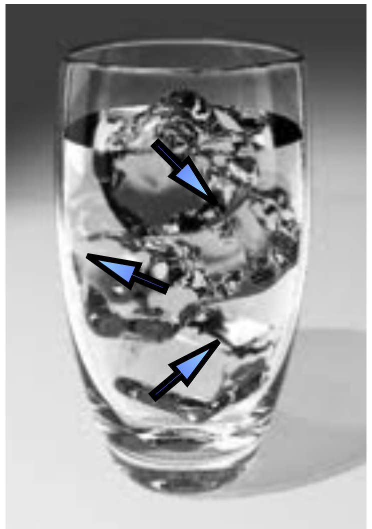


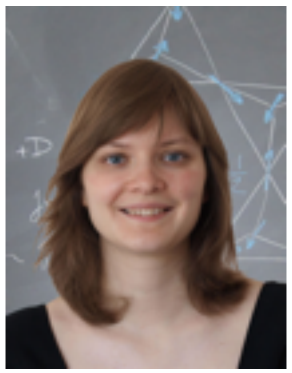
Quantum Spin Liquids

Overview, Quantum Spin Ice, etc.

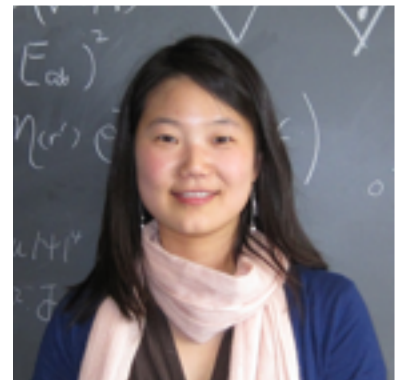
Leon Balents, KITP

Fragnets, KITP, September 2012

Collaborators

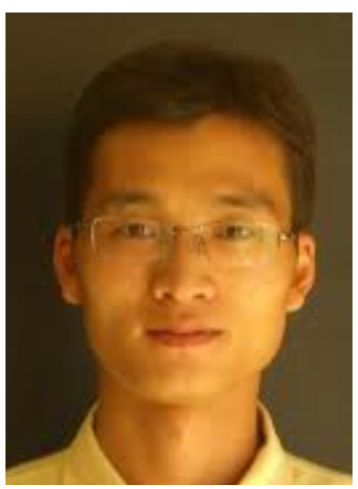


Lucile Savary
UCSB



SungBin Lee
Toronto

Shigeki Onoda
RIKEN



Hong-Chen Jiang
KITP

Zhenghan Wang
Toronto

Kate Ross

Bruce Gaulin

McMaster

A brief history of magnetism

~500BC: Ferromagnetism documented in Greece, India, used in China

sinan, ~200BC

time

1949AD: Antiferromagnetism proven experimentally

A brief history of magnetism

sinan, ~200BC

time

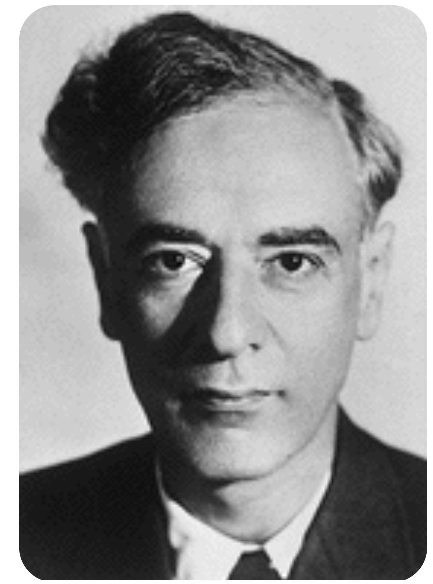
~500BC: Ferromagnetism documented in Greece, India, used in China

Why so long???

1949AD: Antiferromagnetism proven experimentally

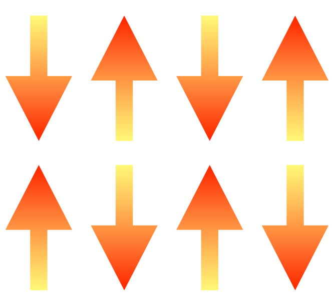
A debate

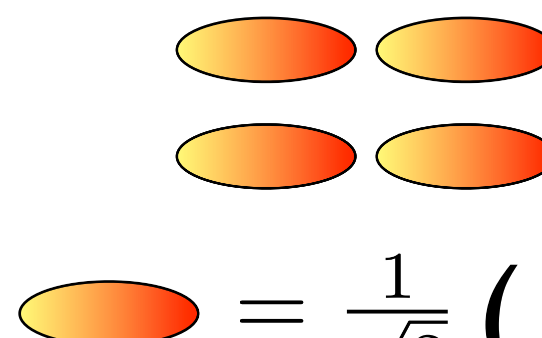
$$H = |J| \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$



Néel

Landau





$$= \frac{1}{\sqrt{2}} \left(\uparrow\downarrow + \downarrow\uparrow \right)$$

antiferromagnet

singlets

The right tool...

The right tool...

- Neutron scattering

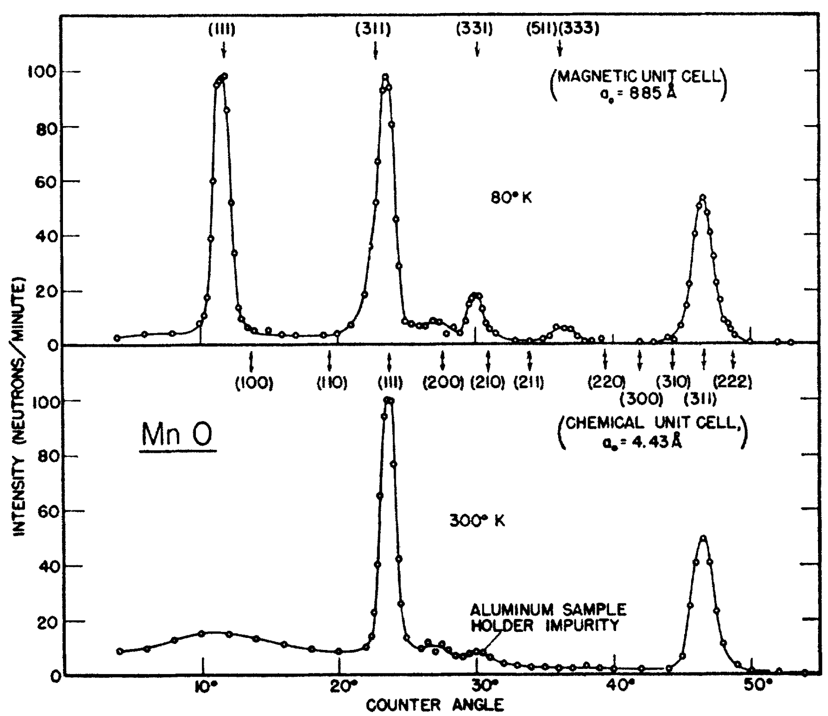


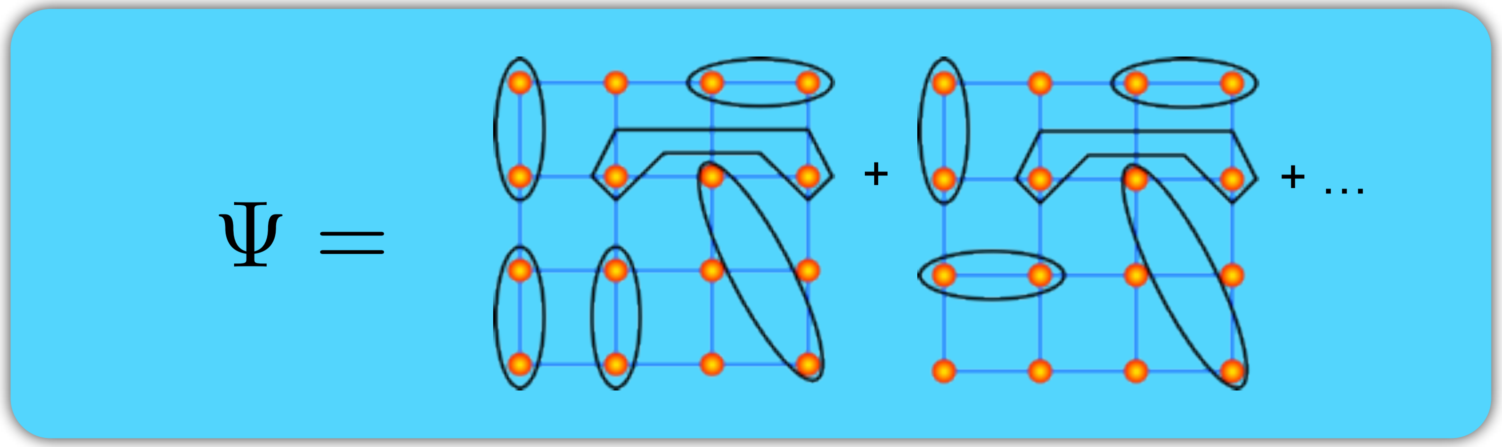
FIG. 1. Neutron diffraction patterns for MnO at room temperature and at 80°K.

Shull and Smart, 1949

- Now we know antiferromagnetism is commonplace

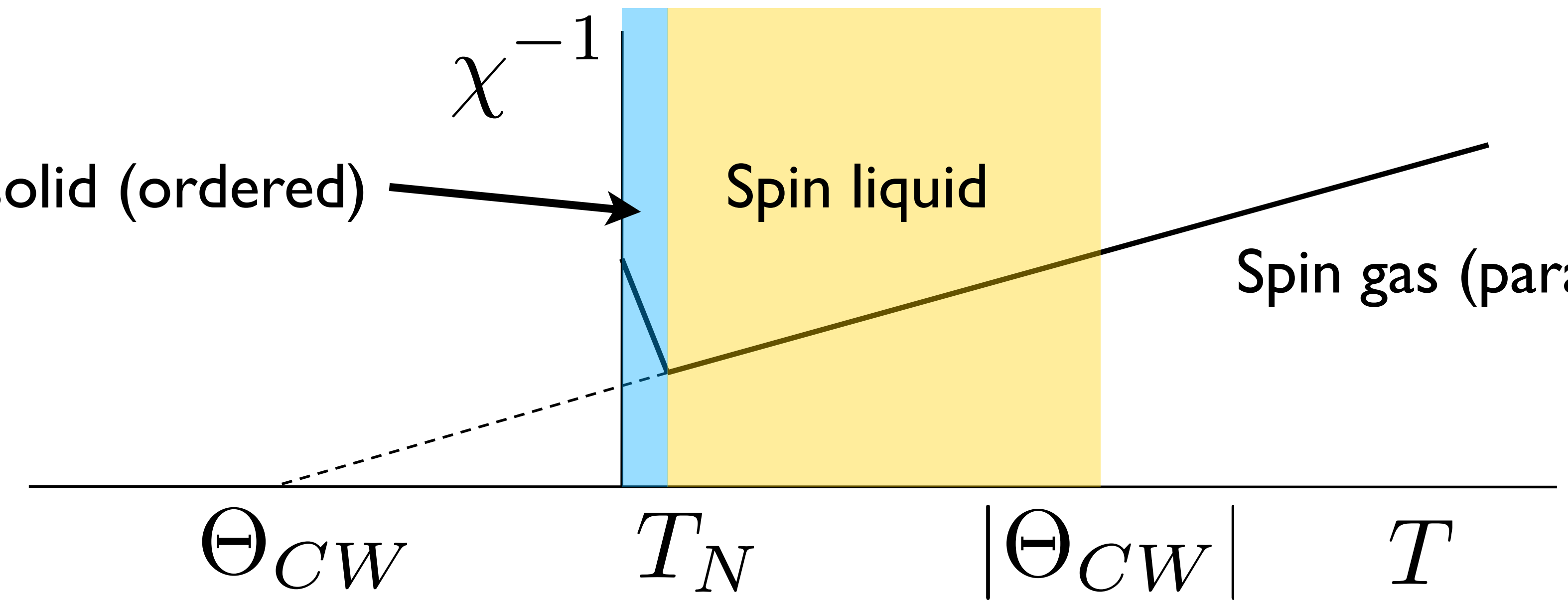
Singlets again

- Anderson (73): revived the idea of singlets in the "Resonating Valence Bond" state



- prototype of the modern QSL

Frustration Parameter



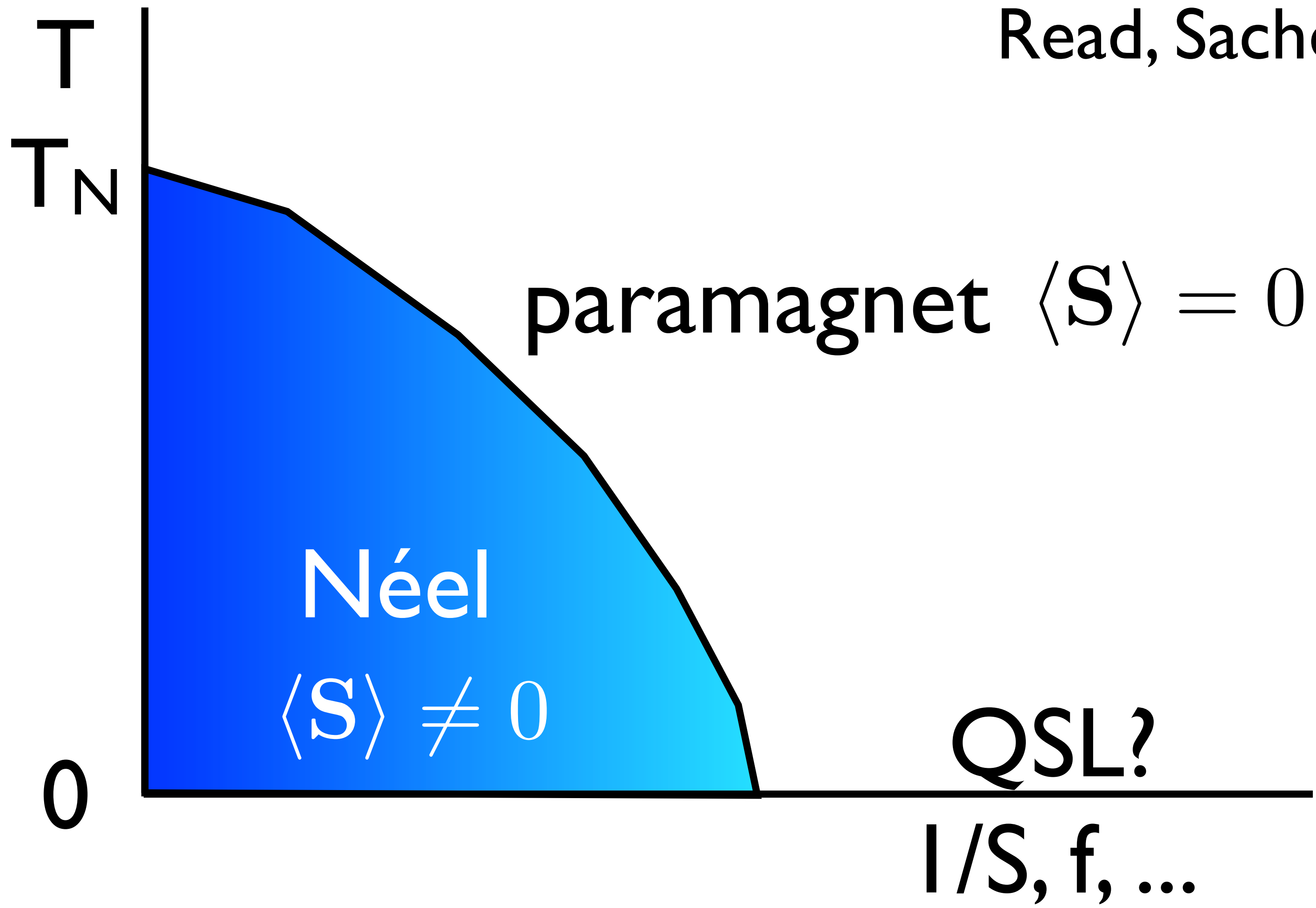
- Local moments: Curie-Weiss law at high T

$$\chi \sim \frac{A}{T - \Theta_{CW}}$$

- Frustration parameter: $f = |\Theta_{CW}|/T_N$
- The empirical search for spin liquids is often just for materials with f >> 1

Quantum Paramagnet

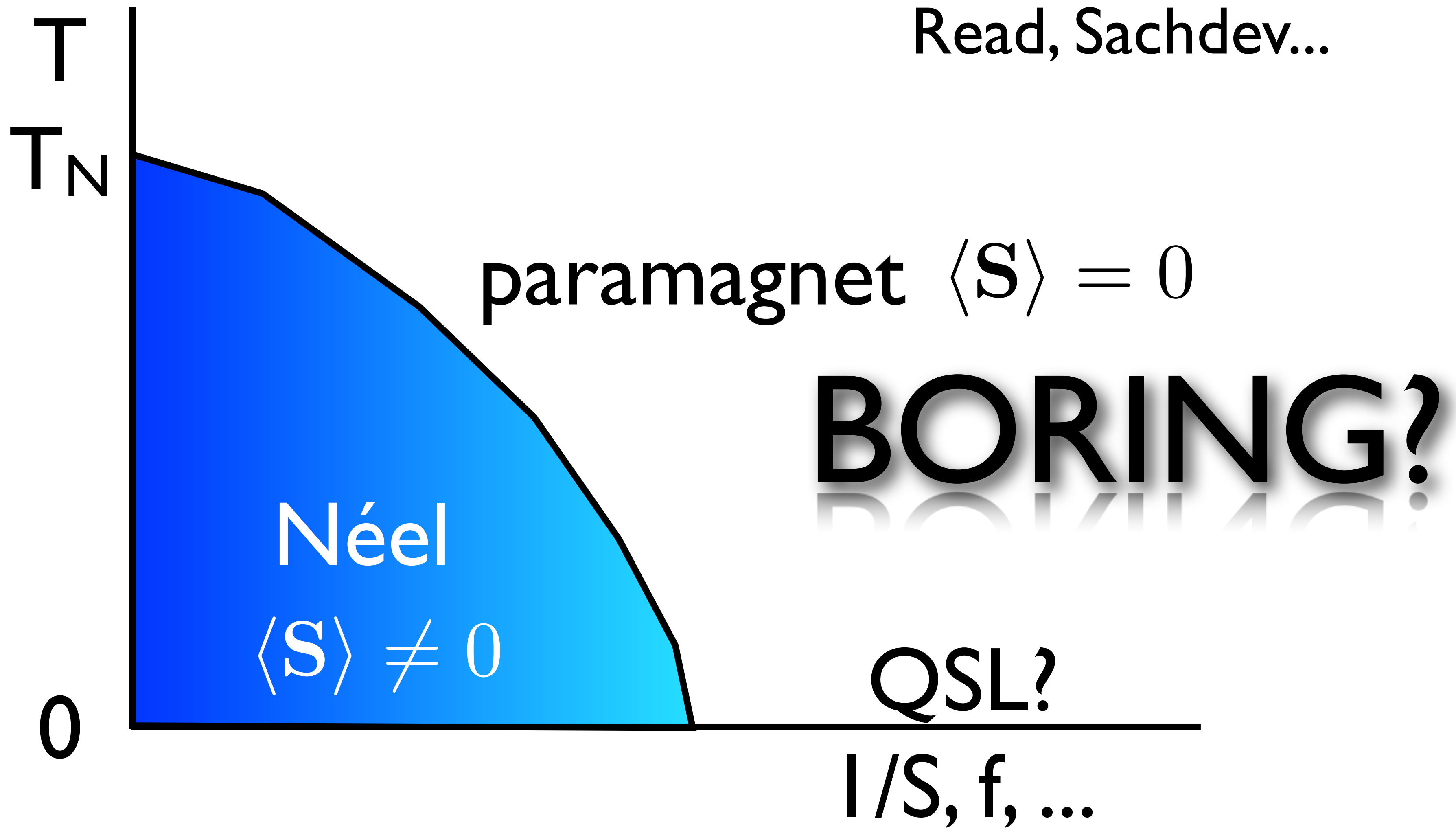
Chakravarty, Halperin, Nelson, 1989
Read, Sachdev...



- Quantum spin liquid = no magnetic order?

Quantum Paramagnet

Chakravarty, Halperin, Nelson, 1989
Read, Sachdev...



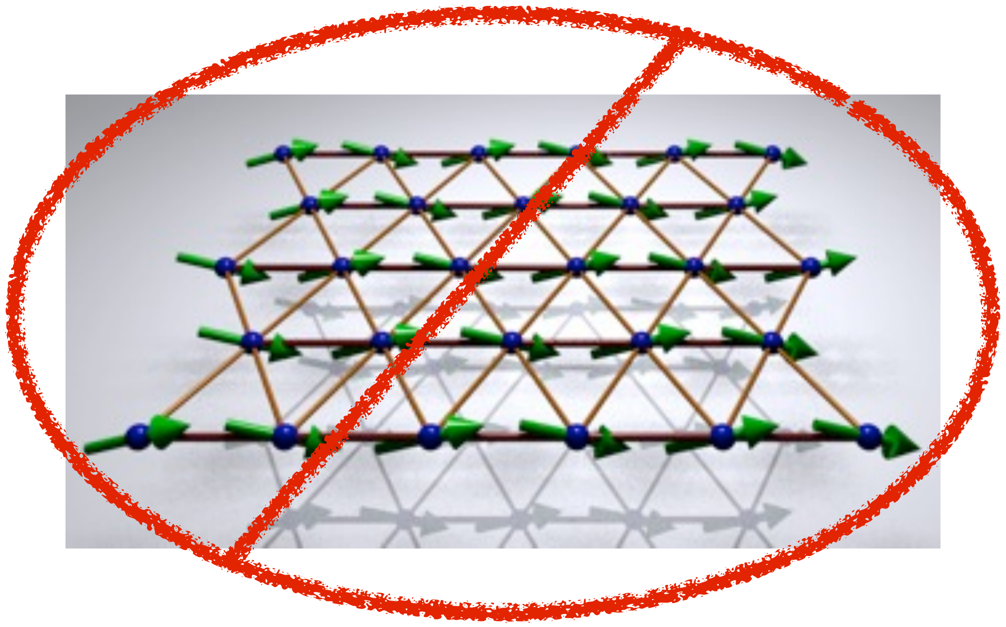
- Quantum spin liquid = no magnetic order?

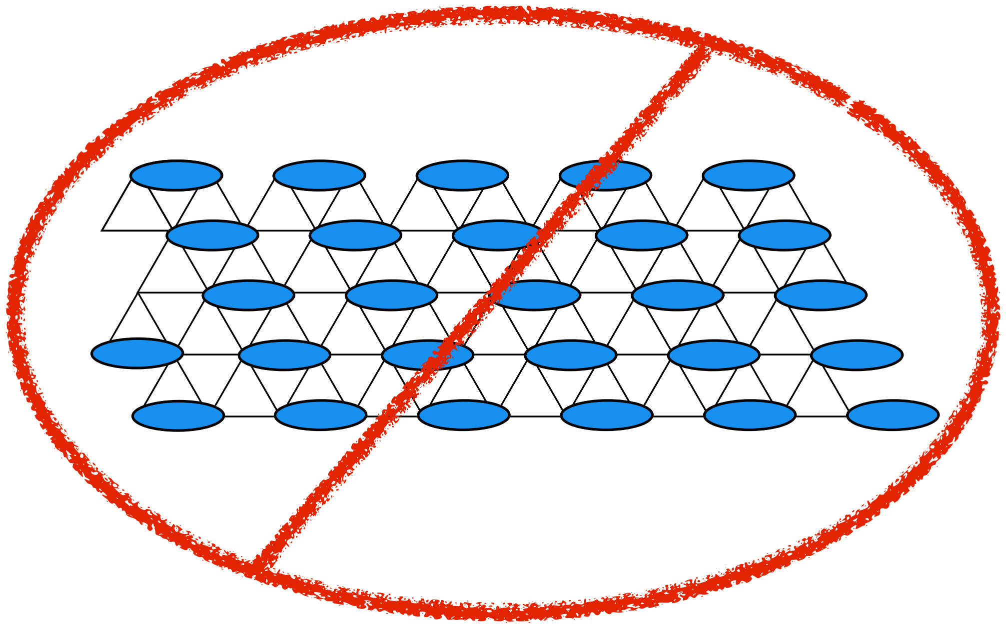
What is a QSL?

- Calling a QSL a quantum paramagnet
  - defines what it isn't!
  - is in itself not interesting!
  - misses the important physics!
- We need a *positive* definition

A Modern View

- Let's call a QSL a ground state of a spin system with *long range entanglement*
- This means a state which cannot be regarded or even approximated as a product state over any finite blocks





A Modern View

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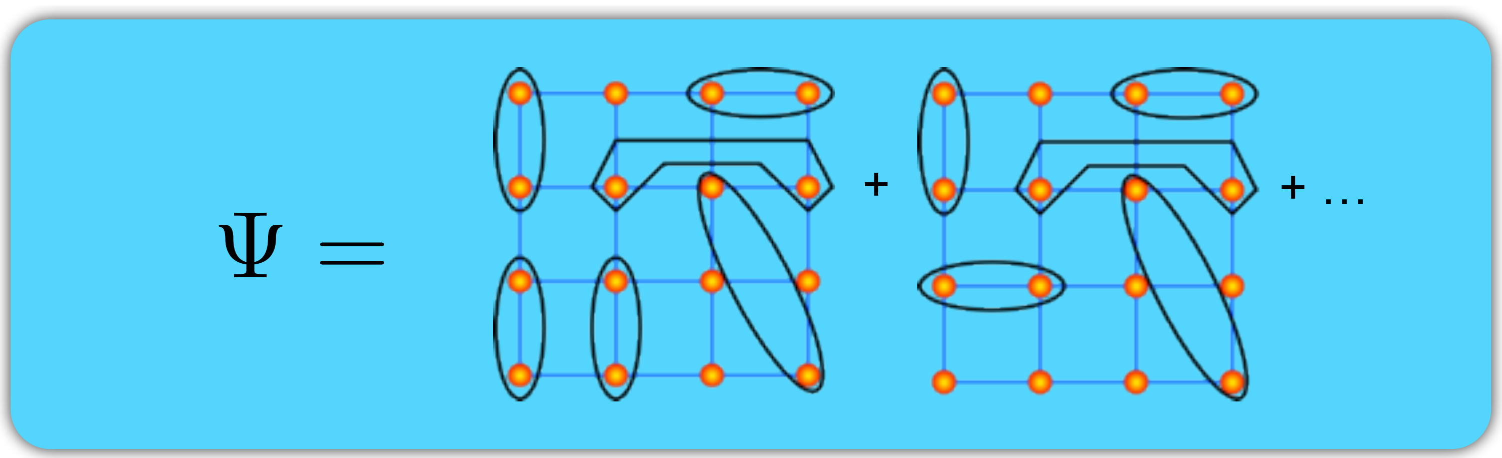
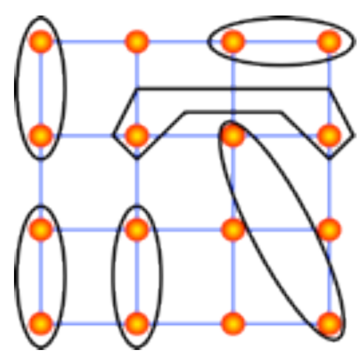
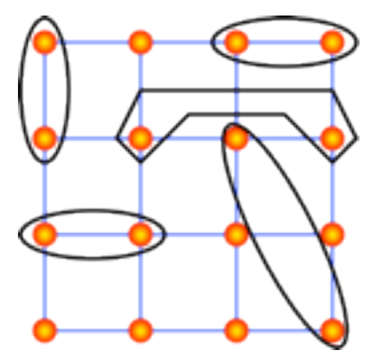


How to describe a QSL?

- A long-range entangled wavefunction is a complicated thing!

