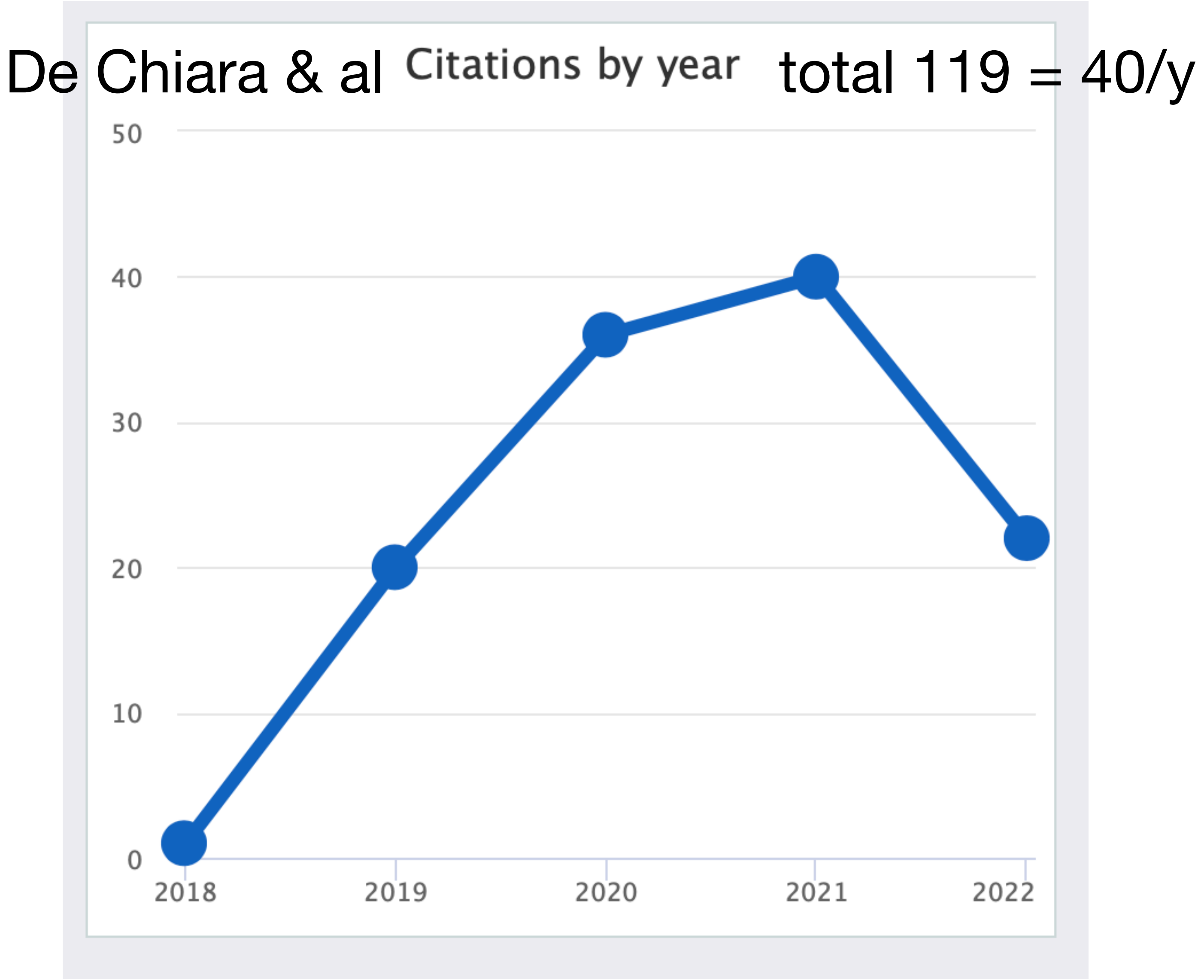
Roie Dann and Ronnie Kosloff
Open system dynamics from thermodynamic compatibility
Phys. Rev. Research **3** (2021) 023006

R Alicki
The quantum open system as a model of the hea
J. Phys. A **12** (1979) L103

H. E. D. Scovil and E. O. Schulz-DuBois
Three-Level Masers as Heat Engines
Phys. Rev. Lett. **2** (1959) 262







Adam Hewgill, Gabriele De Chiara, and Alberto Imparato
Quantum thermodynamically consistent local master equations
Phys. Rev. Research **3** (2021) 013165

... problem solved @ KITP!

talk G. De Chiara in programme 2021
https://www.on.kitp.ucsb.edu/online/info_c21/dechiara/
Energy and Information Transport in Non-Equilibrium Quantum Systems

Gabriele De Chiara, Gabriel Landi, Adam Hewgill, Brendan Reid, Alessandro Ferraro, Augusto J Roncaglia, and Mauro Antezza
Reconciliation of quantum local master equations with thermodynamics
New J. Phys. **20** (2018) 113024

programme 2018
Thermodynamics of Quantum Systems:
Measurement, Engines, and Control

UC SANTA BARBARA
Kavli Institute for
Theoretical Physics

... exactly 20 years ago

QUANTUM LIMITS TO THE SECOND LAW: First International Conference on Quantum Limits to the Second Law

Conference date: 29-31 July 2002
Location: San Diego, California (USA)
ISBN: 0-7354-0098-9
Editors: Daniel P. Sheehan
Volume number: 643
Published: Nov 20, 2002

Modified Feynman ratchet with velocity-dependent fluctuations
Jack Denur
AIP Conference Proceedings 643, 326 (2002); https://doi.org/10.1063/1.1523825

Bartoli's Heat Engine Working With Zero-Point Electromagnetic Radiation
J. J. Mareš, V. Špička, J. Krištofik and P. Hubík
AIP Conference Proceedings 643, 273 (2002); https://doi.org/10.1063/1.1523816

Quantum Brownian motion and its conflict with the second law
Theo M. Nieuwenhuizen and Armen E. Allahverdyan
AIP Conference Proceedings 643, 29 (2002); https://doi.org/10.1063/1.1523777

Dimer as a Challenge to the Second law
V. Čápek
AIP Conference Proceedings 643, 98 (2002); https://doi.org/10.1063/1.1523788

About Perpetuum Mobile without Emotions
Alexey Nikulov
AIP Conference Proceedings 643, 207 (2002); https://doi.org/10.1063/1.1523805

What Does It Mean to Violate the Second Law of Thermodynamics?
James D. Means
AIP Conference Proceedings 643, 420 (2002); https://doi.org/10.1063/1.1523838

Why Do We Believe in the Second Law?
Todd L. Duncan
AIP Conference Proceedings 643, 424 (2002); https://doi.org/10.1063/1.1523839

The Second Law and Karl R. Popper
Guenri E. Norman
AIP Conference Proceedings 643, 442 (2002); https://doi.org/10.1063/1.1523842

What did Carnot say about the Second Law?
Stacy G. Langton
AIP Conference Proceedings 643, 448 (2002); https://doi.org/10.1063/1.1523843

Objectivity of Thermodynamic Quantities
Jorge Berger
AIP Conference Proceedings 643, 456 (2002); https://doi.org/10.1063/1.1523845

https://aip.scitation.org/toc/apc/643/1

Overview

- Status of the 2nd Law (Maxwell demon)
- examples of successful master equations
- concept "strong coupling to bath"
- coupled oscillators (Walls 1970, Levy & Kosloff 2014, Henkel 2021)
- non-Markov/initial slip, non-CP, H(mean force)

2nd Law — Status

fluctuation theorem (Crooks, Evans & Searles, Jarzynski)

$$1 = \langle \exp\Big(-\frac{\Delta S_i}{k_B}\Big)\rangle \geq \exp\Big(-\frac{\langle \Delta S_i \rangle}{k_B}\Big)$$

"intrinsic entropy production" ΔS_i over arb process $i \to f$ (endpoints in eq)

via free energy difference

(another application of Jensen's inequality)

$$\Delta S_i = (S - \beta U)\Big|_i^f$$

$$= \mathrm{tr}\Big(-\rho \,\log \rho + \rho \log \frac{\mathrm{e}^{-\beta H}}{Z}\Big)\Big|_i^f = -D\Big(\rho \| \frac{\mathrm{e}^{-\beta H}}{Z}\Big)\Big|_i^f \geq 0$$

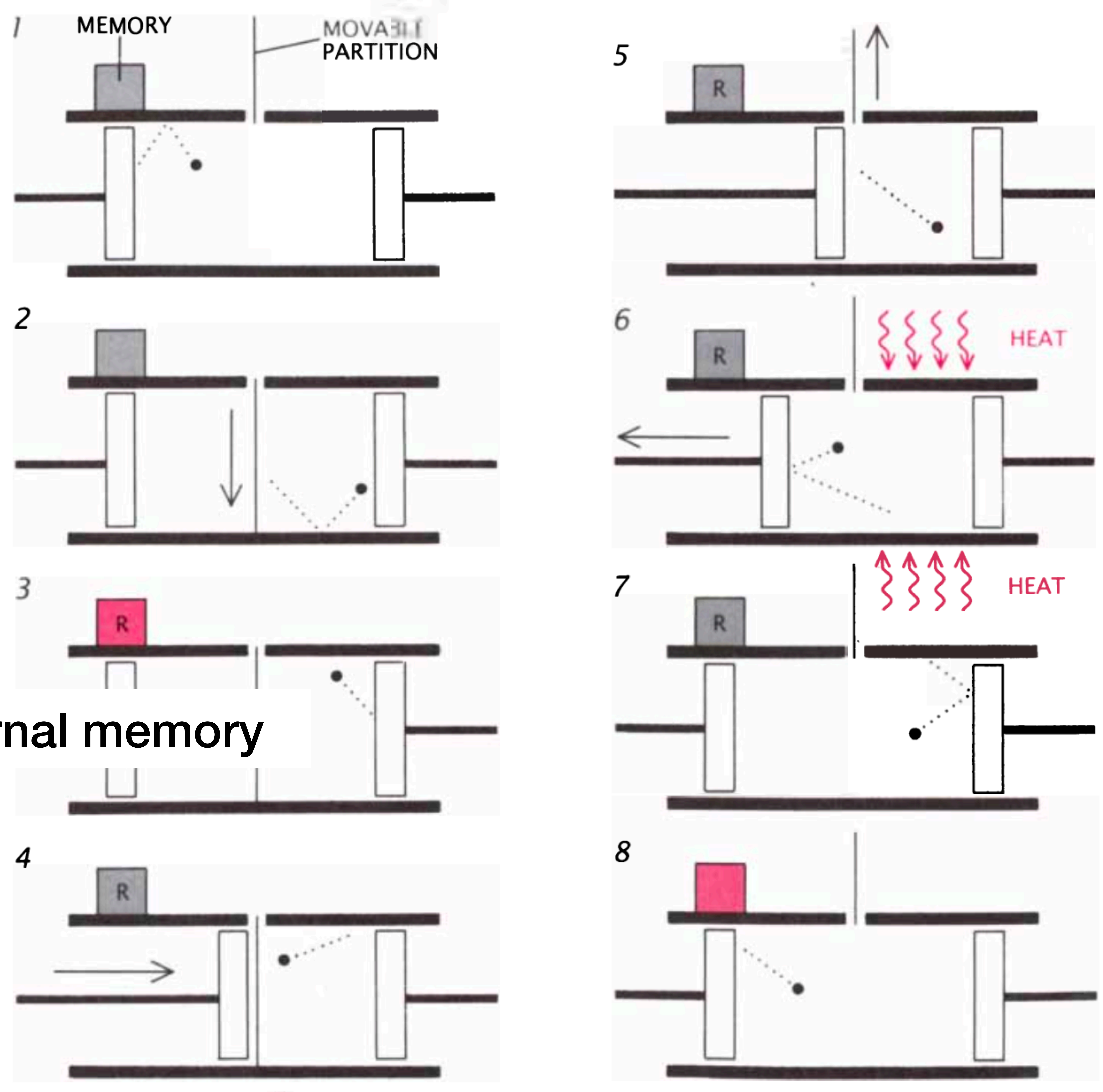
Kullbäck-Leibler distance (not metric)

Maxwell Demon à la Szilard

information
entropy
work

“Left or right?”

overwrite demon internal memory



convert heat into work

bill of overwriting

$\$_{\text{tot}} = k_B \log 2$

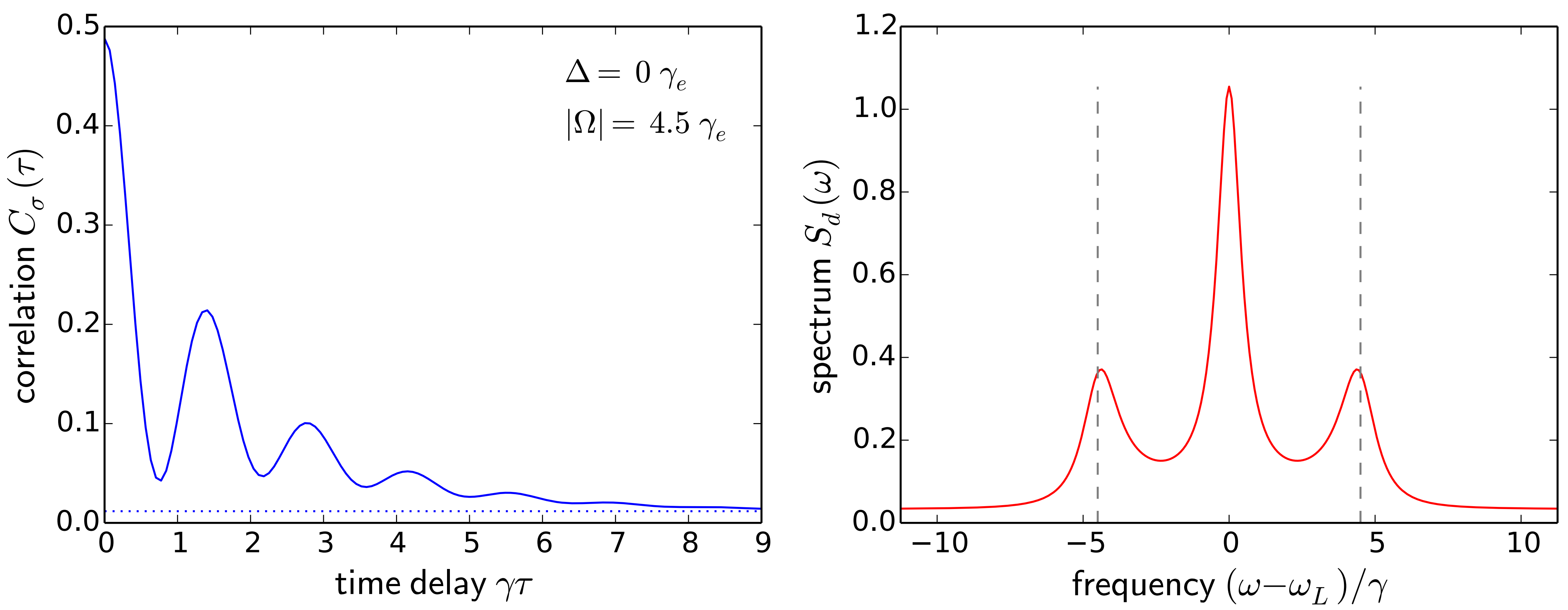
—R. Landauer

Charles H. Bennett
Demons, Engines and the Second Law
Sci. Am. **257** (Nov 1987) 108

