# *Towards a Complete Characterization of Emergent Topological Order From a Microscopic Hamiltonian on the Lattice*

Guifre Vidal

Perimeter Institute

Based on Lukasz Cincio, G. Vidal, arXiv:1208.2623

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theory

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**Exotic Phases of Frustrated Magnets** *KITP, UC Santa Barbara* $8^{th}$ – $12^{th}$ October 2012

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$H$

microscopic Hamiltonian

emergent anyon model

- number of topological fluxes/anyon types

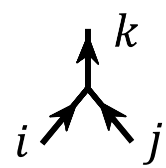
  [toric code: $\mathbb{I}, e, m, \varepsilon$]

  [Ising: $\mathbb{I}, \sigma, \varepsilon$]

- quantum dimensions

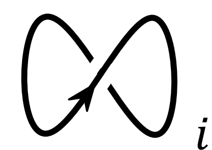
  $d_i$ $\qquad D = \sqrt{\sum_i (d_i)^2}$

- fusion rules $N_{ij}{}^k$

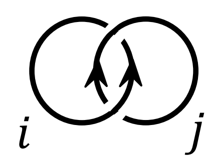



- topological spin $\theta_i$

  topological central charge $c$




- mutual statistics $S_{ij}$




...

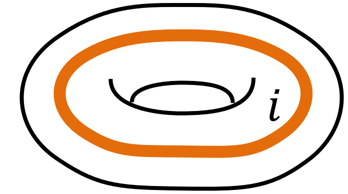
(if gapless edge state)

chiral CFT

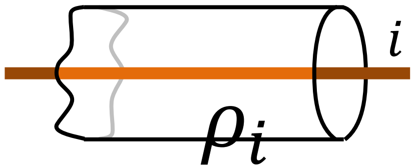
# Background

Infinite cylinder


Finite torus




Entanglement spectrum




→

Spectrum of gapless edge state (chiral CFT)

$$H_i^{(boundary)}$$

H. Li, F. D. M. Haldane, PRL 2008

X.-L. Qi, H. Katsura, A. W. W. Ludwig, PRL 2012

Topological entanglement entropy

$$S_L = aL - \gamma$$

$$\gamma = \log\left(\frac{D}{d_i}\right)$$

→

quantum dimensions

$$\frac{d_i}{D} \qquad D = \sqrt{\sum_i (d_i)^2}$$

A. Kitaev, J. Preskill, PRL 2006

M. Levin, X.-G. Wen, PRL 2006

Modular transformations

→

Topological $S, U$ matrices

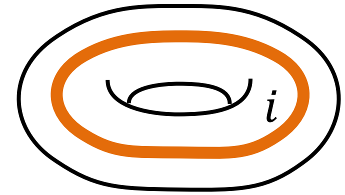
$$V_{ij} = \langle \Psi_i{}^{tor} | R_{\pi/3} | \Psi_j{}^{tor} \rangle \qquad V = DUS^{-1}D^{\dagger}$$

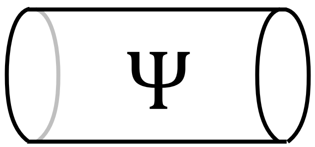
Y. Zhang, T. Grover, A. Turner, M. Oshikawa, A. Vishwanath, PRB 2012

# Background

Infinite cylinder


Finite torus




ground state
on finite cylinder

## 2D DMRG

S. Yan, D. A. Huse, S. R. White, Science 2011

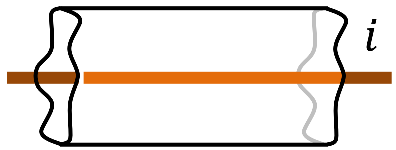
$$D = \sqrt{\sum_i (d_i)^2}$$

H.-C. Jiang, H. Yao, L. Balents, PRB 2012
H.-C. Jiang, Z. Wang, L. Balents, arXiv:1205.4289
S. Depenbrock, I. P. McCulloch, U. Schollwoeck, PRL 2012

OUTLINE

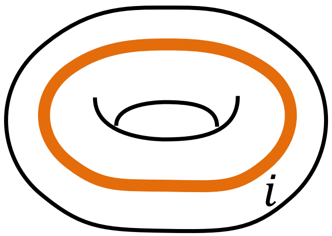
## 1) GROUND STATES

Infinite cylinder




- edge spectrum
  - quantum dimensions
  - chiral CFT

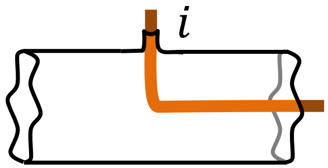
Finite torus




- S matrix
  - mutual statistics
  - quantum dimensions
  - fusion rules
- U matrix
  - central charge
  - topological spins

## 2) QUASIPARTICLE EXCITATIONS

Infinite cylinder

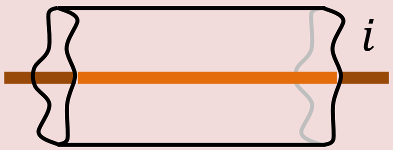



- integer excitations
- fractionalized excitations

# OUTLINE

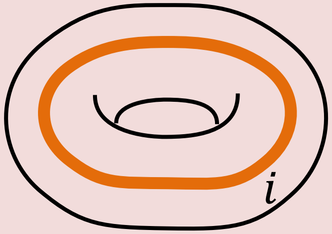
## 1) GROUND STATES

Infinite cylinder




- edge spectrum
  - quantum dimensions
  - chiral CFT

Finite torus




- S matrix
  - mutual statistics
  - quantum dimensions
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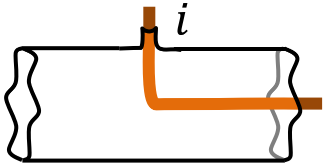
  - central charge
  - topological spins

## 2) QUASIPARTICLE EXCITATIONS

Infinite cylinder

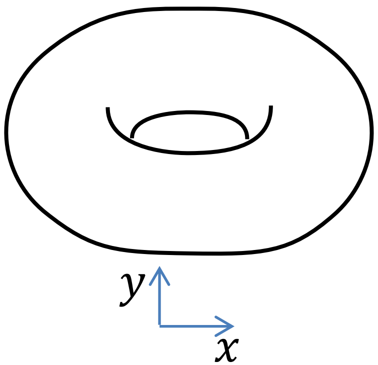



- integer excitations
- fractionalized excitations

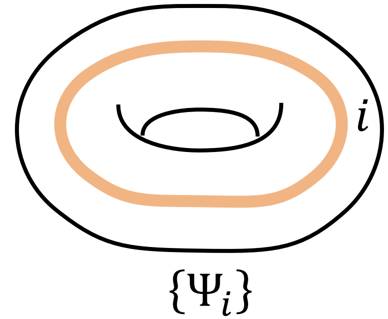
- complete set of ground states of a lattice Hamiltonian $H$

X.-G. Wen, 1989

A) on a torus

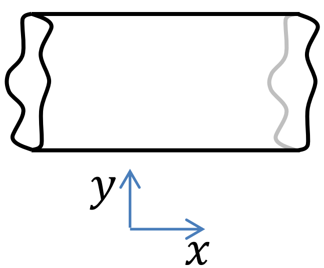



$$L_x \gg L_y \gg \xi$$

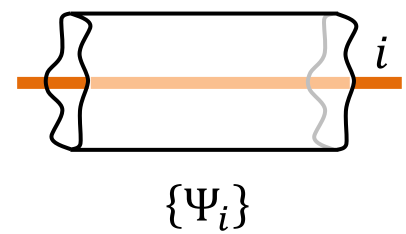



fact: each ground state has a well-defined anyon flux in x-direction

B) on an infinite cylinder




$$L_x = \infty; \;\; L_y \gg \xi$$

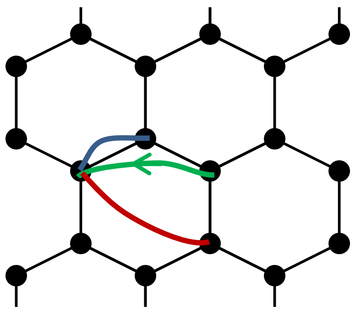



claim: each ‘ground state’ has a well-defined anyon flux in x-direction

# LATTICE MODELS

## Haldane
(hardcore bosonson honeycomb)

F.D.M. Haldane, PRL 1988



$t = 1$

$t' = 0.6$

$\phi = 0.4\pi$

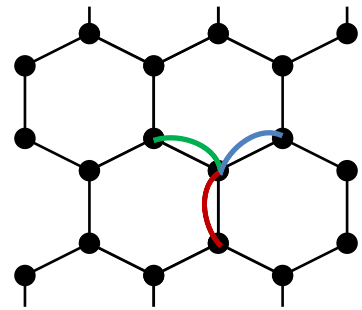
$t'' = -0.58$

$$H = -t \sum_{\langle rr' \rangle} b_r{}^{\dagger} b_{r'} - t' \sum_{\langle\langle rr' \rangle\rangle} b_r{}^{\dagger} b_{r'} e^{i\phi_{rr'}} - t'' \sum_{\langle\langle\langle rr' \rangle\rangle\rangle} b_r{}^{\dagger} b_{r'}$$

