Spin liquid phase in J1-J2 Heisenberg model on the honeycomb lattice

SHOUSHU GONG

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Collaborators:

Donna Sheng
Department of Physics
and Astronomy
California State
University Northridge

Olexei I. Motrunich
Department of Physics
California Institute of Technology

Matthew P. A. Fisher
Department of Physics
University of California, Santa
Barbara

Outline:

- ☐ Background and motivation;
- ☐ DMRG and VMC results;
    - ☐ Phase diagram;
    - ☐ Vanishing magnetic order;
    - ☐ Vanishing dimer order;
    - ☐ Finite spin gap;
    - ☐ Entanglement entropy;
    - ☐ DMRG and VMC comparisons;
- ☐ Summary.

Introduction and motivation:

- Resonating valence bond (RVB) wave function for spin liquid. (P. W. Anderson, 1972).

- Spin liquid: no symmetry breaking even at zero temperature and has fractional excitations. (L. Balents, 2010).

- Heisenberg model on spin-1/2 kagome and J1-J2 square lattice.

  H. C. Jiang, et. al., Phys. Rev. Lett. 101, 117203 (2008).
  S. Yan, et. al., Science 332, 1173 (2011).
  H. C. Jiang, et. al., Phys. Rev. B 86, 024424 (2012).
  L. Wang, et. al., arxiv: 1112.3331, (2011).

  Hubbard model on the honeycomb lattice

  Z. Y. Meng, et. al. Nature (London) 464, 847 (2010).
  S. Sorella, et. al. arxiv: 1207.1783, (2012).

Controversy and questions?

Introduction and motivation:

J1-J2 Heisenberg model on the honeycomb lattice

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j,$$

ED: J. B. Fouet, et. al., EPJB 20, 241 (2001);
A. F. Albuquerque, et. al., PRB 84, 024406 (2011); H. Mosadeq, et. Al., JPCM 23, 226006 (2011).

VMC: B. K. Clark, et. al., PRL 107, 087204 (2011).

Analytic: F. Wang, PRB 82, 024419 (2010); Y. M. Lu, et. al., PRB 84, 024420 (2011).

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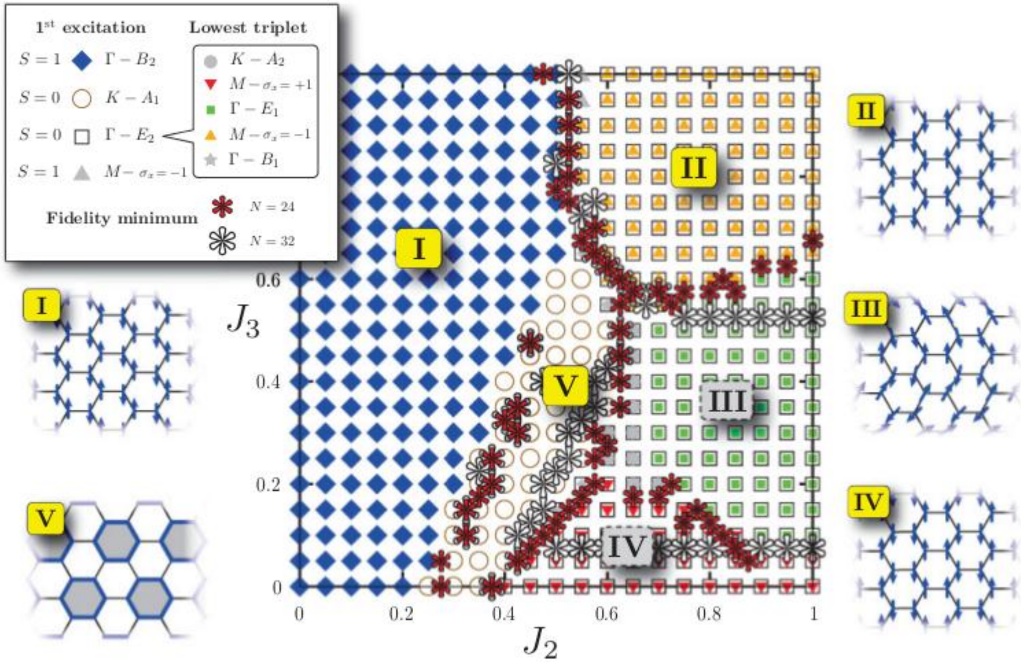


A. F. Albuquerque, et. al. PRB 84, 024406 (2011).

Plaquette VBS or spin liquid

Neel phase

?