Successful Master Equations for Open Quantum Systems

non-th steady states

- driven systems

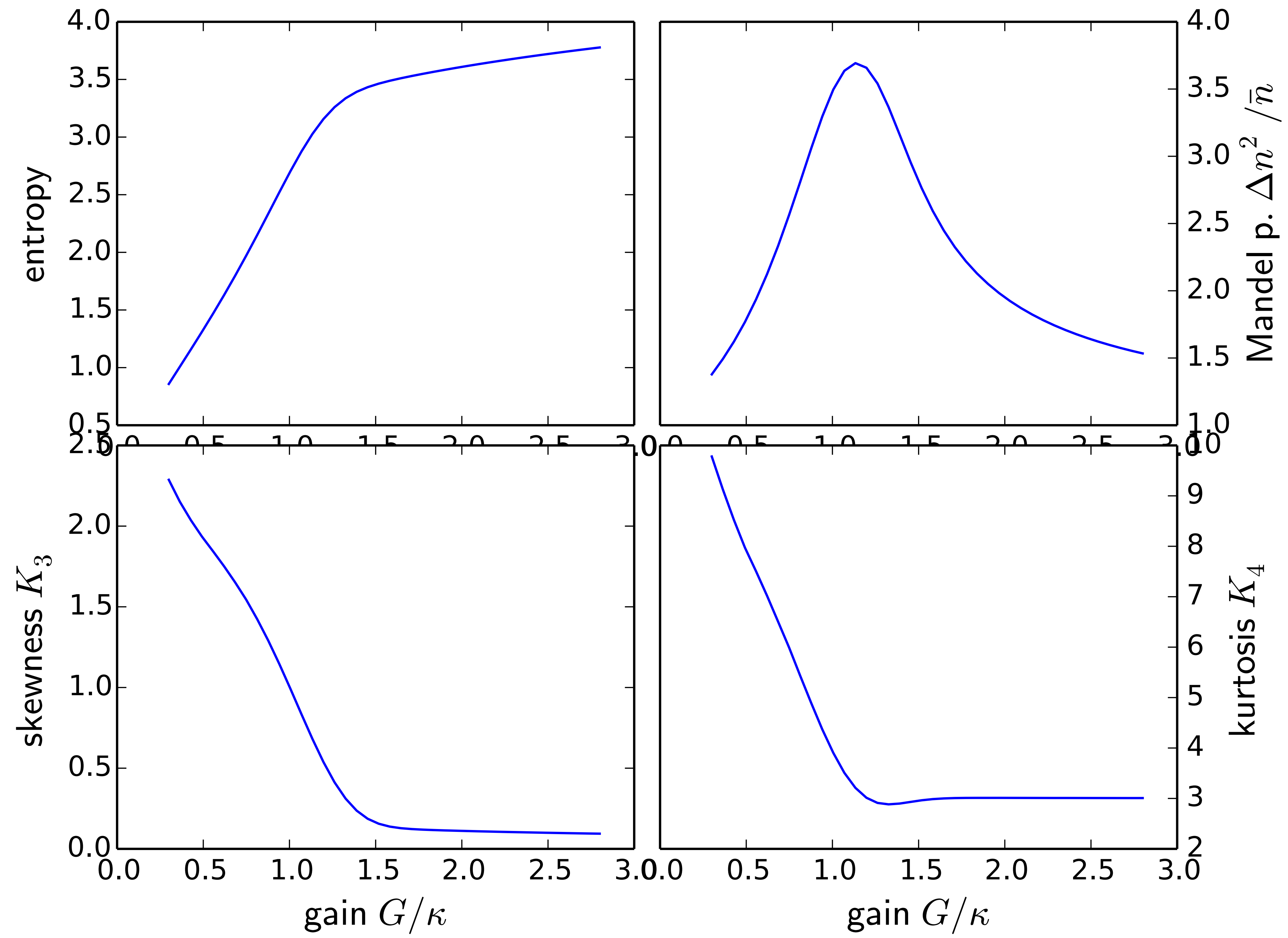


resonance fluorescence (Mollow triplet)

$$H_{\rm rf} = -\hbar\Delta\sigma_z + \frac{\hbar}{2}(\Omega\sigma^\dagger + \Omega^*\sigma) + \text{decay}\ (\gamma_e)$$

www.quantum.physik.uni-potsdam.de/teaching

analysis of photon statistics



single-mode laser (Scully & Lamb)

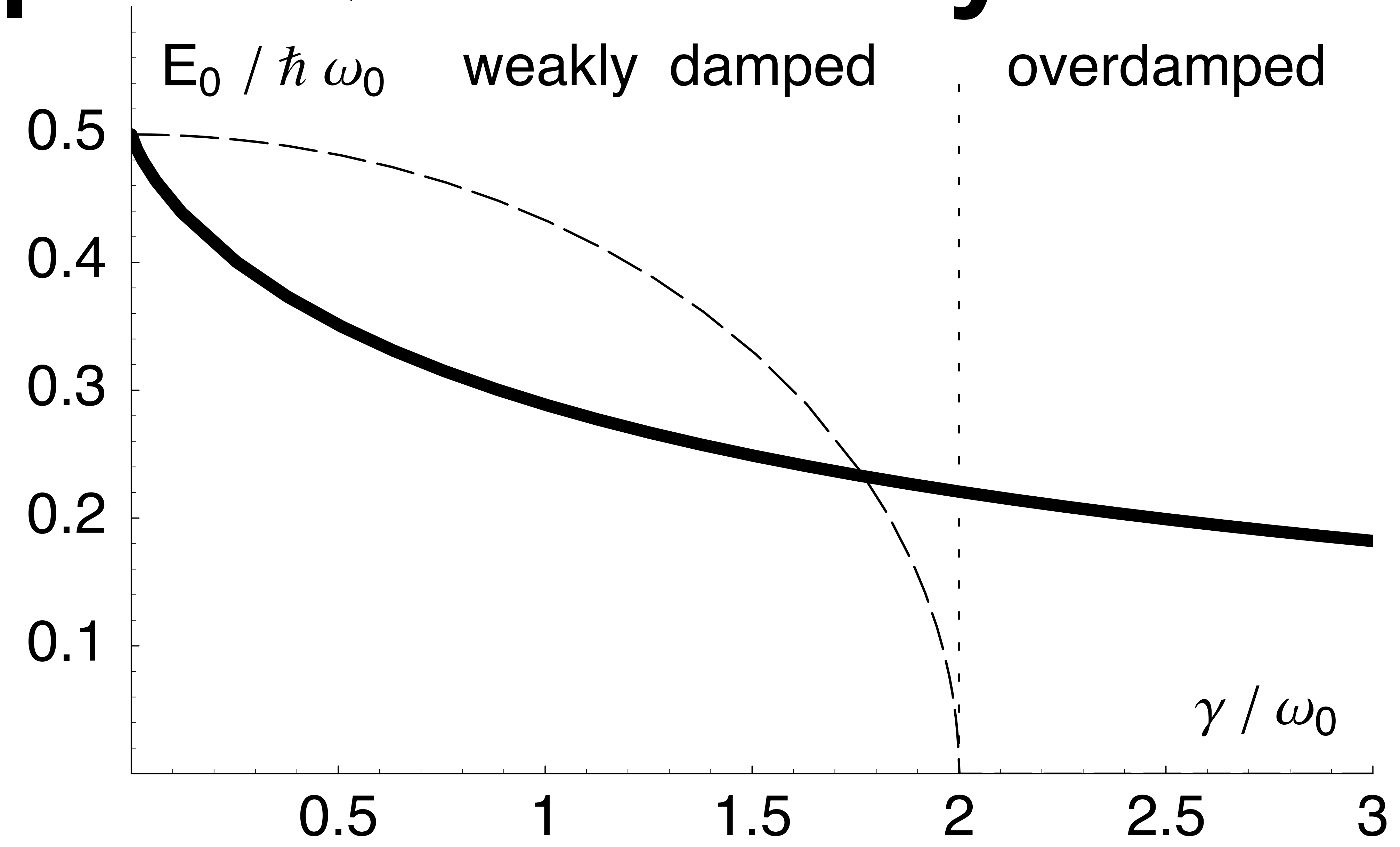
$$L_{\rm g} = \sqrt{G}\, a^\dagger (1 + \beta a^\dagger a)^{-1/2}$$

Successful Master Equations for Open Quantum Systems

non-th steady states

- driven systems
- strong damping

$$\rho_{\mathrm{st}} \propto \exp(-\beta H?)$$



zero-point energy of (over)damped oscillator

$$\coth \frac{\hbar\omega_{\mathrm{eff}}}{2k_B T} = \frac{\Delta p \, \Delta x}{\hbar/2} \approx \left[f(\kappa)\Big((4\kappa/\pi)\log(\omega_D/\omega_0) + (1 - 2\kappa^2) f(\kappa)\Big)\right]^{1/2}$$

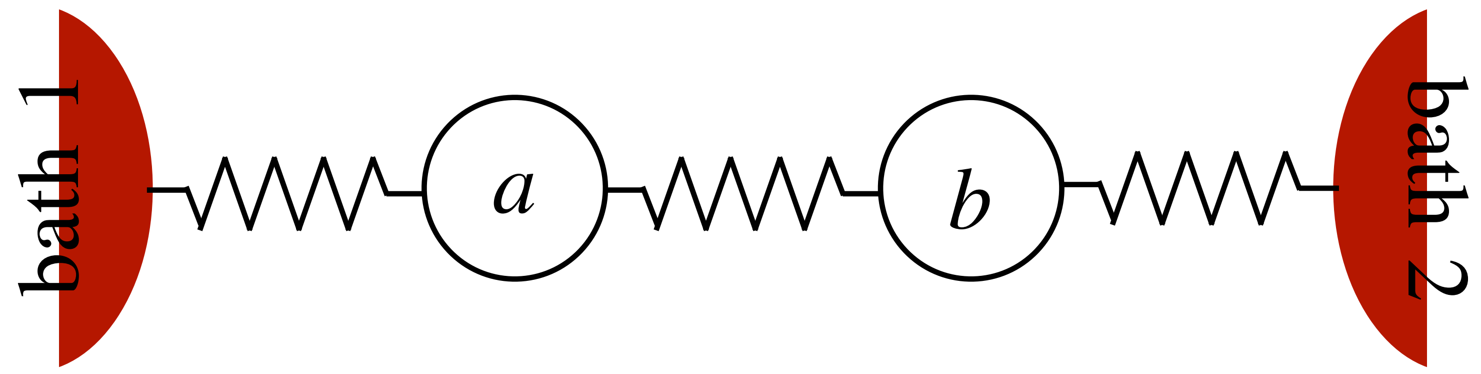
$$\frac{\langle p^2 \rangle}{2M} \neq \frac{M}{2}\omega_0^2 \langle x^2 \rangle$$

$$\kappa = \gamma/2\omega_0 = \cosh(\tau/2) \quad f(\kappa) = \frac{\tau/\pi}{\sinh(\tau/2)}$$

Intravaia & CH (*PRL* 2009)
Grabert, Weiss, Talkner (*Z Phys* B 1984)

Two Oscillators – Local vs. Global Coupling

two heat baths, $T_1 = T_2$



- local coupling: violation of 0th Law

$$H = H_a + H_b + \epsilon(a^\dagger b + b^\dagger a) + \left\{a^\dagger \Gamma_a + b^\dagger \Gamma_b + \mathrm{h.c.}\right\} + H_{B1} + H_{B2}$$

$$\frac{d}{dt} a = -(\mathrm{i}\omega + \gamma_a)\, a - \mathrm{i}\epsilon\, b - \mathrm{i}\Gamma_a$$

$$L_a = \sqrt{\gamma_a (1 + \bar{n}_a)}\, a$$

$$G_a = \sqrt{\gamma_a \bar{n}_a}\, a^\dagger$$

$$\bar{n}_a = \frac{1}{\mathrm{e}^{\beta_1 \omega_a} - 1}$$

$$\rho_{\mathrm{st}} \propto \exp[-\beta\omega(a^\dagger a + b^\dagger b)]$$

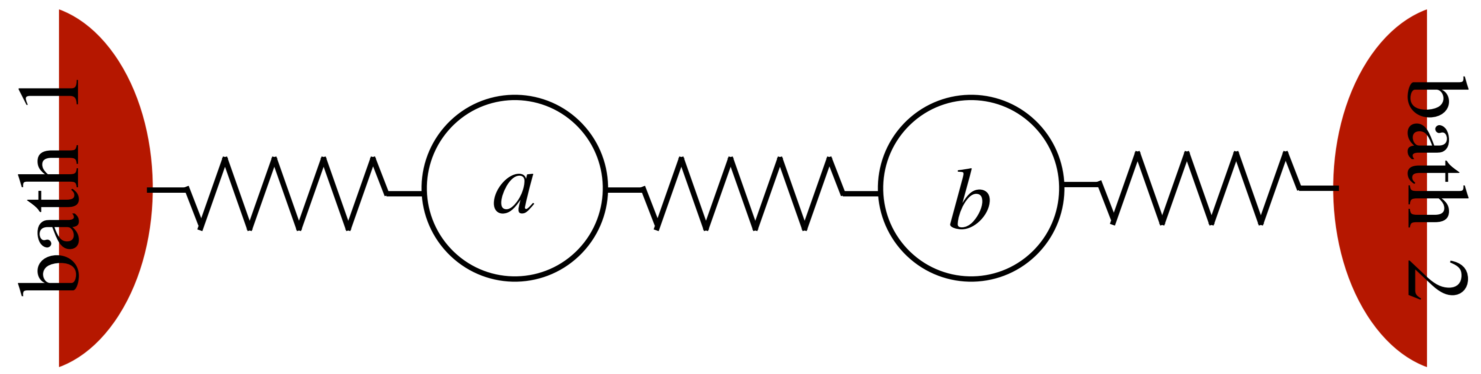
expect equilibrium state: $$\rho_{\mathrm{eq}} \propto \exp[-\beta\omega(a^\dagger a + b^\dagger b) - \beta\epsilon(a^\dagger b + b^\dagger a)]$$

Problem:
Lindblad operators not computed precisely enough
(neglecting *a b* coupling)

Walls (*Z Phys* 1970)

Local vs. Global Coupling

two heat baths, $T_1 = T_2$



- restore conformity with 0th Law

$$H = H_a + H_b + \epsilon(a^\dagger b + b^\dagger a) + \{a^\dagger \Gamma_a + b^\dagger \Gamma_b + \mathrm{h.c.}\} + H_{B1} + H_{B2}$$

$$\frac{d}{dt}a = -(\mathrm{i}\omega + \gamma)\,a - (\mathrm{i}\epsilon - \delta)\,b - \mathrm{i}\Gamma_a$$

$$L_\pm = \sqrt{\gamma_\pm(1+\bar{n}_\pm)/2}\,(a \pm b)$$

$$G_\pm = \sqrt{\gamma_\pm \bar{n}_\pm/2}\,(a^\dagger \pm b^\dagger)$$

supervise your Lindbladians!

normal modes $\omega_\pm = \omega \pm \epsilon - \mathrm{i}(\gamma \pm \delta)$

equilibrium state 👍 $\rho_{\mathrm{st}} \propto \exp[-\beta\omega(a^\dagger a + b^\dagger b) -\beta\epsilon(a^\dagger b + b^\dagger a)]$

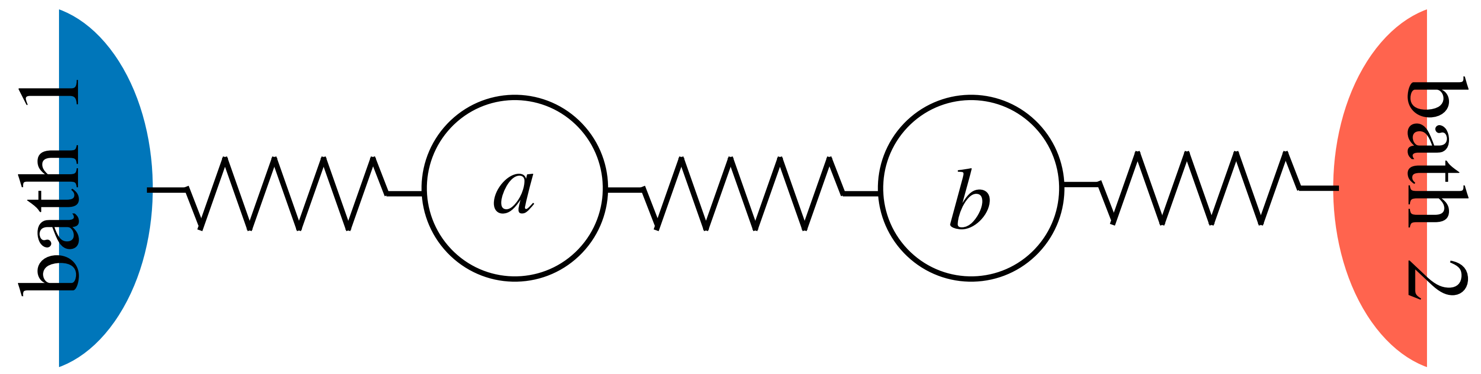
$$a_i^\dagger \Gamma_i = A_j^\dagger \tilde{\Gamma}_j$$