Y.-F. Wang, Z.-C. Gu, C.-D. Gong, D.N. Sheng, PRL 2011

## Kitaev Honeycomb
(non-Abelian phase with magnetic field)

A. Kitaev , Annals of Physics 2006

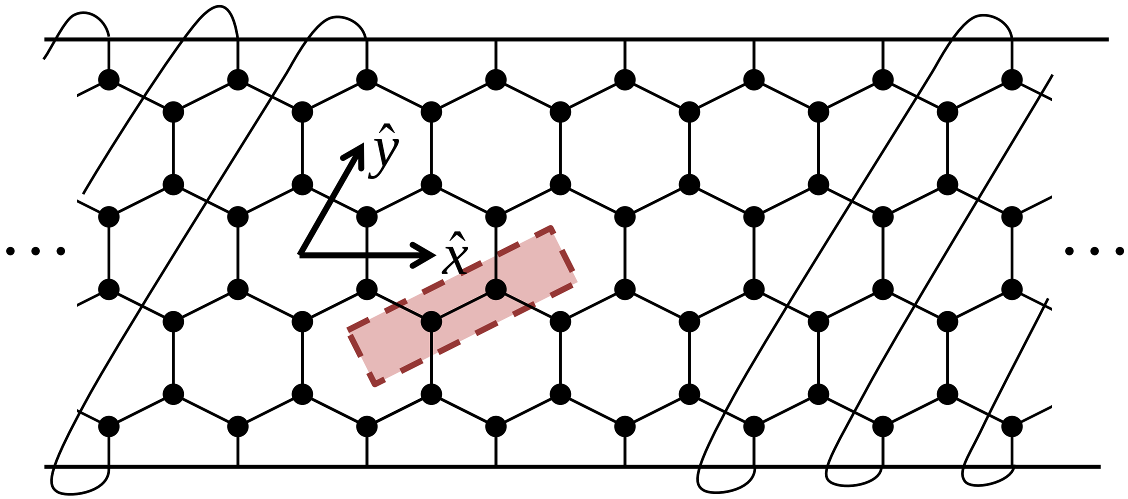


$h$=0.01

$$H = \sum_{\langle rr' \rangle_x} \sigma^x{}_r \sigma^x{}_{r'} + \sum_{\langle rr' \rangle_y} \sigma^y{}_r \sigma^y{}_{r'} + \sum_{\langle rr' \rangle_z} \sigma^z{}_r \sigma^z{}_{r'}$$

$$+h \sum_r (\sigma^X{}_r + \sigma^Y{}_r + \sigma^z{}_r)$$

# VARIATIONAL WAVEFUNCTION




$L_y = 4$

$(XC8-4)$?

$L_x = \infty$

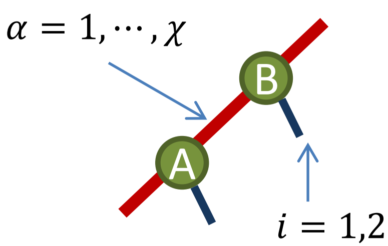
## MPS / 2D DMRG

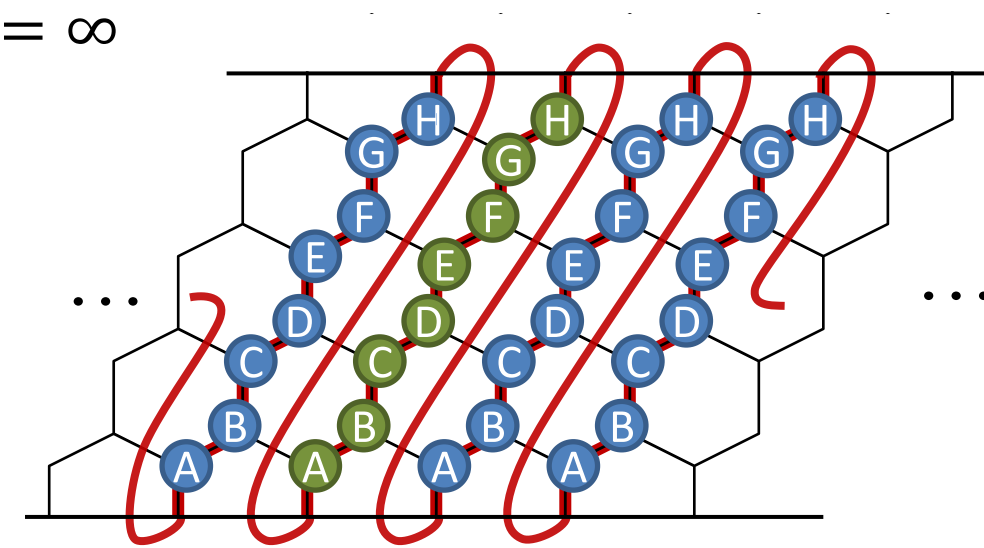
(Matrix Product State)

S. White, PRL 1992

S. Yan, D. A. Huse, S. R. White, Science 2011

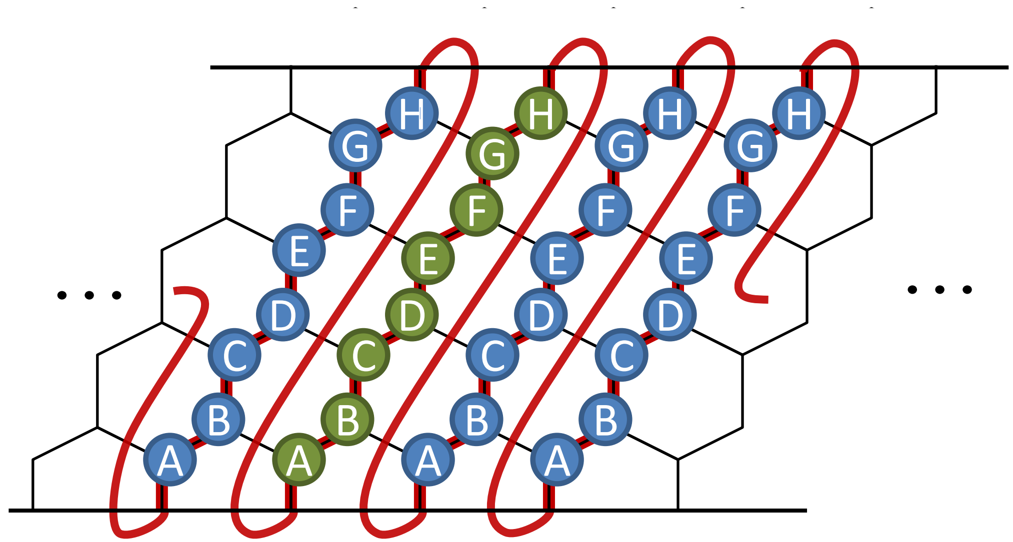
H.-C. Jiang, H. Yao, L. Balents, PRB 2012



