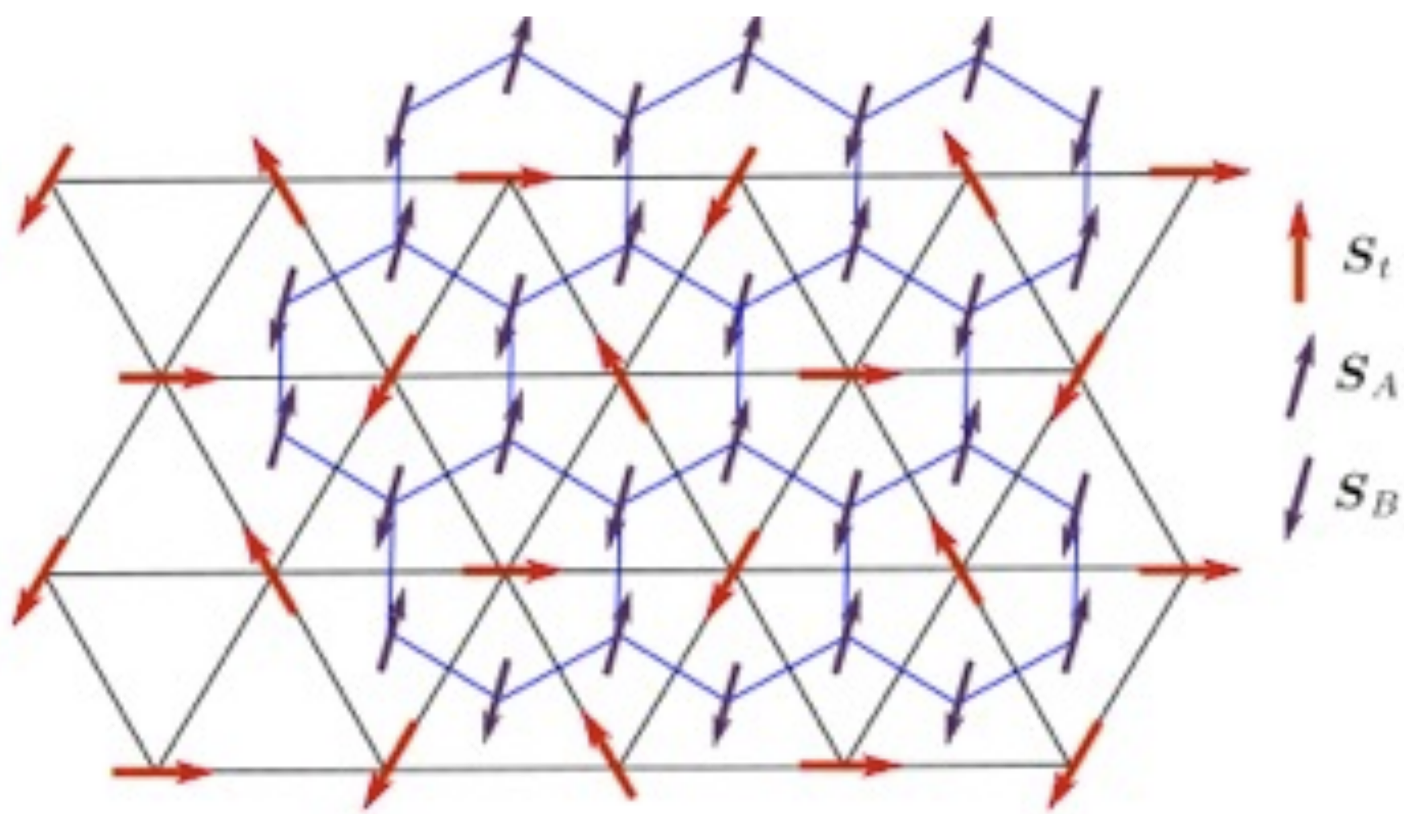
Emergent Critical Phase & Ricci Flow in a 2D Frustrated Heisenberg AFM

arXiv:1206.5740

Peter Orth
Premi Chandra
Piers Coleman
Joerg Schmalian



discussions +
Daniel Friedan

Exotic Phases of Frustrated Magnets, KITP, Oct 8, 2012.

2D Heisenberg Antiferromagnets at Finite Temperature

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Hohenberg-Mermin-Wagner Theorem (1966) $\longrightarrow$ No Long-Range Order at Finite Temperatures

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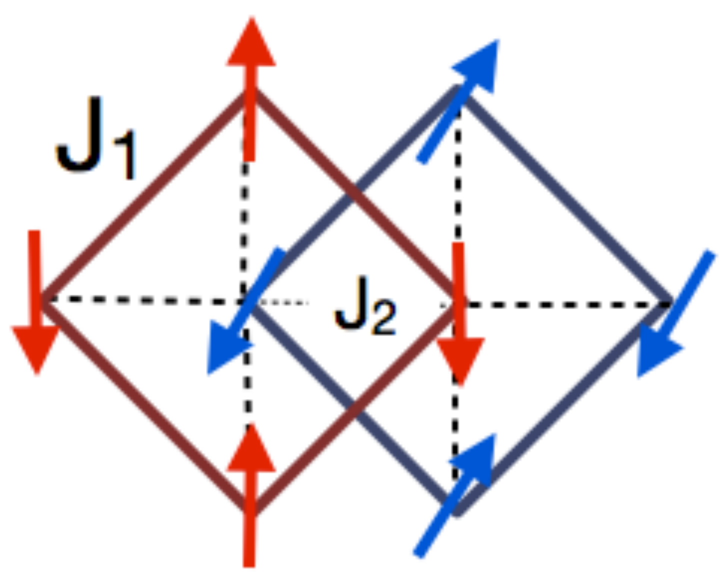
Sublattices classically decoupled



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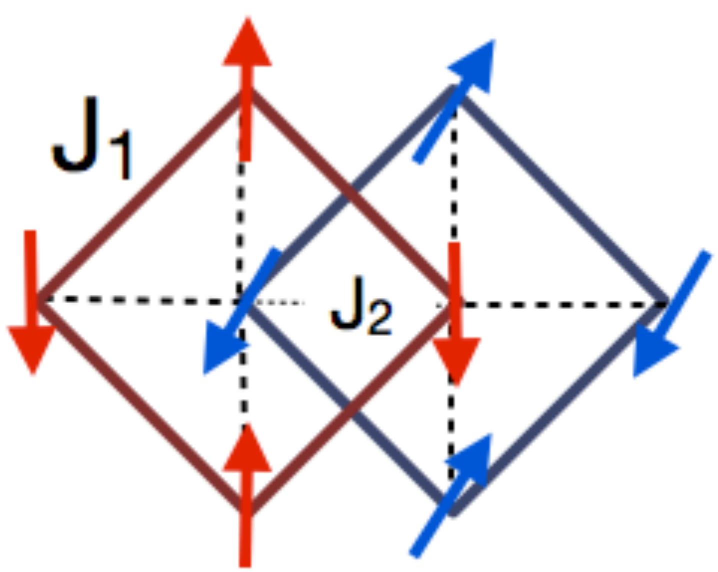

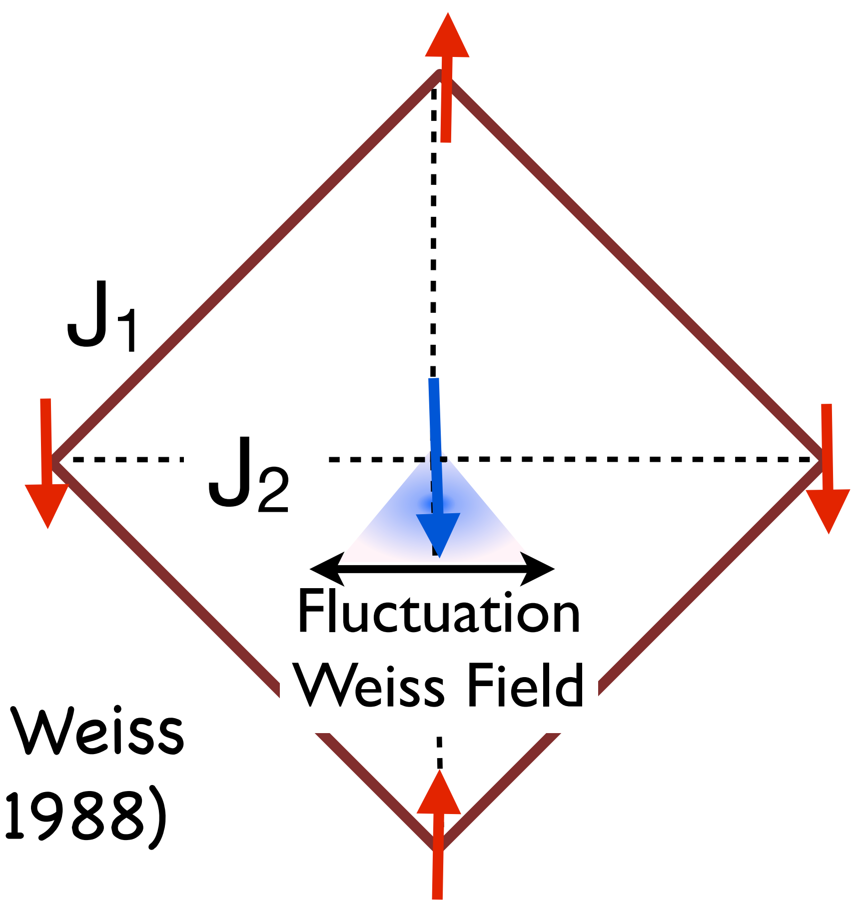

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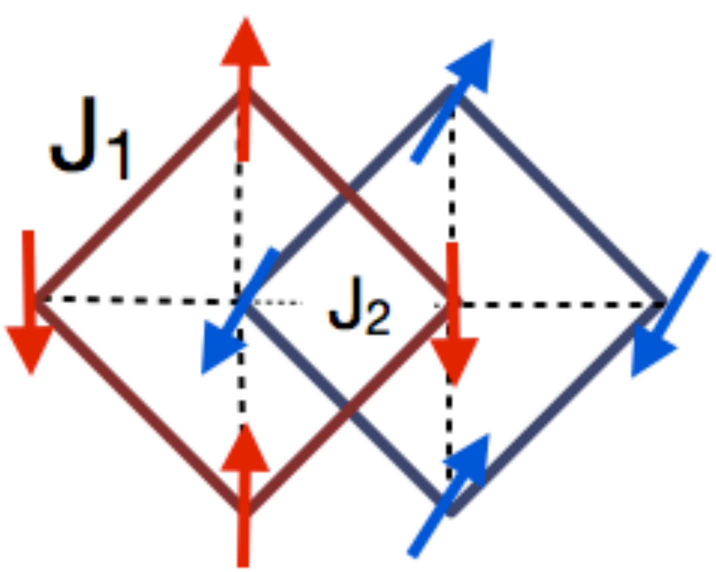
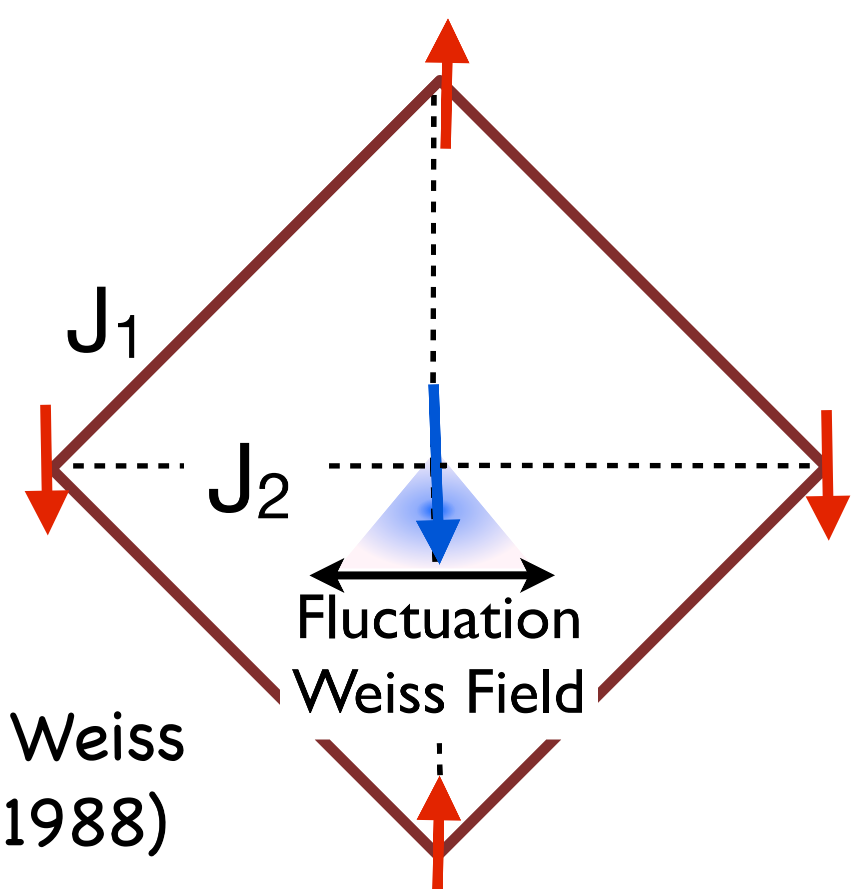


"Order from Disorder" (Villain 1985)



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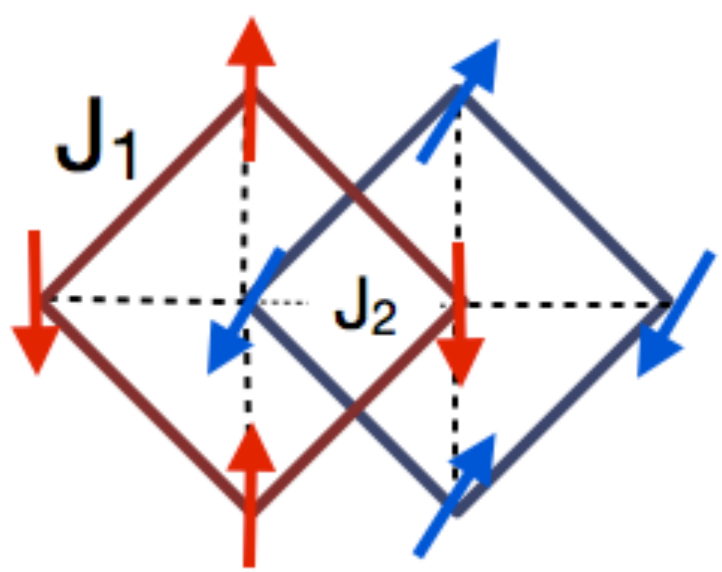

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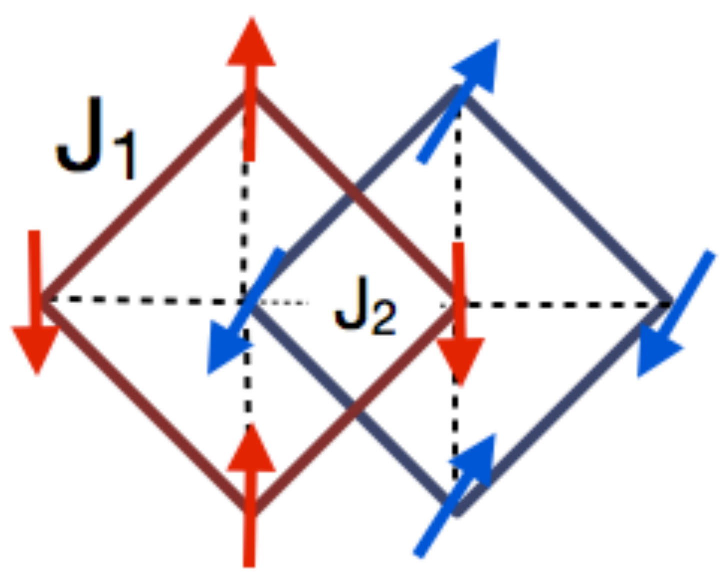

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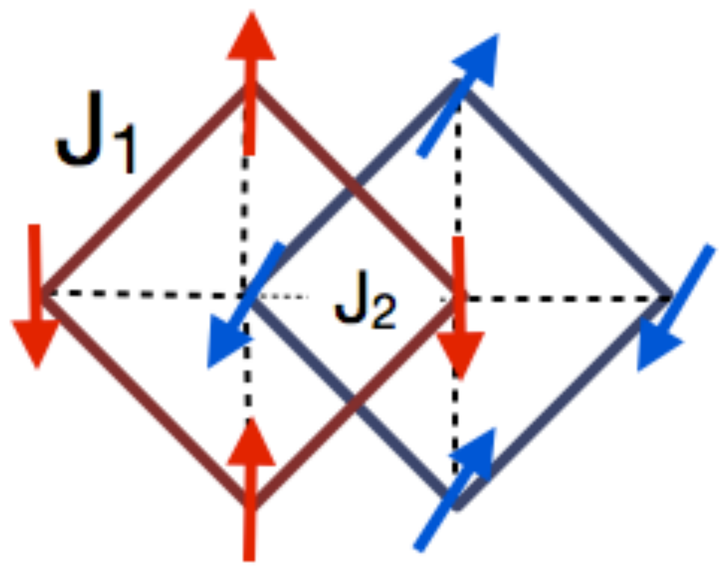

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$\xi_{Z_2} \to \infty$ T_{Z_2} Chandra, Coleman and Larkin (90)

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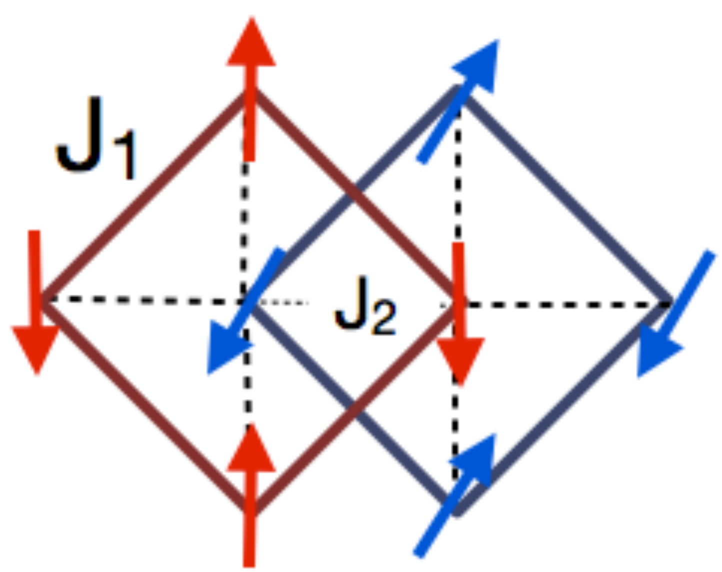

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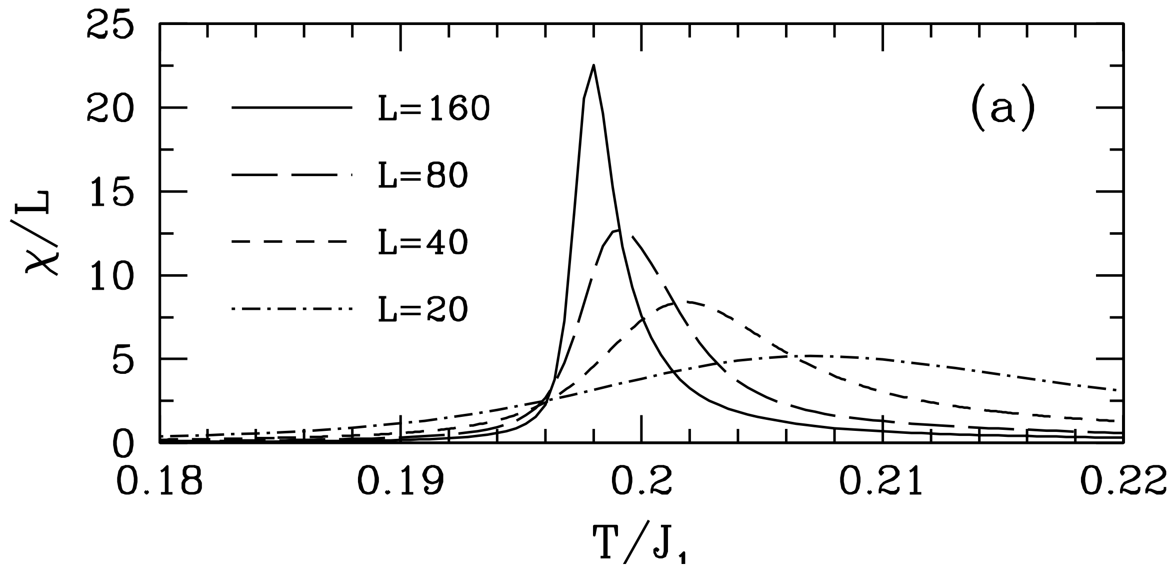
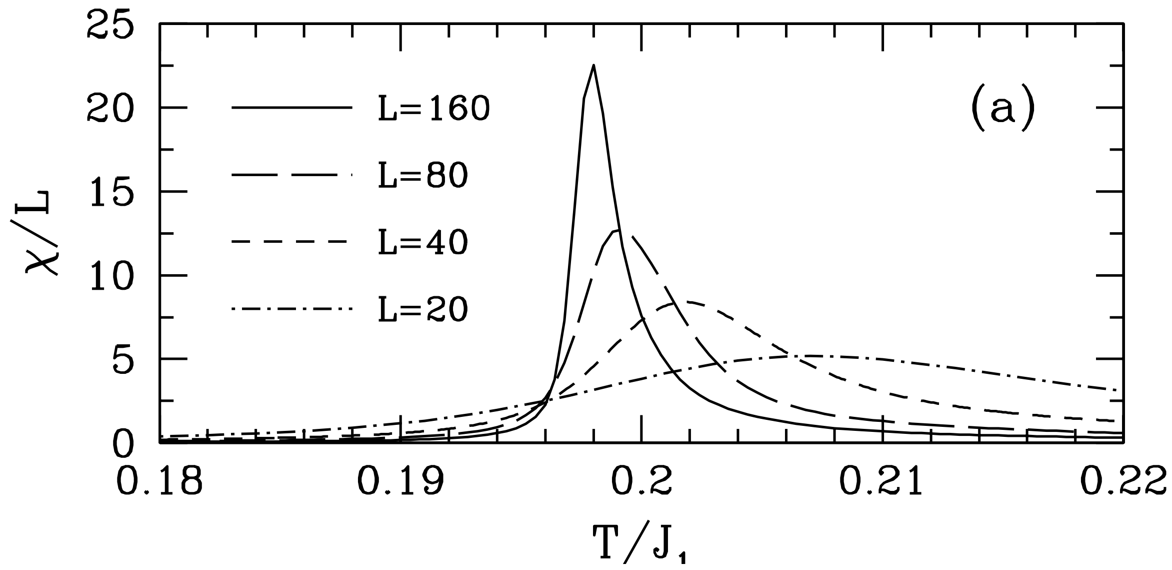
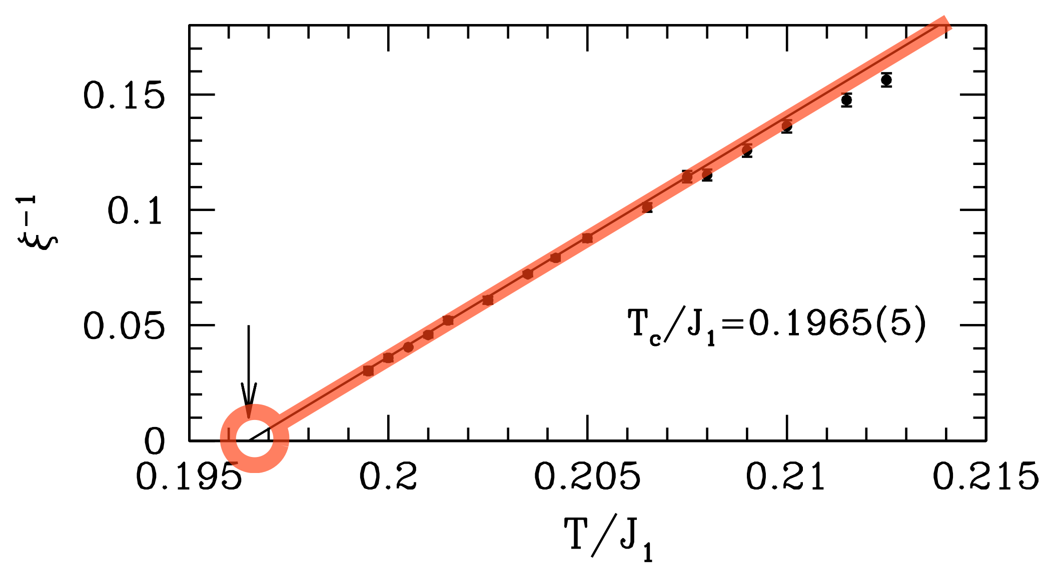
Emergent Z_2 Phase Transition in a disordered Heisenberg System.



Weber et al (2003)





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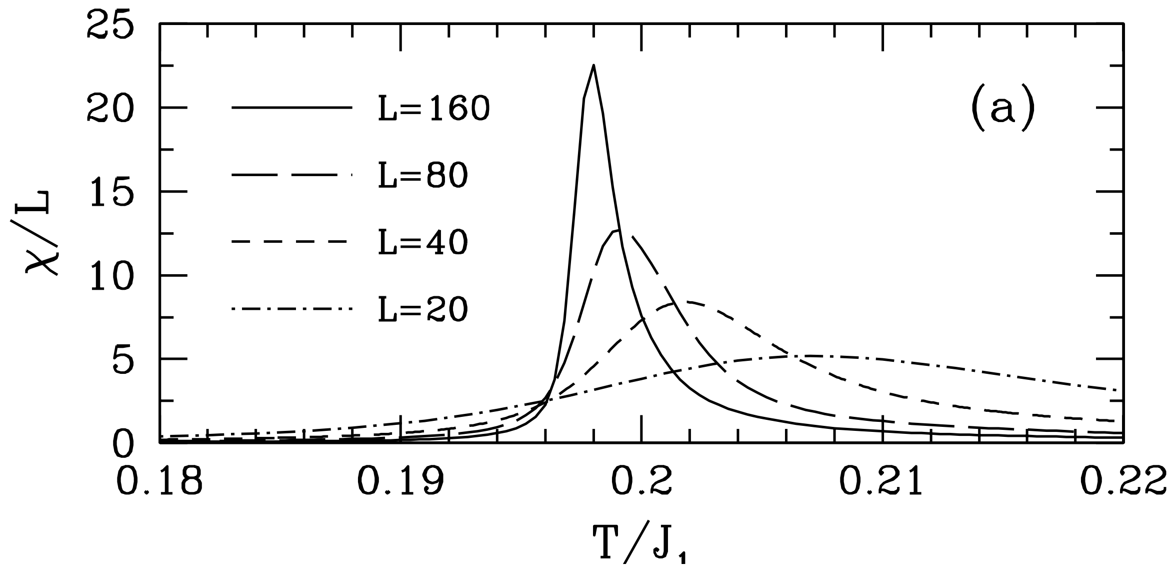

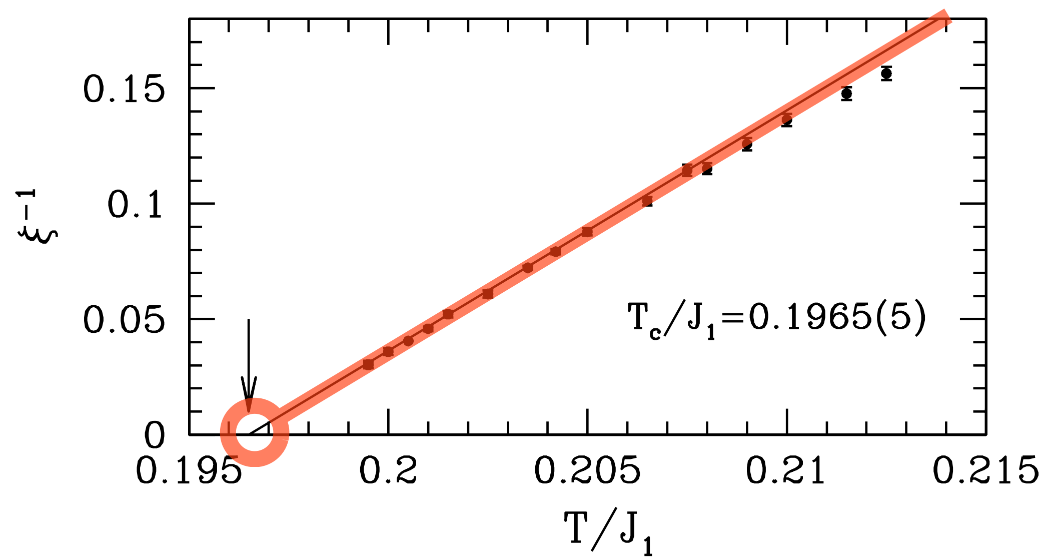

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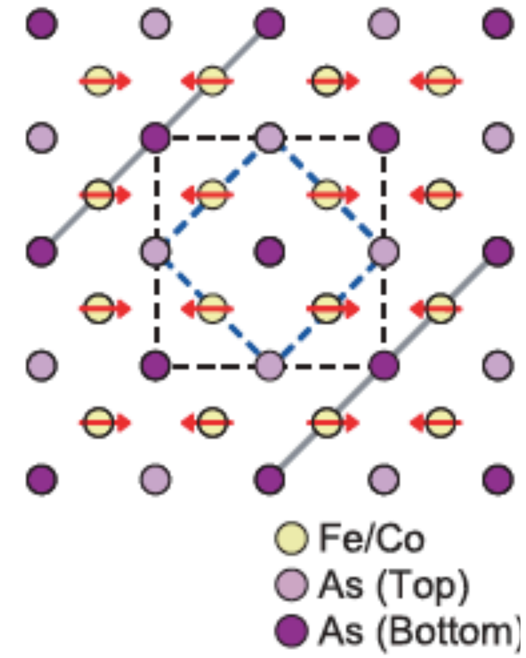

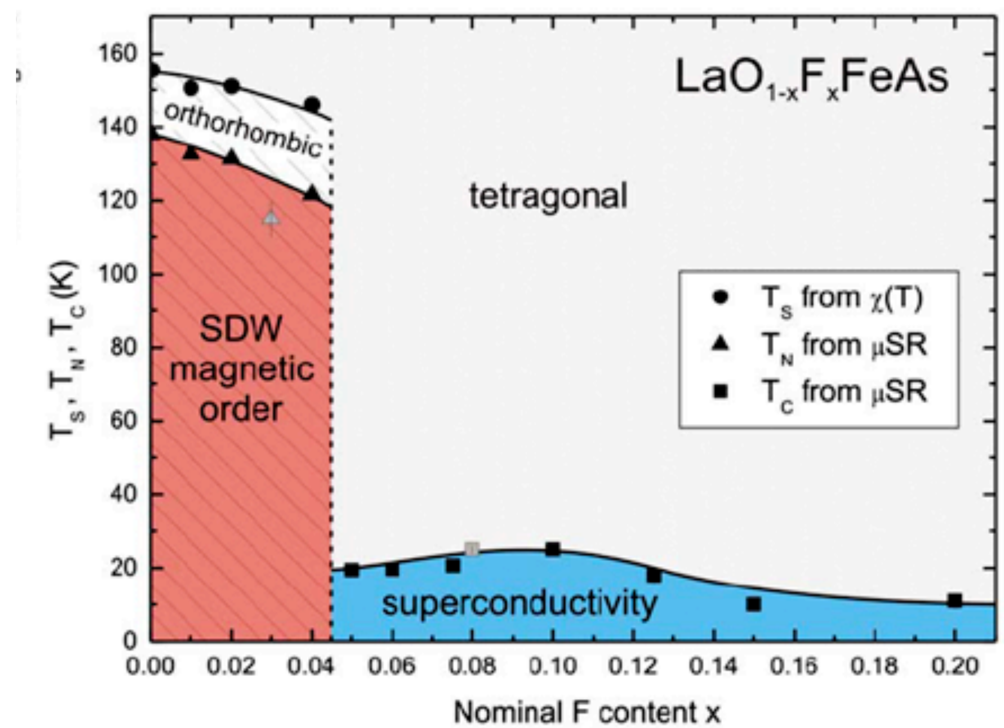

Iron based superconductors (2008).