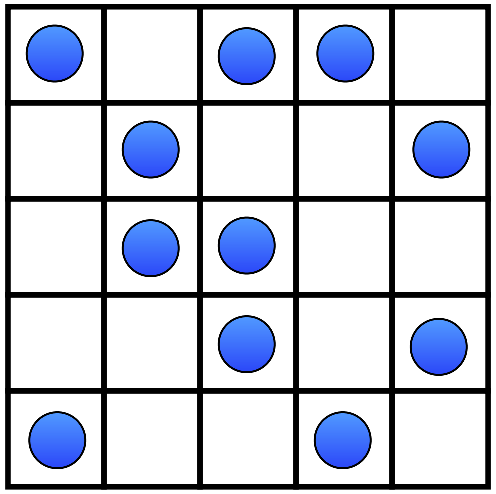
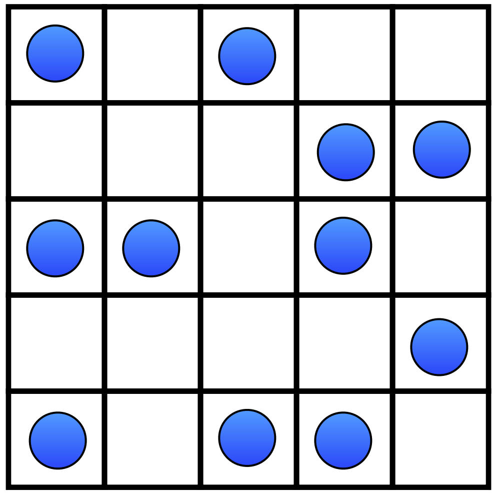
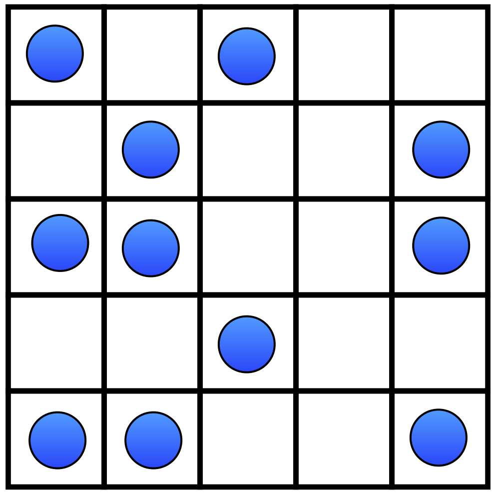
$\Psi = c_1$  + c_2  + …

- Very hard to work directly with all these coefficients - is there another way?

Free Fermions

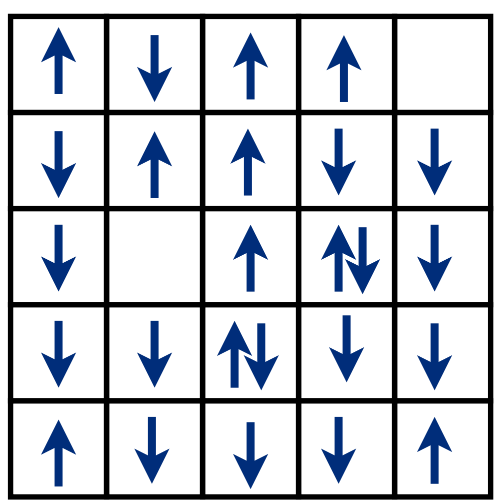
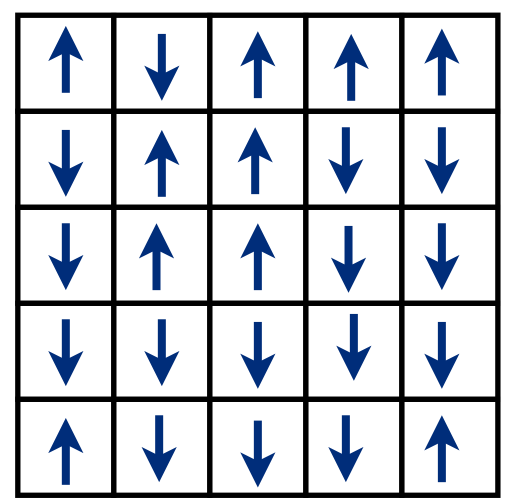
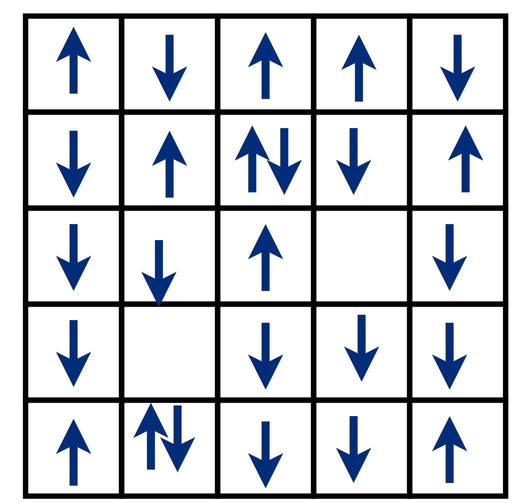
- One useful construction uses a Fermi gas: a product in momentum space rather than real space

$$\Psi = \prod_{k<k_F} c_k^\dagger |0\rangle$$

$= \; c_1$  $+c_2$  $+c_3$  $+\cdots$

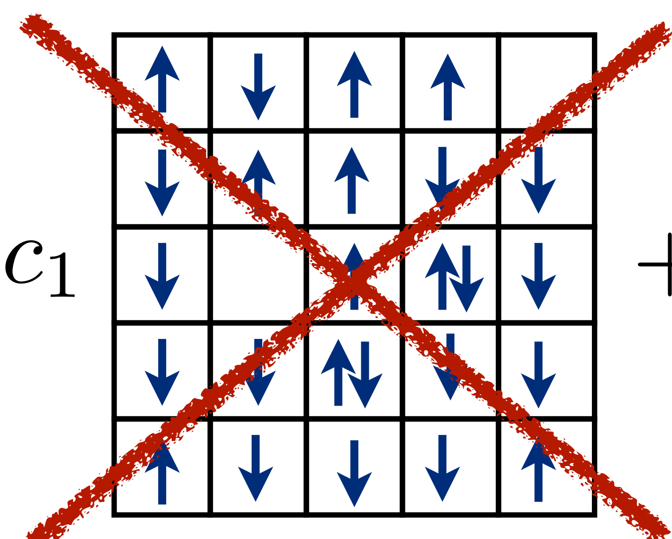
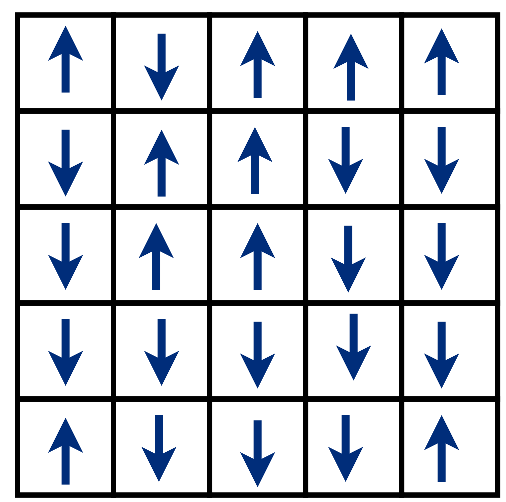
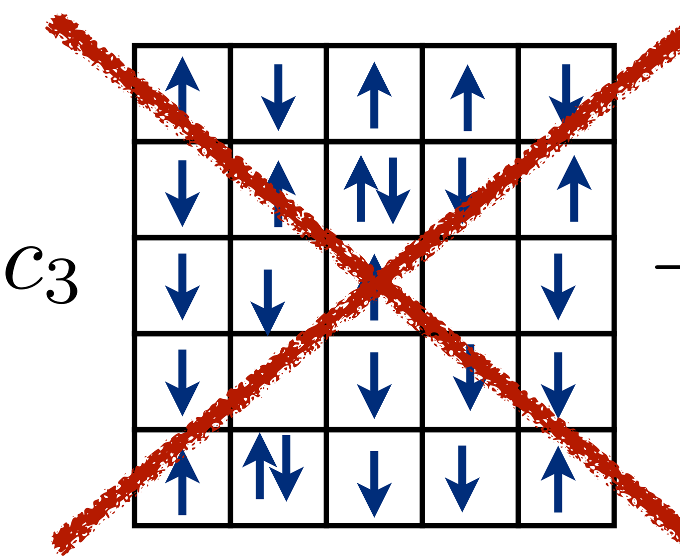
Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site (S=0)

$\Psi_0 = c_1$  $+c_2$  $+c_3$  $+\cdots$

Gutzwiller Construction

- Construct QSL state from free fermi gas with spin, with 1 fermion per site

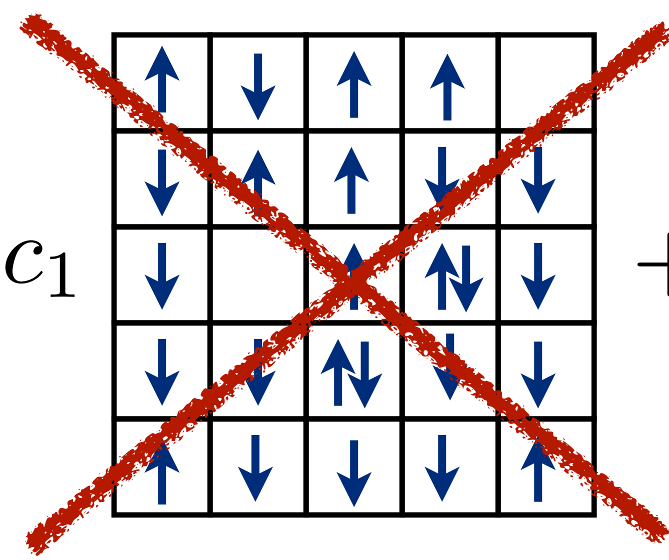
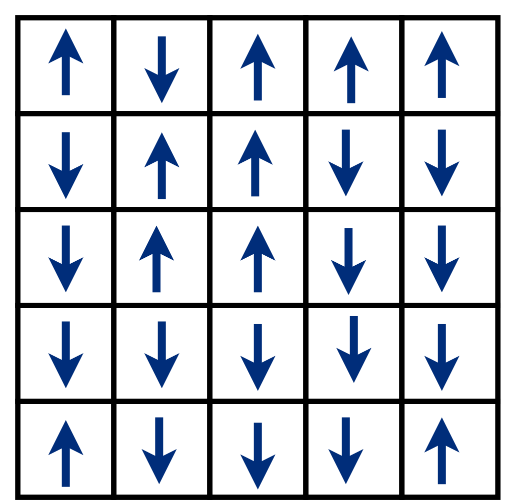
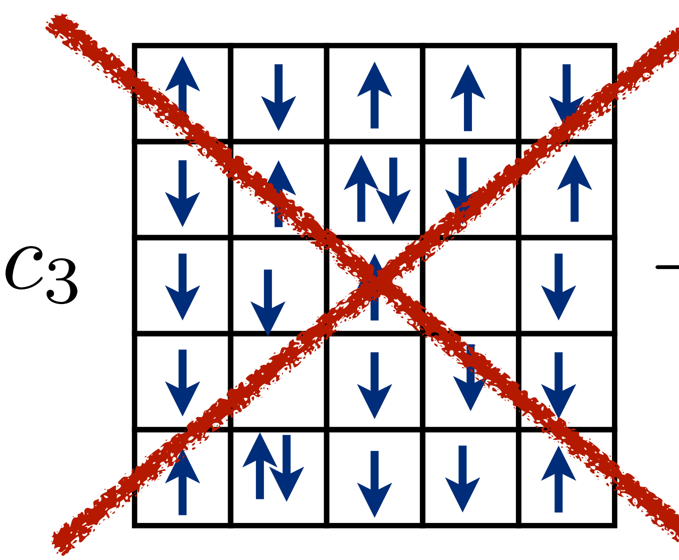
$\Psi = c_1$  $+c_2$  $+c_3$  $+\cdots$

- Projection removes empty and doubly occupied sites

$$\Psi = P_G \Psi_0$$

Gutzwiller Construction

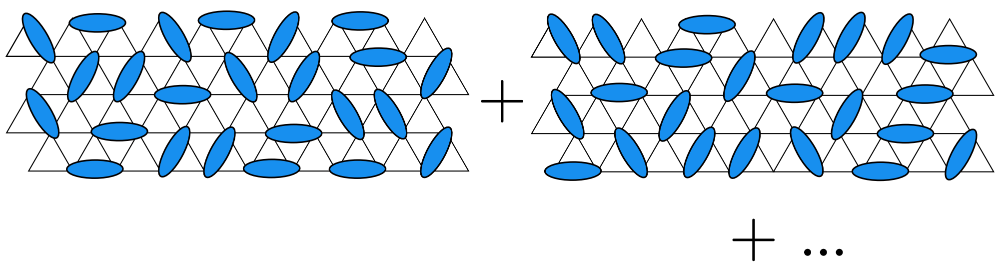
- Construct QSL state from free fermi gas with spin, with 1 fermion per site

$\Psi = c_1$  $+c_2$  $+c_3$  $+\cdots$

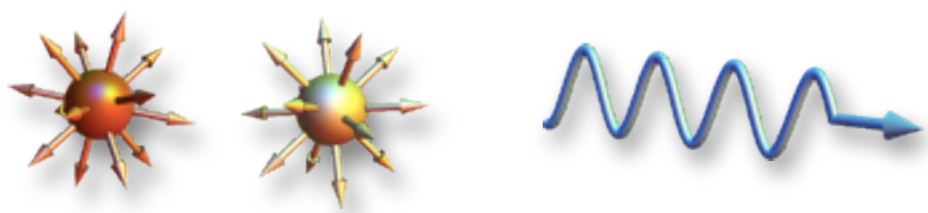
- Belief: low energy physics is described by a gauge theory, with fermion → spinon

Classes of QSLs

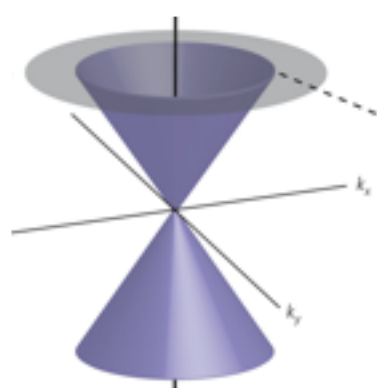
- Topological QSLs
  - full gap



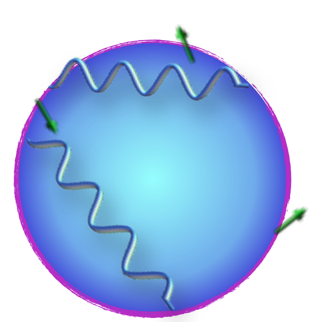
- U(1) QSL
  - gapless emergent "photon"



- Algebraic QSLs
  - Relativistic CFT (power-laws)



- Spinon Fermi surface QSL



Classes of QSLs

- Topological QSLs
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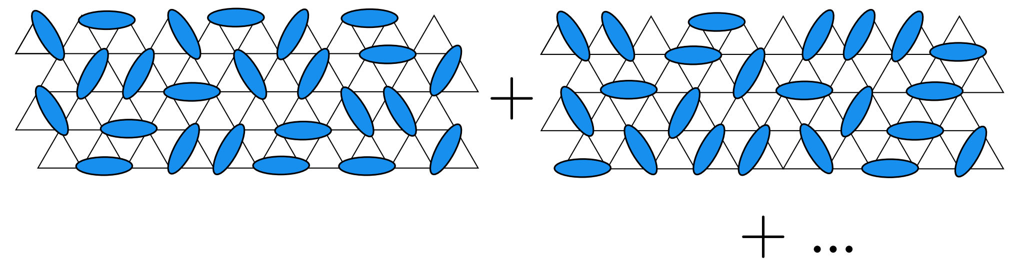
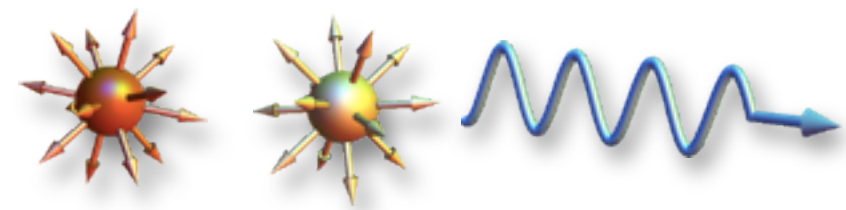


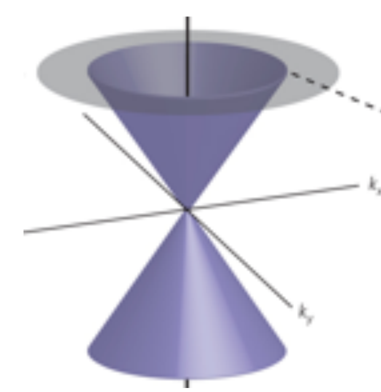
TQFT

- U(1) QSL
  - gapless emergent "photon"



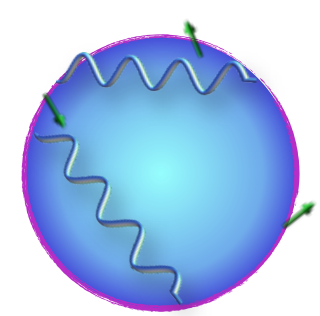
compact U(1) gauge theory

- Algebraic QSLs
  - Relativistic CFT (power-laws)



QED_3

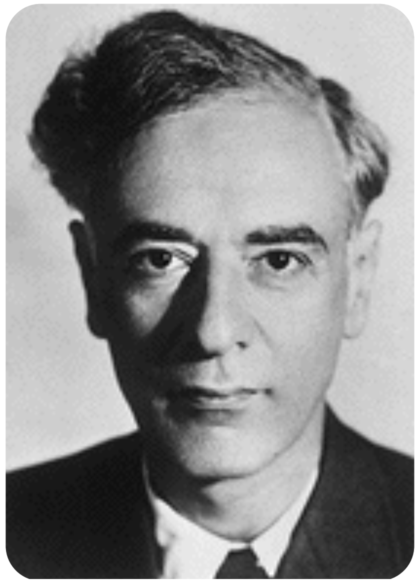
- Spinon Fermi surface QSL

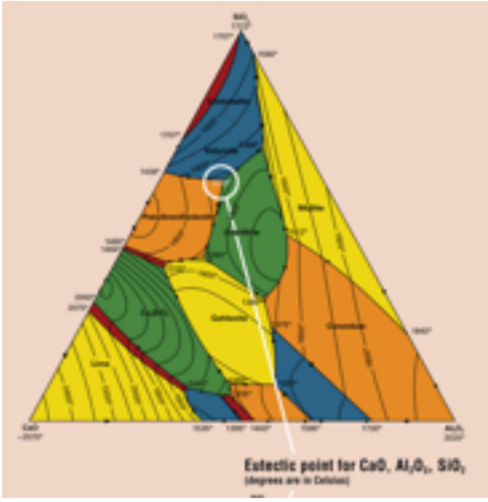


QED_3 w/ μ>0

Why bother?

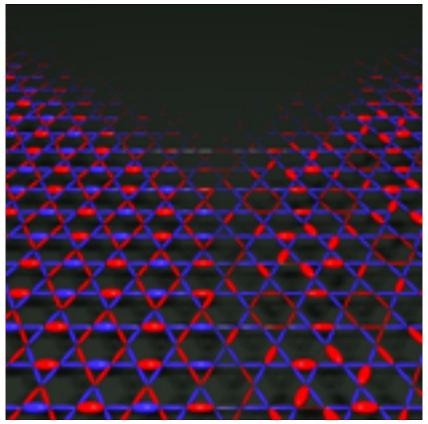
- QSLs are minimal examples of states with quantum order, an entirely new class of phases of matter
  - Perhaps simpler than strongly correlated *conducting* states
- With robust QSLs, qualitatively different quasiparticles would be at our disposal
  - Some would be *very* useful for quantum computing and other applications





Symmetry

- Phases characterized by measurable order parameters
- Phases can "collapse" if symmetry is *explicitly* broken



Long Range Entanglement

- Phases are distinct even in absence of any symmetry
- LRE can be measured directly *non-locally*, e.g. by *entanglement entropy*
- Supports excitations with *exotic quantum numbers and statistics*
- Describable by *emergent gauge structure*

Challenges: theory

- Numerical solution of physically relevant models is very challenging, and it is also hard to extract QSL behavior from finite systems
- Many QSLs are described by strongly coupled gauge theories
- The hardest part is connecting to real materials!

Challenges: theory

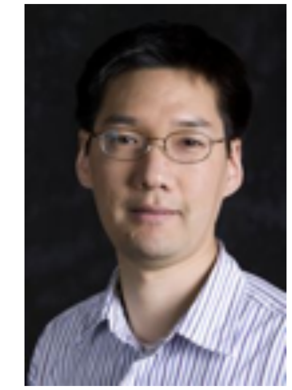
- Numerical solution of physically relevant models is very challenging, and it is also hard to extract QSL behavior from finite systems

- Many QSLs are described by strongly

  materials!

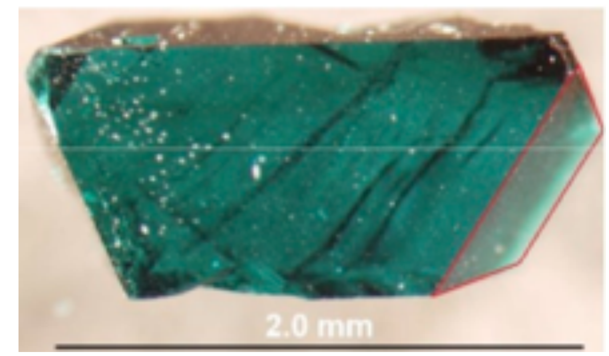
Density Matrix Renormalization Group (DMRG) is now able to accurately solve realistic 2d quantum spin models

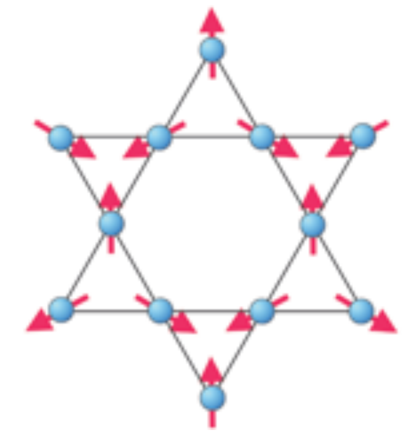
S=1/2 kagomé AF



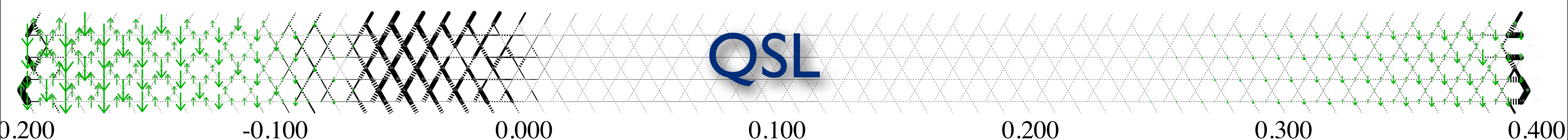
- e.g. "Herbertsmithite"

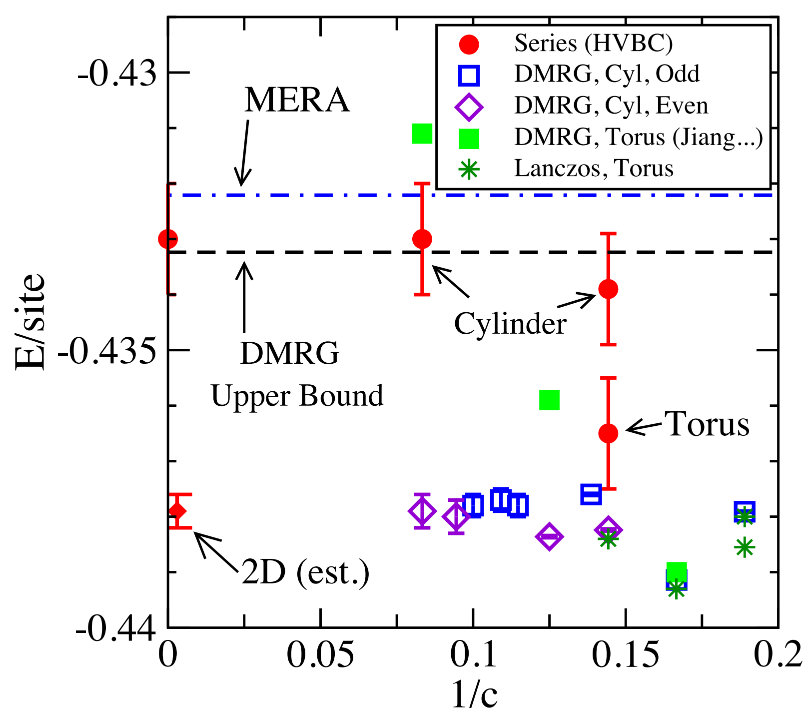
  $ZnCu_3(OH)_6Cl_2$





YS Lee group

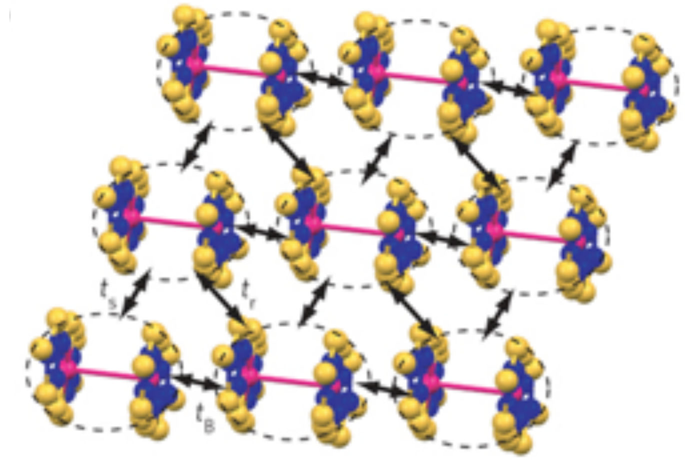


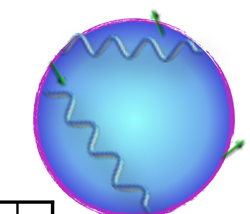


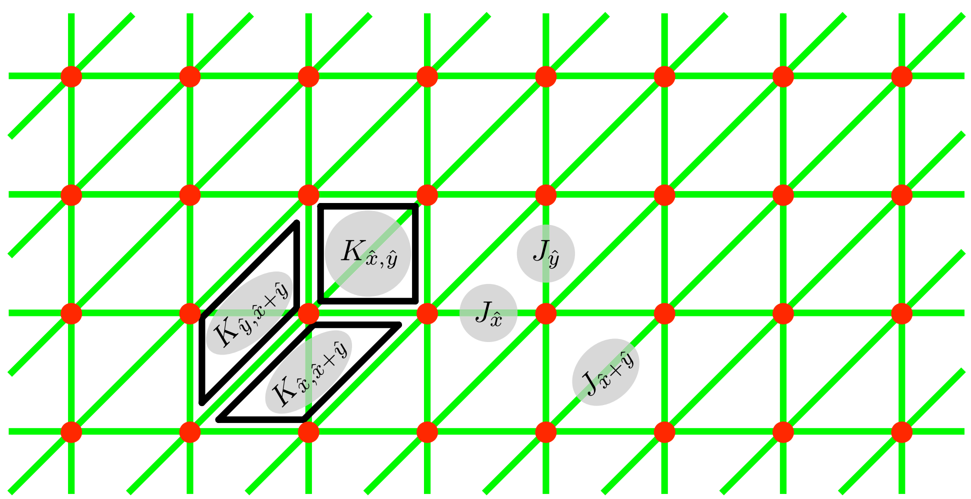
S. Yan *et al*, 2010

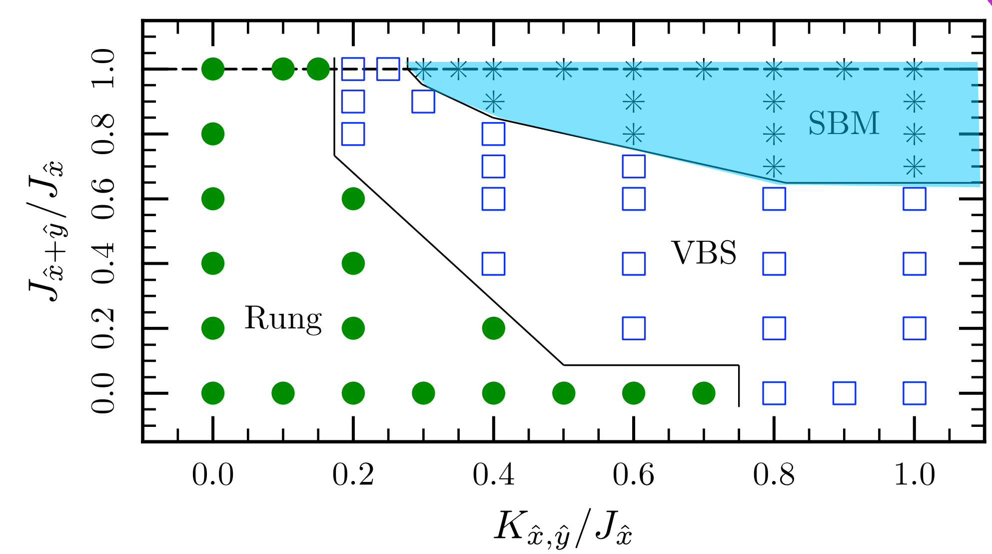
Ring Exchange Model

- Organic $EtMe_3Sb[Pd(dmit)_2]_2$









MS Block *et al*, 2011

Challenges: theory

- Numerical solution of physically relevant models is very challenging, and it is also hard to extract QSL behavior from finite systems
- Many QSLs are described by strongly

Entanglement entropy gives “smoking gun” evidence for some QSLs

- The hardest part is connecting to real materials!