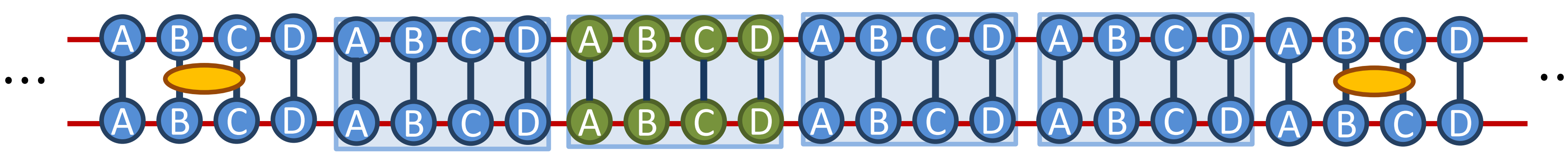



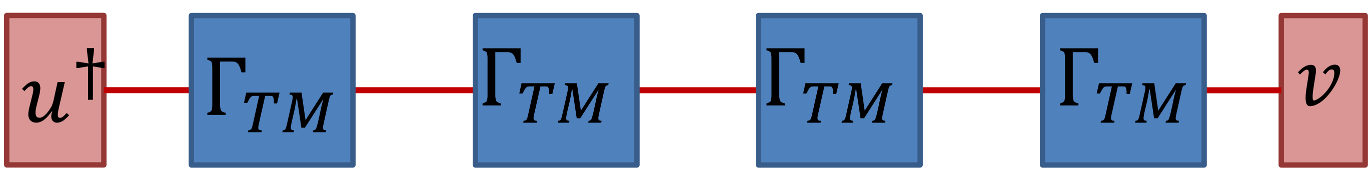

Computational cost $O(\chi^3 = e^{L_y})$ $L_y \gg \xi$

# CORRELATION LENGTH




$$\langle\Psi|o(0,0)o(x,y)|\Psi\rangle =$$



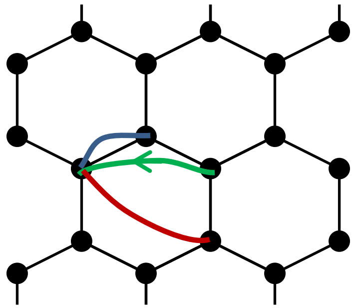




$$\approx \lambda^x = e^{-x/\xi_{TM}}$$

$$\xi_{TM} \stackrel{\text{def}}{=} -\frac{1}{\log(\lambda)}$$

## Haldane model (hardcore bosons)

F.D.M. Haldane, PRL 1988



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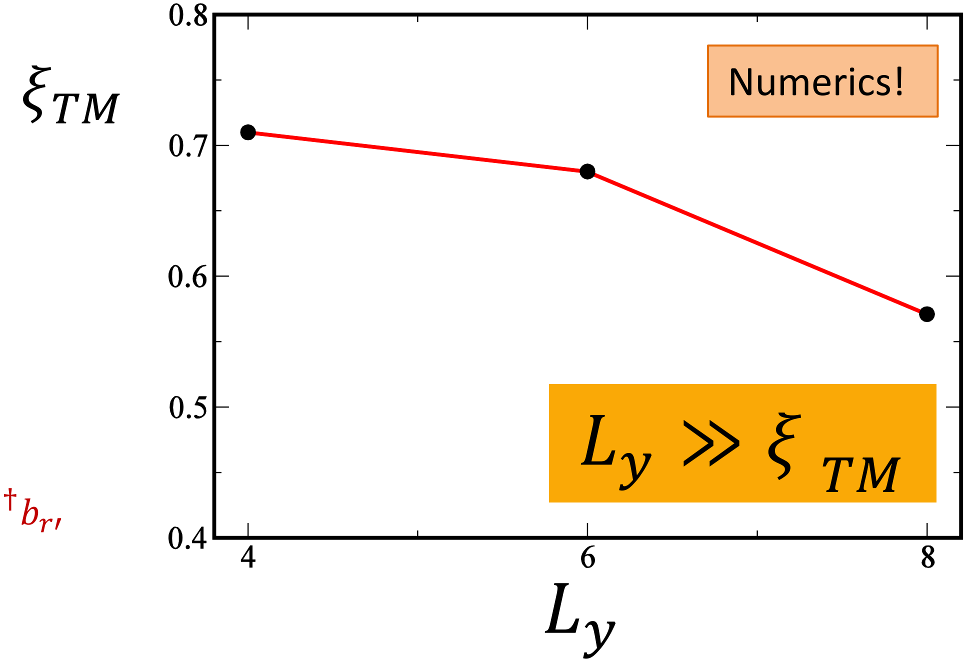
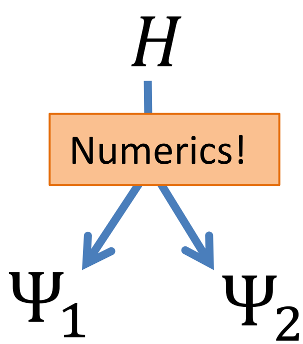
Y.-F. Wang, Z.-C. Gu, C.-D. Gong, D.N. Sheng, PRL 2011

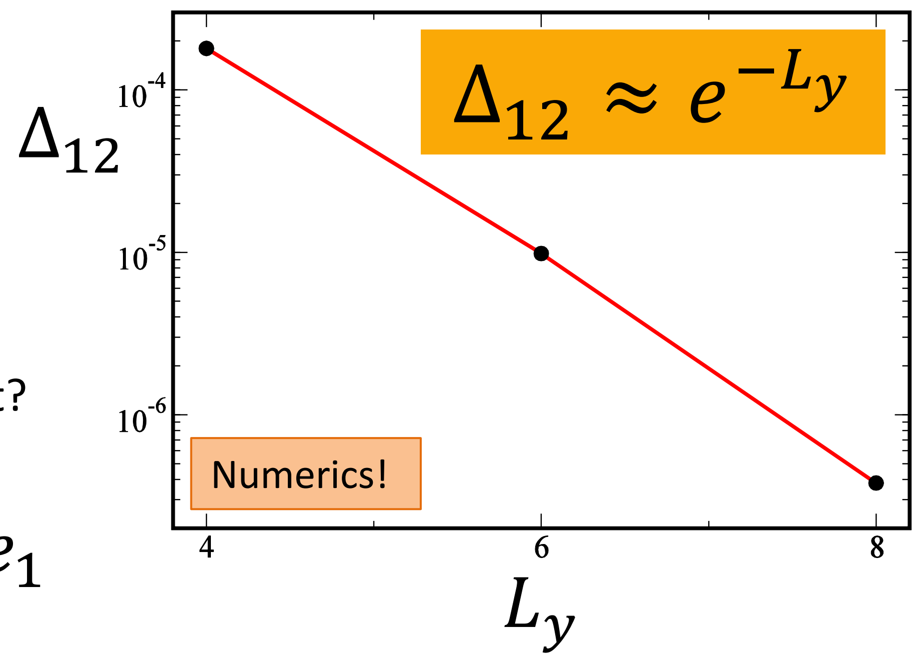



We find 2 'ground states':



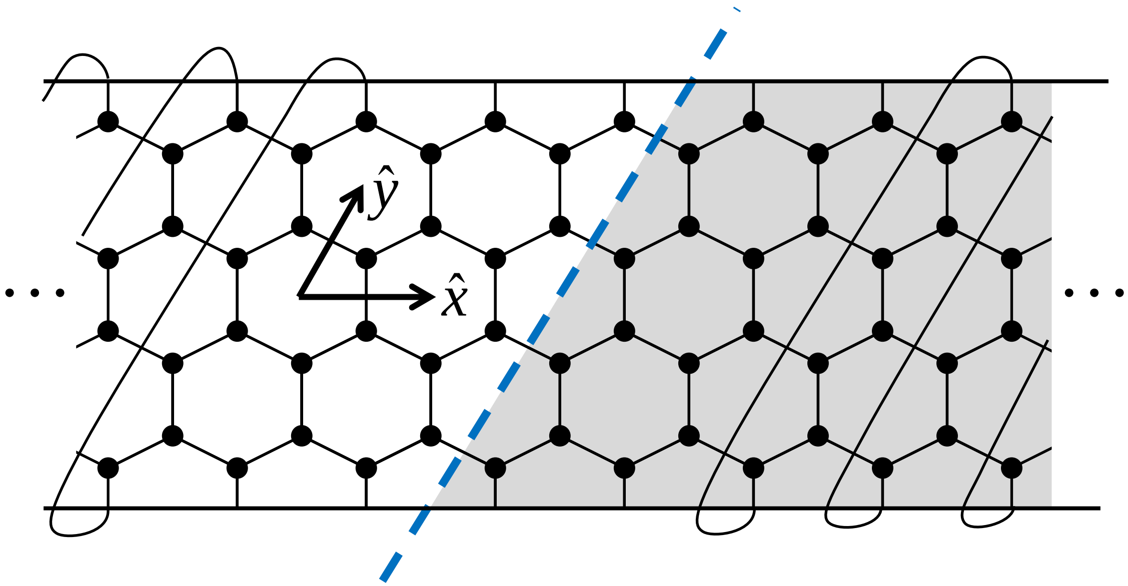

complete set?

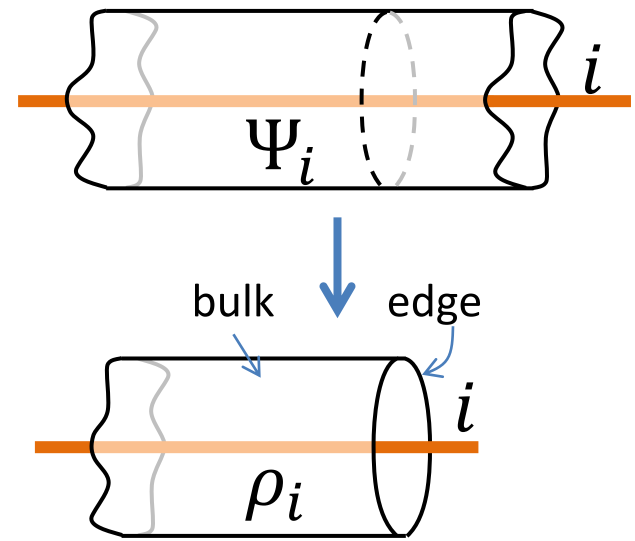
$$e_i \stackrel{\text{def}}{=} \frac{E_i}{L_x L_y} \qquad \Delta_{12} \stackrel{\text{def}}{=} e_2 - e_1$$



Haldane model
(hardcore bosons)

# ENTANGLEMENT SPECTRUM (I)




$$H \rightarrow \Psi_1 \rightarrow \rho_1 \rightarrow \{p_{1,\alpha}\}$$

$$H \rightarrow \Psi_2 \rightarrow \rho_2 \rightarrow \{p_{2,\alpha}\}$$

'ground states'
infinite cylinder

density matrices
semi-infinite cylinder

spectra

$$\rho_i |p_{i,\alpha}\rangle = p_{i,\alpha} |p_{i,\alpha}\rangle$$

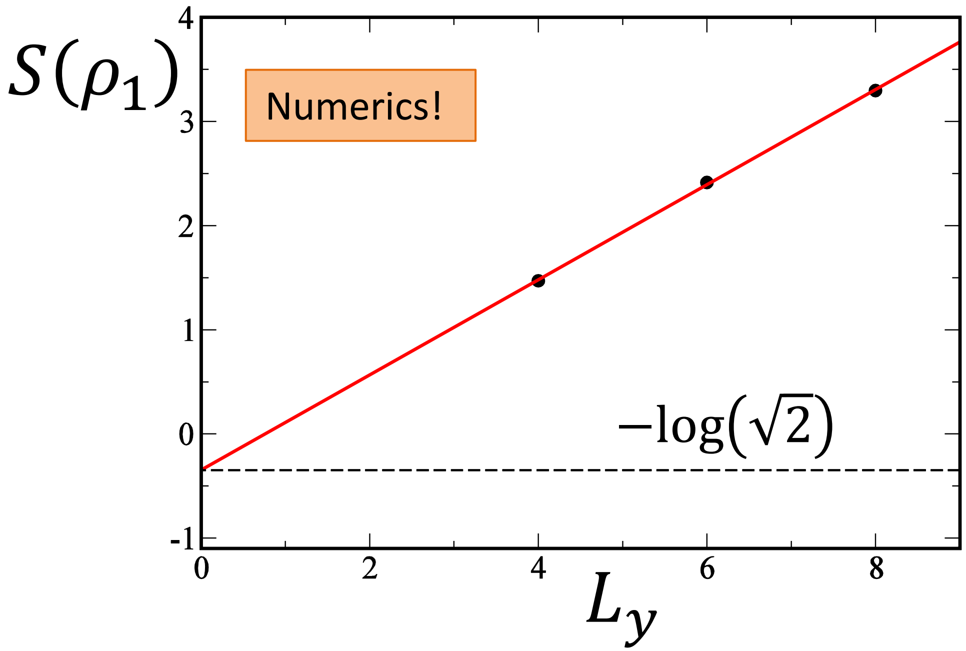
Haldane model
(hardcore bosons)

