Exact Topological Degeneracies on Finite Kagome Clusters

- The current case for Z2
- Topological states on cylinders
- Narrow Connections
- Exact degeneracies

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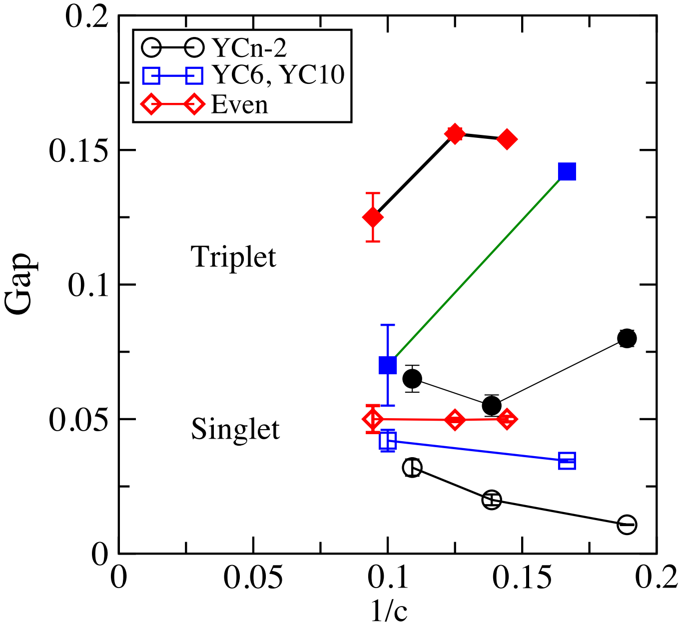
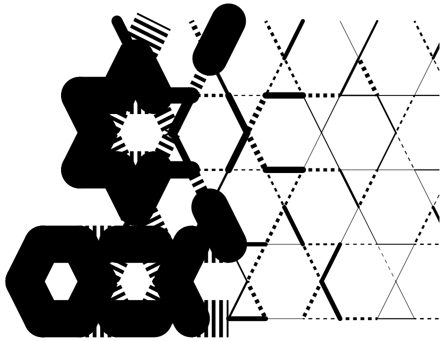
KITP, Oct. 9, 2012 *Spin Liquids Conference*

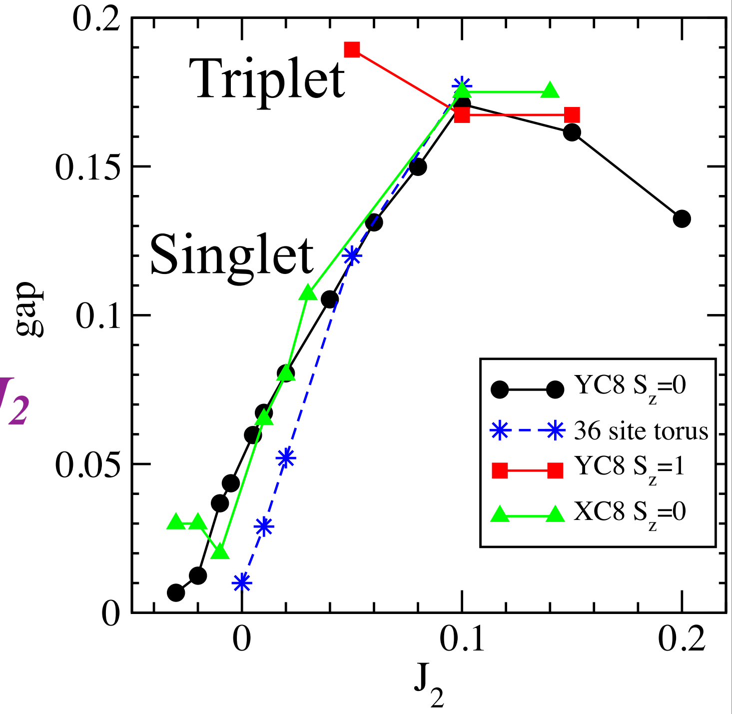
Evidence for Z_2 on the kagome



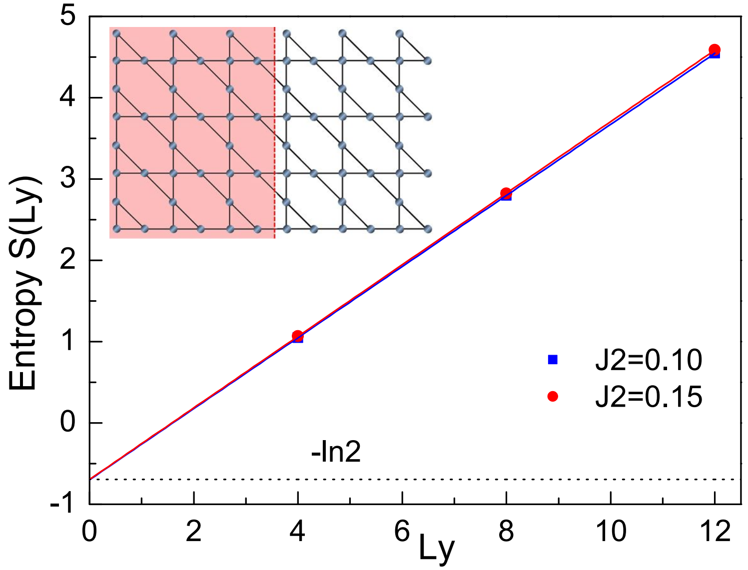
Gaps



Gaps with J_2



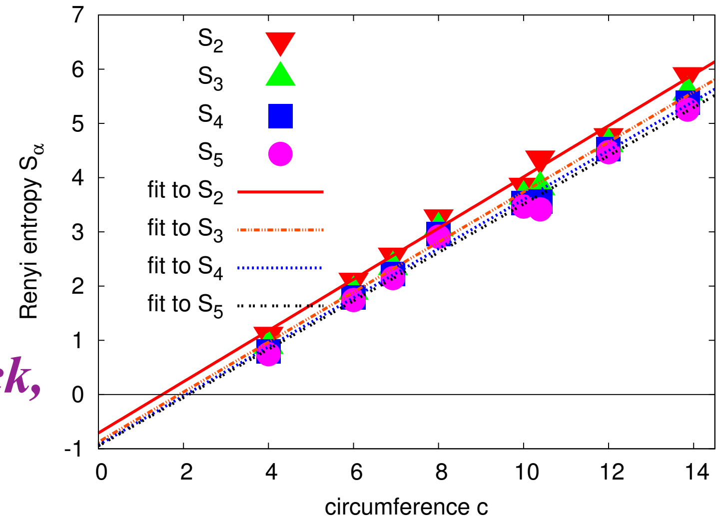
Very short correlation lengths ~ 1



Topological Entanglement Entropy

Jiang, et al

Dependbrock, et al

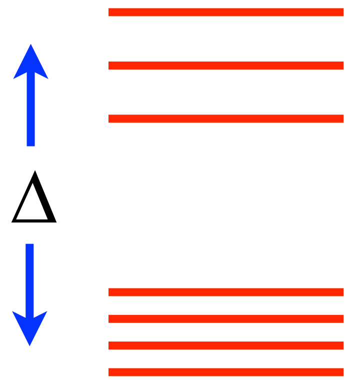


Also: Even/odd cylinder effects (next slides)