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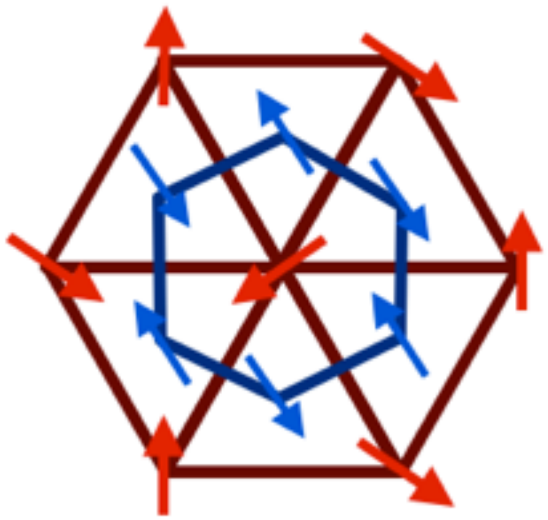
Can we find a model that has an emergent critical phase even though its underlying Heisenberg degrees of freedom have a finite correlation length?

2D Heisenberg AFM Hamiltonian



$$H = H_{hh} + H_{tt} + H_{th}$$

$$H_{\alpha\beta} = J_{\alpha\beta} \sum_{j=1}^{N_L} \sum_{\delta_{\alpha\beta}} S_\alpha(j) S_\beta(j + \delta_{\alpha\beta})$$

$J_{tt} = J_{hh} = 1$

$J_{th} \ll 1$

$\alpha, \beta \in \{t, A, B\}$

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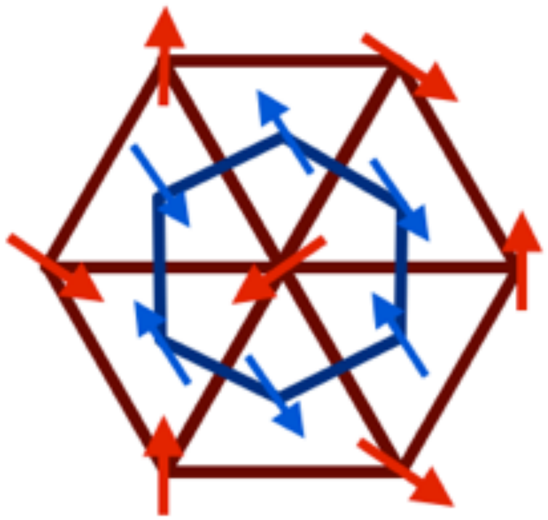

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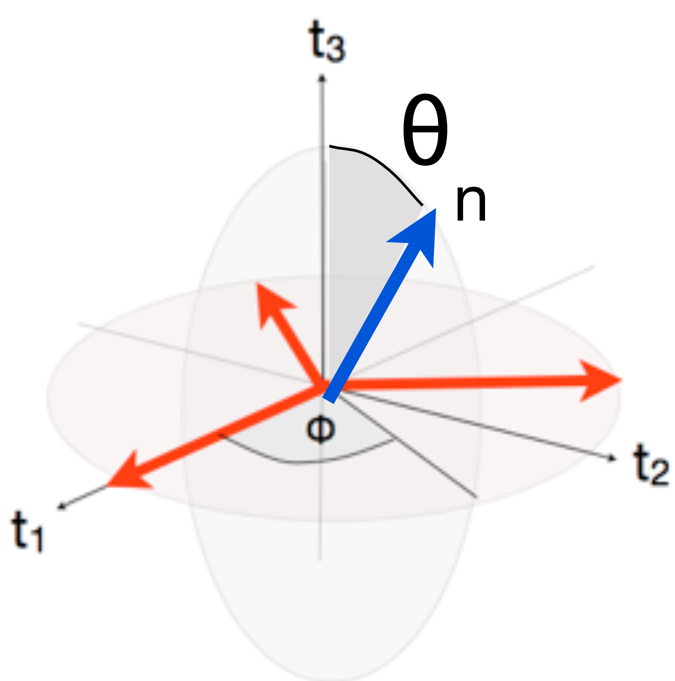
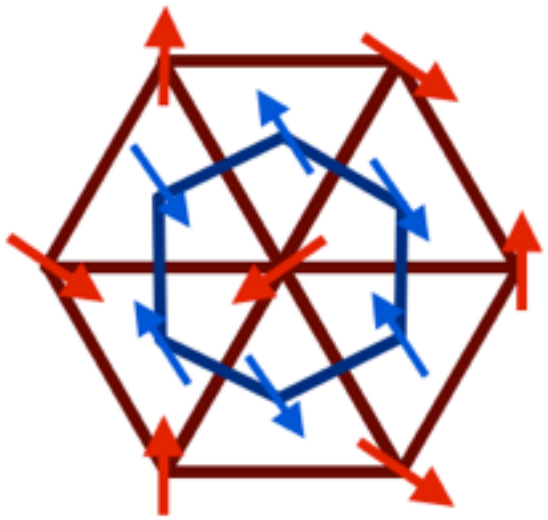


Classically: two decoupled sublattices.

2D Heisenberg AFM Hamiltonian



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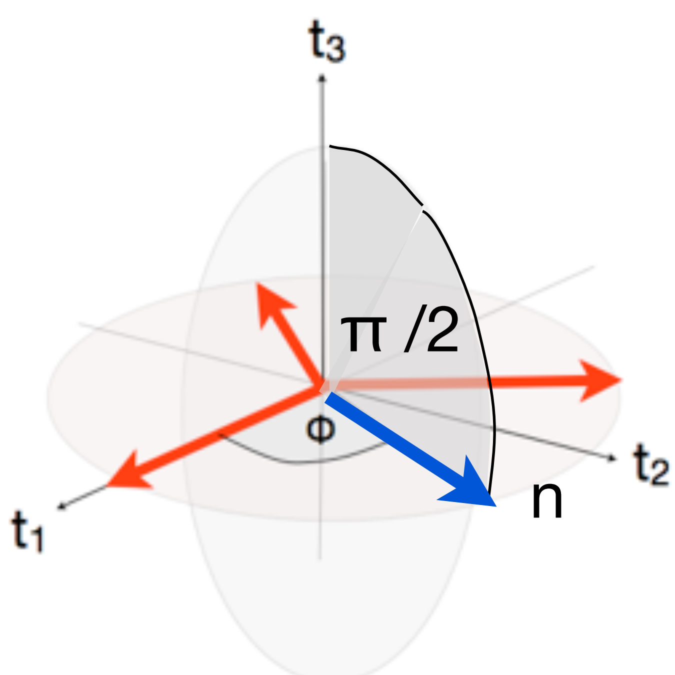

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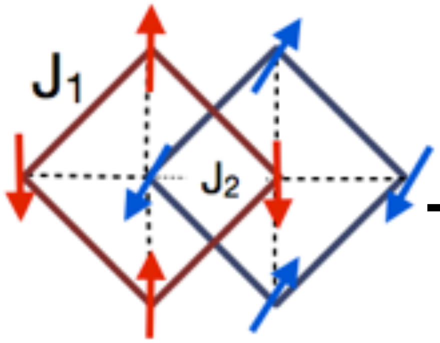
Order from disorder drives coplanarity.



T

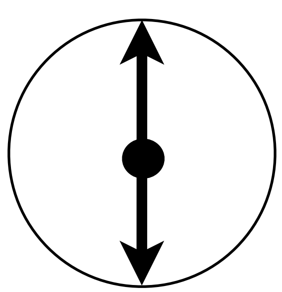
free moments

J_1

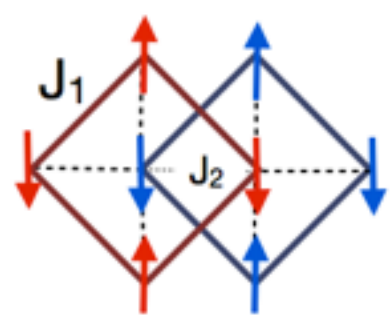
decoupled sublattices

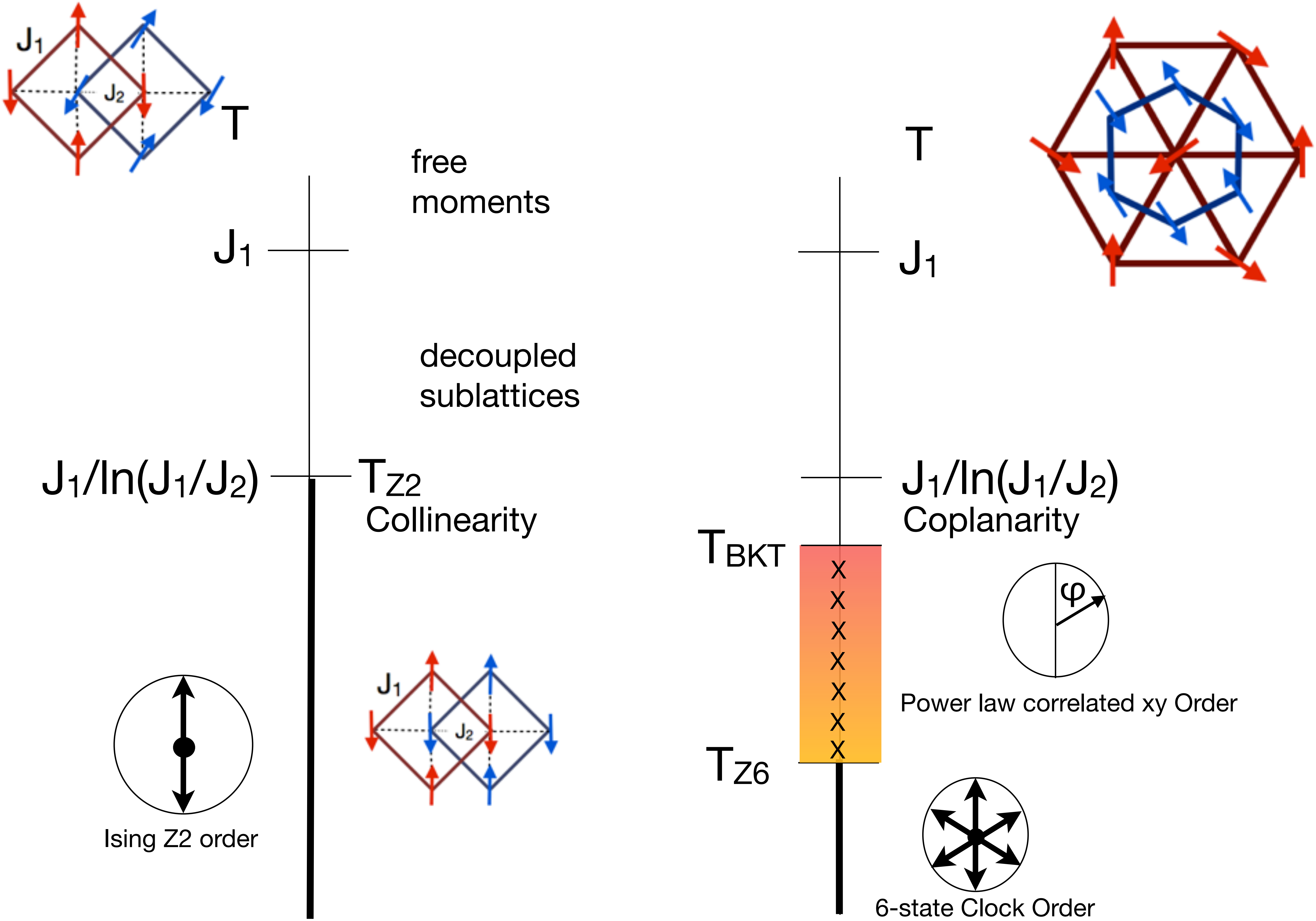
$J_1/\ln(J_1/J_2)$ T_{Z2}

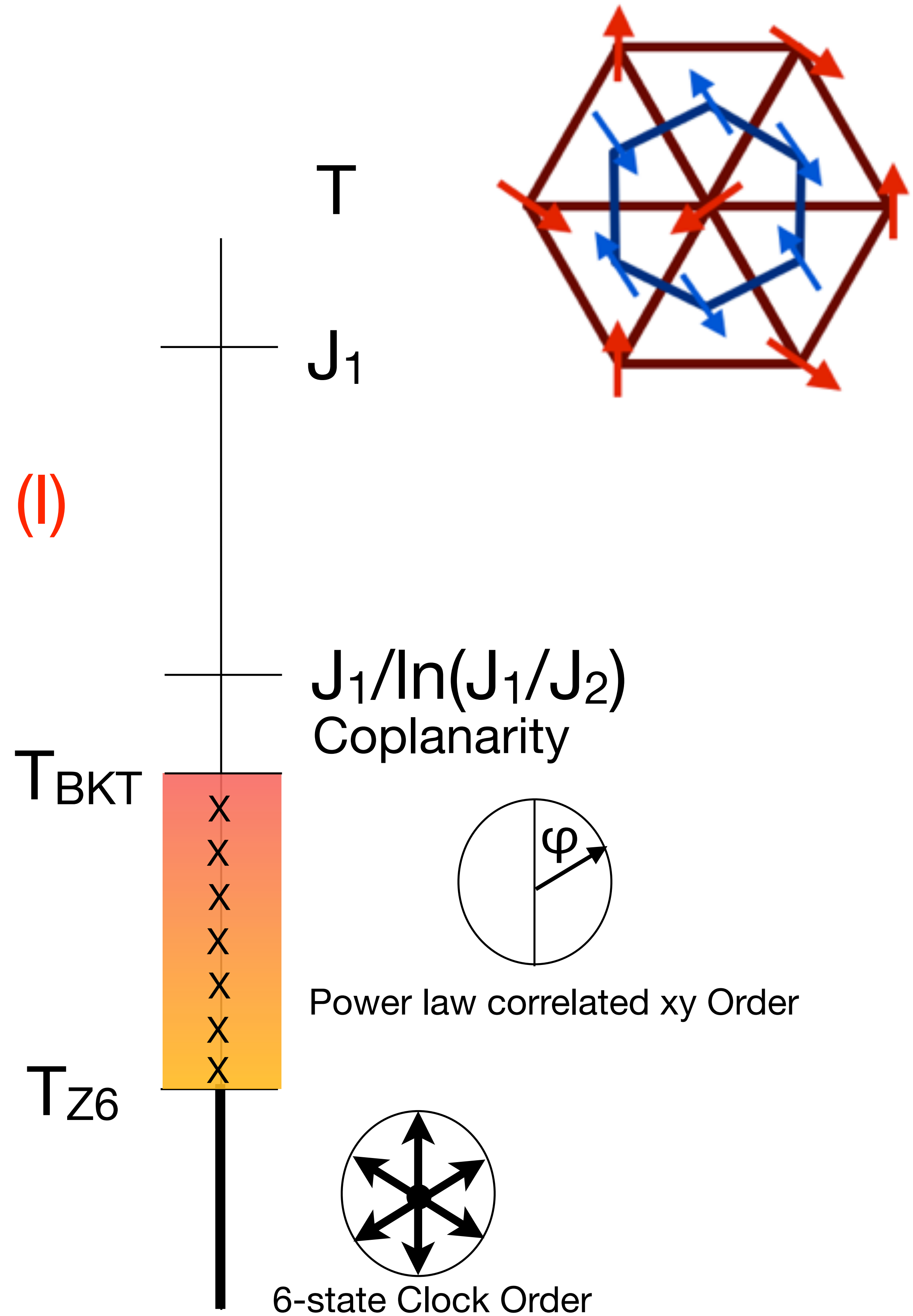
Collinearity



Ising Z2 order




J1
J2
T
free
moments
J1
decoupled
sublattices
J1/ln(J1/J2)
TZ2
Collinearity
J1
J2
Ising Z2 order
T
J1
J1/ln(J1/J2)
Coplanarity
TBKT
X
X
X
X
X
X
X
φ
Power law correlated xy Order
TZ6
6-state Clock Order



T
J1
(I)
J1/ln(J1/J2)
Coplanarity
TBKT
φ
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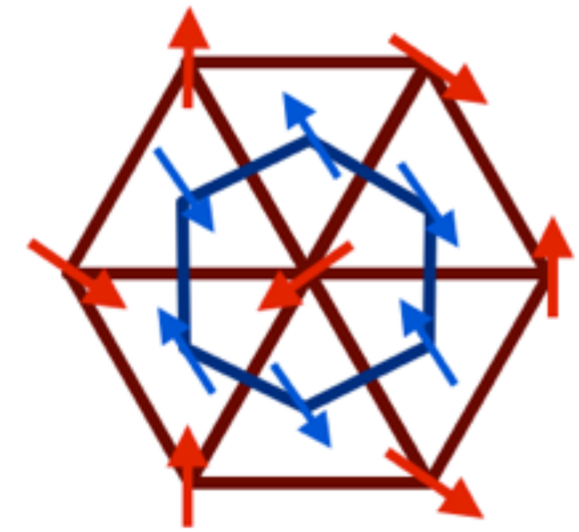

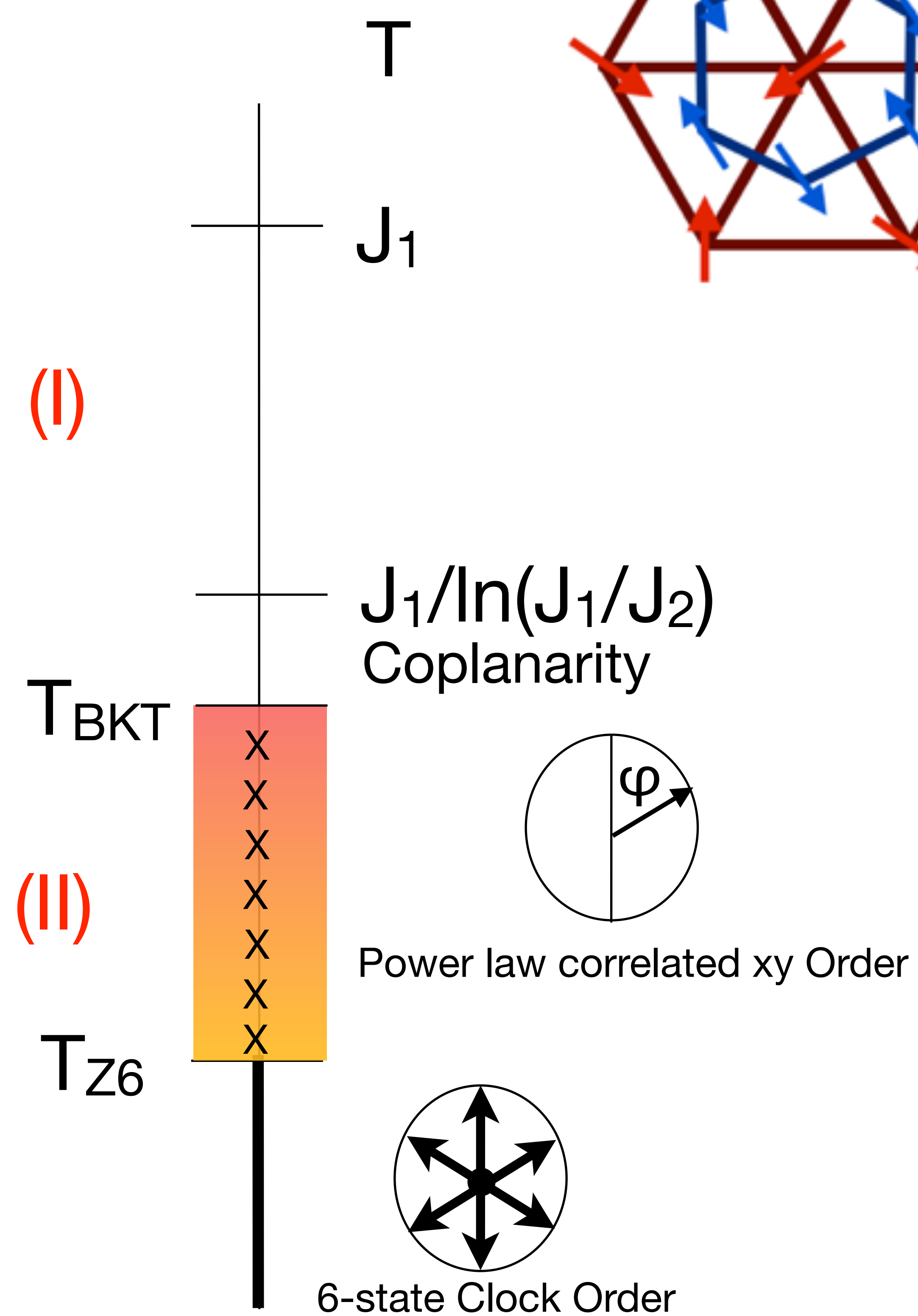
T
J_1
(I)
$J_1/\ln(J_1/J_2)$
Coplanarity
T_{BKT}
φ
(II)
Power law correlated xy Order
T_{Z6}
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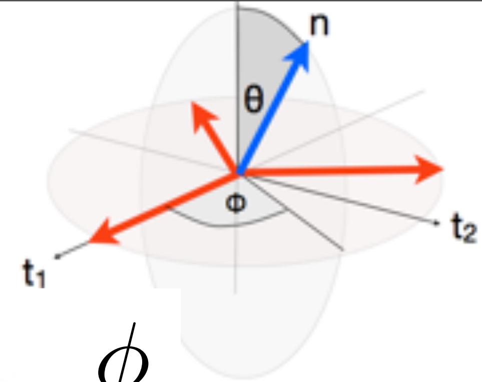
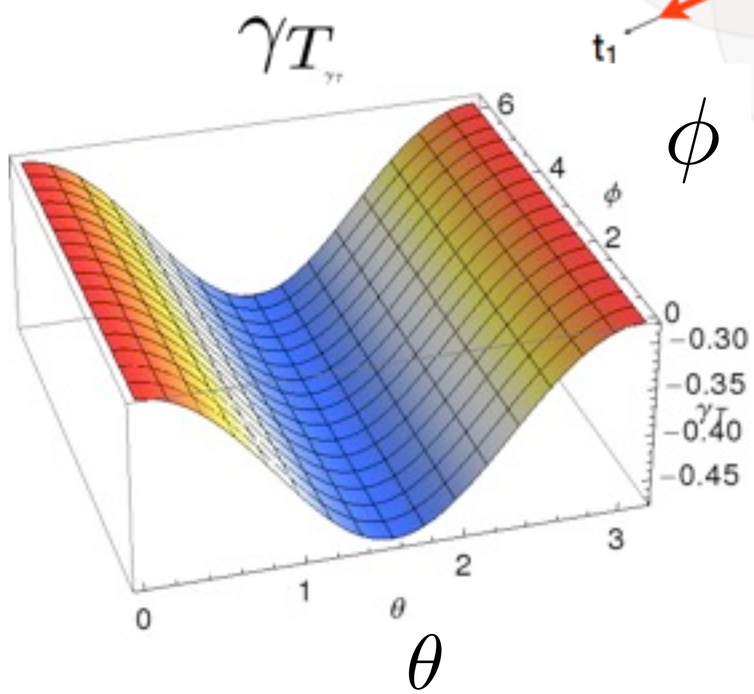
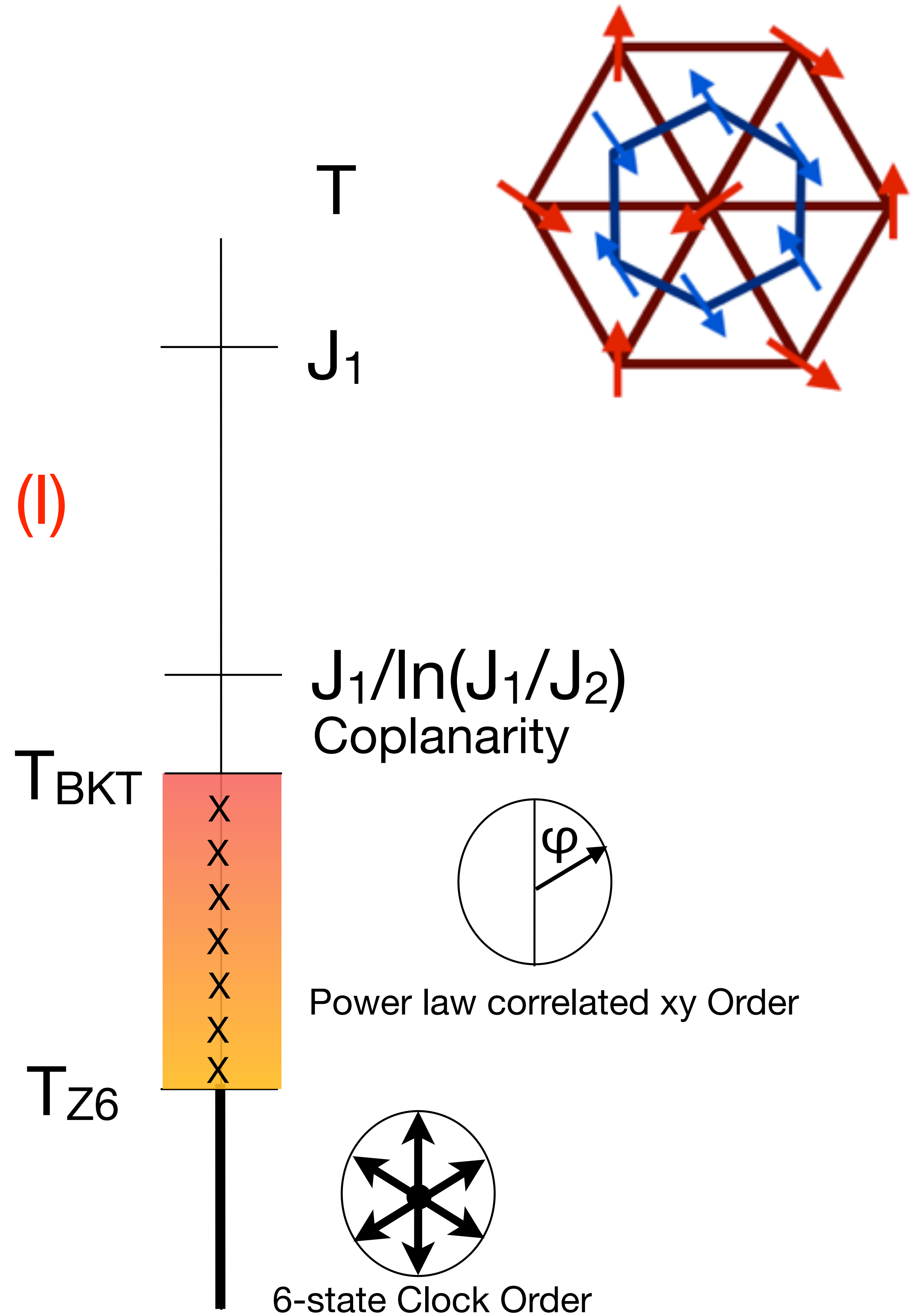
Coplanarity