0

J_{c1}~0.2

J_{c2}~0.4

?

Paramagnetic

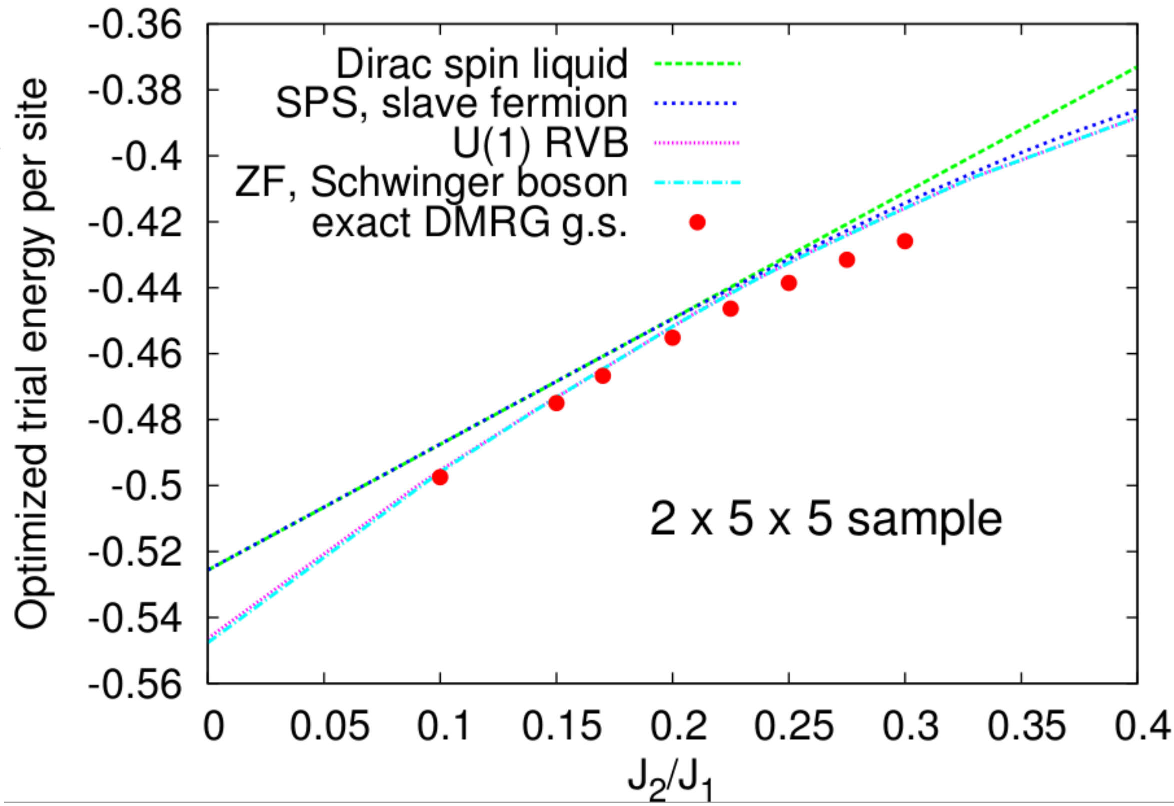
Stripe or staggered dimer phase

Introduction and motivation:

Motivation: close energy between Z2 spin liquid and U(1) RVB.

F. Wang, PRB 82, 024419 (2010);
Y. M. Lu, et. al., PRB 84, 024420 (2011);
B. K. Clark, et. al., PRL 107, 087204 (2011).

Gapped Z2 spin liquid and U(1) RVB both have low variational energies close to the ED results