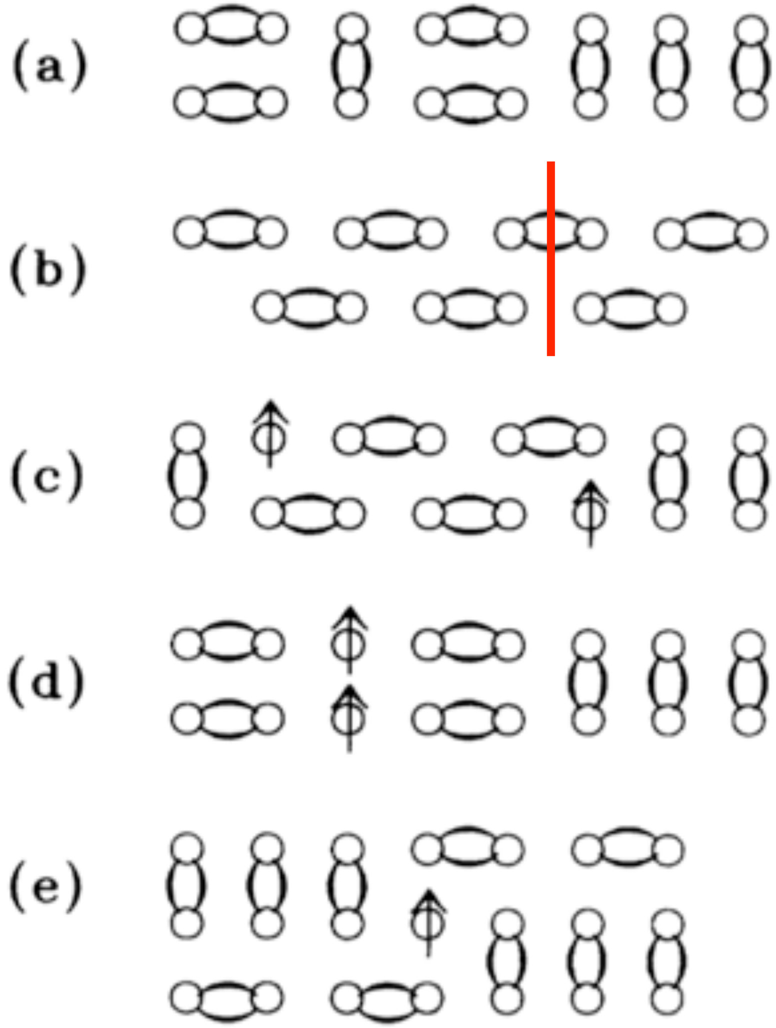
What's been missing: ***Topological Ground State Degeneracies***

Topological Degeneracies

- Degeneracy signatures of a Z_2 spin liquid:
  - Sphere or open disk: no degeneracies
  - Cylinder: 2-fold degeneracy, with gap between states falling exponentially with width
  - Torus (usual periodic BCs): 4-fold degeneracy
  - Degenerate states have identical *local* properties; deviations and splittings fall exponentially with the size
- The torus degeneracy test seems ideal, but for the kagome, no one has pulled it off
  - Finite size effects are large
    - $\Delta E \sim L_x L_y \exp(-L_y/\xi)$ or $\Delta E \sim L_x L_y \exp(-L_x/\xi)$
    - Estimates with RVB/PEPS (Poilblanc talk) need YC20?
    - Requires fully periodic BCs



RVB on Odd vs Even Ladders/Cylinders



Ground state of an even ladder

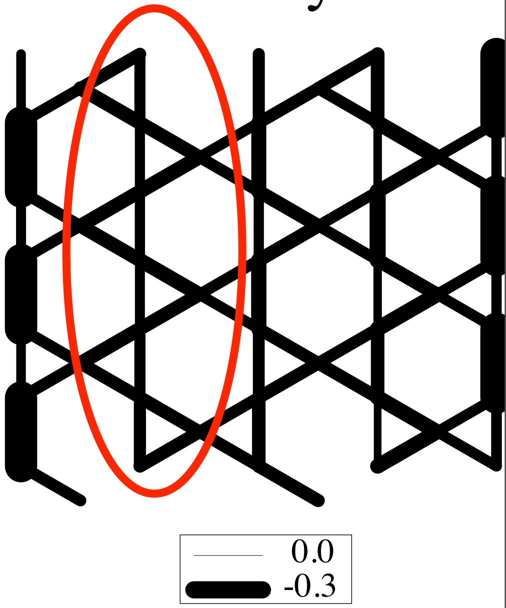
Topologically odd state of an even ladder--the "staggered state"

Two spinons

Bound spinons

Odd ladder

YC6: an odd cylinder

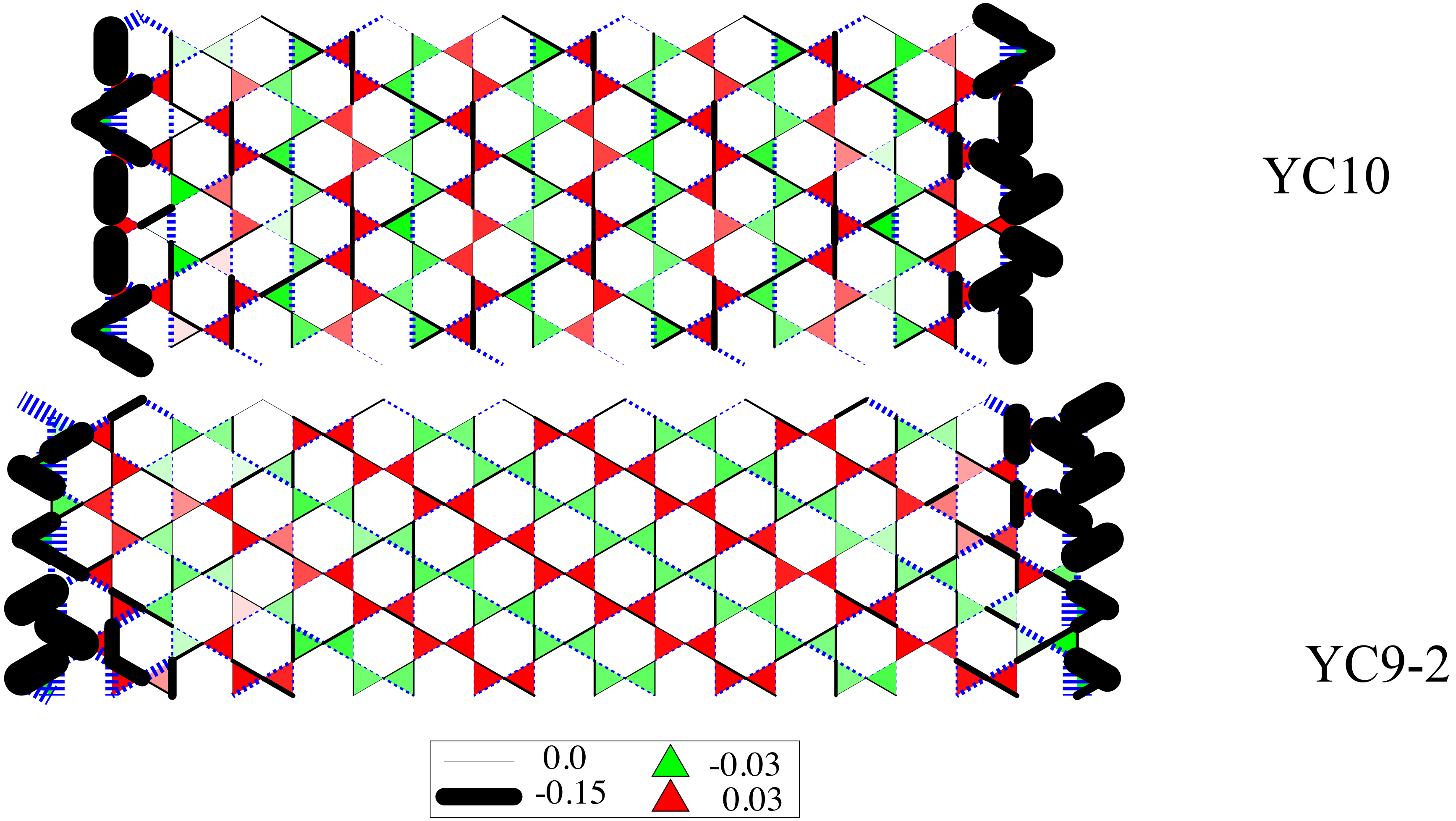


Exchange Energy

White, Noack, Scalapino, PRL 73, 886 (1994).