$\{p_{1,\alpha}\}, \{p_{2,\alpha}\} \longrightarrow S(\rho_1), S(\rho_2)$

spectrum

Scaling of entanglement entropy

A. Kitaev, J. Preskill, PRL 2006
M. Levin, X.-G. Wen, PRL 2006




Region with flux $i$

$$S_L = aL - \log\left(\frac{D}{d_i}\right)$$

*For one ground state in large finite cylinder, H.-C. Jiang, H. Yao, L. Balents, PRB 2012,
H.-C. Jiang, Z. Wang, L. Balents, arXiv:1205.4289

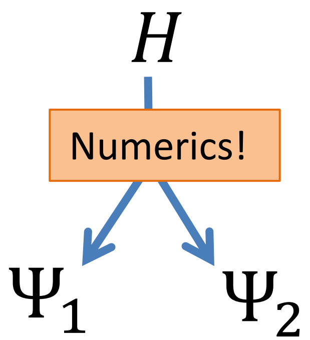
Numerics!

$$\frac{d_1}{D} = 0.7079 \approx \frac{1}{\sqrt{2}} \ (0.1\%)$$

$$S(\rho_1) - S(\rho_2) = \log\left(\frac{d_1}{d_2}\right)$$

Numerics! $d_1/d_2 = 1.005$

We found 2 'ground states':




complete set?

$$D \stackrel{\text{def}}{=} \sqrt{\sum_i (d_i)^2}$$

$$\Downarrow$$

$$\sum_i \left(\frac{d_i}{D}\right)^2 = 1$$

Numerics!

$$\sum_i \left(\frac{d_i}{D}\right)^2 = 1.007$$

$\Longrightarrow$ complete set

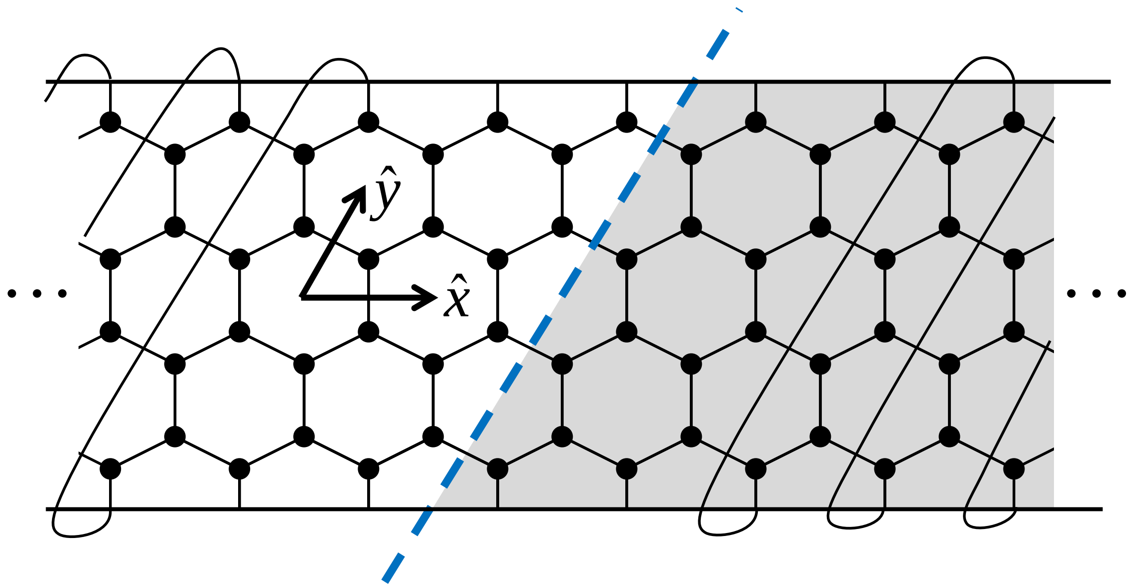
Any anyon model has identity $i = \mathbb{I}$, with quantum dimension $d_{\mathbb{I}} = 1$

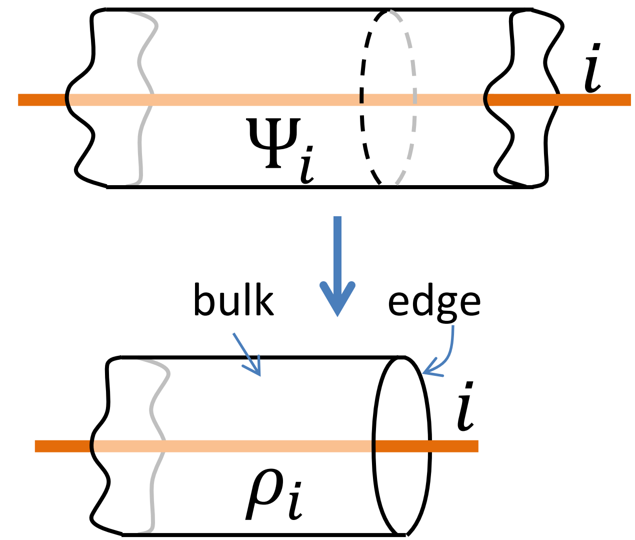
$$d_1 = 1, \quad \Longrightarrow \quad d_2 = 1.005 \approx 1, \; D = 1.413 \approx \sqrt{2}\,(0.1\%),$$

Numerics!

Haldane model
(hardcore bosons)

# ENTANGLEMENT SPECTRUM (II)




$$H \nearrow \Psi_1 \rightarrow \rho_1 \rightarrow \{p_{1,\alpha}; k_{1,\alpha}\}$$

$$H \searrow \Psi_2 \rightarrow \rho_2 \rightarrow \{p_{2,\alpha}; k_{2,\alpha}\}$$

'ground states'
infinite cylinder

density matrices
semi-infinite cylinder

spectra

entanglement energies

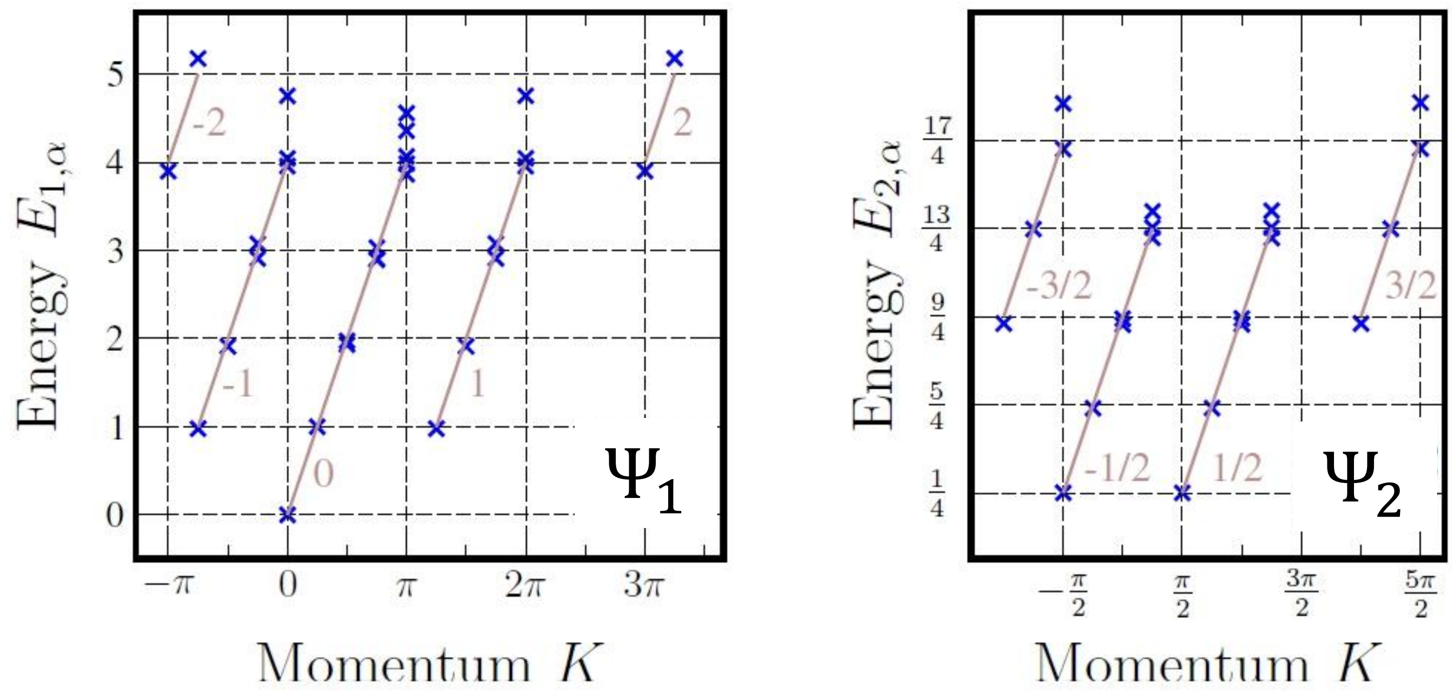
$$\rho_i|p_{i,\alpha}; k_{i,\alpha}\rangle = p_{i,\alpha}|p_{i,\alpha}; k_{i,\alpha}\rangle$$

$$E_i \stackrel{\text{def}}{=} -\log(p_{i,\alpha})$$

$$T_{y1}|p_{i,\alpha}; k_{i,\alpha}\rangle = e^{-i\frac{2\pi}{L_y}k_{i,\alpha}}|p_{i,\alpha}; k_{i,\alpha}\rangle$$

momentum in $y$-direction

H. Li, F. D. M. Haldane, PRL 2008
X.-L. Qi, H. Katsura, A. W. W. Ludwig, PRL 2012




- Spectrum organized as multiplets of emergent SU(2) [lattice model is only U(1) symmetric]

