field-induced thermal transport in BEC antiferromagnets

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experimental data generously provided by: **Alex Sologubenko, Vivien Zapf, Marcelo Jaime**

PRL **106,** 037203 (2011) PHYSICAL REVIEW LETTERS week ending 21 JANUARY 2011

Thermal Transport and Strong Mass Renormalization in $NiCl_2$-$4SC(NH_2)_2$

Y. Kohama,[1] A. V. Sologubenko,[2] N. R. Dilley,[3] V. S. Zapf,[1] M. Jaime,[1] J. A. Mydosh,[4] A. Paduan-Filho,[5] K. A. Al-Hassanieh,[6] P. Sengupta,[7] S. Gangadharaiah,[8,9] A. L. Chernyshev,[8,9] and C. D. Batista[6]

PRL **106**, 037203 (2011)

Minoru Yamashita, *et al.*

Highly Mobile Gapless Excitations in a Two-Dimensional Candidate Quantum Spin Liquid

Science **328**, 1246 (2010).

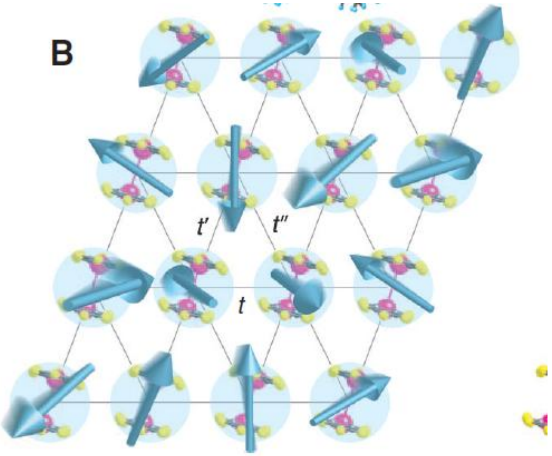
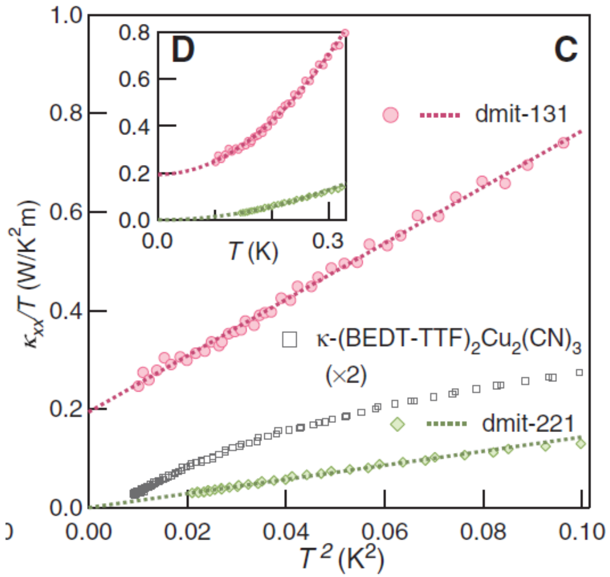
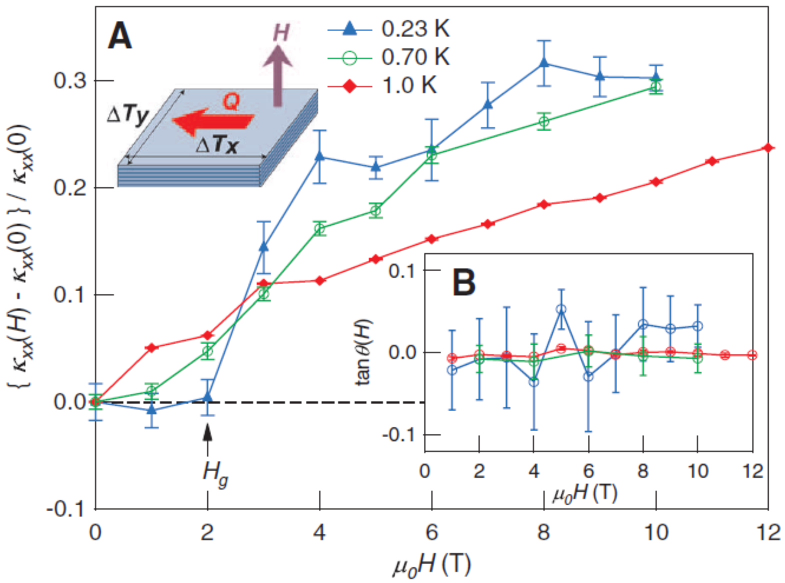


Fig. 1. The crystal structure of $EtMe_3Sb[Pd(dmit)_2]_2$





Chinese Terracotta Warriors (479-221 BC)

Elisabeth West FitzHugh & Lynda A. Zycherman
Studies in Conservation **37**, 145 (1992)

Heinz Berke
Angew. Chem. Int. Ed. **41**, 2483 (2002)

Calcite - $CaCO_3$ 

Weddellite - $Ca(CO_3)_2(OH)_2$ 

Hydrocerussite – $2Pb(CO_3)_2 \cdot Pb(OH)_2$ 

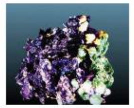
Soot - carbon black

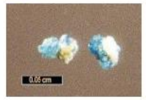
Han Purple - $BaCuSi_2O_6$

Cinnabar - HgS 

Hematite - Fe_2O_3 

Minium – Pb_3O_4 

Malachite – $Cu_2CO_3(OH)_2$ 

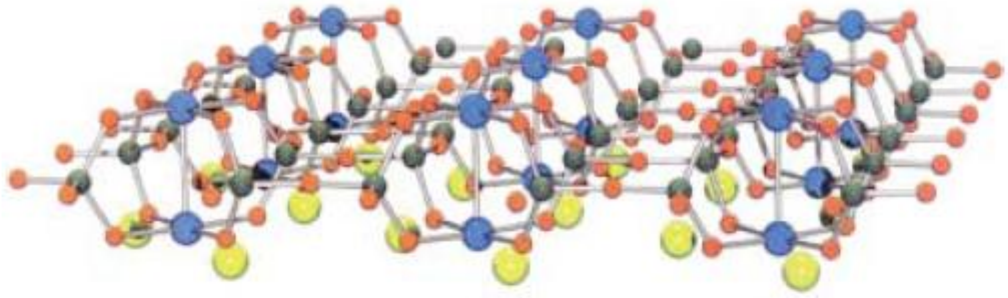
Han Blue – $BaCuSi_4O_{10}$ (cuprorivaite) 

J. Zuo et al., *J. Raman Spectrosc. 34, 121 (2003)*
H. Langhals & D. Bathelt, *Angew. Chem. Int. Ed.* **42**, 5676 (2003)

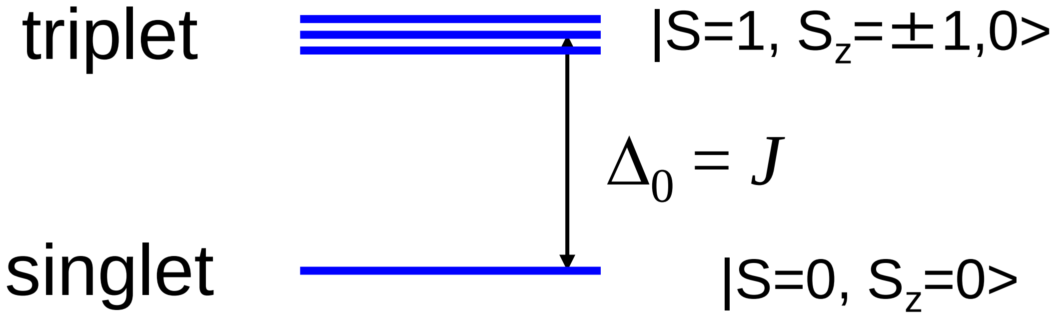
BEC: coupled dimers

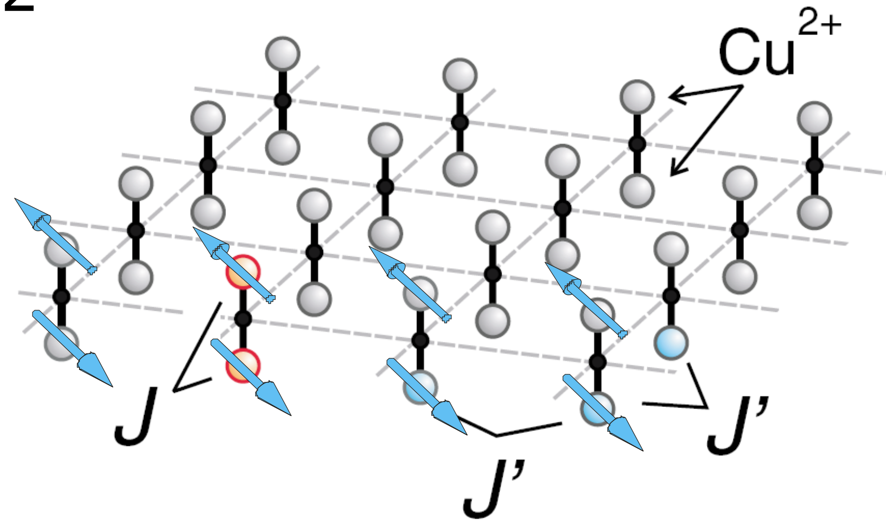
- Ba-Cu silicate, Han Purple = $BaCuSi_2O_6$

$$\boldsymbol{H} = J \sum \mathbf{S}_i \mathbf{S}_{i'} + J' \sum \mathbf{S}_i \mathbf{S}_j$$



dimer = antiferromagnetic pair of S=1/2

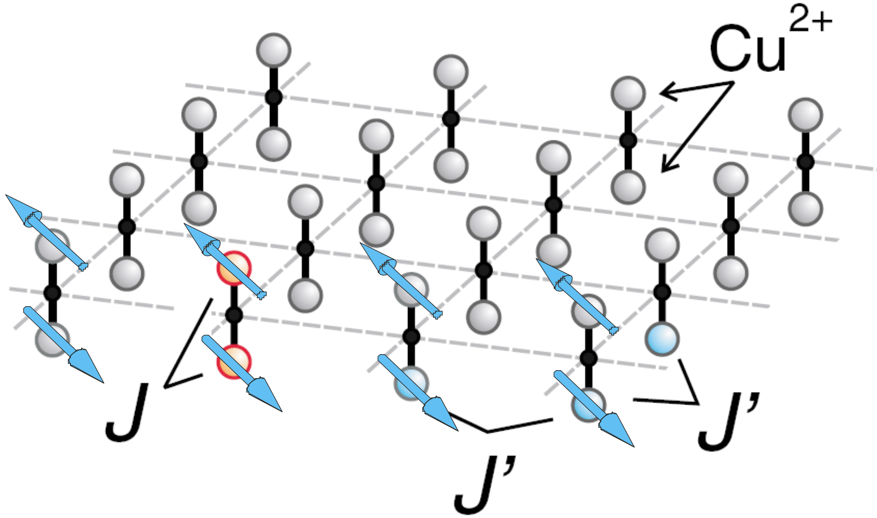




- array of coupled spin dimers

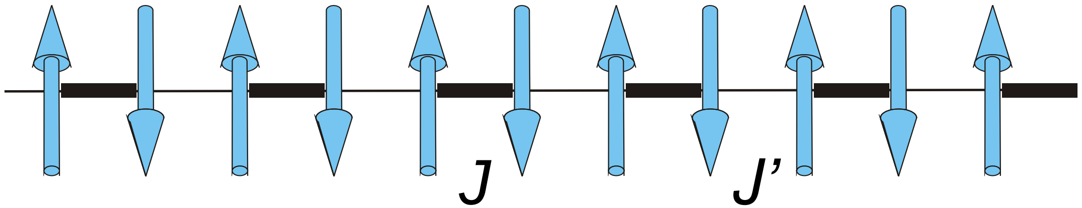
BEC: coupled dimers

- Han Purple = $BaCuSi_2O_6$



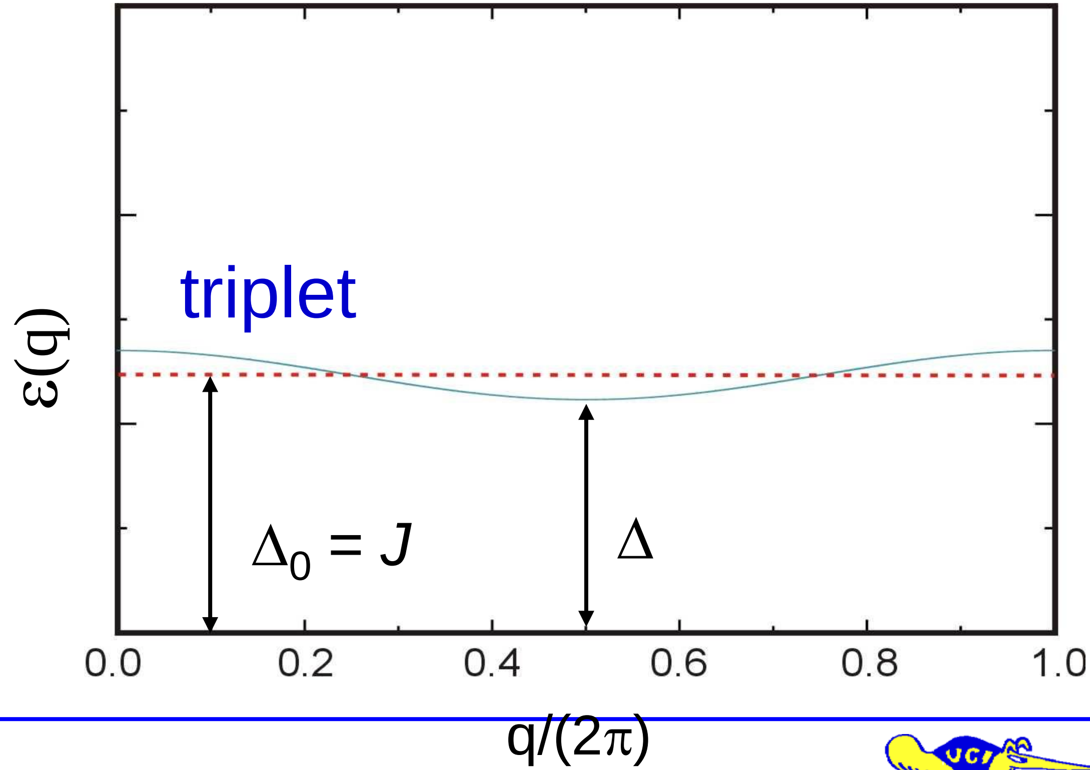
$$\boldsymbol{H} = J \sum \mathbf{S}_i \mathbf{S}_{i'} + J' \sum \mathbf{S}_i \mathbf{S}_j$$

weakly coupled dimers



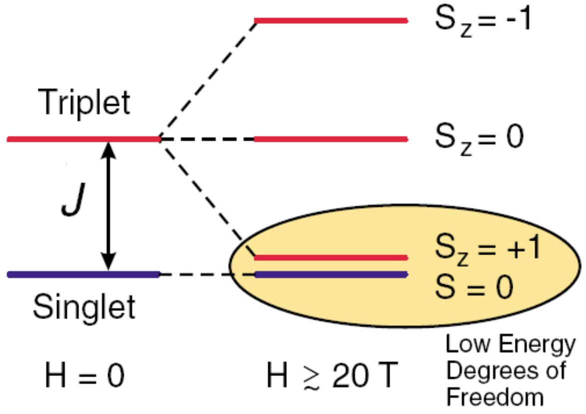
dispersion = "bands of triplets"