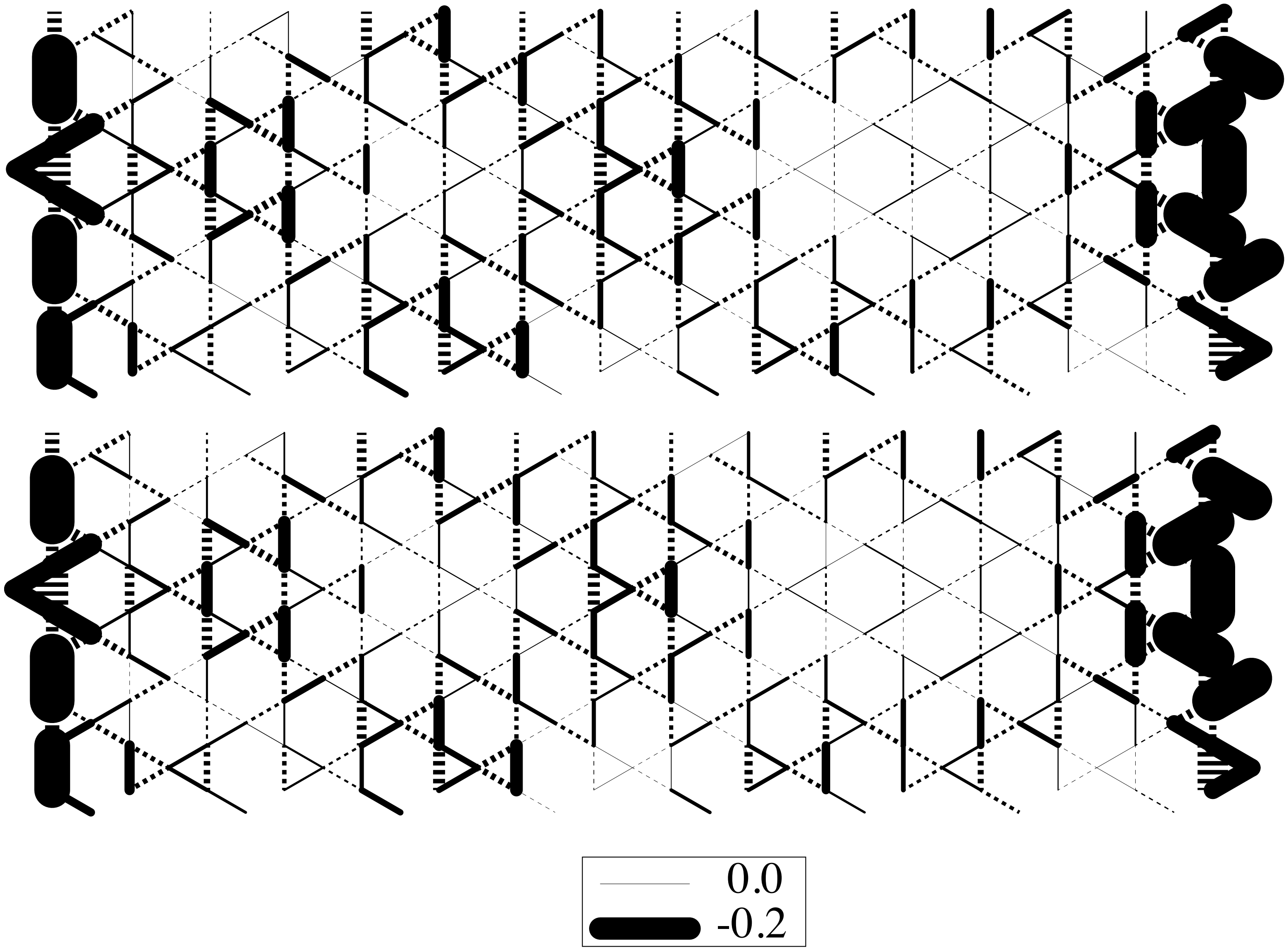
1) Both even and odd cyls have two topo states set by BCs
2) On even cyls, the states are not degenerate except in the limit of wide cylinders
3) Odd cyls have degenerate topo states, each with a "Valence Bond Density Wave"

Odd Cylinders: Bond density wave patterns



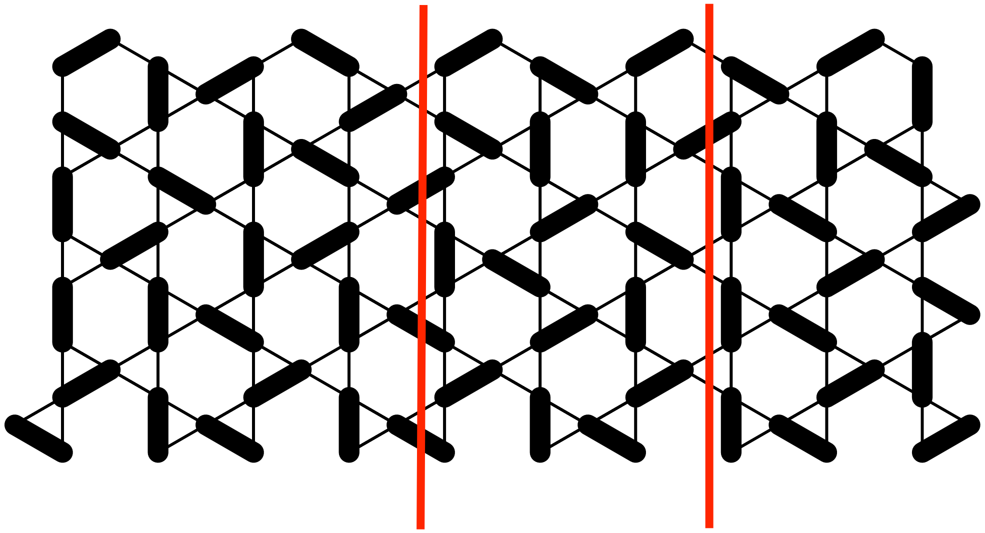
Here a good test of RVB/Z_2 is that the strength of the density wave falls exponentially with the width