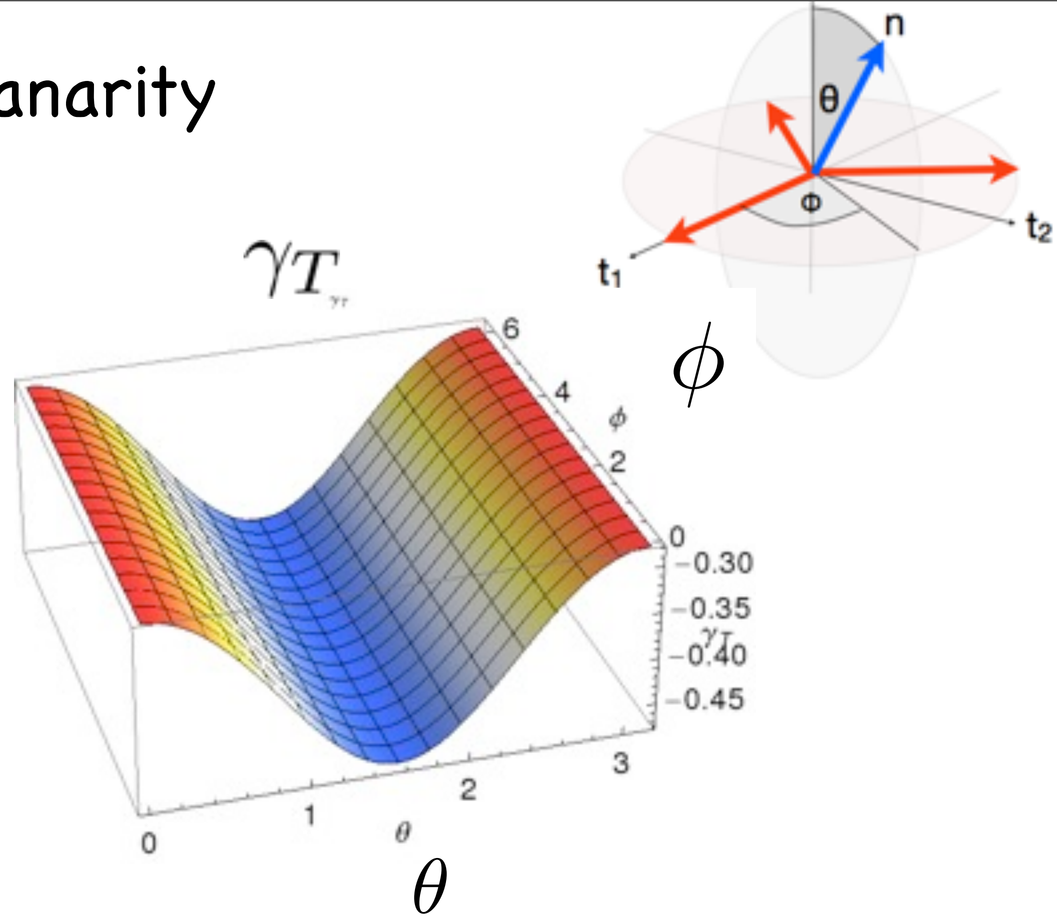


$$\delta F(\theta,\phi) = \sum_{\alpha,p\in MBZ} E_\alpha(p)\Big(\langle B^\dagger_{\alpha,p} B_{\alpha,p}\rangle + \frac{1}{2}\Big)$$

$$\delta F(\theta,\phi) = \gamma_T(\theta,\phi) T$$



Coplanarity



Minimum occurs at $\theta = \dfrac{\pi}{2}$

→ Coplanar Spin Ordering

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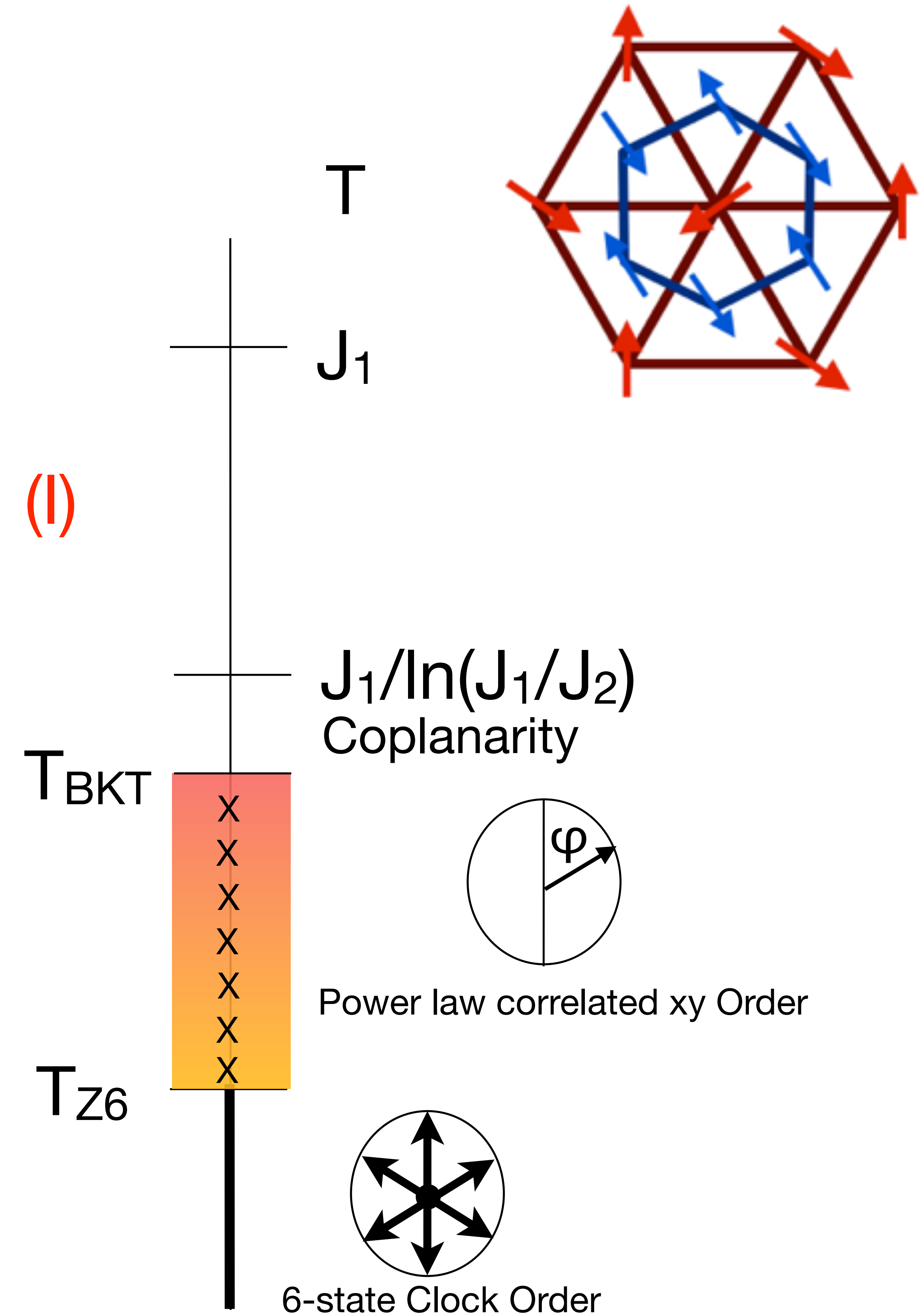

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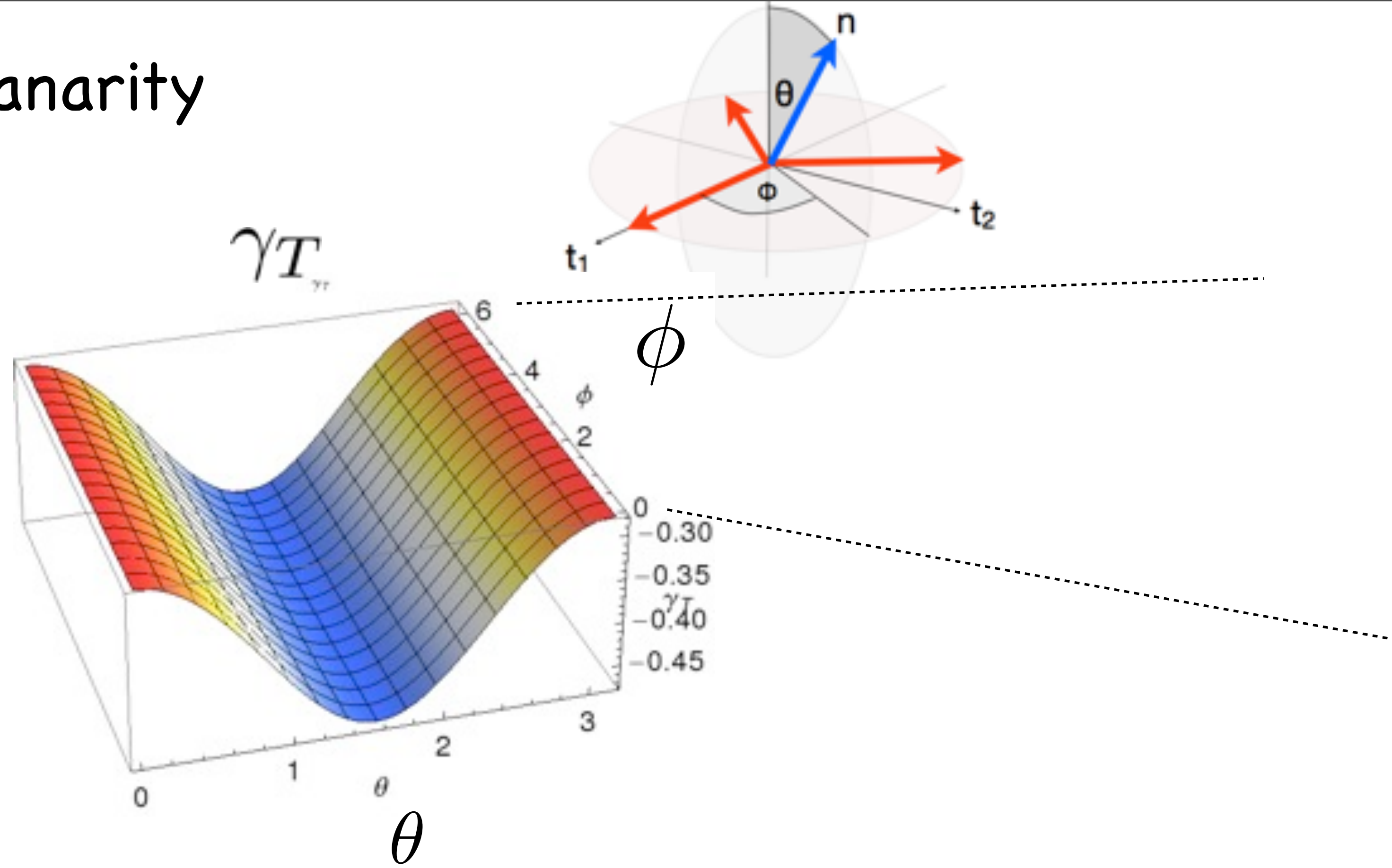

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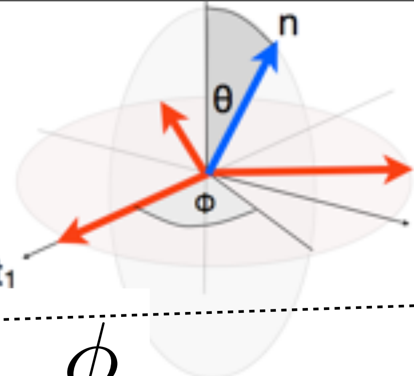

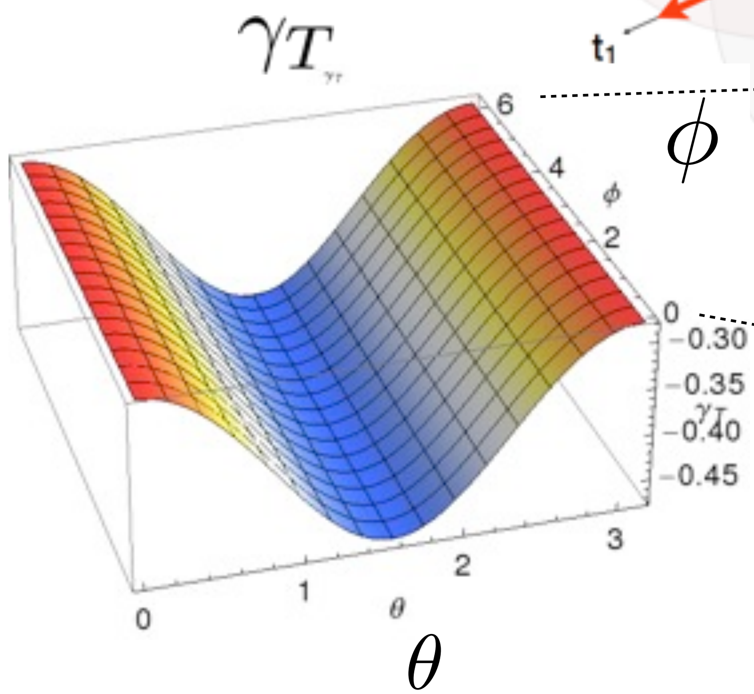

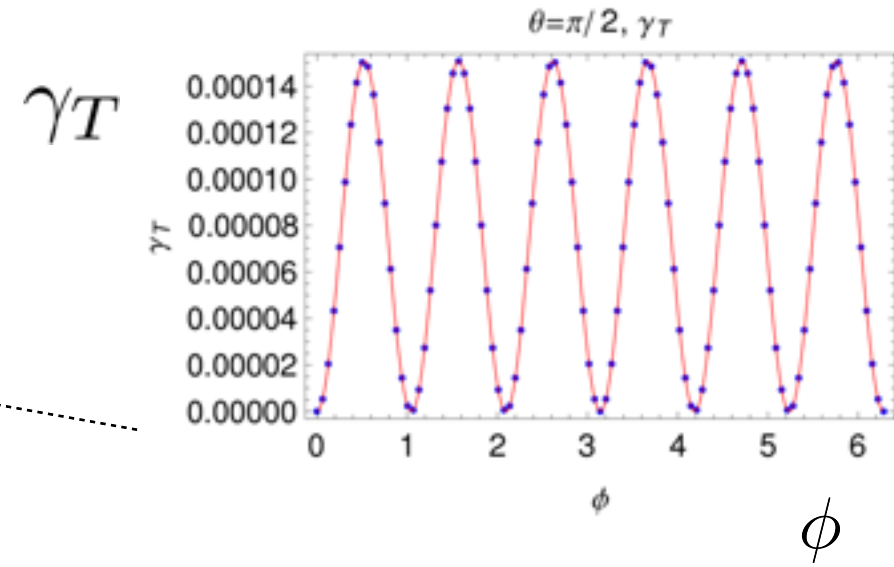

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Six Degenerate Minima at $\phi = \frac{2\pi n}{6}$

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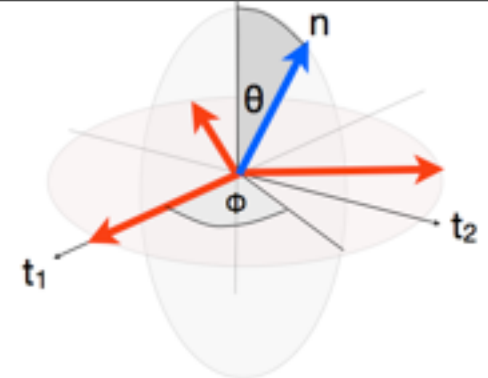

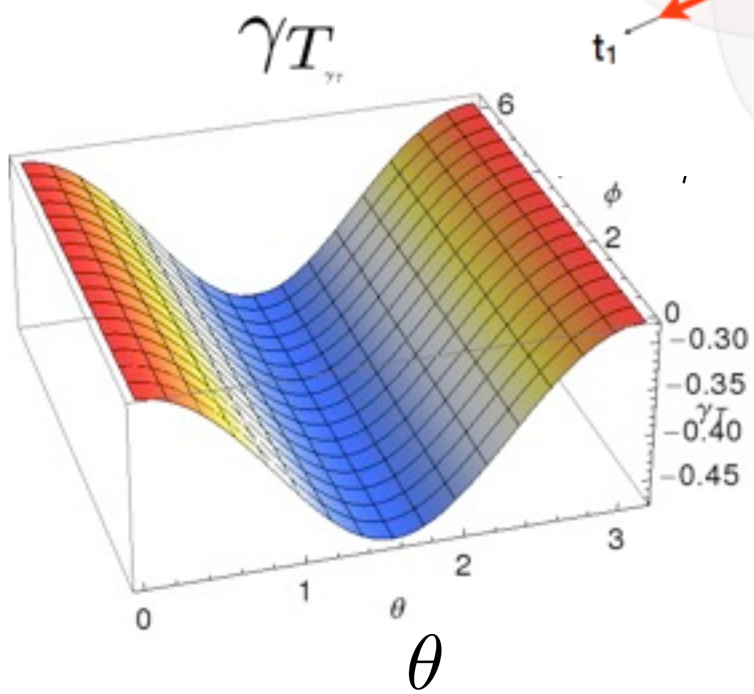

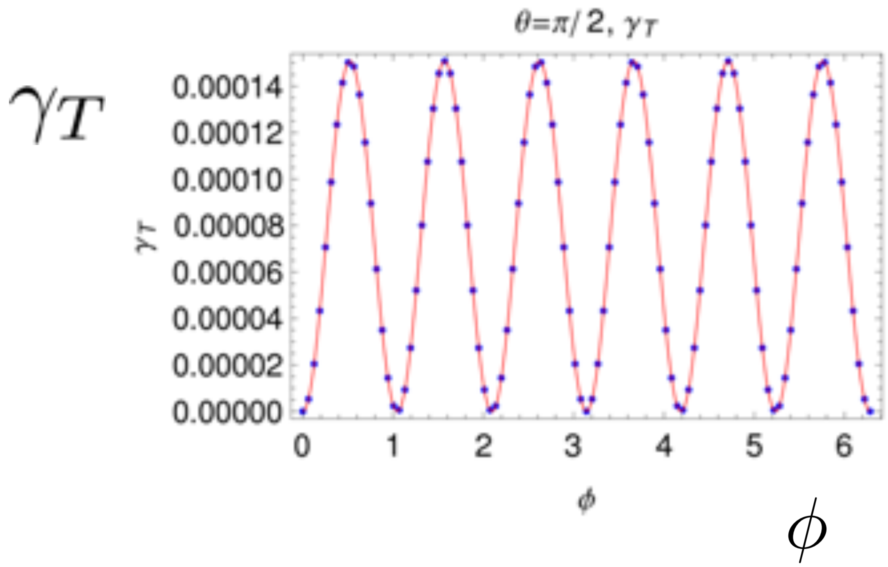

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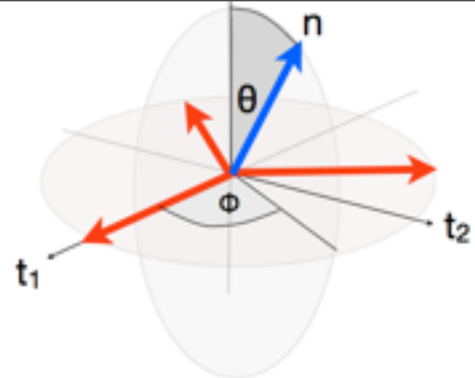
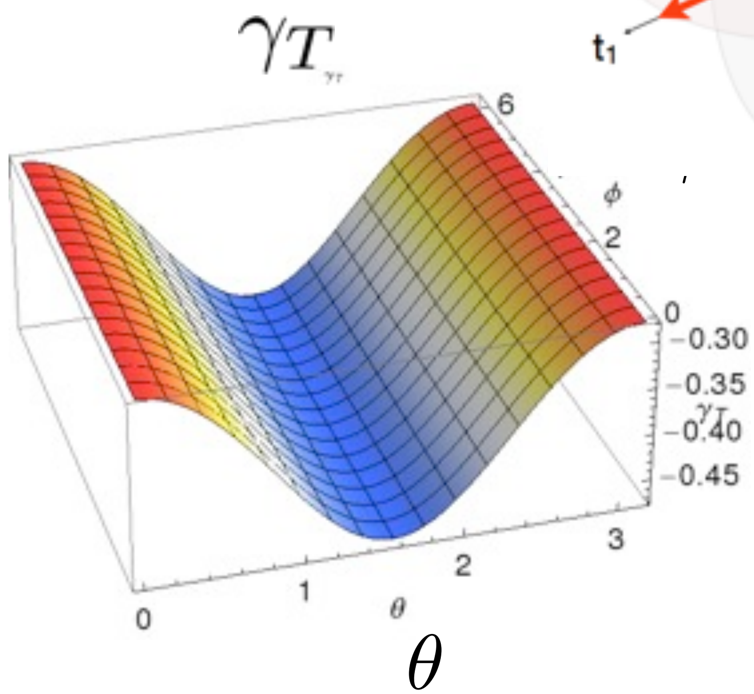
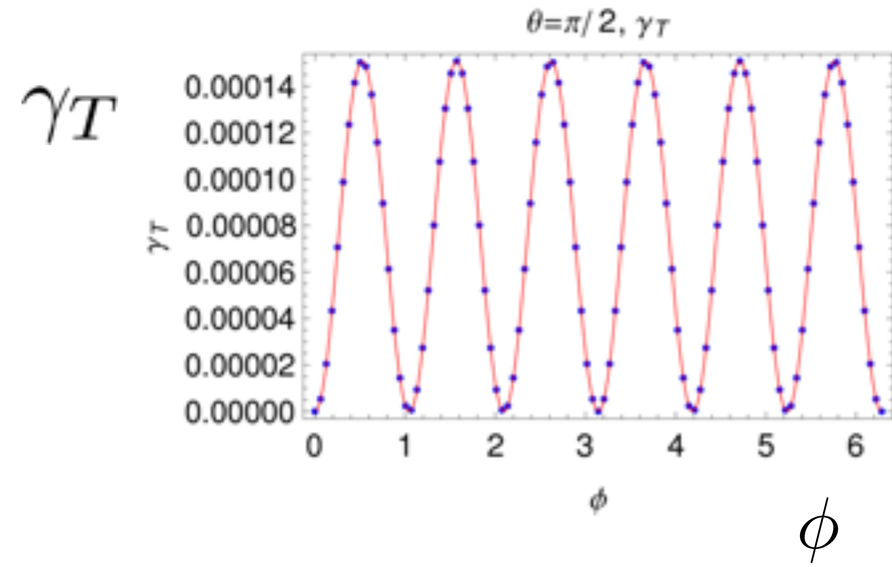
→ Coplanar Spin Ordering

Coplanarity







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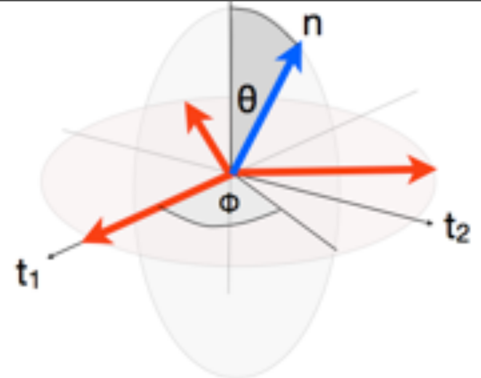
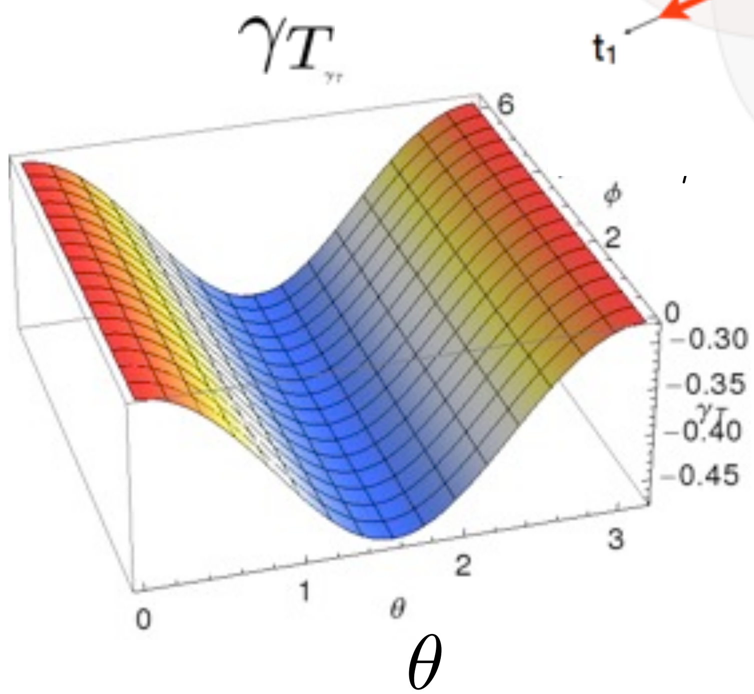
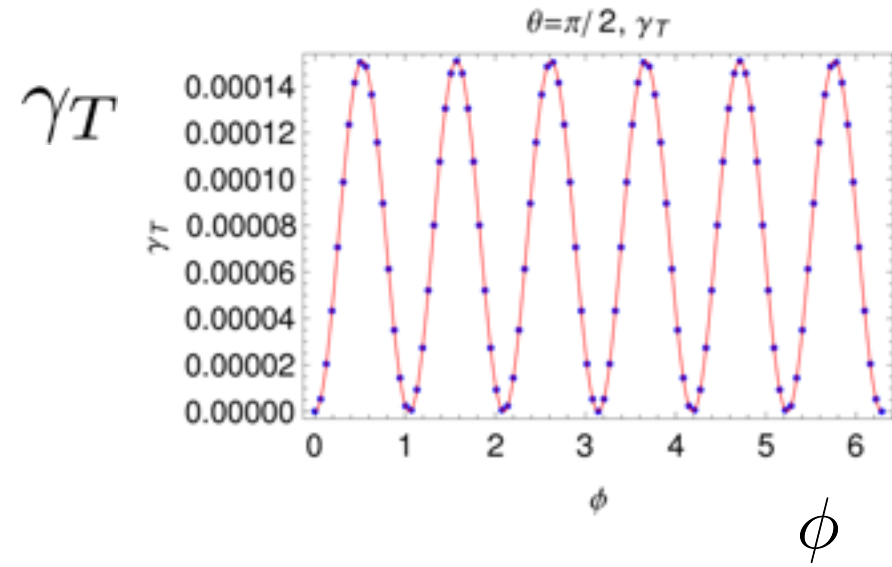
$\longrightarrow$ Coplanar Spin Ordering

$$S = S_h + S_t + S_{th}$$

Coplanarity







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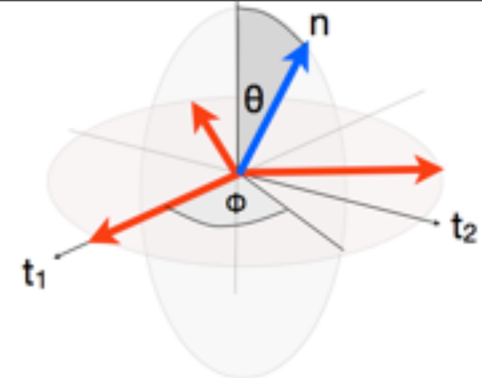
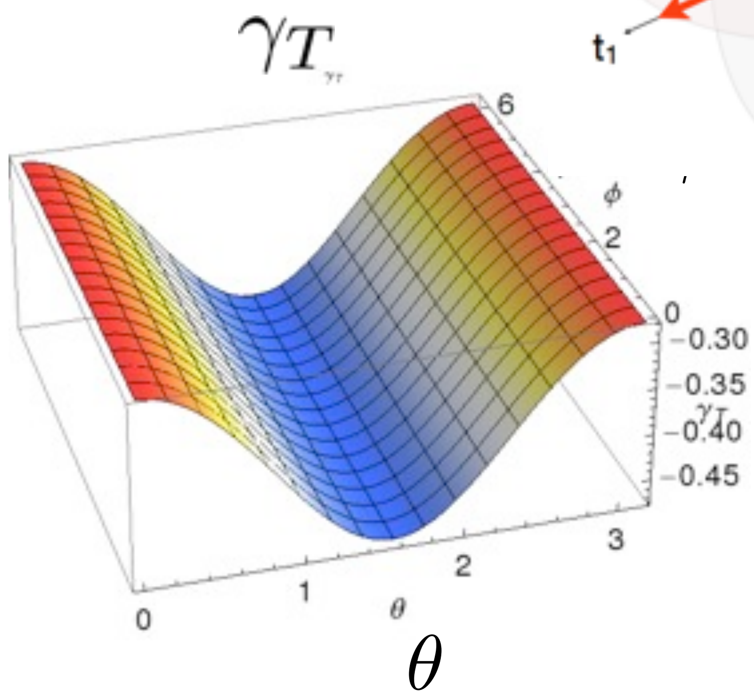
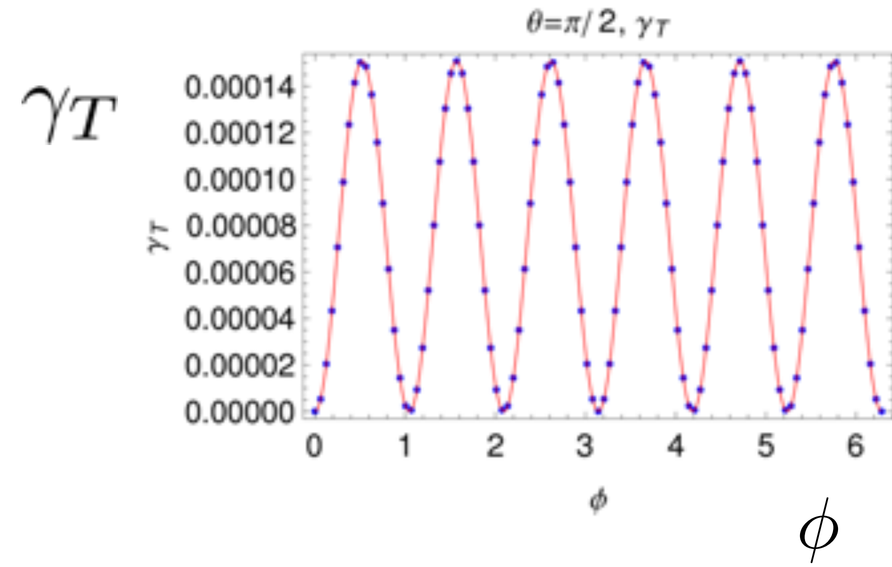
Coplanarity