$A_j = U_{ji}\, a_i$ $\quad [\tilde{\Gamma}_i, \tilde{\Gamma}_j^\dagger] \neq \delta_{ij}$ if $\gamma_a \neq \gamma_b$

Walls (*Z Phys* 1970)

Heat Transport – Local vs. Global Coupling

two heat baths, $T_1 \neq T_2$



- local coupling: violation of 2nd Law

$$H = H_a + H_b + \epsilon(a^\dagger b + b^\dagger a) + \left\{a^\dagger \Gamma_a + b^\dagger \Gamma_b + \mathrm{h.c.}\right\} + H_{B1} + H_{B2}$$

$$\frac{d}{dt}a = -(\mathrm{i}\omega + \gamma_a)\,a - \mathrm{i}\epsilon\, b - \mathrm{i}\Gamma_a$$

$$L_a = \sqrt{\gamma_a(1+\bar{n}_a)}\,a$$

$$G_a = \sqrt{\gamma_a \bar{n}_a}\,a^\dagger$$

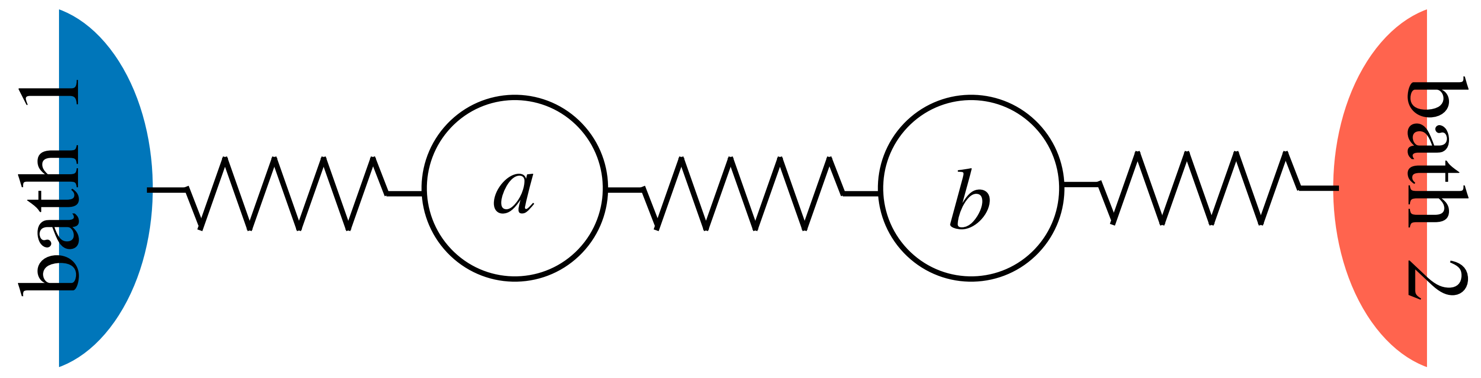
heat current ... sign not fixed by $T_1 - T_2$

$$\dot{Q}_{a\to b} = -\mathrm{i}\epsilon\omega_a\langle a^\dagger b - b^\dagger a\rangle = 2\gamma\omega_a \frac{\epsilon^2(\bar{n}_a - \bar{n}_b)}{(\omega_a-\omega_b)^2 + 4\epsilon^2 + \gamma^2}$$

Levy & Kosloff (*Europhys Lett* 2014)
De Chiara & al (*New J Phys* 2018)

Heat Transport – Local vs. Global Coupling

two heat baths, $T_1 < T_2$



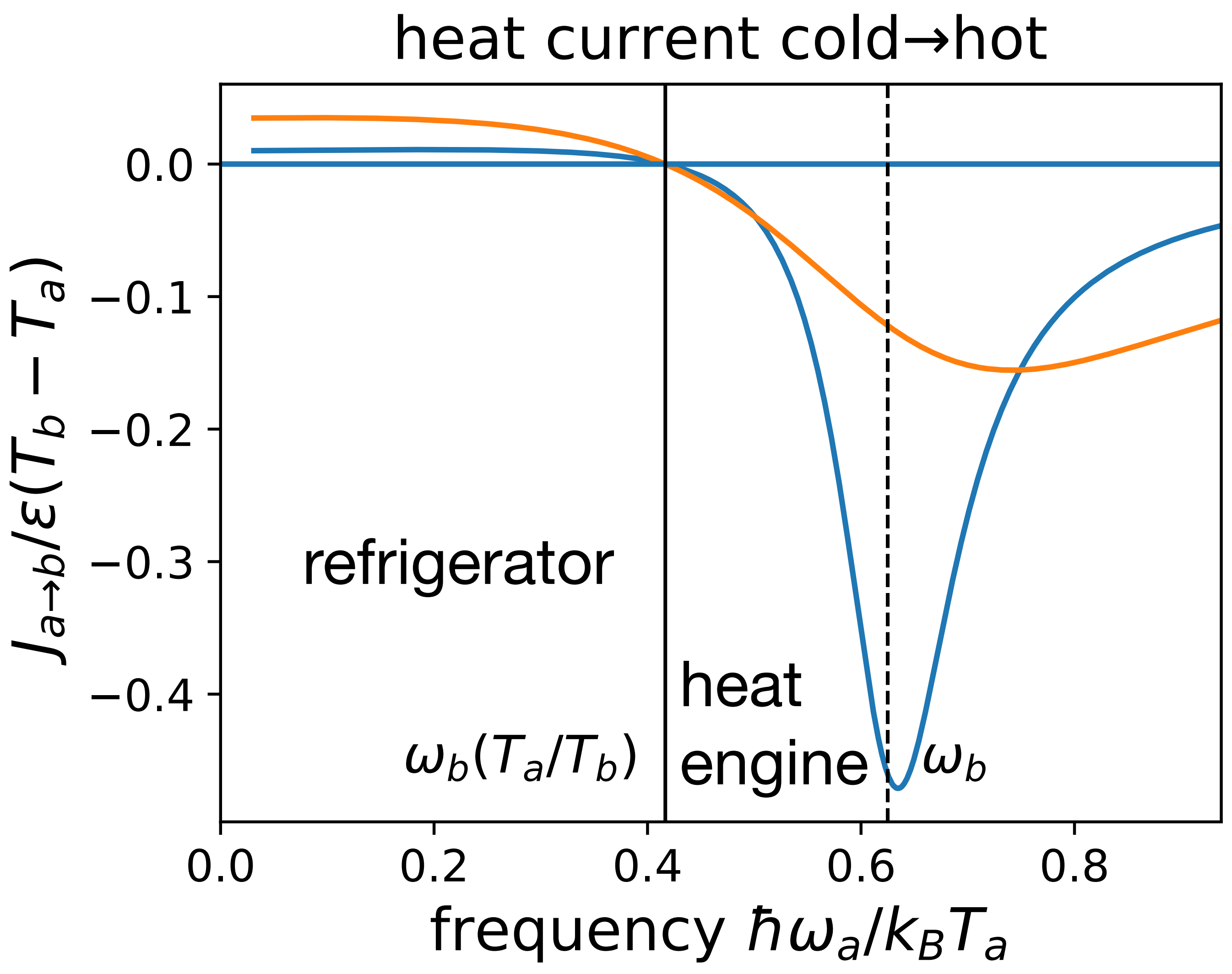
- local coupling: violation of 2nd Law ... no!

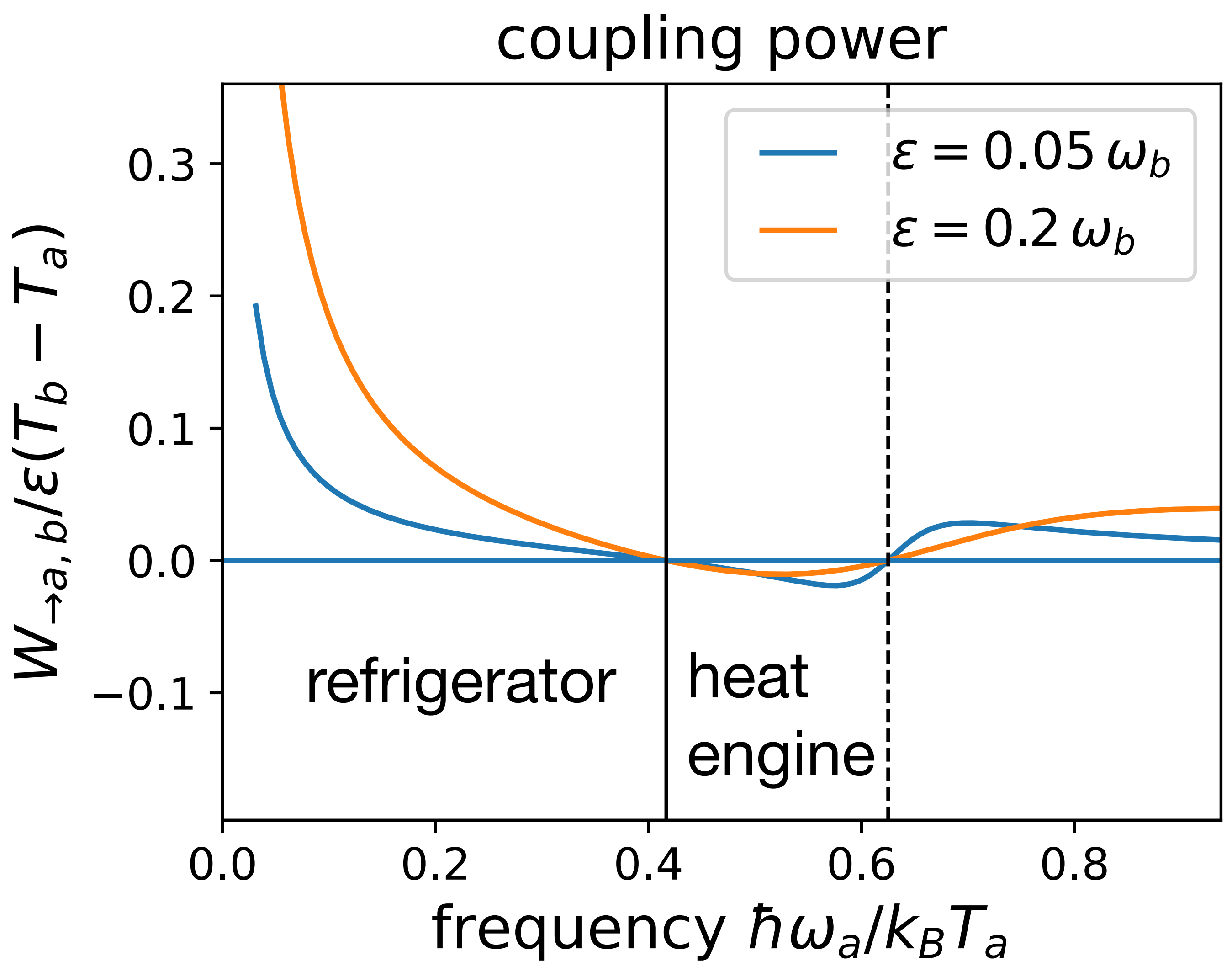
heat current cold → hot

$$\dot{Q}_{a\to b} = -\mathrm{i}\epsilon\omega_a\langle a^\dagger b - b^\dagger a\rangle = 2\gamma\omega_a \frac{\epsilon^2(\bar{n}_a - \bar{n}_b)}{(\omega_a - \omega_b)^2 + 4\epsilon^2 + \gamma^2}$$

rate of work to de/couple bath

$$\dot{W}_{\to a,b} = \frac{\omega_b - \omega_a}{\omega_a}\dot{Q}_{a\to b}$$





Levy & Kosloff (*Europhys Lett* 2014)
De Chiara & al (*New J Phys* 2018)

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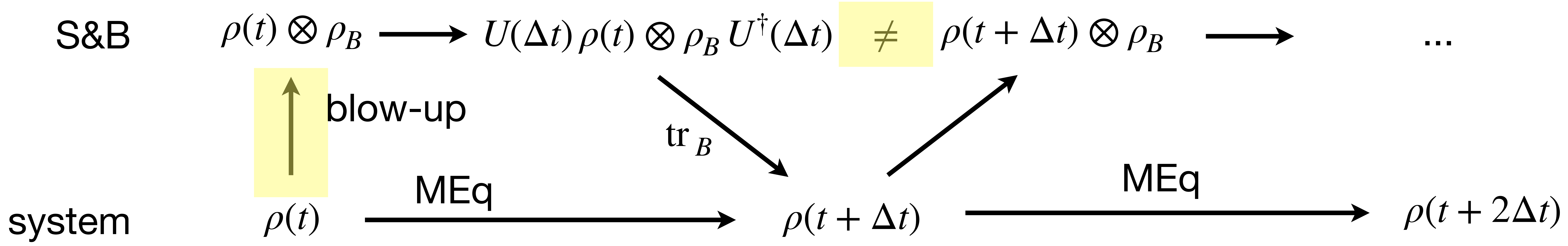
System & Bath: “blow-up Map”

replace by “fresh bath”, throw away correlations



rate of de/coupling work

$$\dot{W}_{\to a,b} = \frac{\omega_b - \omega_a}{\omega_a}\dot{Q}_{a\to b} = \langle D_a^*(V_{ab}) + D_b^*(V_{ab})\rangle$$

$$D(\rho) = \frac{1}{2}[L\rho, L^\dagger] + \frac{1}{2}[L, \rho L^\dagger]$$

$$D^*(V) = \frac{1}{2}L^\dagger[V, L] + \frac{1}{2}[L^\dagger, V]\,L \qquad \mathrm{tr}\{D(\rho)\,V\} = \mathrm{tr}\{\rho\, D^*(V)\}$$

De Chiara & al (*New J Phys* 2018)