The odd topological state on YC8

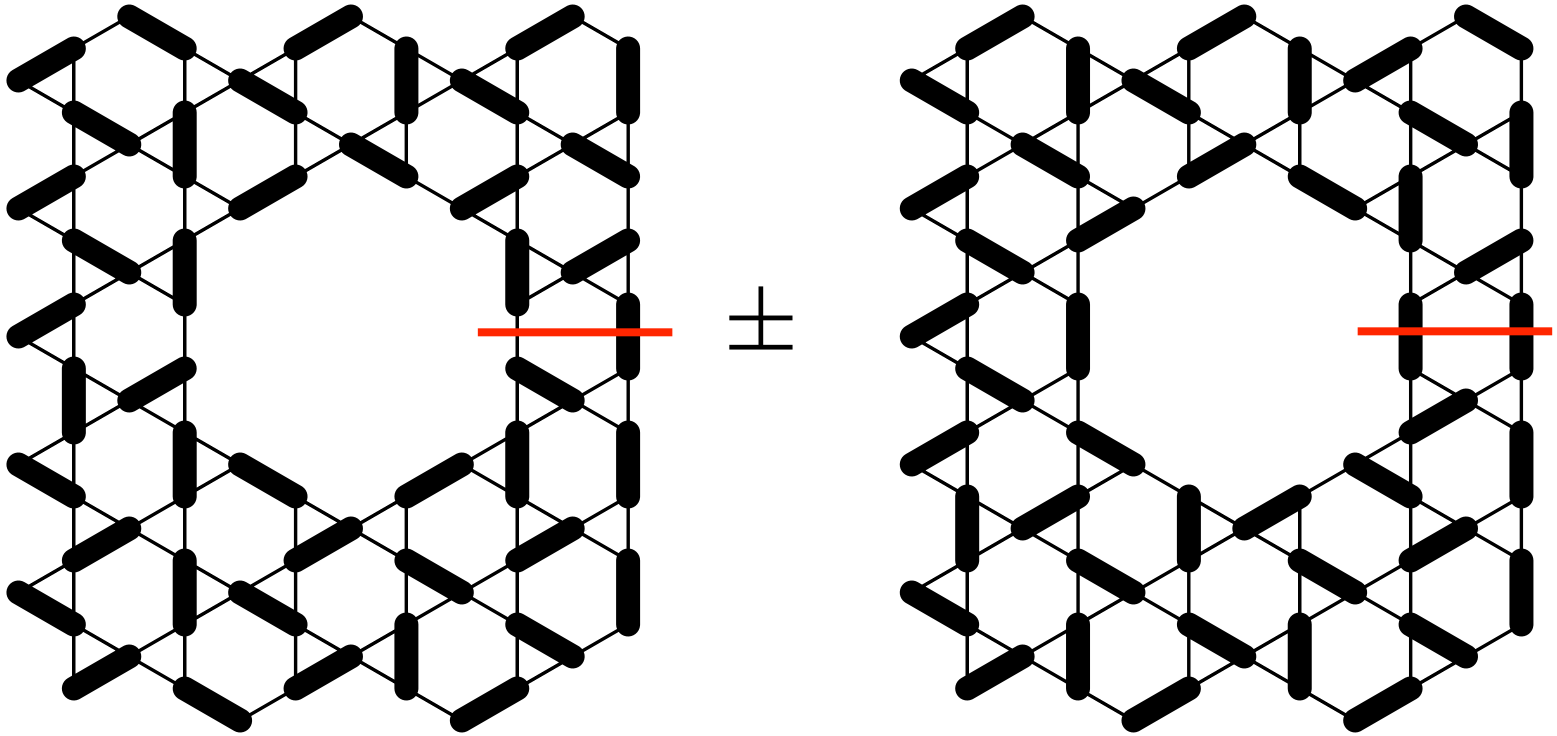


Two completely different runs, different initial states, but the same system, up to m=5000: identical irregular pattern!

This odd topo state is only higher by 0.00069(3) per site. But the small bond pattern is not understood, and the singlet gap for this state is small: ~0.01 for this length (versus 0.05 for the even state)



This constraint (1 or 3 bonds versus 2) is a big effect. Does this lead to surprisingly large finite size effects? (different ξ ?)



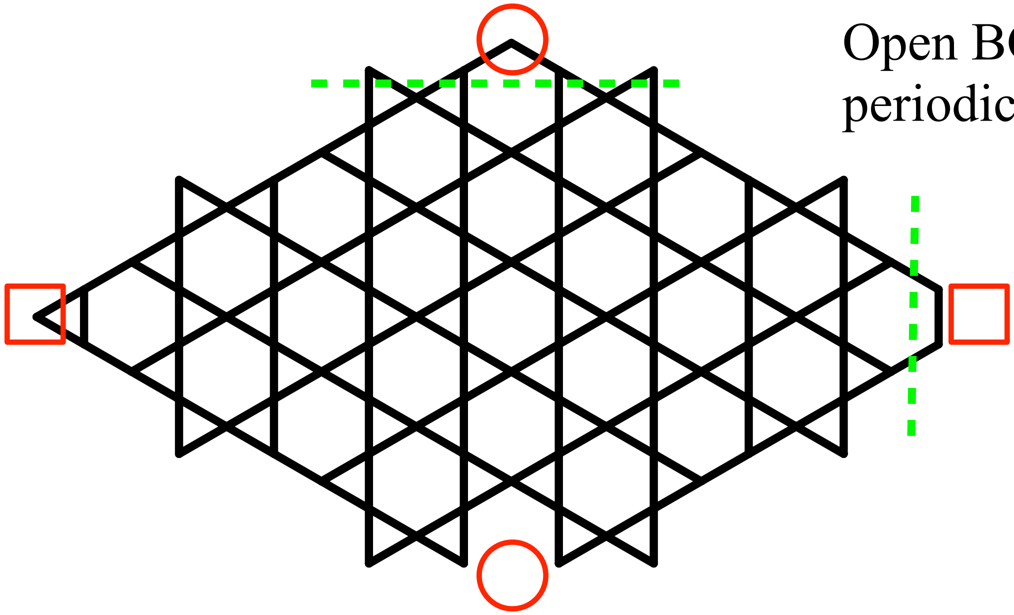
A disk with a hole is topologically equivalent to a cylinder

"+" : No vison state

"-" : Vison state (not yet seen numerically)

RVB pictures for cylinders

Topological Degeneracies on "Quasi-tori"