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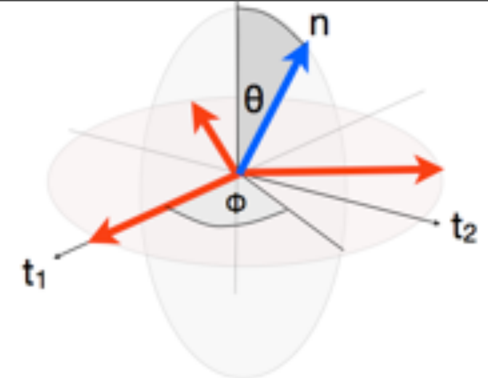
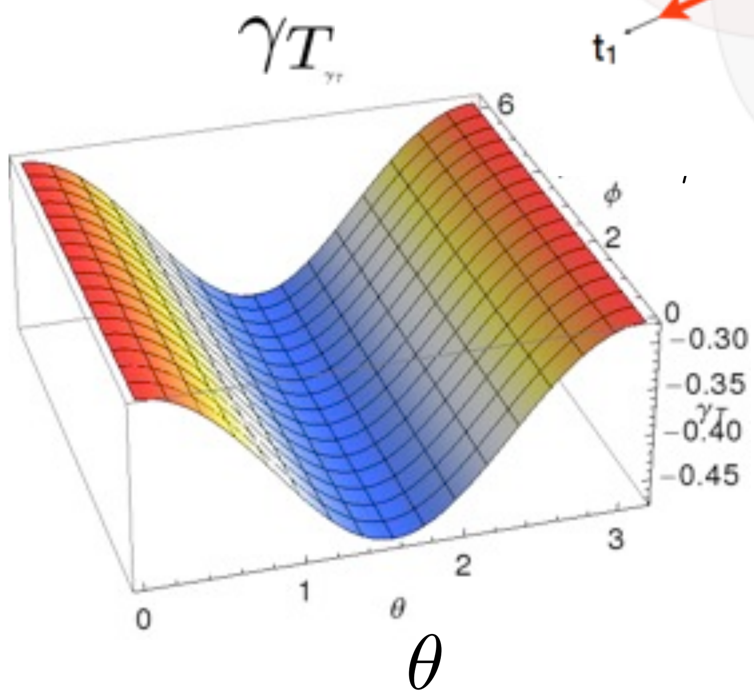
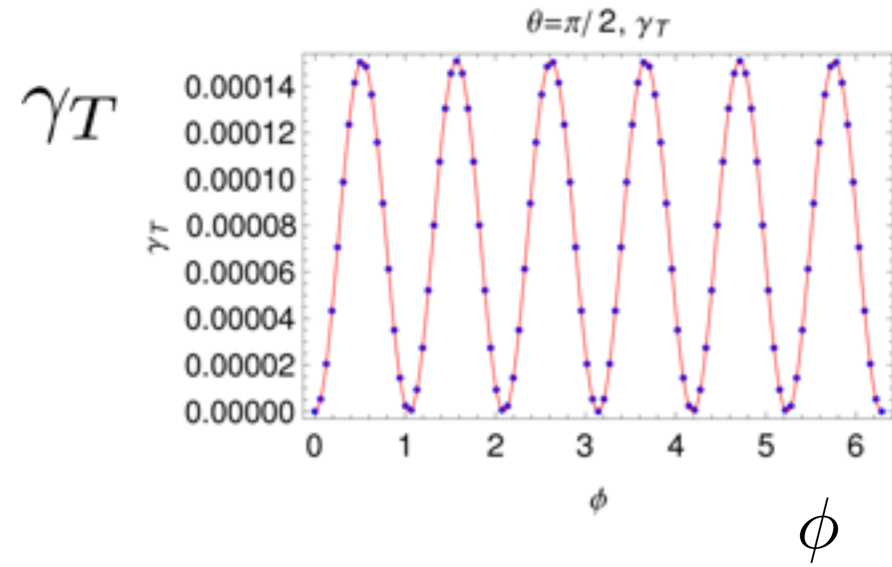
$$S_t = \frac{K_t}{2} \int d^2x \left((\nabla t_1)^2 + (\nabla t_2)^2 + (\nabla t_3)^2 \right)$$

Coplanarity







Minimum occurs at $\theta = \frac{\pi}{2}$

Six Degenerate Minima at $\phi = \frac{2\pi n}{6}$

→ Coplanar Spin Ordering

$$S = S_h + S_t + S_{th}$$

$$S_h = \frac{K_h}{2} \int d^2x (\nabla n)^2 \qquad S_t = \frac{K_t}{2} \int d^2x \left((\nabla t_1)^2 + (\nabla t_2)^2 + (\nabla t_3)^2 \right)$$

$$S_{th} = \int d^2x \left(\alpha \cos^2\theta + \lambda \cos 6\phi \right)$$

$O(J_{th}^2)$ $O(J_{th}^6)$

Both terms relevant
at high temperatures

The Coplanarity Cross-over Temperature

$$\xi = \text{coherence length of coplanar flucs}$$

$$a e^{\frac{2\pi J_1}{T}} = a \left(\frac{J_1}{J_2} \right)$$

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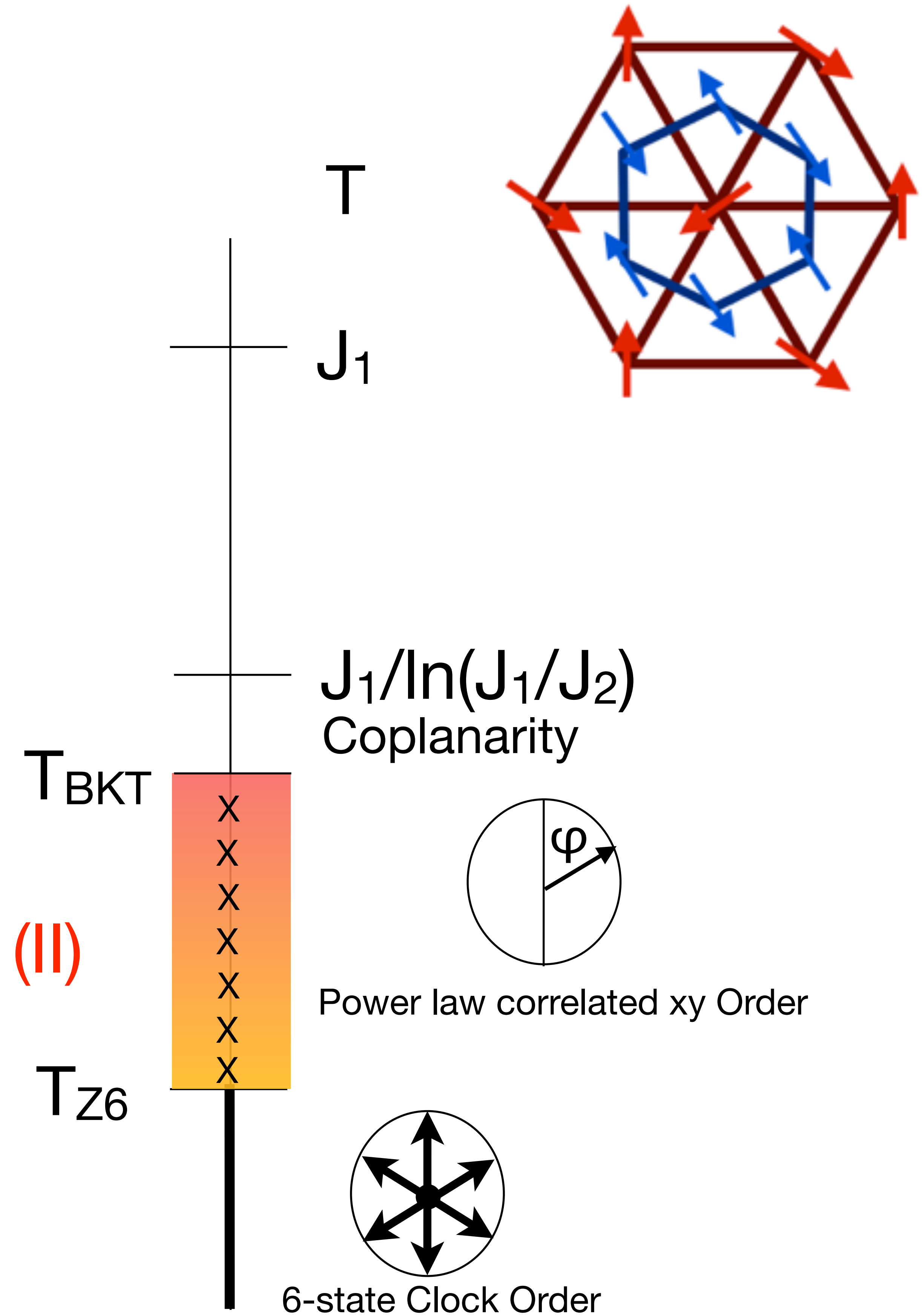
$$ae^{\frac{2\pi J_1}{T}} = a\left(\frac{J_1}{J_2}\right)$$

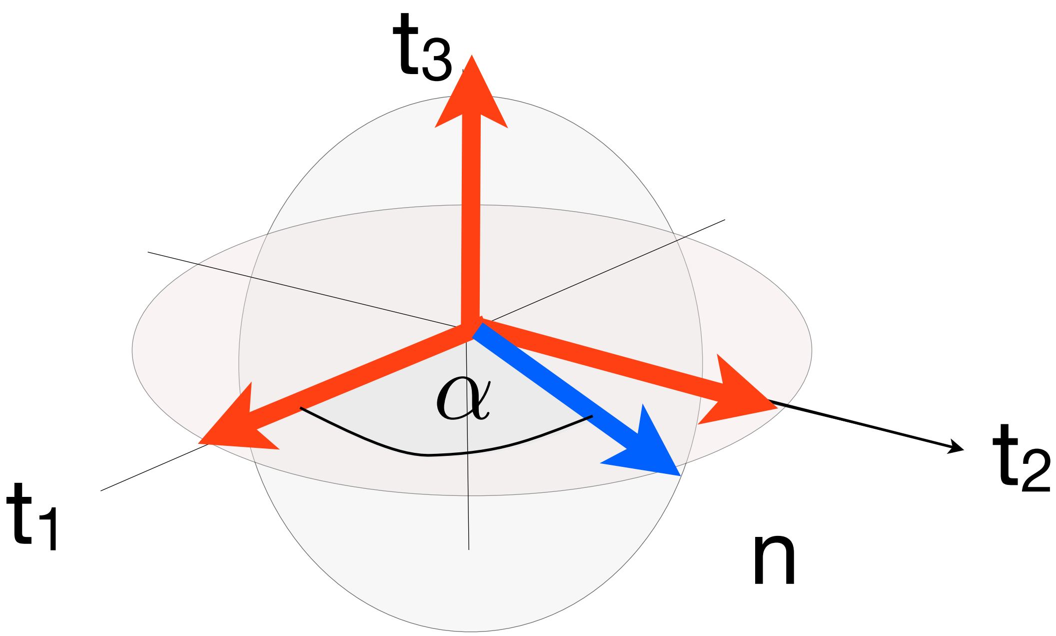
$$\downarrow$$

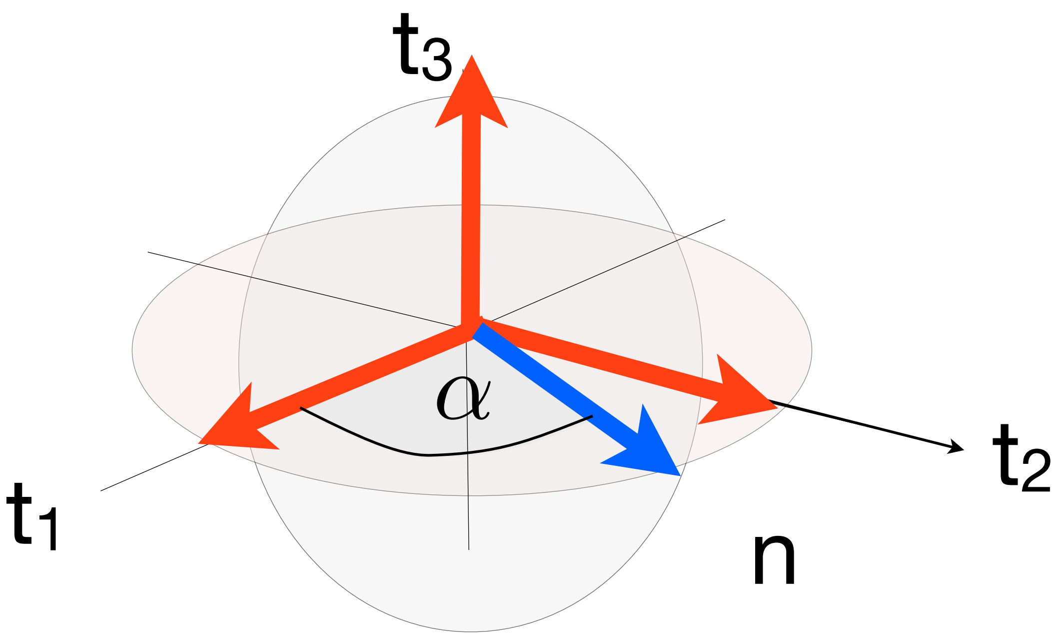
$$T_{coplanarity} \sim \frac{J_1}{\ln\left(\frac{J_1}{J_2}\right)}$$


T
J1
J1/ln(J1/J2)
Coplanarity
TBKT
φ
(II)
Power law correlated xy Order
TZ6
6-state Clock Order

13

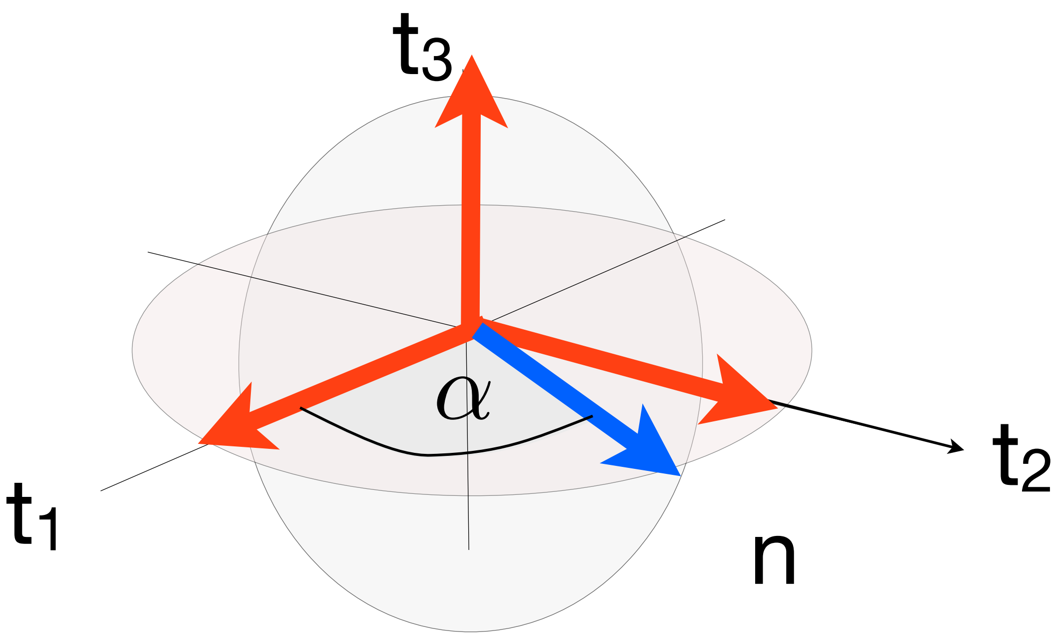
Scaling in the Coplanar State





Scaling in the Coplanar State

Order parameter: SO(3) x U(1):



Scaling in the Coplanar State

Order parameter: SO(3) x U(1):

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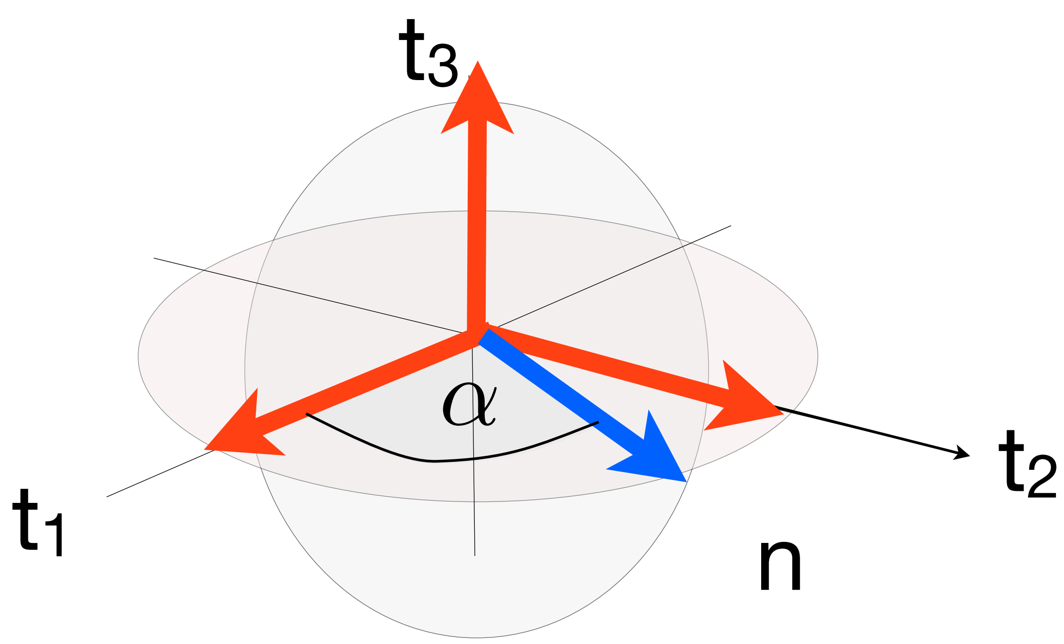

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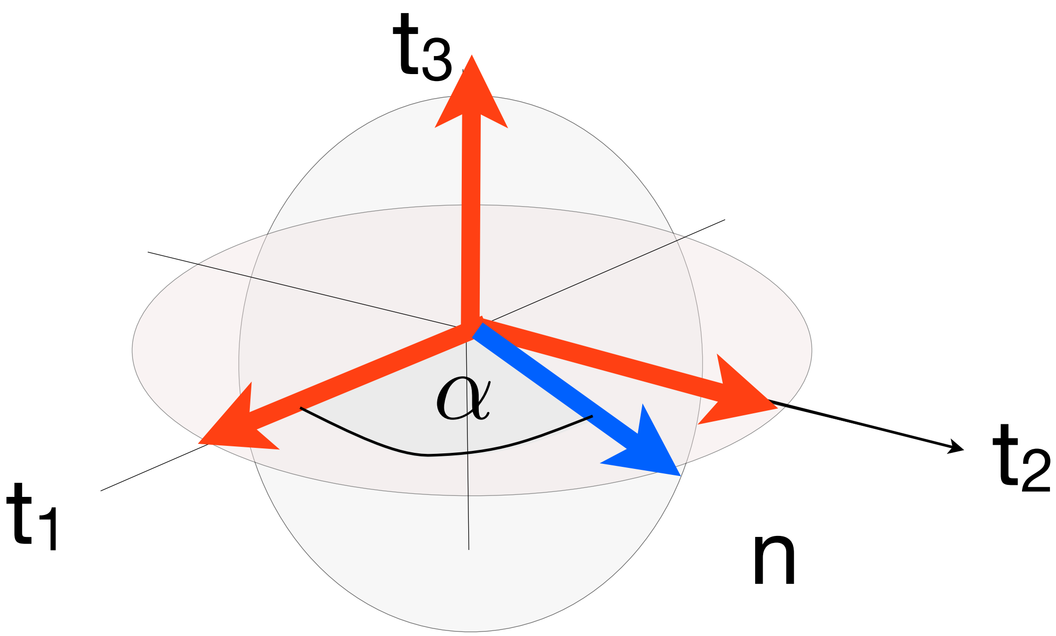

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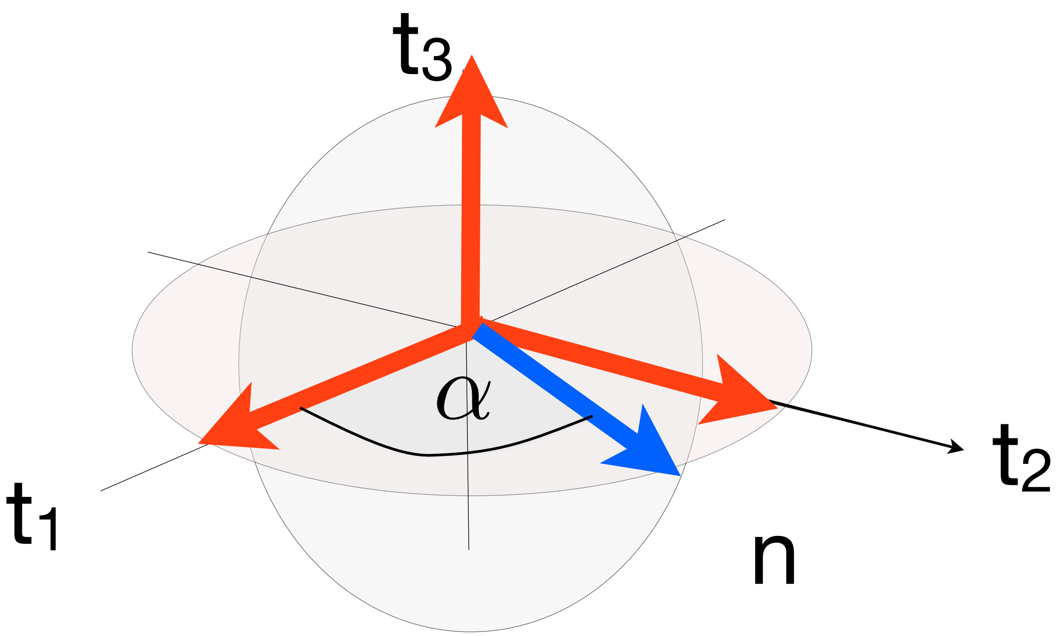

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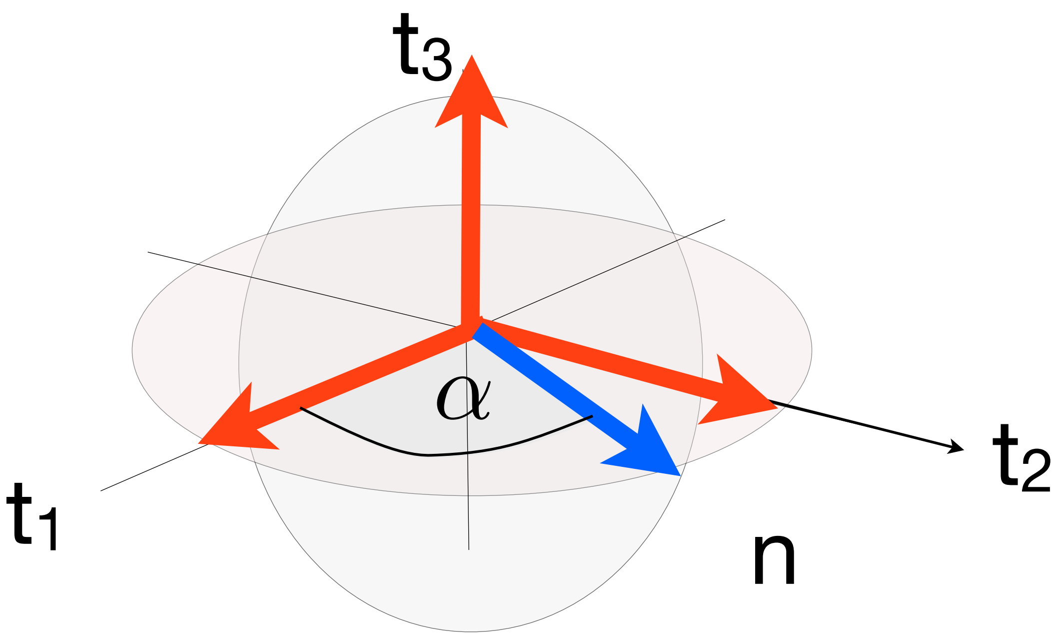
Scaling: confirms decoupling of U(1) Relative Degree of Freedom to form a phase with topologically stable vortices. Binding of the vortices leads to a power law phase in which the 6-fold anisotropy is irrelevant.



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$$r = \kappa/2I_3$$

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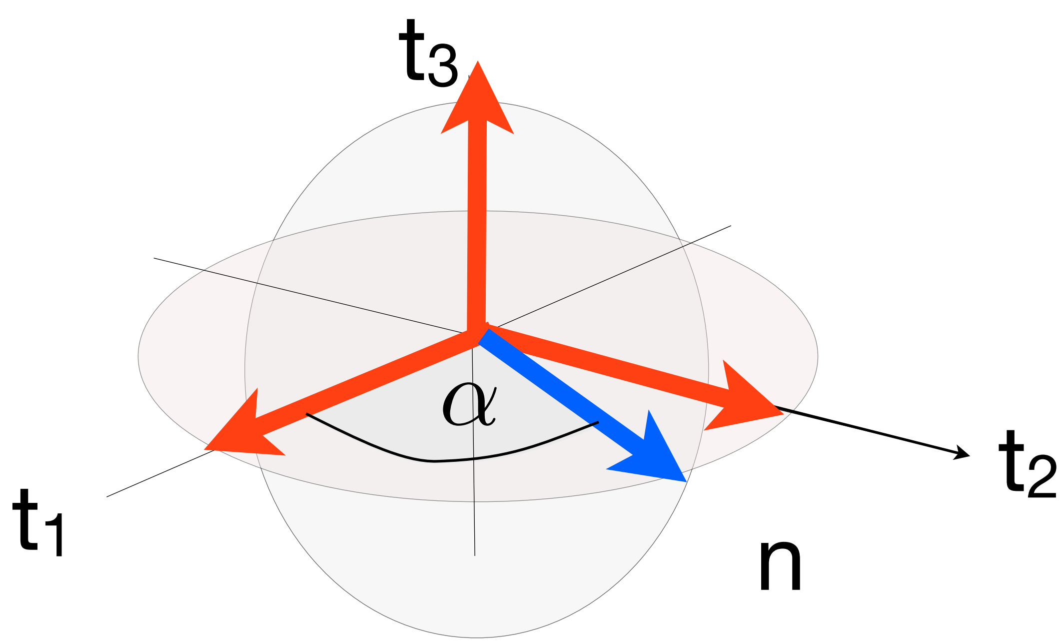

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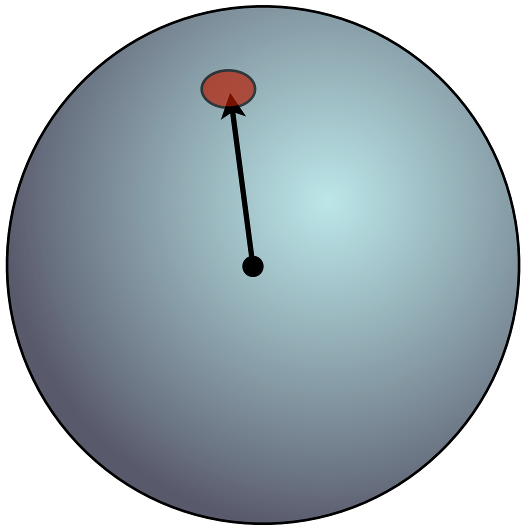
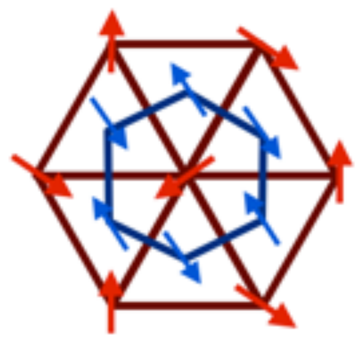
Magnetism, Gravity and String Theory.