L. S. Schulman and B. Gaveau
Ratcheting Up Energy by Means of Measurement
Phys. Rev. Lett. **97** (2006) 240405

“Blow-up” in action: Micromaser

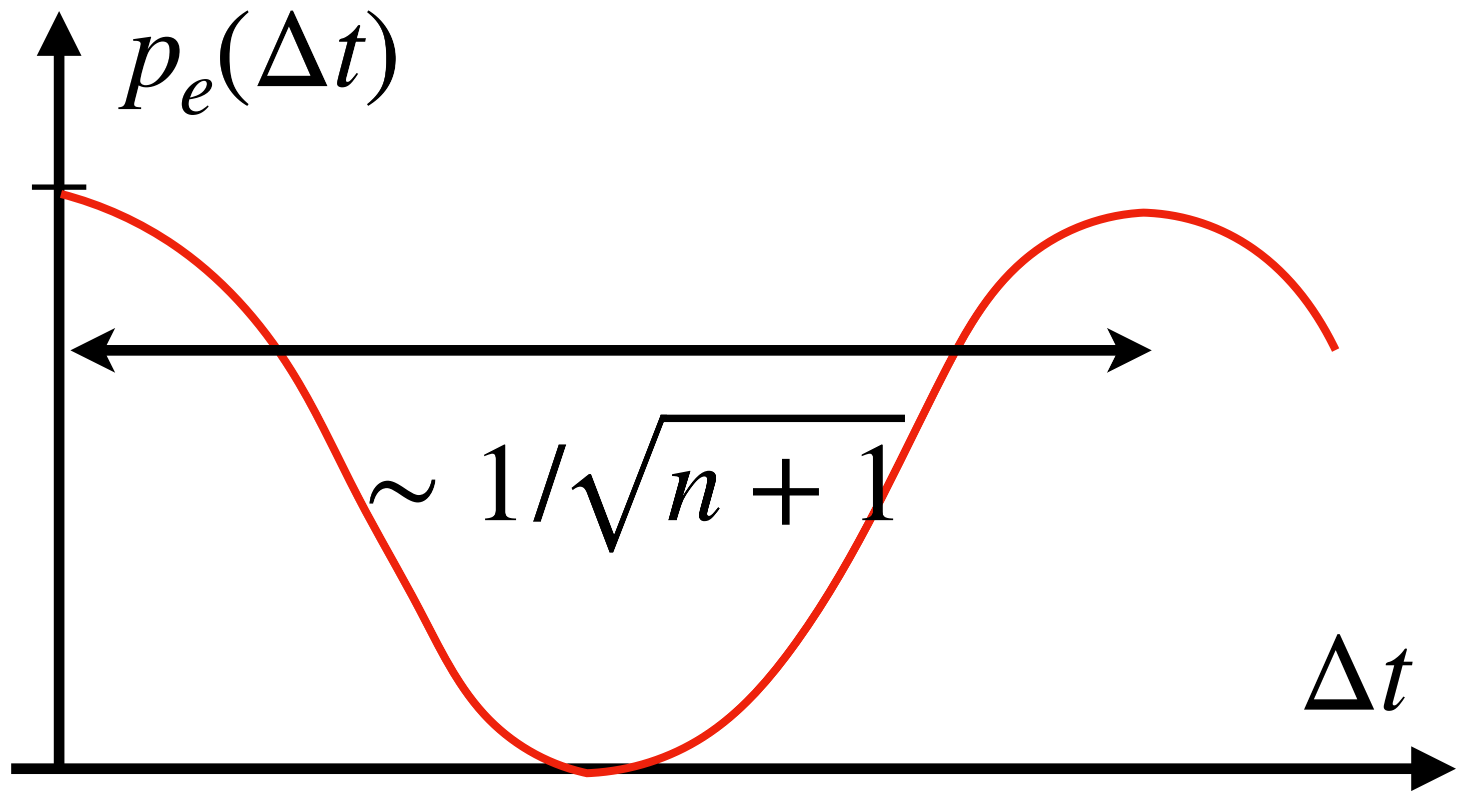
“fresh atom”

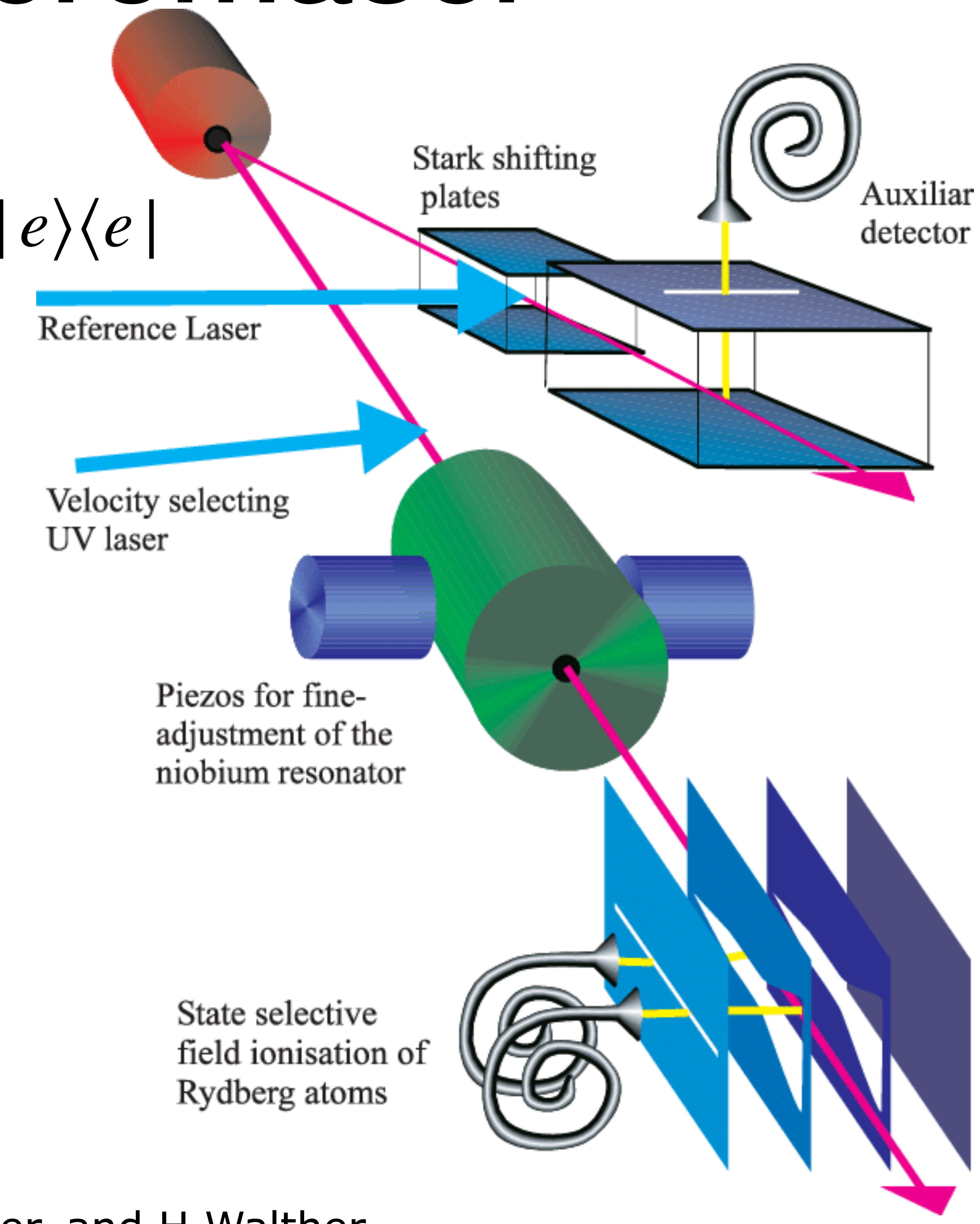
S&B $\rho(t) \otimes |e\rangle\langle e| \longrightarrow U(\Delta t) \cdots U^\dagger(\Delta t)$ $\qquad \rho(t+\Delta t) \otimes |e\rangle\langle e|$

blow-up (↑) $\qquad \mathrm{tr}_A$ (↘) $\qquad$ (↑)

system $\rho(t)$ $\xrightarrow{\text{MEq}}$ $\rho(t+\Delta t)$

resonant Rabi oscillation





B T H Varcoe, S Brattke, M Weidinger, and H Walther
Preparing pure photon number states of the radiation field
Nature **403** (2000) 743

C H, **Laser theory in manifest Lindblad form** [*J. Phys. B* **40** (2007) 2359]

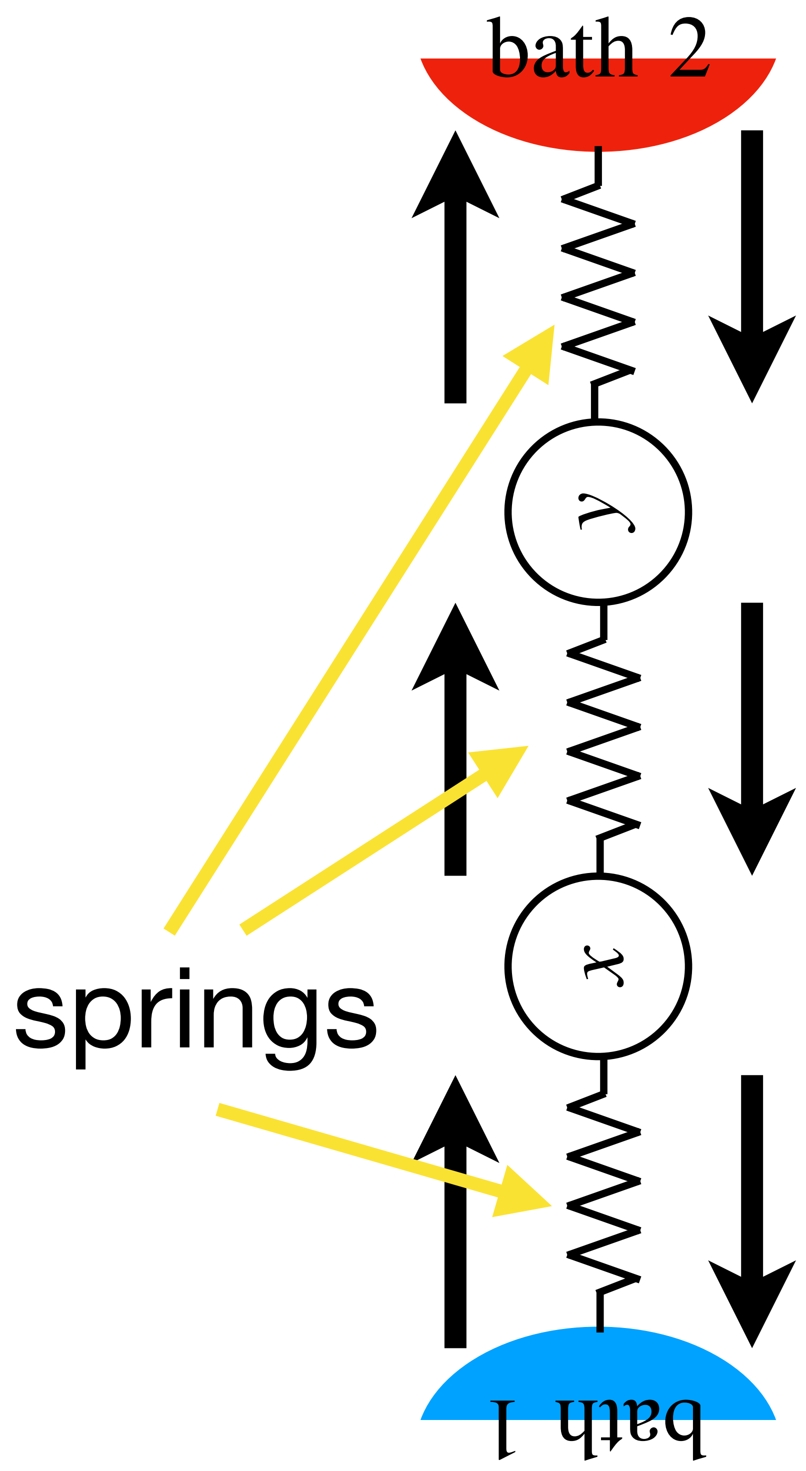
Californian Odyssey

Toronto 2015 exhibition [sunwardhobbies.ca]

Paradise Beach

Entanglement of Fluctuations

finite T : two mode-entanglement and EPR paradox



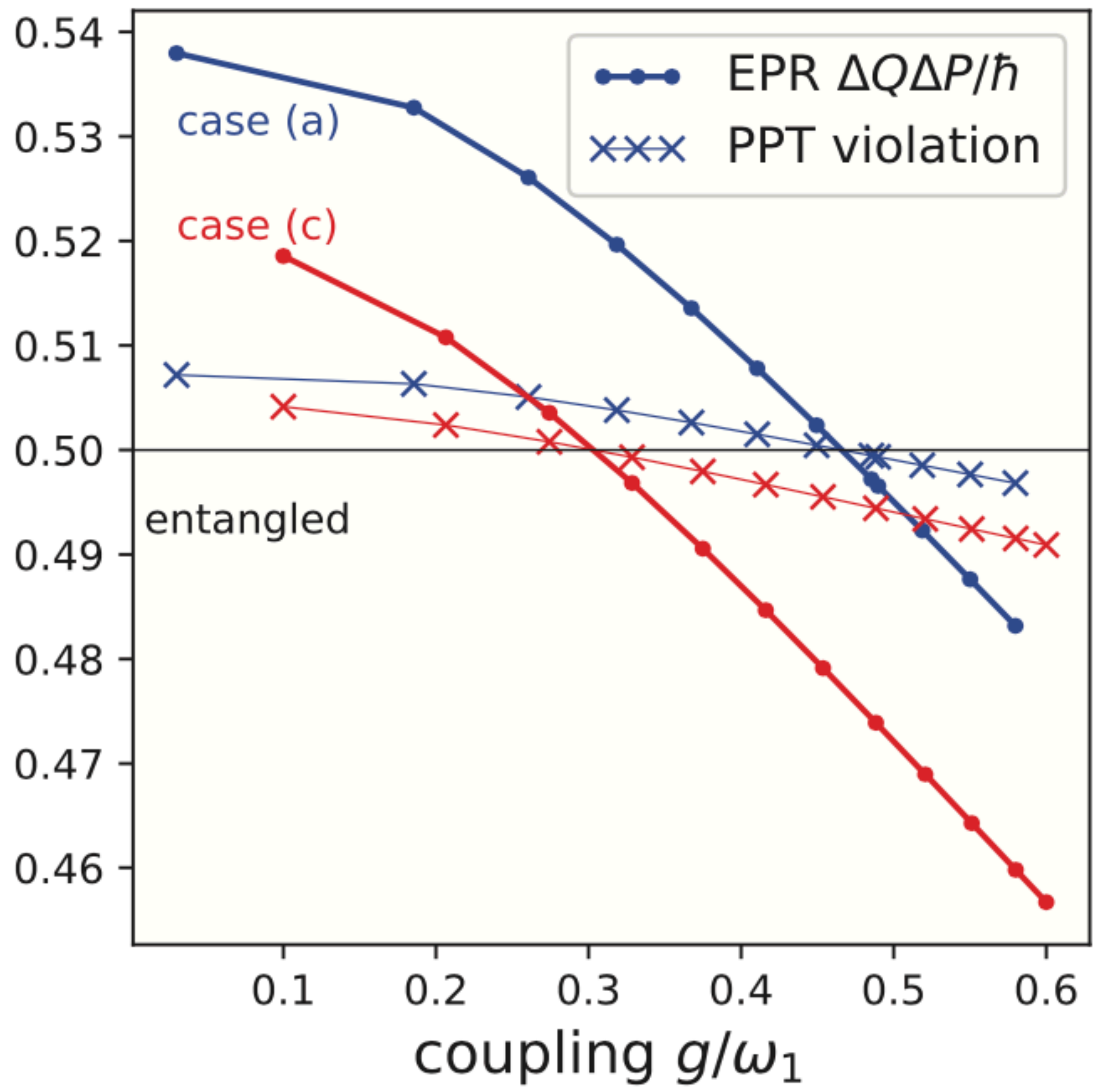


Figure 4. Covariances that test the entanglement between the two oscillators in the stationary state. Baths at low temperatures $T_1 = 0.1\,\hbar\omega_{10}$, $T_2 = 0.15\,\hbar\omega_{10}$. The curves marked case (a), case (c) correspond to the parameters of Figure 1, only the coupling between the oscillators is varied, expressed as $g = (\lambda/m_1)^{1/2}$. The crosses visualize the PPT criterion for the partially transposed covariance matrix C^Γ: when the smallest (ordinary) eigenvalue of the hermitean matrix $C^\Gamma + i\hbar\sigma/2$ falls below zero, the state is entangled.[40] (Our plot shifts this eigenvalue up by $\hbar/2$ so that entanglement appears below the same line.) The dots and curves give the uncertainty product $\Delta Q \Delta P$ of the EPR coordinate combinations (59) in units of $\hbar$, constructed from the smallest symplectic eigenvalue of C^Γ.

correlation entropy (à la Klich)