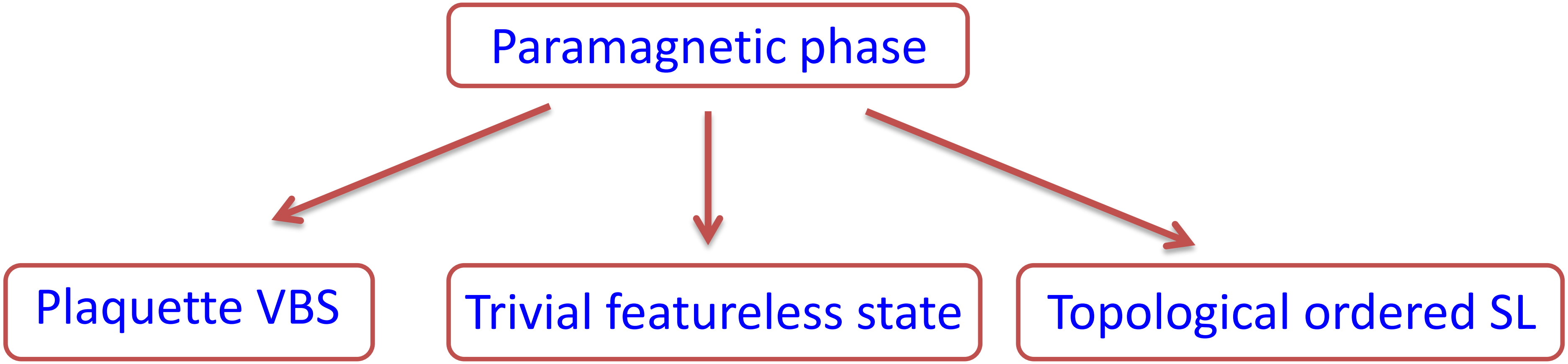
- **Both the slave fermion and Schwinger boson lead to a gapped spin liquid for J2≥0.2, or U(1) RVB instable toward VBS.**
- **We need DMRG to find the exact ground state.**

Introduction and motivation:

J1-J2 Heisenberg model on the honeycomb lattice

I. Kimchi, et. al., arxiv: 1207. 0498.

Featureless bose insulator without topological order on the honeycomb lattice.
If gapped, detect the topological order.



Large scale DMRG (with SU(2) symmetry) and VMC study this model!

Strong candidate for spin liquid

Outline:

- Background and motivation;
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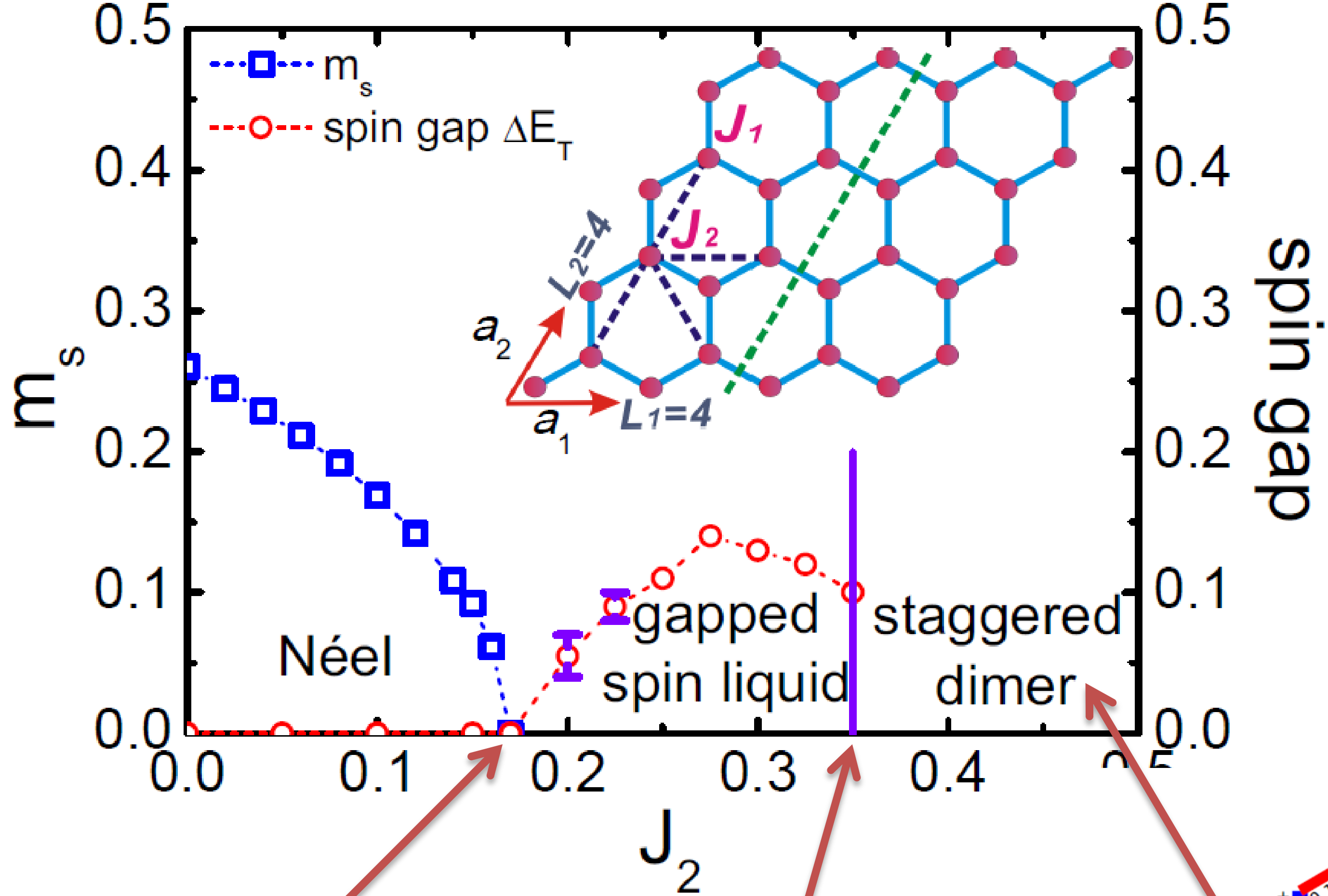
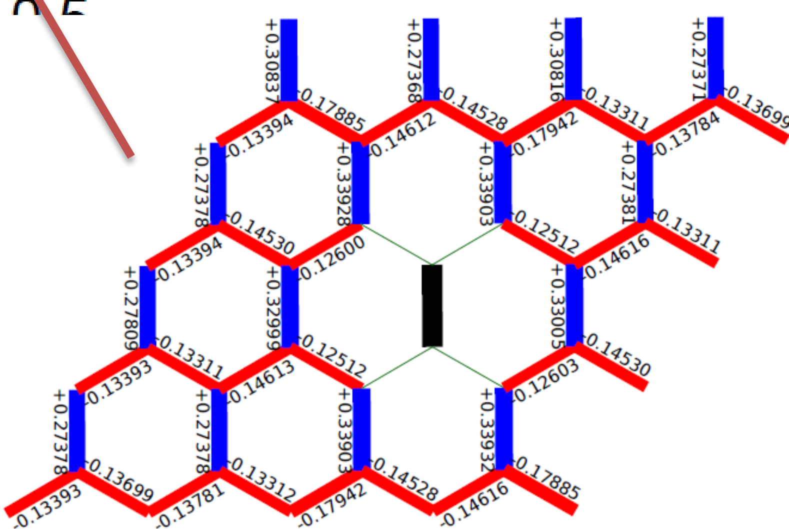
  - Vanishing magnetic order;
  - Vanishing dimer order;
  - Finite spin gap;
  - Entanglement entropy;
  - DMRG and VMC comparisons;
- Summary.

Phase diagram:



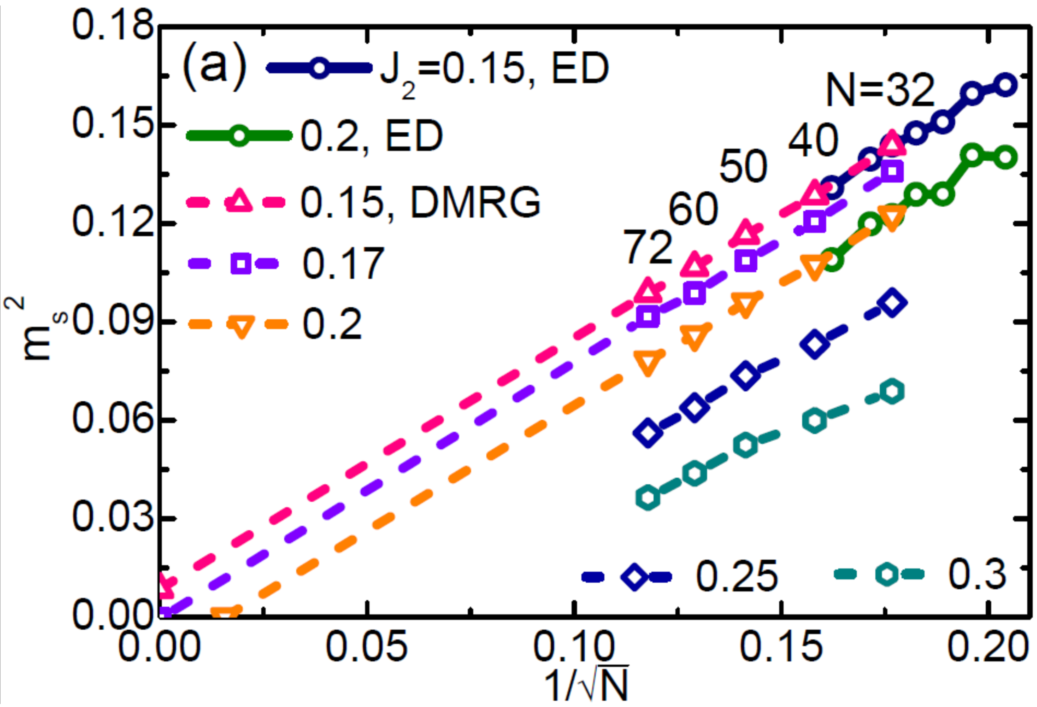
Staggered dimer



$J_{c1}=0.17J_1$

$J_{c2}=0.35J_1$

Staggered magnetic moment, spin structure factor:

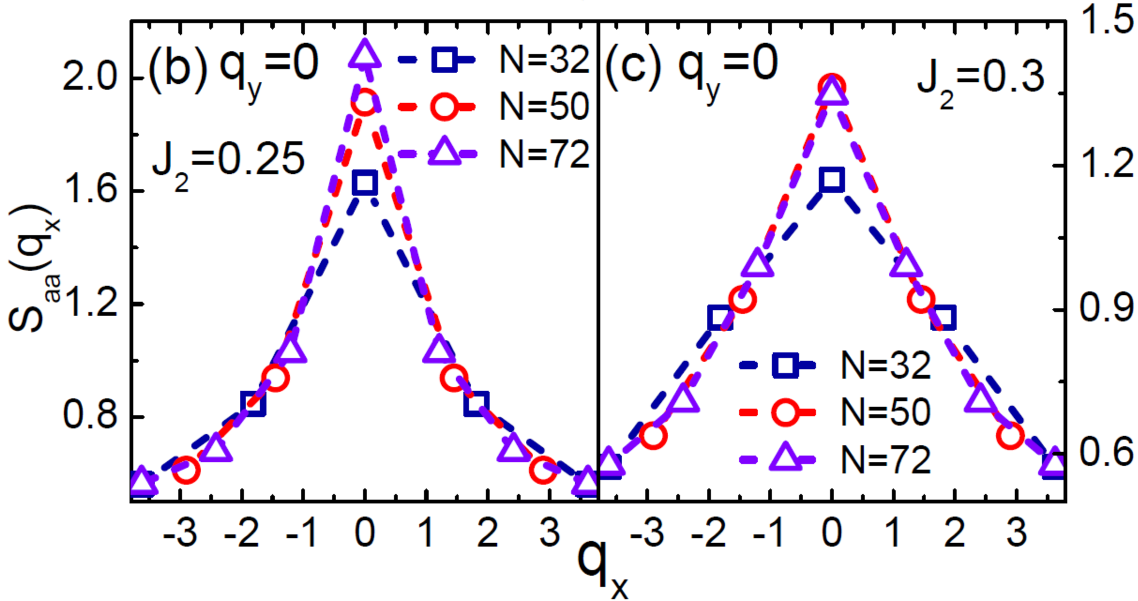


$$m_s^2 = \frac{1}{N(N+2)}\langle(\sum_i(-1)^i\mathbf{S}_i)^2\rangle$$

$J_{c1}=0.17J_1$

ED data is from A. F. Albuquerque, et. al. PRB 84, 024406 (2011).

$$S_{aa}(\vec{q}) = \frac{1}{L_1\times L_2}\sum_{i,j}\langle\mathbf{S}_i\cdot\mathbf{S}_j\rangle e^{i\vec{q}\cdot(\vec{r}_i-\vec{r}_j)}$$