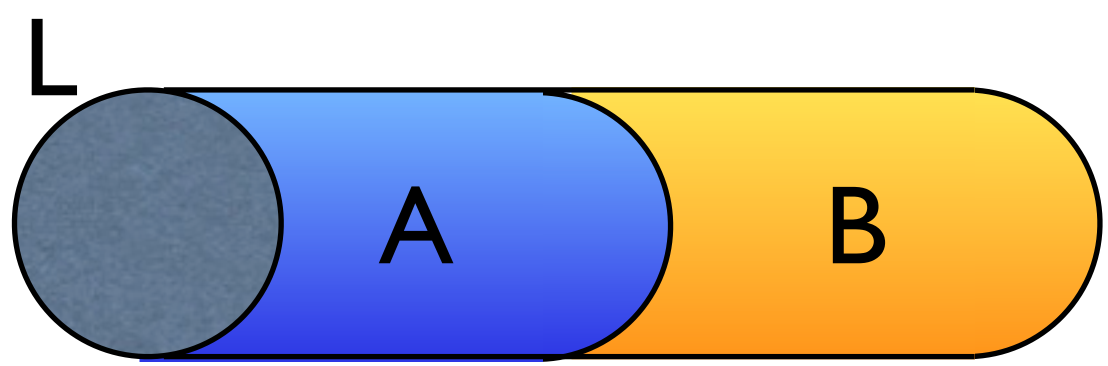
Topological Entanglement Entropy

- For gapped QSLs, can define a quantitative measure of long-range entanglement

$$S(L) \sim \alpha L - \gamma$$




2006



$$\rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|$$

$$S = -\mathrm{Tr}_A[\rho_A \ln \rho_A]$$

Topological Entanglement Entropy

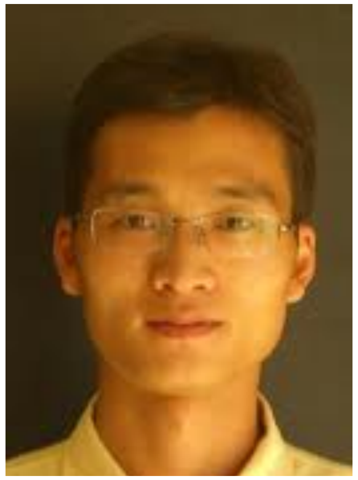
- For gapped QSLs, can define a quantitative measure of long-range entanglement

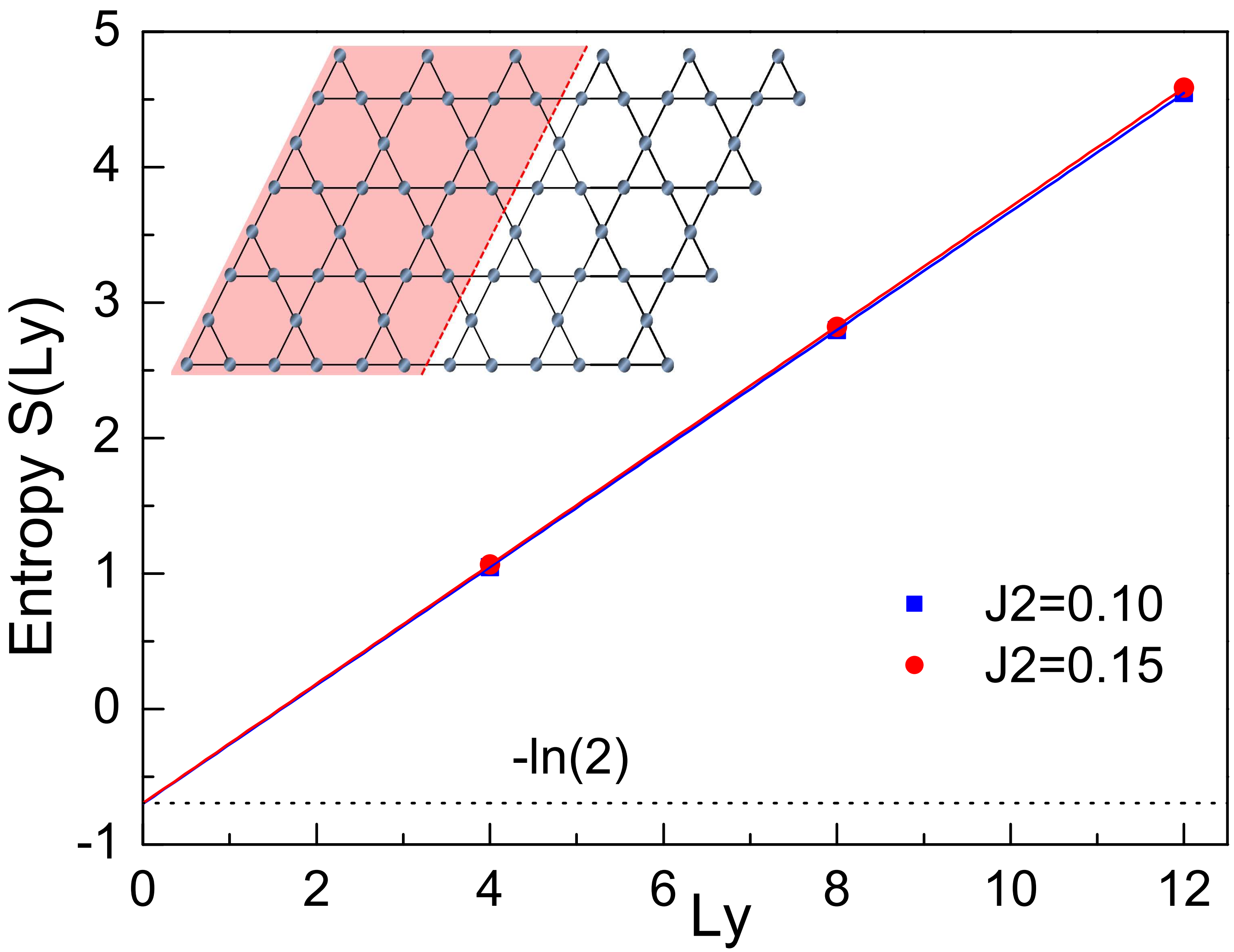
$$S(L) \sim \alpha L - \gamma$$

γ_{DMRG}=0.698(8)

γ_{th}=ln(2)=0.693

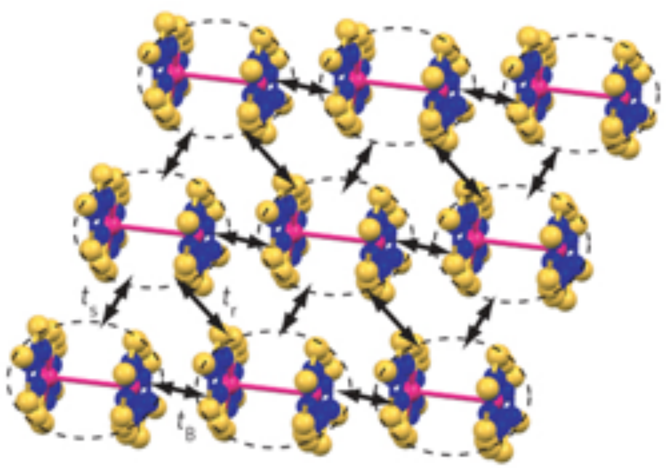
H.C. Jiang, Z. Wang, LB
arXiv:1205.4289

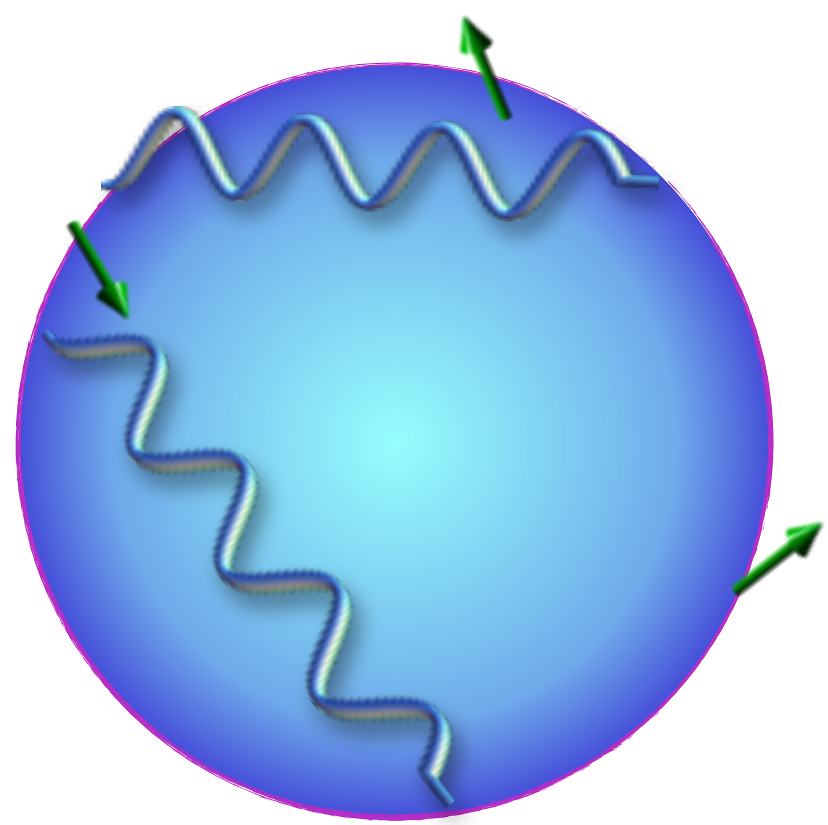




Spinon Fermi Surface

- Proposed to be realized in some organics

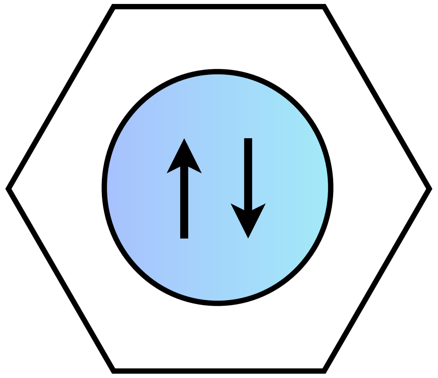




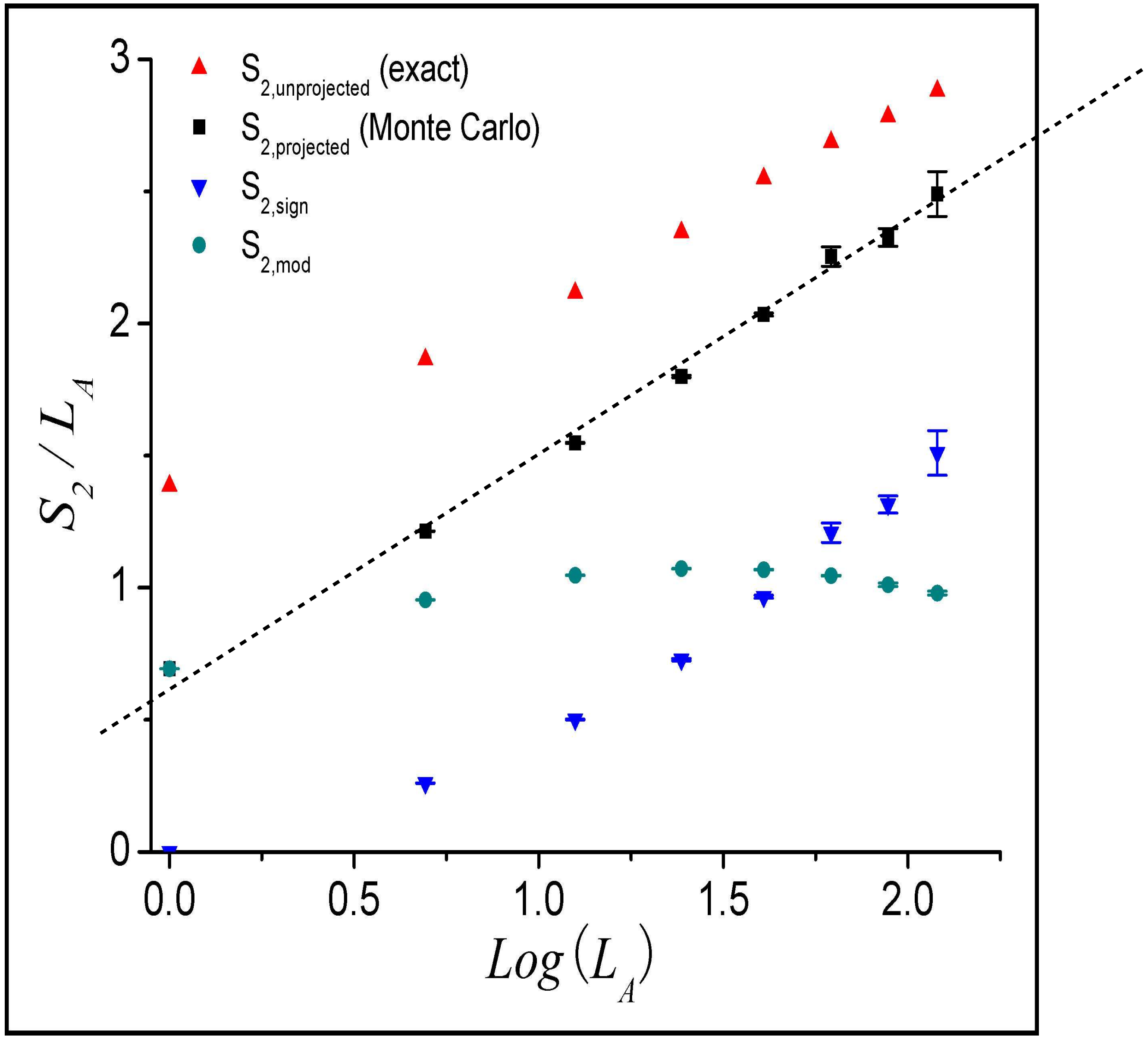
Spinon Fermi Surface

- Anomalous entanglement S ~ L ln (L)

$$\Psi = P_G$$



Yi Zhang *et al*, 2011



Challenges: theory

- [illegible] ca[illegible] on[illegible] ica[illegible] evant [illegible] is [illegible] all[illegible] an[illegible] also [illegible] ex[illegible] QSL behavior [illegible] finite systems

- Many QSLs are described by strongly coupled gauge theories

Recent progress!
Controlled expansion for field theory
Models within AdS/CFT?

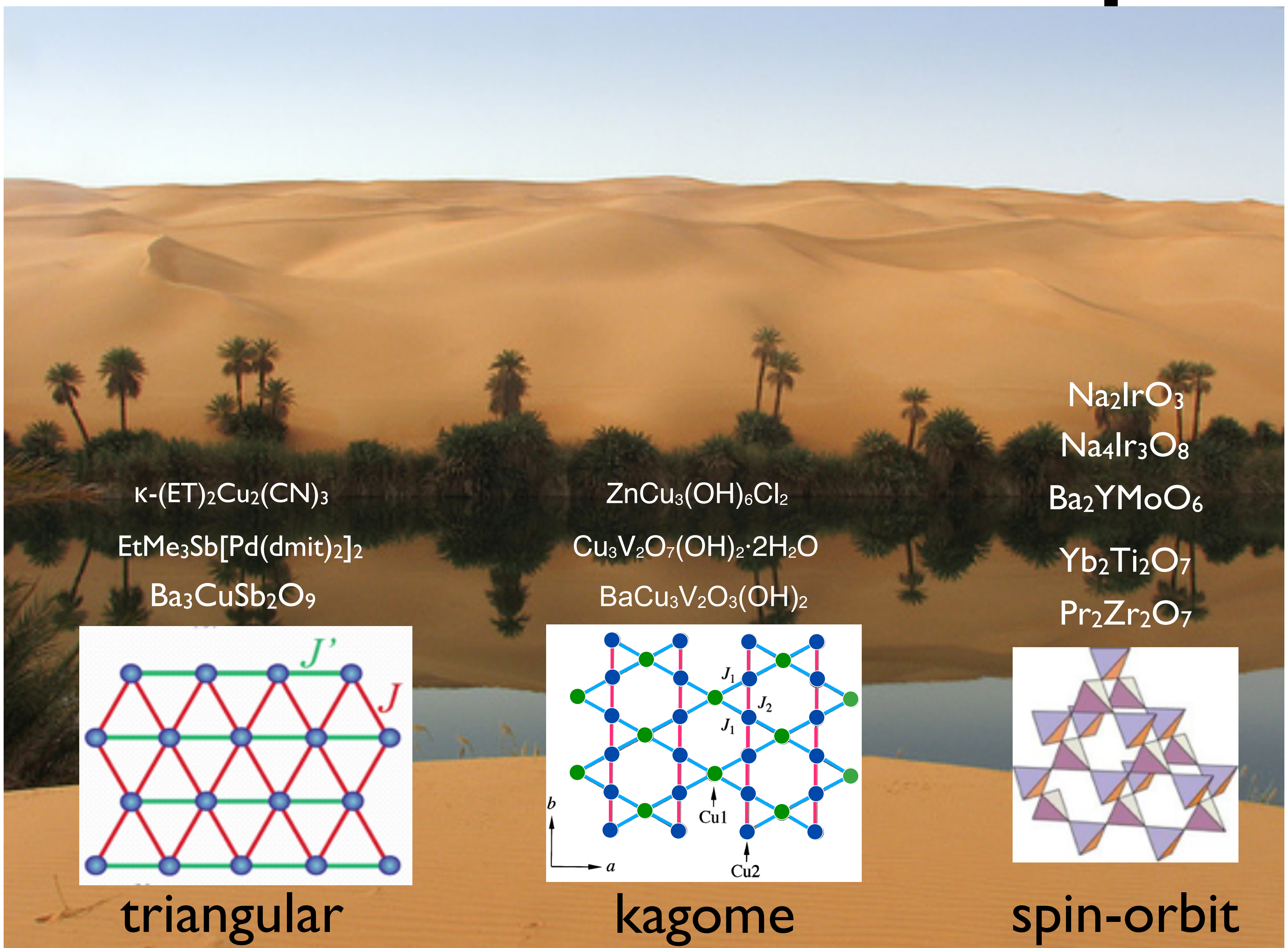
Challenges: theory

- Numerical solution of physically relevant models is very challenging, and it is also hard to extract QSL behavior from finite systems
- Many QSLs are described by strongly coupled gauge theories
- The hardest part is connecting to real materials!

The QSL Landscape

10 years ago

The QSL Landscape

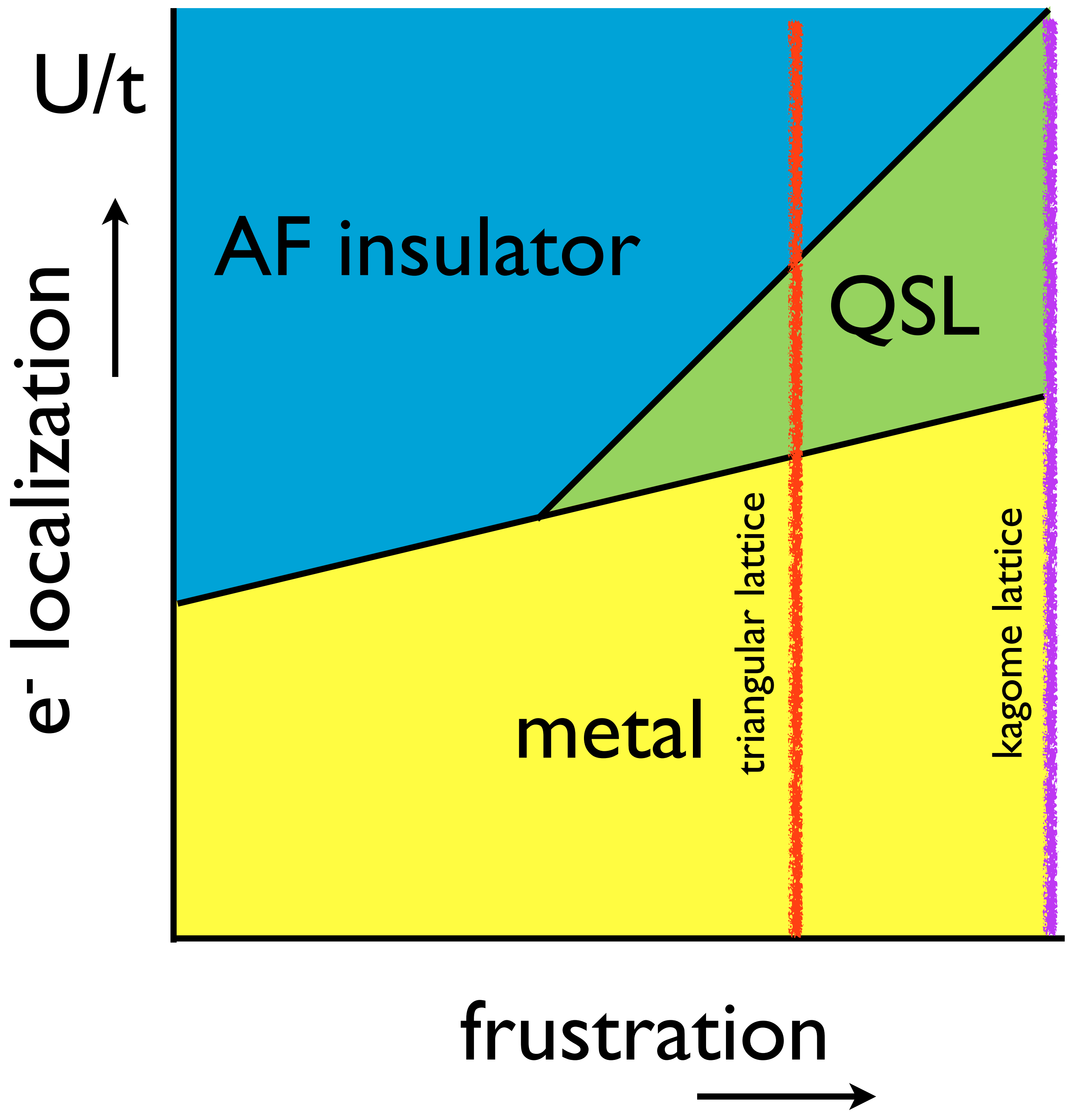


now

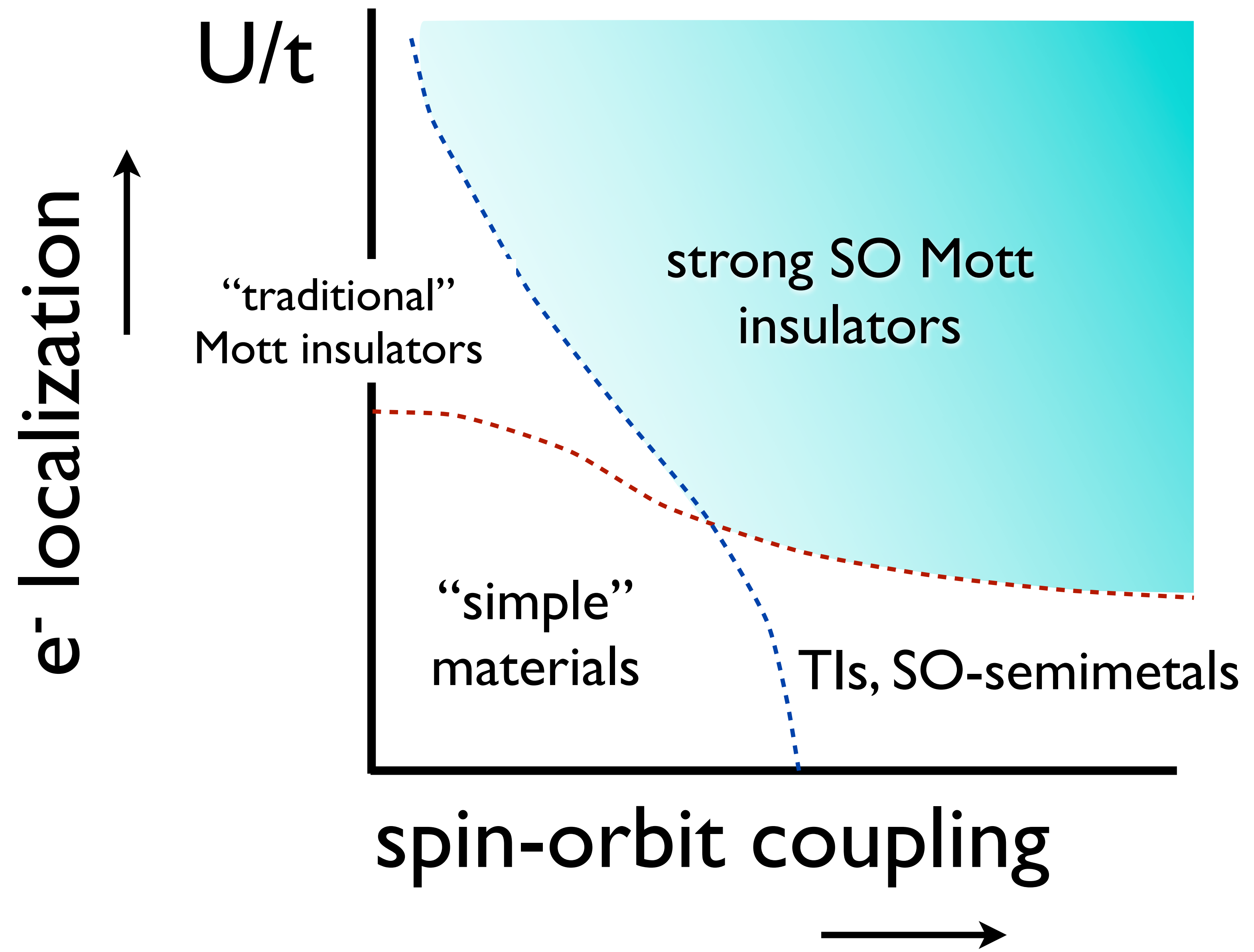
Where to look?