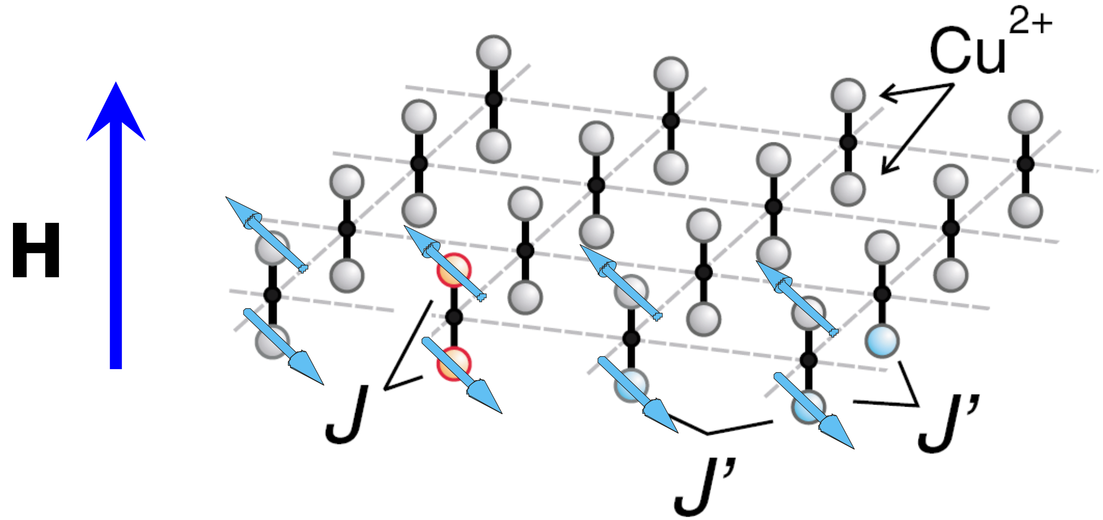
$$\varepsilon(q) \sim J + J' \cos(q)$$



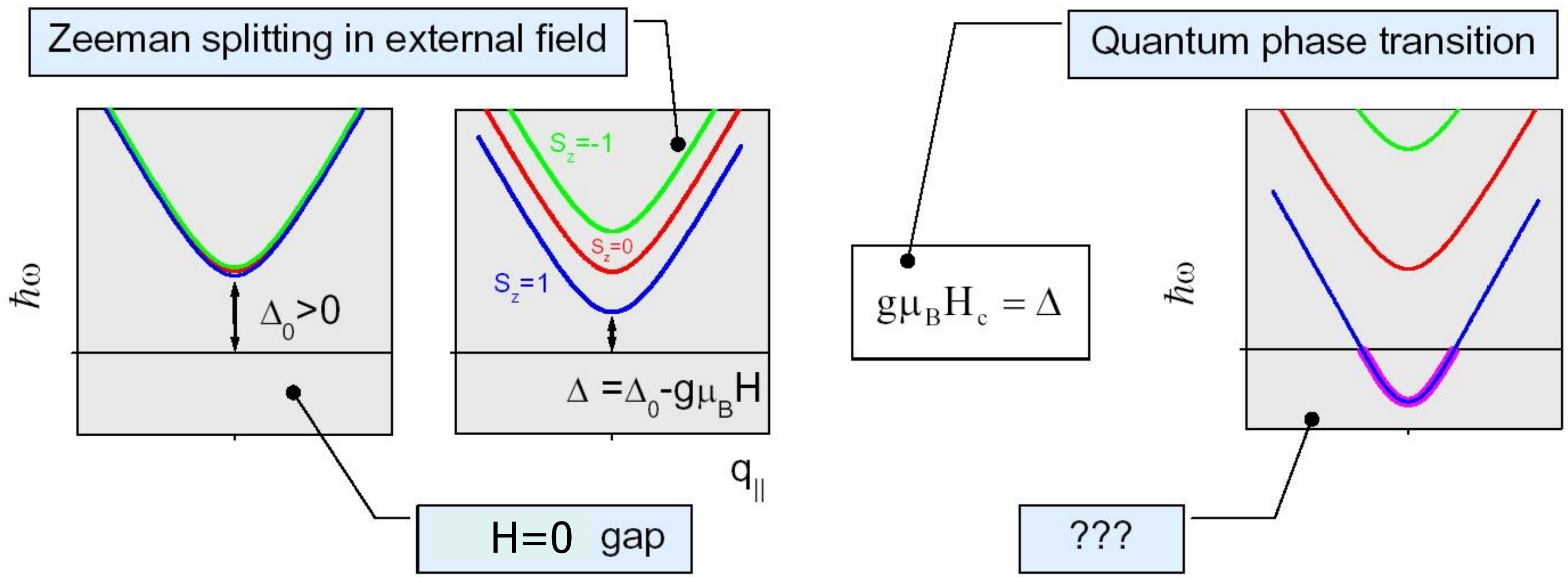
BEC … in a magnetic field

single spin dimer

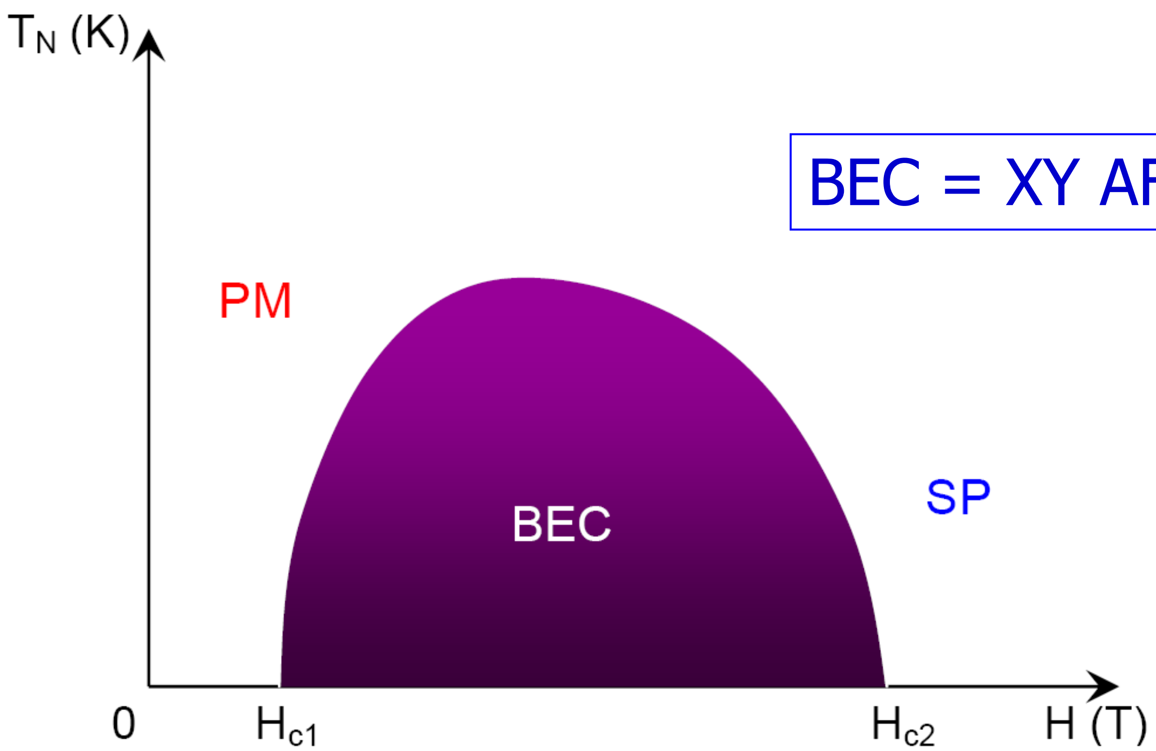




coupled spin dimers

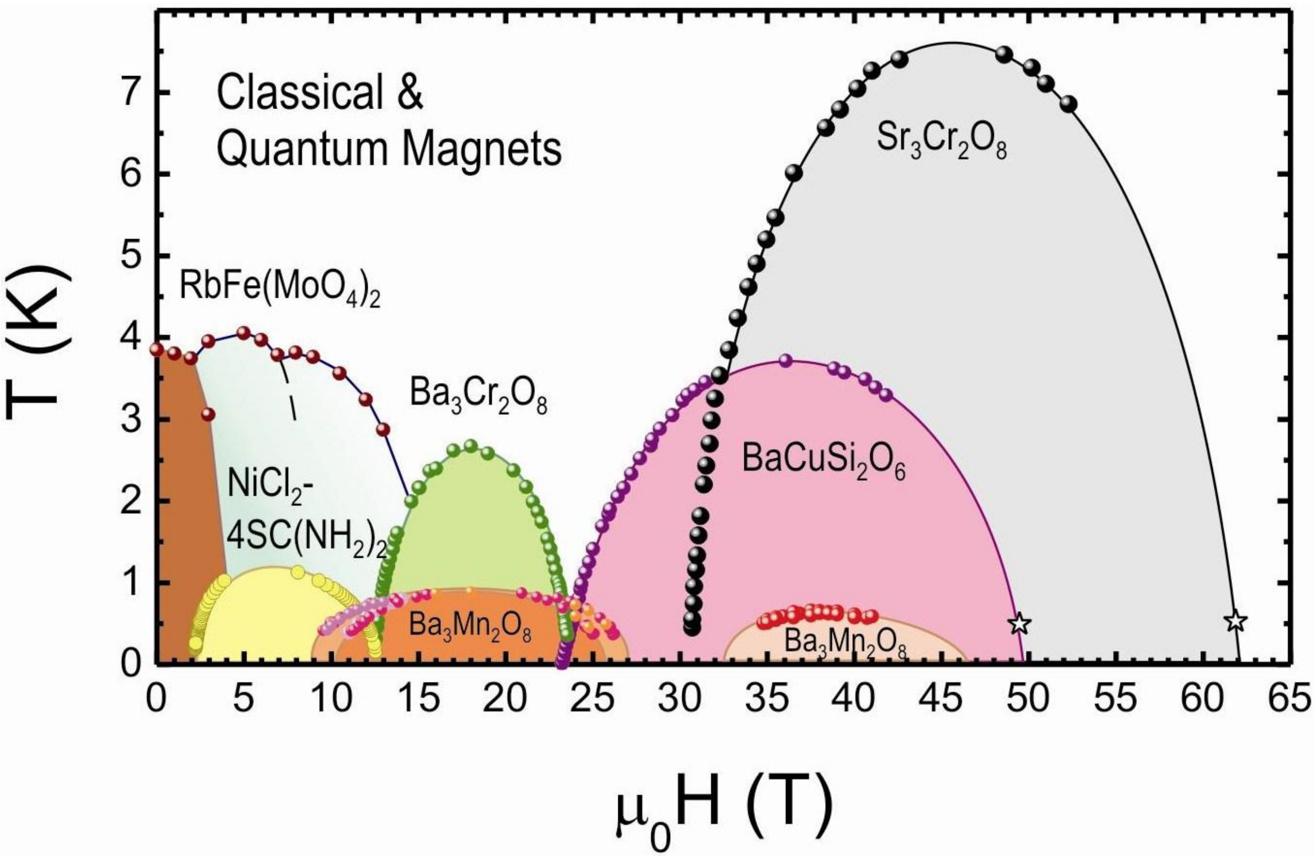


BEC, III: PhD



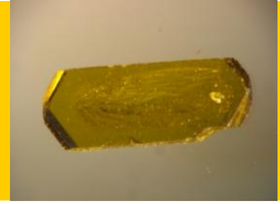
BEC = XY AF

BEC, III: PhD



courtesy of Marcelo Jaime,
Netsu Sokutei **37**, 26 (2010).

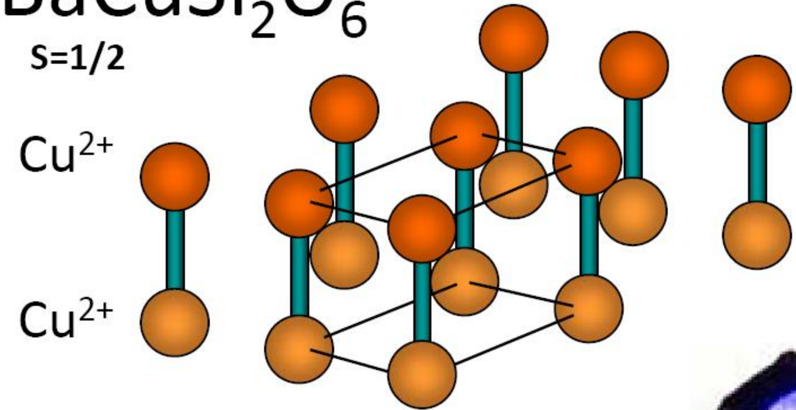
BEC, IV: DTN



dichloro*tetrakis*thiourea-nickel (II) = DTN

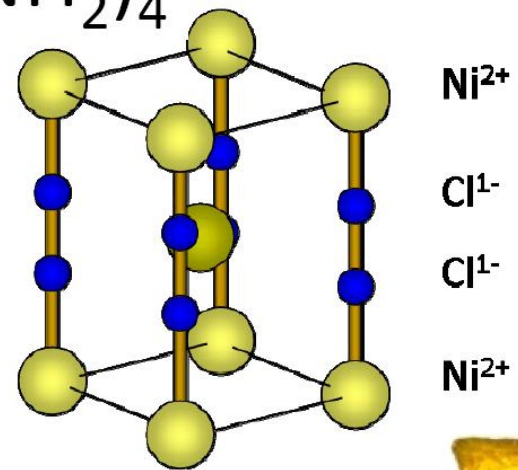
$BaCuSi_2O_6$

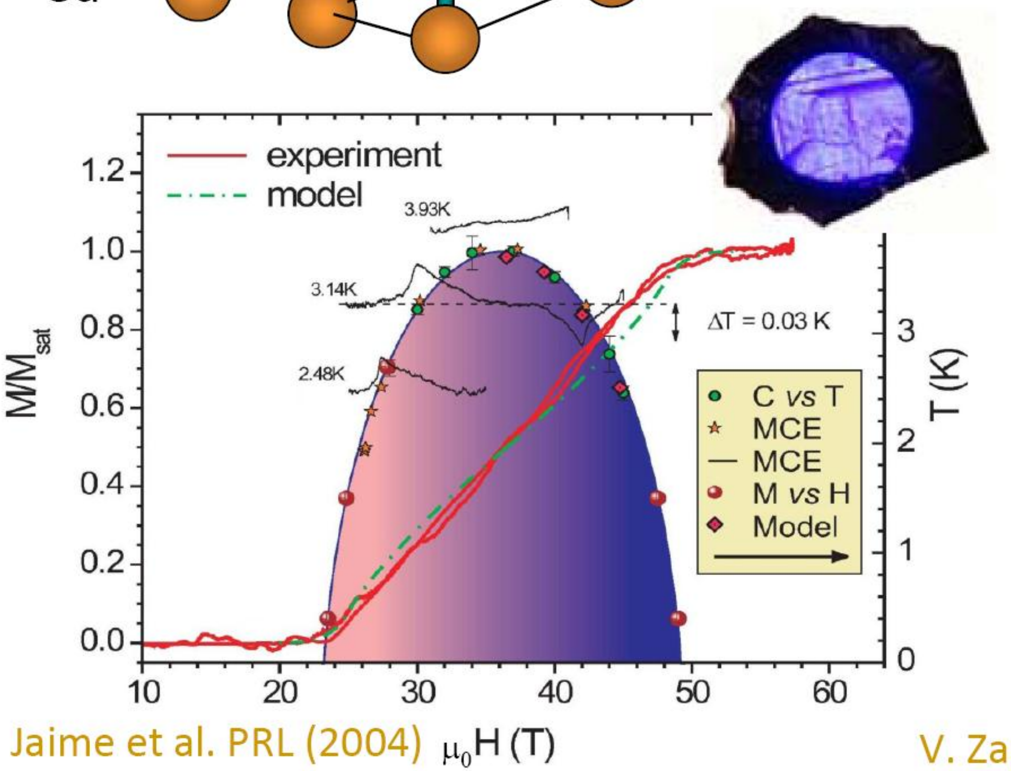
S=1/2



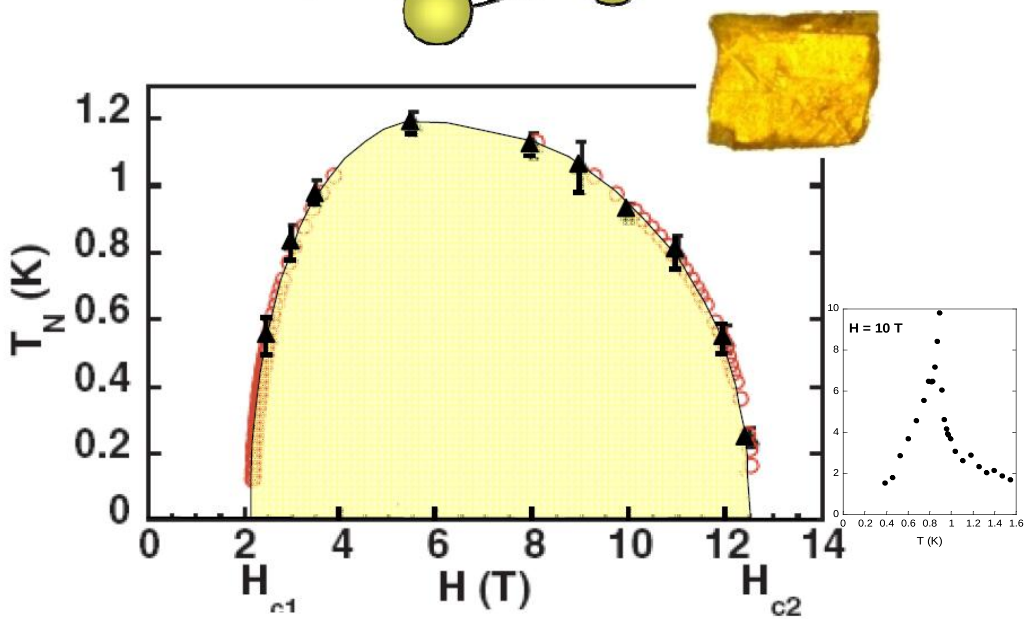
$NiCl_2$-$4SC(NH_2)_4$

S=1





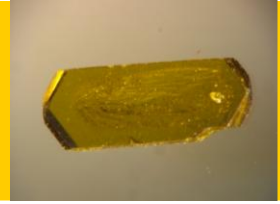
Jaime et al. PRL (2004)



V. Zapf et al. PRL 96, 77204 (2006)

courtesy of Marcelo Jaime

DTN



Field-induced magnetic ordering in uniaxial nickel systems: A second example
Armando Paduan-Filho — J. Chem. Phys. 74, 4103 (1981).

PHYSICAL REVIEW B 69, 020405(R) (2004)
Field-induced magnetic ordering in $NiCl_2 \cdot 4SC(NH_2)_2$
A. Paduan-Filho,* X. Gratens, and N. F. Oliveira, Jr.