$\Psi_1$ $m_z = \cdots -2, -1, 0, 1, 2 \cdots$

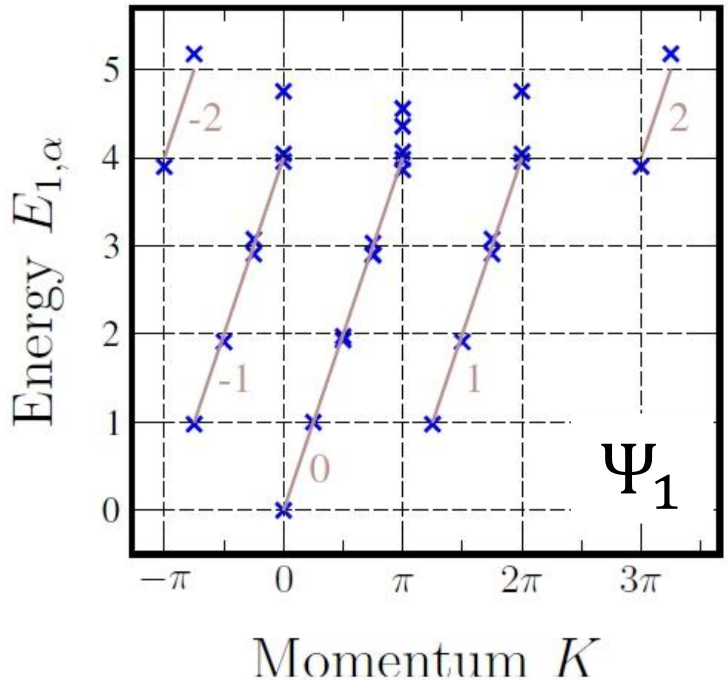
integer irreps $s = 0,1,2,\cdots$

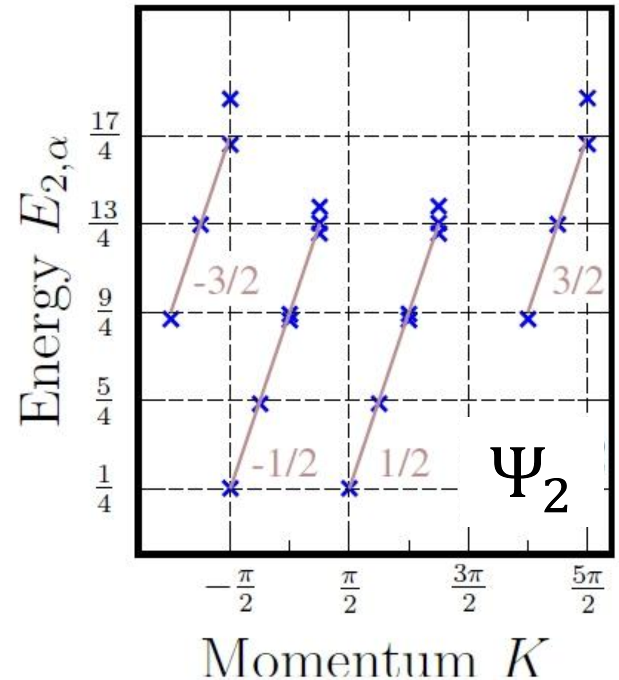
$\Psi_2$ $m_z = \cdots \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{3}{2}, \cdots$

integer irreps $s = 0,1,2,\cdots$

- Degeneracy pattern: $\{1,1,2,3,5,\cdots\}$

Xiao-Gang: “bosonic Gaussian theory”

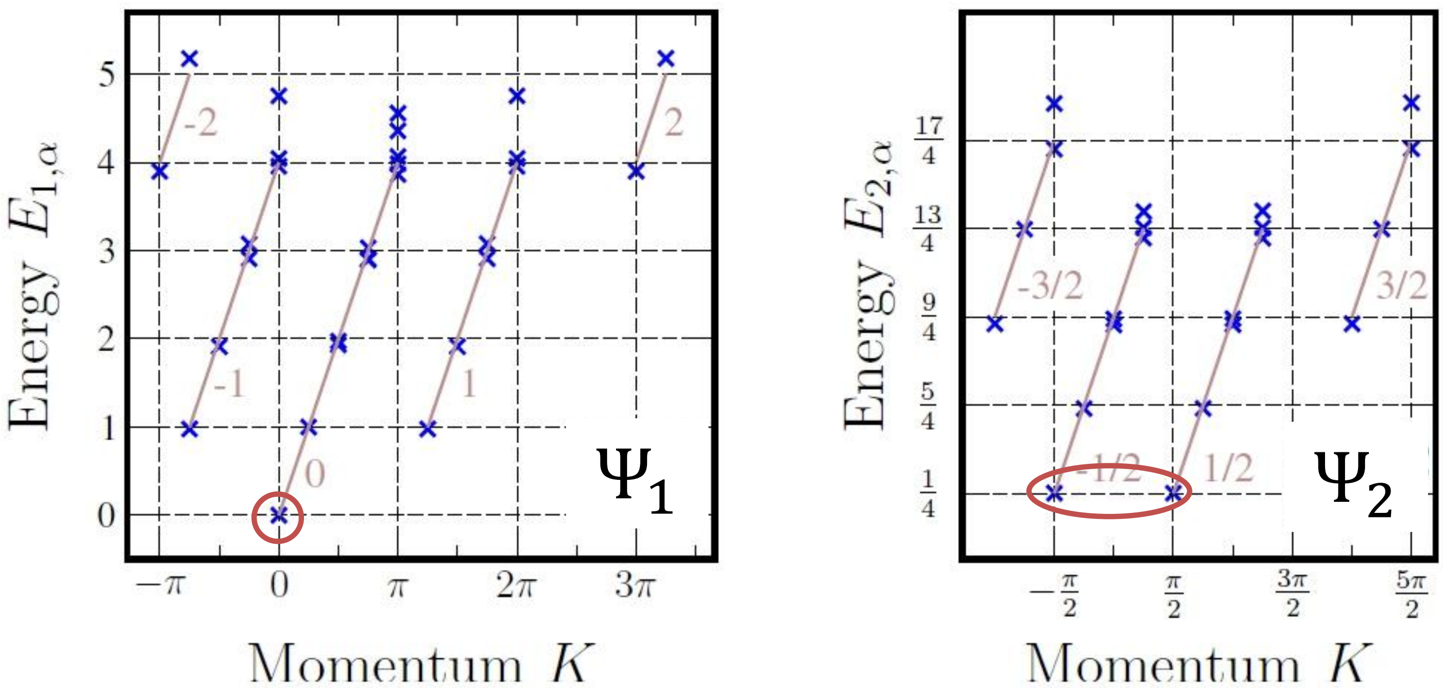



| $L_0$ | $m$: −2 | −1 | 0 | 1 | 2 | $su(2)$ decomposition |
|---|---|---|---|---|---|---|
| 0 | | | 1 | | | (0) |
| 1 | | 1 | 1 | 1 | | (2) |
| 2 | | 1 | 2 | 1 | | (2)+(0) |
| 3 | | 2 | 3 | 2 | | 2(2)+(0) |
| 4 | 1 | 3 | 5 | 3 | 1 | (4)+2(2)+2(0) |
| 5 | 1 | 5 | 7 | 5 | 1 | (4)+4(2)+2(0) |
| 6 | 2 | 7 | 11 | 7 | 2 | 2(4)+5(2)+4(0) |

| $L_0$ | $m$: −2 | −1 | 0 | 1 | 2 | 3 | $su(2)$ decomposition |
|---|---|---|---|---|---|---|---|
| $\frac{1}{4}$ | | | 1 | 1 | | | (1) |
| $\frac{5}{4}$ | | | 1 | 1 | | | (1) |
| $\frac{9}{4}$ | | 1 | 2 | 2 | 1 | | (3)+(1) |
| $\frac{13}{4}$ | | 1 | 3 | 3 | 1 | | (3)+2(1) |
| $\frac{17}{4}$ | | 2 | 5 | 5 | 2 | | 2(3)+3(1) |
| $\frac{21}{4}$ | | 3 | 7 | 7 | 3 | | 3(3)+4(1) |
| $\frac{25}{4}$ | 1 | 5 | 11 | 11 | 5 | 1 | (5)+4(3)+6(1) |