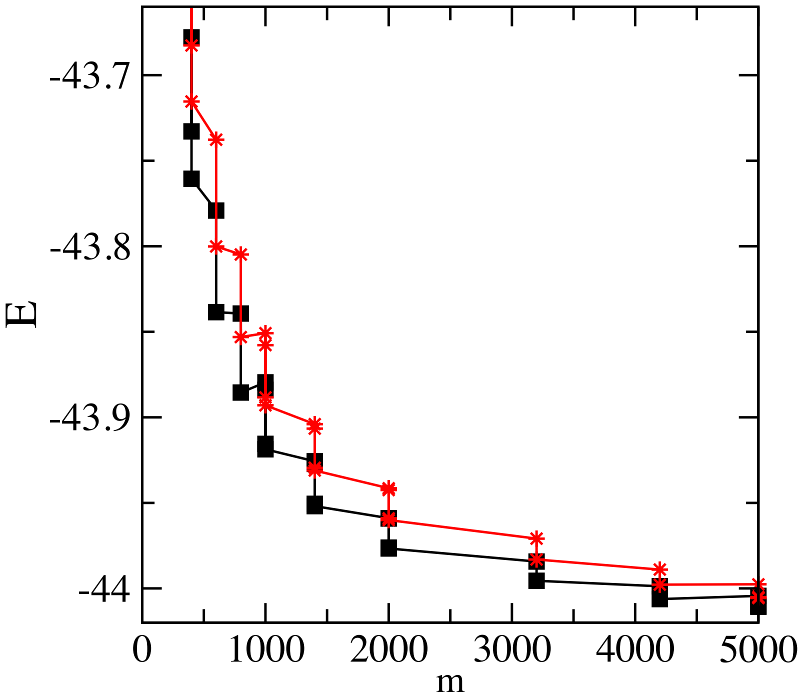


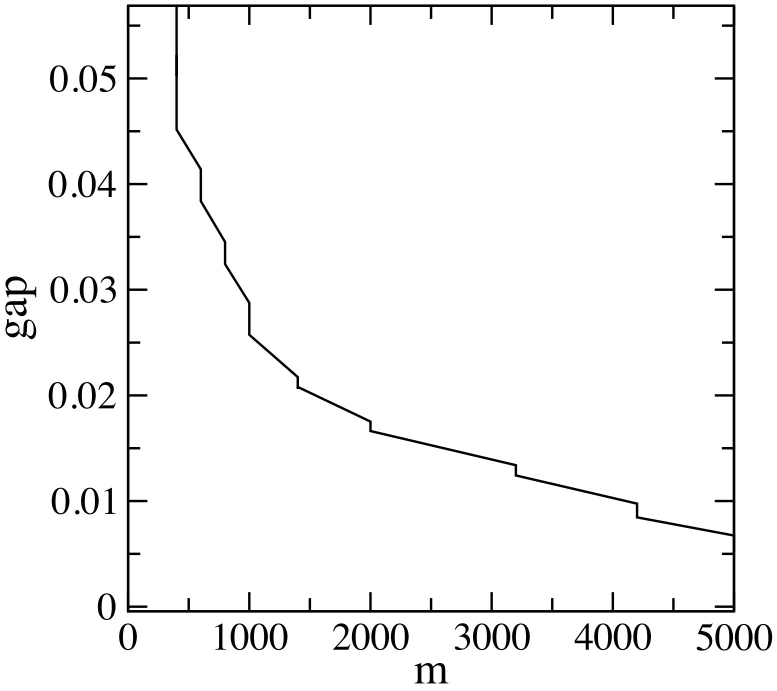
Open BCs except for periodic connections

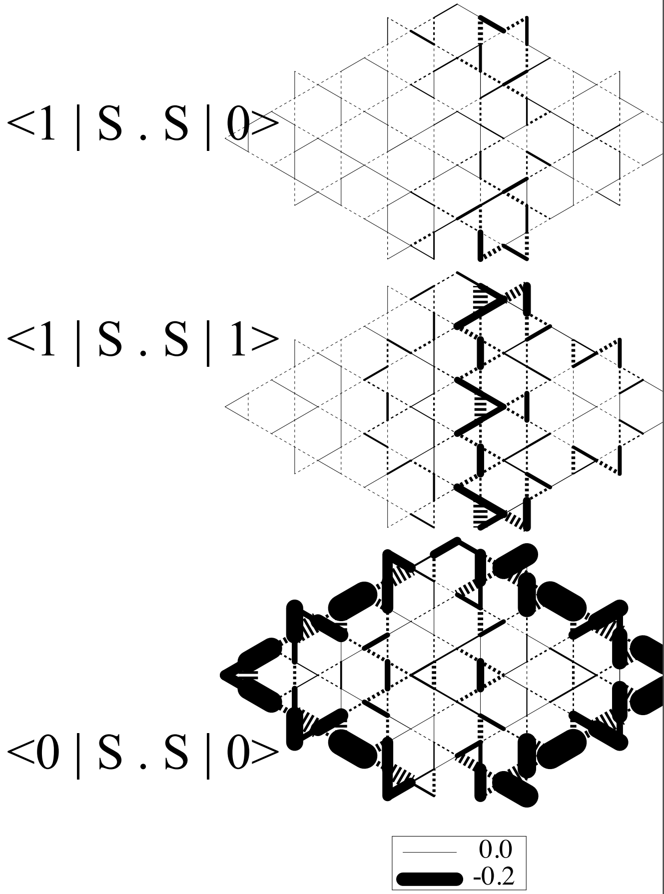
- Expect smaller topo splittings because of fewer perturbing periodic connections
- Entanglement only slightly greater than open; small correction to area law (good for DMRG)

- It looks like a torus with a hole, but 1D connections mean only 4 topo states
- Small connections split even/odd winding number states--except with carefully chosen reflection symmetry
- Reflection operators identify topo states

Topological Degeneracies on “Quasi-tori”