PRL 96, 077204 (2006) — PHYSICAL REVIEW LETTERS — week ending 24 FEBRUARY 2006
Bose-Einstein Condensation of S ... Freedom in $NiCl_2$-$4SC(NH_2)_2$
V. S. Zapf,[1] D. Zocco,[1] B. R ... M. Kenzelmann,[2,3] C. Niedermayer,[3]

PRL 98, 047205 (2007) — PHYSICAL REVIEW LETTERS — week ending 26 JANUARY 2007
Magnetic Excitations in the Spin-1 Anisotropic Heisenberg Antiferromagnetic Chain System $NiCl_2$-$4SC(NH_2)_2$
S. A. Zvyagin,[1] J. Wosnitza,[1] C. D. Batista,[2] M. Tsukamoto,[3] N. Kawashima,[3] J. Krzystek,[4] V. S. Zapf,[5] ... N. F. Oliveira, Jr.,[6] and A. Paduan-Filho[6]

... field

JOURNAL OF APPLIED PHYSICS 101 ...
Magnetostriction in the Bose-Einste... $NiCl_2-4SC(NH_2)_2$ (Invited)
V. S. Zapf[a)]

PRL 106, 037203 (2011) — PHYSICAL REVIEW LETTERS — week ending 21 JANUARY 2011
Thermal Transport and Strong Mass Renormalization in $NiCl_2$-$4SC(NH_2)_2$
Y. Kohama,[1] A. V. Sologubenko,[2] N. R. Dilley,[3] V. S. Zapf,[1] M. Jaime,[1] J. A. Mydosh,[4] A. Paduan-Filho,[5] K. A. Al-Hassanieh,[6] P. Sengupta,[7] S. Gangadharaiah,[8,9] A. L. Chernyshev,[8,9] and C. D. Batista[6]

... PHYSICAL REVIEW LETTERS — week ending 31 OCTOBER 2008
...se-Einstein Condensation Universality Class in $NiCl_2$-$4SC(NH_2)_2$ at Ultralow Temperatures
L. Yin,[1] J. S. Xia,[1] V. S. Zapf,[2] N. S. Sullivan,[1] and A. Paduan-Filho[3]

V. S. Zapf,[1] V. F. ... P. Egan,[1,5] C. Pantea,[1] ... Filho[6]

PHYSICAL REVIEW B 78, 094406 (2008)
Character of magnetic excitations in a quasi-one-dimensional antiferromagnet near the quantum critical points: Impact on magnetoacoustic properties
O. Chiatti,[1] A. Sytcheva,[1] J. Wosnitza,[1] S. Zherlitsyn,[1] A. A. Zvyagin,[2,3] V. S. Zapf,[4] M. Jaime,[4] and A. Paduan-Filho[5]

... (2008)
Spin d... $)_2$ in the field-induced ordered phase
S. A. Zvyagin,[1] J. Wosnit... ...uk,[2,*] V. S. Zapf,[3] M. Jaime,[3] A. Paduan-Filho,[4] V. N. Glazkov,[5] S. S. Sosin,[5] and A. I. Smirnov[5]

PRL 101, 087602 (2008) — PHYSICAL REVIEW LETTERS — week ending 22 AUGUST 2008
Unusual Magneto-Optical Phenomenon Reveals Low Energy Spin Dispersion in the Spin-1 Anisotropic Heisenberg Antiferromagnetic Chain System $NiCl_2 - 4SC(NH_2)_2$
S. Cox,[1] R. D. McDonald,[1] M. Armanious,[2] P. Sengupta,[3] and A. Paduan-Filho[4]

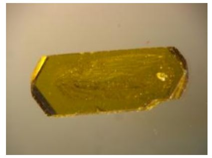
PRL 102, 077204 (2009) — PHYSICAL REVIEW LETTERS — week ending 20 FEBRUARY 2009
Critical Properties at the Field-Induced Bose-Einstein Condensation in $NiCl_2$-$4SC(NH_2)_2$
A. Paduan-Filho,[1] K. A. Al-Hassanieh,[2] P. Sengupta,[2,3] and M. Jaime[3]

PHYSICAL REVIEW B 83, 140405(R) (2011)
Magnetoelectric effects in an organometallic quantum magnet
V. S. Zapf,[1] P. Sengupta,[2] C. D. Batista,[3] F. Nasreen,[1,4] F. Wolff-Fabris,[1,*] and A. Paduan-Filho[5]

DTN, I: model

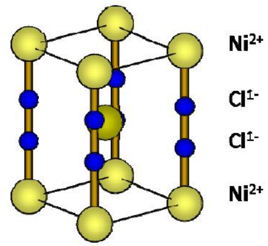
$$\mathrm{H} = D\sum_i \left(S_i^z\right)^2 - g\mu_B H \sum_i S_i^z + \sum_{<i,j>\nu} J_\nu \vec{S}_i \cdot \vec{S}_j$$

Spin-Orbit coupling Zeeman term AFM exchange



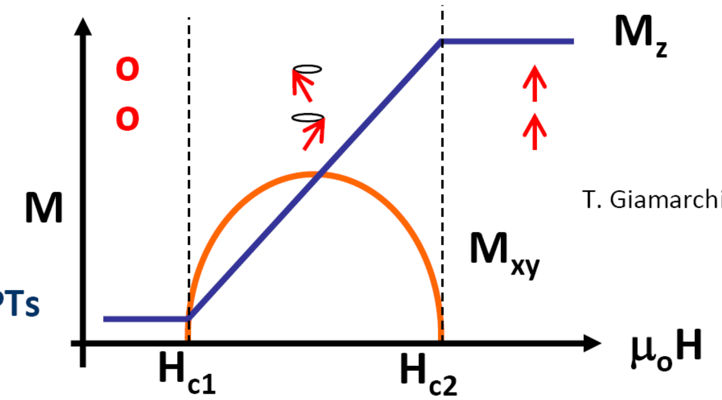
Mapping:

$$\mathcal{H} = \sum_i (J_0 - h) a_i^\dagger a_i + \sum_{i,j} t_{ij} a_i^\dagger a_j + \frac{1}{2}\sum_{i,j} U_{ij} a_i^\dagger a_j^\dagger a_j a_i$$



$D = 8.9$ K
$J_c = 2.2$ K
$J_a = 0.18$ K

Spin	Bosons
State $\vert S_z = 1\rangle$ (S=1 qp)	Occupied boson state $\vert 1\rangle$ (particle)
State $\vert S_z = 0\rangle$ (S=0 qp)	Empty boson state $\vert 0\rangle$ (hole)
Rotational invariance *O(2)*	Particle conservation *U(1)*
Staggered Magnetization $M_{xy} = \langle S^x_i + iS^y_i\rangle$	Condensate wavefunction $\psi(r)$
Magnetic Field	Chemical Potential ($H \sim N \sim \mu$)



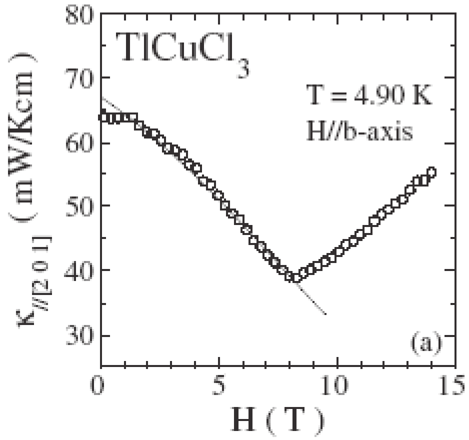
@T=0
amplitude driven QPTs

T. Giamarchi, C. Ruegg, O.Tchernyshyov
Nature Phys. 4, 198 (2008).

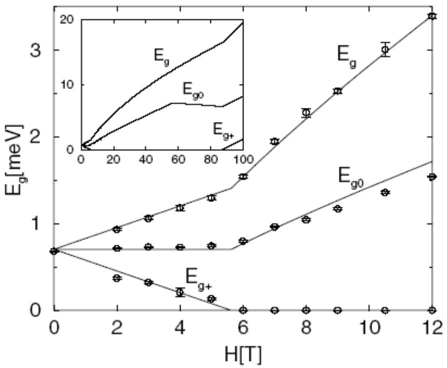
courtesy of Marcelo Jaime

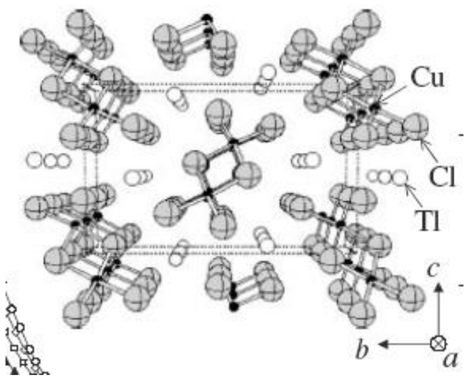
thermal conductivity, other BECs

spin-dimer AF $TlCuCl_3$



K. Kudo *et.al.*, JPSJ **73**, 2358 (2004)

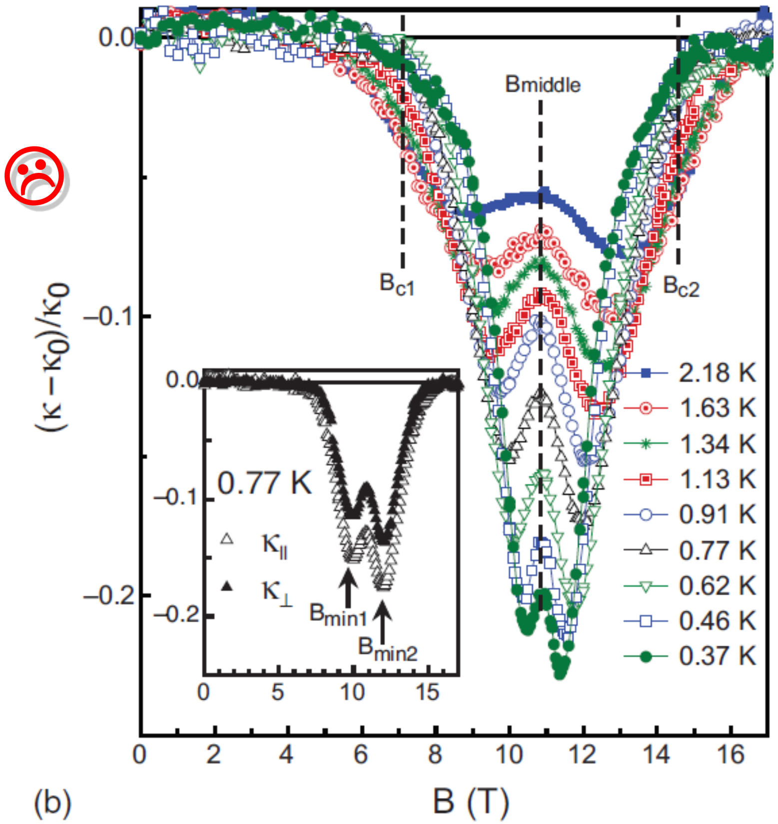




M. Matsumoto *et.al.*, PRL **89**, 077203 (2002)

phonons only ☹

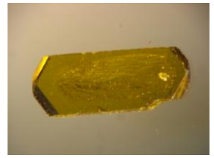
spin-1/2 ladder system $(C_5H_{12}N)_2CuBr_4$=PCB



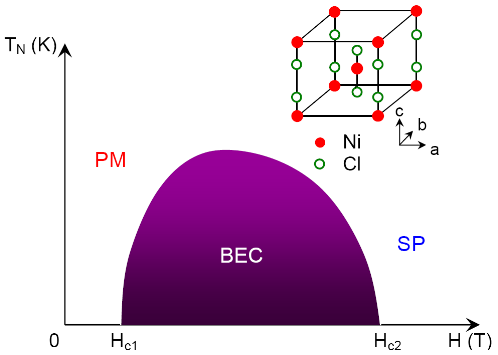
Sologubenko etal., PRB **80**, 220411 (2009).

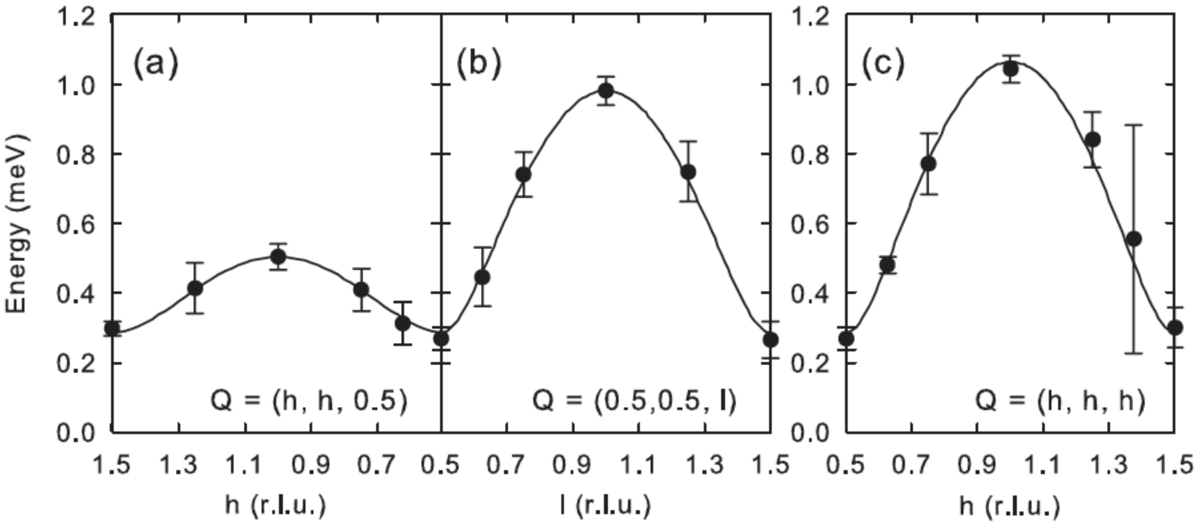
thermal conductivity experiments, DTN

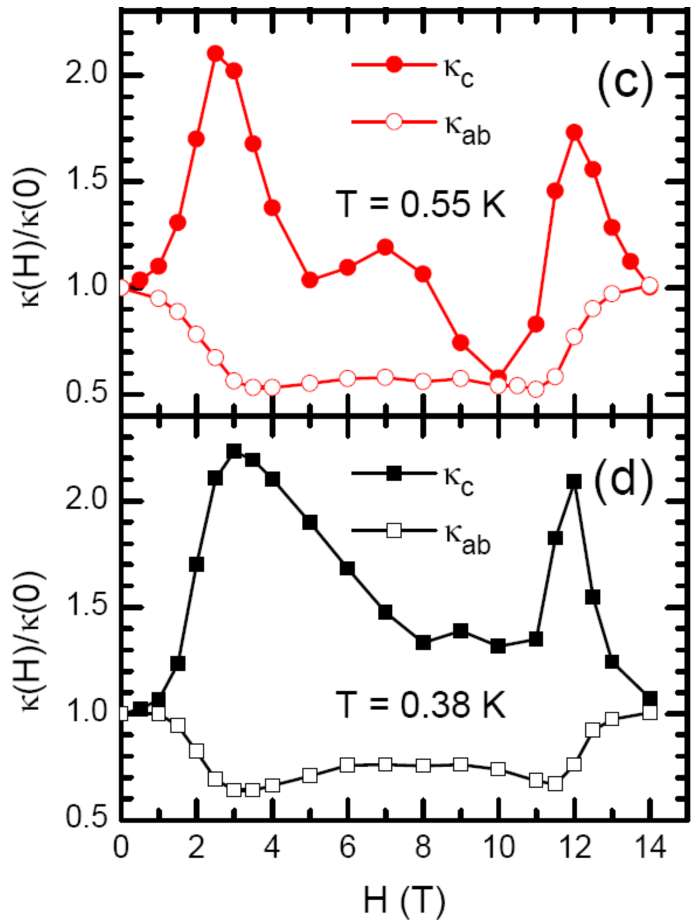
- thermal transport in BEC antiferromagnets vs magnetic field



S=1 AF $NiCl_2$-$4SC(NH_2)_2$







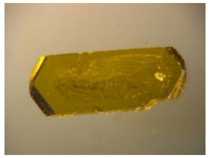
V. S. Zapf etal., PRL **96**, 077204 (2006).

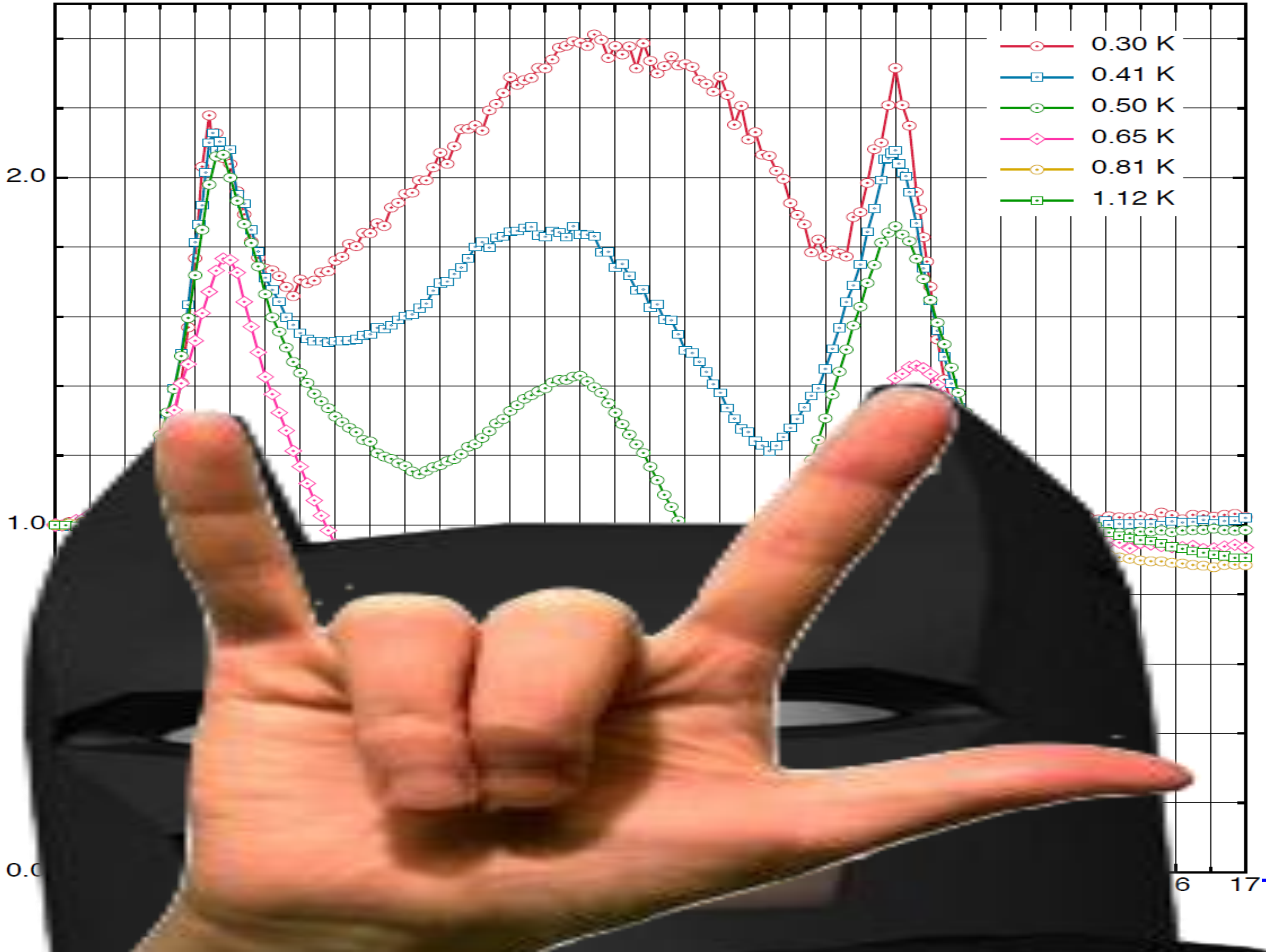
X. F. Sun etal., PRL **102**, 167202 (2009).

thermal conductivity experiments, DTN

data by: Alex Sologubenko, John Mydosh (UCologne)