- Materials with
  - S=1/2 spins
  - Frustration
  - Significant charge fluctuations
  - Exotic interactions (c.f. Spin-orbit coupling)

Where to look?



Spin Orbit?



QSLs here?

$Na_4Ir_3O_8$

Ba_2YMoO_6

$Yb_2Ti_2O_7$

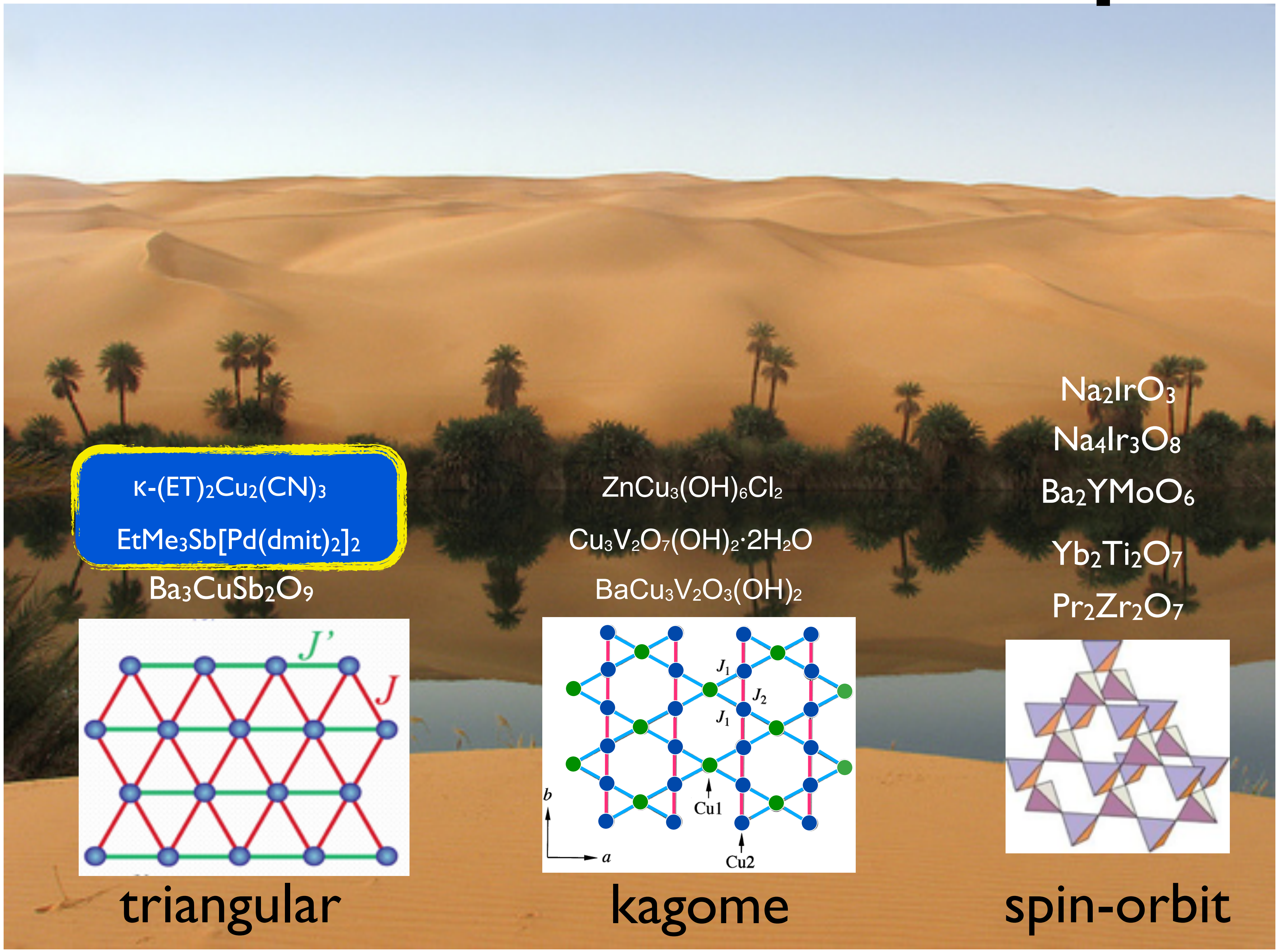
$Pr_2Zr_2O_7$

Na_2IrO_3

Challenges: experiment

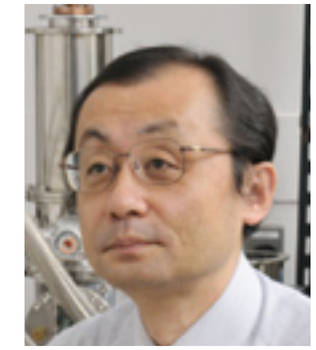
- Quantum order of the ground state is intrinsically *non-local*: not visible to local or spatially averaged probes
- Signatures of quantum order are mainly in the *excitations*
  - We can see these through *thermodynamics* or directly through *scattering*

The QSL Landscape

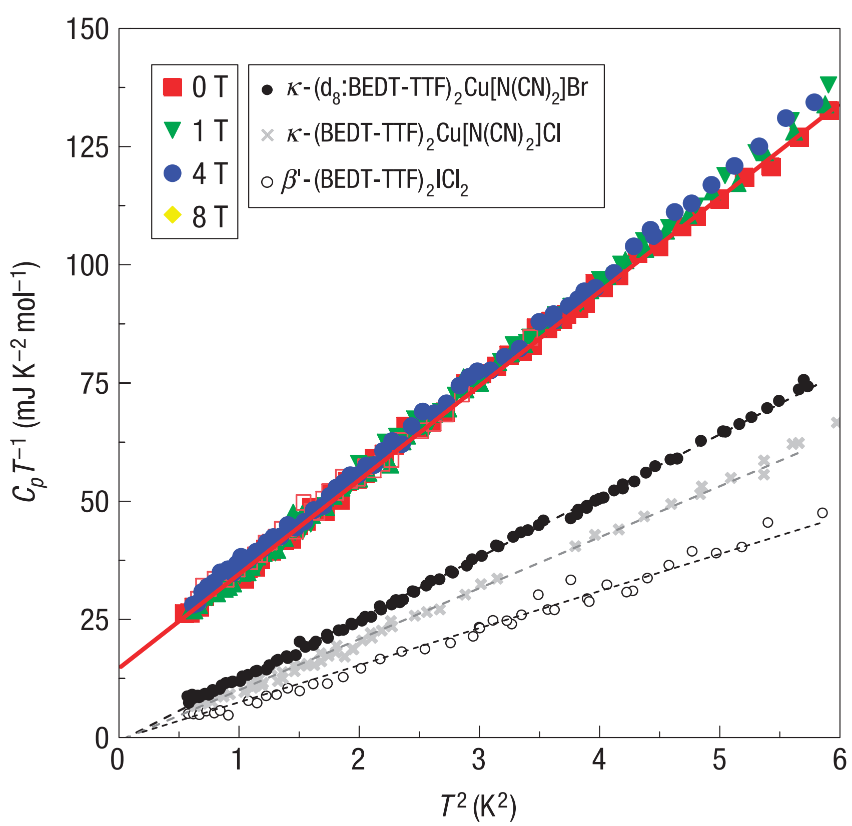


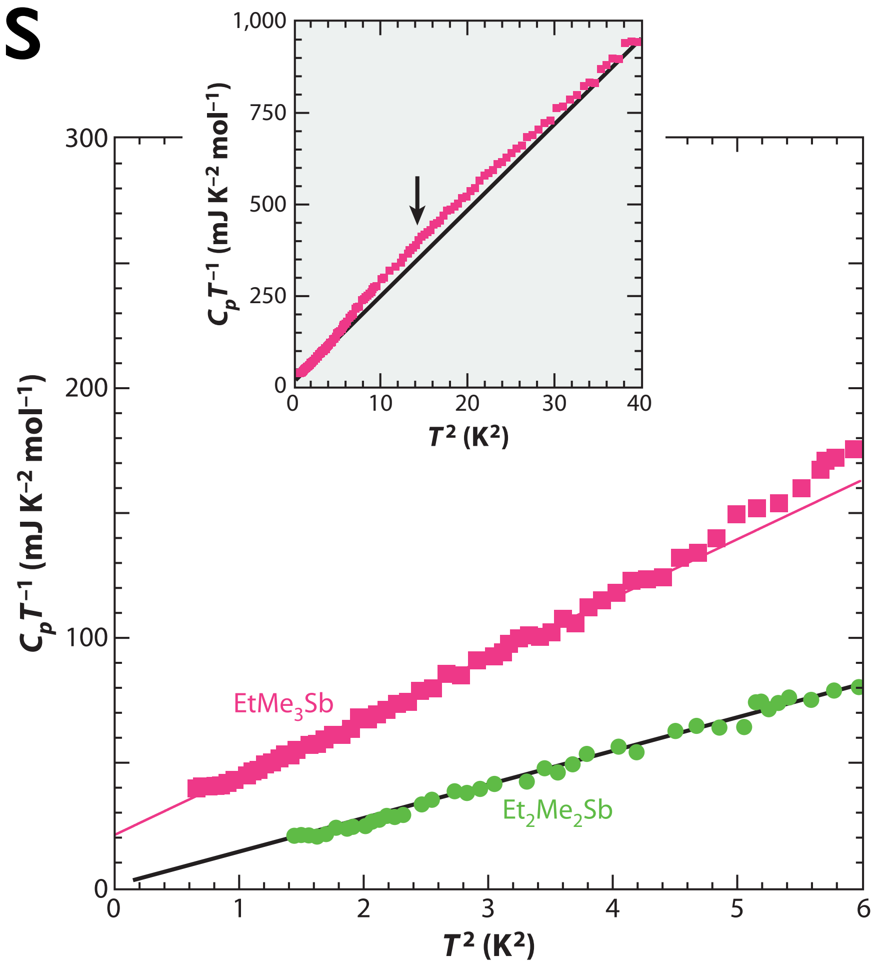
now

Specific Heat



- C ~ γT indicates gapless behavior with large density of states



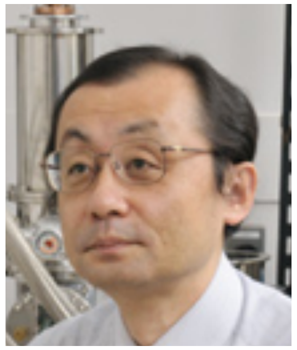


κ-$(ET)_2Cu_2(CN)_3$

β'-Pd$(dmit)_2$

S. Yamashita *et al*, 2008

Specific Heat



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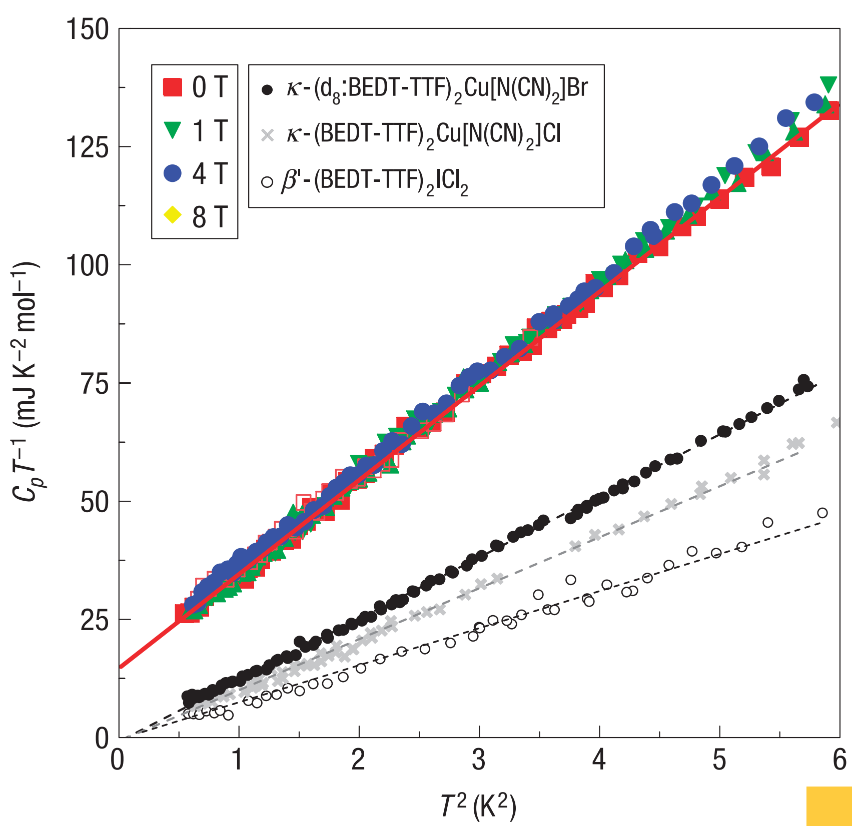
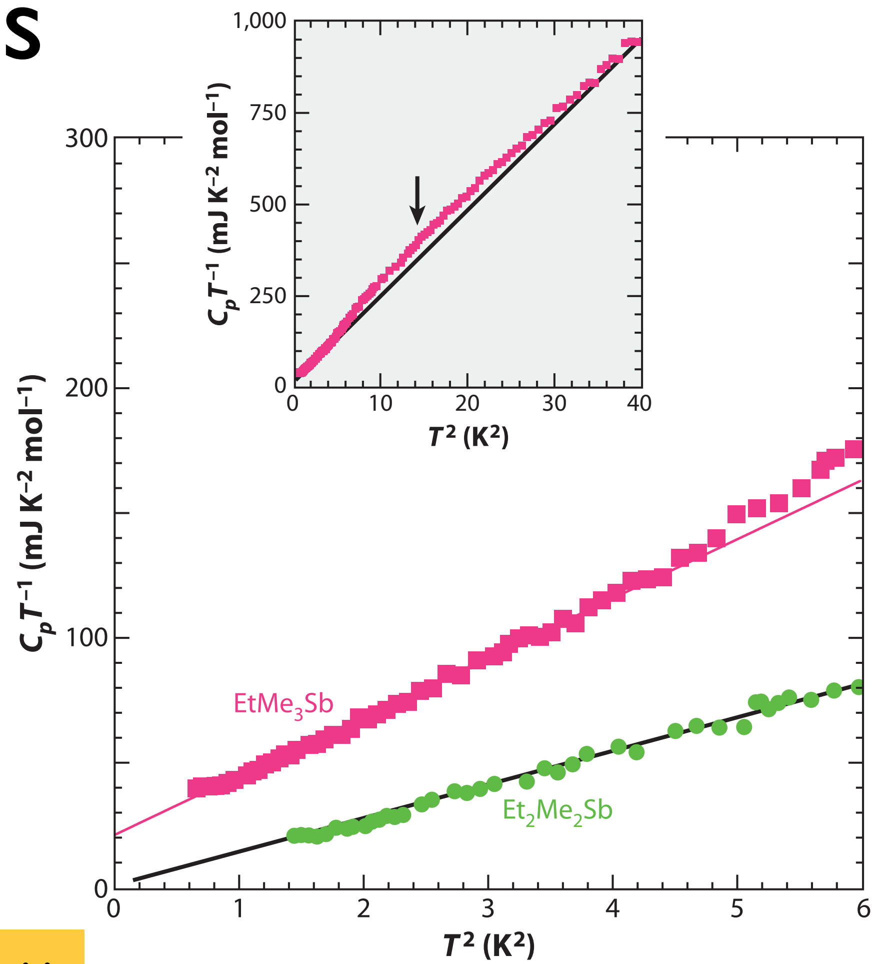




γ_{Cu} ~ 0.7 !!

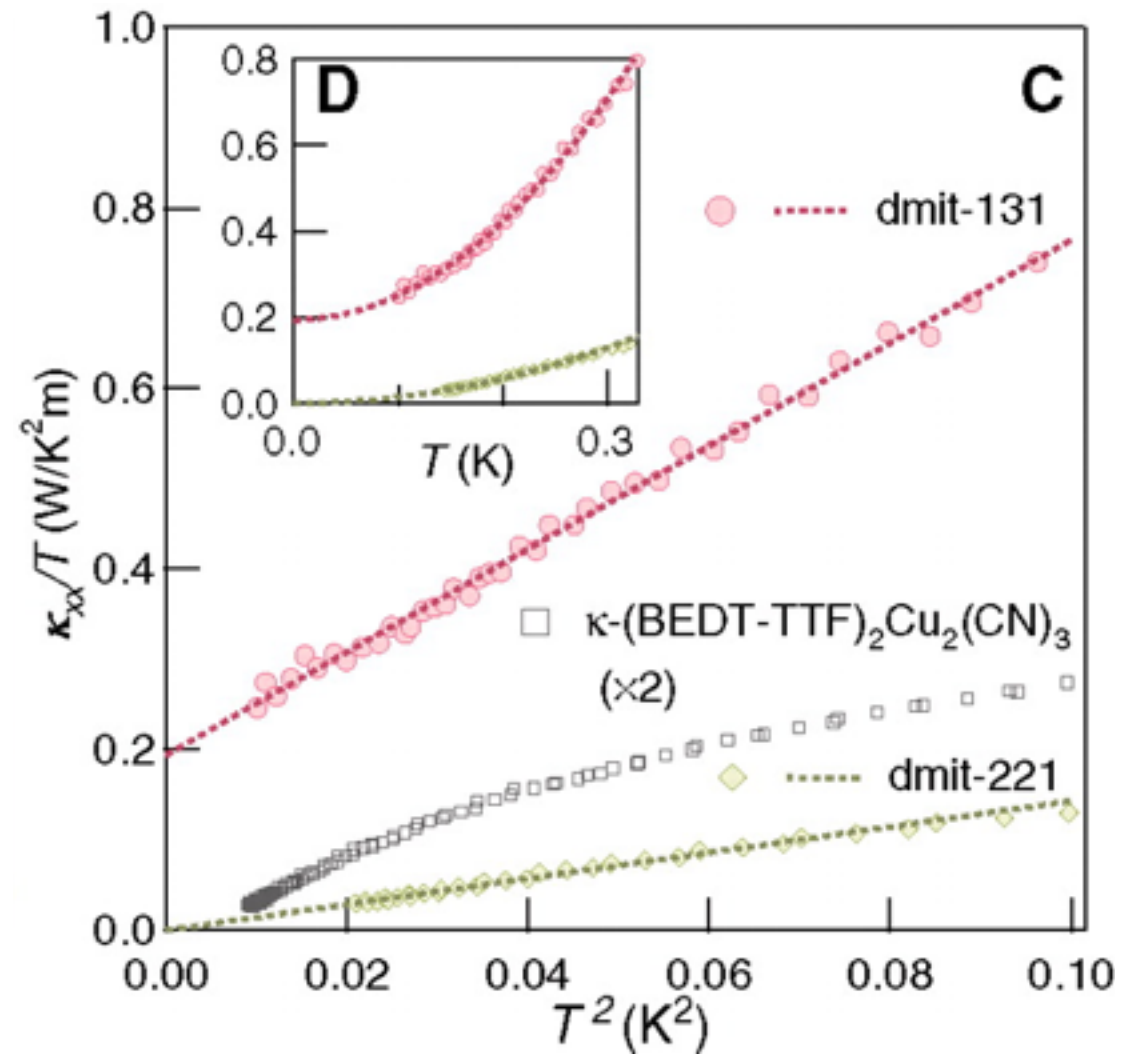
κ-$(ET)_2Cu_2(CN)_3$

β'-$Pd(dmit)_2$

S. Yamashita *et al*, 2008

Thermal conductivity

- Huge linear thermal conductivity indicates the gapless excitations are propagating, at least in dmit
- Estimate for a *metal* would correspond to a mean free path $l \sim 1$ μm ≈ 1000 a !



M. Yamashita *et al*, 2010

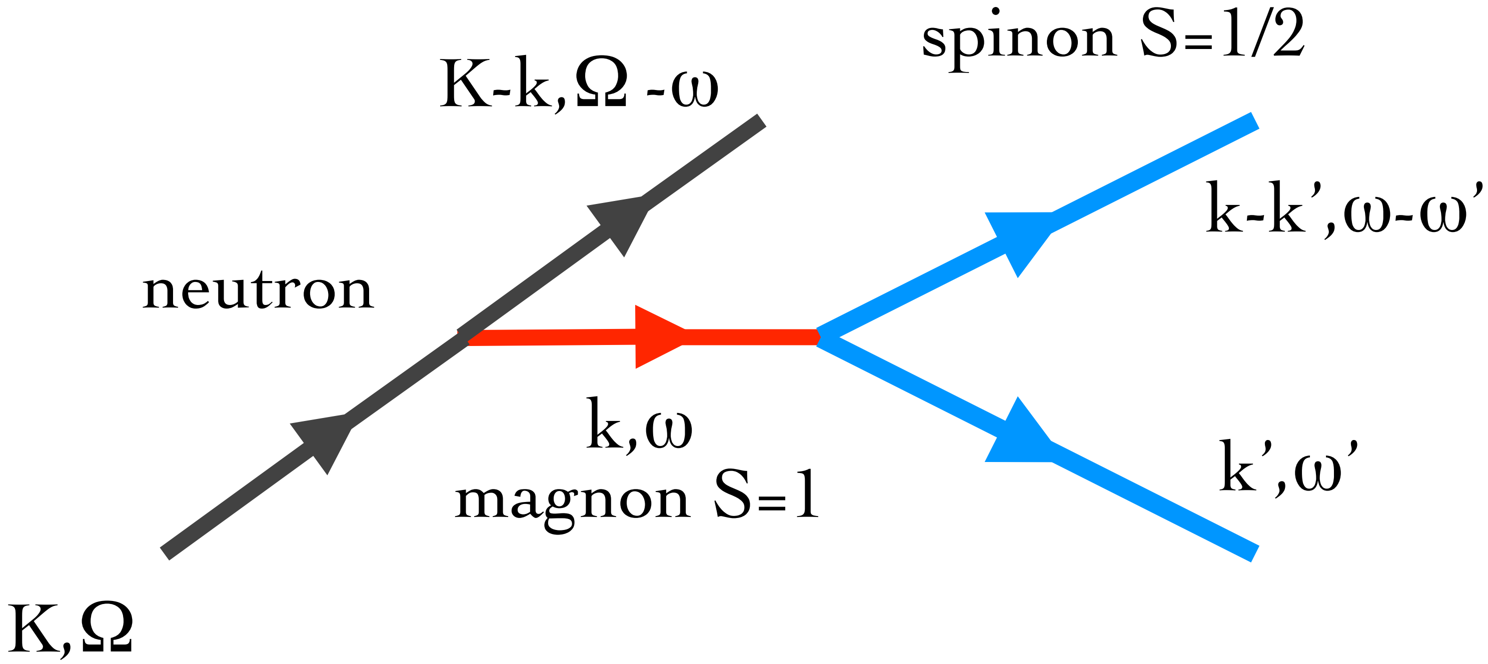
Similar to expectations for spinon Fermi surface

Challenges: experiment

- Quantum order of the ground state is intrinsically *non-local*: not visible to local or spatially averaged probes
- Signatures of quantum order are mainly in the *excitations*
  - ***Can we probe them directly?***

Neutron scattering

- In a quantum spin liquid, the elementary spin excitations are *fractional, S=1/2 spinons*



broad peak with
$\omega=\varepsilon(k')+\varepsilon(k-k')$

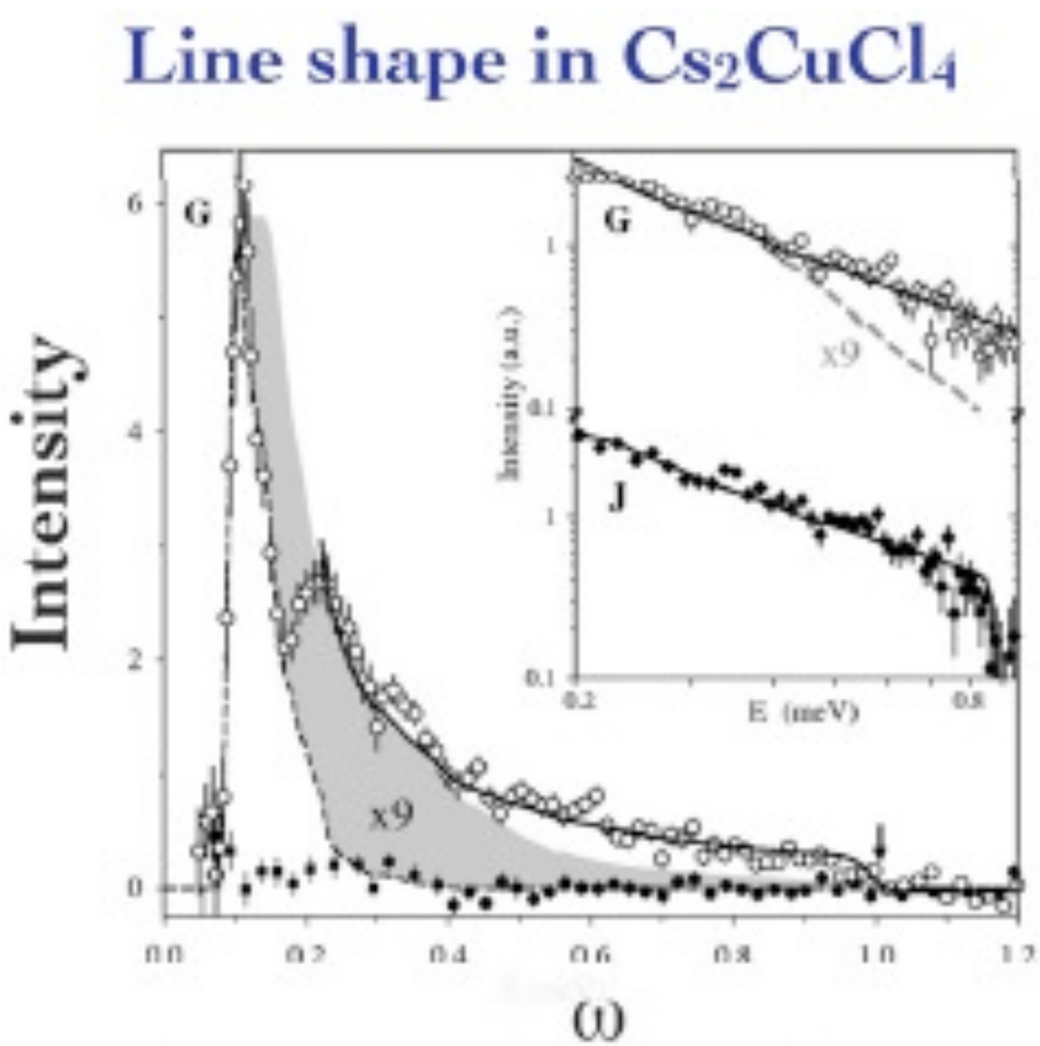
- Most of the information is in the continuum!