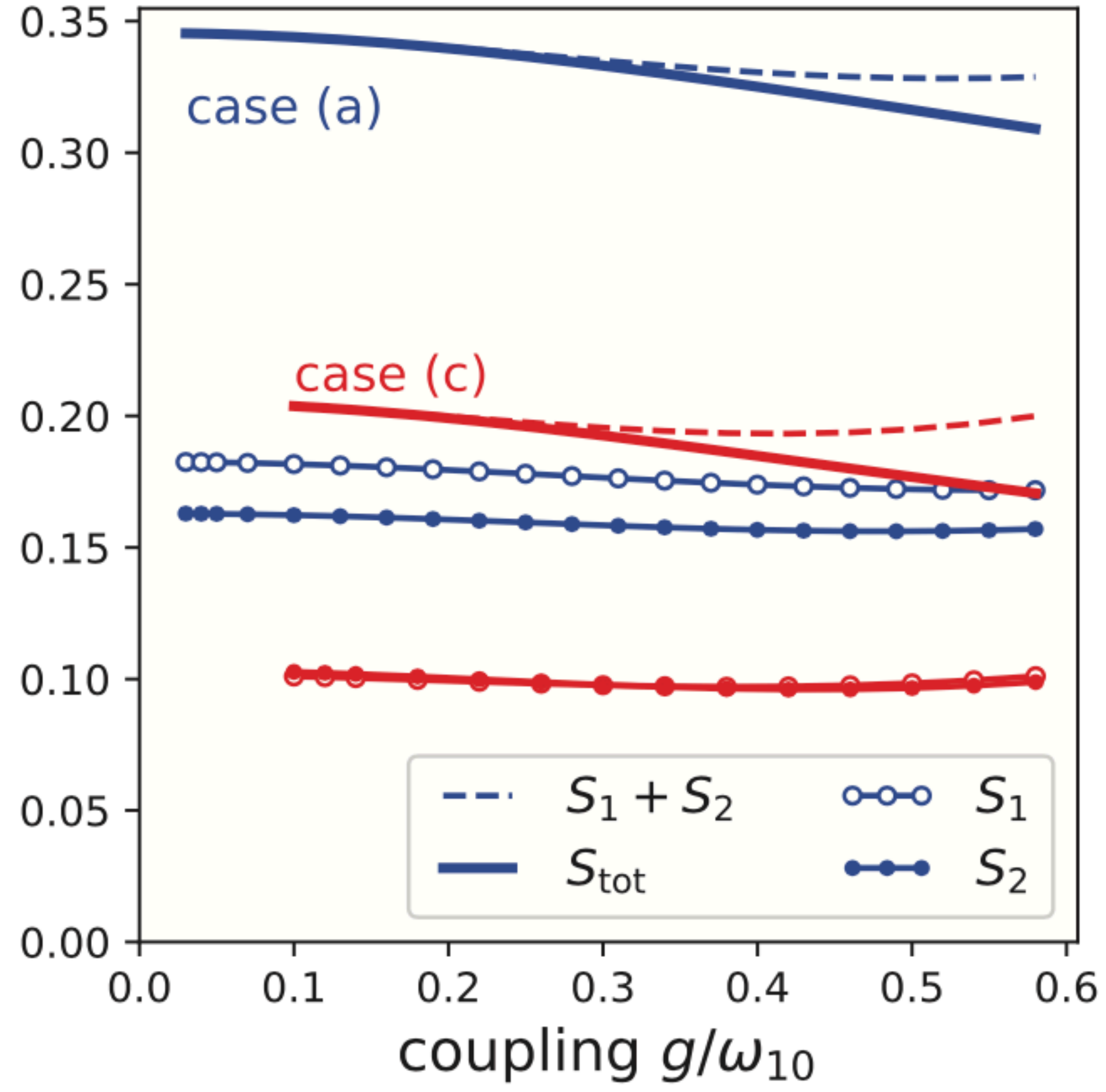
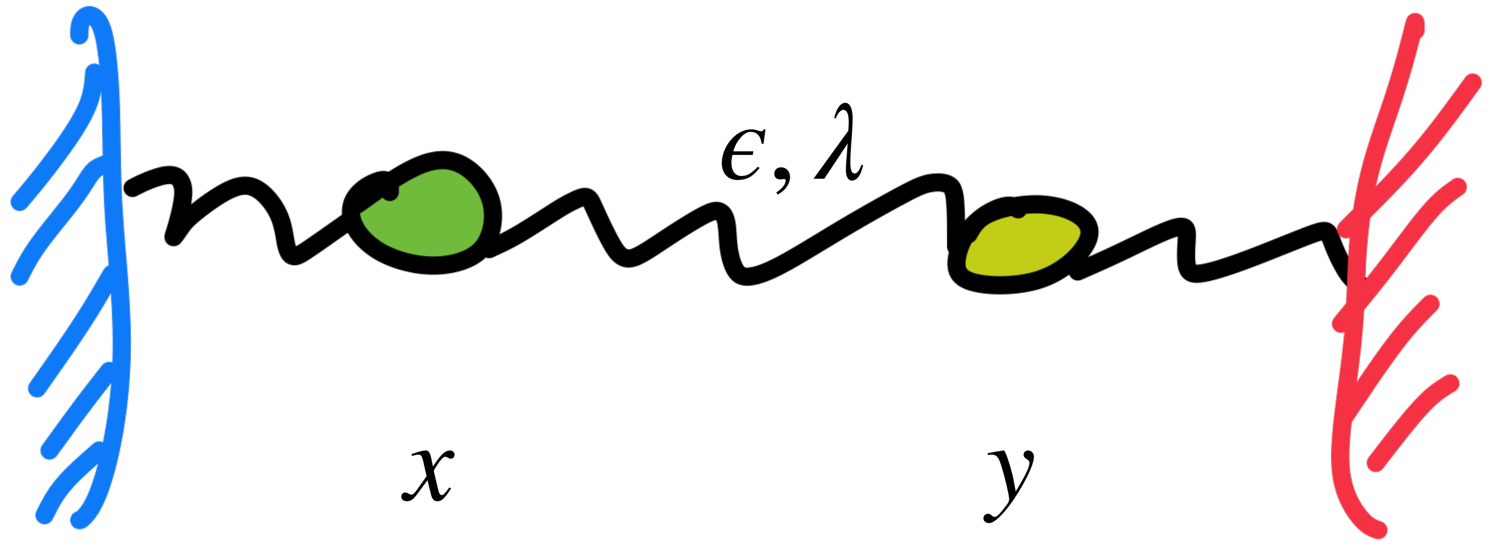


Figure 5. Entropies of the reduced oscillator states versus the coupling $g = (\lambda/m_1)^{1/2}$. The total entropy S_{tot} (Equation (60)) is computed from the symplectic eigenvalues of the full covariance matrix C (Equation (52)), while the partial entropies S_1, S_2 arise from its sub-blocks A, B. The difference with respect to additivity is a simple measure of correlations, not sensitive to entanglement, however (compare to Figure 4). The sets of curves marked case (a) (blue) and case (c) (red) correspond to the parameters of Figure 1. Distinct temperatures chosen as in Figure 4.

Henkel (*Ann. Phys. (Berlin)* 2021)

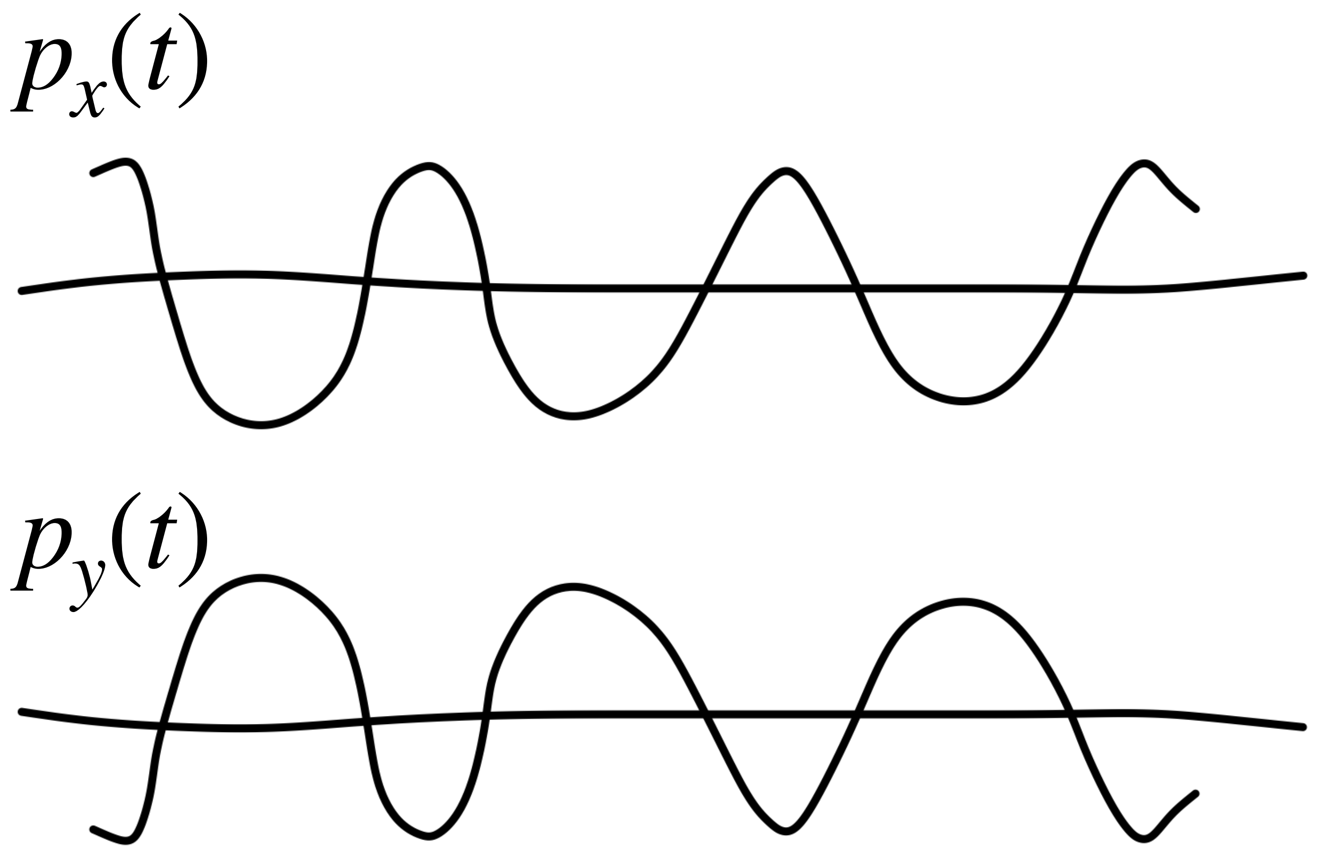
“Entanglement Spectra” of two Oscillators



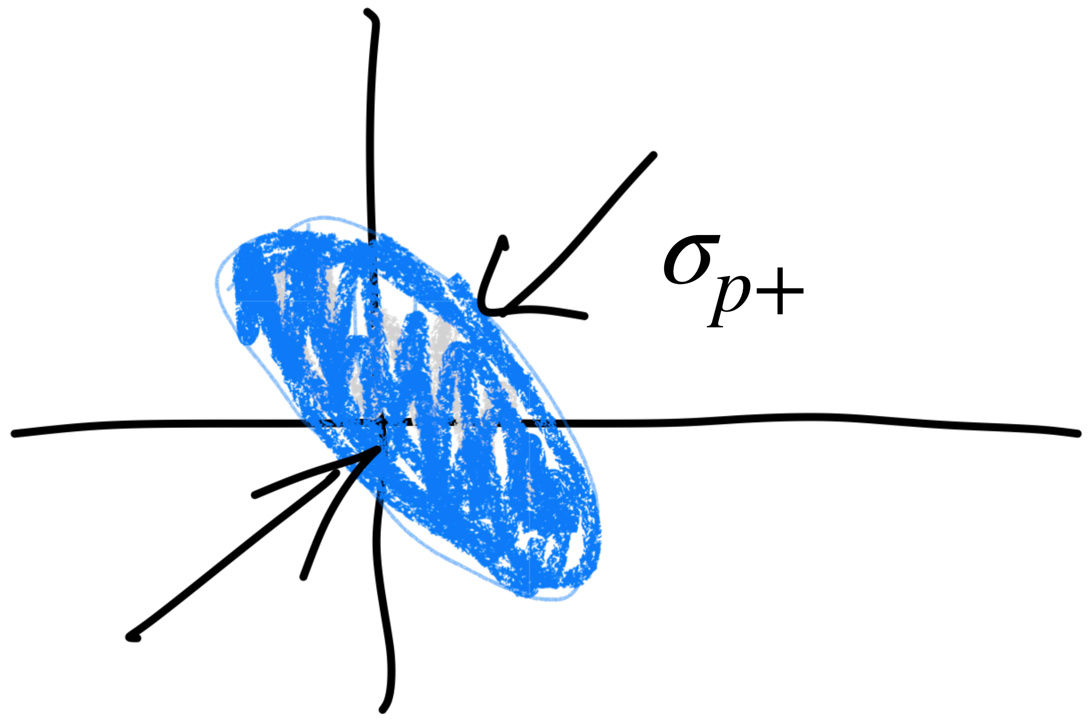
$$H = \frac{p_x^2}{2m_1} + \frac{k_1}{2}x^2 + H_2(p_y, y) + \frac{\epsilon}{2}(x-y)^2 + H_{B1} + H_{B2}$$

bosonic heat bath $H_{B1} = \sum_j \frac{1}{2m_j}\left[p_j^2 + \omega_j^2(x_j - c_j x)^2\right]$