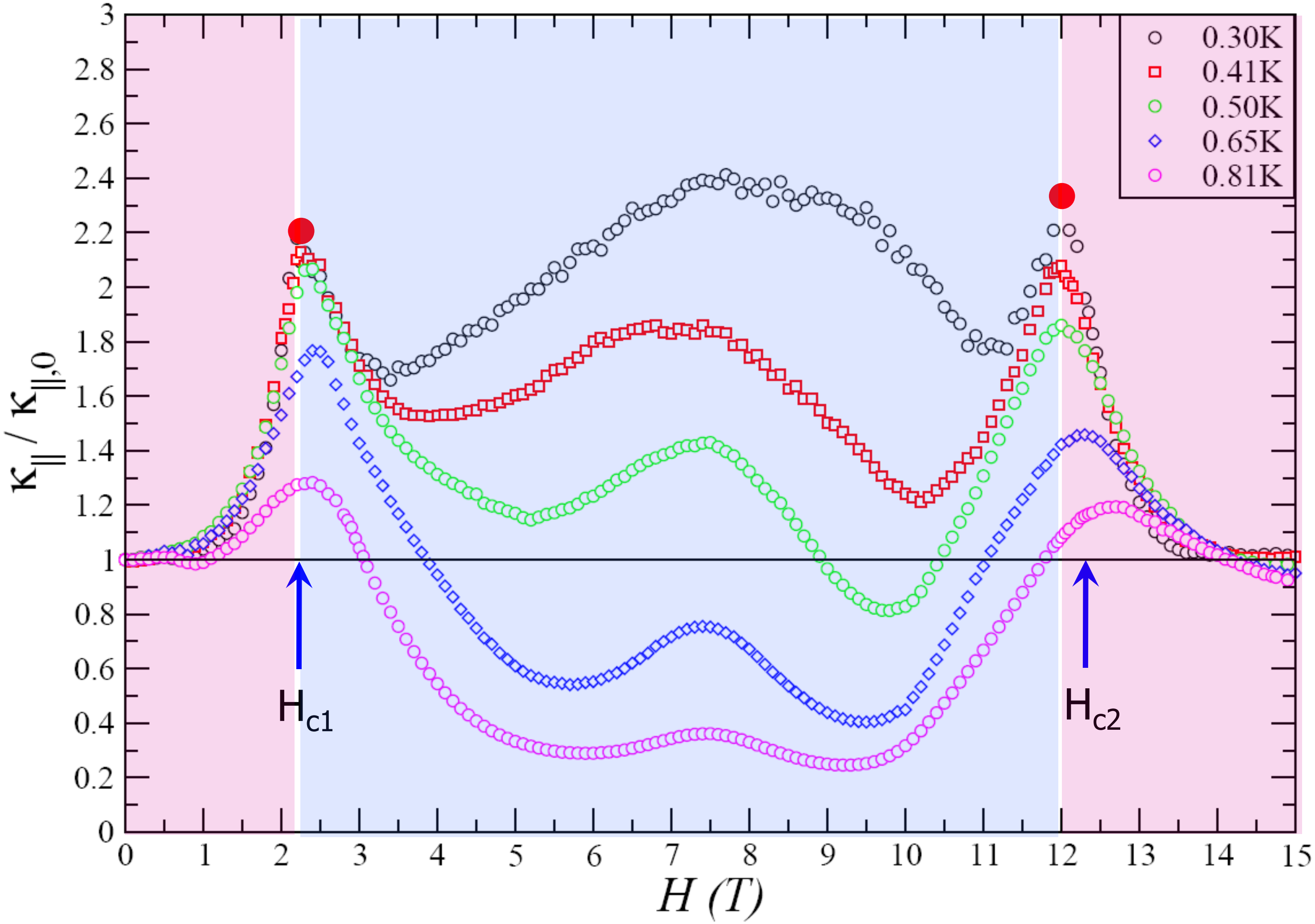
PRL **106**, 037203 (2011)





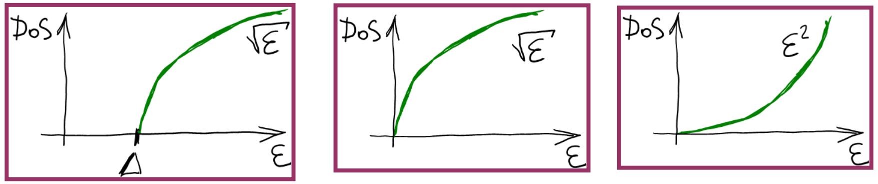
quick analysis

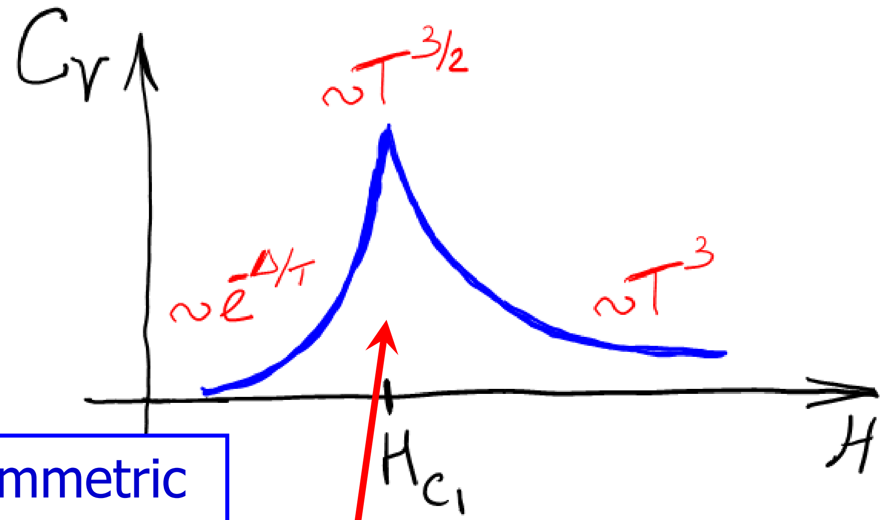
- ☑ at $H=0$, gaps → conductivity is due to phonons alone (small $T \ll \Delta \sim 3K$)
- ☑ excess of the conductivity is due to spin excitations
- ☑ peaks are near H_{c1} and H_{c2}, κ/κ_0 increases as T decreases
- ☑ at larger T, suppression of κ in the BEC phase is due to phonon scattering on magnons



puzzles, I: "no asymmetry" problem

DoS argument for the specific heat, C_v, ➔

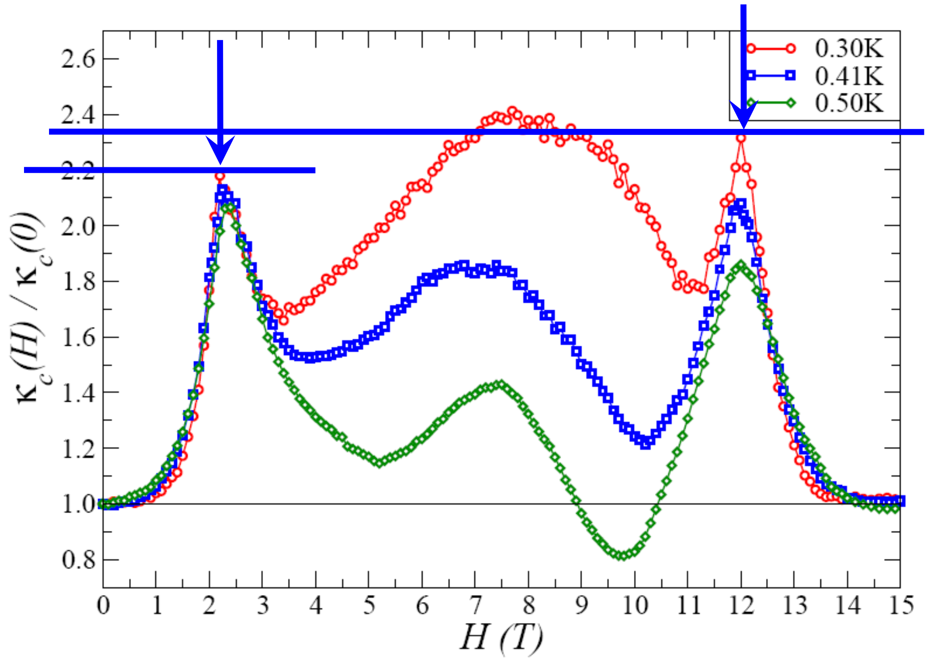


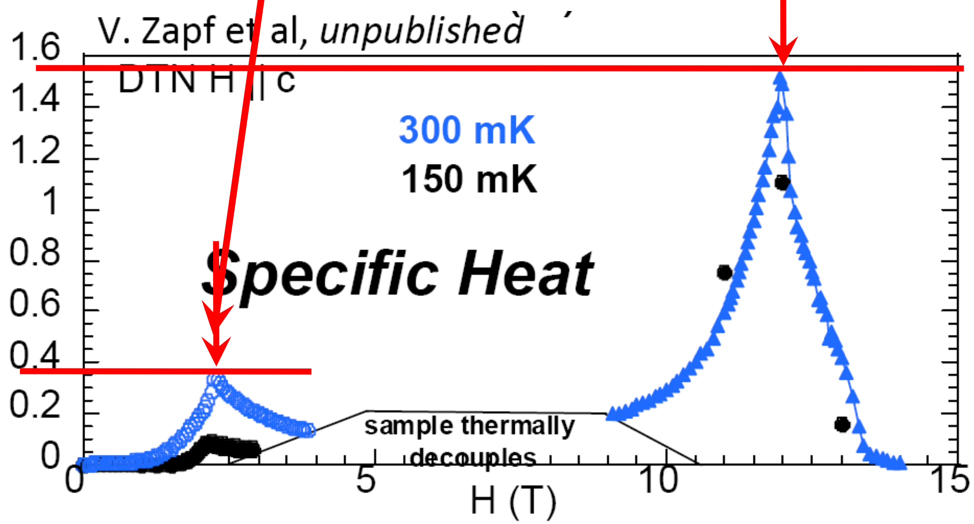


☑ peaks in C_v at H_{c1} and H_{c2}, are very asymmetric

☑ not so for the thermal conductivity → ??

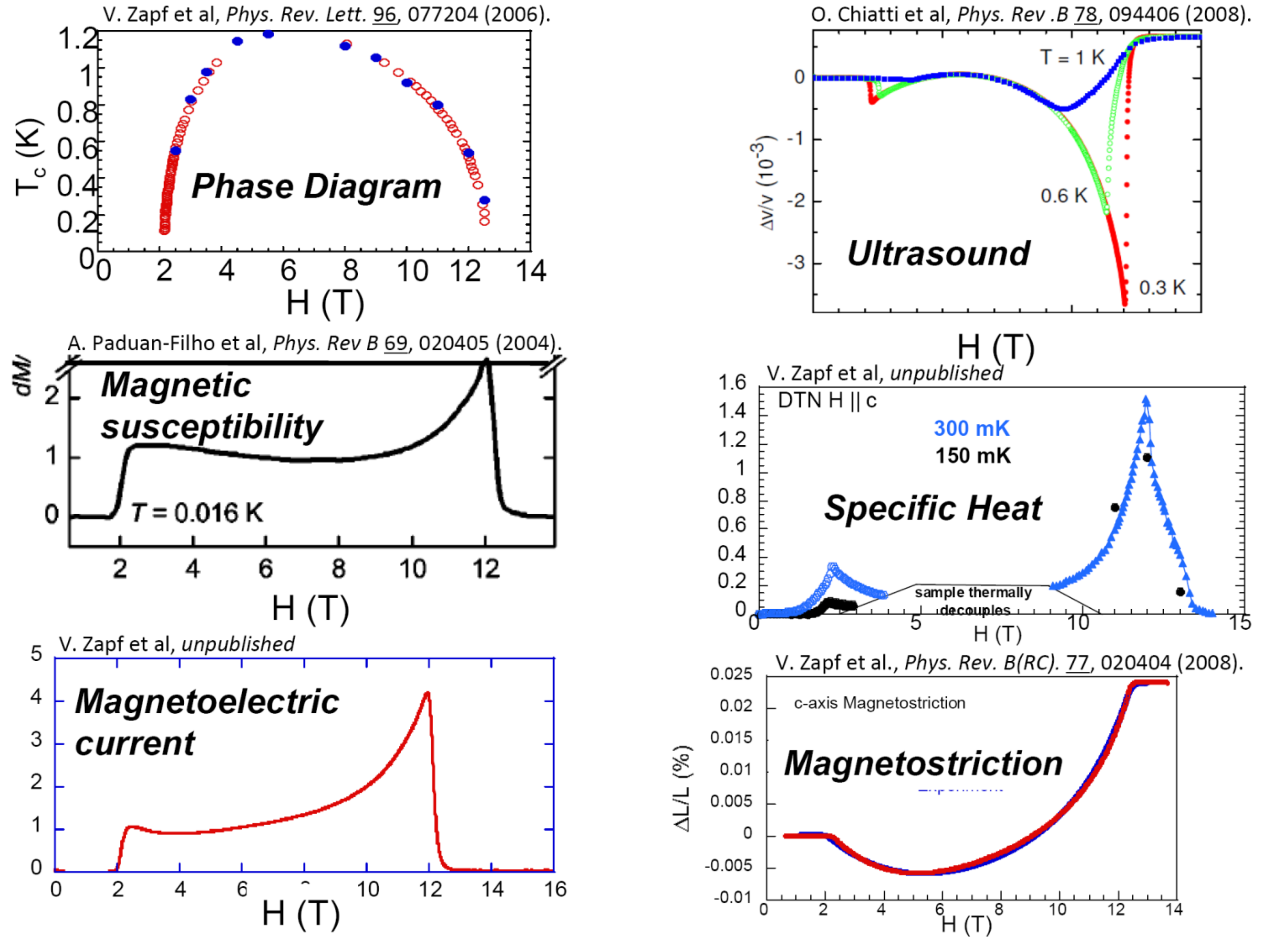
agrees well with experiment

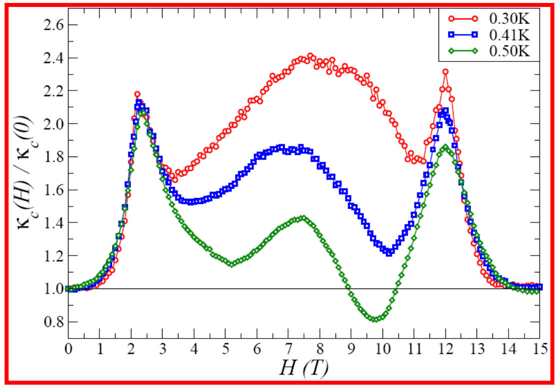




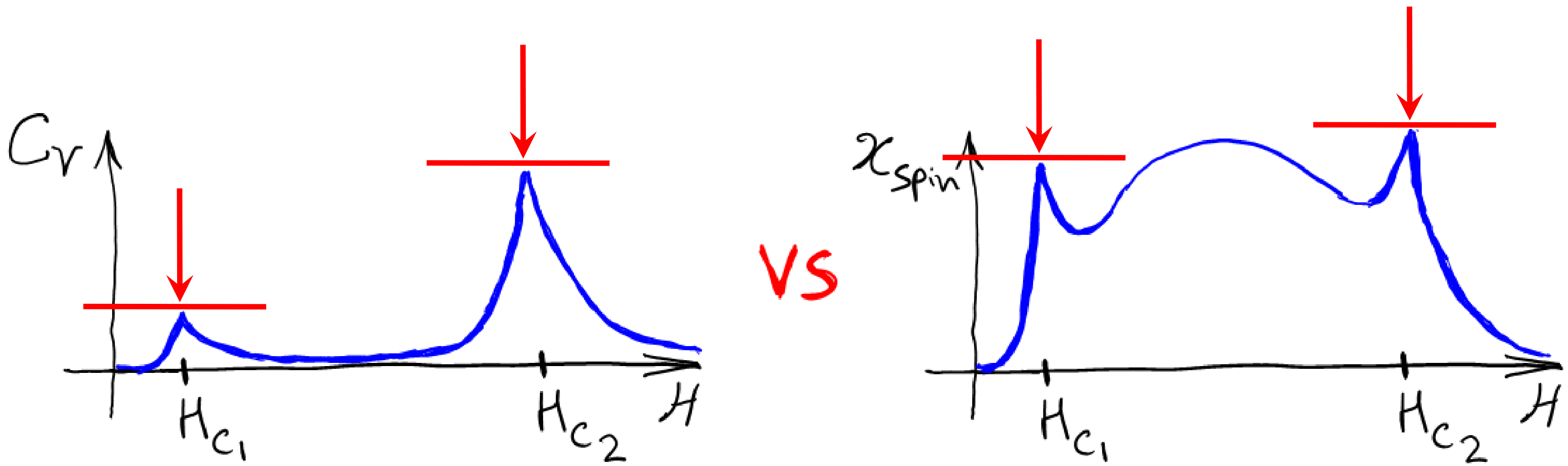
puzzles, I: “no asymmetry” problem

This asymmetry affects many properties**, but not thermal conductivity …**





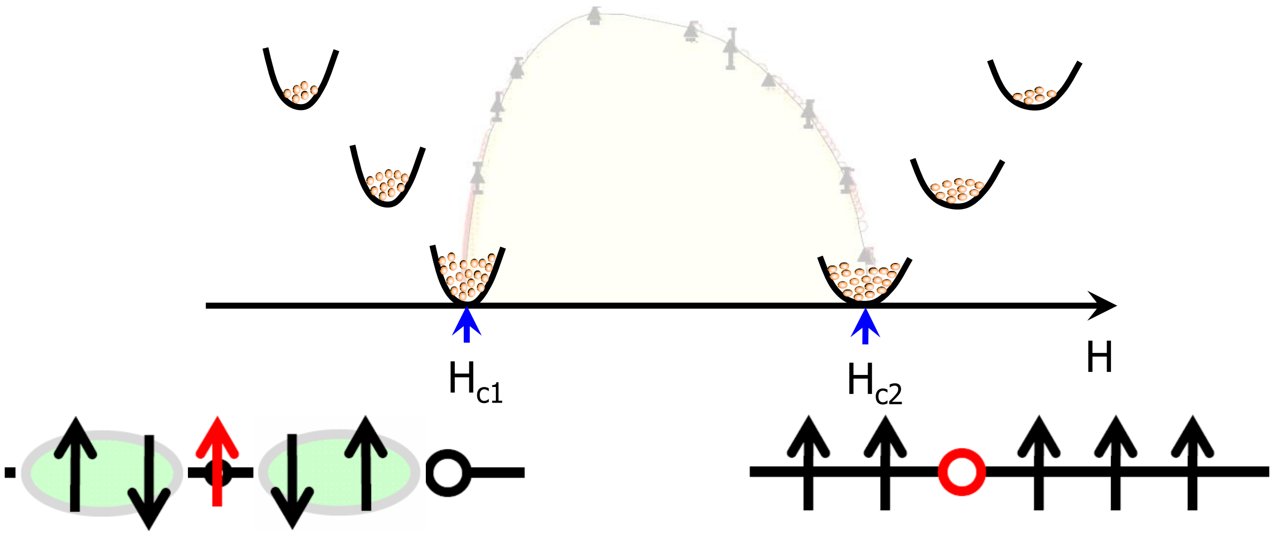
“no asymmetry” problem

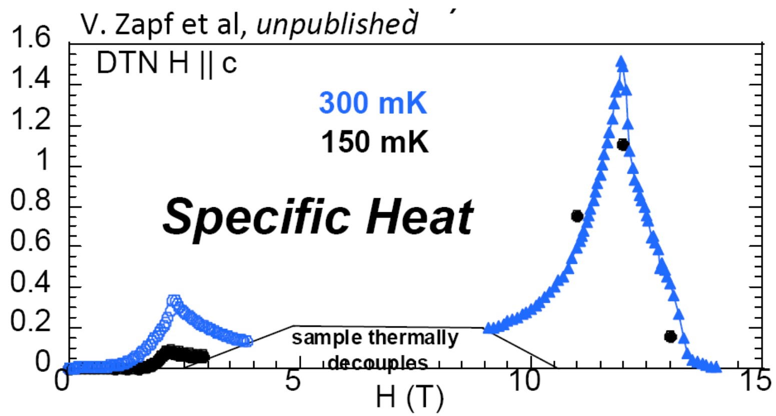


specific heat

-- at the QCP, specific heat: $C_v(H_c) \propto (mT)^{3/2}$

-- asymmetry of the peaks = "asymmetry" of masses





-- m_2 is "bare", $m_1 < m_2$ is renormalized by fluctuations

PRL **106**, 037203 (2011)