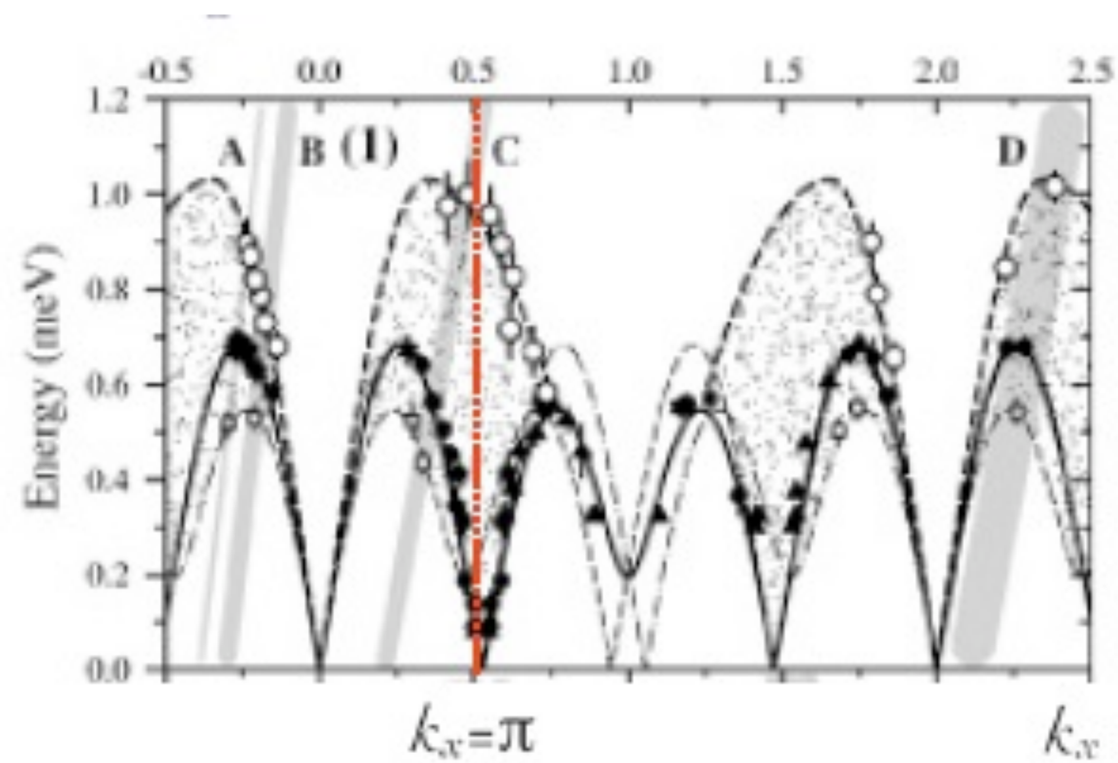
Oleg Starykh

Cs_2CuCl_4

Masanori Kohno

- Proof of principle: 1d spinons





R. Coldea, 2000

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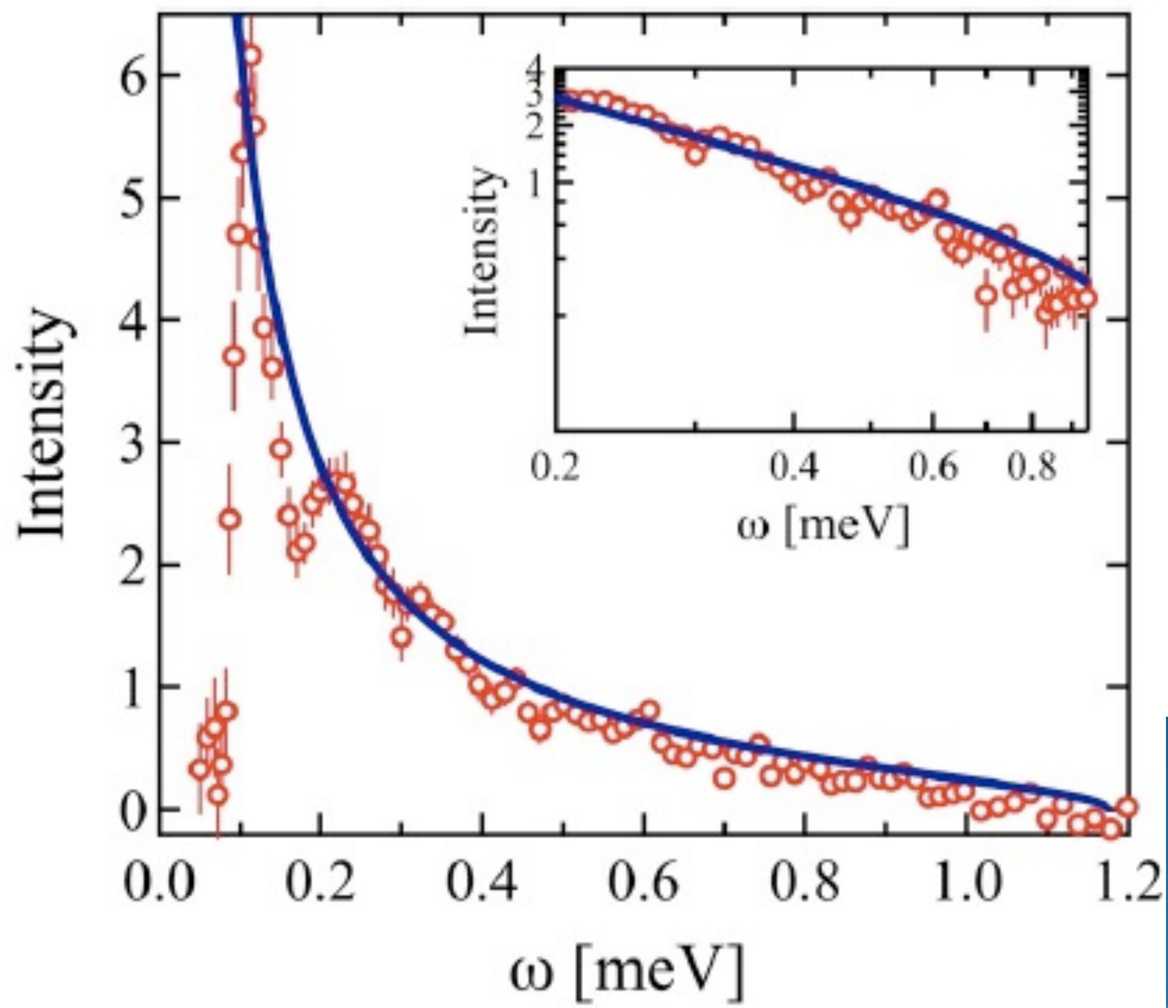
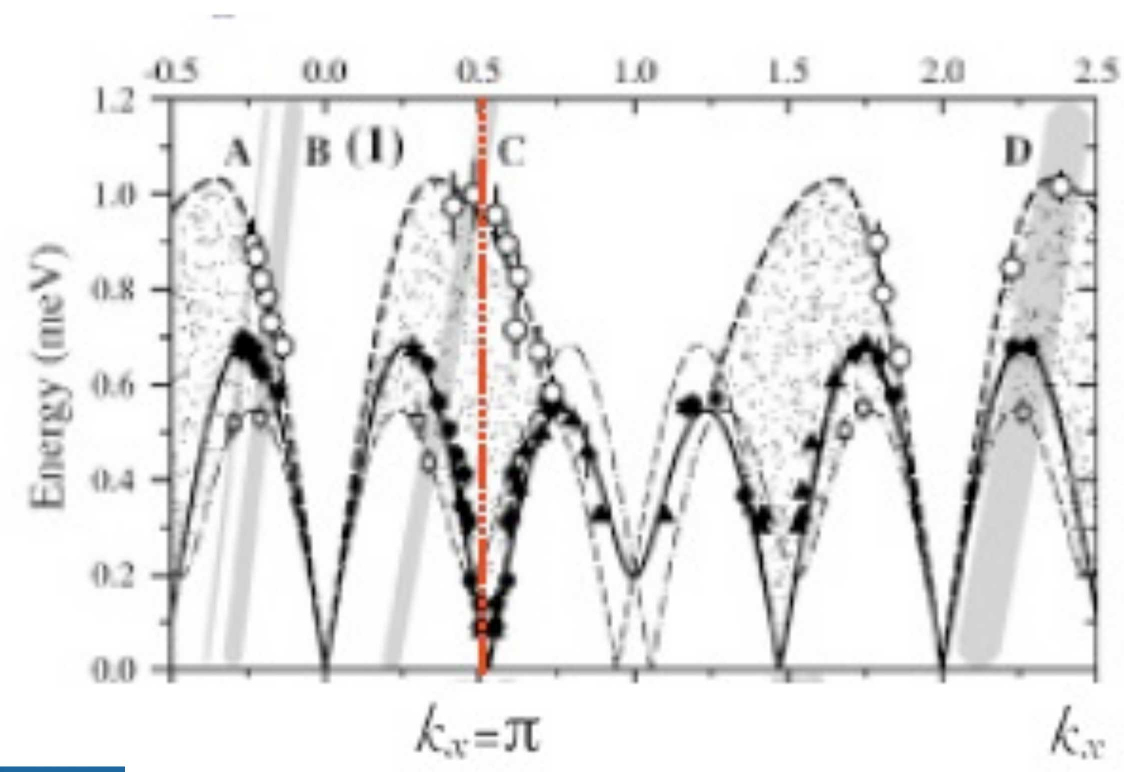
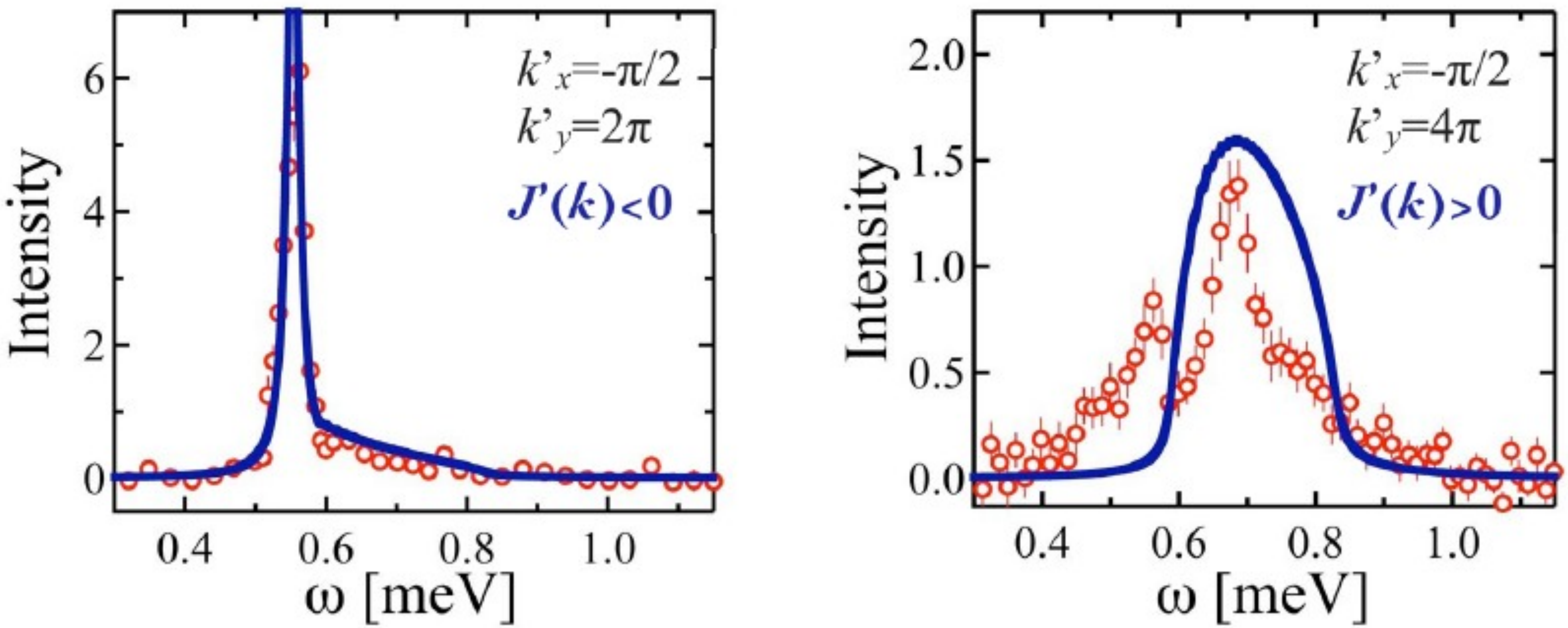




R. Coldea, 2000

Spinon interactions

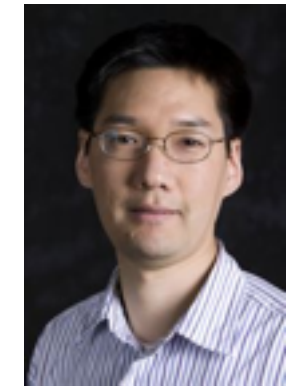
- For each k, spinons may be bound or not



- Curves: 4-spinon theory w/ experimental resolution

Convincing understand required quantitative theory

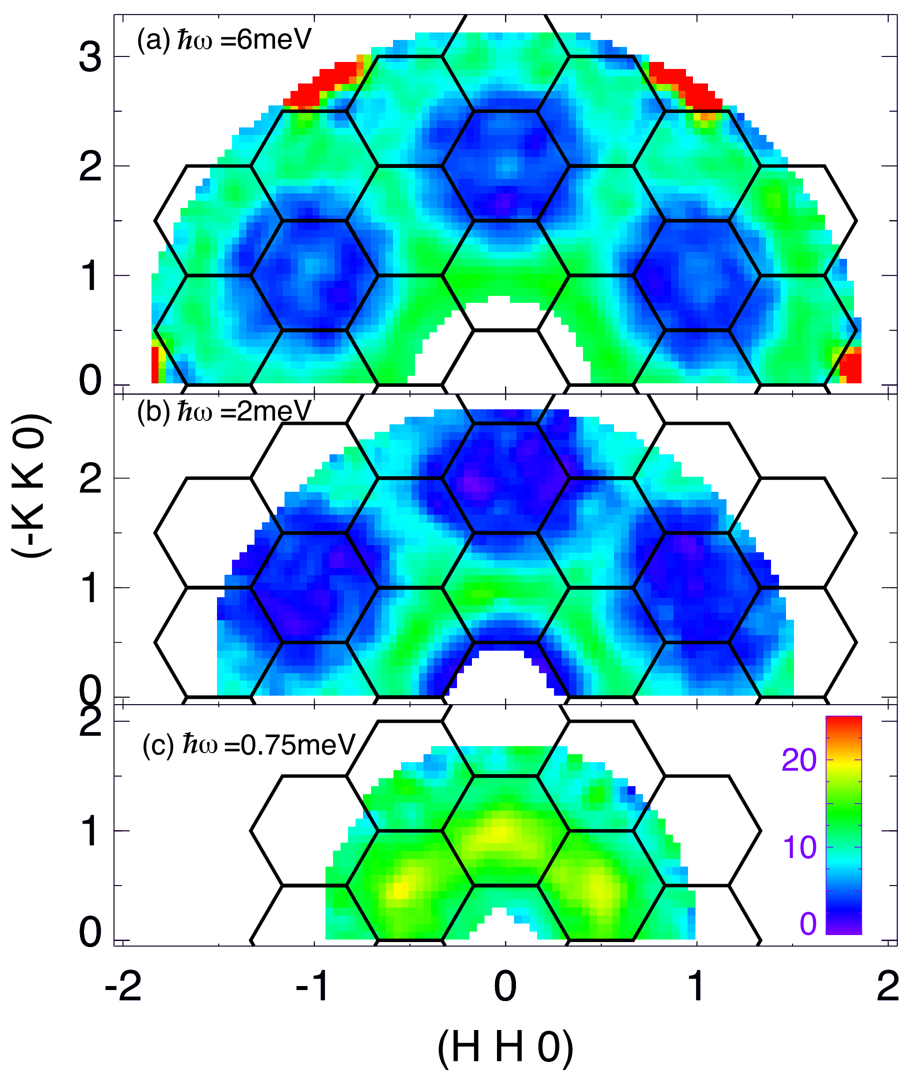
Herbertsmithite



$ZnCu_3(OH)_6Cl_2$

- S=1/2 kagome material does not order to 50mK with exchange J ~ 200K

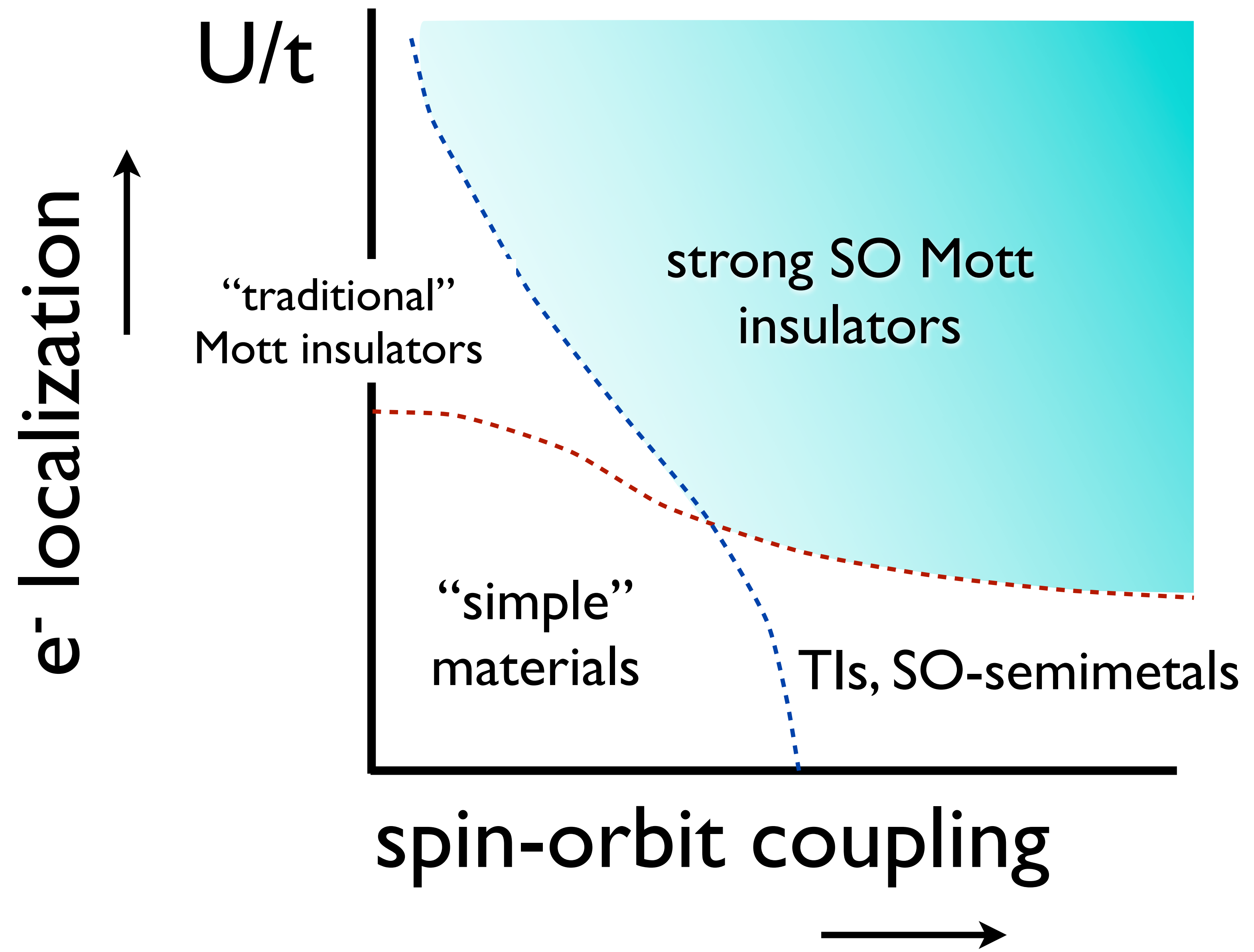
T-H Han *et al*, 2012



spinon continua?
or disorder?

MACS, NIST

Spin Orbit



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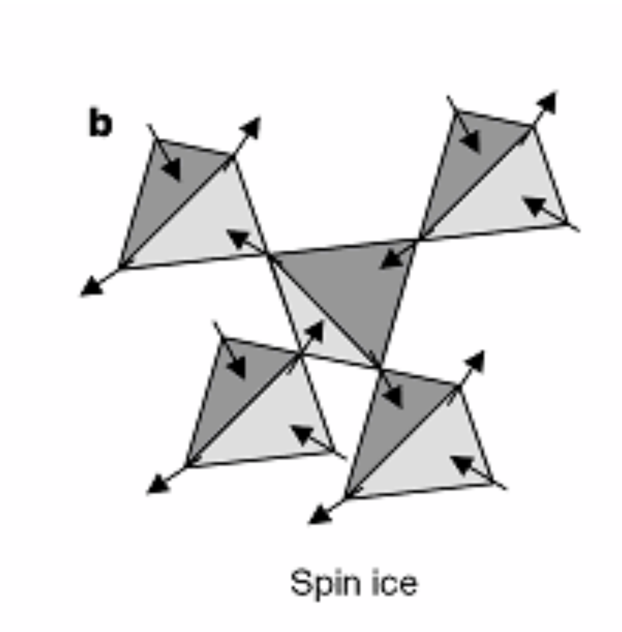
quantum spin ices

Spin ice

- Spins in $Ho_2Ti_2O_7$, $Dy_2Ti_2O_7$ have dominant NN Ising coupling J_{zz} enforcing classical 2in-2out "ice rules" for T < few K



$$H \approx J_{zz} \sum_{\langle ij \rangle} S_i^z S_j^z$$

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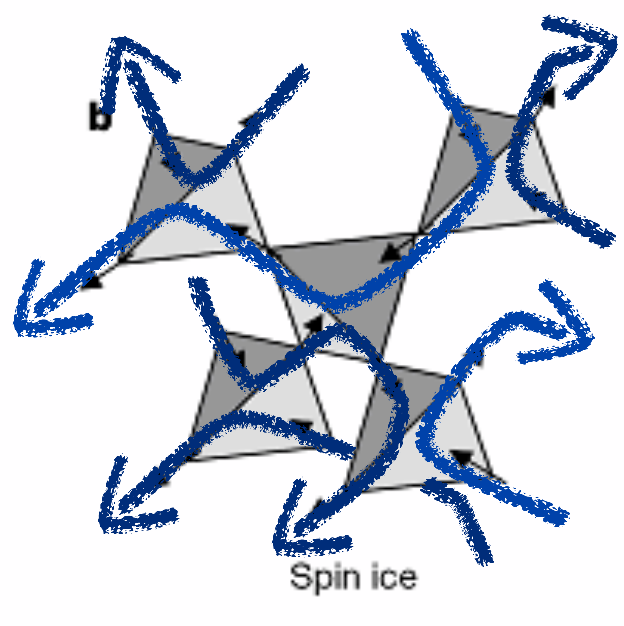


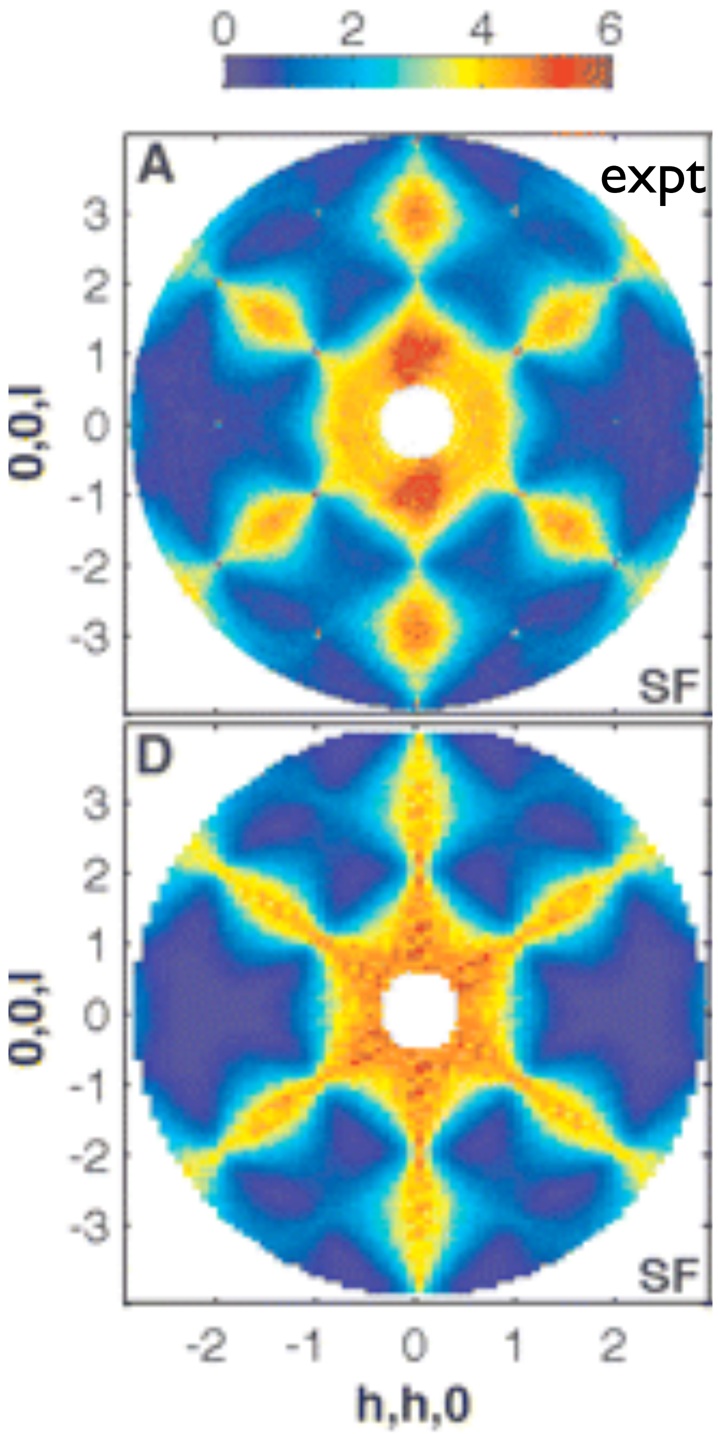
$$\vec{S} \sim \vec{b}$$

$$\vec{\nabla} \cdot \vec{b} = 0$$

artificial magnetostatics: spins map to field lines

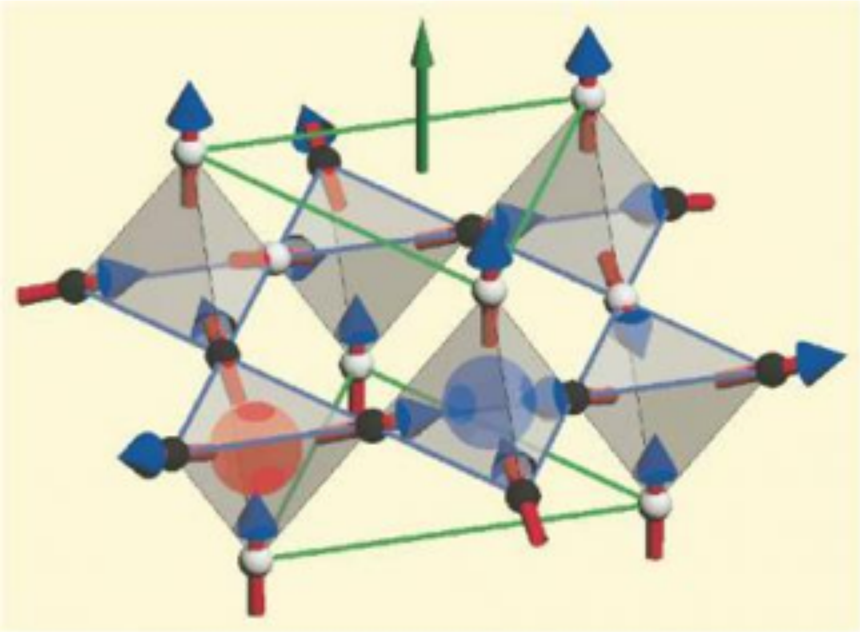
Spin ice

T. Fennell *et al*, 2009

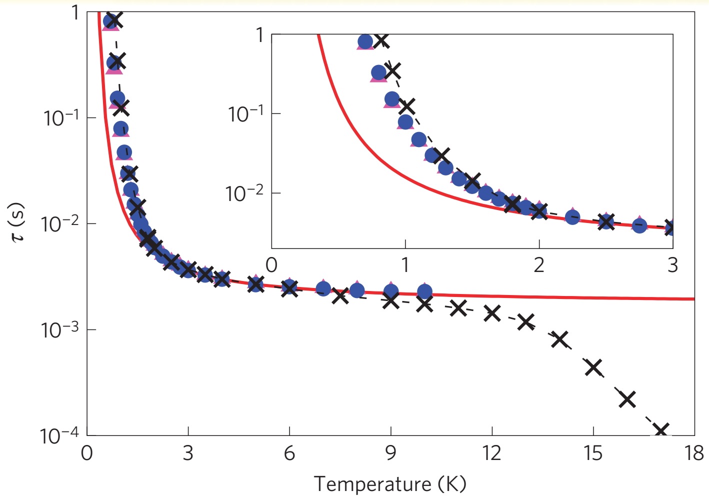


pinch
points

$$\vec{\nabla} \cdot \vec{b} = 0$$



Castelnovo
et al, 2008

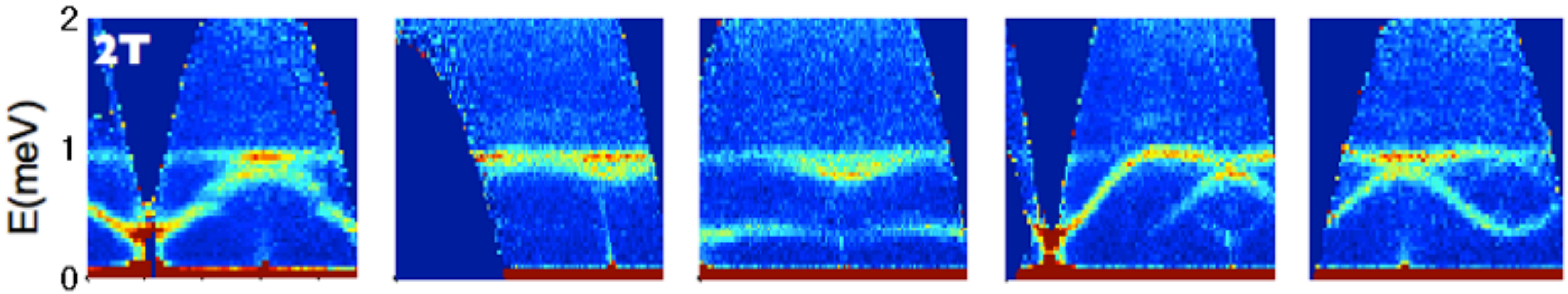


Jaubert and
Holdsworth

magnetic monopoles
behave like diffusing
ions in a
polyelectrolyte

$Yb_2Ti_2O_7$

- INS: spin waves with bandwidth ~ 1meV ~ 10K!
  - indicates *ballistic quantum* spin dynamics