15

Magnetism, Gravity and String Theory.

$$X(x, y) = (\phi, \theta, \psi, \alpha)$$





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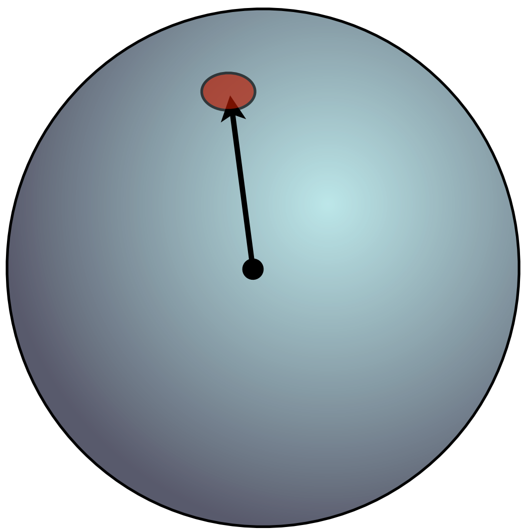

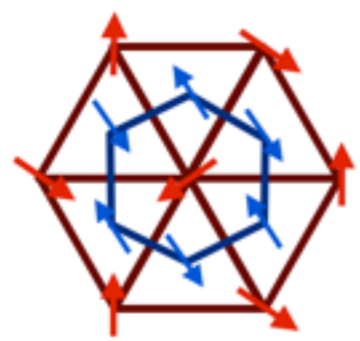

Magnetization = 4D vector

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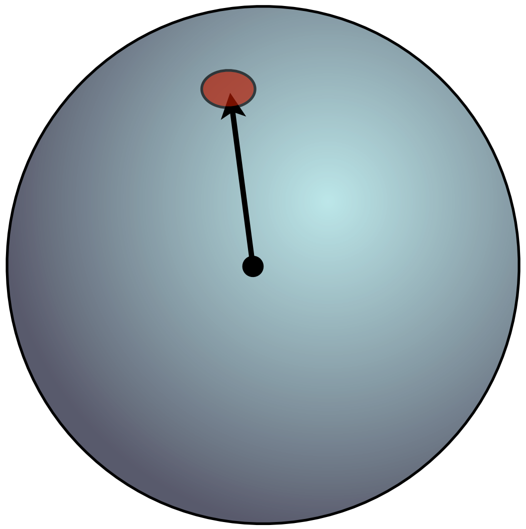

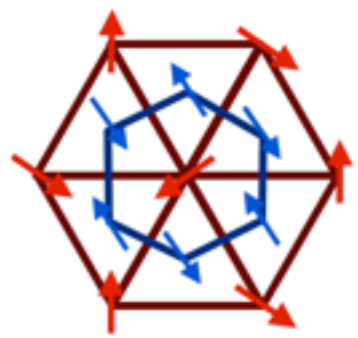

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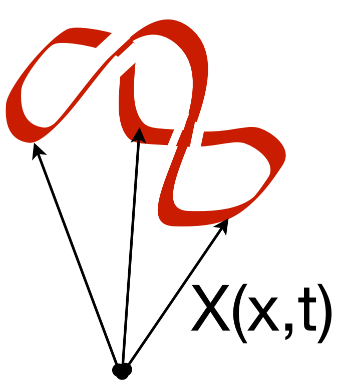

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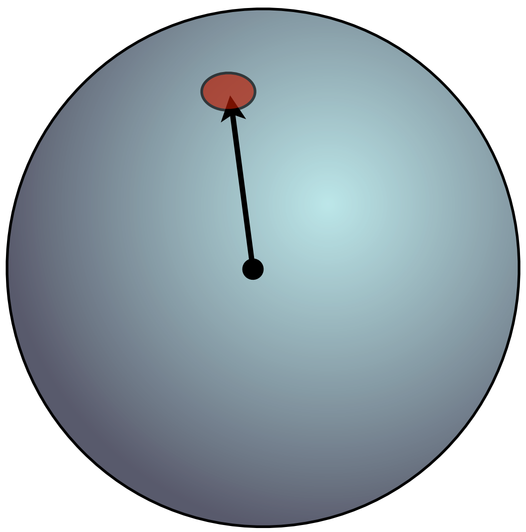

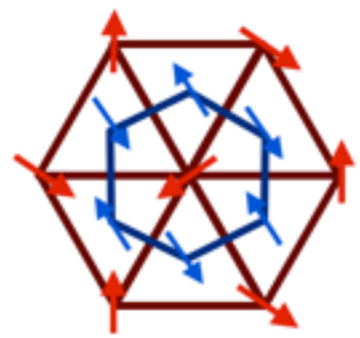

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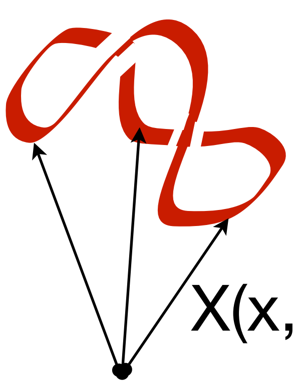

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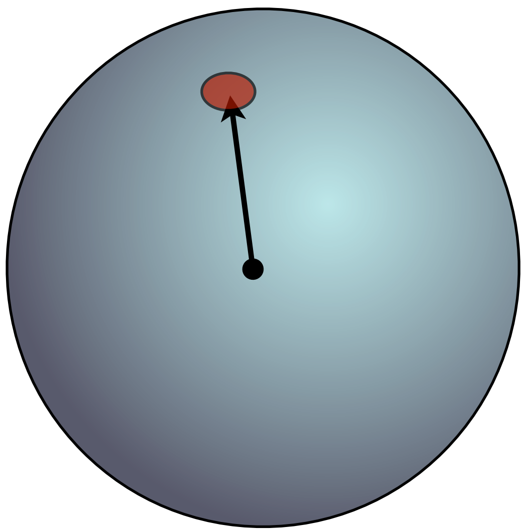

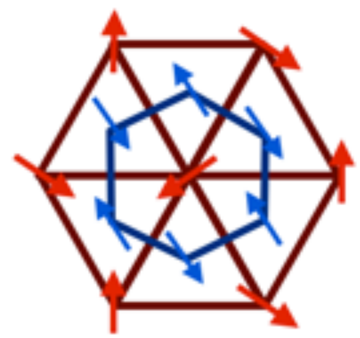

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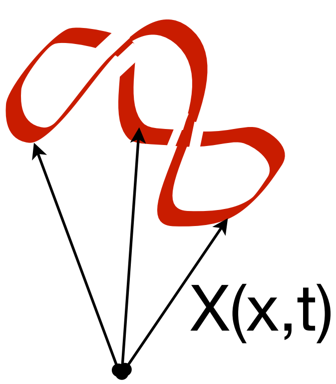

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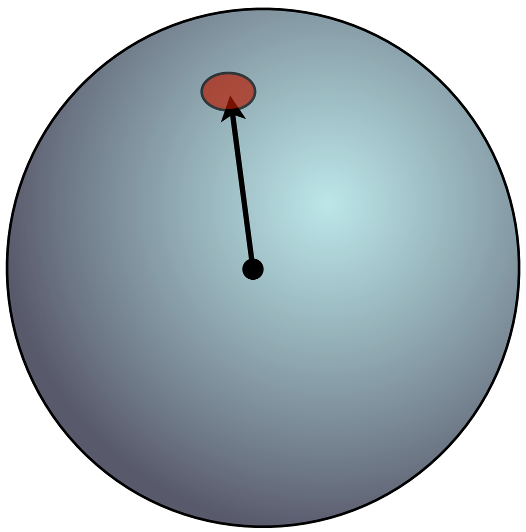

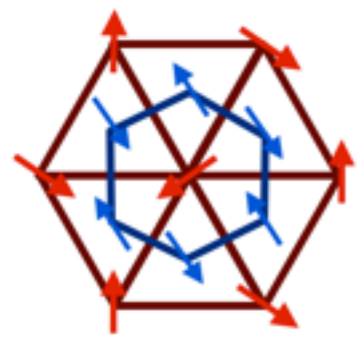

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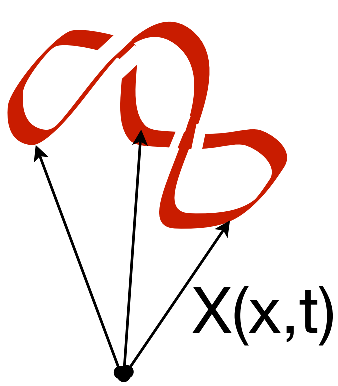

Spinwave stiffness = metric tensor

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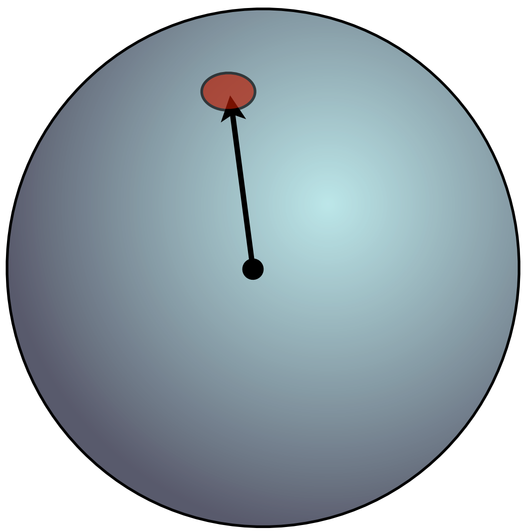

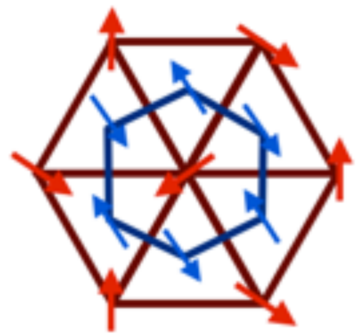

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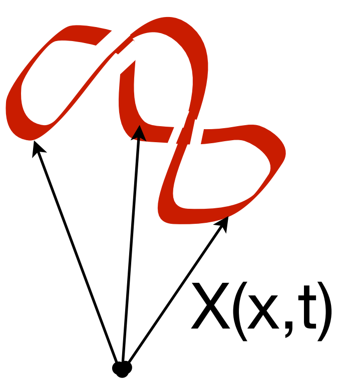

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Friedan ’80, Hamilton ’81, Perelmann ’06

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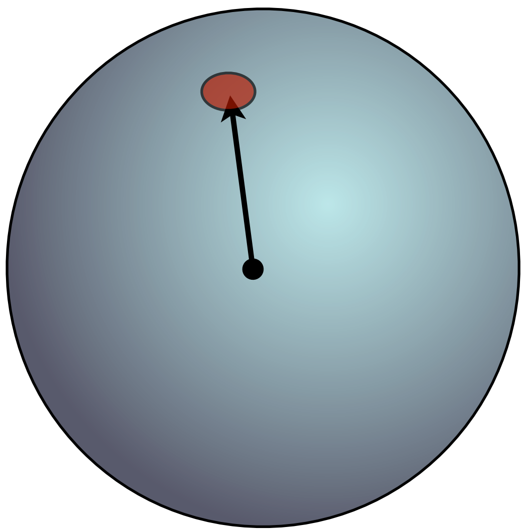

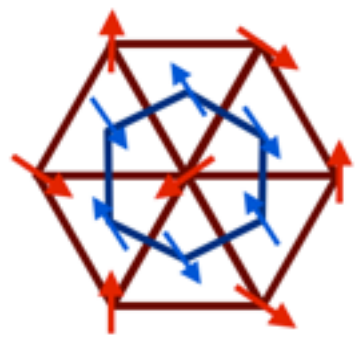

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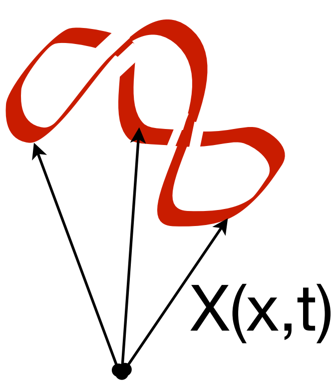

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"Ricci Flow"

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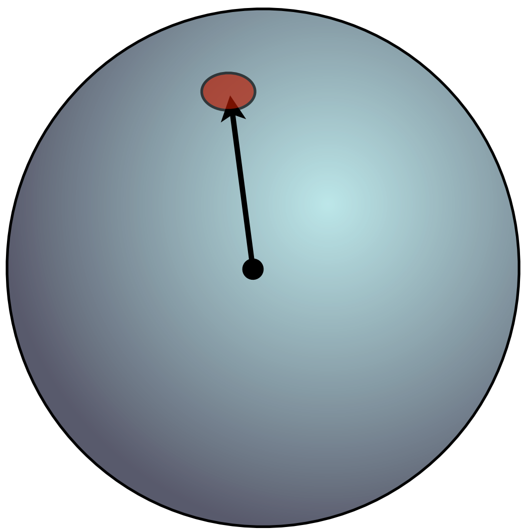

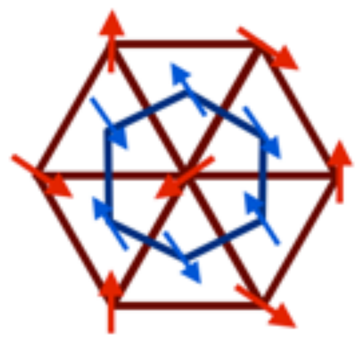

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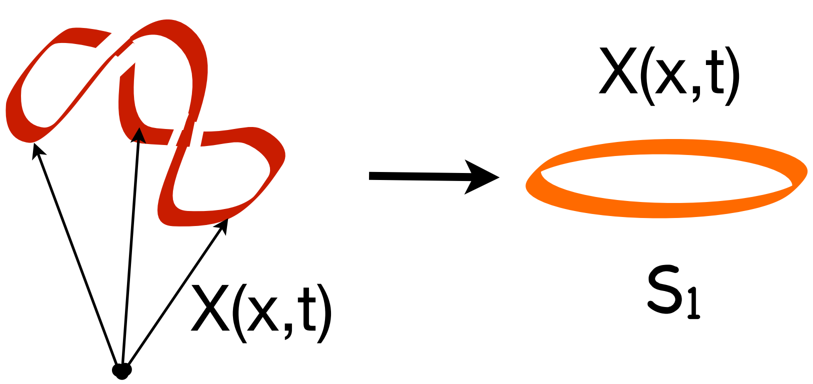

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The decoupling of the U(1) degrees of freedom from the SO(3) degrees of freedom is a kind of compactification from a four to a one dimensional universe.

15

Scaling details.

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$$
\begin{aligned}
S = &-\frac{1}{2}\int d^2x\left(I_1\Omega^2_{\mu,1} + I_2\Omega^2_{\mu,1} + I_3\Omega^2_{\mu,1}\right)\\
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\end{aligned}
$$

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Small mathematica code to calculate the Ricci tensor

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Metric Tensor

$$\text{gl} = \begin{pmatrix} \sin^2(\theta)\left(\text{I1}\sin^2(\psi) + \text{I2}\cos^2(\psi)\right) + \text{I3}\cos^2(\theta) & \sin(\theta)\,(\text{I1}-\text{I2})\sin(\psi)\cos(\psi) & \text{I3}\cos(\theta) & \frac{1}{2}\kappa\cos(\theta) \\ \sin(\theta)\,(\text{I1}-\text{I2})\sin(\psi)\cos(\psi) & \text{I1}\cos^2(\psi) + \text{I2}\sin^2(\psi) & 0 & 0 \\ \text{I3}\cos(\theta) & 0 & \text{I3} & \frac{\kappa}{2} \\ \frac{1}{2}\kappa\cos(\theta) & 0 & \frac{\kappa}{2} & \text{I}\varphi \end{pmatrix};$$

Cristoffel Symbol

```
For[i = 1, i ≤ 4, i++,
 For[k = 1, k ≤ 4, k++,
  For[l = 1, l ≤ 4, l++,
   Γ[[i, k, l]] =
    Sum[1/2 * gu[[i, j]] *
      (D[gl[[j, k]], x[[l]]] + D[gl[[j, l]], x[[k]]] -
       D[gl[[k, l]], x[[j]]]), {j, 1, 4}]]]]
```

$$\frac{dg_{ij}}{dl} = -\frac{1}{2\pi} R_{ij}$$

Small mathematica code to calculate the Ricci tensor

Metric Tensor

$$gl = \begin{pmatrix} \sin^2(\theta)\left(I1\sin^2(\psi) + I2\cos^2(\psi)\right) + I3\cos^2(\theta) & \sin(\theta)\,(I1 - I2)\sin(\psi)\cos(\psi) & I3\cos(\theta) & \frac{1}{2}\kappa\cos(\theta) \\ \sin(\theta)\,(I1 - I2)\sin(\psi)\cos(\psi) & I1\cos^2(\psi) + I2\sin^2(\psi) & 0 & 0 \\ I3\cos(\theta) & 0 & I3 & \frac{\kappa}{2} \\ \frac{1}{2}\kappa\cos(\theta) & 0 & \frac{\kappa}{2} & I\varphi \end{pmatrix};$$

$$(\Gamma^i)_{kl} = \frac{1}{2} g^{ij} \left(\nabla_l g_{jk} + \nabla_k g_{jl} - \nabla_j g_{kl}\right)$$

Ricci Tensor

```
For[i = 1, i ≤ 4, i++,
  For[k = 1, k ≤ 4, k++,
   Riccill[[i, k]] =
    Sum[(D[Γ[[l, i, k]], x[[l]]] - D[Γ[[l, i, l]], x[[k]]] +
       Sum[(Γ[[m, l, m]] * Γ[[l, i, k]] - Γ[[m, i, l]] * Γ[[l, k, m]]), {m, 1, 4}]), {l, 1, 4}]]];
```

$$R^k{}_{ijl} = \Gamma^k{}_{il,j} - \Gamma^k{}_{ij,l} + \Gamma^k{}_{jn}\Gamma^n{}_{li} - \Gamma^k{}_{ln}\Gamma^n{}_{ij}$$

$$R_{ij} = R^k{}_{ikj}$$

$$\frac{dg_{ij}}{dl} = -\frac{1}{2\pi}R_{ij}$$

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Small mathematica code to calculate the Ricci tensor

This is the renormalization of I3

```
-FullSimplify[1/(2 π) Riccill[[3, 3]]]
```

$$-\frac{I3^2-(I1-I2)^2}{4\pi\, I1\, I2}$$

Metric Tensor

$$gl = \begin{pmatrix} \sin^2(\theta)\left(I1\sin^2(\psi)+I2\cos^2(\psi)\right)+I3\cos^2(\theta) & \sin(\theta)\,(I1-I2)\sin(\psi)\cos(\psi) & I3\cos(\theta) & \frac{1}{2}\kappa\cos(\theta) \\ \sin(\theta)\,(I1-I2)\sin(\psi)\cos(\psi) & I1\cos^2(\psi)+I2\sin^2(\psi) & 0 & 0 \\ I3\cos(\theta) & 0 & I3 & \frac{\kappa}{2} \\ \frac{1}{2}\kappa\cos(\theta) & 0 & \frac{\kappa}{2} & I\varphi \end{pmatrix};$$

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```

$$\frac{dg^{ab}}{dl} = -\frac{1}{2\pi}R^{ab}$$

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```

$$-\frac{\text{I3}^2 - (\text{I1} - \text{I2})^2}{4\,\pi\,\text{I1}\,\text{I2}}$$

$$\frac{dI_1}{dl} = \frac{(I_2 - I_3)^2 - I_1^2}{4\pi I_2 I_3} - \frac{\left(I_1^2 - I_2^2\right)\kappa^2}{16\pi I_2 I_3^2 \left(I_\varphi - \frac{\kappa^2}{4I_3}\right)}$$

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$$S = -\frac{1}{2}\int d^2x\Big(I_1\Omega^2_{\mu,1} + I_2\Omega^2_{\mu,1} + I_3\Omega^2_{\mu,1}\Big)$$
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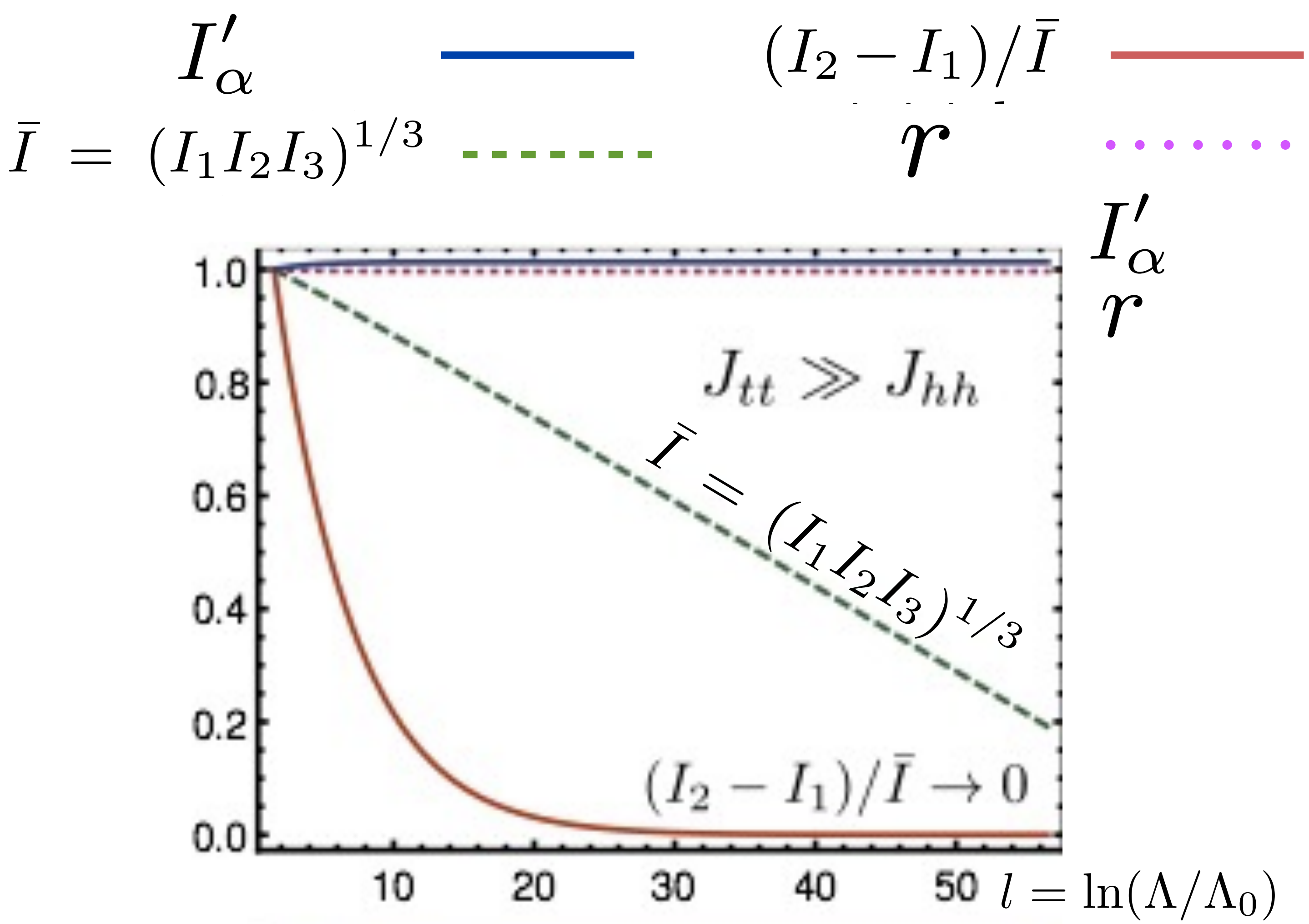

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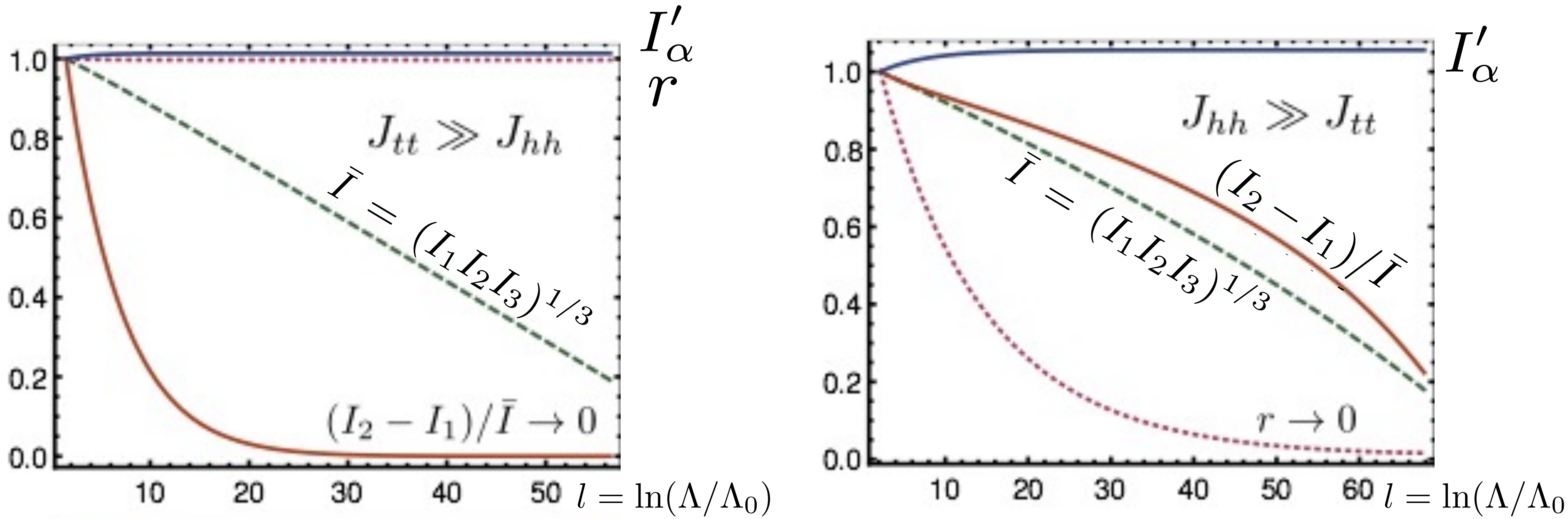
I'_α (solid blue line) $\quad (I_2 - I_1)/\bar{I}$ (solid red line)

$\bar{I} = (I_1 I_2 I_3)^{1/3}$ (dashed green line) $\quad r$ (dotted magenta line)

Decoupling becomes complete as $I_1 \to I_2$



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I'_α (blue solid line) $(I_2 - I_1)/\bar{I}$ (red solid line)

$\bar{I} = (I_1 I_2 I_3)^{1/3}$ (green dashed line) r (magenta dotted line)