constraint $\sum_m b^{\dagger}_{\mathbf{r}m} b_{\mathbf{r}m} = 1$. The subscript $m = \{\downarrow, 0, \uparrow\}$ labels the eigenstates of $S^z_{\mathbf{r}}$ with the eigenvalues $\{-1, 0, 1\}$. The spin operators in this representation are:

$$S^z_{\mathbf{r}} = n_{\mathbf{r}\uparrow} - n_{\mathbf{r}\downarrow}, \; S^+_{\mathbf{r}} = \left(S^-_{\mathbf{r}}\right)^\dagger = \sqrt{2}\left(b^\dagger_{\mathbf{r}\uparrow} b_{\mathbf{r}0} + b^\dagger_{\mathbf{r}0} b_{\mathbf{r}\downarrow}\right), \quad (2)$$

with $n_{\mathbf{r}m} = b^\dagger_{\mathbf{r}m} b_{\mathbf{r}m}$. We enforce the constraint by introducing spatially uniform Lagrange multiplier μ

$$\hat{\mathcal{H}} = \mathcal{H} + \mu \sum_{\mathbf{r}} \left(b^\dagger_{\mathbf{r}\uparrow} b^\dagger_{\mathbf{r}\uparrow} + b^\dagger_{\mathbf{r}\downarrow} b^\dagger_{\mathbf{r}\downarrow} + b^\dagger_{\mathbf{r}0} b^\dagger_{\mathbf{r}0} - 1\right). \quad (3)$$

The lowest energy state in the $H < H_{c1}$ paramagnetic regime is $b^\dagger_{\mathbf{r}0}|0\rangle$ and the ground state corresponds to a non-zero expectation value of the $S^z = 0$ boson: $b^\dagger_{\mathbf{r}0} = b_{\mathbf{r}0} = s$. By using the spin representation (2) with the mean-field value for $b_0^{(\dagger)}$ and neglecting higher-order terms in powers of $b^{(\dagger)}_{\uparrow(\downarrow)}$, we obtain the Hamiltonian in the harmonic approximation

$$\hat{\mathcal{H}} = E_0 + \sum_{\mathbf{k},\sigma} \left[A_{\mathbf{k}\sigma} \hat{b}^\dagger_{\mathbf{k}\sigma} \hat{b}_{\mathbf{k}\sigma} + \frac{B_{\mathbf{k}}}{2}\left(\hat{b}^\dagger_{\mathbf{k}\sigma} \hat{b}^\dagger_{-\mathbf{k}\bar{\sigma}} + \text{H.c.}\right)\right], \quad (4)$$

with $A_{\mathbf{k}\sigma} = (\mu + s^2 \epsilon_{\mathbf{k}} - h_\sigma)$ and $B_{\mathbf{k}} = s^2 \epsilon_{\mathbf{k}}$, where $E_0 = N(\mu - D)(s^2 - 1)$ is the bare ground-state energy, N is the number of sites, $\sigma = \{\uparrow, \downarrow\}$, $h_\sigma = \pm h$, $\bar{\sigma} = -\sigma$, $\hat{b}^{(\dagger)}_{\mathbf{k}\sigma}$ are the Fourier transformed bosonic operators, and $\epsilon_{\mathbf{k}} = 2\sum_\nu J_\nu \cos k_\nu$. The anomalous terms indicate that bosons with opposite S^z are created and annihilated in the ground state. These are the quantum fluctuations that lead to renormalization of the quasi-particle dispersion relation. The Hamiltonian (4) is diagonalized by the Bogolyubov transformation

$$\hat{b}_{\mathbf{k}\sigma} = u_{\mathbf{k}} \beta_{\mathbf{k}\sigma} + v_{\mathbf{k}} \beta^\dagger_{-\mathbf{k}\bar{\sigma}}, \quad (5)$$

$H \geq H_{c2}$ spins are fully polarized and the spectrum can be computed exactly. Since there are no quantum fluctuations for $H \geq H_{c2}$, the exact value of h_{c2} is $h_{c2} = g\mu_B H_{c2} = D - 2\epsilon_{\mathbf{Q}}$, while the unrenormalized excitation spectrum is $\widetilde{\omega}^>_{\mathbf{k}} \equiv \epsilon_{\mathbf{k}} - \epsilon_{\mathbf{Q}} + h - h_{c2}$, which also has a minimum at $\mathbf{Q}$ with the gap $\Delta^> = h - h_{c2}$. Since only the excitations near $\mathbf{k} = \mathbf{Q}$ are important at low temperatures, we define the mass tensors for $H < H_{c1}$ and $H > H_{c2}$ as:

$$\frac{1}{m^*_{\nu\nu}} = \left.\frac{\partial^2 \widetilde{\omega}^<_{\mathbf{k}}}{\partial k^2_\nu}\right|_{\mathbf{k}=\mathbf{Q}}, \qquad \frac{1}{m_{\nu\nu}} = \left.\frac{\partial^2 \widetilde{\omega}^>_{\mathbf{k}}}{\partial k^2_\nu}\right|_{\mathbf{k}=\mathbf{Q}}. \quad (9)$$

Then the mass renormalization factor is given by

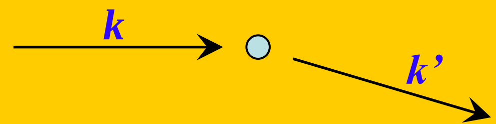
$$\frac{m_{\nu\nu}}{m^*_{\nu\nu}} = s^2 \frac{\mu}{\omega^0_{\mathbf{Q}}} \approx \frac{H_{c2}}{4H_{c1}} \cdot \left(1 + \sqrt{1 + \frac{8H^2_{c1}}{H^2_{c2}}}\right). \quad (10)$$

For the parameters of the Hamiltonian from Ref. [8], we obtain $m_{\nu\nu}/m^*_{\nu\nu} \simeq 3.2$. Such a large difference of masses must readily demonstrate itself in the strong asymmetry of the C_v vs H curves near H_{c1} and H_{c2} as well as in the slopes of the ... ere C_v ... ted by ... .

$$\frac{m_2}{m_1} \approx \frac{H_{c2}}{2H_{c1}} \qquad \text{for} \quad H_{c1} \ll H_{c2}$$

The C_p was measured in single crystals of DTN grown from aqueous solutions of thiourea and nickel chloride, with magnetic field applied along the crystalline c-axis. The experimental C_p vs H was obtained using an AC technique [17], while sweeping the magnetic field in a ^{3}He fridge furbished with a 17 T superconducting magnet system at the National High Magnetic Field Laboratory (NHMFL) and the Los Alamos National Laboratory. We also used the standard thermal relaxation method to obtain C_v vs T with a dilution re-

impurity scattering

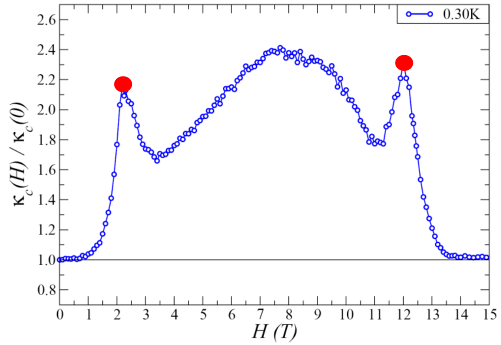


$$\varepsilon_{\mathbf{k}} = k^2/2m$$

-- at the QCP impurities are effective scatterers:

-- mean-free path:

$$\ell_{\mathrm{imp}}^{-1} = \frac{1}{|\mathbf{v_k}|\tau_{\mathbf{k}}} = \frac{n_i}{2\pi} m^2 \left|V_{\mathrm{imp}}\right|^2$$



-- ratio of thermal conductivities:

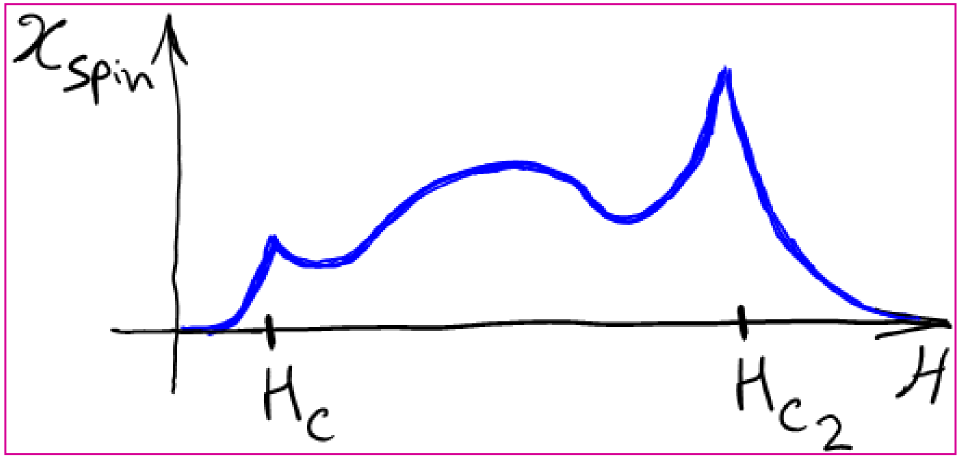
$$\frac{\kappa(H_{c2})}{\kappa(H_{c1})} = \left(\frac{m_2}{m_1}\right) \cdot \frac{\ell_2}{\ell_1}$$

$\mathrm{DoS} \cdot v_k^2$

-- however, V_{imp} is also renormalized by fluctuations (!)

-- it is **most natural** if renormalization of V_{imp} is the **same** as for $1/m$,

then $\ell_1 = \ell_2$ and $\frac{\kappa_2}{\kappa_1} = \frac{m_2}{m_1}$



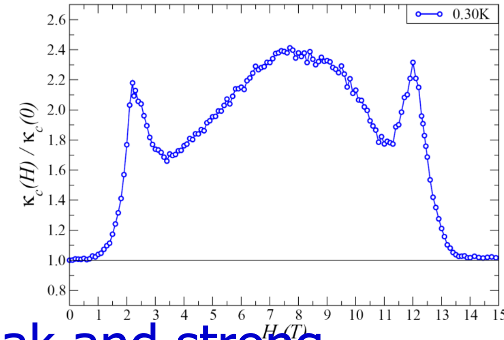
PRL **106**, 037203 (2011)

which impurity: bond/site, weak/strong?

-- ratio of thermal conductivities:

$$\frac{\kappa(H_{c2})}{\kappa(H_{c1})} = \left(\frac{m_2}{m_1}\right) \cdot \frac{\ell_2}{\ell_1}$$

DoS · v_k^2



-- **impurities:** modulations of *D* and *J* ($V_{imp}=\delta D$ and $V_{imp}=\delta J$), weak and strong

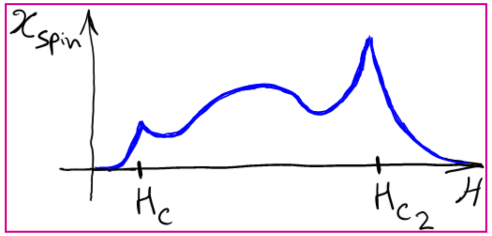
-- which is the leading effect?

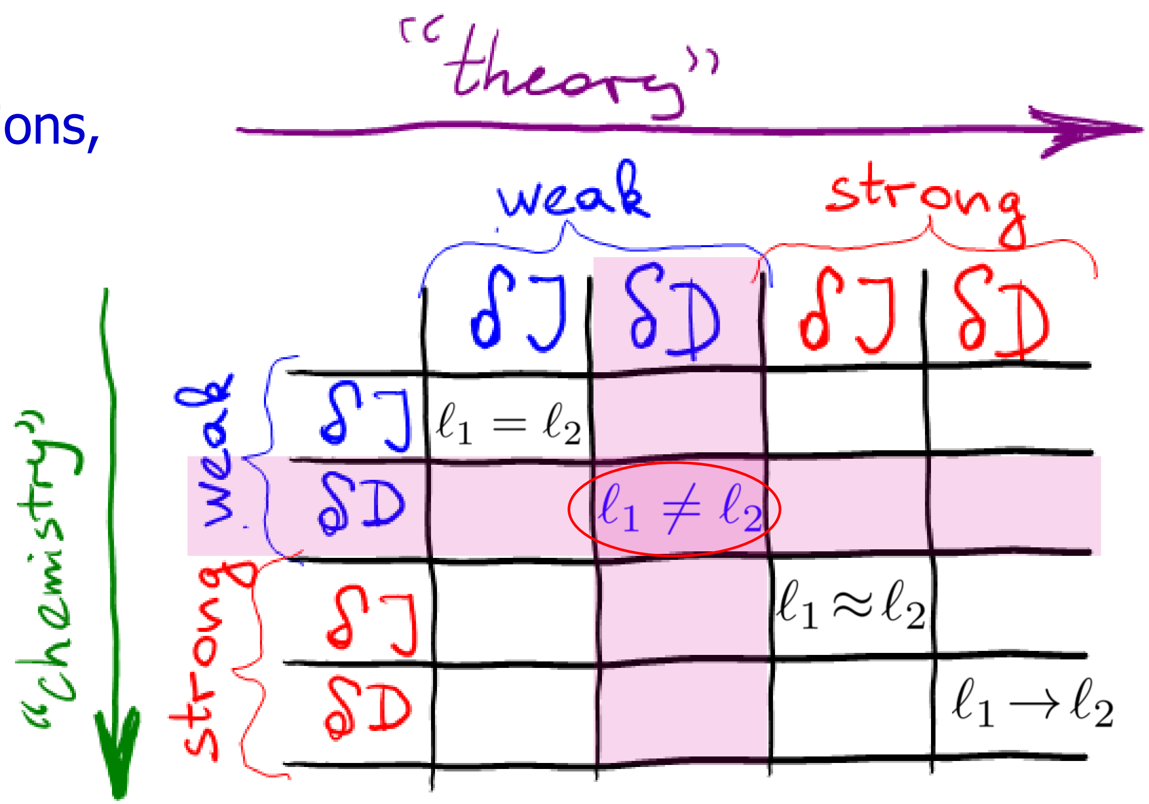
-- DTN=clean material → structural distortions, *D* is the leading term → weak δD

-- "direct" check: all but one, same renormalization of V_{imp} as for *1/m*

$$\ell_1 = \ell_2$$

$$\frac{\kappa_2}{\kappa_1} = \frac{m_2}{m_1}$$





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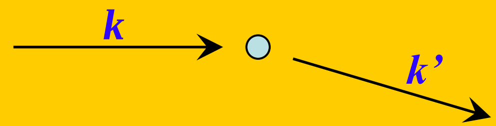
site-disorder



-- thermal conductivities:

$$\frac{\kappa(H_{c2})}{\kappa(H_{c1})} = \left(\frac{m_2}{m_1}\right) \cdot \frac{\ell_2}{\ell_1}$$

-- δD at H_{c1}:

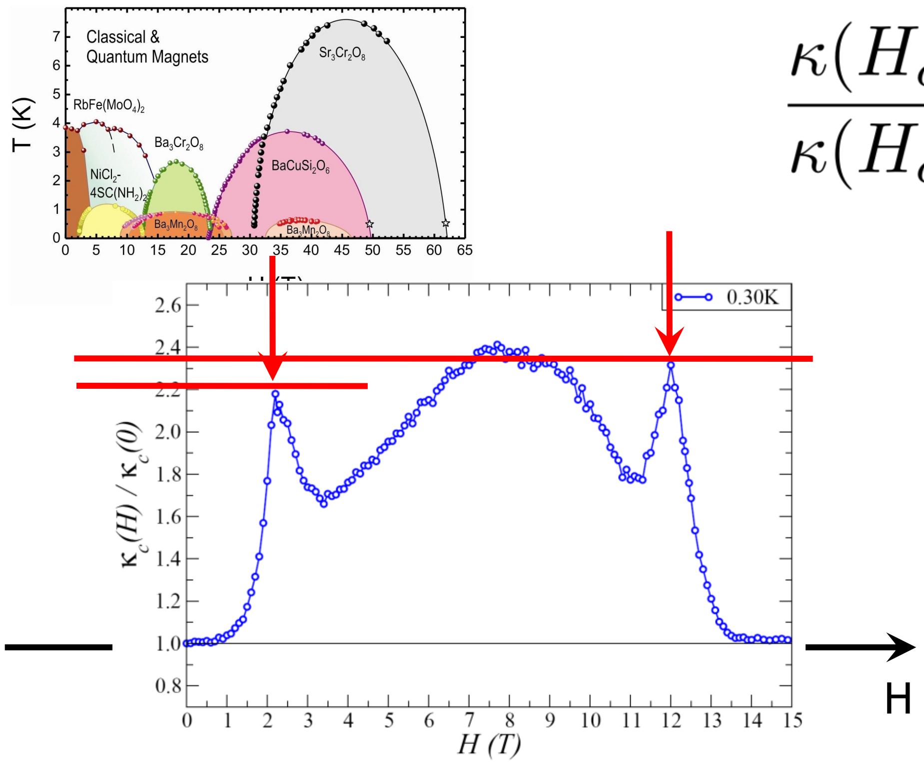
$$\mathcal{H}^D_{\rm imp} = \delta D \sum_{\sigma,\mathbf{k},\mathbf{k}'} e^{i\mathbf{R}_\ell(\mathbf{k}-\mathbf{k}')} \left(u_\mathbf{k} u_{\mathbf{k}'} + v_\mathbf{k} v_{\mathbf{k}'}\right) \beta^\dagger_{\mathbf{k}\sigma} \beta_{\mathbf{k}'\sigma}$$