K.A. Ross *et al* (2009,2011)

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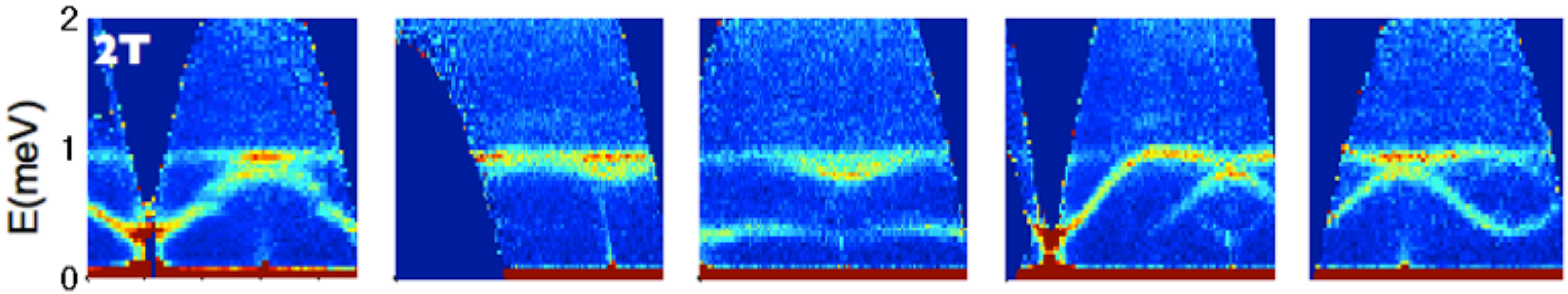
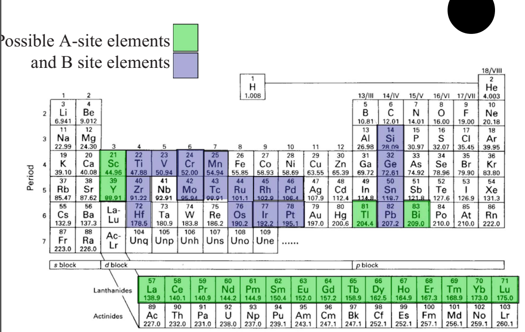


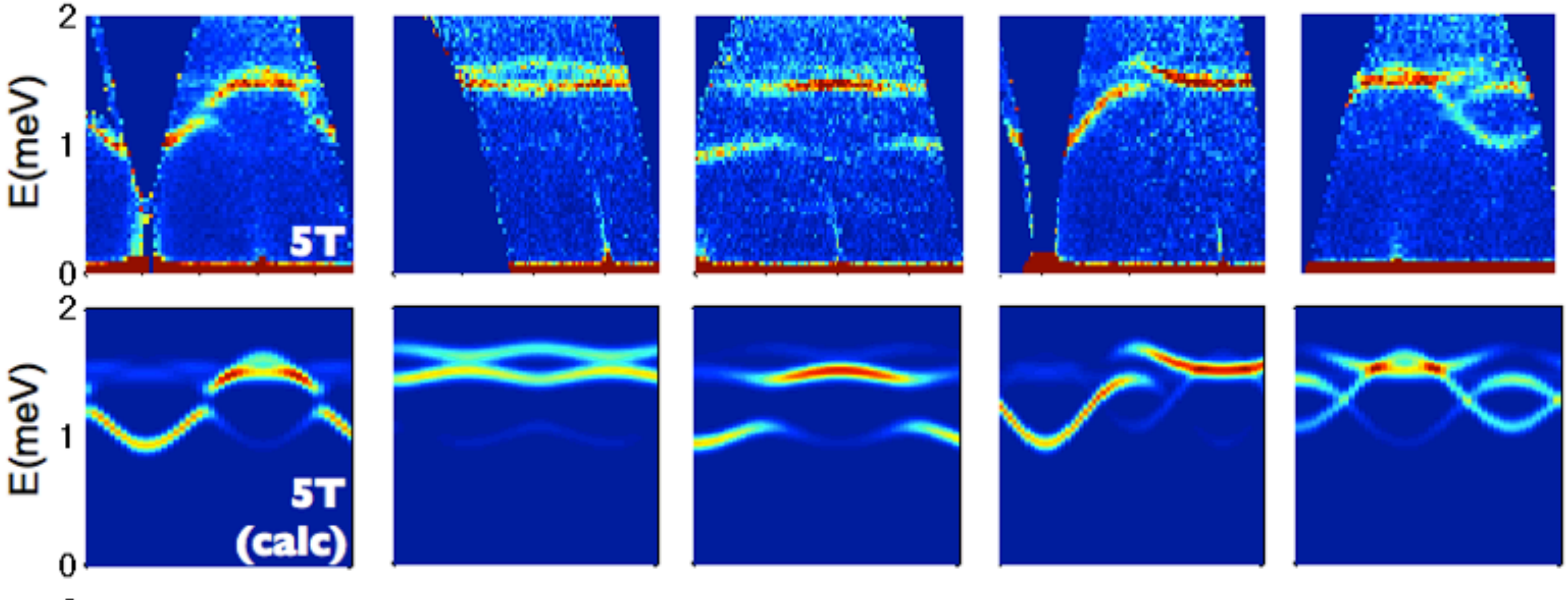
K.A. Ross *et al* (2009,2011)

- Other quantum pyrochlores:
  - $Er_2Ti_2O_7$, $Tb_2Ti_2O_7$, $Pr_2Zr_2O_7$, ...spinels?



$Yb_2Ti_2O_7$

- Complete phenomenological Hamiltonian extracted from INS with B=5T



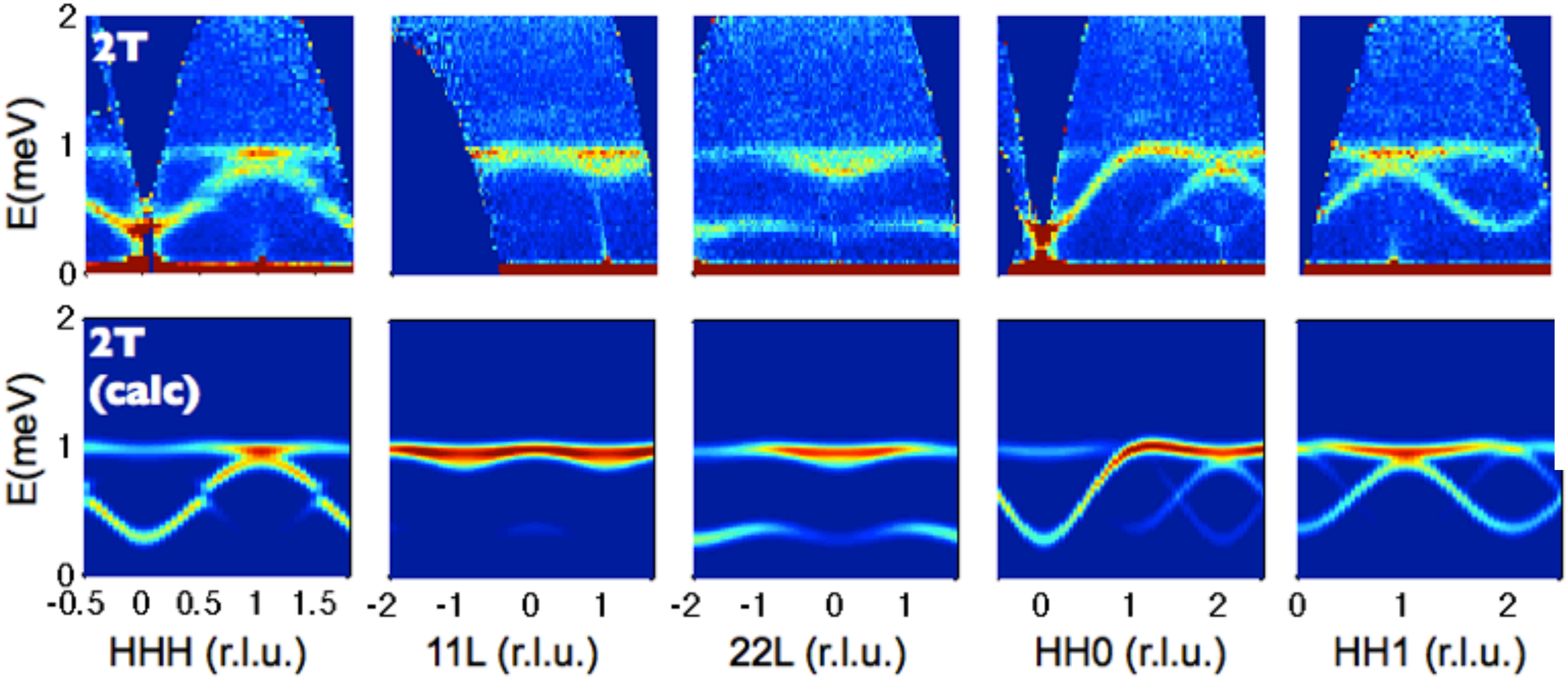
K. Ross, L. Savary, B. Gaulin, and LB, PRX **1**, 021002 (2011)

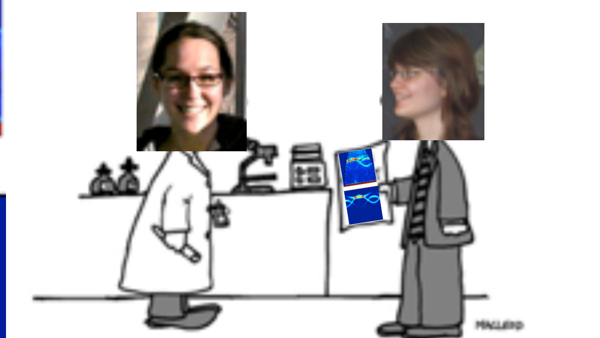
J_{zz} = 0.17±0.04 meV

$J_{\pm}$ = 0.05±0.01 meV $J_{z\pm}$ = 0.14±0.01 meV $J_{\pm\pm}$ = 0.05±0.01 meV

Spin interactions

- Same parameters reproduce B=2T data





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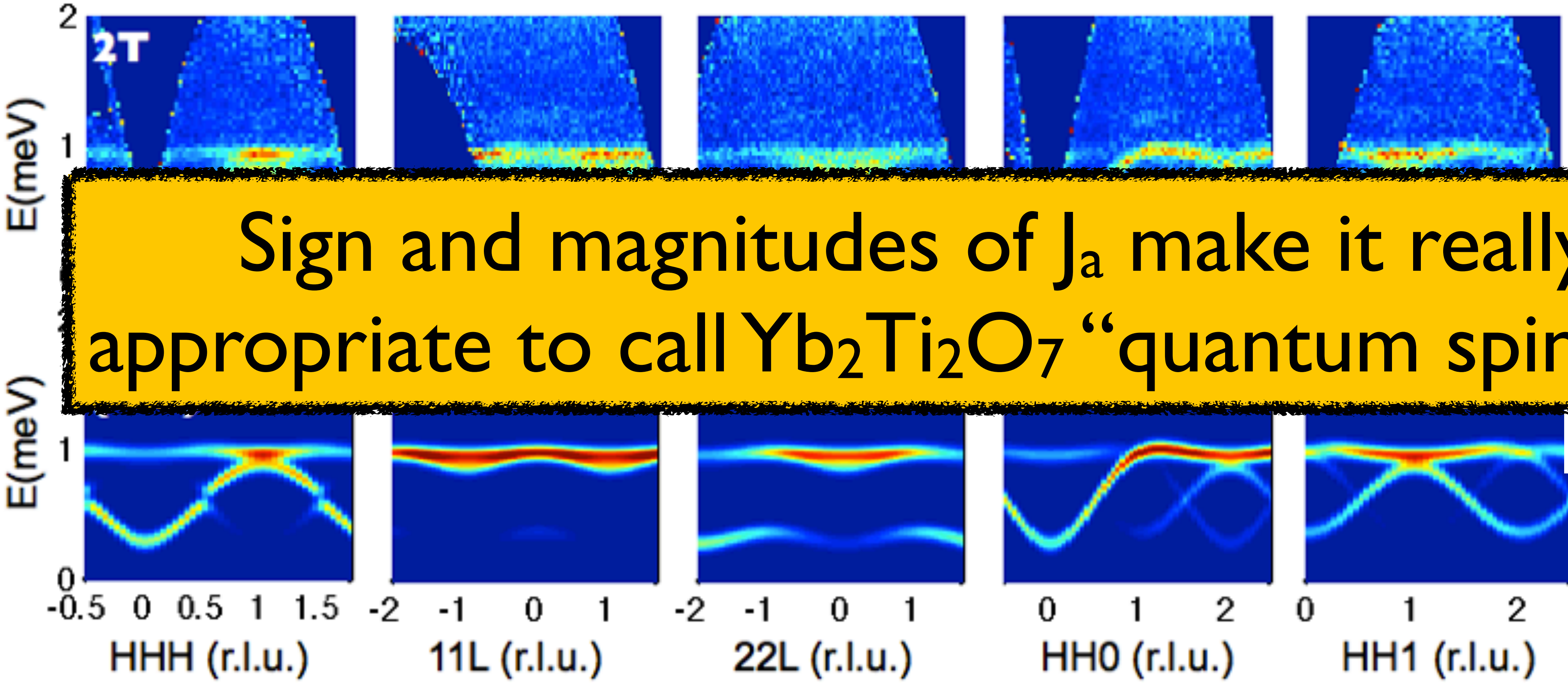

Sign and magnitudes of J_a make it really appropriate to call $Yb_2Ti_2O_7$ "quantum spin ice"

K. Ross, L. Savary, B. Gaulin, and LB, PRX **1**, 021002 (2011)

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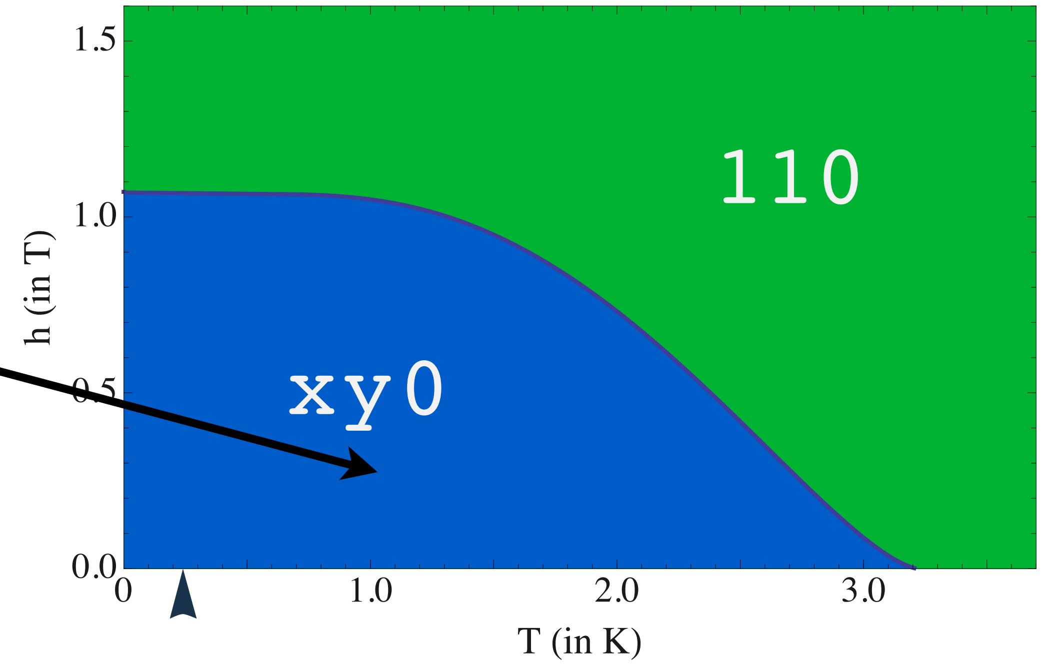
Fluctuations

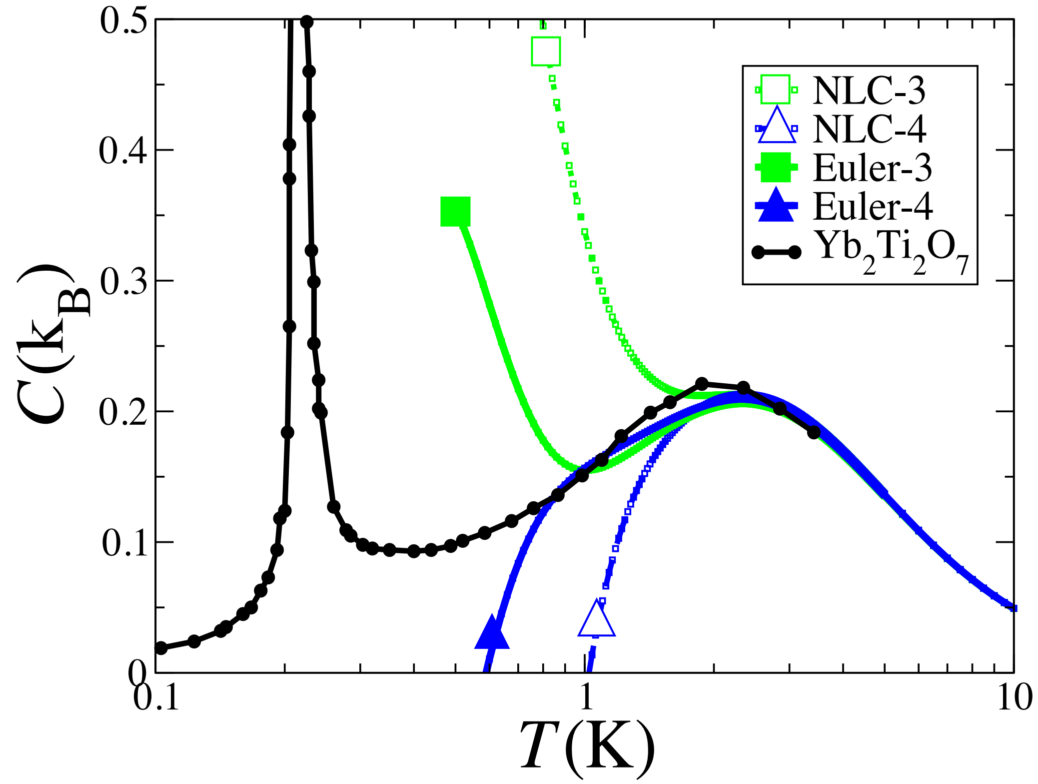
- Comparison with mean field theory *fails badly* at low field

MFT predicts an ordered ferromagnet here



thermodynamic phase transition observed here: 14 times lower temperature than mean field T_c

Model Check?

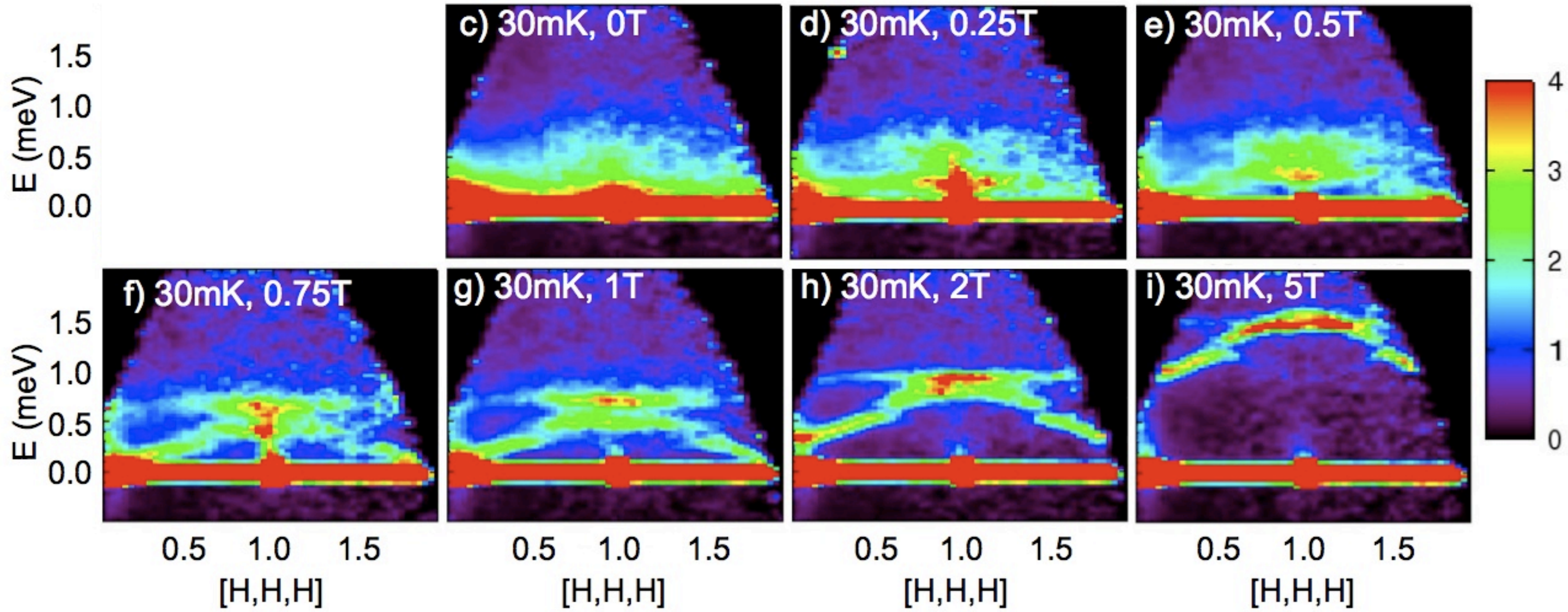


Applegate *et al*, arXiv:1203.4569

- Zero parameter fit to intermediate temperature specific heat!

spin liquid?

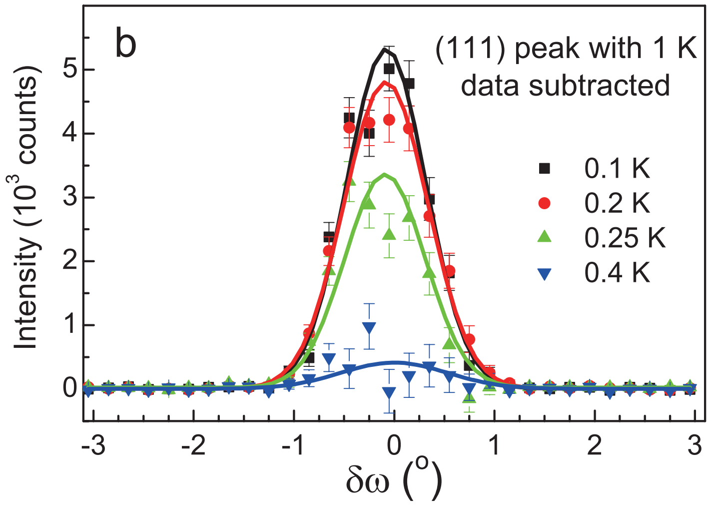
K.A. Ross *et al* (2009)



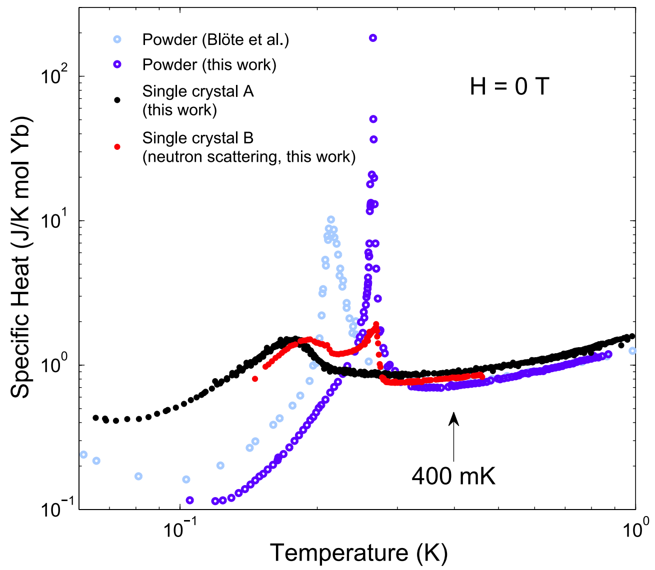
- Spin waves appear absent in low field, but emerge for B>0.5T
  - a low field QSL?