temperature $T_1 > T_2$ steady state $t \to \infty$

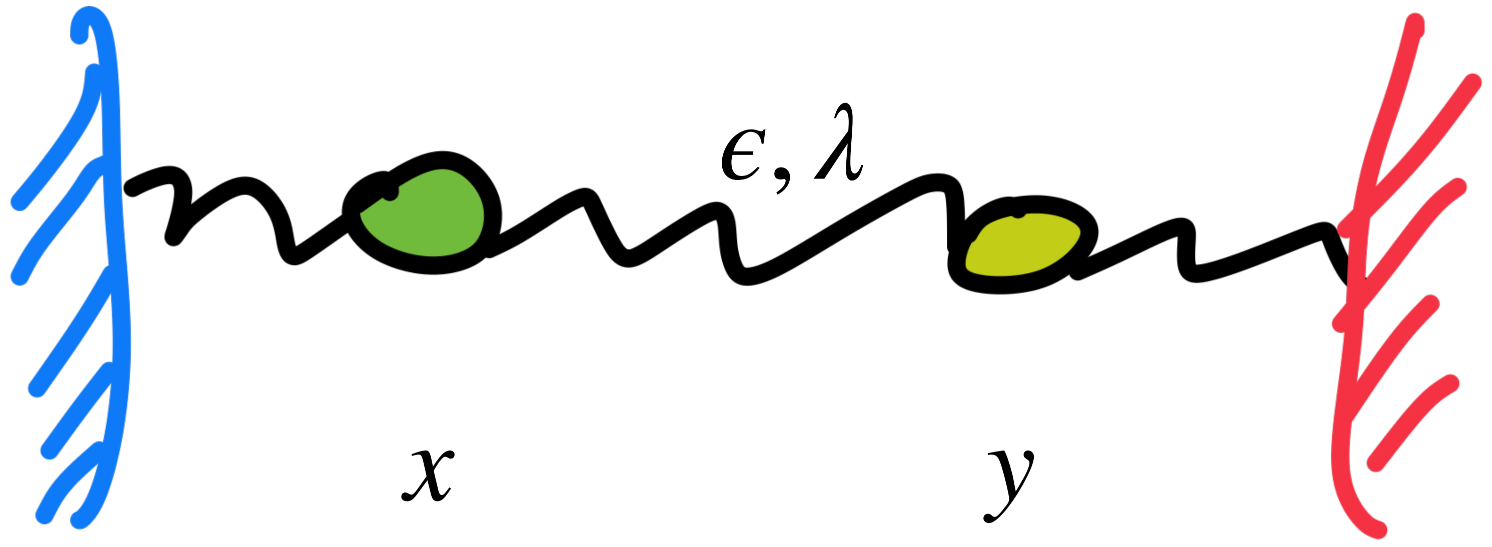


(cross) correlations $\langle p_x(t)\, p_y(t') + p_y(t')\, p_x(t) \rangle$



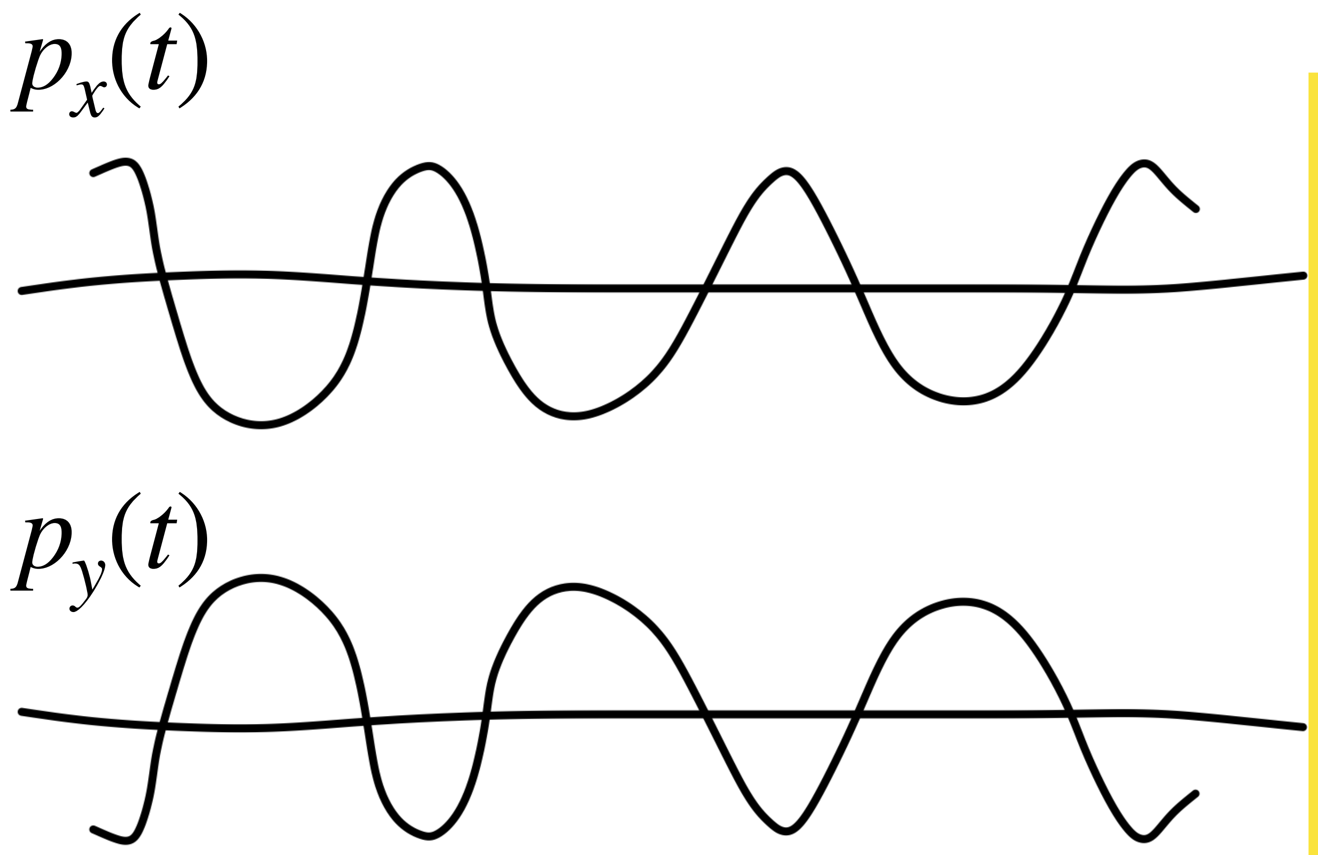
Dhar, Saito & Hänggi, *Phys Rev E* **85** (2012) 011126
Ghesquière, Sinayskiy & Petruccione, *Phys Scr* **2012**, 014017
Dorofeyev, *Can J Phys* **91** (2013) 537
Henkel, *Ann Phys* (*Berlin*) **533** (2021) 2100089

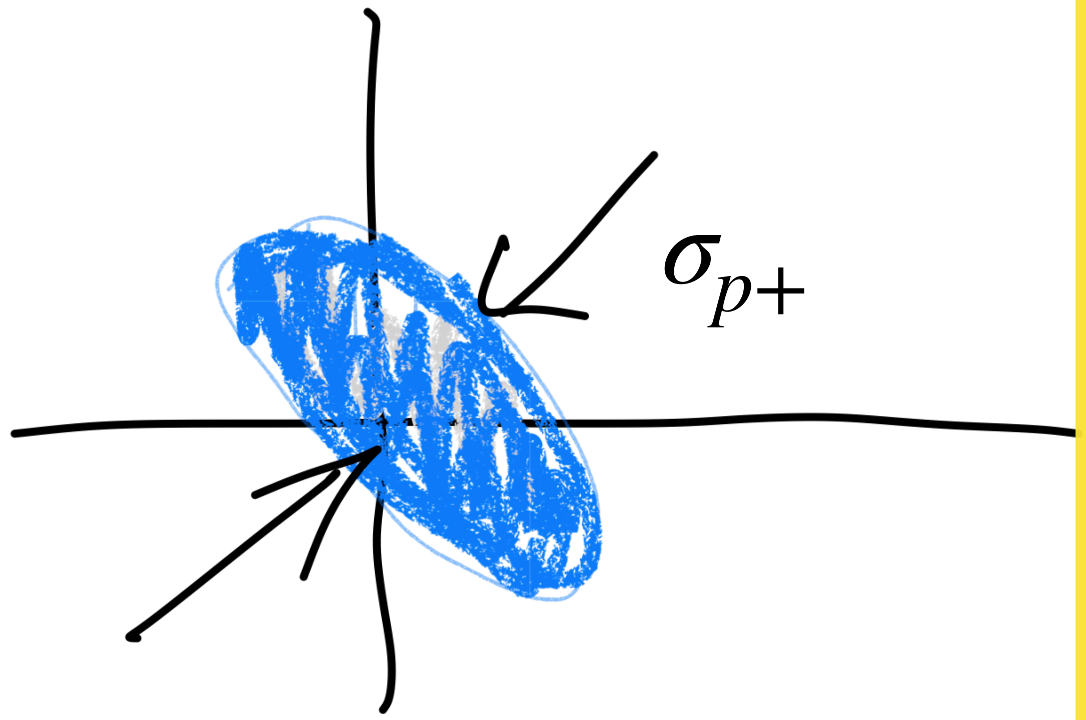
“Entanglement Spectra” of two Oscillators



$$H = \frac{p_x^2}{2m_1} + \frac{k_1}{2}x^2 + H_2(p_y, y) + \frac{\epsilon}{2}(x-y)^2 + H_{B1} + H_{B2}$$

bosonic heat bath $H_{B1} = \sum_j \frac{1}{2m_j}\left[p_j^2 + \omega_j^2(x_j - c_j x)^2\right]$





EPR Correlations (two-Mode Squeezing)

$\sigma_{p+} = \Delta(p_x + p_y) <$ **Heisenberg:** $\sqrt{\hbar m \omega}$

$[p_x + p_y,\ x - y] = 0$

Duan-Simon Criterion: entangled continuous Variables

- positive partial Transpose
- “optimal” Quadrature $Q_{\text{EPR}} = \alpha x + \beta y + \gamma p_x + \eta p_y$

Oscillators driven out of Equilibrium

Langevin equations

$$\dot{p}_x + k_1' x = \lambda y - \mu_1 * \dot{x} + F_1(t)$$

$$\dot{p}_y + k_2' y = \lambda x - \mu_2 * \dot{y} + F_2(t)$$

Langevin forces

damping

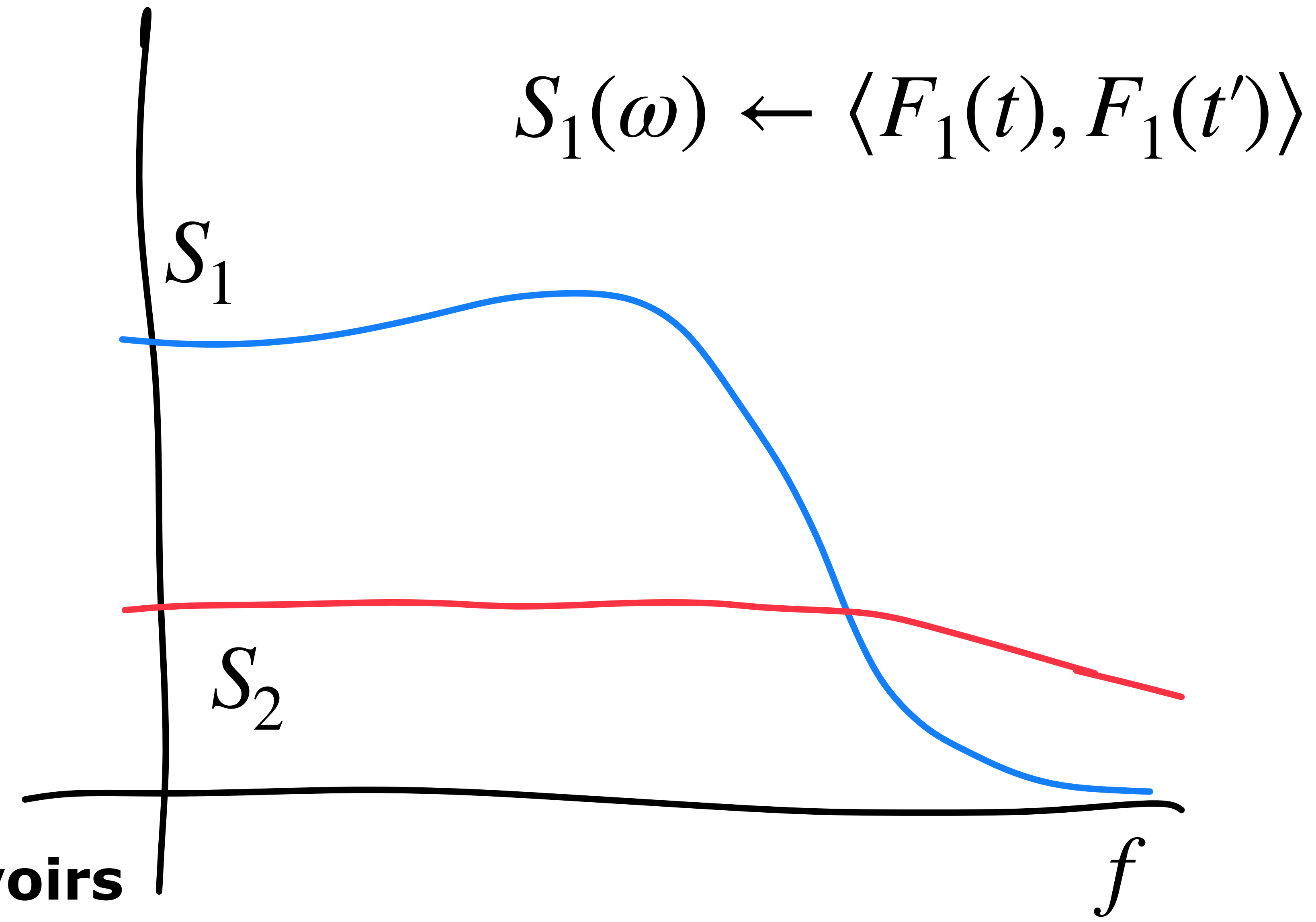
Fourier solution

$$k_i' = k_i + \lambda$$

thermal spectrum

$$\langle (x - y)^2 \rangle_{12}$$

$$= \int_0^\infty \frac{d\omega}{2\pi} \left\{ \left| \frac{K_2(\omega) - \lambda}{D(\omega)} \right|^2 S_{F1}(\omega) + \left| \frac{K_1(\omega) - \lambda}{D(\omega)} \right|^2 S_{F2}(\omega) \right\}$$



$$K_i(\omega) = k_i + \lambda - m_i \omega^2 - i\omega\mu_i(\omega)$$ resonant denominator

$$D(\omega) = K_1(\omega) K_2(\omega) - \lambda^2$$

Coupled quantum oscillators within independent quantum reservoirs
I. Dorofeyev, *Can. J. Phys.* **91** (2013) 537

Oscillators driven out of Equilibrium

Langevin equations

$$\dot{p}_x + k_1' x = \lambda y - \mu_1 * \dot{x} + F_1(t)$$

$$\dot{p}_y + k_2' y = \lambda x - \mu_2 * \dot{y} + F_2(t)$$

Langevin forces

damping

Fourier solution

$$k_i' = k_i + \lambda$$

$$\langle (x-y)^2 \rangle_{12}$$

$$= \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \left\{ \left| \frac{K_2(\omega) - \lambda}{D(\omega)} \right|^2 S_{F1}(\omega) + \left| \frac{K_1(\omega) - \lambda}{D(\omega)} \right|^2 S_{F2}(\omega) \right\}$$

$$K_i(\omega) = k_i + \lambda - m_i \omega^2 - \mathrm{i}\omega\mu_i(\omega)$$

resonant denominator

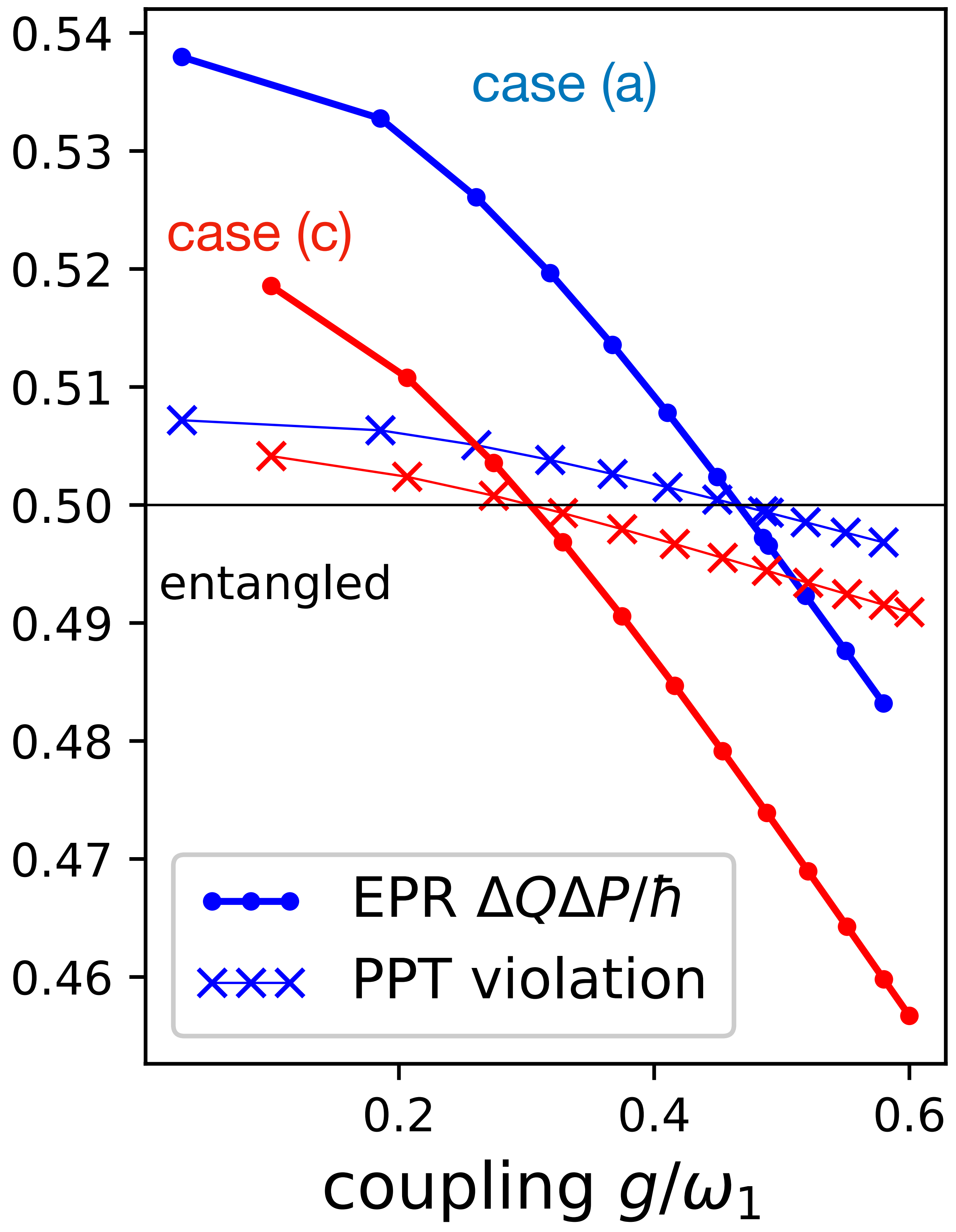
$$D(\omega) = K_1(\omega) K_2(\omega) - \lambda^2$$

cross-correlations

$$\begin{pmatrix} \dots & \langle xy \rangle & -\langle x p_y \rangle \\ & \langle p_x y \rangle & -\langle p_x p_y \rangle \\ \dots & & \dots \end{pmatrix}$$