P. Di Francesco, P. Mathieu, D. Senechal, *Conformal Field Theory*, 1997

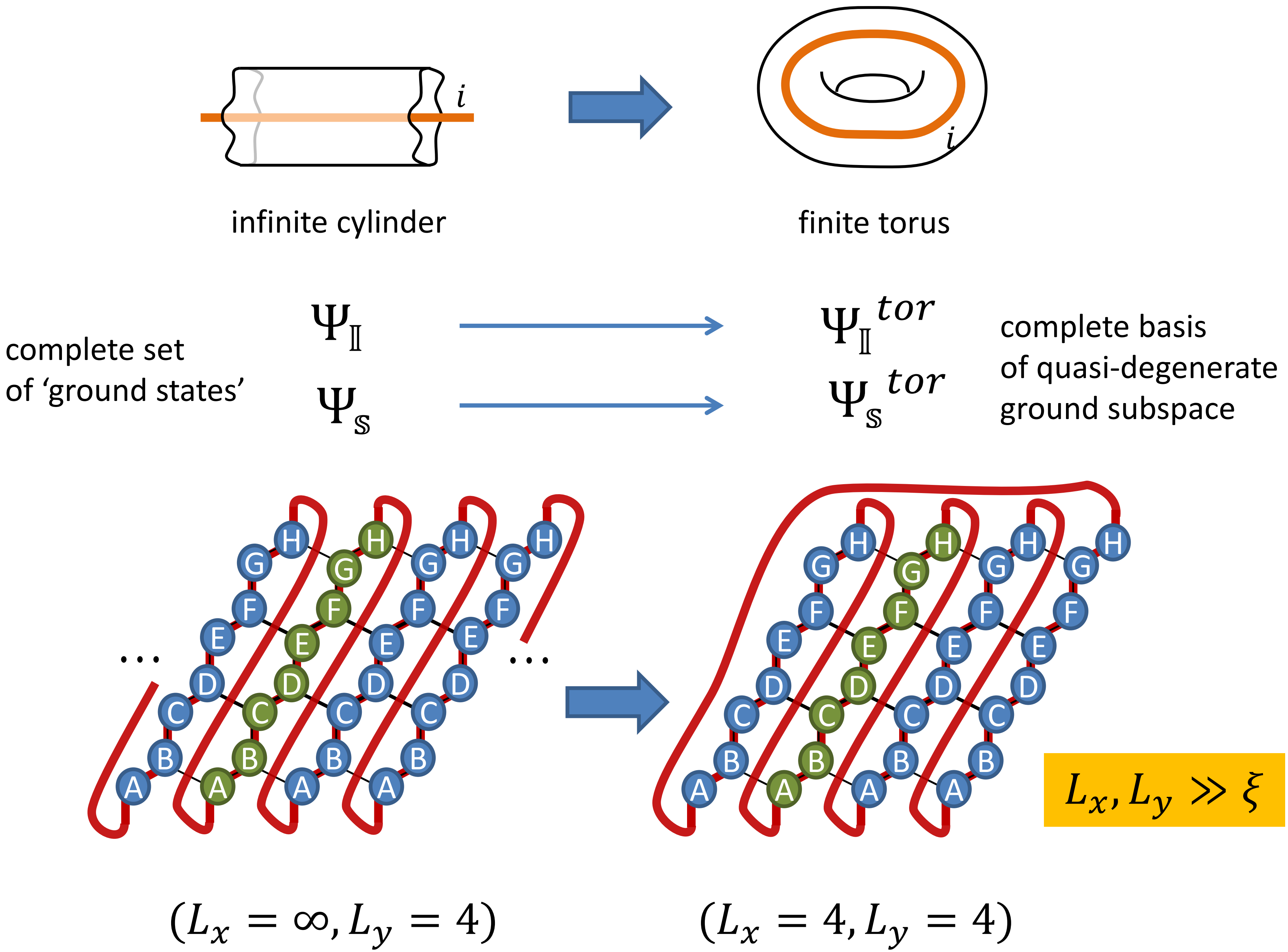



chiral $SU(2)_1$ Wess-Zumino-Witten CFT

$\Psi_i$ primary field + tower of (Virasoro and Kac-Moody) descendants

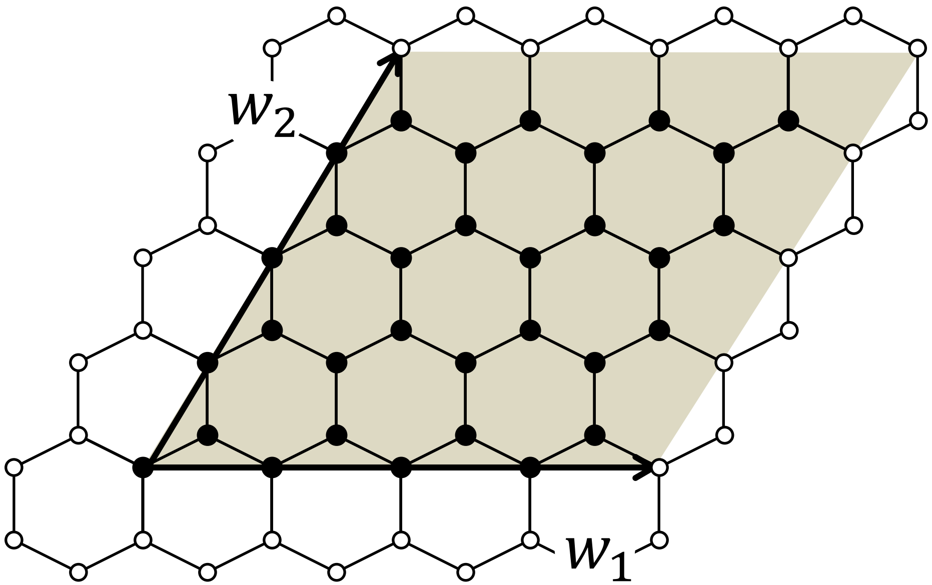
$\Psi_1$ identity $I$, $SU(2)$ singlet → $\Psi_{\mathbb{I}}$ identity

$\Psi_2$ chiral vertex operator $e^{i\varphi/\sqrt{2}}$, $SU(2)$ doublet → $\Psi_{\mathbb{S}}$ semion



i
infinite cylinder
i
finite torus
complete set
of ‘ground states’
$\Psi_{\mathbb{I}}$
$\Psi_{\mathbb{S}}$
$\Psi_{\mathbb{I}}^{tor}$
$\Psi_{\mathbb{S}}^{tor}$
complete basis
of quasi-degenerate
ground subspace
…
…
$L_x, L_y \gg \xi$
$(L_x = \infty, L_y = 4)$
$(L_x = 4, L_y = 4)$

Y. Zhang, T. Grover, A. Turner, M. Oshikawa, A. Vishwanath, PRB 2012




- torus: two vectors $w_1, w_2$
- modular transformations $SL(2,\mathbb{Z})$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \longrightarrow \begin{bmatrix} w_1{}' \\ w_2{}' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

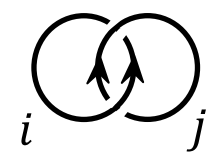
$$a, \mathrm{b}, \mathrm{c}, \mathrm{d} \in \mathbb{Z}; \quad ad - bc = 1$$

- generators

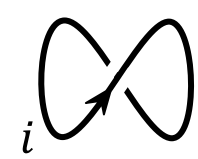
$$\mathcal{s} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \mathcal{u} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

- ground space of $H$ is a representation of the modular group

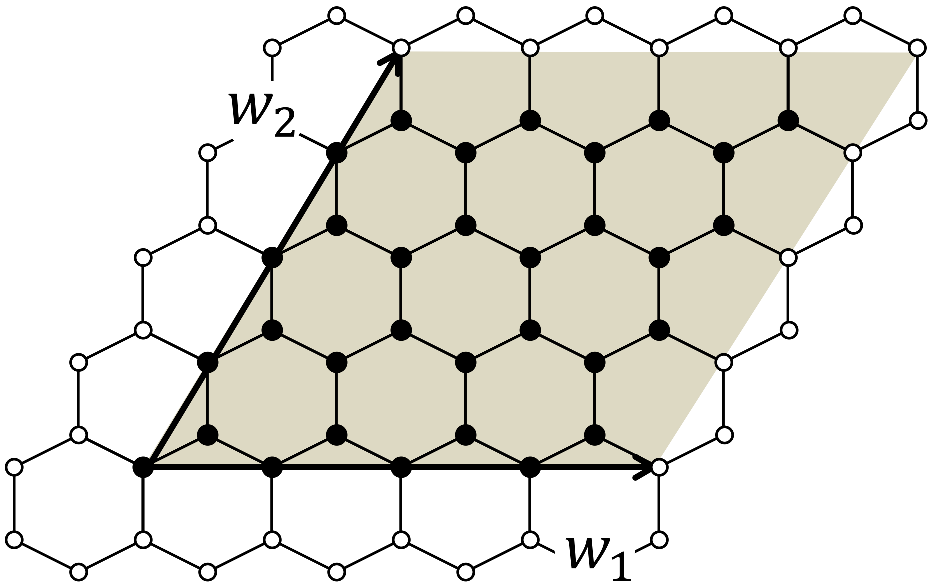
$\mathcal{s} \longrightarrow \mathrm{S}$ topological $S$ matrix $S_{ij} = \frac{1}{D}$




$\mathcal{u} \longrightarrow \mathrm{U}$ topological $U$ matrix $U_{ii} = \frac{1}{d_i}$



Y. Zhang, T. Grover, A. Turner, M. Oshikawa, A. Vishwanath, PRB 2012




- $\pi/_3$ rotation $R_{\pi/_3}$ is a symmetry of $H$ on torus
- it corresponds to $US^{-1}$
- matrix of overlaps

$$V_{ij} = \langle \Psi_i^{tor} | R_{\pi/_3} | \Psi_j^{tor} \rangle$$

$$V = DUS^{-1}D^{\dagger}$$

$$S = \begin{bmatrix} S_{\mathbb{II}} & S_{\mathbb{I}\mathbb{s}} \\ S_{\mathbb{s}\mathbb{I}} & S_{\mathbb{ss}} \end{bmatrix} \qquad U = e^{-i\frac{2\pi}{24}c} \begin{bmatrix} 1 & 0 \\ 0 & \theta_{\mathbb{s}} \end{bmatrix} \qquad D = \begin{bmatrix} e^{i\phi_{\mathbb{I}}} & 0 \\ 0 & e^{i\phi_{\mathbb{s}}} \end{bmatrix}$$

$e^{i\phi_j}$ freedom in defining $\Psi_j^{tor}$

$S_{\mathbb{I}i}, S_{i\mathbb{I}} > 0$

$L_x = L_y = 6$

Numerics!