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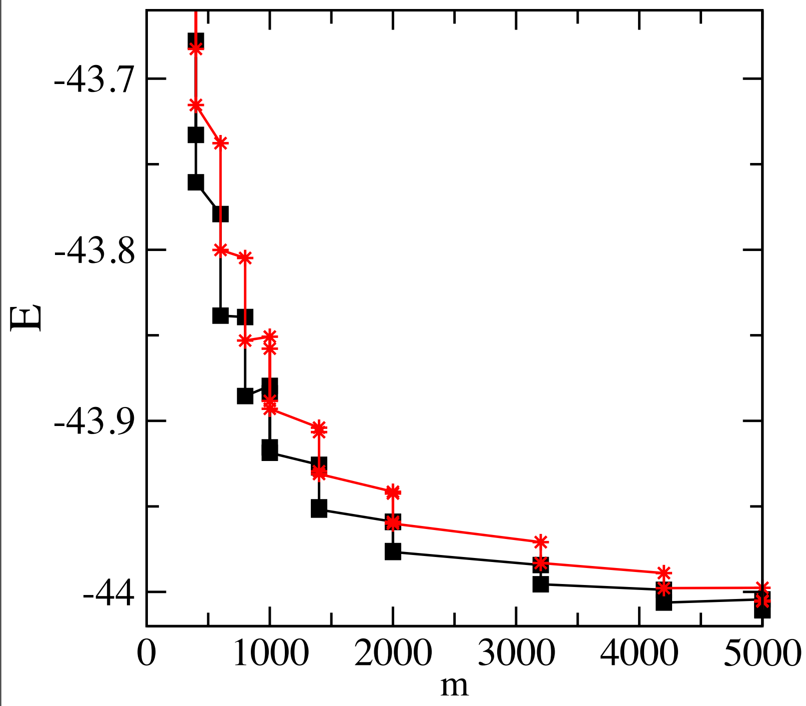
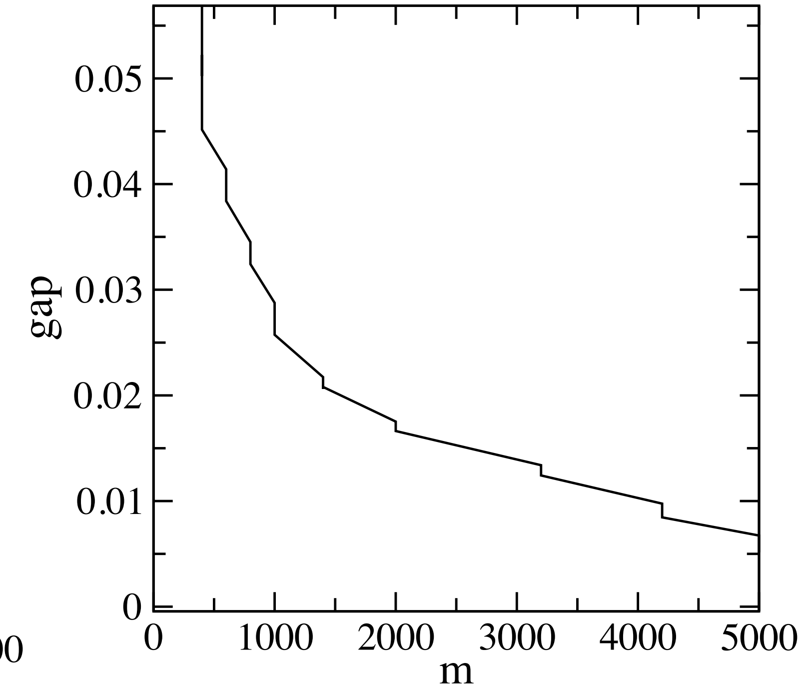
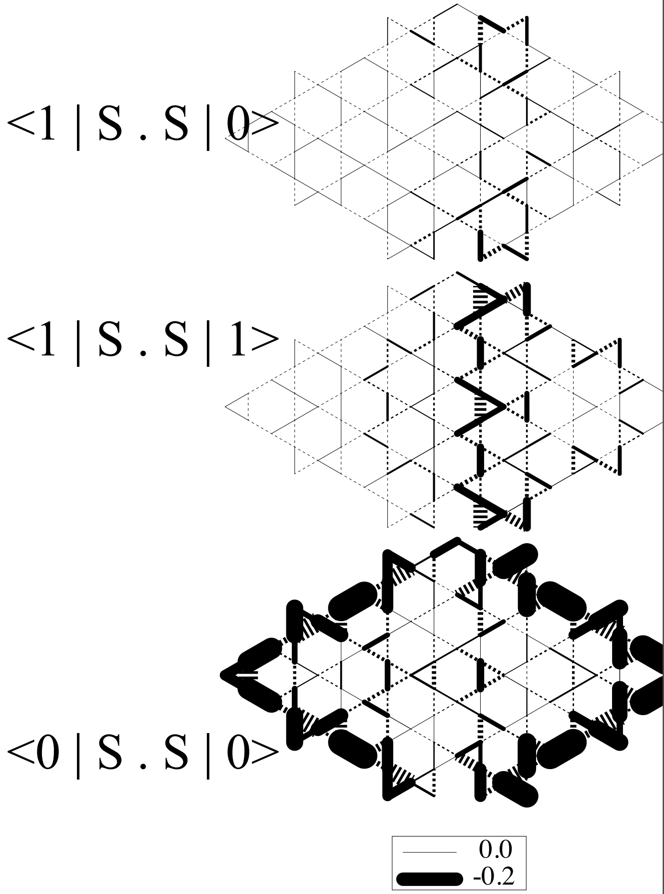
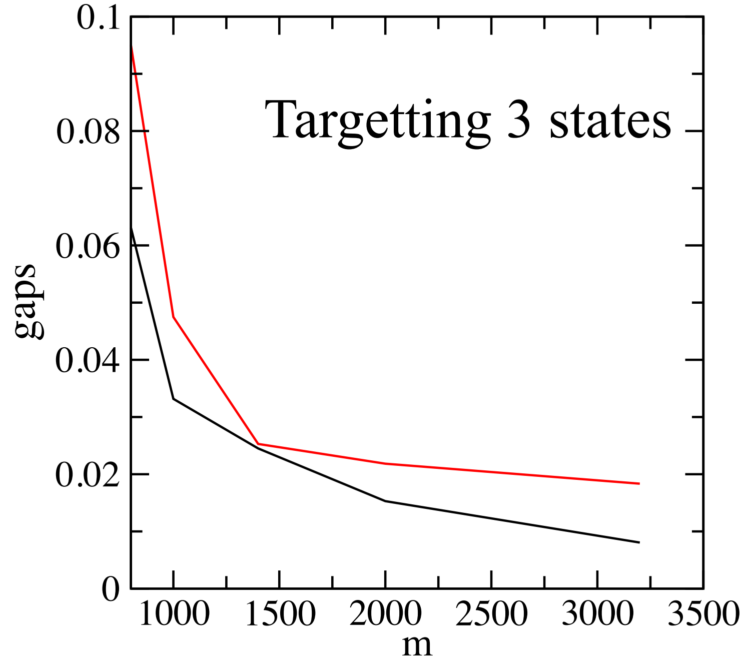