$Yb_2Ti_2O_7$ controversy

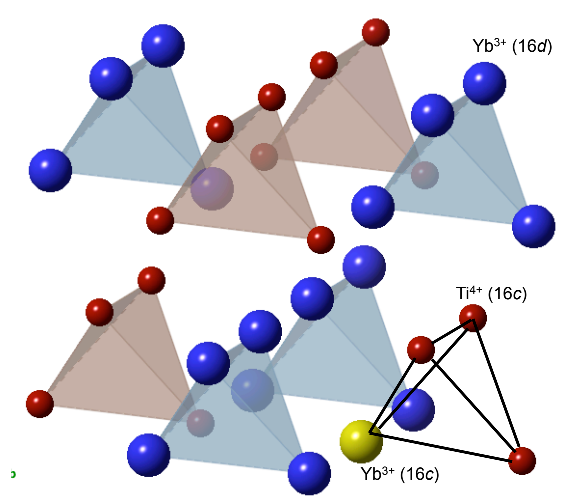
- Is low-T phase ferromagnetic?
    - Majority: no...but
    - Y. Yasui group: yes
- Likely this is due to strong sample dependence



LJ Chang *et al*, 2011 (pub2012)

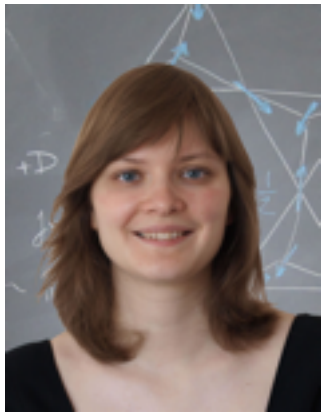


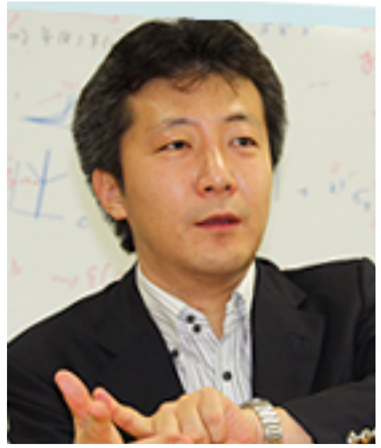
KA Ross et al, 2012

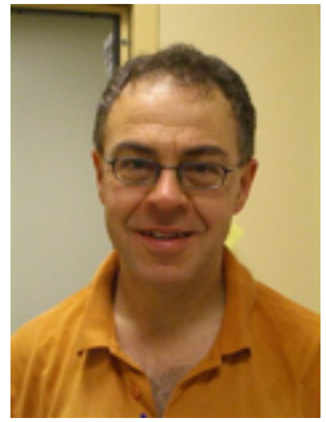


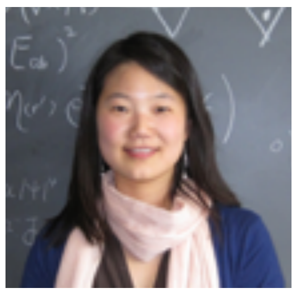
$Yb_{2+x}Ti_{2-x}O_7$

"Stuffed" quantum spin ice?



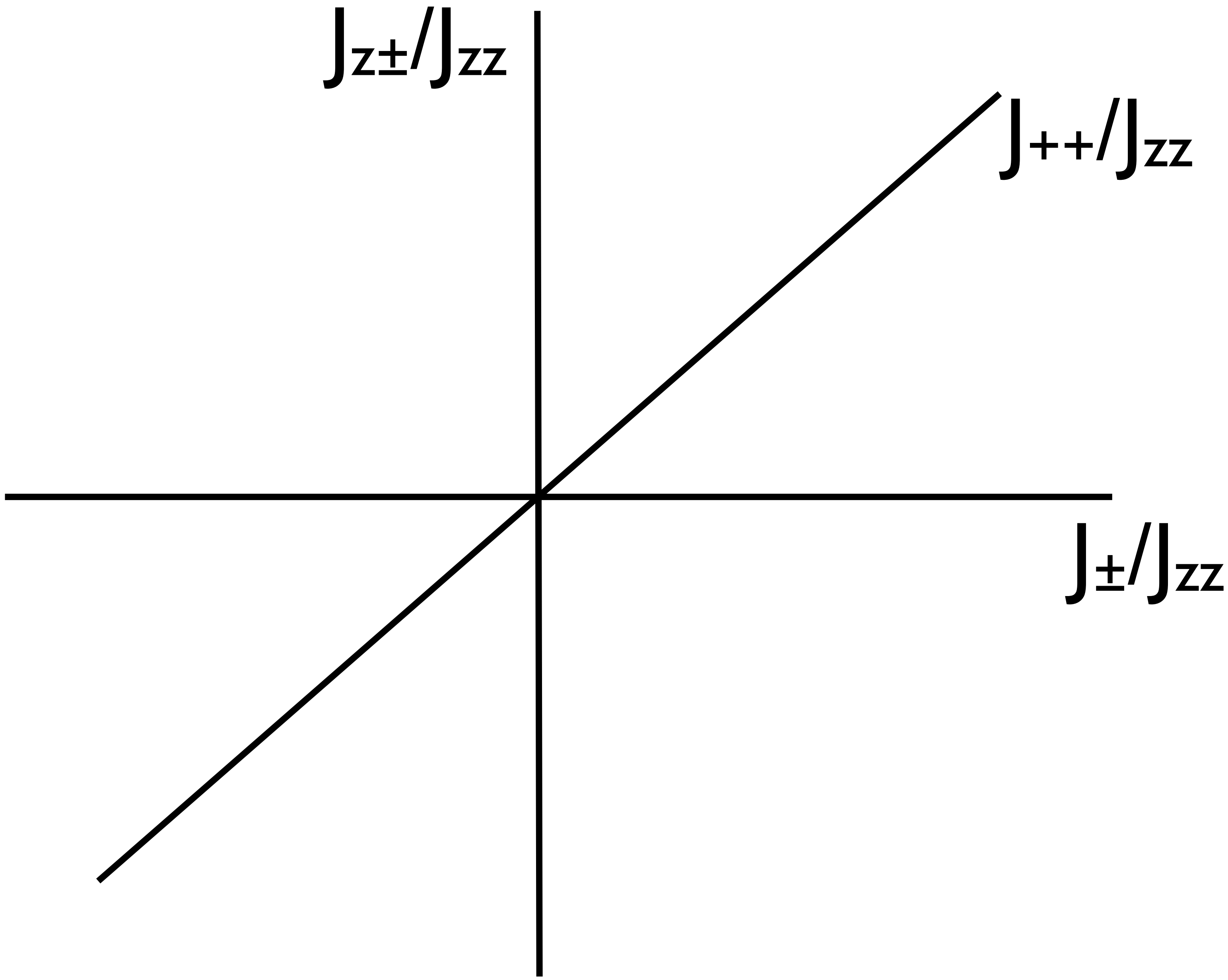




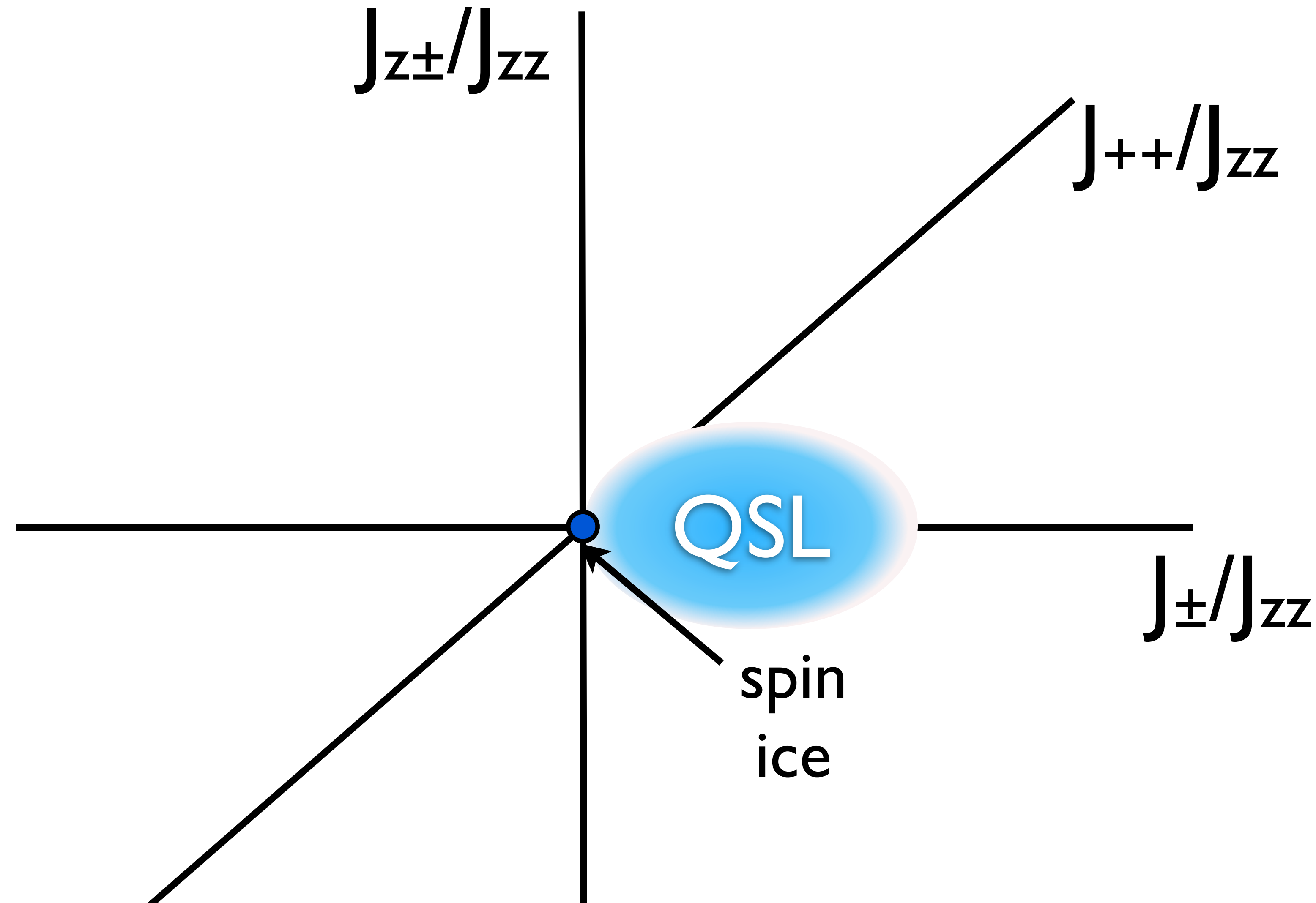



ICE
AGE
2
THE QUANTUM

T=0 Phase Diagram



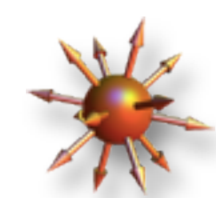
T=0 Phase Diagram



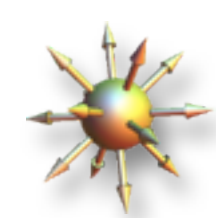
U(1) QSL = emergent compact QED
M. Hermele, MPA Fisher, L. Balents, 2004
A. Banerjee *et al*, 2008

Excitations

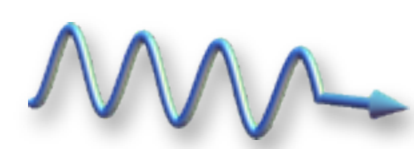
- Where spin ice realizes "emergent magnetostatics", the U(1) QSL is "emergent compact quantum electrodynamics"
  - *coherent propagating* monopoles = "spinons"

  - dual (electric) monopoles

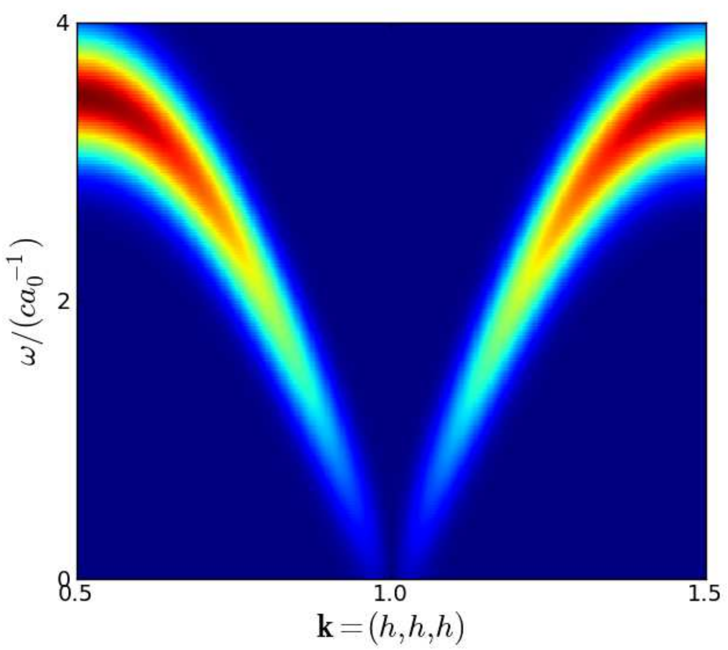
  - artificial photon: gapless!

- Consistent with observed continuum?

Emergent “Photon”

- Some QSLs may have sharp collective excitations, such an “emergent photon” in a 3d U(1) QSL



O. Benton *et al*, 2012

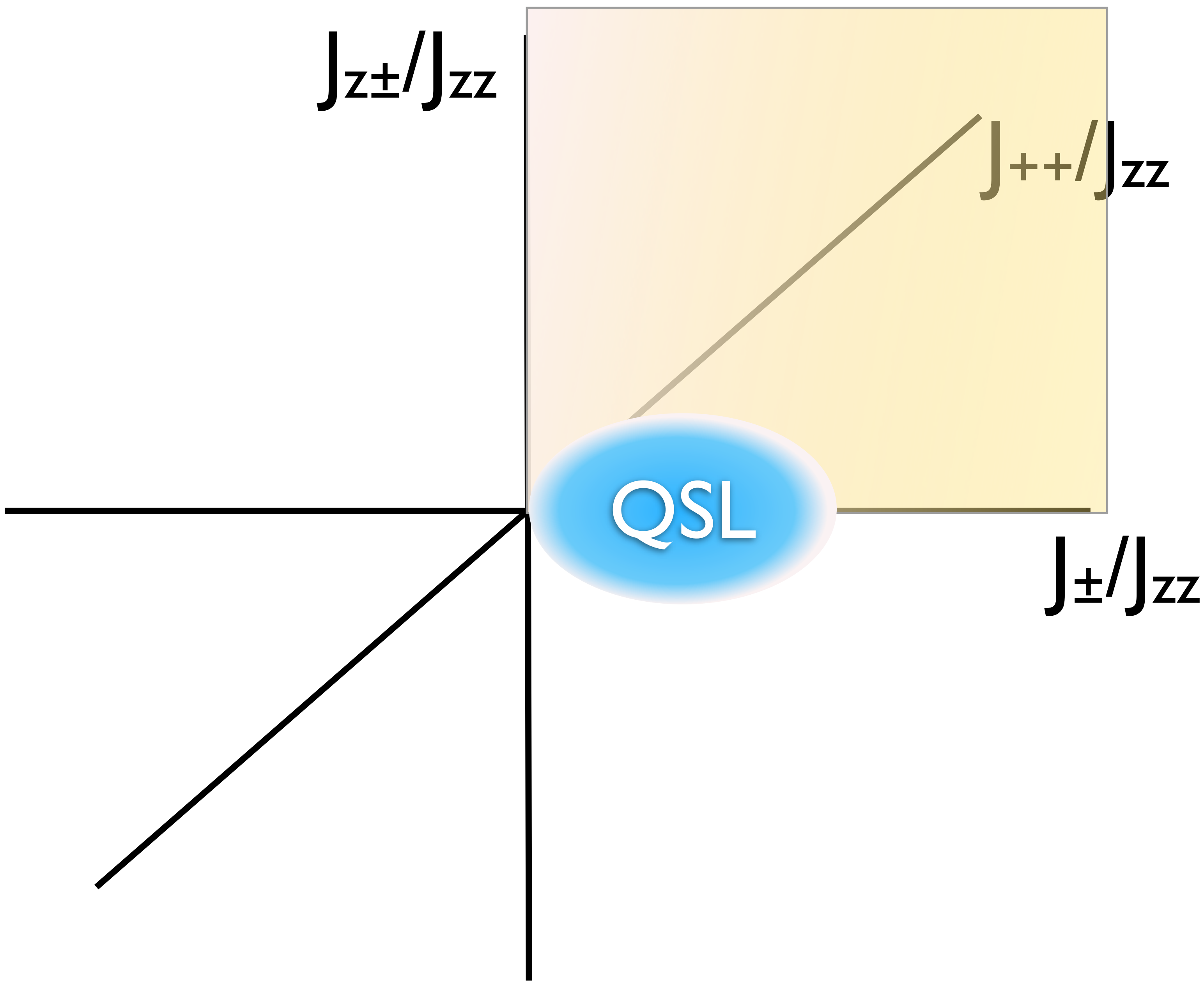
Similar to a spin wave but:

- Is purely transverse
- Intensity vanishes as $\omega \to 0$
- Has no anisotropy gap

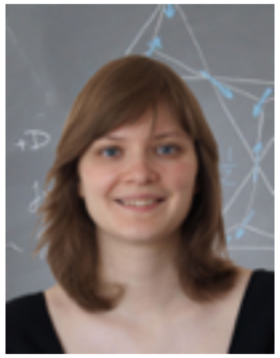
+ gapped spinon continuum

L. Savary + LB, 2012

T=0 Phase Diagram



non-perturbative approach?



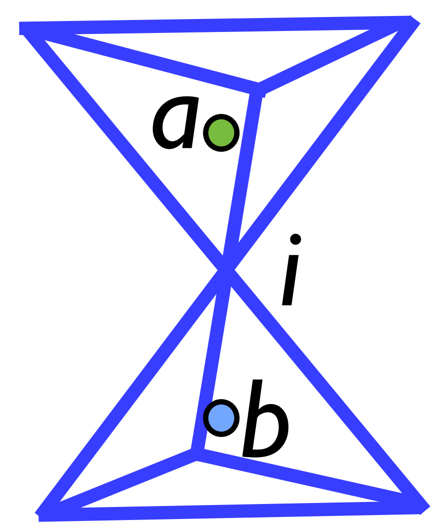
Slave rotor formulation

- Exact reformulation of Hamiltonian

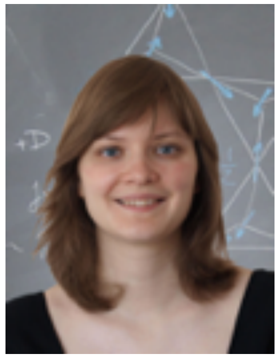
$$Q_a = (-1)^a \sum_{i \in a} S_i^z$$

$$S_i^{\pm} = \Phi_a^\dagger \Phi_b s_{ab}^{\pm}$$

$$S_i^z = s_{ab}^z$$



- Meaning:
  - Q_a is gauge (monopole/spinon) charge
  - $\Phi_a = e^{i\gamma_a}$ is spinon annihilation operator



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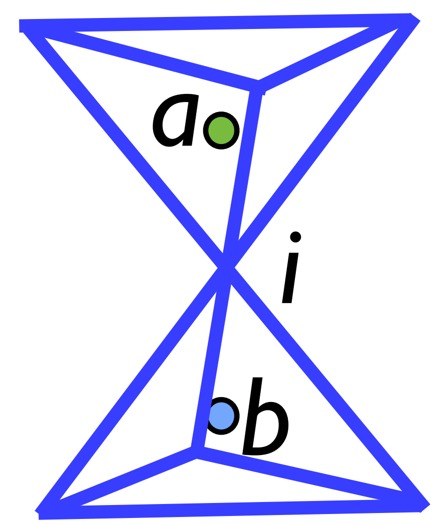
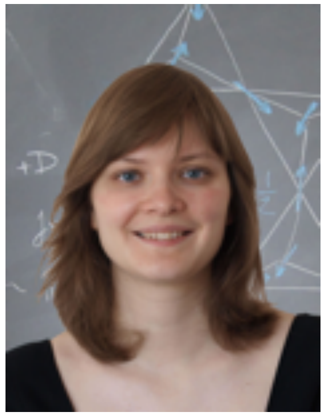


- Gauge symmetry

$$\Phi_a \to \Phi_a e^{i\theta_a} \qquad s_{ab}^{\pm} \to s_{ab}^{\pm} e^{i(\theta_a - \theta_b)}$$

- $s_{ab}^{\pm} = e^{iA_{ab}}$ play the role of gauge fields



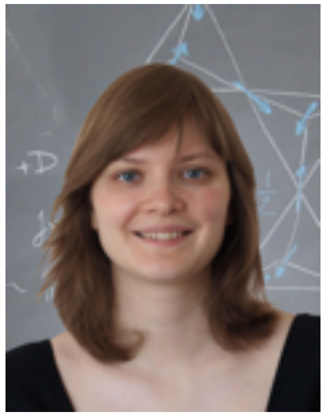
Gauge Theory

$$H = \sum_{r\in A,B} \frac{J_{zz}}{2} Q_r^2$$

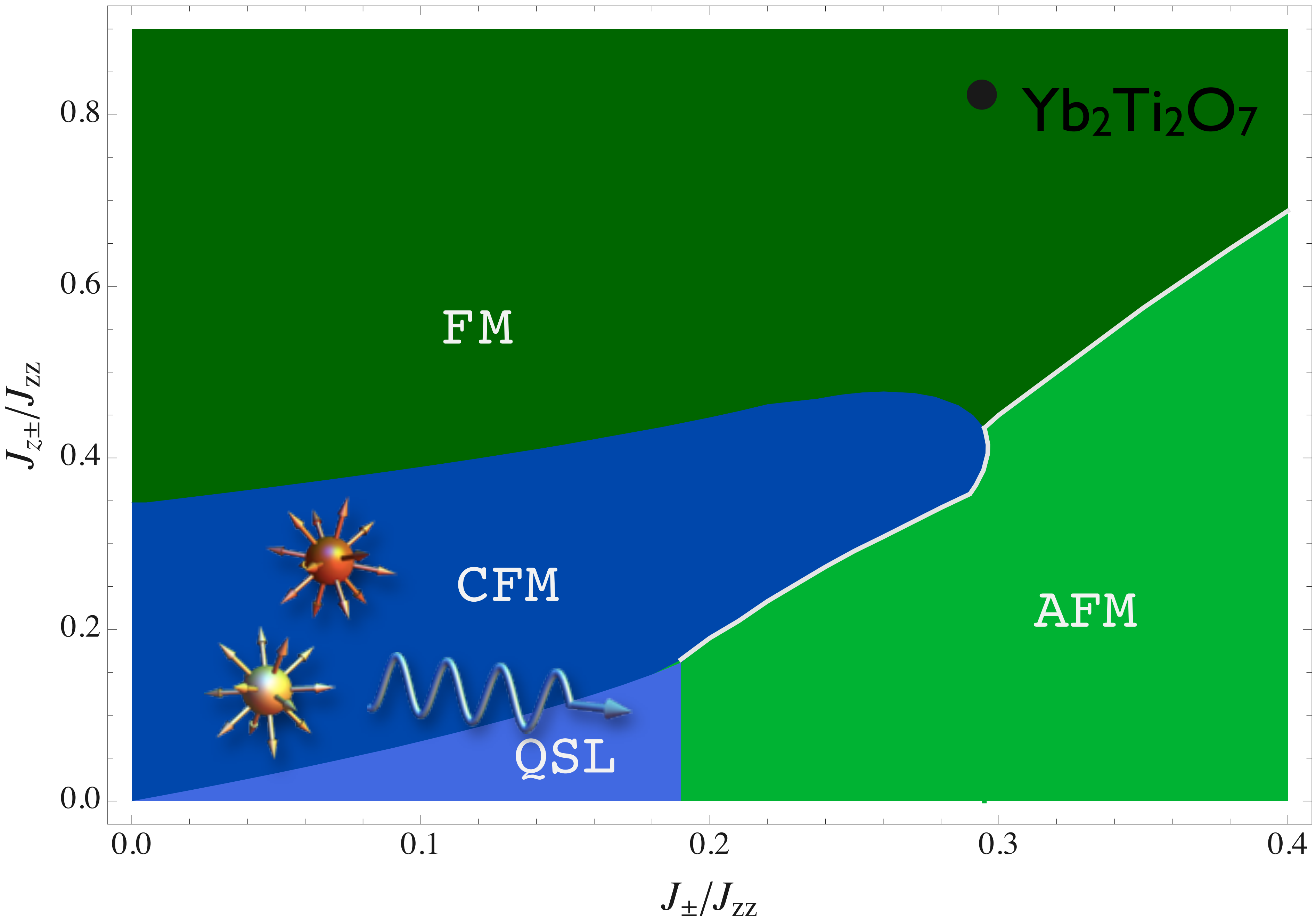
$$-J_{\pm}\left\{\sum_{r\in B}\sum_{\mu>\nu}\left(\Phi^{\dagger}_{r-e_\mu}\Phi_{r-e_\nu}\mathsf{s}^{+}_{r,r-e_\mu}\mathsf{s}^{-}_{r,r-e_\nu} + \text{h.c.}\right) + \sum_{r\in A}\sum_{\mu>\nu}\left(\Phi^{\dagger}_{r+e_\mu}\Phi_{r+e_\nu}\mathsf{s}^{-}_{r,r+e_\mu}\mathsf{s}^{+}_{r,r+e_\nu} + \text{h.c.}\right)\right\}$$

$$-J_{z\pm}\left\{\sum_{r\in A}\sum_{\mu\neq\nu}\left(\gamma^{*}_{\mu\nu}\Phi^{\dagger}_{r}\Phi_{r+e_\nu}\mathsf{s}^{z}_{r,r+e_\mu}\mathsf{s}^{+}_{r,r+e_\nu} + \text{h.c.}\right) + \sum_{r\in B}\sum_{\mu\neq\nu}\left(\gamma^{*}_{\mu\nu}\Phi^{\dagger}_{r-e_\nu}\Phi_{r}\mathsf{s}^{z}_{r,r-e_\mu}\mathsf{s}^{+}_{r,r-e_\nu} + \text{h.c.}\right)\right\}$$

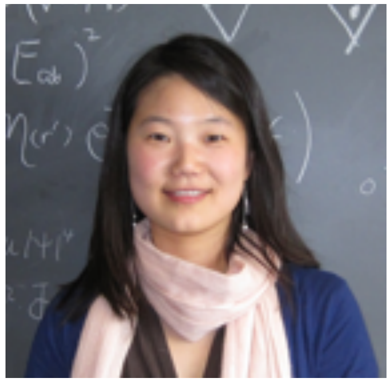
- Problem is exactly reformulated as a *lattice compact abelian Higgs theory*
- Can apply standard mean-field methods for lattice gauge theory (Wilson 1974...)