-- different set of "coherence factors" than in *m*

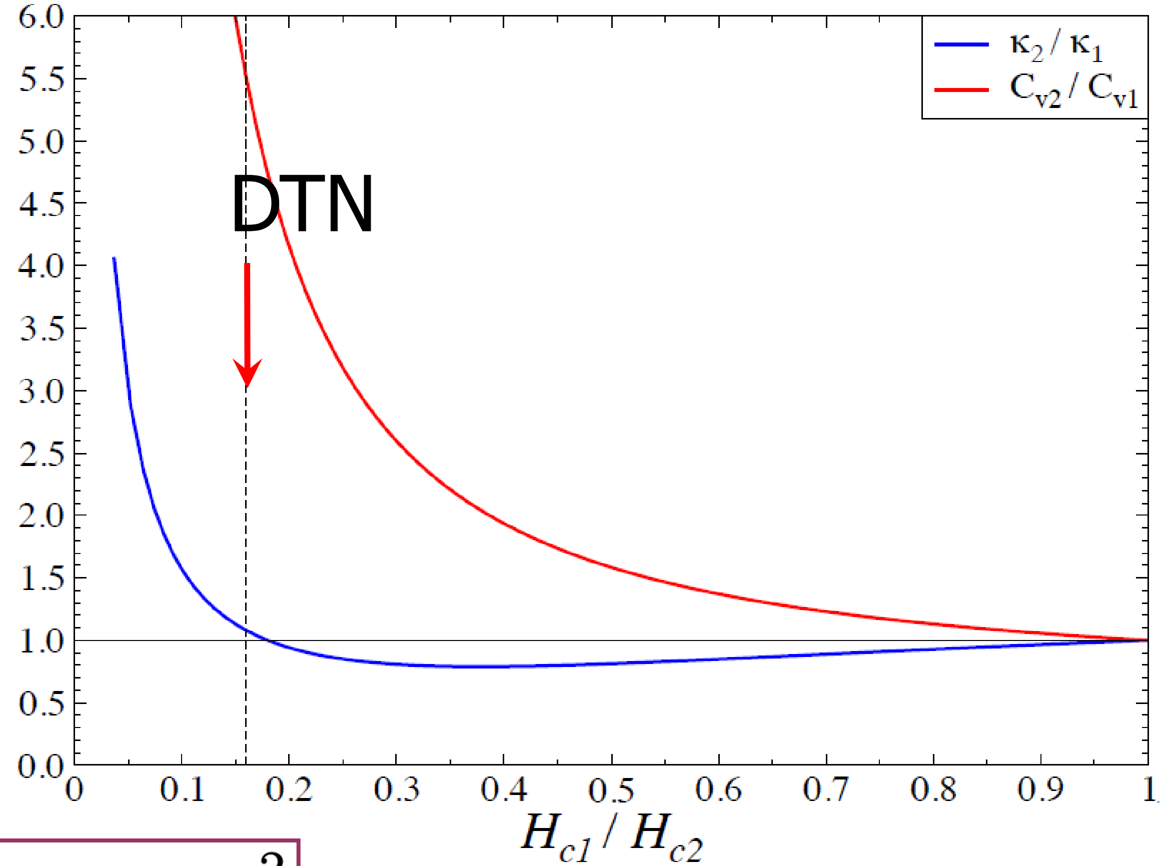
$$\frac{\kappa(H_{c2})}{\kappa(H_{c1})} = \left(u_\mathbf{Q} + v_\mathbf{Q}\right)^2 \cdot \frac{\left(u^2_\mathbf{Q} + v^2_\mathbf{Q}\right)^2}{\left(u_\mathbf{Q} + v_\mathbf{Q}\right)^4}$$

$$\Rightarrow \quad \frac{\kappa(H_{c2})}{\kappa(H_{c1})} \approx \underbrace{\left(\frac{m_2}{m_1}\right)}_{\text{DoS}\cdot v_k^2} \cdot \frac{1}{4} \left(1 + \left(\frac{m_1}{m_2}\right)^2\right)^2$$

thermal conductivity vs H_c's ratio



$$\frac{\kappa(H_{c2})}{\kappa(H_{c1})}$$



$$\frac{\kappa(H_{c2})}{\kappa(H_{c1})} \approx \left(\frac{m_2}{m_1}\right) \cdot \frac{1}{4}\left(1 + \left(\frac{m_1}{m_2}\right)^2\right)^2 \approx 1.1$$

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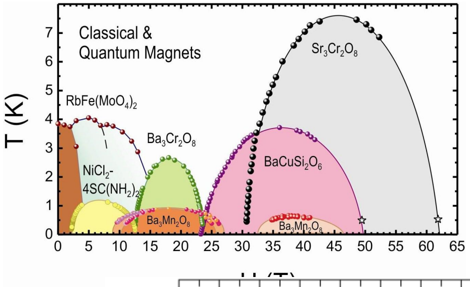
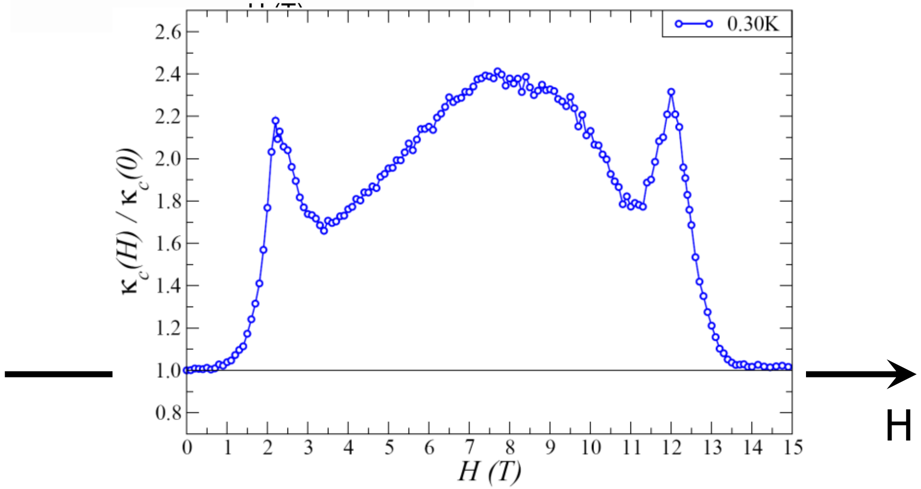
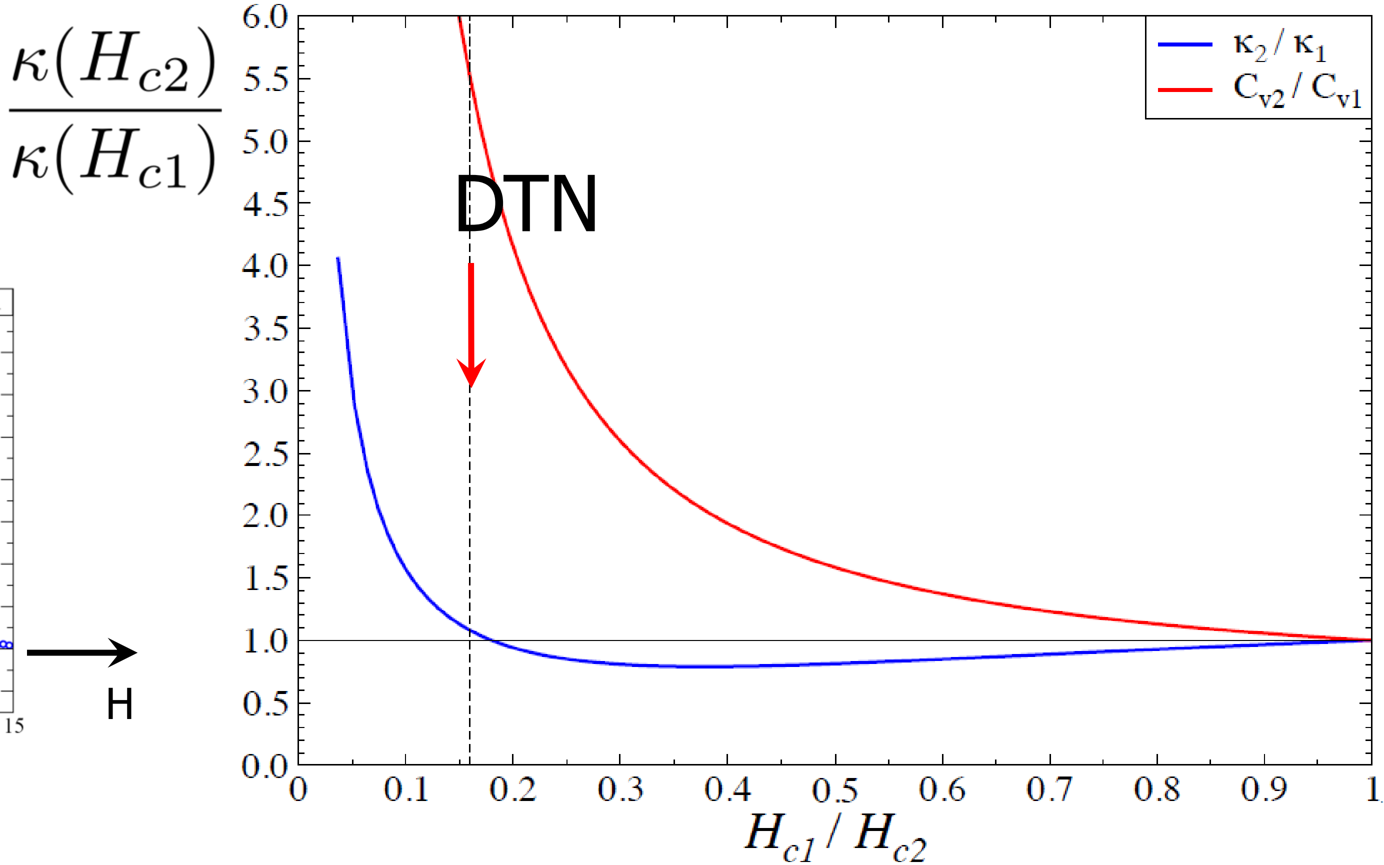
"no asymmetry" ~~problem~~





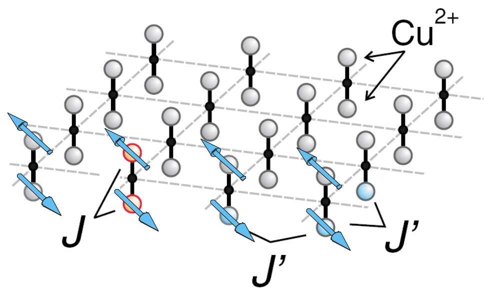


-- unless $H_{c1} << H_{c2}$, $\kappa_2 \approx \kappa_1$ for any BEC system at low enough T

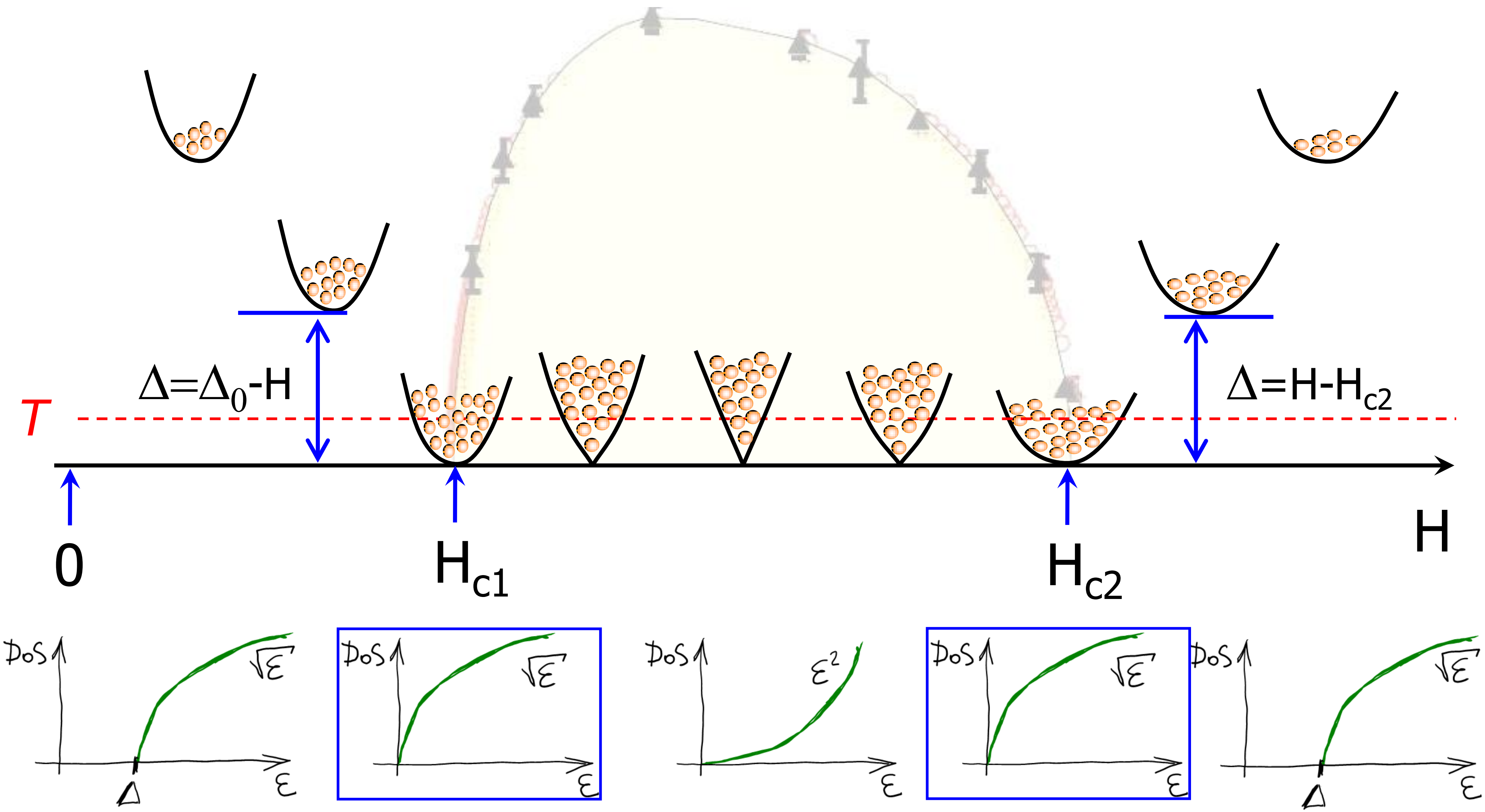
-- valid for the **dimer-based systems** as well:

modulations of **intra-dimer** J lead to the same scattering as δD

-- proposal: pressure experiment on DTN, will reduce κ_1



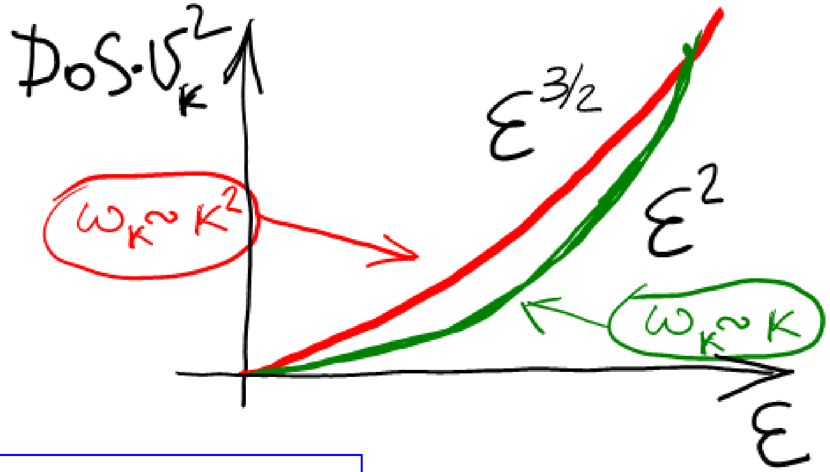
away from QCPs, dispersion, DOS



puzzles, III: minima

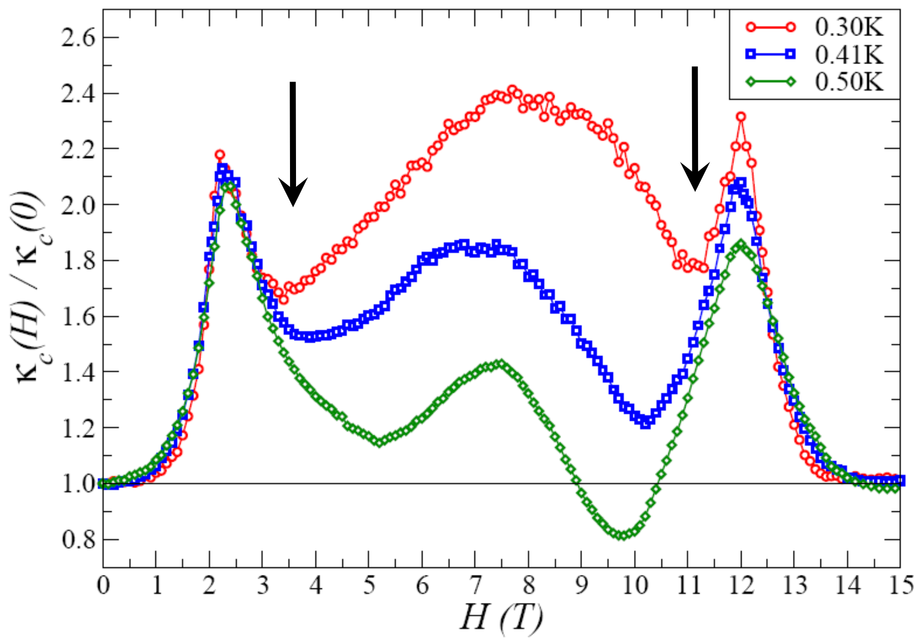
$$\kappa \propto \int_0^T k^2\, dk \cdot (\mathbf{v_k})^2 \cdot \tau_\mathbf{k}$$