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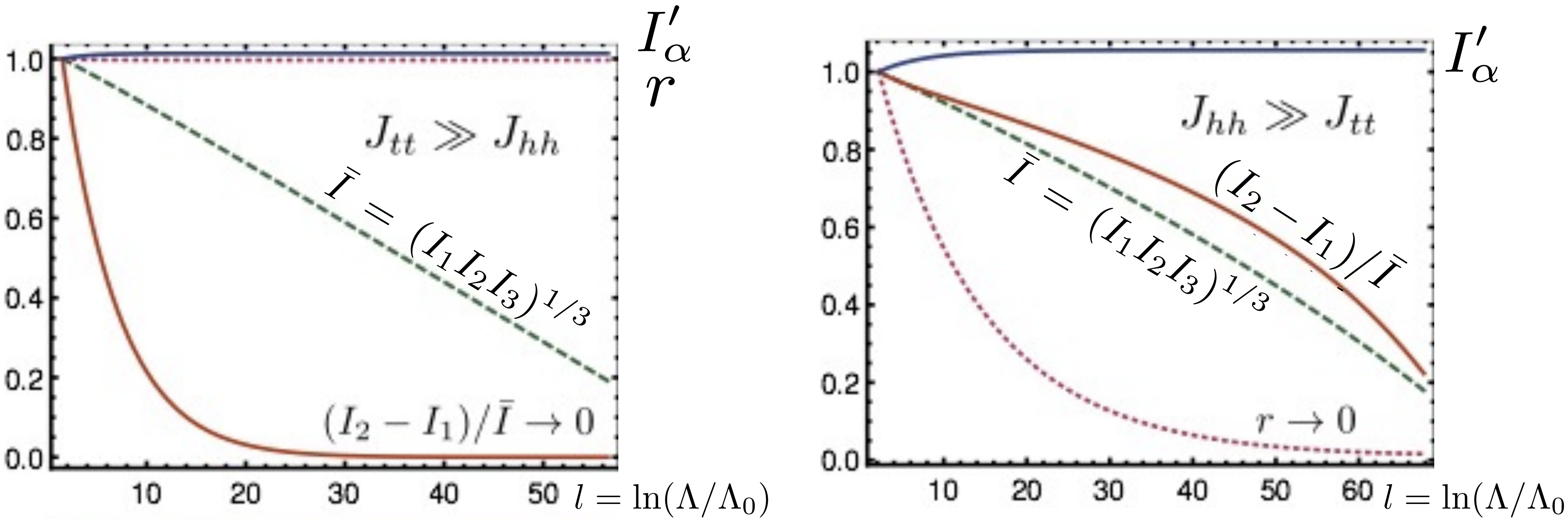

Decoupling of U(1) Degrees of Freedom in Both Parameter Regimes

$$S_{\mathbb{Z}_6} = \frac{1}{2} \int d^2x \left[(I'_\alpha (\partial_\mu \alpha)^2 + \lambda \cos(6\alpha) \right] .$$

21

$$S_{\mathbb{Z}_6} = \frac{1}{2} \int d^2x \left[(I'_\alpha (\partial_\mu \alpha)^2 + \lambda \cos(6\alpha) \right] .$$

$$\frac{d \ln(\lambda)}{dl} = 2 - \frac{n^2/2}{2\pi I'_\alpha}$$

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n>4, @ T_{BKT}

Anisotropy Irrelevant

(Jose et al,1977)

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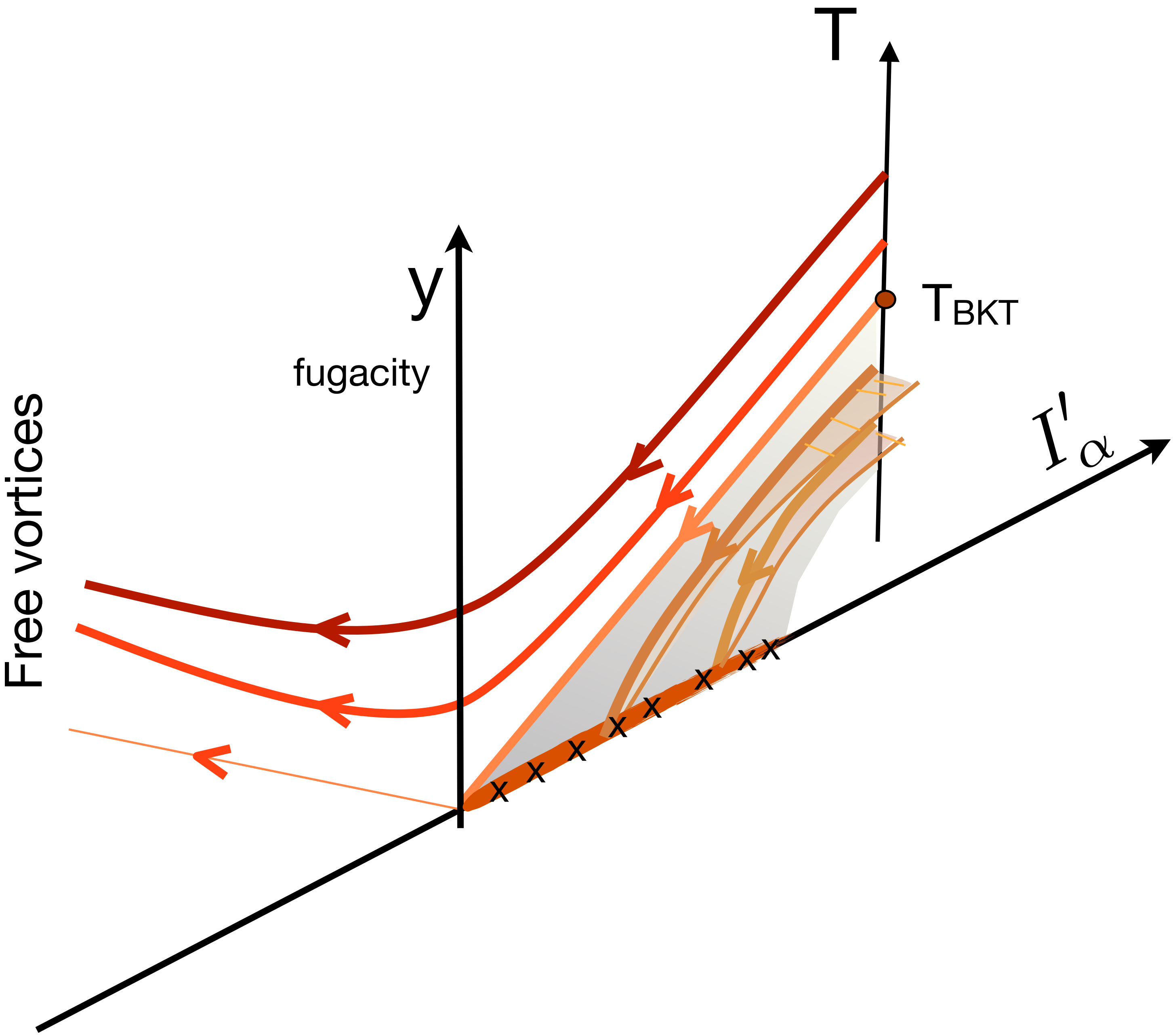

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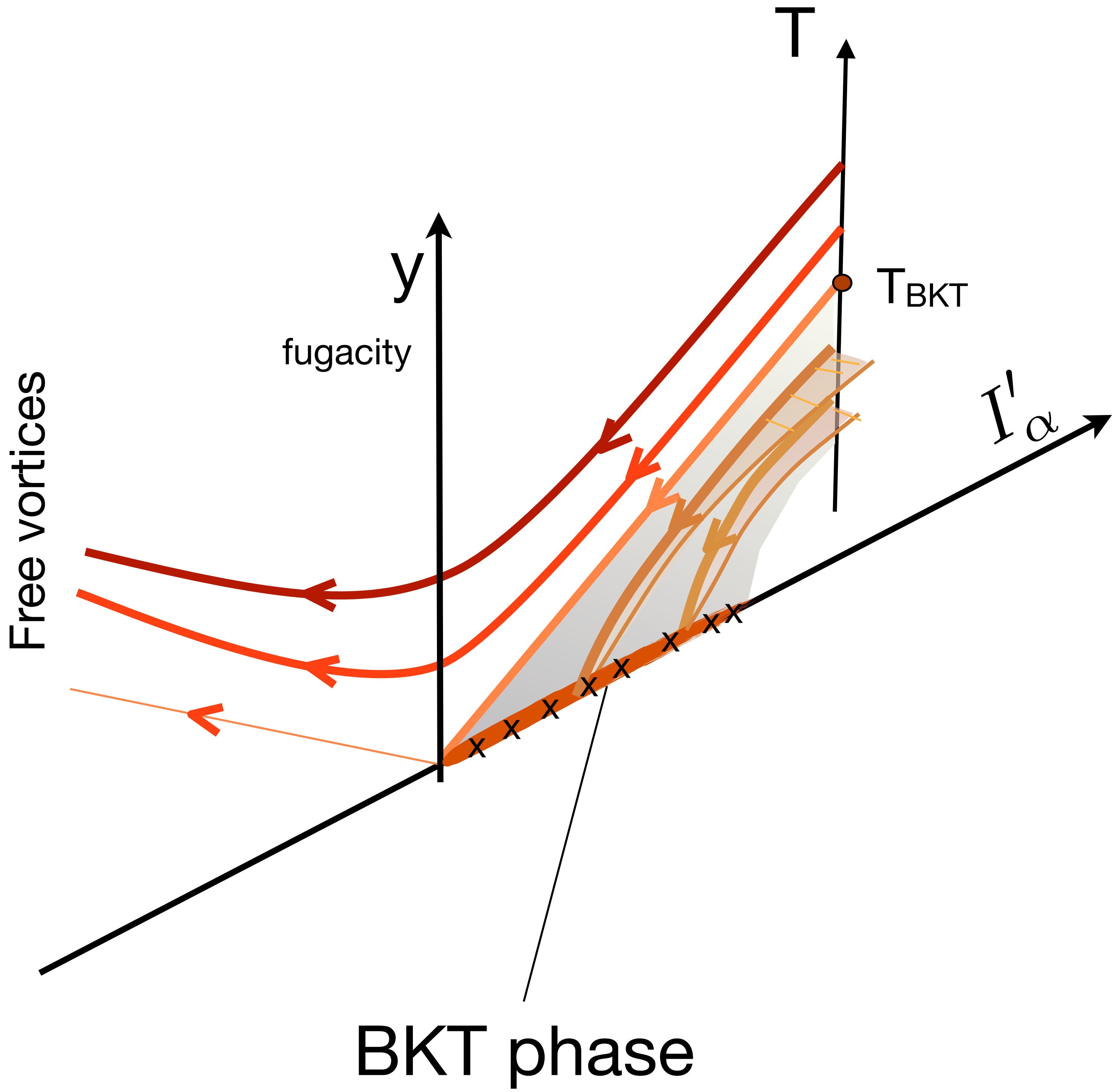

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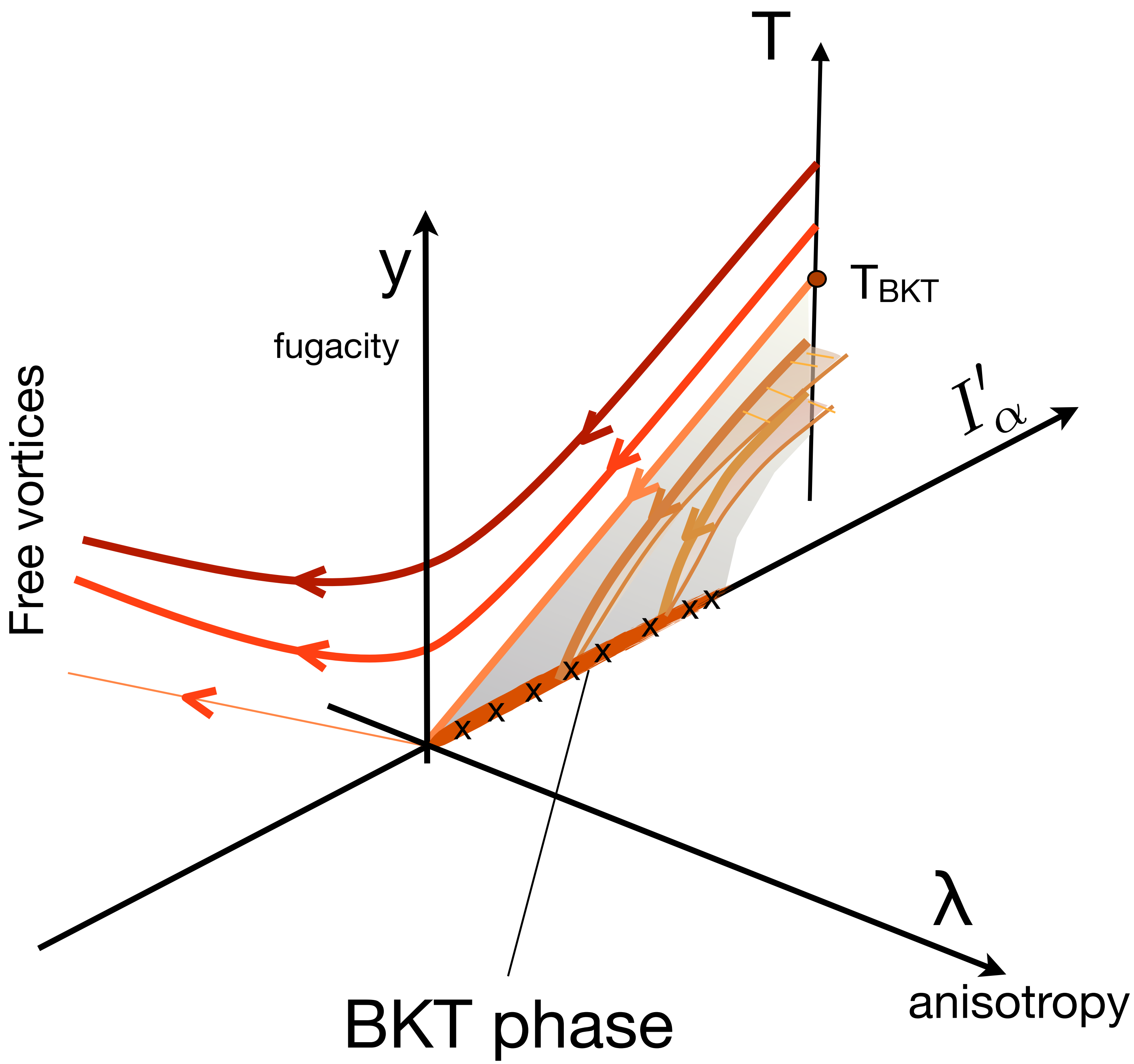

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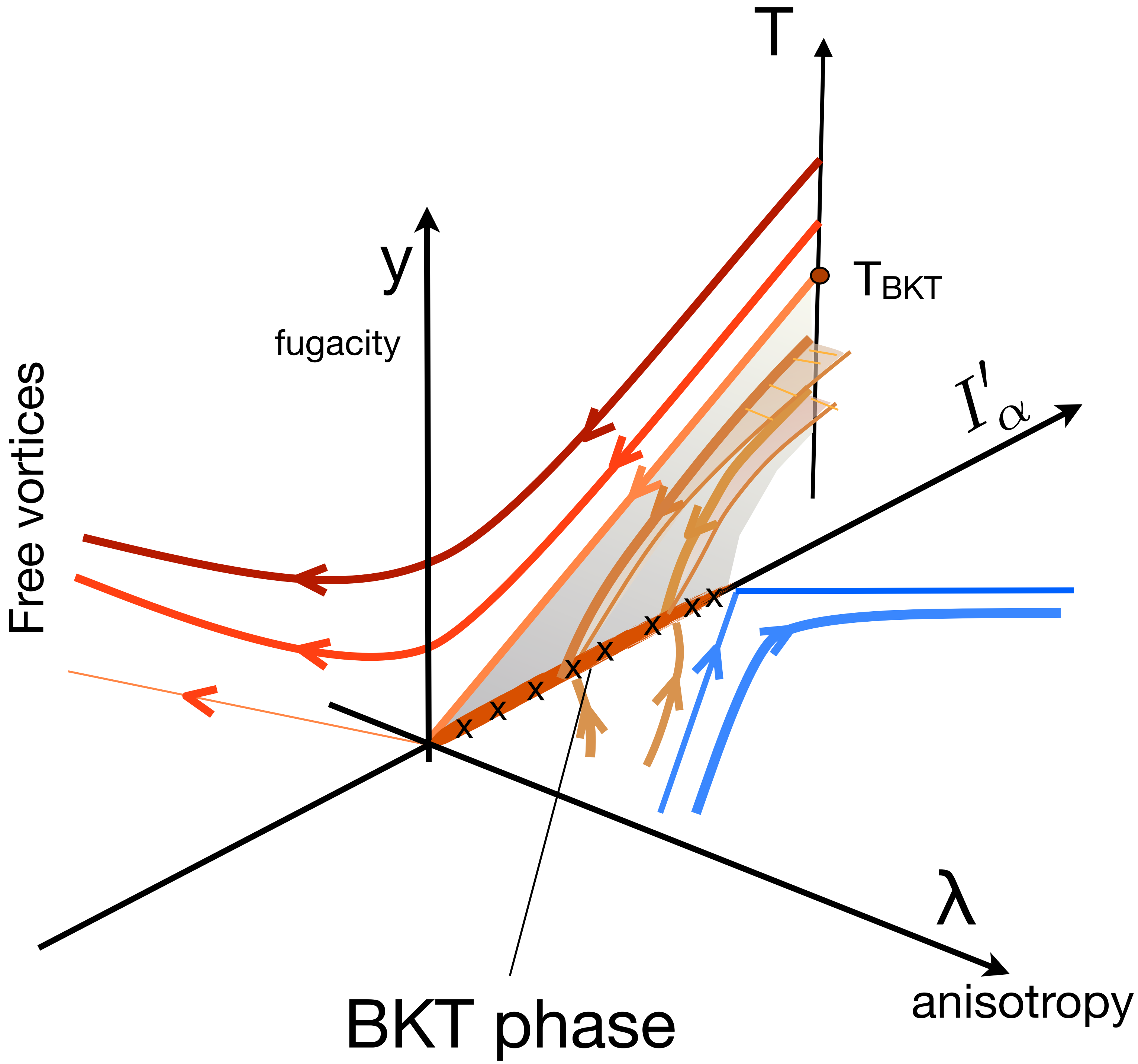

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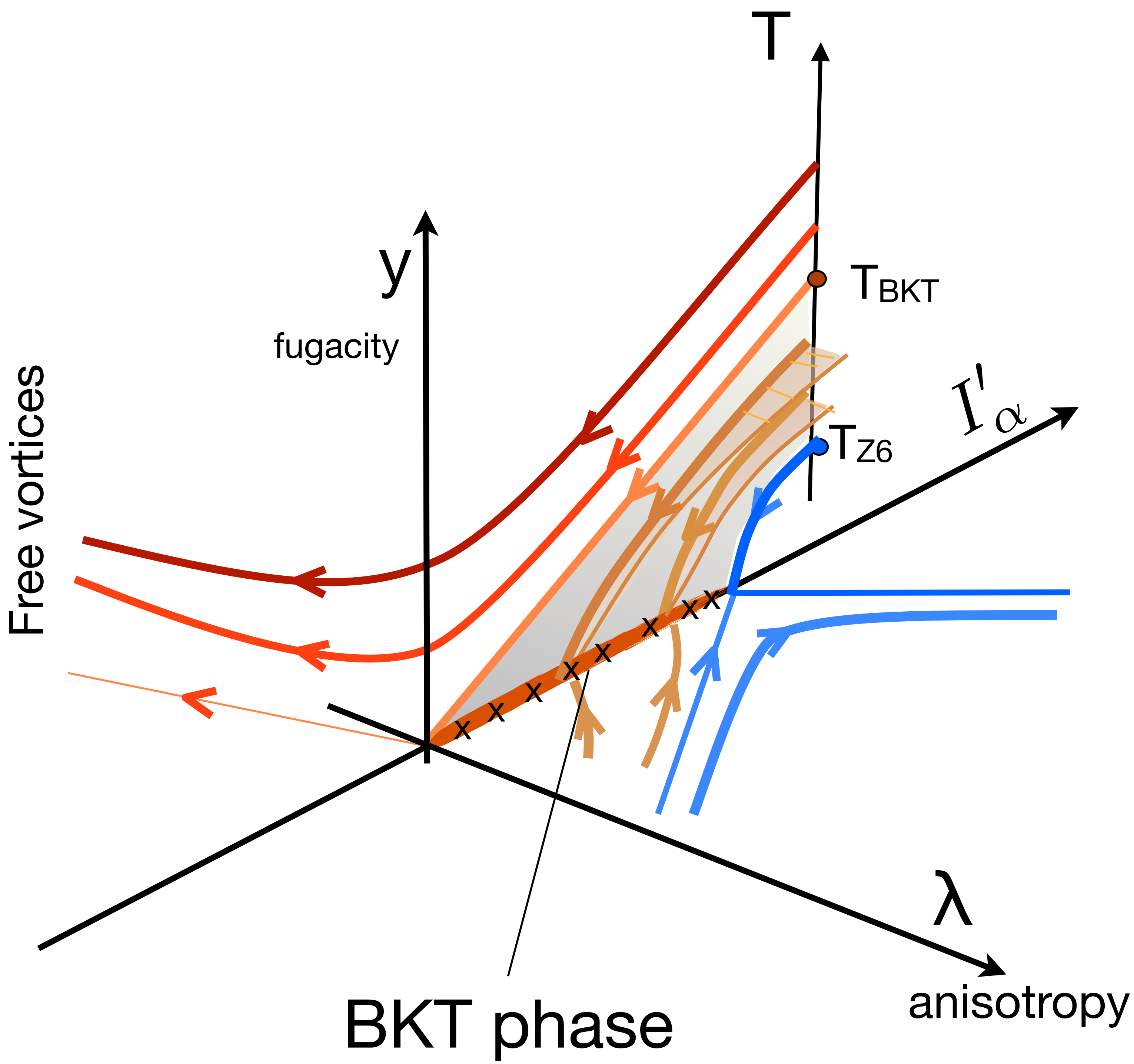

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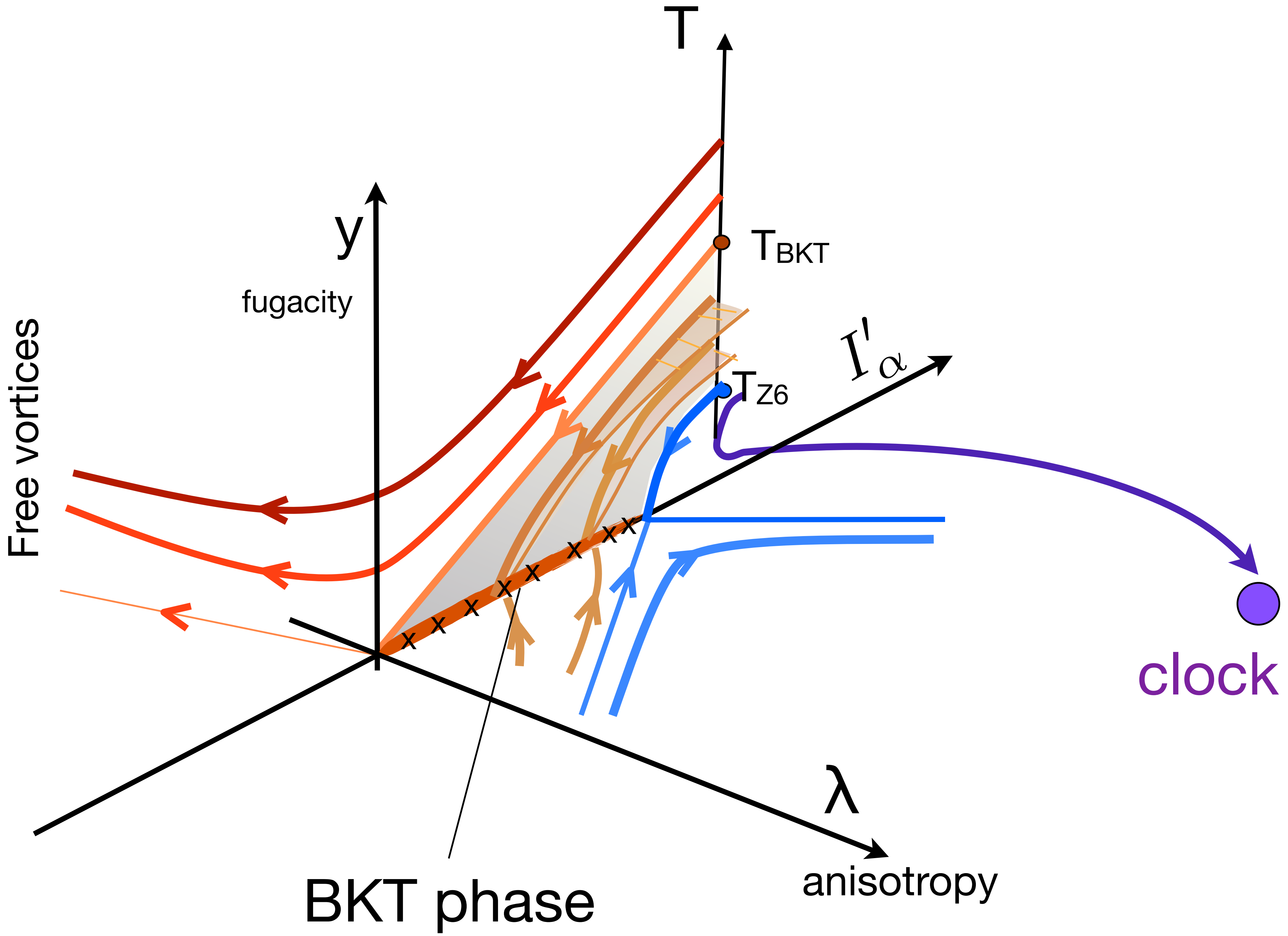

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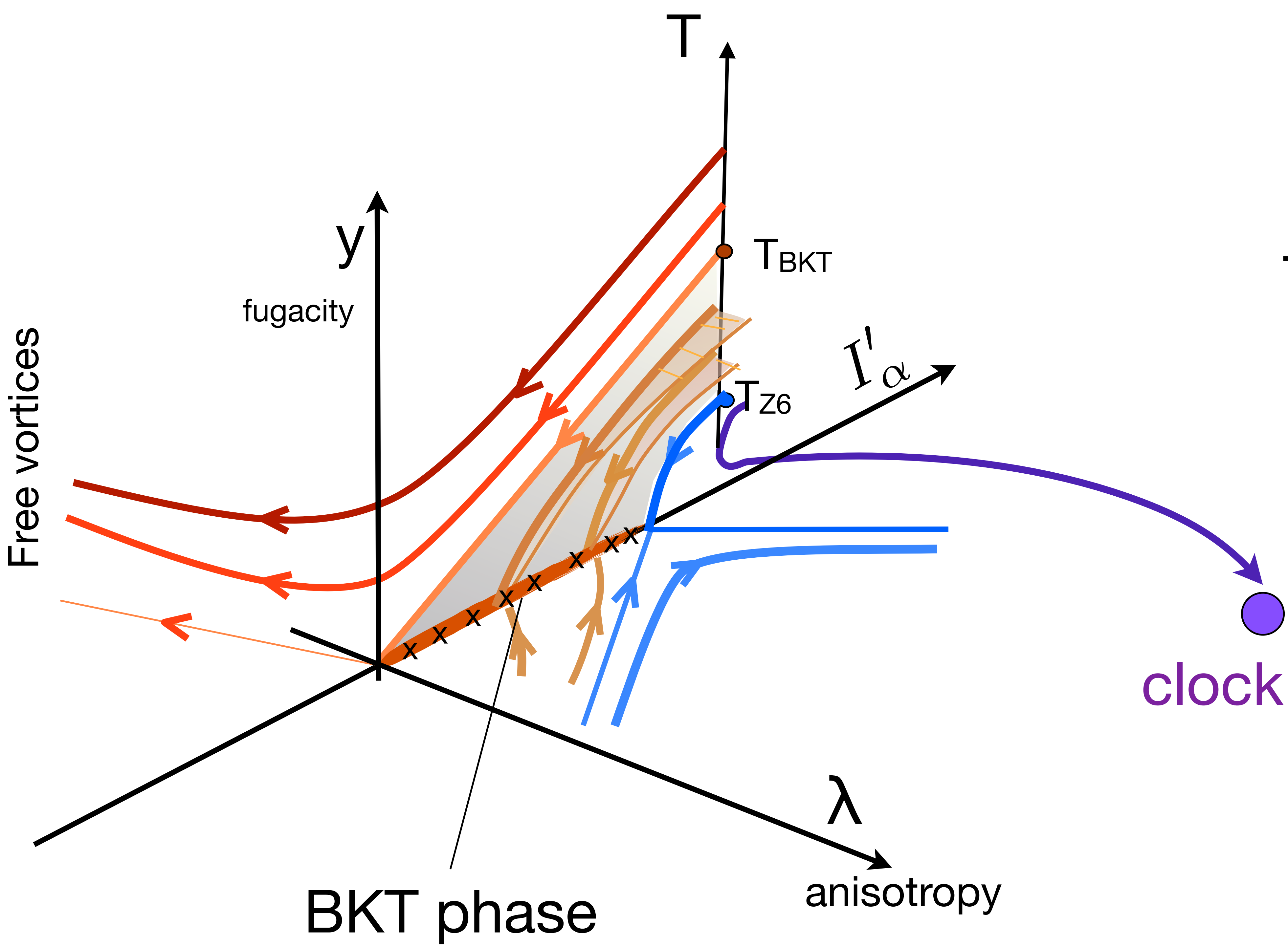