(MF) Phase diagram

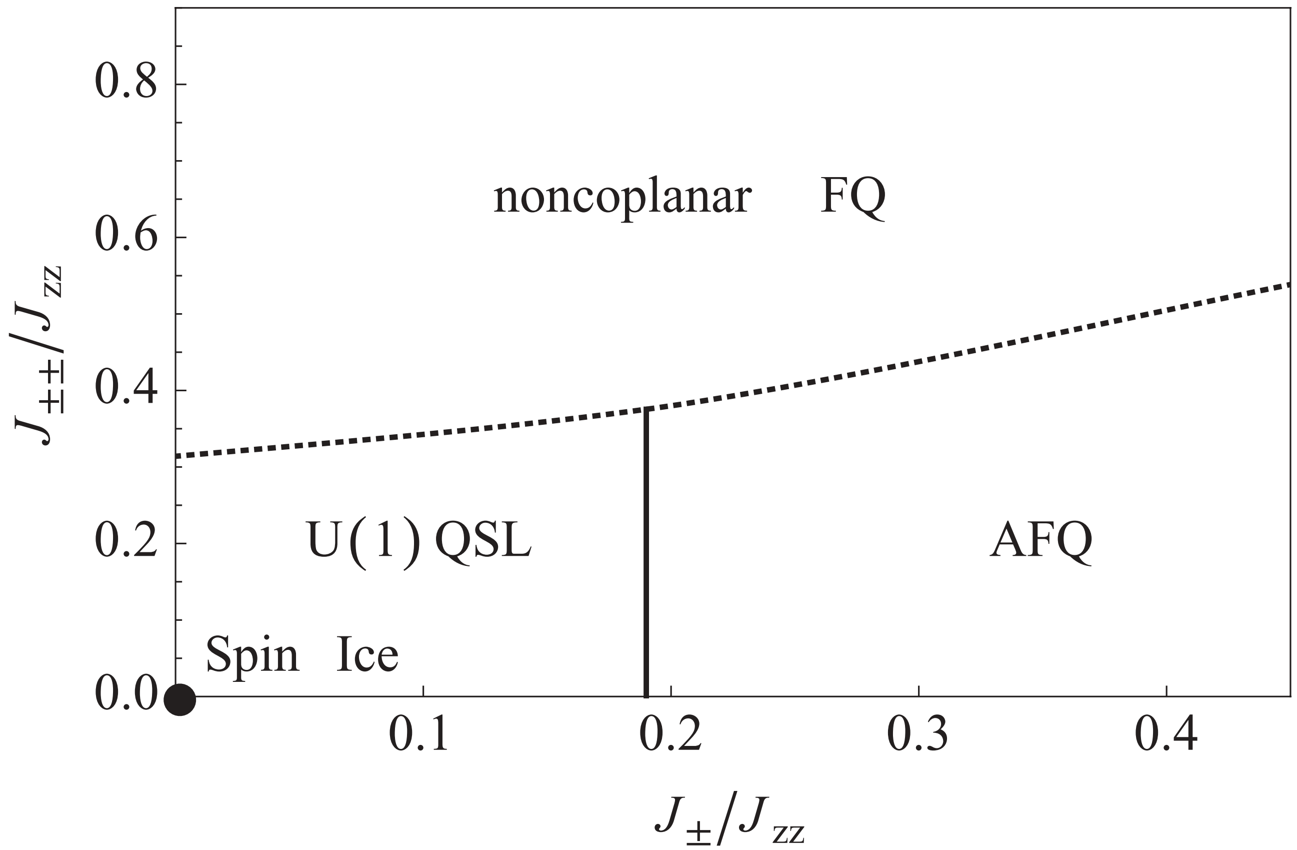


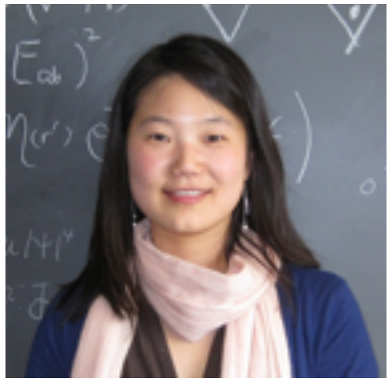
L. Savary + LB, PRL 2012



Phase Diagram'

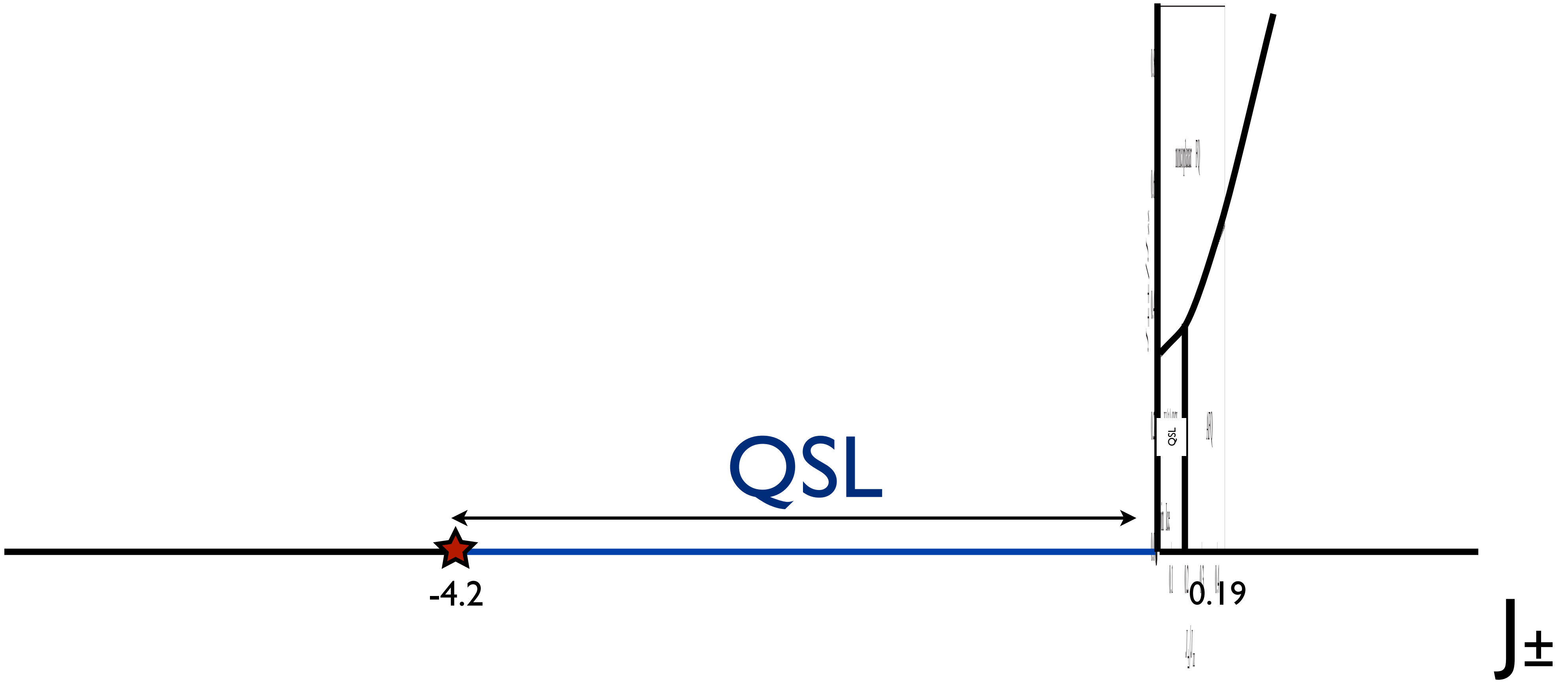
- Non-Kramers ion: $J_{z\pm}=0$
- $S^{\pm}$ = quadrupolar operator

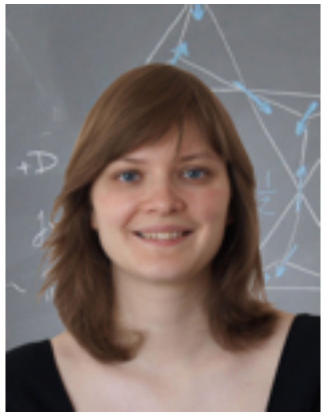




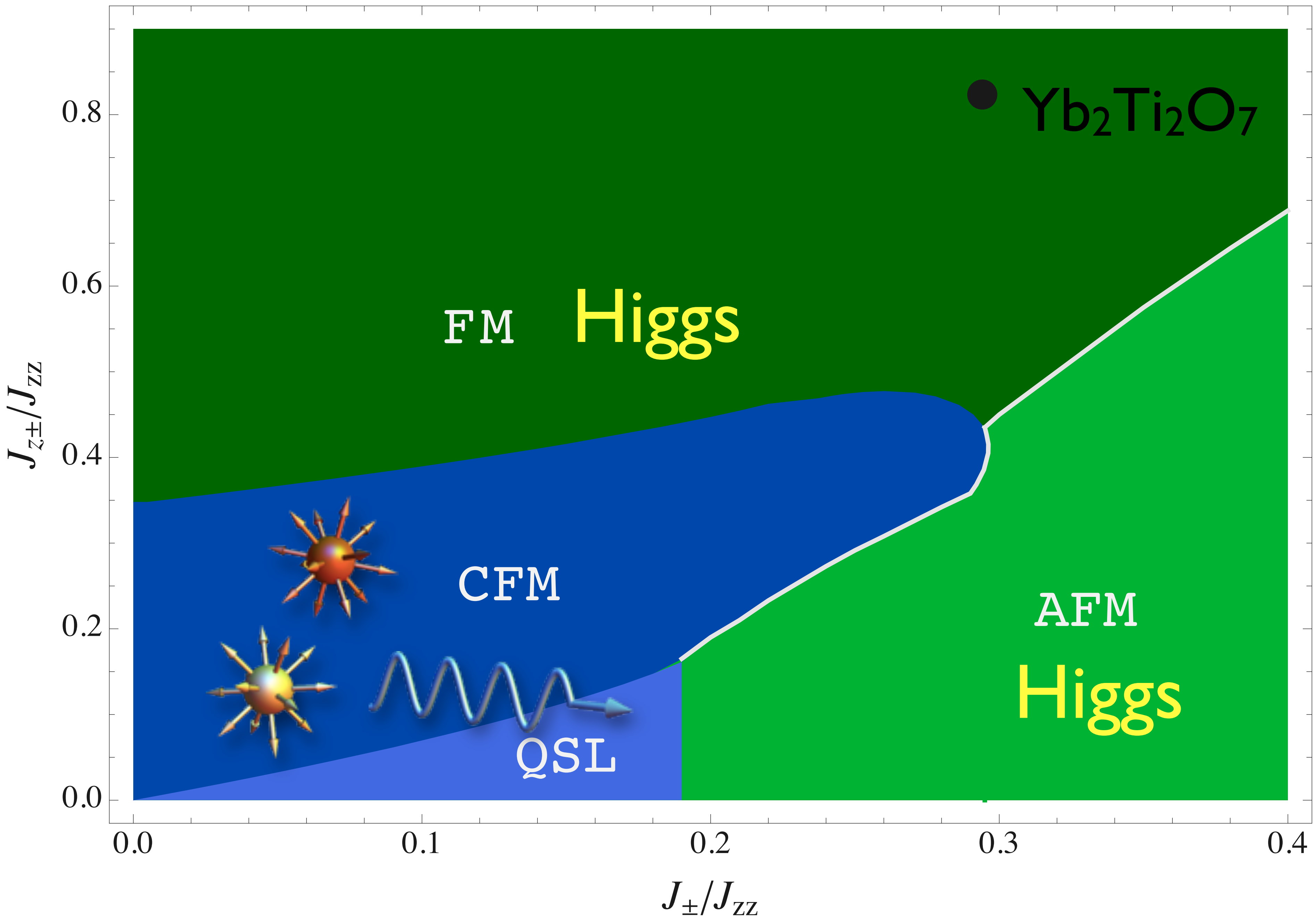
AF Case

- $J_\pm < 0$ (AF) may be relevant?
    - *much* stabler QSL expected



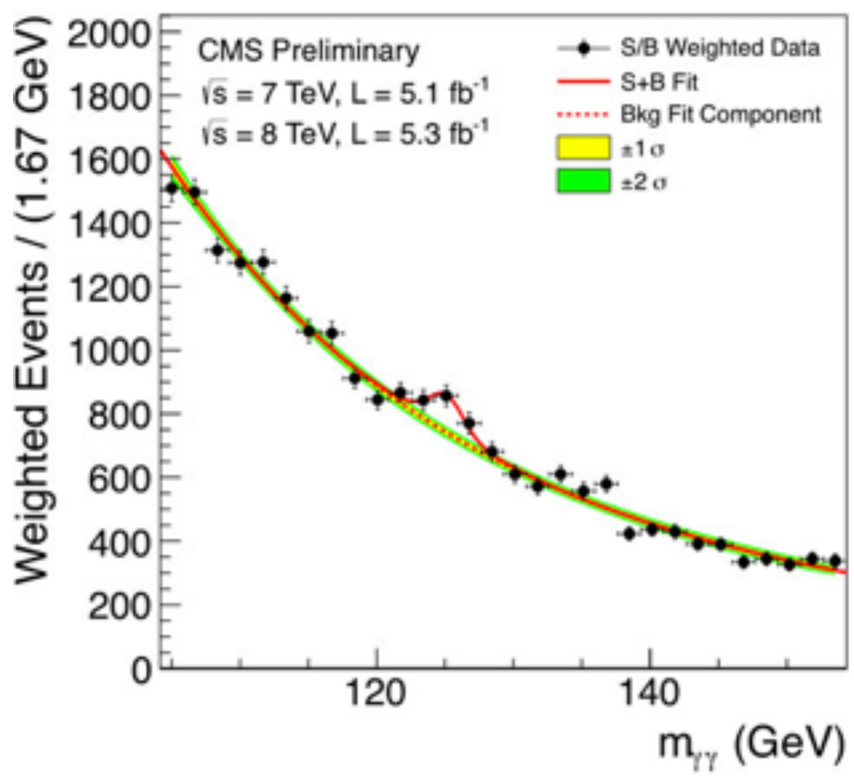


(MF) Phase diagram



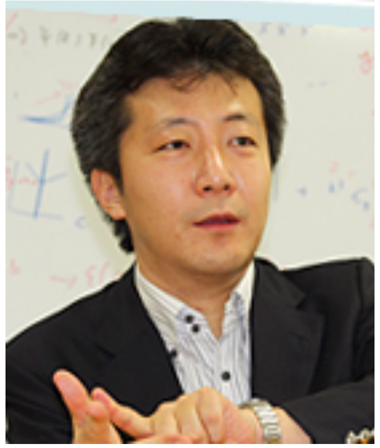
L. Savary + LB, PRL 2012

Discovery?



Higgs transition from a magnetic Coulomb liquid to a ferromagnet in $Yb_2Ti_2O_7$

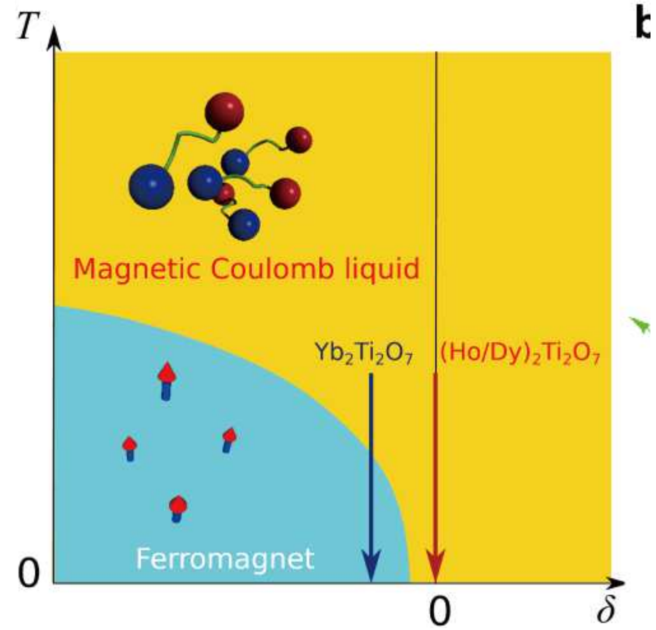
Lieh-Jeng Chang[1,2], Shigeki Onoda[3], Yixi Su[4], Ying-Jer Kao[5], Ku-Ding Tsuei[6], Yukio Yasui[7,8], Kazuhisa Kakurai[2] & Martin Richard Lees[9].



July 4, 2012

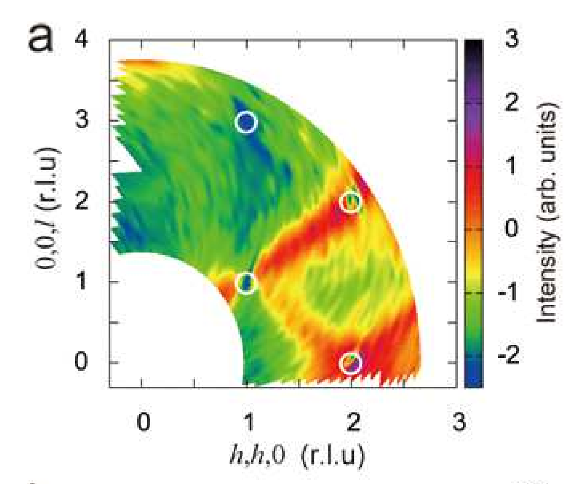
Nat. Comm., Aug. 7 2012

Higgs Transition?

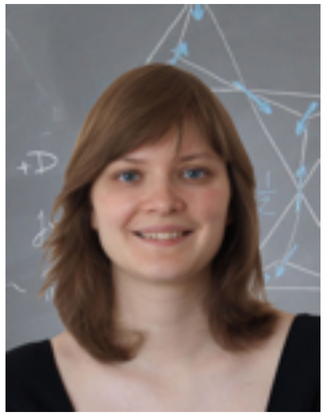


- Not sure what it means
- They observe:
  - transition is first order
  - quasi-pinch points above T_c

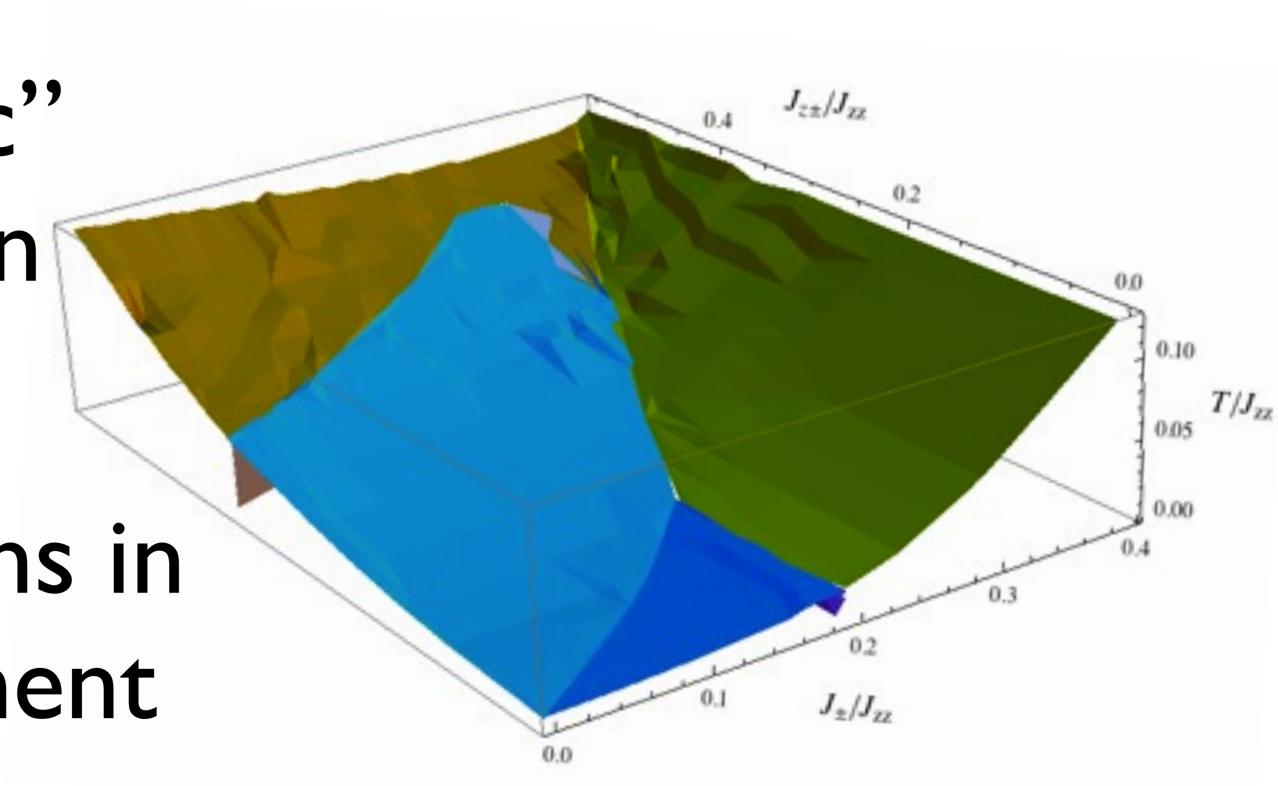
T=0.3K



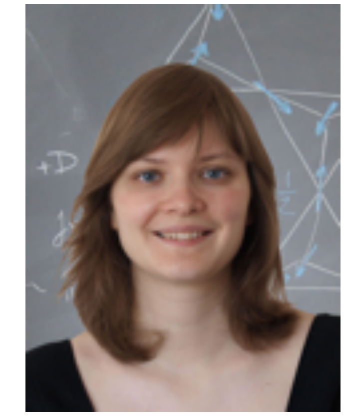
Possible picture



- Paramagnetic phase is somewhat like classical spin ice, $T \ll J_{zz}$
  - Lots of residual entropy
- Can be a "catastrophic" collapse of QSL to gain the spin ice entropy
  - Precisely this happens in gMFT: T>0 confinement

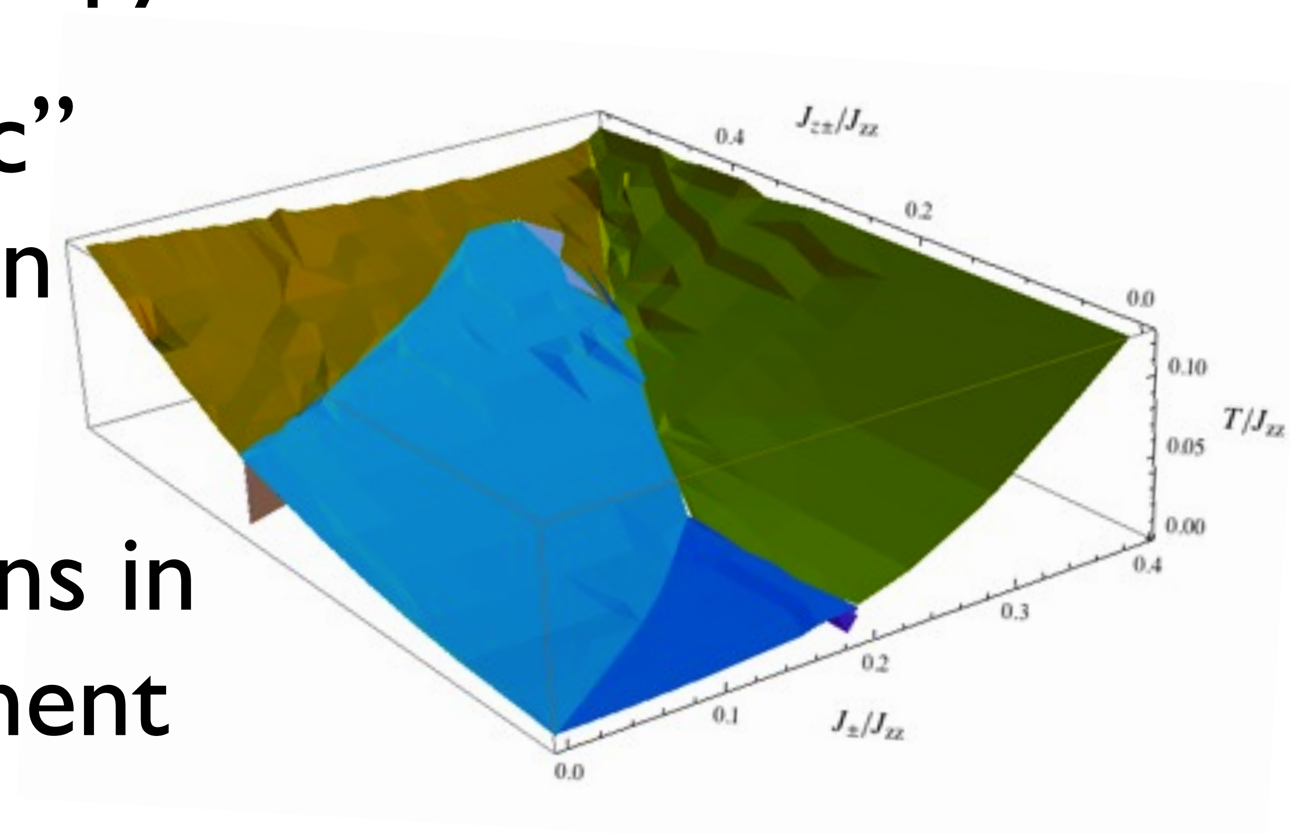


Possible picture



- Paramagnetic phase is somewhat like classical spin ice, T << J_{zz}
  - Lots of residual entropy
- Can be a "catastrophic" collapse of QSL to gain the spin ice entropy
  - Precisely this happens in gMFT: T>0 confinement

picture of classical spin ice above T_c might explain both pinch points and order of transition



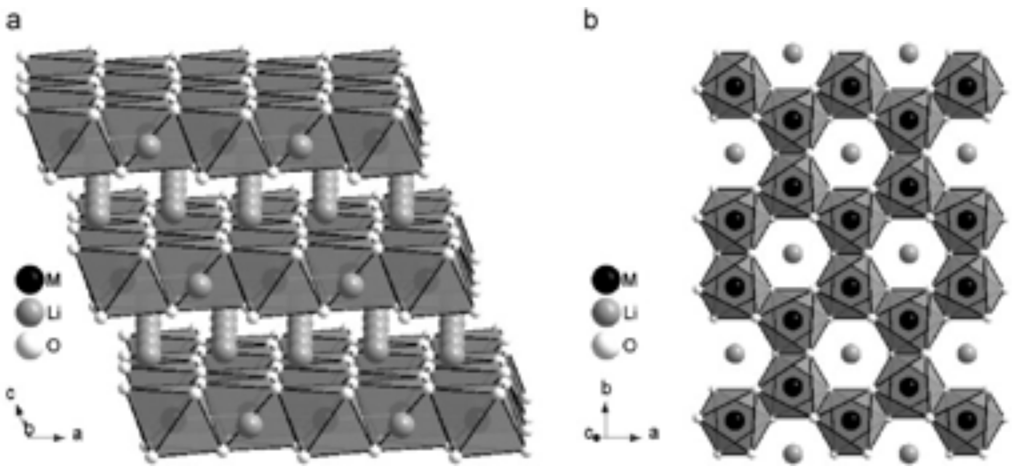
Outlook on QSI?

Pros	Cons
A substantial family of quantum $S_{eff}=1/2$ frustrated magnets	May be materials issues (but when are there not?)
Interactions can be measured	Interactions are complicated
J~1K means can be manipulated by laboratory fields	May be a microkelvin problem to observe QSL?
Rich phase diagram	Perhaps hard to find material which hits a QSL state?
Detailed INS measurements possible. Photon could be directly observed	Tough test of theory! and not so many single crystal materials available
We can expect many more experiments	it makes me impatient!

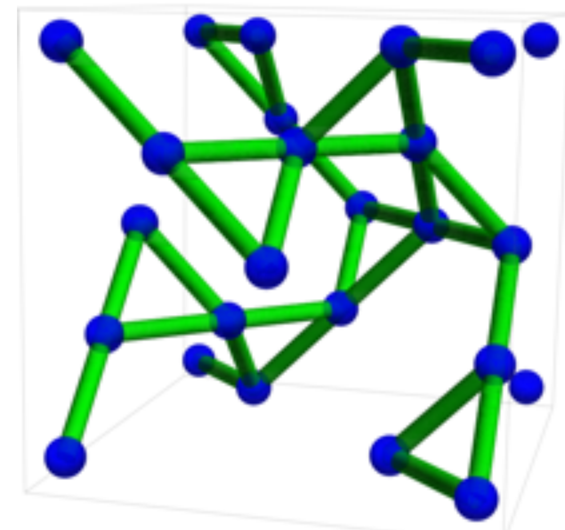
Directions

- Understand pyrochlores in intermediate correlation regime (iridates?)
- Metallic spin liquids and connection to heavy fermions
  - Ties to QSI: $Pr_2Ir_2O_7$?
- Application of DMRG technology to more realistic models of 2d QSL materials

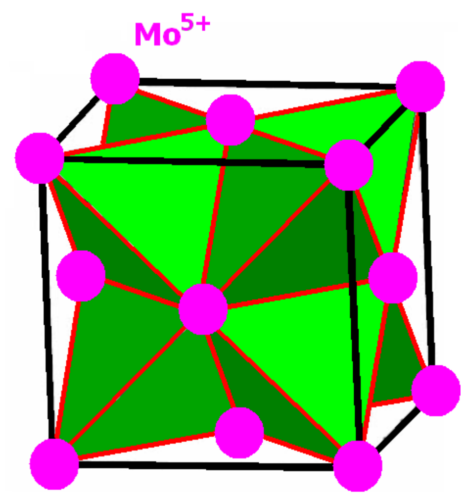
Other QSLs?



A_2IrO_3: Kitaev model?



$Na_4Ir_3O_8$: hyperkagomé QSL



Ba_2YMoO_6 : frustrated FCC lattice

A future history of magnetism

sinan, ~200BC

time

~500BC: Ferromagnetism documented in Greece, India, used in China

1949AD: Antiferromagnetism proven experimentally

~2016AD: Conclusive experiments on quantum spin liquids?