$6 \times 6 \times 2 = 72$ sites

$$V = e^{-i\frac{2\pi}{24}c} \begin{bmatrix} S_{\mathbb{II}} & S_{\mathbb{I}\mathbb{s}} e^{i(\phi_{\mathbb{s}} - \phi_{\mathbb{I}})} \\ S_{\mathbb{s}\mathbb{I}} e^{i(\phi_{\mathbb{I}} - \phi_{\mathbb{s}})} & \theta_{\mathbb{s}} (S_{\mathbb{ss}})^* \end{bmatrix} = \begin{bmatrix} 0.685 + 0.181i & -0.229 + 0.669i \\ -0.693 - 0.138i & -0.183 + 0.681i \end{bmatrix}$$

$$S = \begin{bmatrix} S_{\mathbb{I}\mathbb{I}} & S_{\mathbb{I}\mathbb{s}} \\ S_{\mathbb{s}\mathbb{I}} & S_{\mathbb{s}\mathbb{s}} \end{bmatrix} \qquad U = e^{-i\frac{2\pi}{24}c} \begin{bmatrix} 1 & 0 \\ 0 & \theta_{\mathbb{s}} \end{bmatrix} \qquad D = \begin{bmatrix} e^{i\phi_{\mathbb{I}}} & 0 \\ 0 & e^{i\phi_{\mathbb{s}}} \end{bmatrix}$$

$$S_{\mathbb{I}i}, S_{i\mathbb{I}} > 0$$

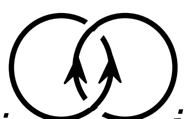
Numerics!

$$V = e^{-i\frac{2\pi}{24}c} \begin{bmatrix} S_{\mathbb{I}\mathbb{I}} & S_{\mathbb{I}\mathbb{s}} e^{i(\phi_{\mathbb{s}} - \phi_{\mathbb{I}})} \\ S_{\mathbb{s}\mathbb{I}} e^{i(\phi_{\mathbb{I}} - \phi_{\mathbb{s}})} & \theta_{\mathbb{s}} (S_{\mathbb{s}\mathbb{s}})^* \end{bmatrix} = \begin{bmatrix} 0.685 + 0.181i & -0.229 + 0.669i \\ -0.693 - 0.138i & -0.183 + 0.681i \end{bmatrix}$$

$$\Downarrow$$

topological $S$ matrix

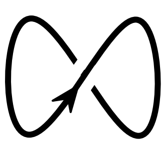
$$S_{ij} = \frac{1}{D}$$



$i$ $j$

topological $U$ matrix

$$U_{ii} = \frac{1}{d_i}$$



$i$

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} + \frac{10^{-3}}{\sqrt{2}} \begin{bmatrix} -1.4 & 0.2 \\ -1.4 & 4 + 4i \end{bmatrix}$$

$$U = e^{-i\frac{2\pi}{24}1} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \times e^{-i\frac{2\pi}{24}0.01} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i0.007} \end{bmatrix}$$

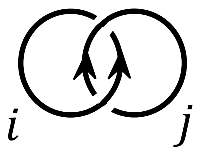
chiral semion

(Monte Carlo statistical error $< 10^{-4}$)

Numerics!

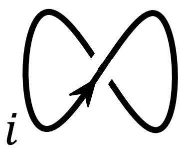
topological $S$ matrix

$$S_{ij} = \frac{1}{D}$$




topological $U$ matrix

$$U_{ii} = \frac{1}{d_i}$$




$$S = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} + \frac{10^{-3}}{\sqrt{2}}\begin{bmatrix} -1.4 & 0.2 \\ -1.4 & 4+4i \end{bmatrix}$$

$$U = e^{-i\frac{2\pi}{24}1}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \times e^{-i\frac{2\pi}{24}0.01}\begin{bmatrix} 1 & 0 \\ 0 & e^{-i0.007} \end{bmatrix}$$

chiral semion

(Monte Carlo statistical error $< 10^{-4}$)

Numerics!

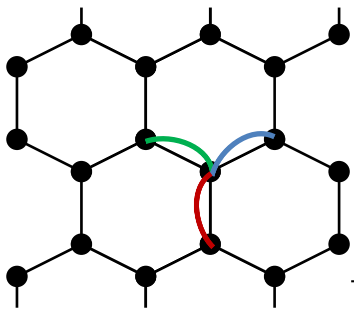
- from topological $S$ matrix
  - quantum dimensions $d_{\mathbb{I}} = d_{\mathbb{s}} = 1$
  - $\mathbb{Z}_2 \times \mathbb{Z}_2$ fusion rules

| | |
|---|---|
| $\mathbb{I} \times \mathbb{I} = \mathbb{I}$ | $\mathbb{I} \times \mathbb{s} = \mathbb{s}$ |
| $\mathbb{s} \times \mathbb{I} = \mathbb{s}$ | $\mathbb{s} \times \mathbb{s} = \mathbb{I}$ |

- from topological $U$ matrix
  - central charge $c = 1$
  - topological spin $\Theta_{\mathbb{s}} = i$ (semion!)

Kitaev Honeycomb
(non-Abelian phase with magnetic field)

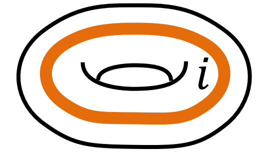
A. Kitaev , Annals of Physics 2006



$h$=0.01

$$+h\sum_r(\sigma^X{}_r+\sigma^Y{}_r+\sigma^z{}_r)$$

$$H=\sum_{\langle rr'\rangle_x}\sigma^x{}_r\sigma^x{}_{r'}+\sum_{\langle rr'\rangle_y}\sigma^y{}_r\sigma^y{}_{r'}+\sum_{\langle rr'\rangle_z}\sigma^z{}_r\sigma^z{}_{r'}$$


$H$ Numerics → $\Psi_1$, $\Psi_2$, $\Psi_3$ → $\Psi_1{}^{tor}$, $\Psi_2{}^{tor}$, $\Psi_3{}^{tor}$

'ground states'
infinite cylinder

'ground states'
finite torus

Numerics!

$L_x = L_y = 4$

$$S=\frac{1}{2}\begin{bmatrix}1.02 & 1.40 & 1.01\\ 1.41 & 0.03 & -1.41\\ 1.04 & -1.36 & 1.04\end{bmatrix}$$

$$\approx\frac{1}{2}\begin{bmatrix}1.00 & 1.41 & 1.00\\ 1.41 & 0.00 & -1.41\\ 1.00 & -1.41 & 1.00\end{bmatrix}$$

$\sqrt{2}\approx 1.41$

5% !

Ising anyon model

$$\begin{matrix} & \mathbb{I} & \sigma & \varepsilon \end{matrix}$$
$$S=\frac{1}{2}\begin{bmatrix}1 & \sqrt{2} & 1\\ \sqrt{2} & 0 & -\sqrt{2}\\ 1 & -\sqrt{2} & 1\end{bmatrix}$$

- quantum dimensions

$d_{\mathbb{I}}=1 \quad d_\sigma=\sqrt{2} \quad d_\varepsilon=1; \quad D=2$

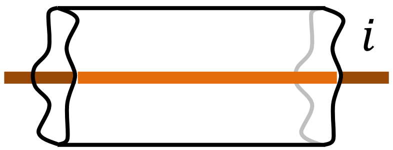
- fusion rules

$\sigma\times\varepsilon=\sigma \quad \sigma\times\sigma=\mathbb{I}+\varepsilon \quad \varepsilon\times\varepsilon=\mathbb{I}$

OUTLINE

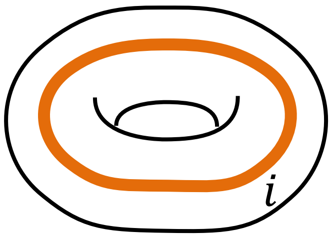
## 1) GROUND STATES

Infinite cylinder




- edge spectrum
  - quantum dimensions
  - chiral CFT

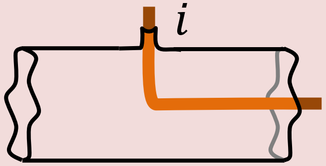
Finite torus




- S matrix
  - mutual statistics
  - quantum dimensions
  - fusion rules
- U matrix
  - central charge
  - topological spins