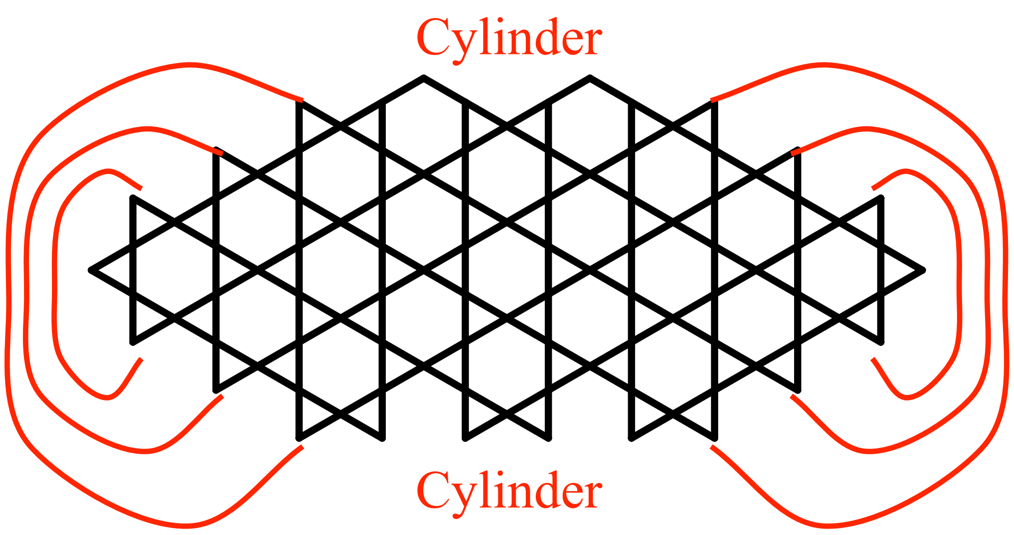


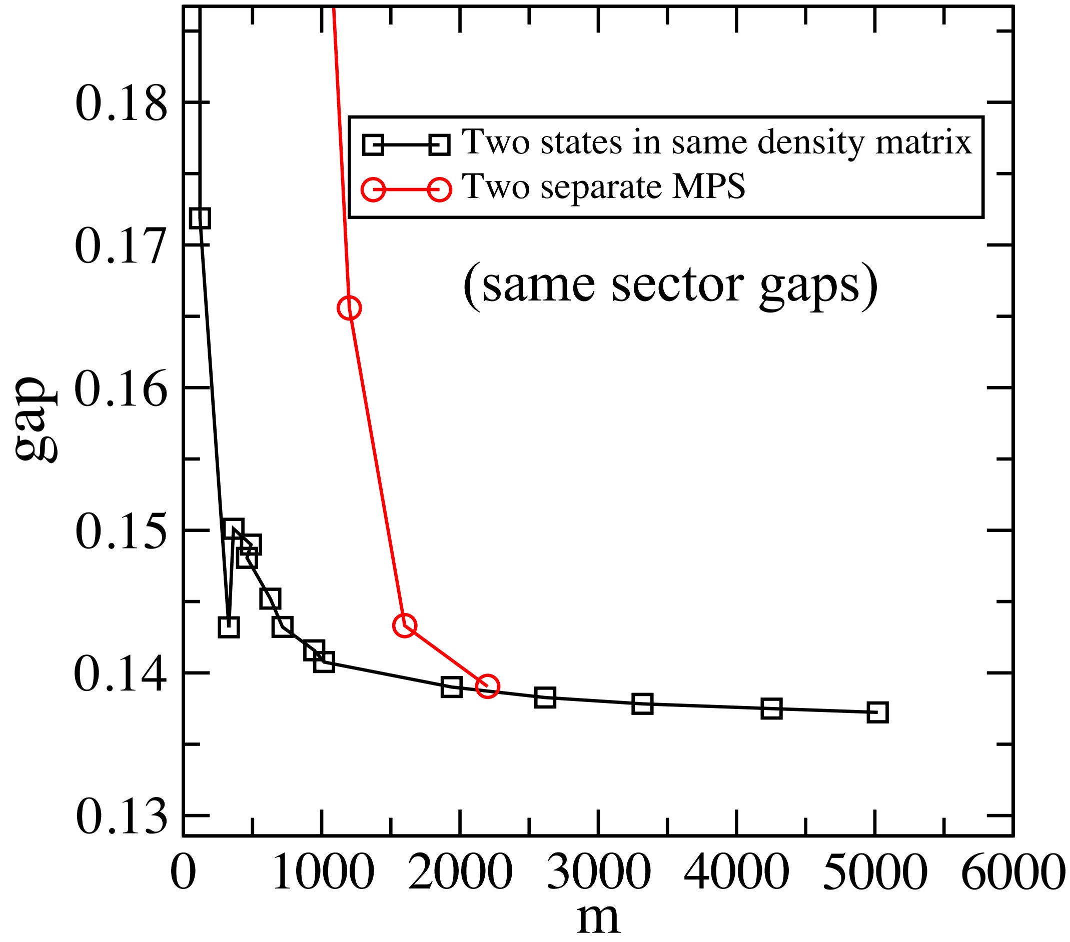


At least two low lying excited states (but there should only be one!

What is going on?

Excited states of a "sphere" or capsule





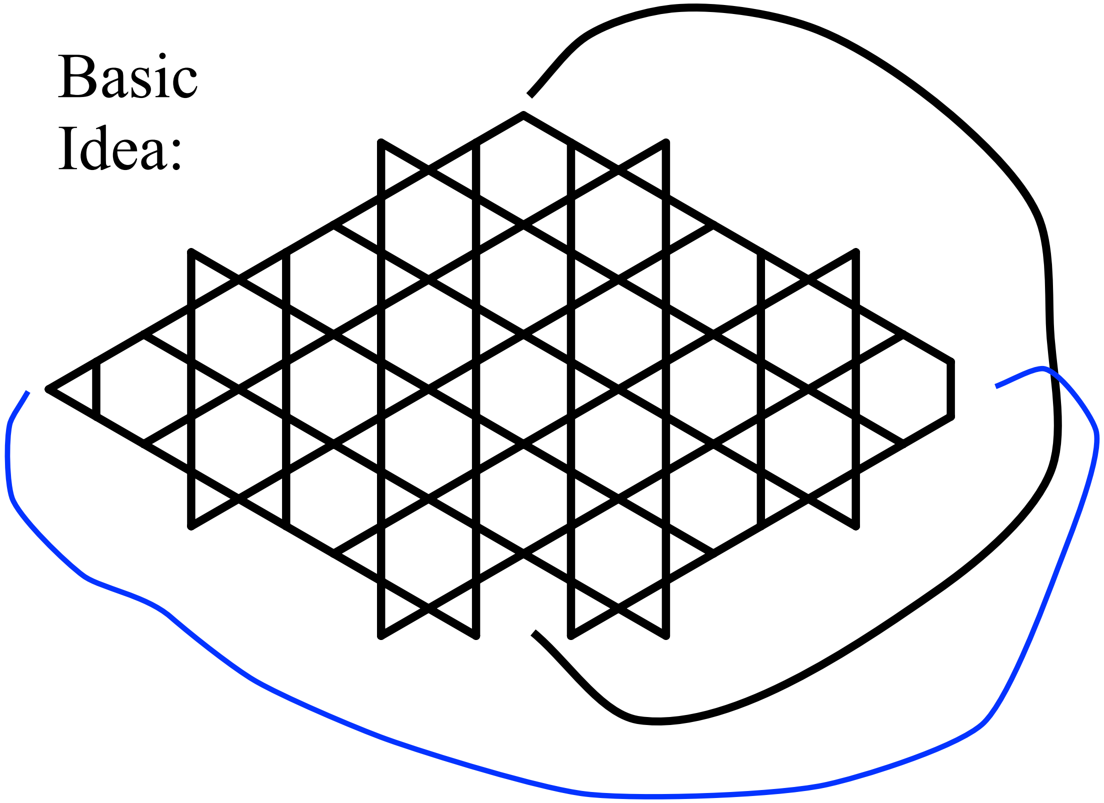
The spherical geometry has gaps similar to the bulk gaps.

Now: erase all red connecting lines, redo DMRG:
get gap of about 0.014 !

Conclusions: 1) it is hard to avoid low energy edge states
2) there may be unavoidable single vison states any time we have an open edge

Exact Topological Degeneracies

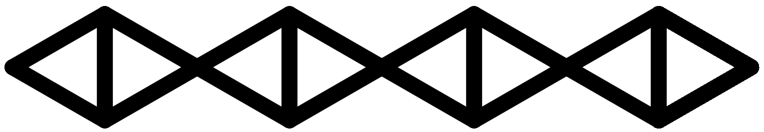
Basic Idea:



Lines are 1D gapped S=1/2 systems which carry the topo connection

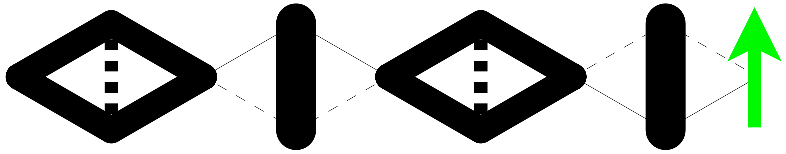
The "topological wires" make the loop resonances longer, decreasing the gap.

A "perfect" topological wire:
1) Correlation length is 0
2) Finite Gap (Increase vertical J to 1.5)



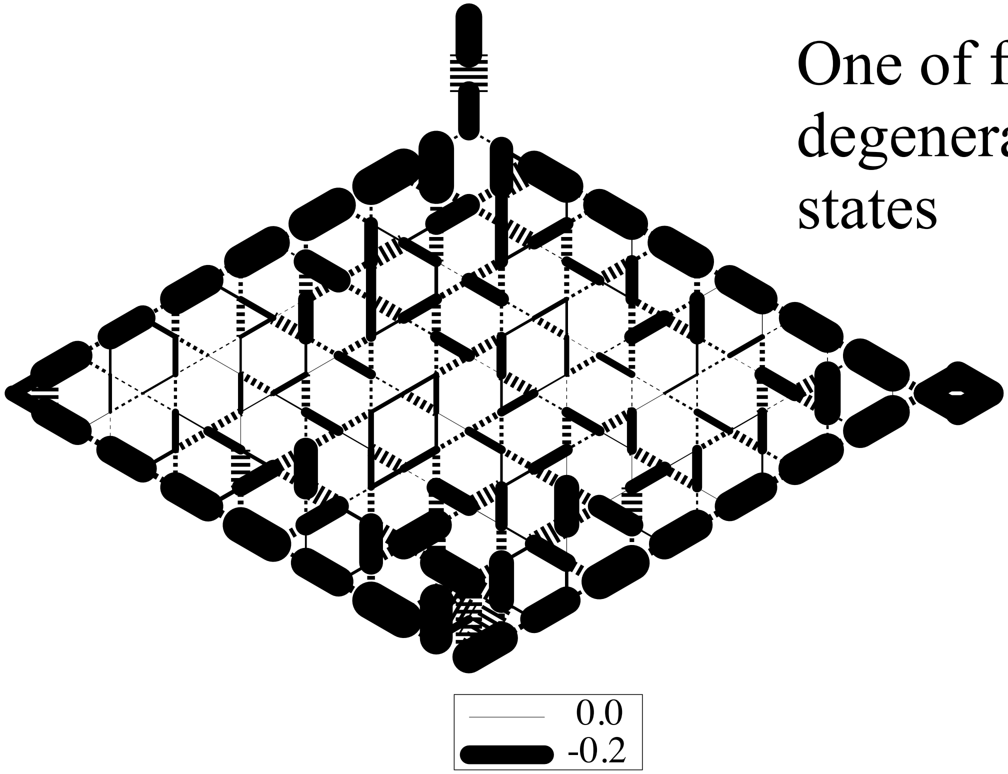
Exchanging the sites in any vertical bond is a symmetry!
So vertical bonds are exactly S=0 or S=1