Coupled quantum oscillators within independent quantum reservoirs
I. Dorofeyev, *Can. J. Phys.* **91** (2013) 537

Entangled Oscillators



$$g = (\lambda^2/m_1 m_2)^{1/4}$$
$$T_1, T_2 = (0.1, 0.15)\hbar\omega_1$$

covariance matrix

$$(q_i) = (x, p_x, y, p_y)$$
$$C_{ij} = \lim_{t\to\infty} \langle q_i(t)\,, q_j(t)\rangle$$

partial transpose: $-p_y$

$$C^\Gamma = \begin{pmatrix} \langle x^2\rangle & \langle x p_x\rangle & \langle xy\rangle & -\langle x p_y\rangle \\ & \langle p_x^2\rangle & \langle p_x y\rangle & -\langle p_x p_y\rangle \\ & & \langle y^2\rangle & -\langle y p_y\rangle \\ \ldots & & & \langle p_y^2\rangle \end{pmatrix}$$

symplectic eigenvalues/-vectors

$$S C^\Gamma S^\top = \mathrm{diag}(\eta_1, \eta_1, \eta_2, \eta_2)$$
$$\eta_1 = \Delta Q_1^2 = \Delta P_1^2$$

Optimal quadrature, e.g.,

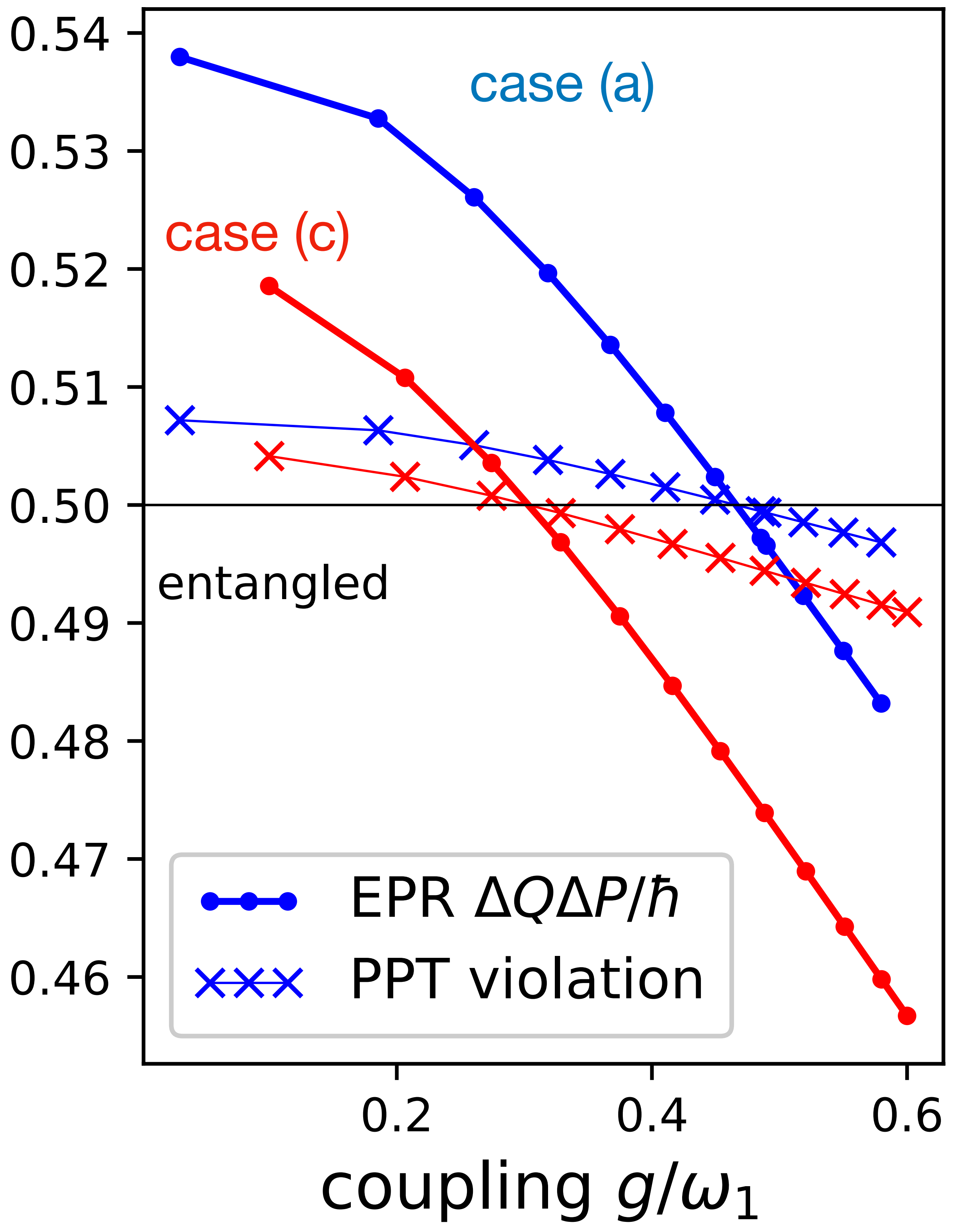
$Q_1 = x - y,\ P_1 = p_x + p_y$

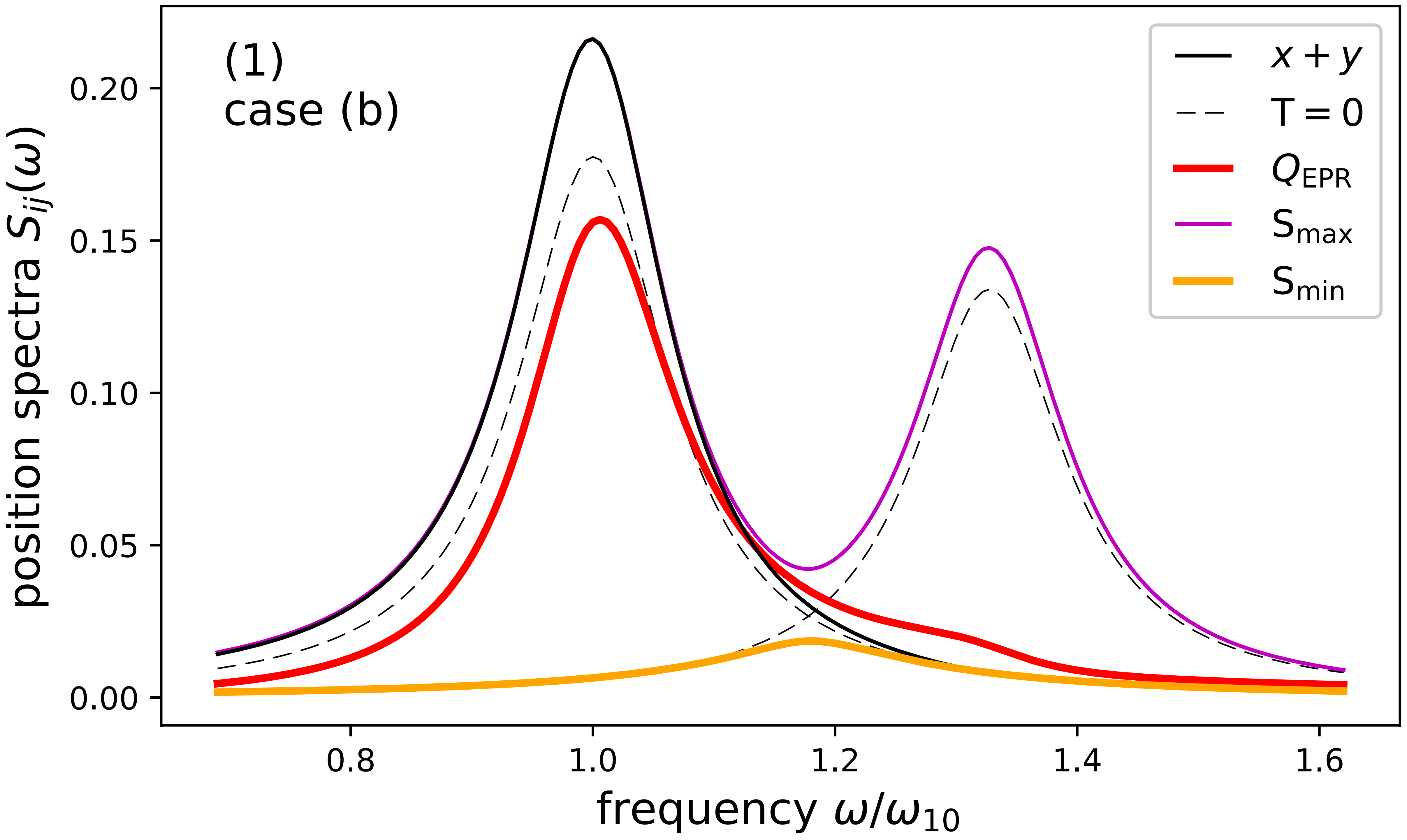
EPR entanglement: $\min(\eta_1, \eta_2) < \hbar/2$

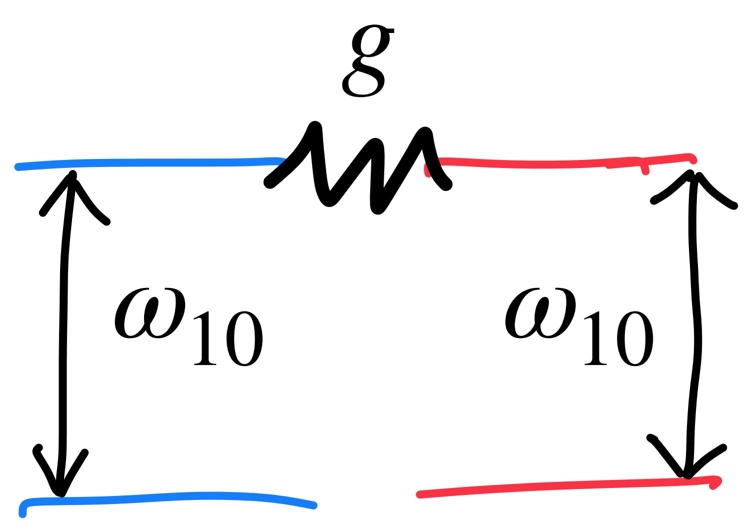
Peres (1996), Horodecki³ (1996)
Simon (2000), Duan, Giedke, Cirac & Zoller (2000)

Henkel, *Ann Phys* (*Berlin*) **533** (2021) 2100089

Entanglement Spectrum





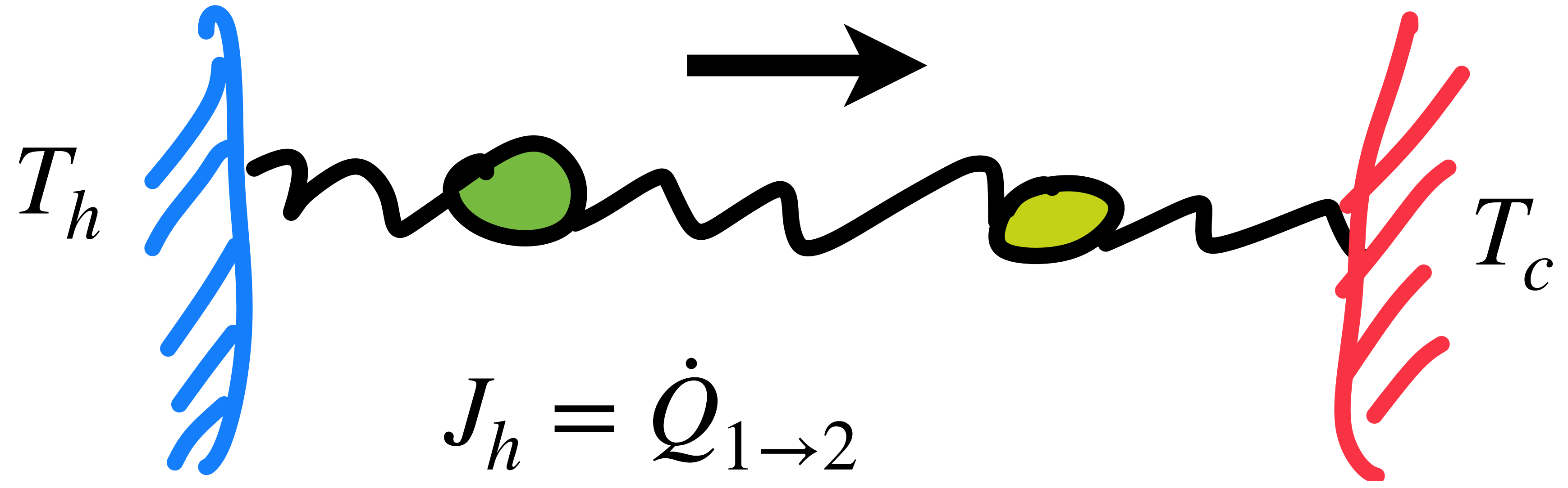


$- - -\ T = 0$ limits for $S_{x\pm y}(\omega)$ from fluctuation-dissipation theorem