DoS · v_k^2



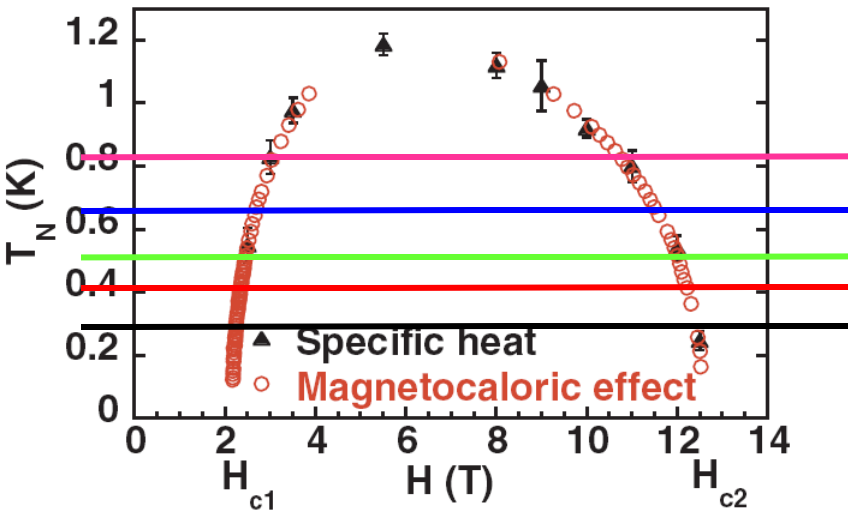
"effective" DoS is not that different between $\omega \sim k$ and $\omega \sim k^2$ regimes

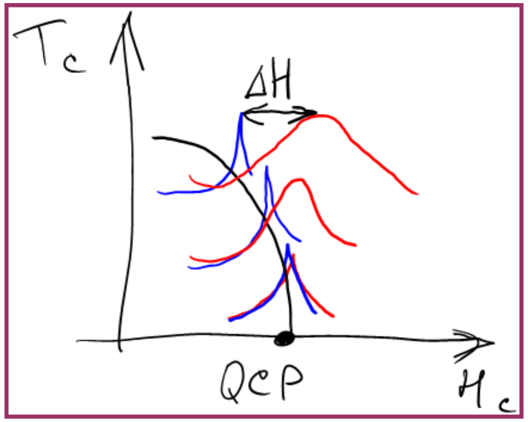
☑ what are the minima in the BEC phase?
☑ strongly field-dependent scattering?

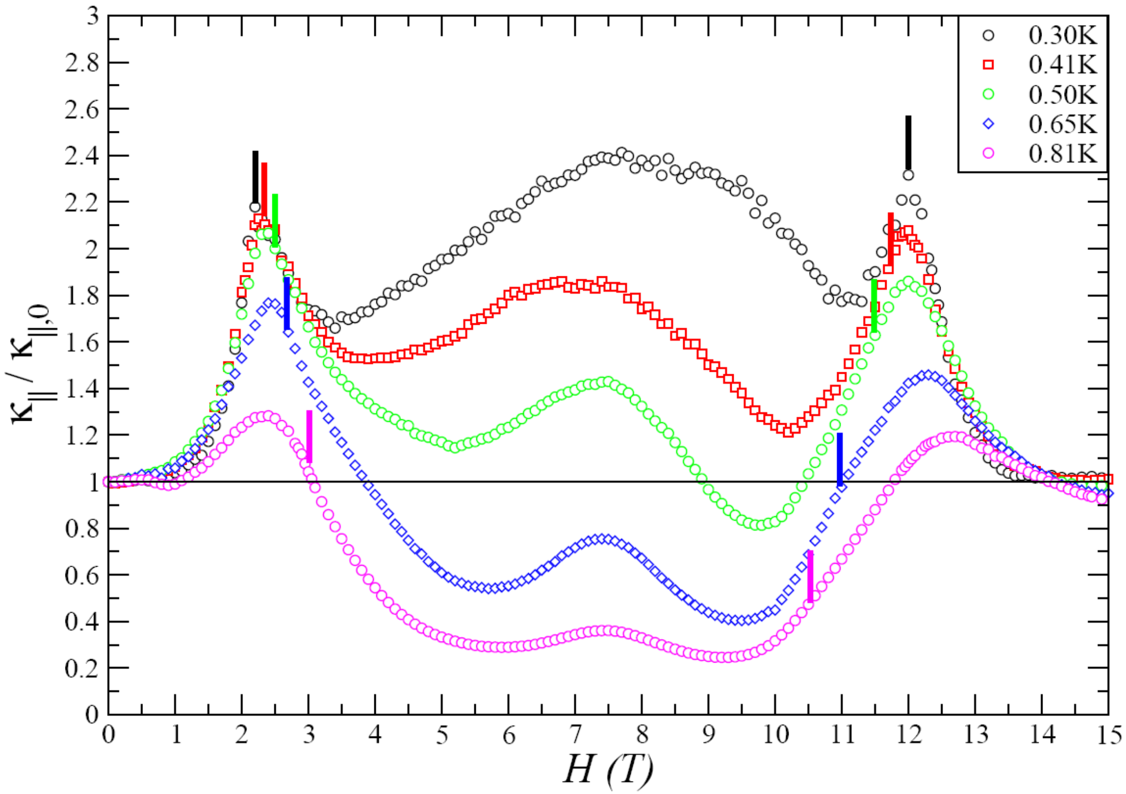


puzzles, II: "peak migration" problem

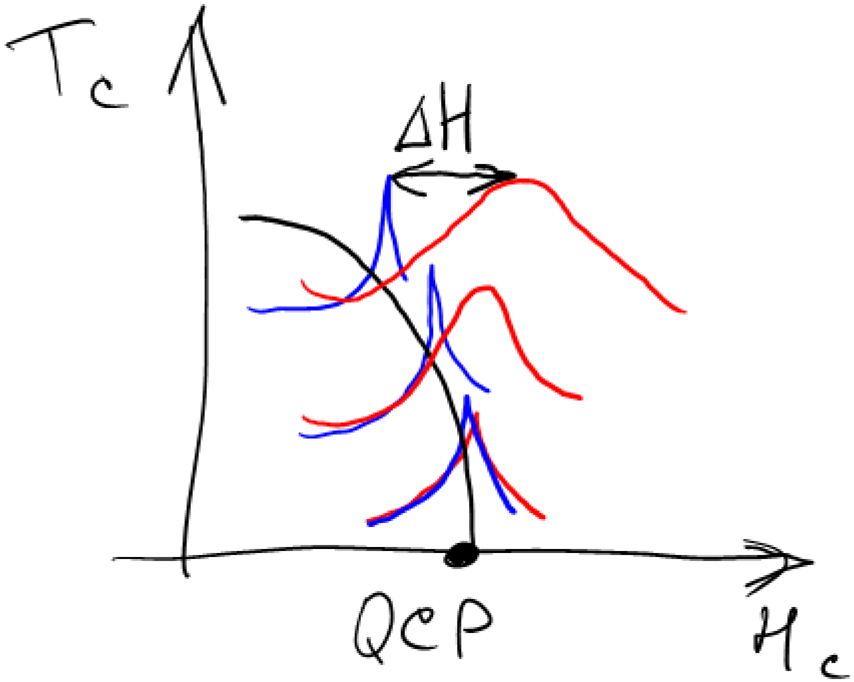
☑ peaks/maxima in κ "migrate" **away** from H_c's as T increases → ??







migration of the peaks



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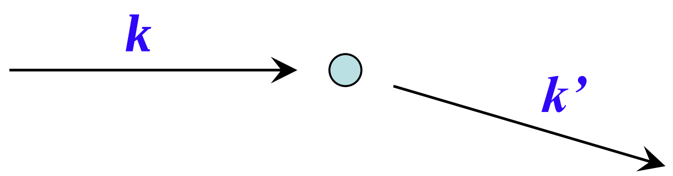
scatterings in the paramagnetic phase

☑ both b-b and impurity scattering are important for $\omega_{\mathbf{k}}=\Delta+k^2/2m$ band

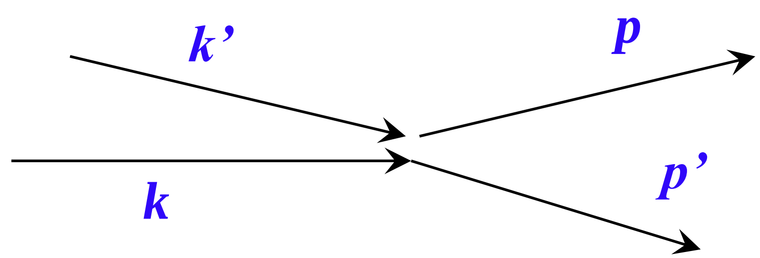
$$\mathcal{H} = \sum_{\mathbf{k}} (\varepsilon_{\mathbf{k}} - \mu_0) b^\dagger_{\mathbf{k}} b_{\mathbf{k}} + g_0 \sum_{\mathbf{k},1,2} b^\dagger_{\mathbf{k}+2-1} b^\dagger_1 b_2 b_{\mathbf{k}}$$

$$\mathcal{H}^D_{\text{imp}} = \delta D \sum_{\mathbf{i}} b^\dagger_{\mathbf{i}} b_{\mathbf{i}} = \delta D \sum_{\mathbf{k},\mathbf{k}'} e^{i\mathbf{R}_{\mathbf{i}}(\mathbf{k}-\mathbf{k}')}\, b^\dagger_{\mathbf{k}} b_{\mathbf{k}'}$$

☑ *impurity scattering:* $\mathbf{k} \neq \mathbf{k}'$



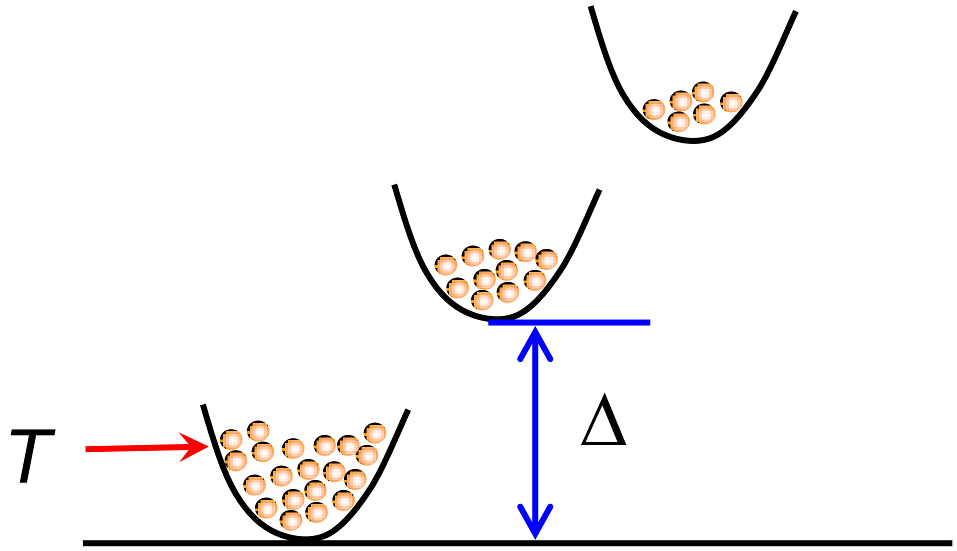
☑ *b-b scattering:* $\mathbf{k} + \mathbf{k}' = \mathbf{p} + \mathbf{p}'$



impurity scattering

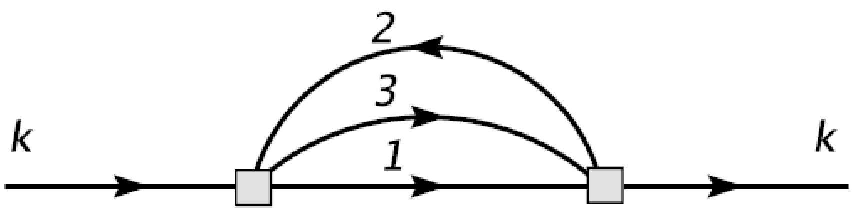
☑ impurity scattering does not depend on the gap value, only velocity of the boson

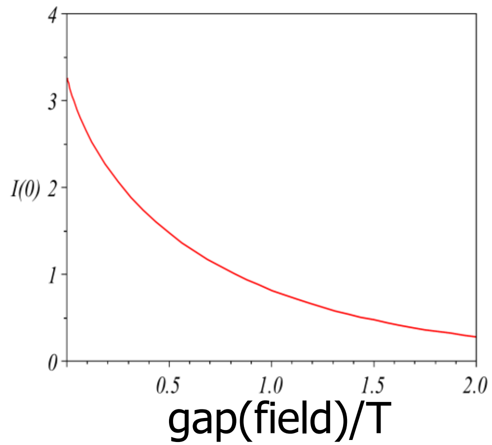
$$\frac{1}{\tau_{\mathbf{k}}^{\text{imp}}} \propto n_i |V_i|^2 \, m \, |\mathbf{k}|$$



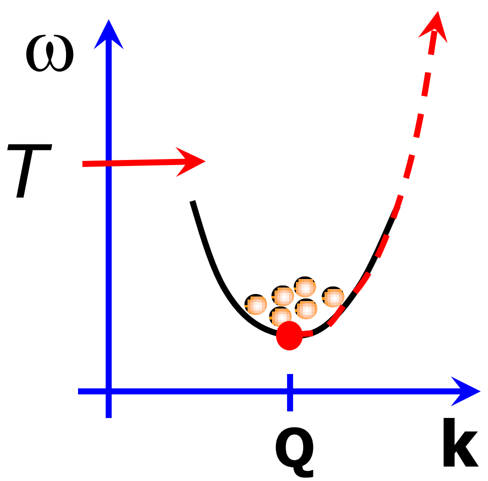
b-b scattering, on-shell

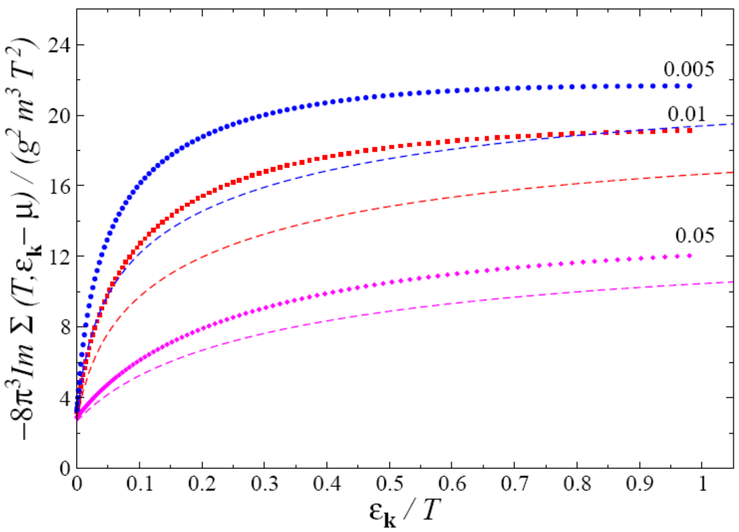
- ☑ ~ constant vs energy
- ☑ decreases exponentially vs gap(field) [no bosons to scatter on]





$$\frac{1}{\tau_{\mathbf{k}=0}} = -\mathrm{Im}\Sigma(\mathbf{k}, \varepsilon_{\mathbf{k}} - \mu)\Big|_{\mathbf{k}=0} \approx \frac{g_0^2 m^3 T^2}{8\pi^3} \cdot \frac{\pi^2}{3} \cdot \exp(-\Delta/T)$$