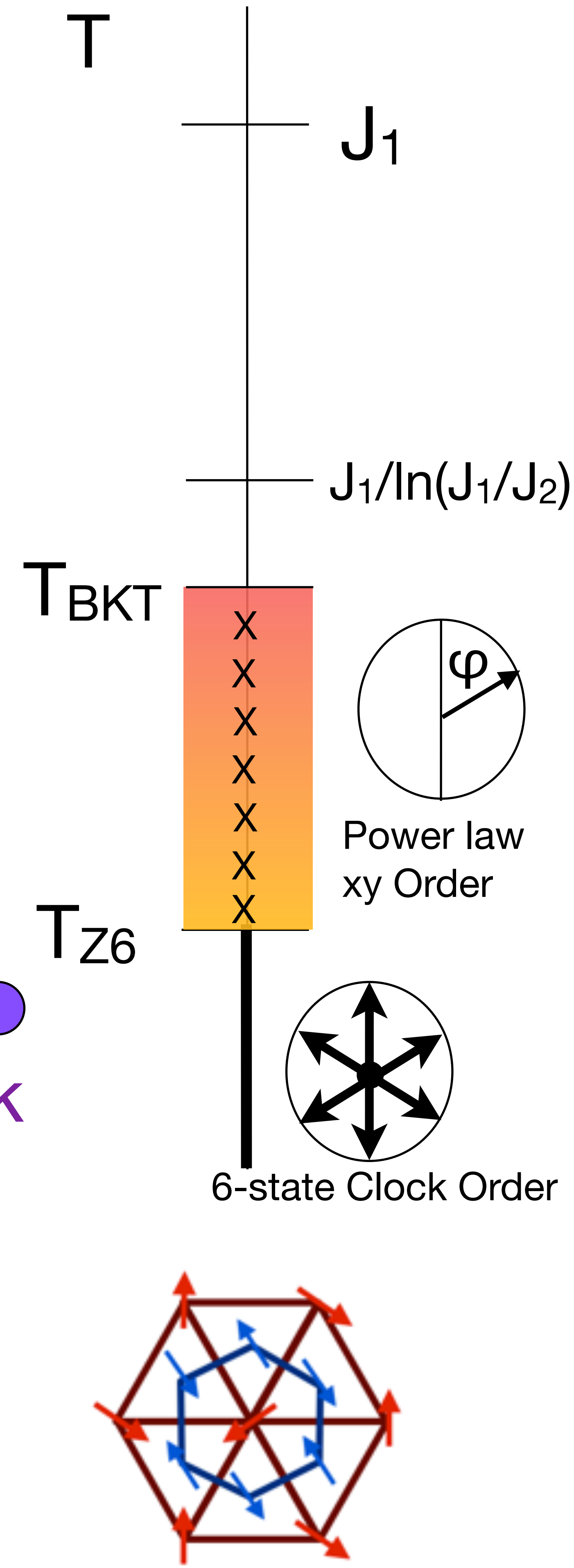

PHYSICAL REVIEW B **82**, 012402 (2010)

Real space observation of spin frustration in Cr on a triangular lattice

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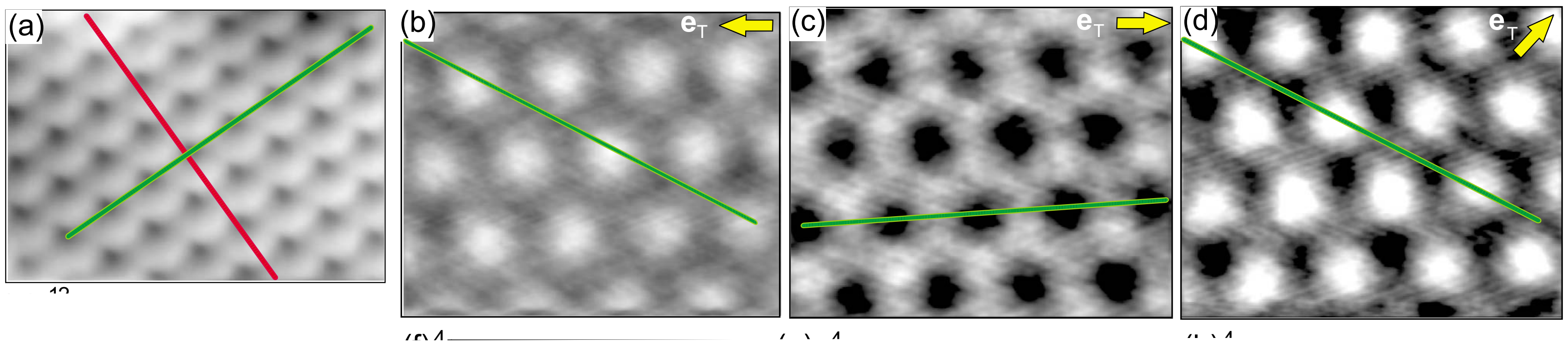
(Received 1 February 2010; revised manuscript received 9 May 2010; published 2 July 2010)

Single layer Cr on Pd 111 surface

Spin Polarized STM (SP-STM)



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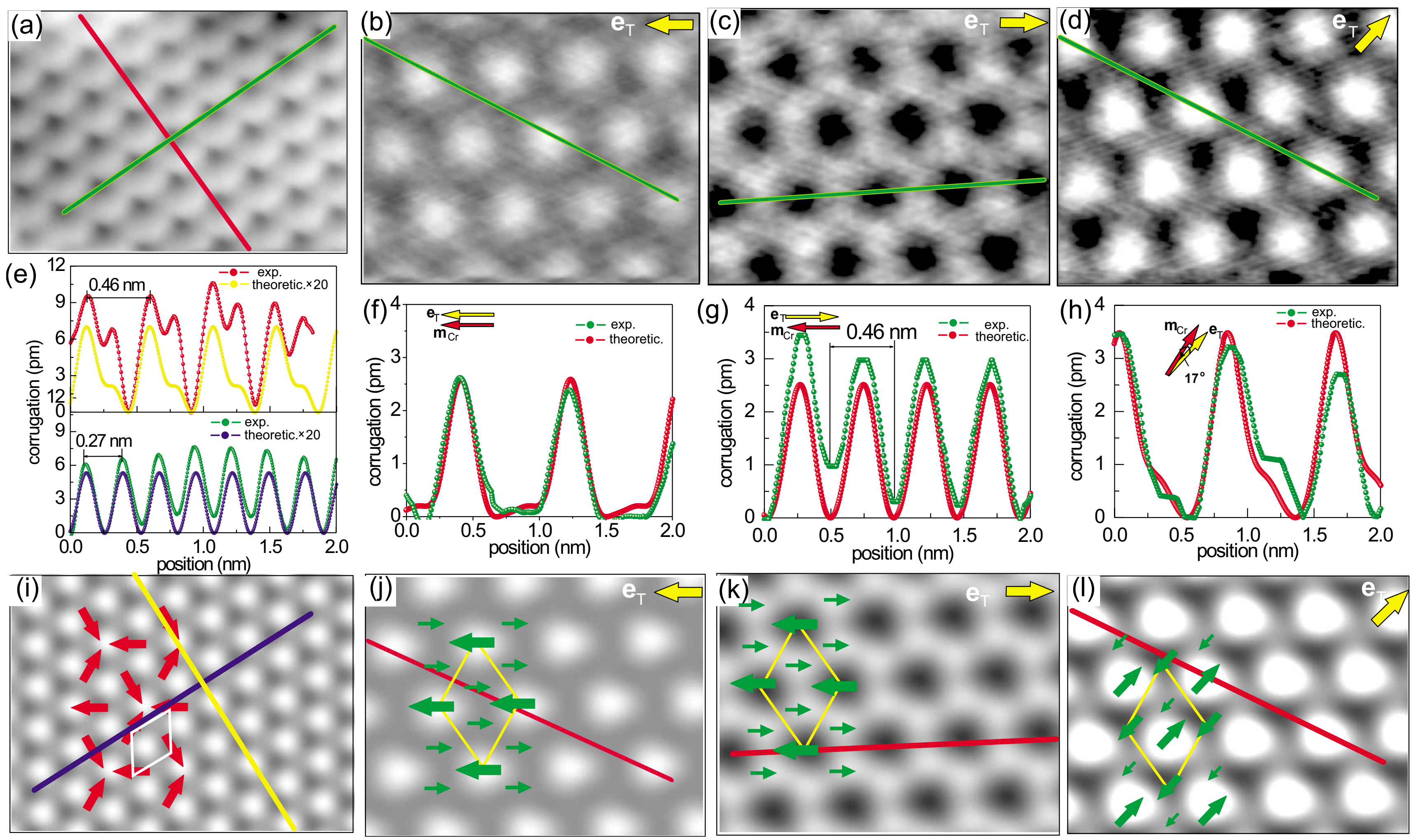

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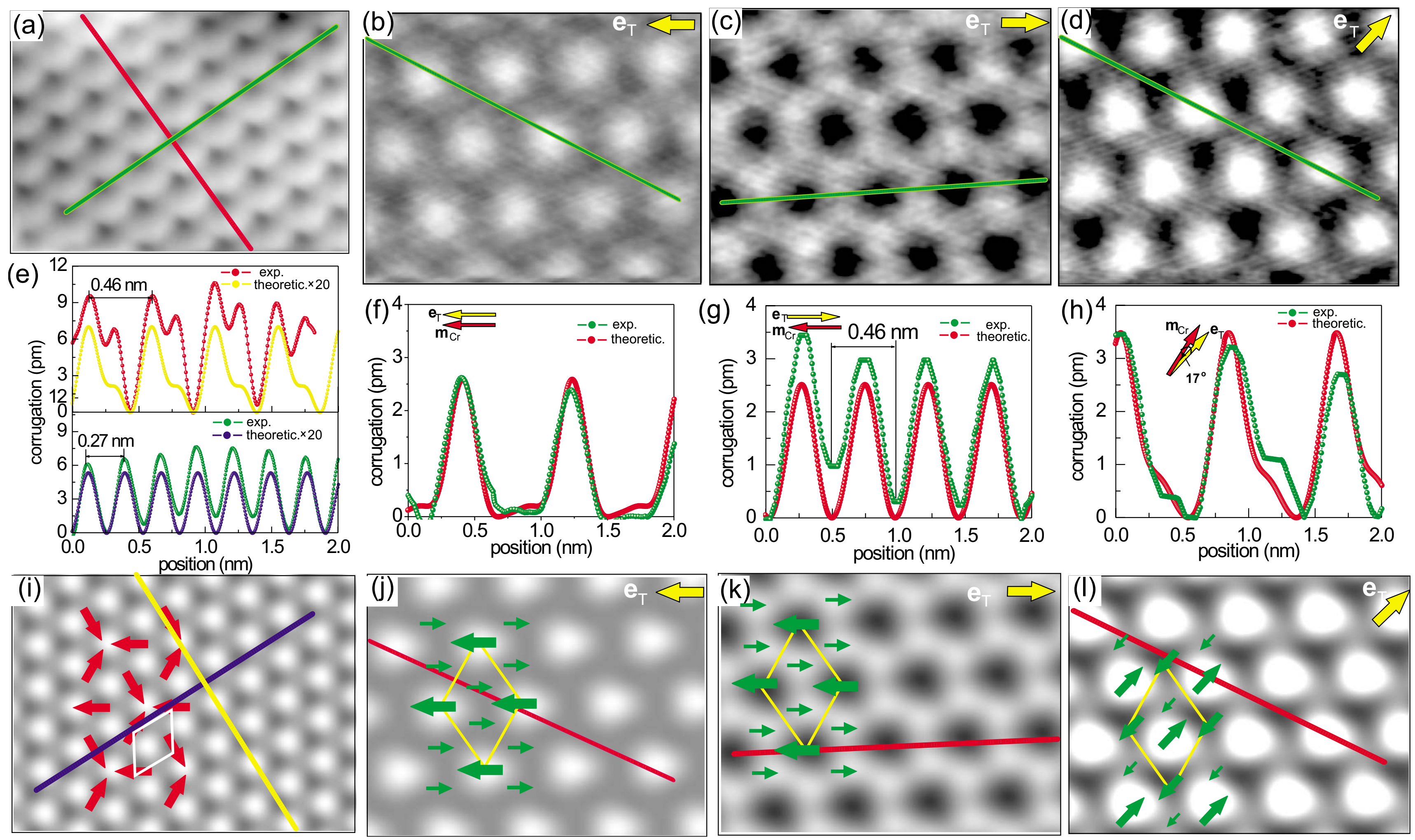
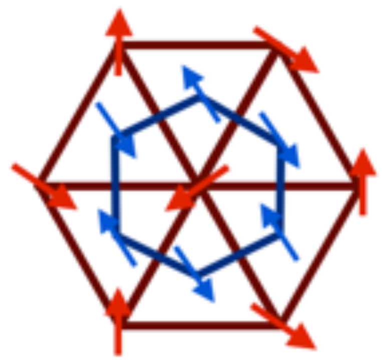


Bilayer: possible candidate for powerlaw phase?

S and Q



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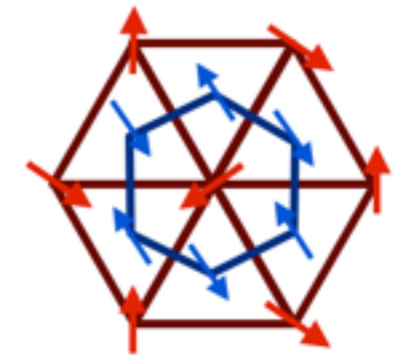

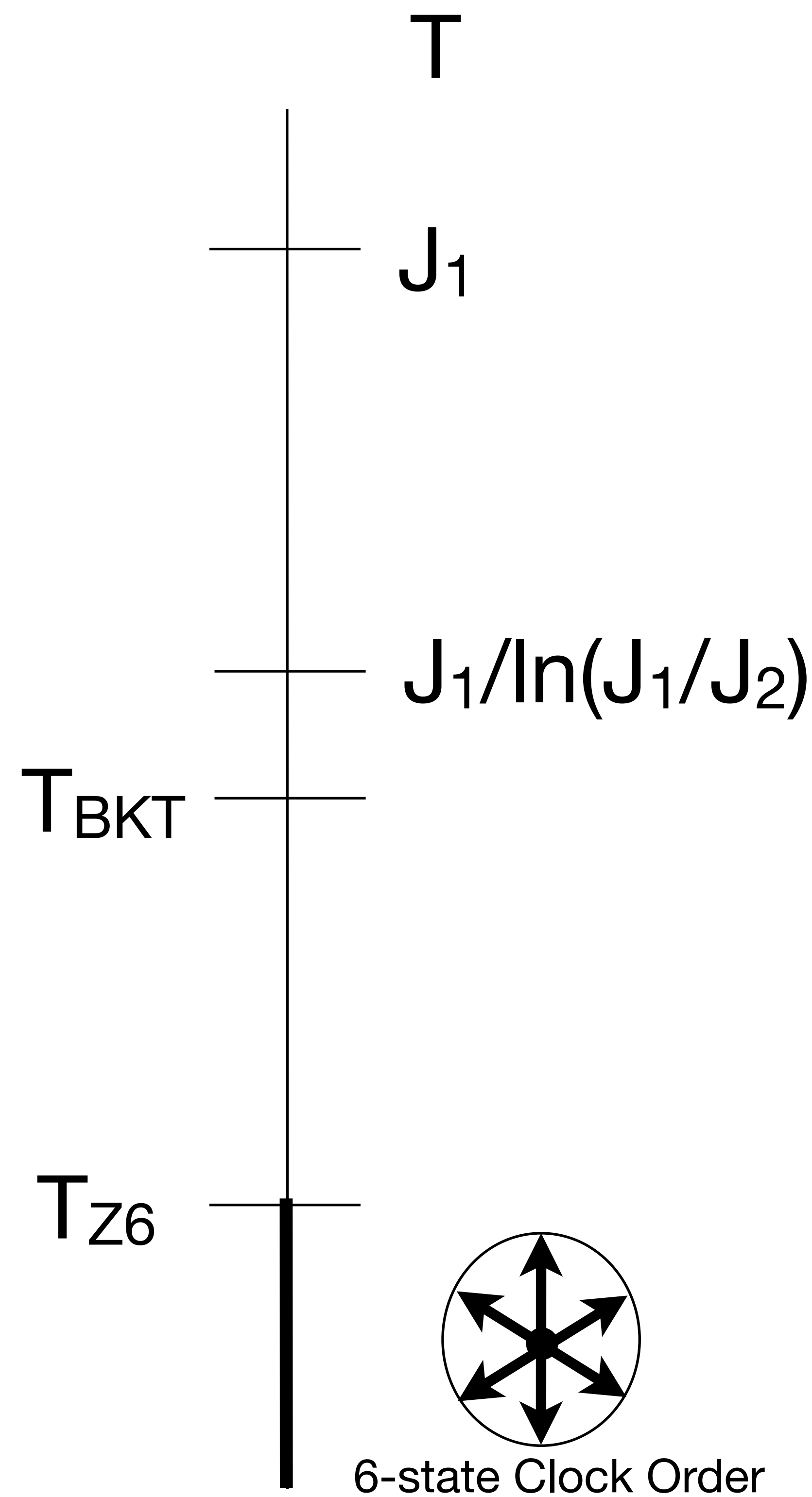

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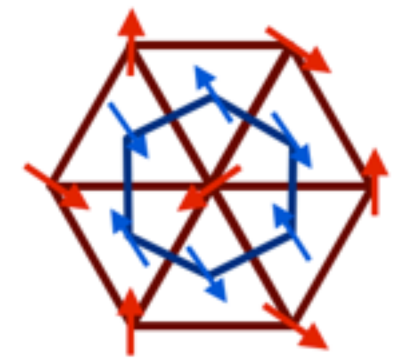

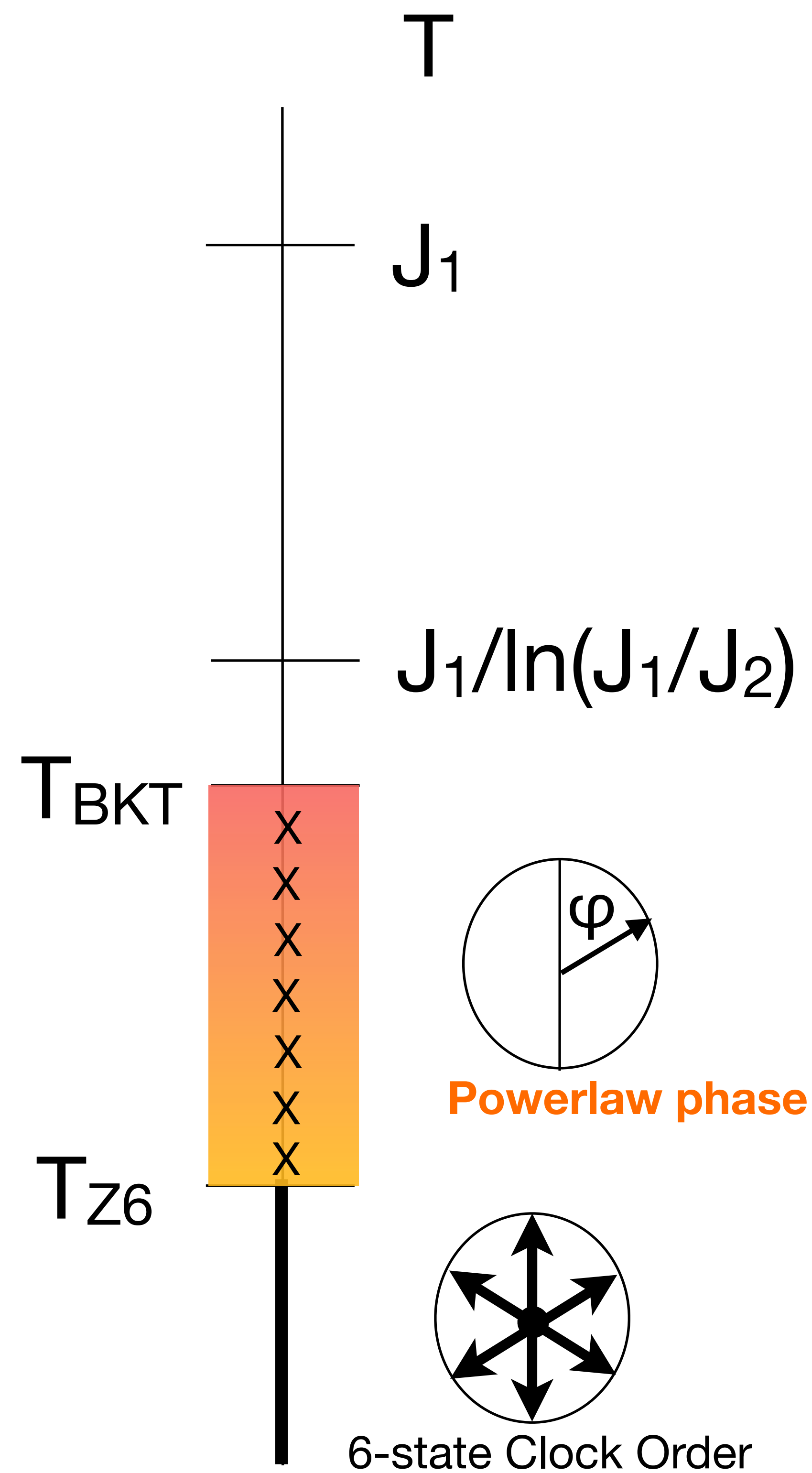

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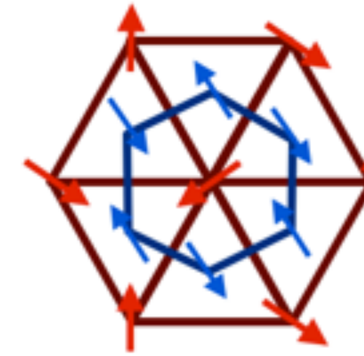

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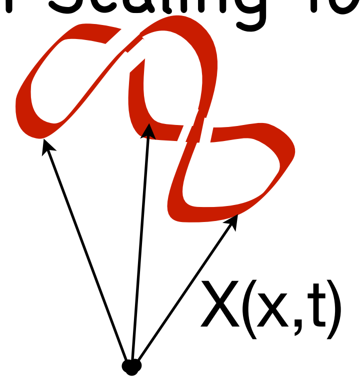

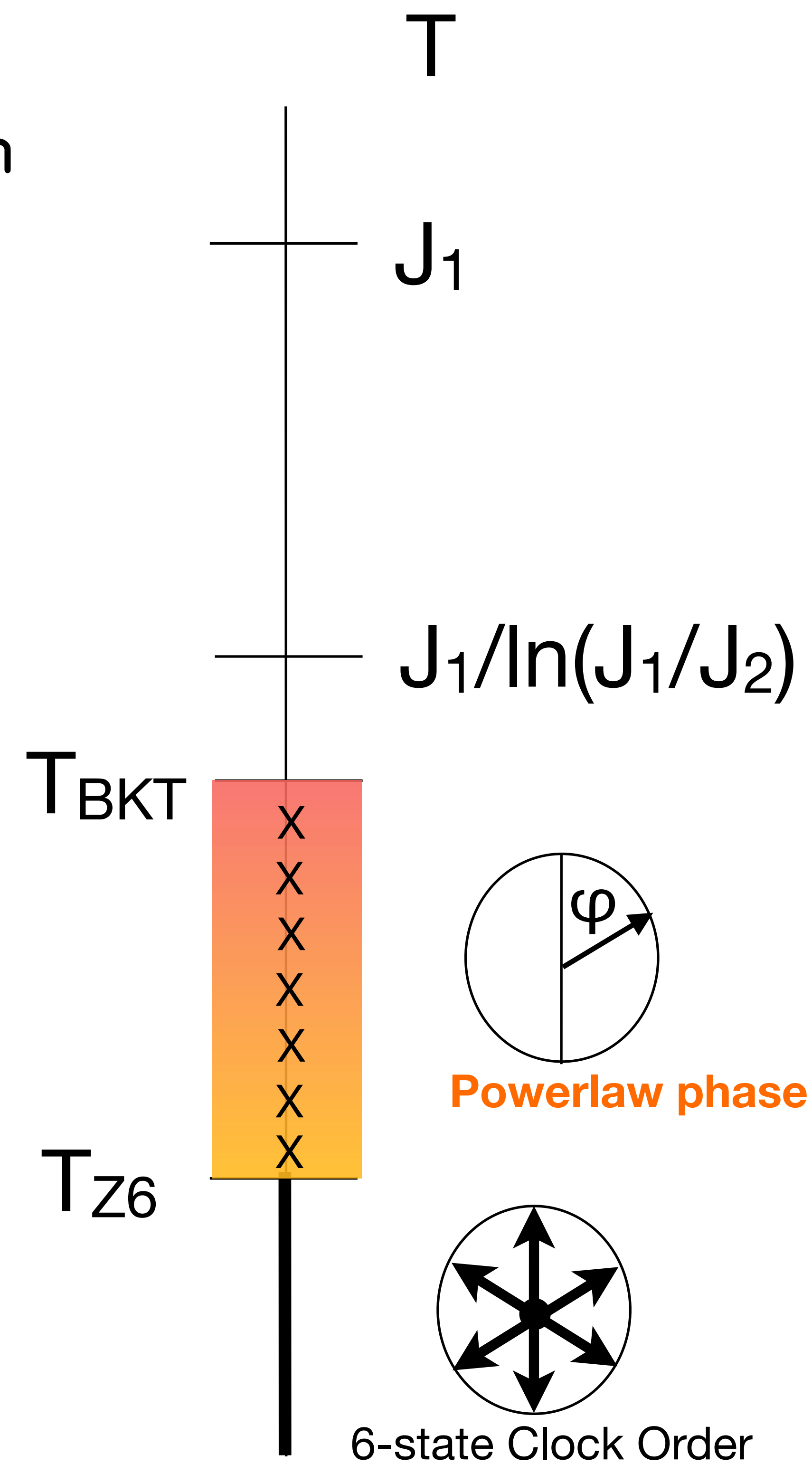

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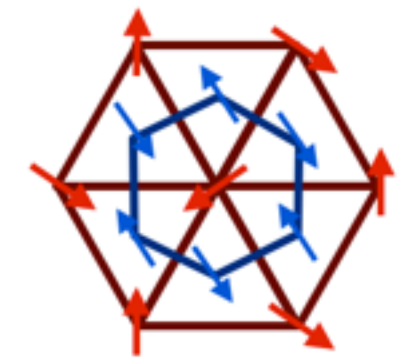

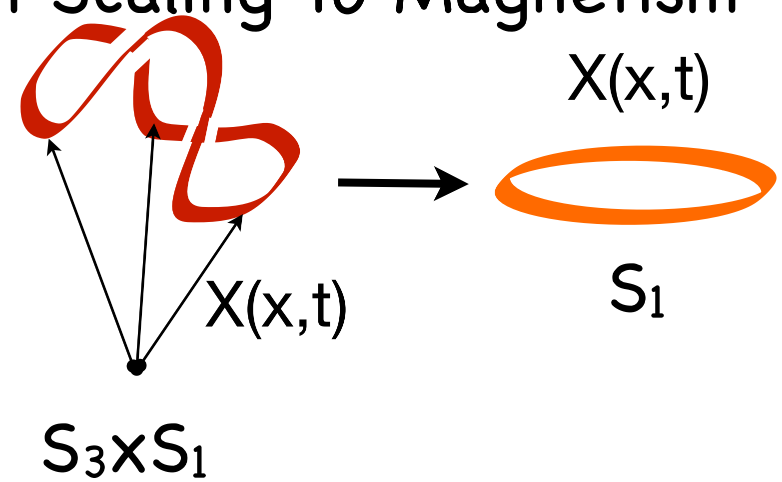

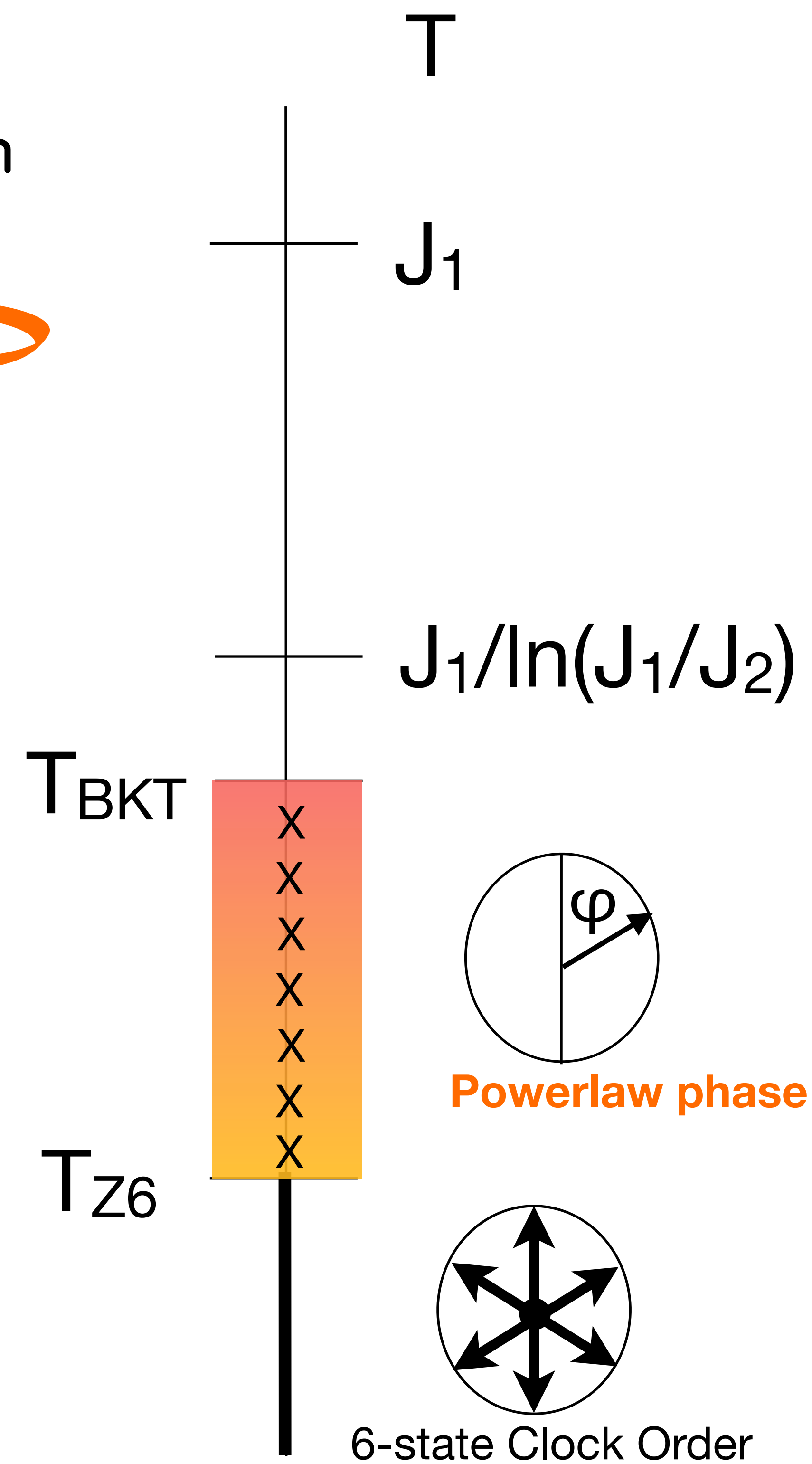

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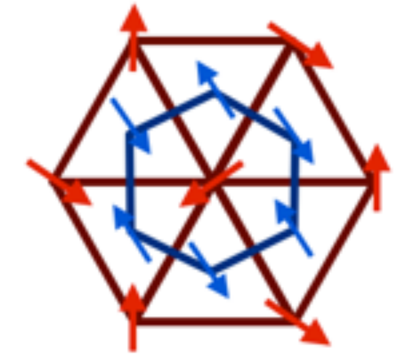