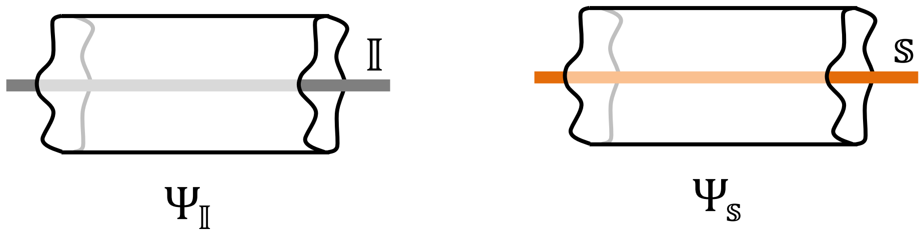
## 2) QUASIPARTICLE EXCITATIONS

Infinite cylinder

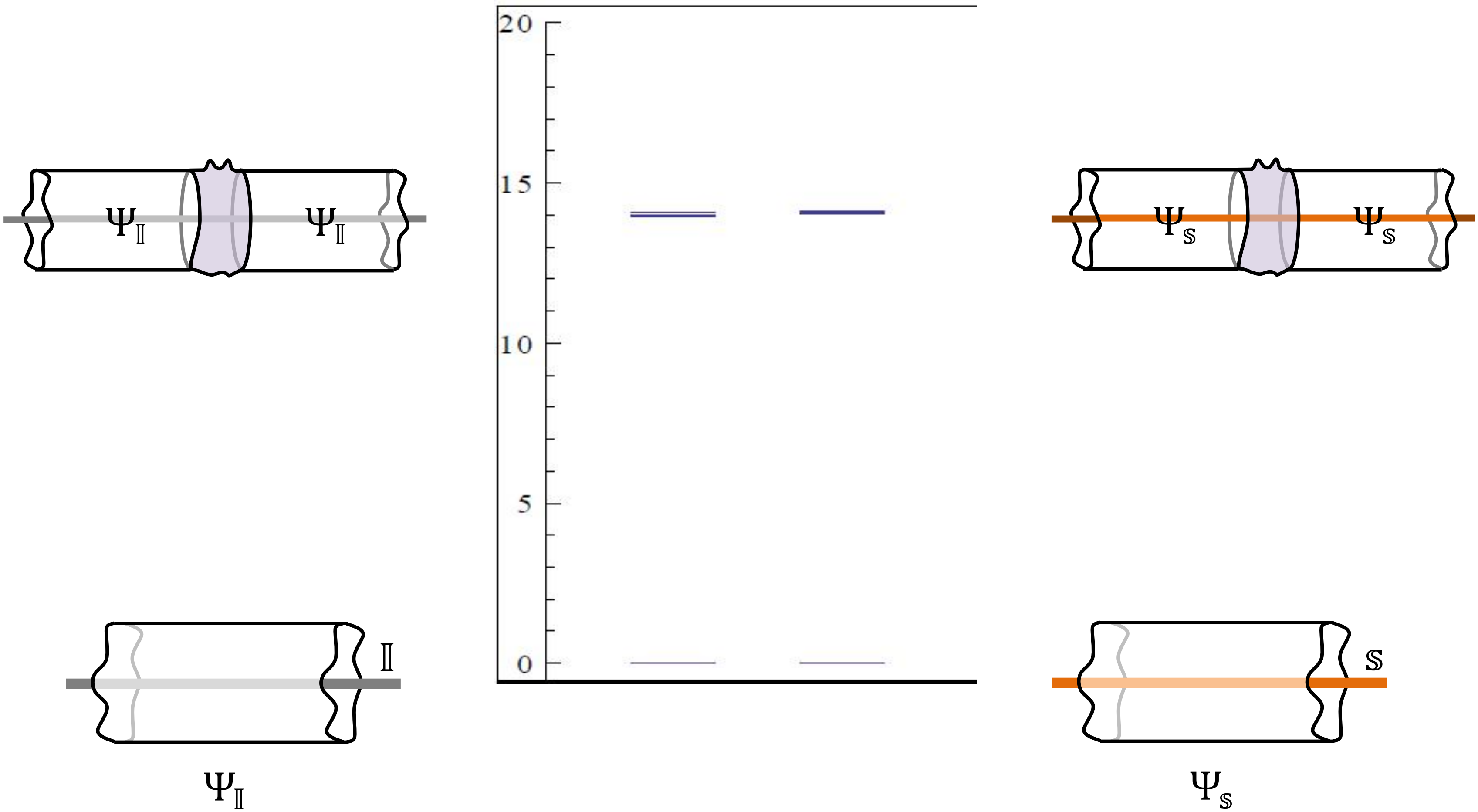



- integer excitations
- fractionalized excitations

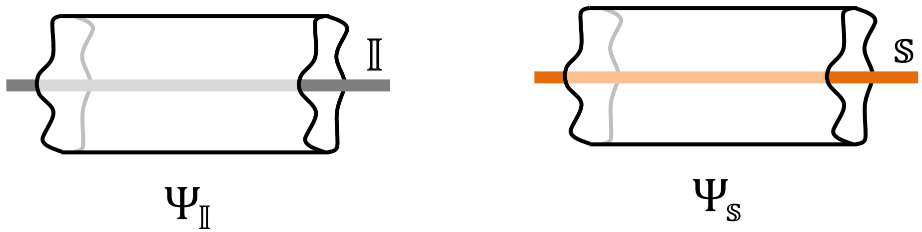
- ground states:



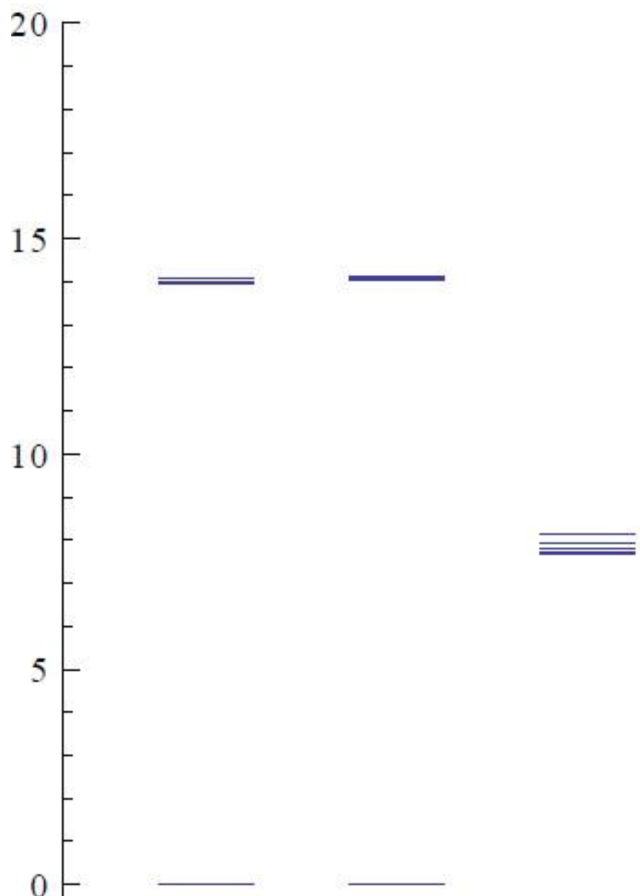

- integer excitations

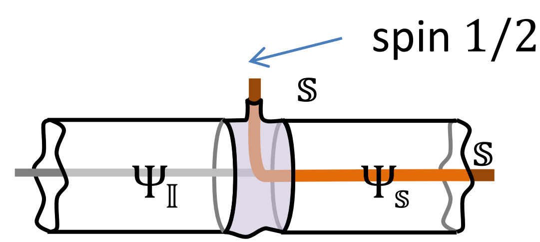


- ground states:




- fractionalized excitations



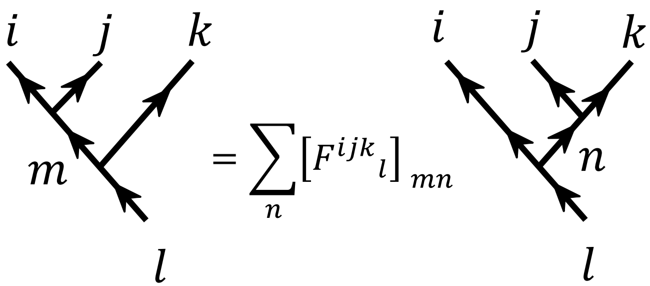



ultimate goal:
complete characterization

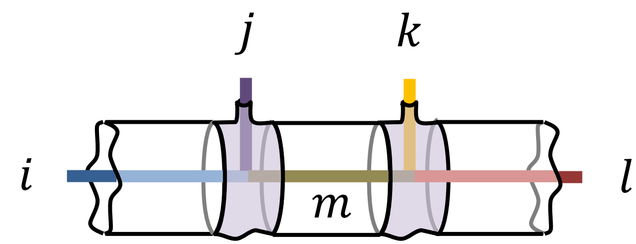
microscopic Hamiltonian

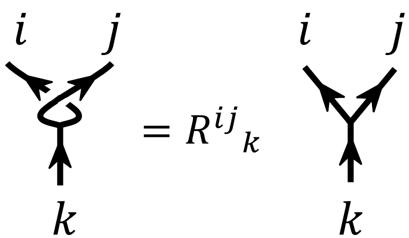
$H$

on infinite cylinder

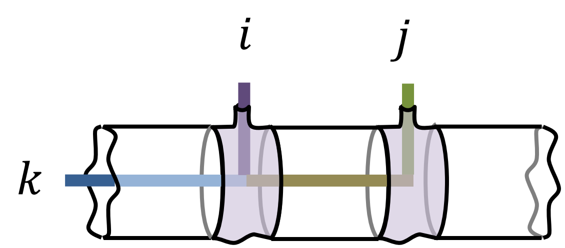



- F – symbols




- R – symbols



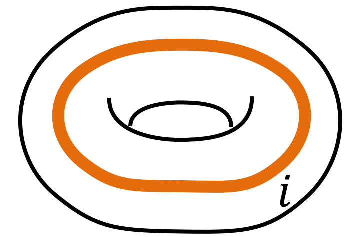
SUMMARY

microscopic Hamiltonian

$H$

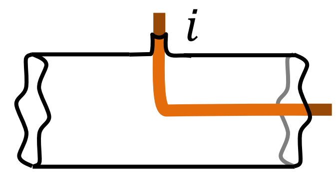
on infinite cylinder


$\{\Psi_i\}$ complete set of 'ground states'




$\{\Psi_i^{tor}\}$ complete basis in quasi-degenerate ground space

- integer excitations
- fractionalized excitations




FAQs:

Q: Why on the cylinder (and not on the torus)?

A1: Cost of DMRG/Tensor networks is much lower
A2: Simpler entanglement spectrum
A3: Single fractionalized excitation

Q: Why on the infinite cylinder (and not on a large cylinder)?

A1: complete set of 'ground states'
A2: translation invariance/unit cell: map to torus

Q: Why DMRG (and not 2D tensor network, e.g. PEPS)?

A: DMRG is better understood and more reliable; but $L_y$ limited. PEPS is next.