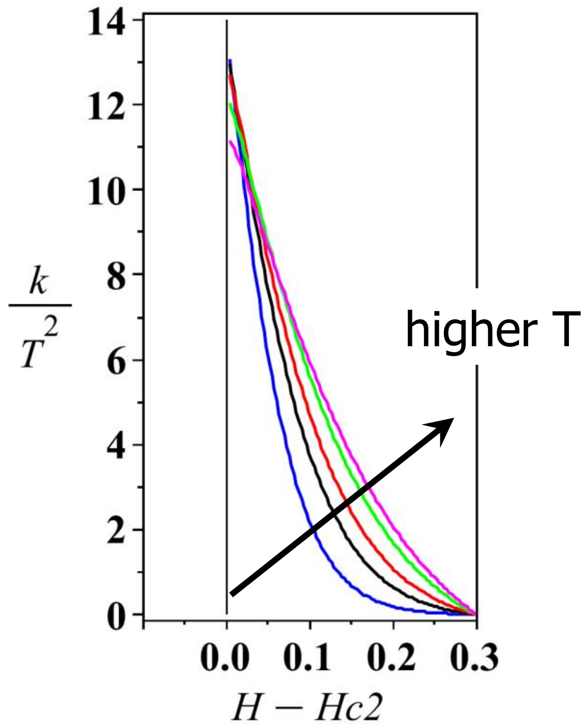


thermal conductivity, impurity only

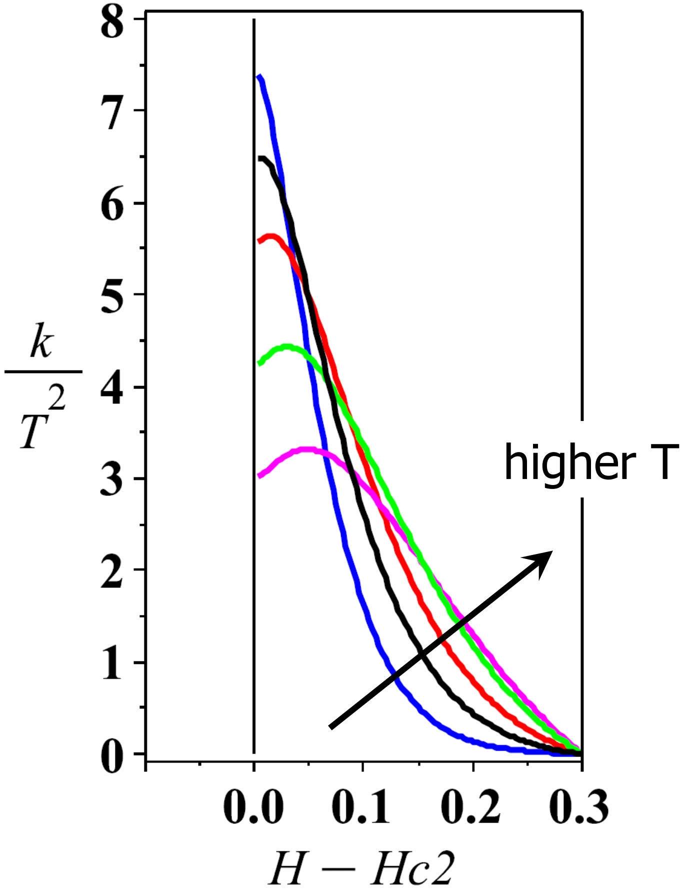
$$\kappa \sim \int_0^\infty k^2 dk \cdot \frac{k^2}{m^2} \cdot \frac{\omega_{\mathbf{k}}^2}{T^2} \cdot \frac{e^{\omega_{\mathbf{k}}/T}}{(e^{\omega_{\mathbf{k}}/T}-1)^2} \cdot \tau_{\mathbf{k}}$$

$$\omega_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \mu$$

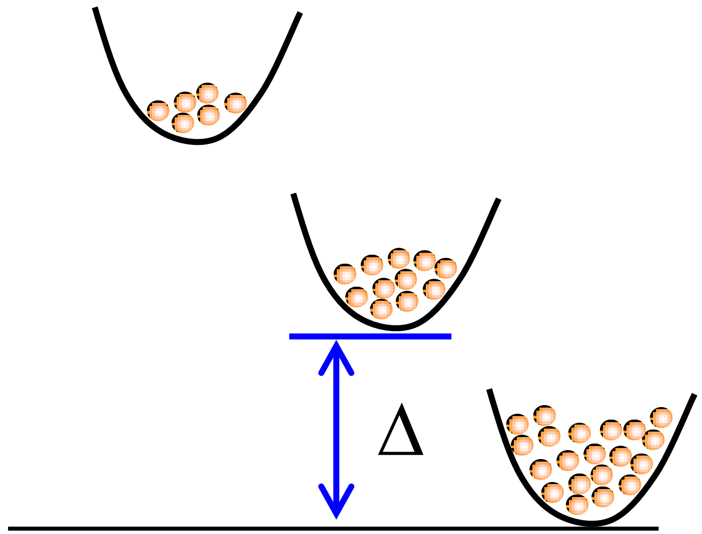
- ☑ impurity only:
  - ☑ some T-dependence, change in shape



thermal conductivity, Goldilocks gap



☑ impurity and b-b together:
- ☑ **"migrating" peaks!**
- ☑ reason → bosons provide extra scattering
- ☑ stronger for heavier particles (at > H_{c2})



☑ more heat carriers, but also more scatterers → "optimal" gap

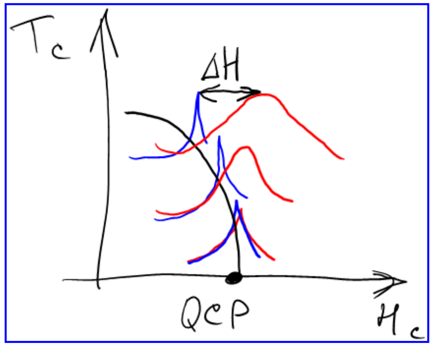
“peak migration” problem

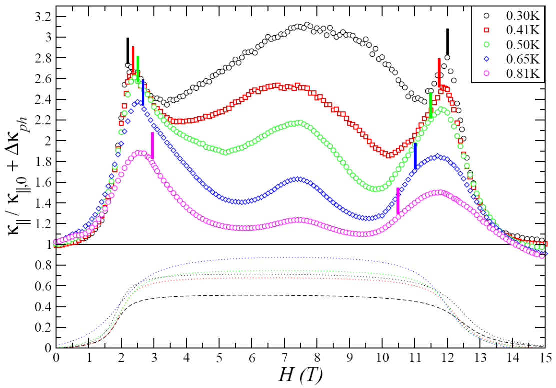
- ☑ what is “optimal” gap (optimal ΔH)?
- ☑ when the impurity and b-b mean-free paths are equal (only b-b part knows about the gap $\Delta = \Delta H$)

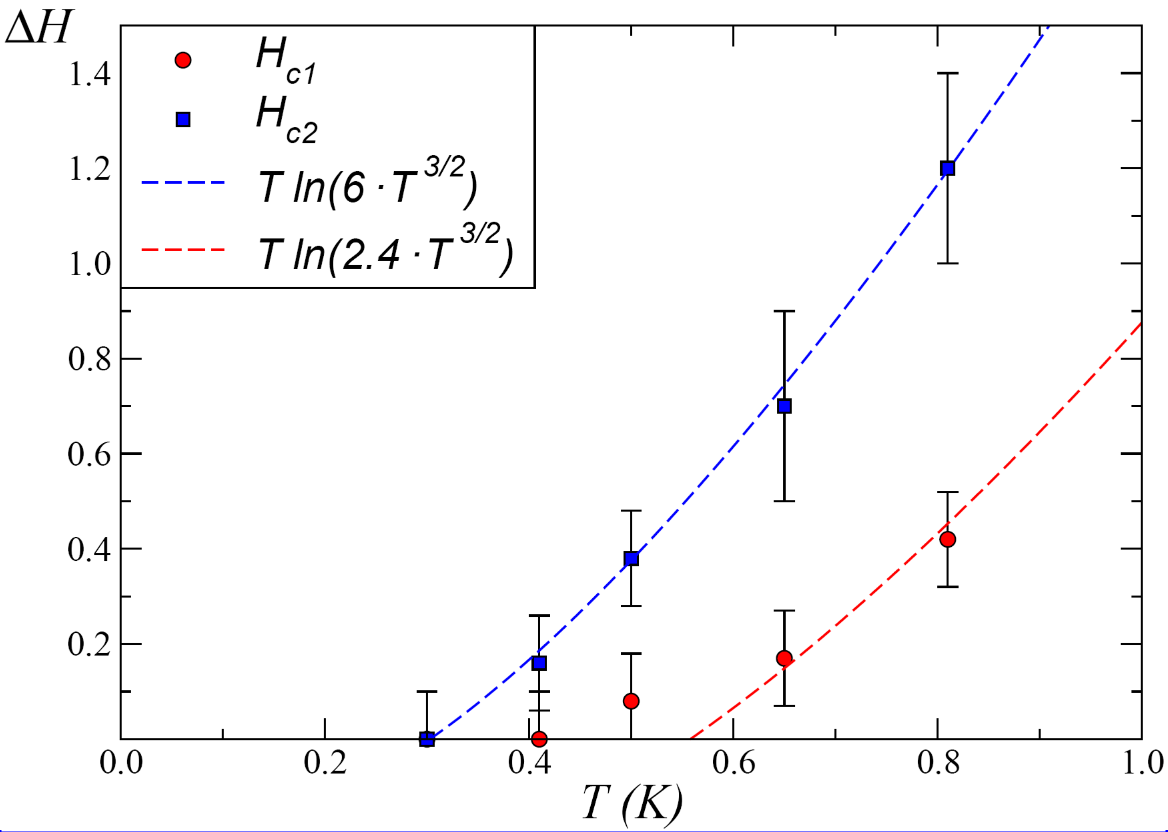
$$\ell_{\rm imp} \approx \ell_{bb}$$

$$n_i \approx (mT)^{3/2} e^{-\Delta/T}$$

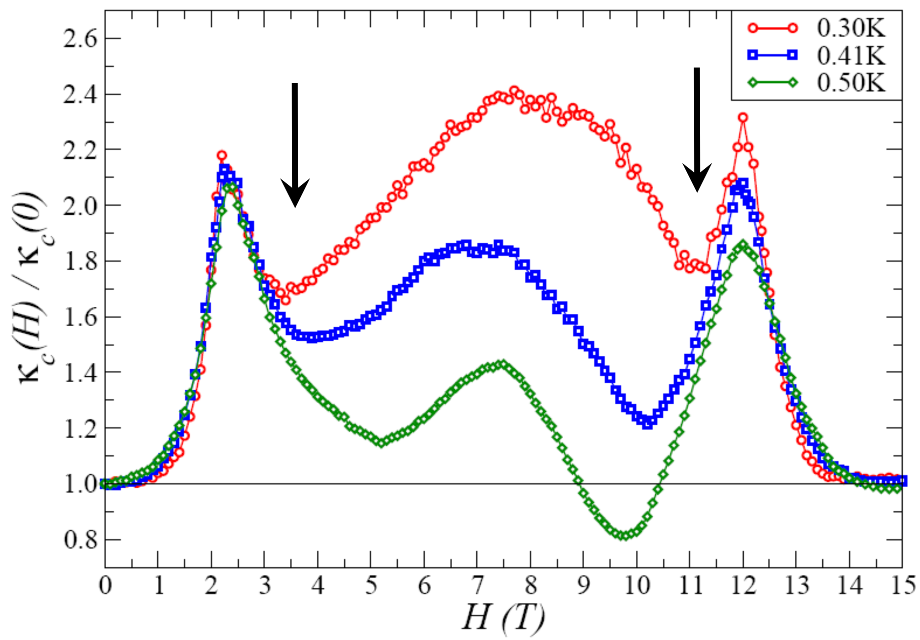
$$\Delta = T \cdot \ln\left(\frac{(mT)^{3/2}}{n_{\rm imp}}\right)$$







minima "inside" of the BEC phase

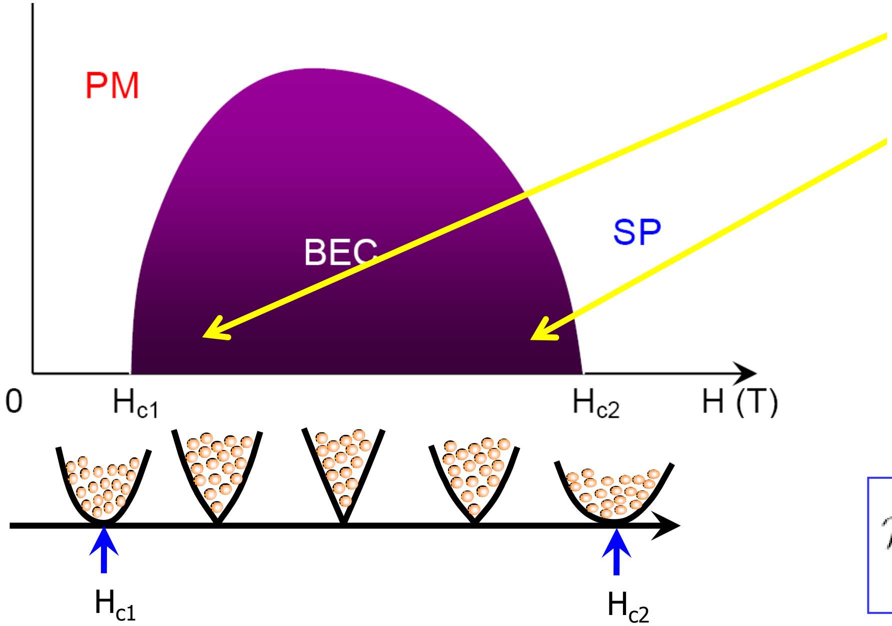


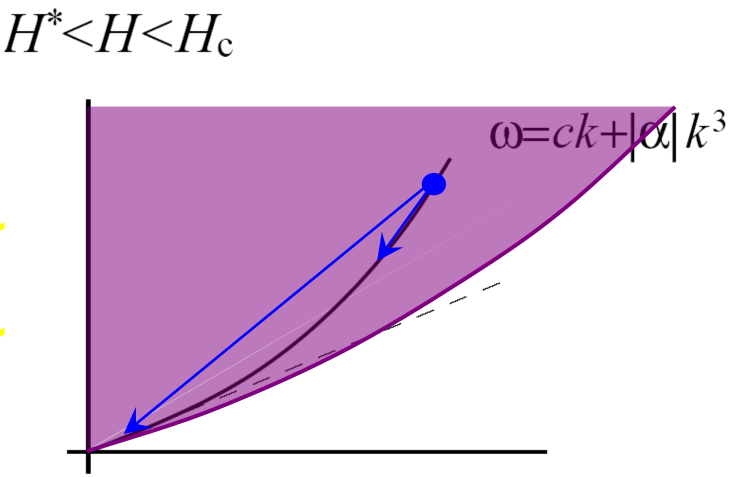
☑ strongly field-dependent scattering?

unpublished

kinematics and interactions

- ☑ concave spectrum
- ☑ XY AF in a field = canted AF





$$\hat{\mathcal{H}} = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} b^{\dagger}_{\mathbf{k}} b_{\mathbf{k}} + \frac{1}{2} \sum_{\mathbf{k},\mathbf{q}} V_{\mathbf{k},\mathbf{q}} \left(b^{\dagger}_{\mathbf{k}-\mathbf{q}} b^{\dagger}_{\mathbf{q}} b_{\mathbf{k}} + \mathrm{h.c.} \right) + \ldots$$

non-collinearity → transverse-longitudinal coupling → 3-boson terms

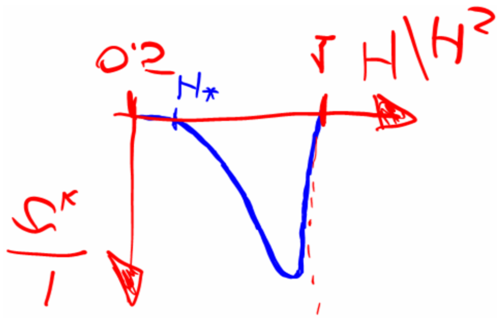
- three-boson terms are necessary for 1-in-2 and 2-in-1 decay/recombination processes
- "kinematic" conditions (*E*- and *k*- conservations) make it sufficient
- forbidden for the convex spectrum and outside of the symmetry-broken phase

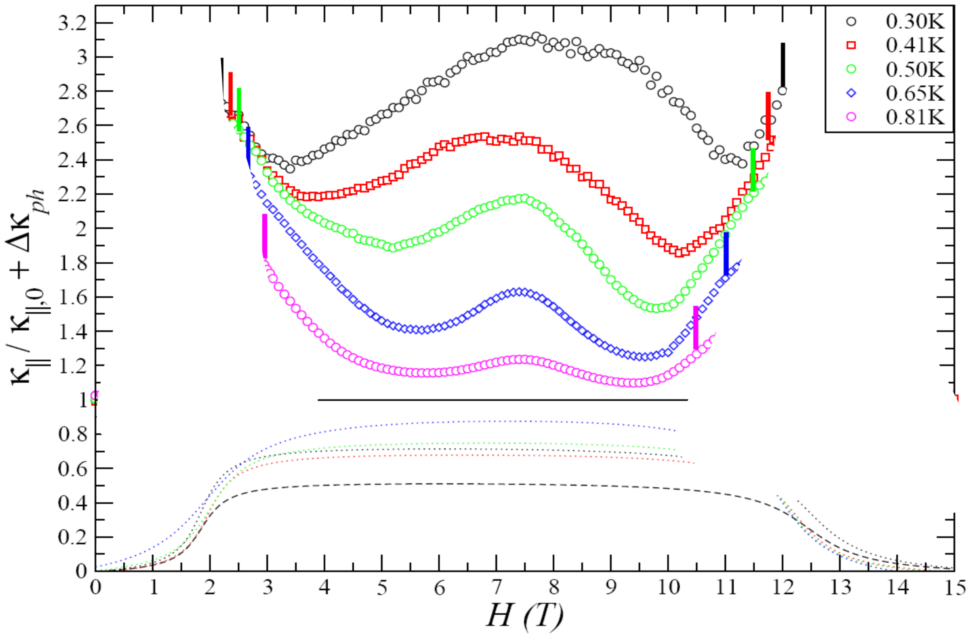
two-particle continuum energy: $E^{(2)}_{\mathbf{k}}(\mathbf{q}) = \omega(\mathbf{q}) + \omega(\mathbf{k}-\mathbf{q})$

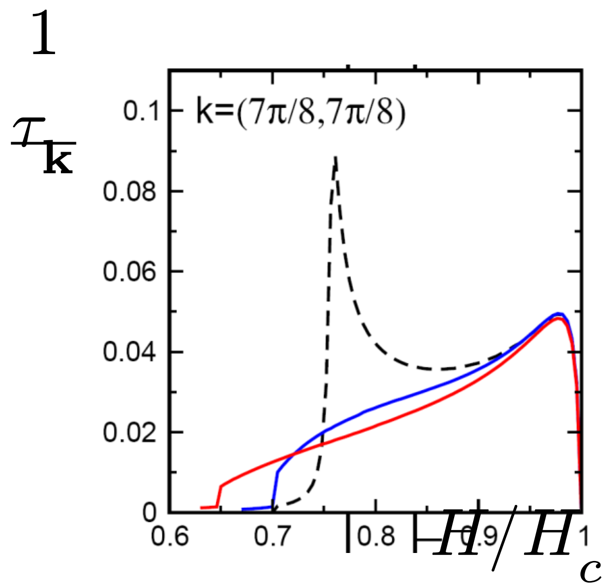
possible role of 3-boson processes …

preliminary result:

$\kappa \sim v^2 C_v \cdot \tau$

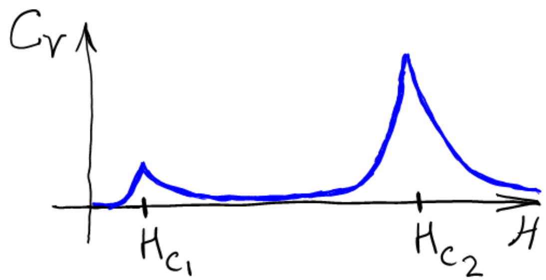






conclusions

- <u>clear</u> field-induced thermal current by spins in DTN
  - "no asymmetry" – intriguing compensation of *m*, δD renorm.
  - "migrating peaks" – interplay of imp. and b-b, "Goldilocks gap"
  - "minima" in the ordered state – 3-boson decays
- experiments in other BEC's needed (clean and low *T*!!)
- more to come



vs