$$g = (\lambda^2/m_1 m_2)^{1/4}$$
$$T_1, T_2 = (0.1, 0.15)\hbar\omega_1$$

$$g \approx 0.51\,\omega_1$$
$$T_1, T_2 = (0.5, 0.25)\hbar\omega_1$$

Heat Transfer Spectrum



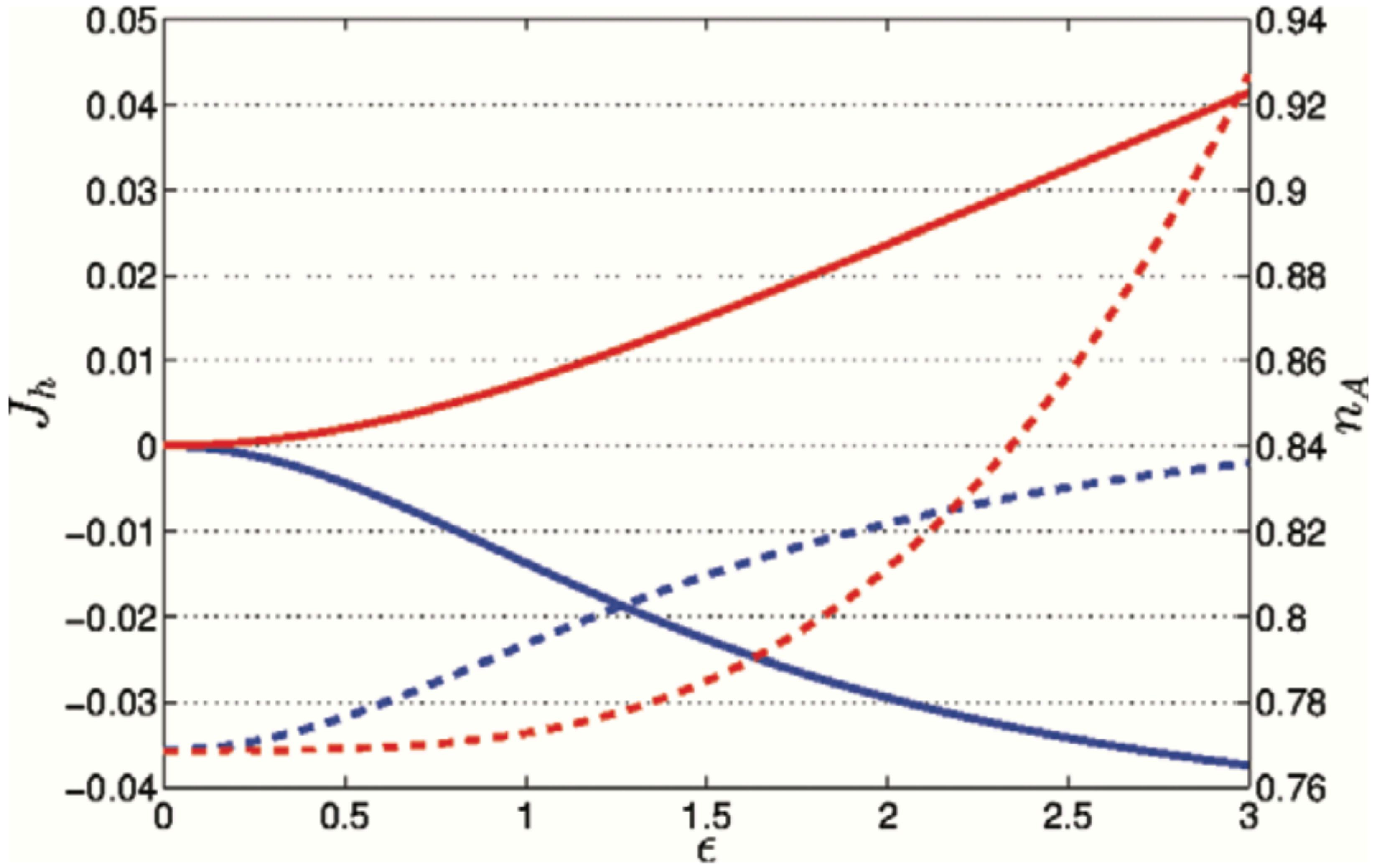
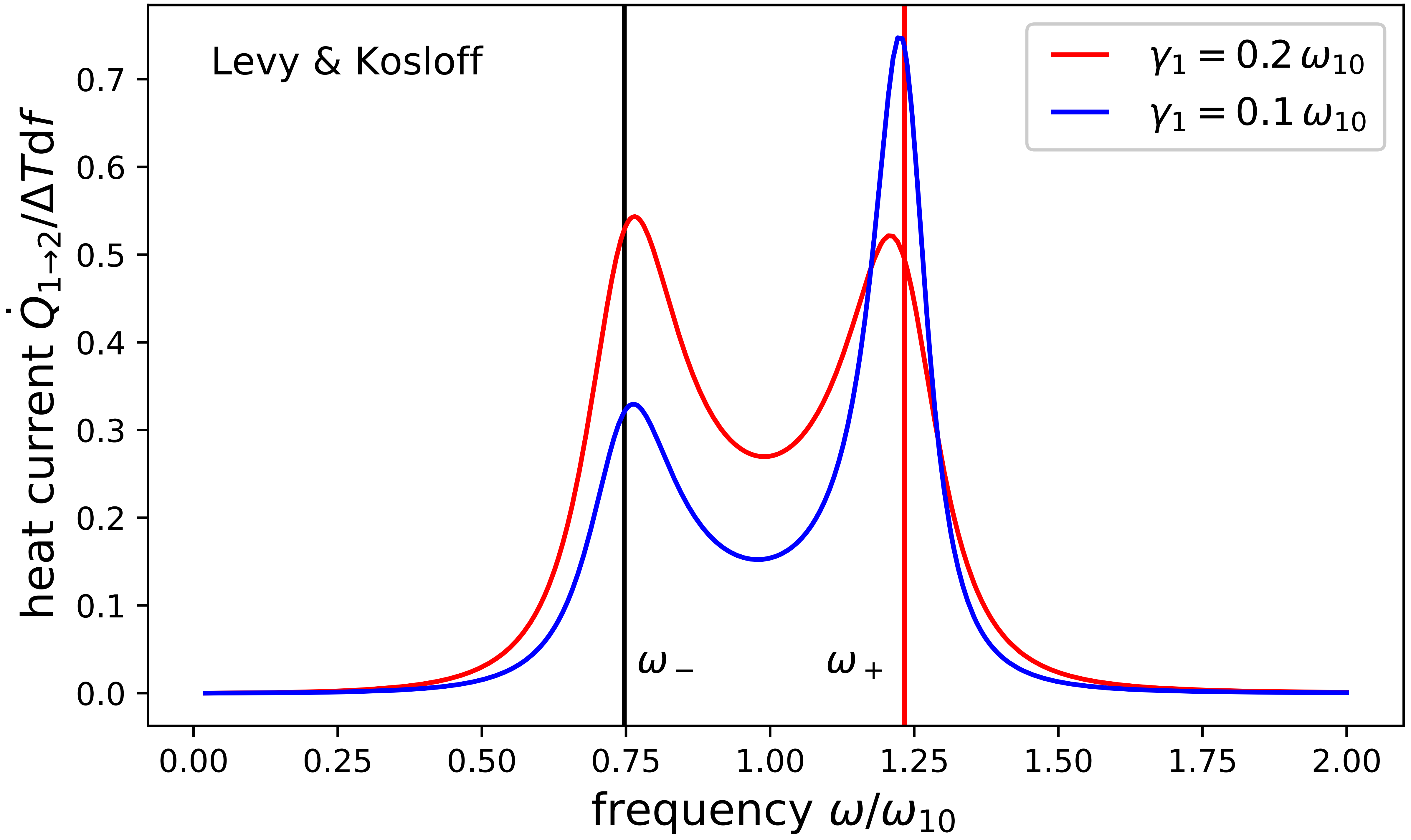


Fig. 3: (Colour on-line) The heat current $\mathcal{J}_h$ and the population as a function of the coupling parameter ϵ evaluated in the local (blue line) and the global (red line) approaches. The population of subsystem A (dashed line), and the heat flow from the hot bath $\mathcal{J}_h$ (solid line). Here $T_h = 12$, $T_c = 10$, $\omega_h = 10$, $\omega_c = 5$ and $\kappa = 10^{-4}$. γ(ω_c) = 0.01

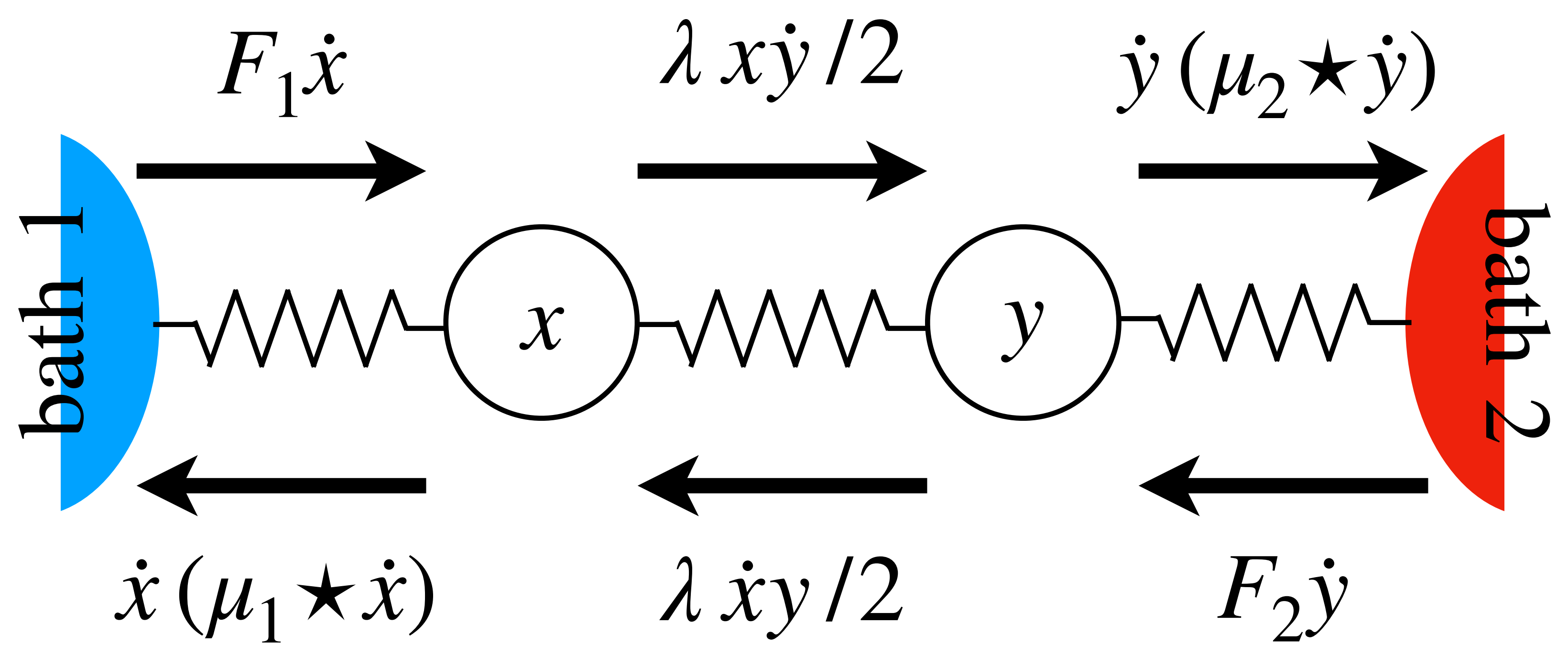


$$V \sim a^\dagger b + \text{h.c.}$$

The local approach to quantum transport may violate the second law of thermodynamics
Levy & Kosloff [*Europhys Lett* **107** (2014) 20004]

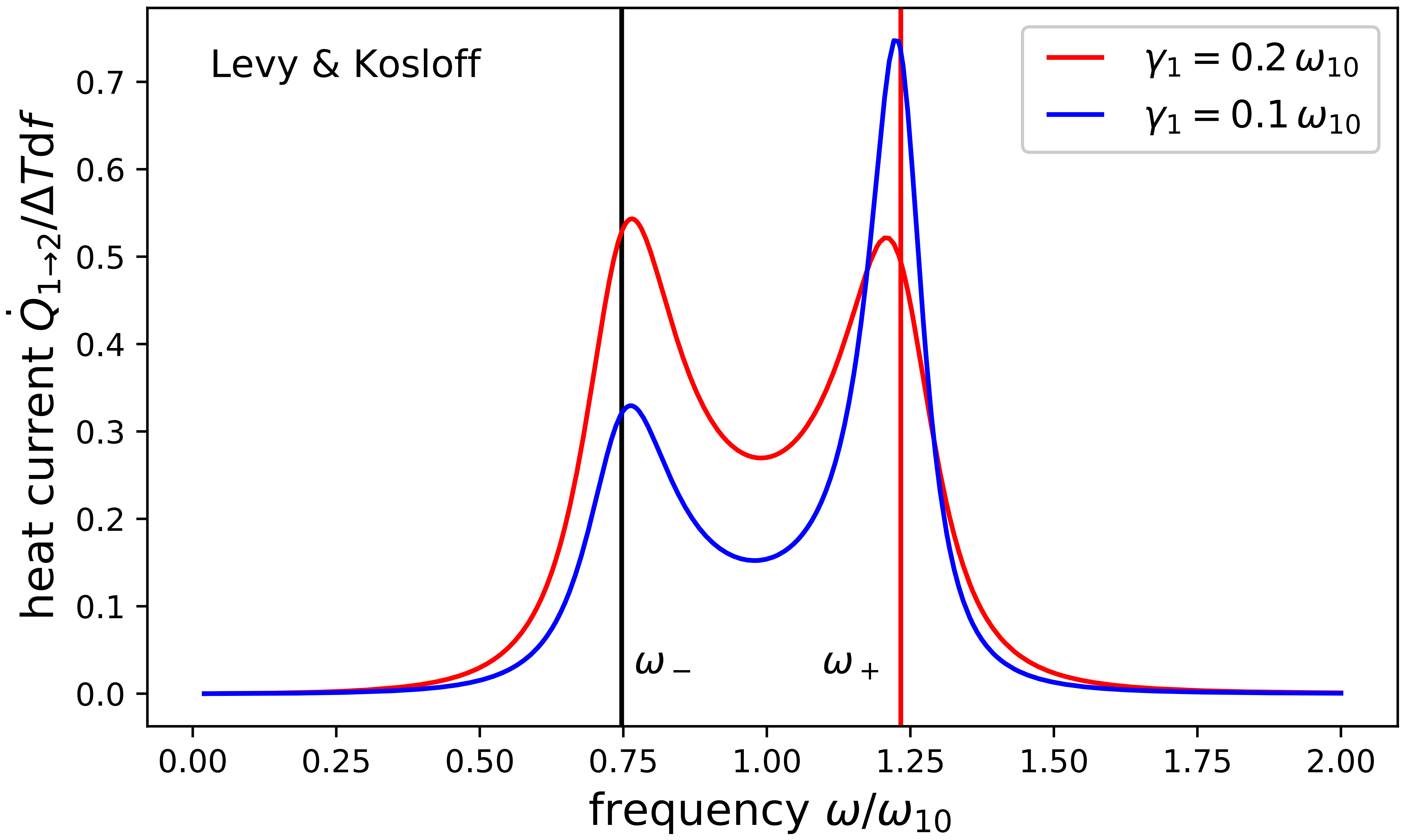
critique: Hewgill, De Chiara & Imparato
[*Phys Rev Research* **3** (2021) 013165]

Heat Transfer Spectrum



$$\overline{\left\langle \frac{\mathrm{d}H_1}{\mathrm{d}t} \right\rangle} = \lambda\, \overline{\langle \dot{x}(y - x)\rangle} - \overline{\langle \dot{x}(\mu_1 * \dot{x})\rangle} + \overline{\langle F_1 \dot{x}\rangle} \qquad (41)$$

The first term is the power transferred by the connecting spring. The second term is negative definite (for any friction kernel) and can be interpreted as the power dissipated into heat bath 1. Finally, the last term is the rate of work performed by the Langevin force F_1 on the oscillator (x, p_x).



$$\dot{Q}_{1\to 2} = \frac{\lambda}{2}\overline{\langle x\dot{y} - \dot{x}y\rangle}$$

$$\sim -\mathrm{i}(\omega_a + \omega_b)\langle a^\dagger b - b^\dagger a\rangle + \mathrm{i}(\omega_a - \omega_b)\langle ab - a^\dagger b^\dagger\rangle$$

$$\dot{Q}_{1\to 2} = 4\lambda^2 \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\omega^2 \frac{\rho_1\rho_2}{|D|^2}(\vartheta_1 - \vartheta_2)$$

sign OK for
all parameters
(weak/strong damping,
Markov or not)

$$\vartheta(\omega) = \frac{\hbar\omega}{2}\coth\frac{\hbar\omega}{2T} = \hbar\omega\left[\bar{n}(\omega) + \tfrac{1}{2}\right] \qquad S_{F1}(\omega) = 4\rho_1(\omega)\vartheta_1(\omega)$$

Henkel, *Ann Phys* (*Berlin*) **533** (2021) 2100089

Master Equations – acquitted?

- versatile in weak coupling: coherent evol'n (H) & losses (L) compete
- thermalisation OK with detailed balance (Lindblad rates)
- heat transport OK with full work balance
- correct entropy production if completely positive*
- "strong damping" challenge – blurred system | bath boundary

*R Alicki
The quantum open system as a model of the heat engine
J. Phys. A **12** (1979) L103

... thanks to

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