Four degenerate ground states



With this perfect wire, only need 7 sites

Exact Topological Degeneracies



One of four exactly degenerate topological states

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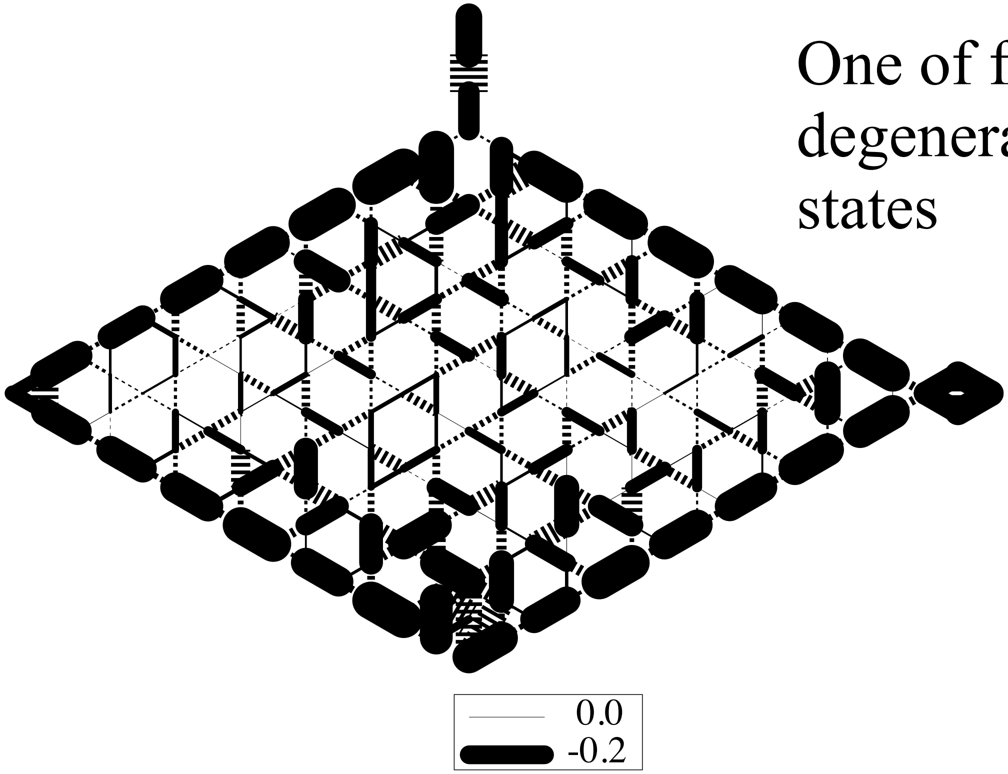
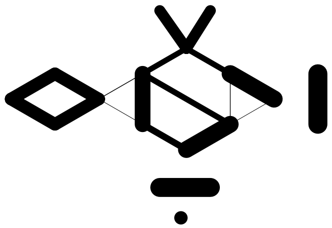
Exact Topological Degeneracies

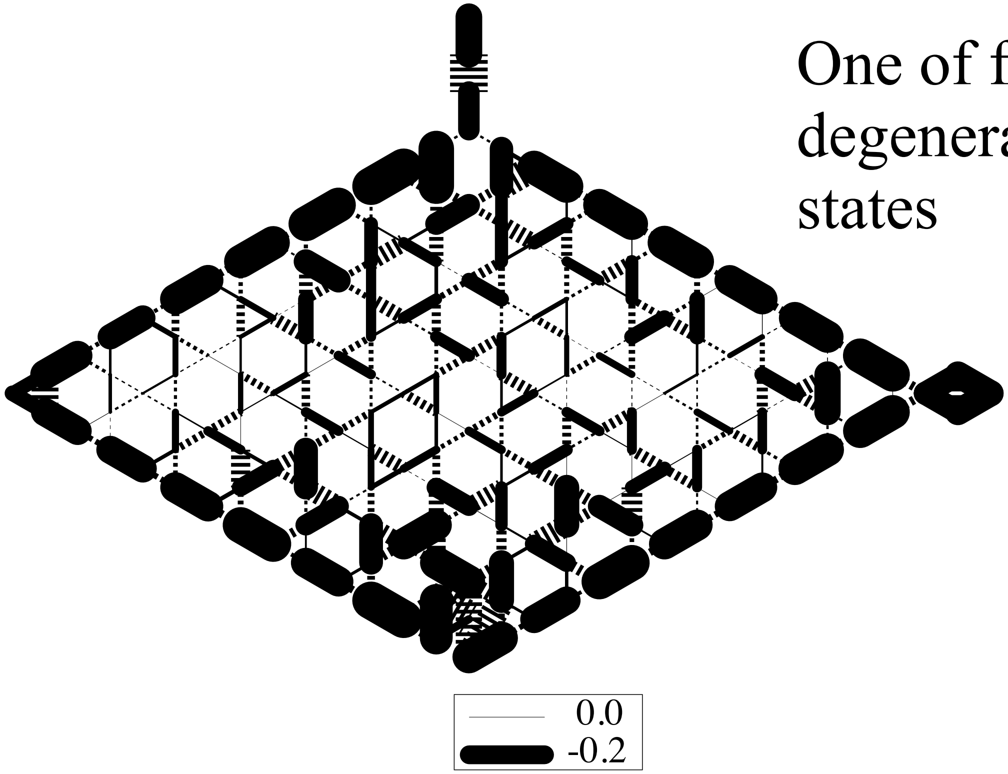


One of four exactly degenerate topological states

Unfortunately, this construction produces four degenerate “topological” states even for this micro-cluster!!



Exact Topological Degeneracies



One of four exactly degenerate topological states

So: does this construction tell us anything about whether the kagome is Z_2?

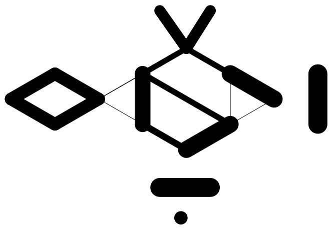
A little:

1) It shows multiple topo ground states from one cluster or from different clusters is not so different

2) We can adiabatically turn the perfect wires into standard periodic connections and track the topo states.

3) It suggests tests for Z_2 using open clusters.

Unfortunately, this construction produces four degenerate “topological” states even for this micro-cluster!!



Summary

- The kagome system passes some of the tests for a Z_2 spectacularly well (gaps, correlation lengths, TEE)
- Some of the properties/tests still don't work, apparently because of bigger finite size effects.
- Seeing a quadruple degeneracy on a torus is still out of reach, but we have had partial success with "quasi-tori".