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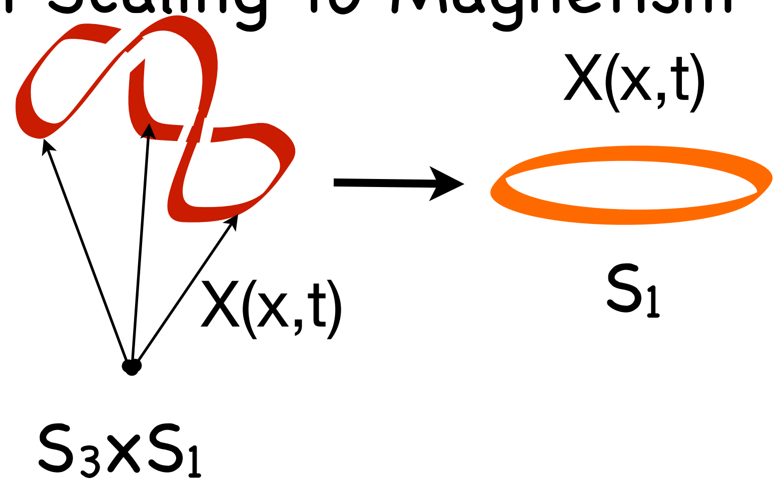
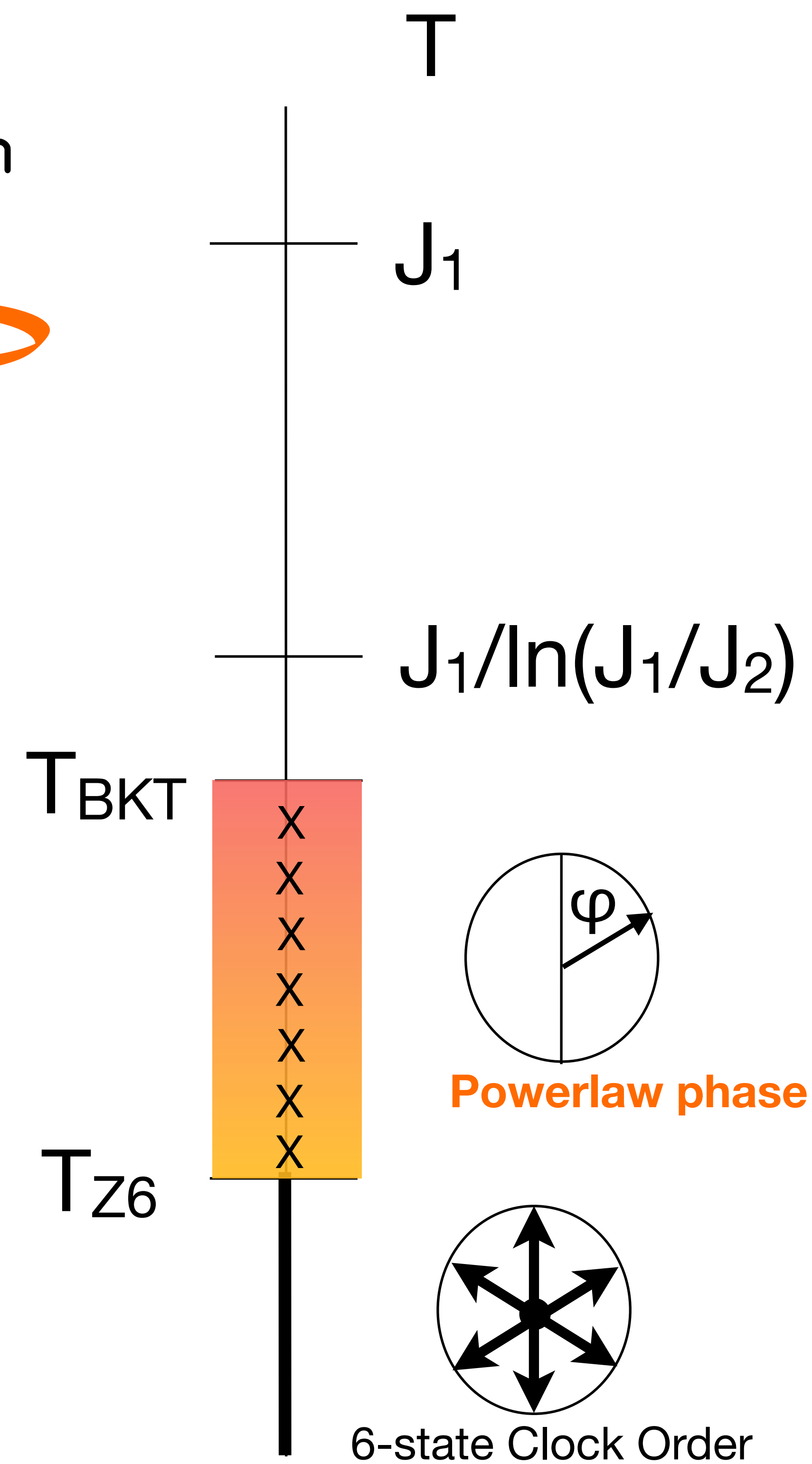


- Realization of mapping of RG into time. (cf AdSCFT)



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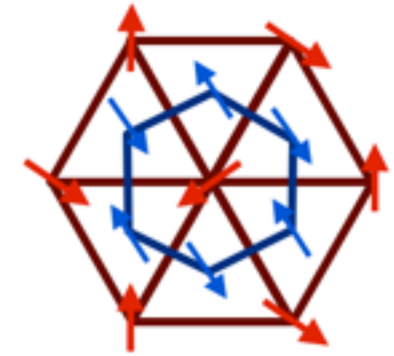

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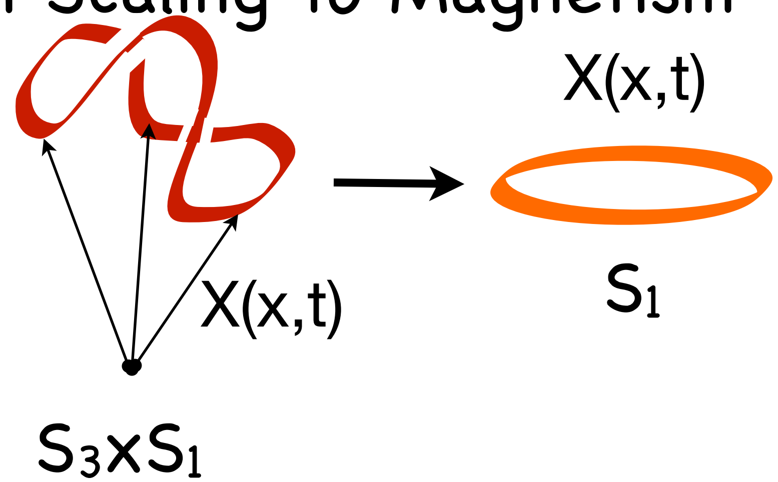

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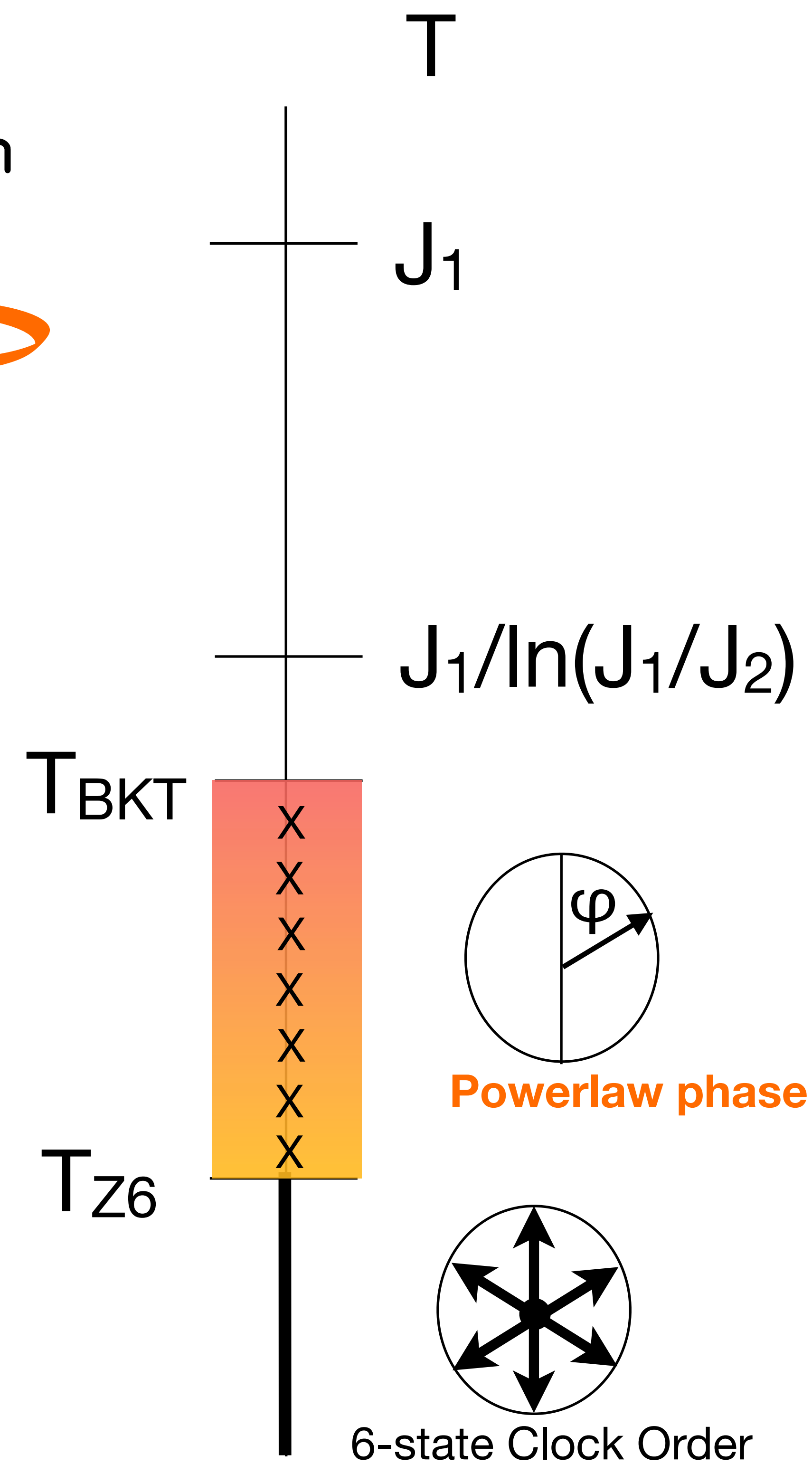

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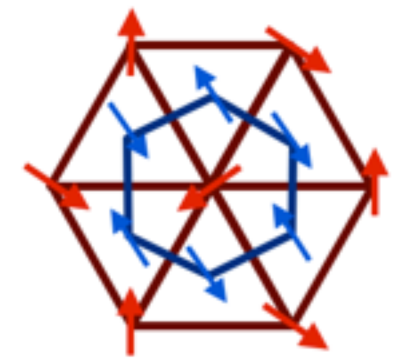

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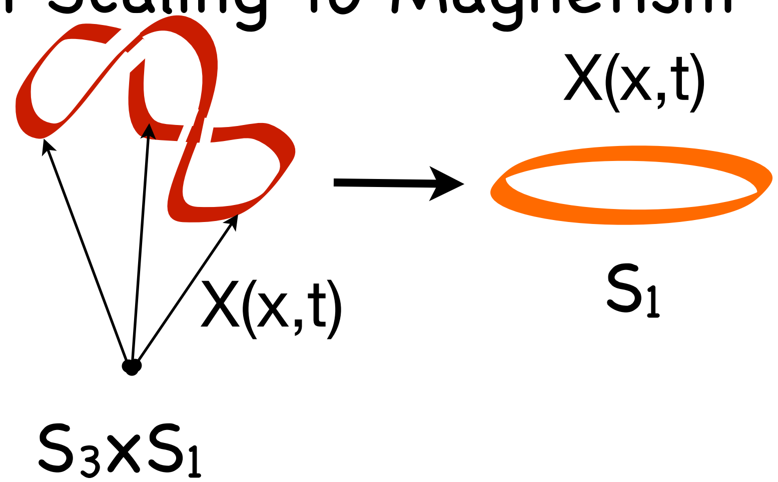

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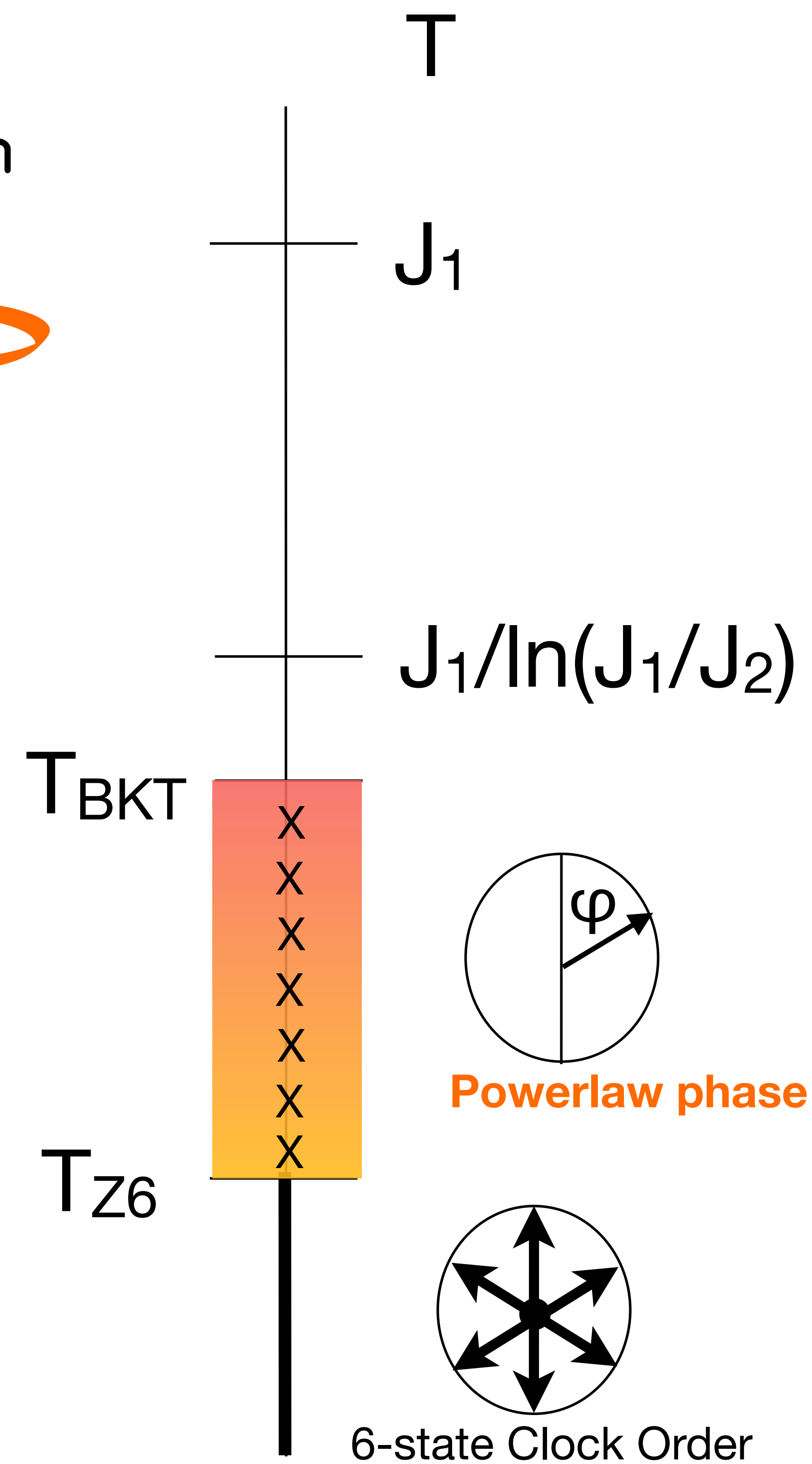
Thanks: D. Friedan



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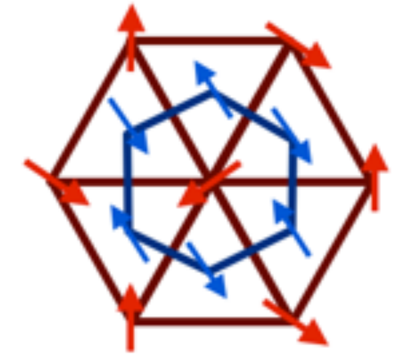

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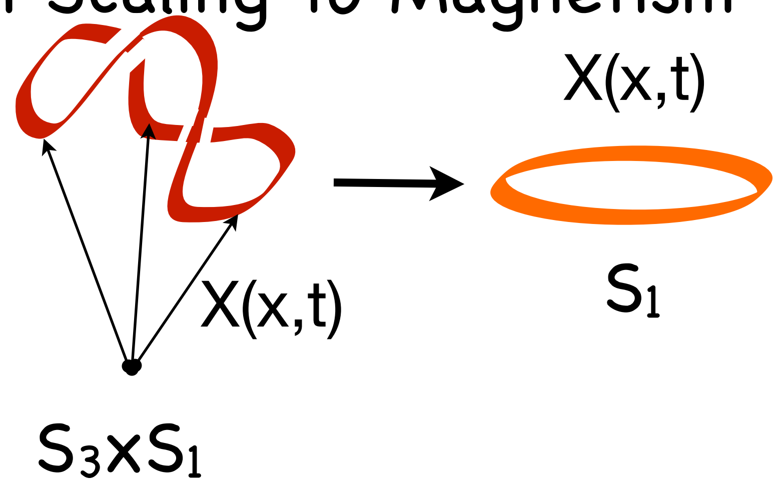

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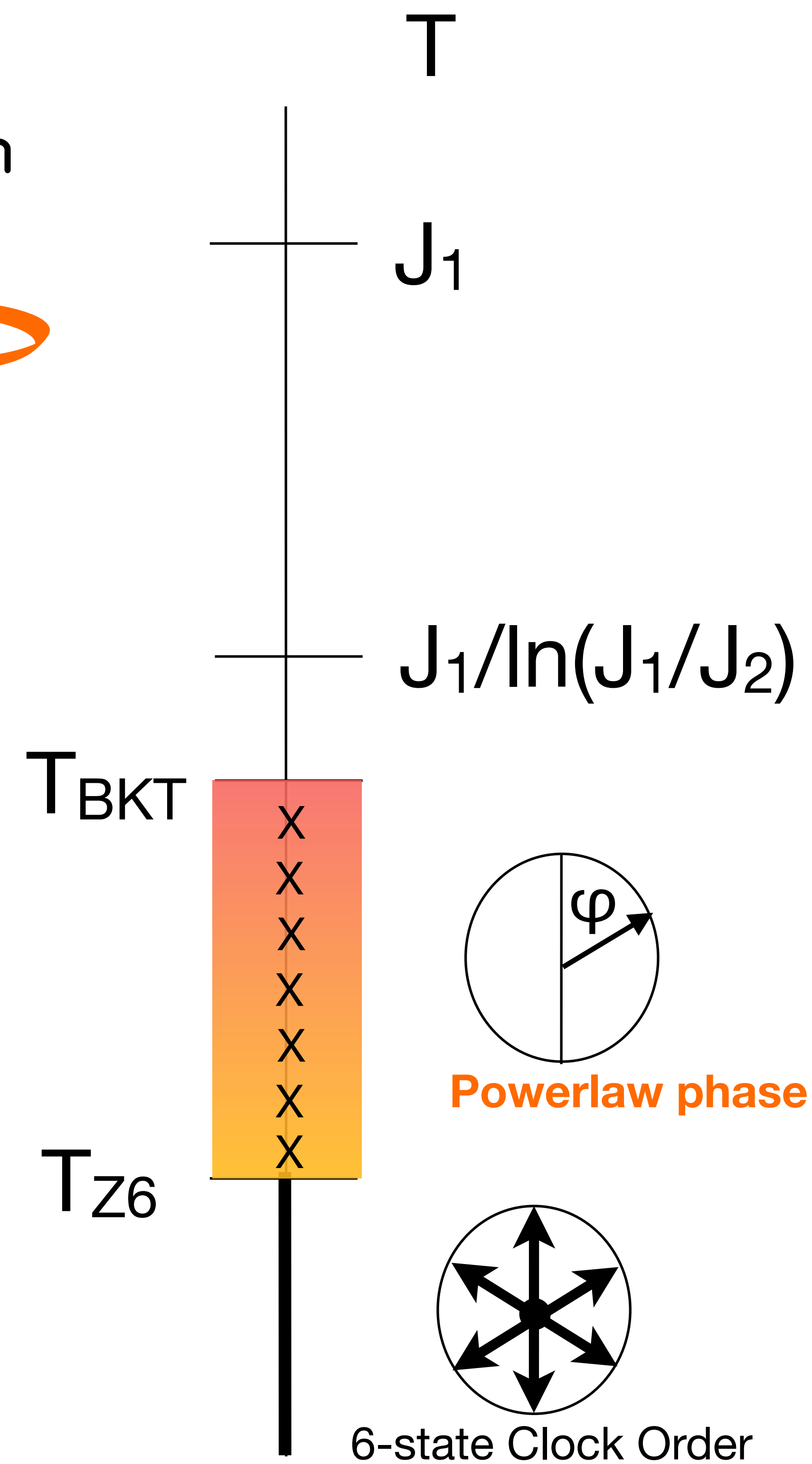

- Quantum version?

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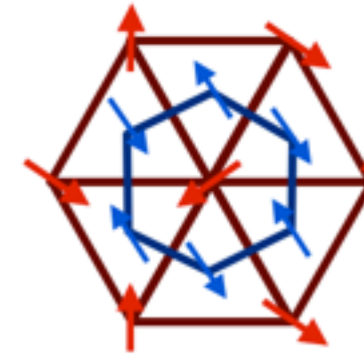

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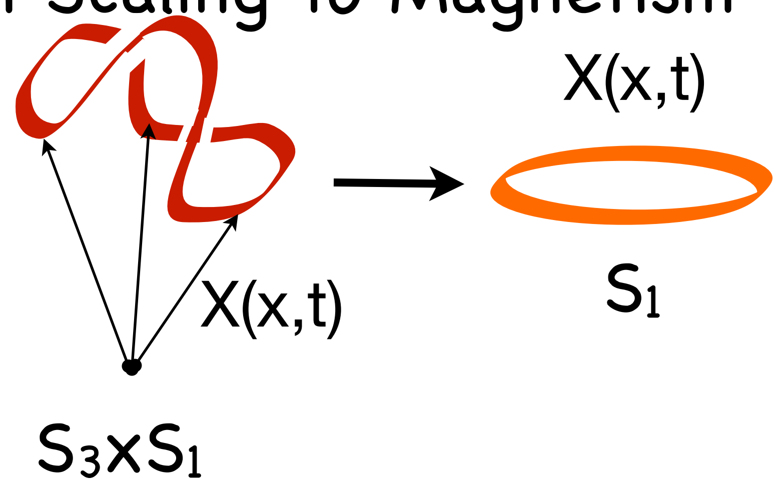

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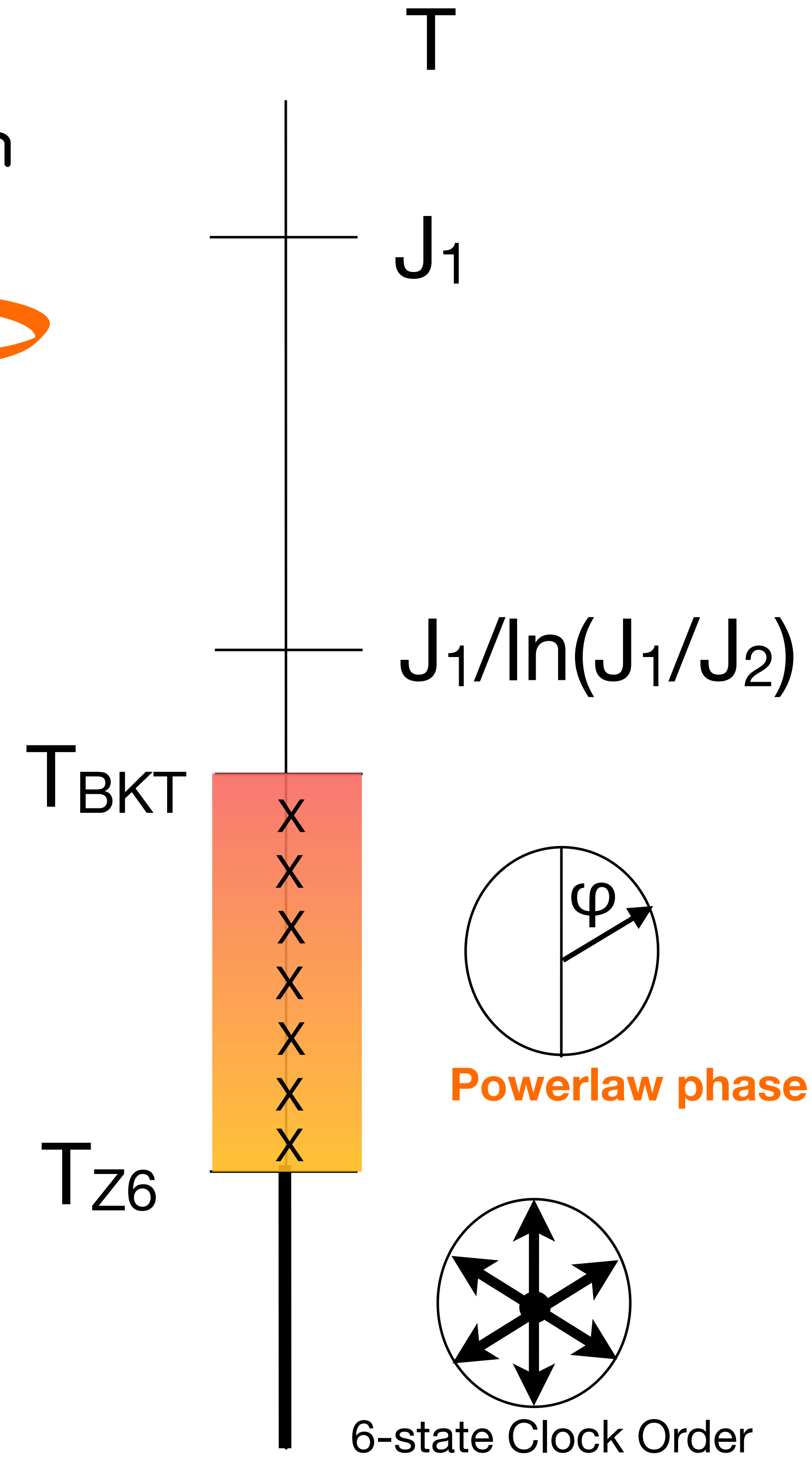

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Can one suppress T_{Z6} to zero : power law spin-liquid?

Thank you!