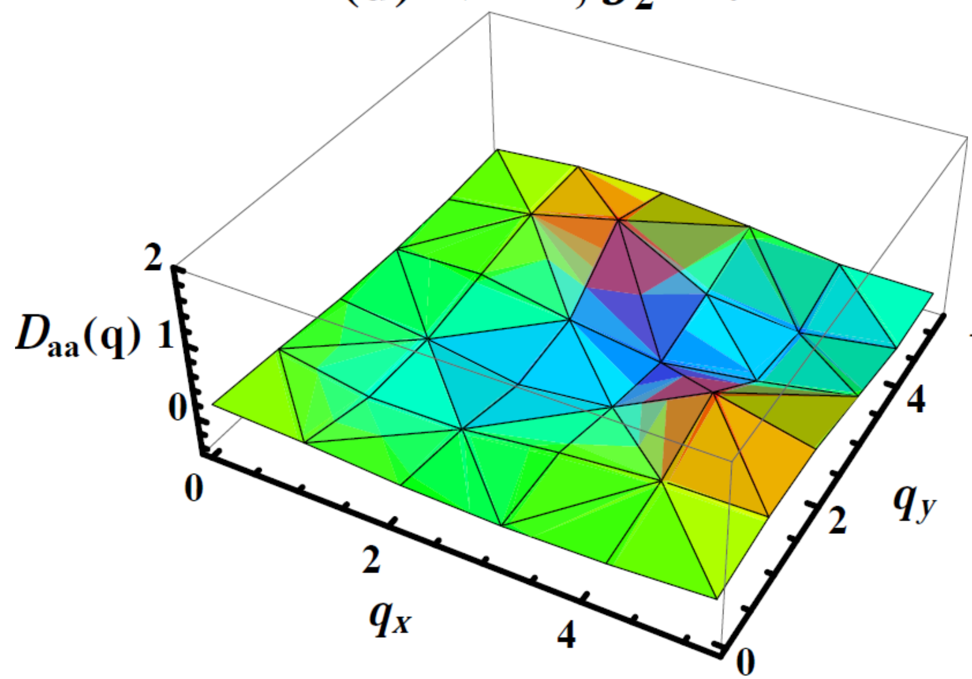
Neel peak saturates with increasing size.

DMRG in PBC, kept 20000~40000 U(1) equivalent states, truncation error<1.0^{-6} .

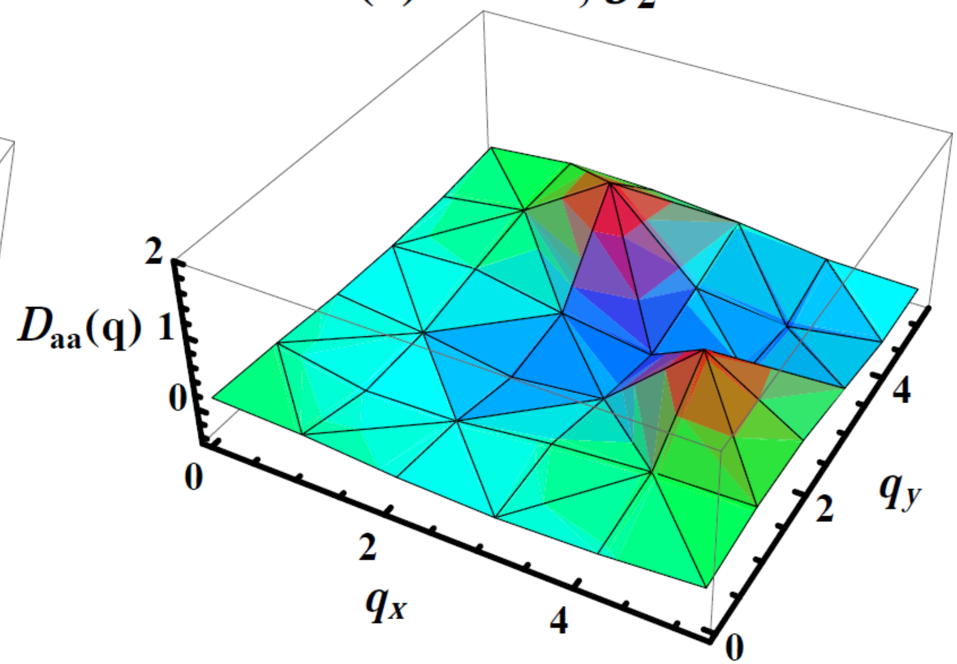
I. P. MuCulloch, et. al., (2002), (2007).

Dimer structure factor:

(d) N=72, J_2=0.25



(e) N=72, J_2=0.3

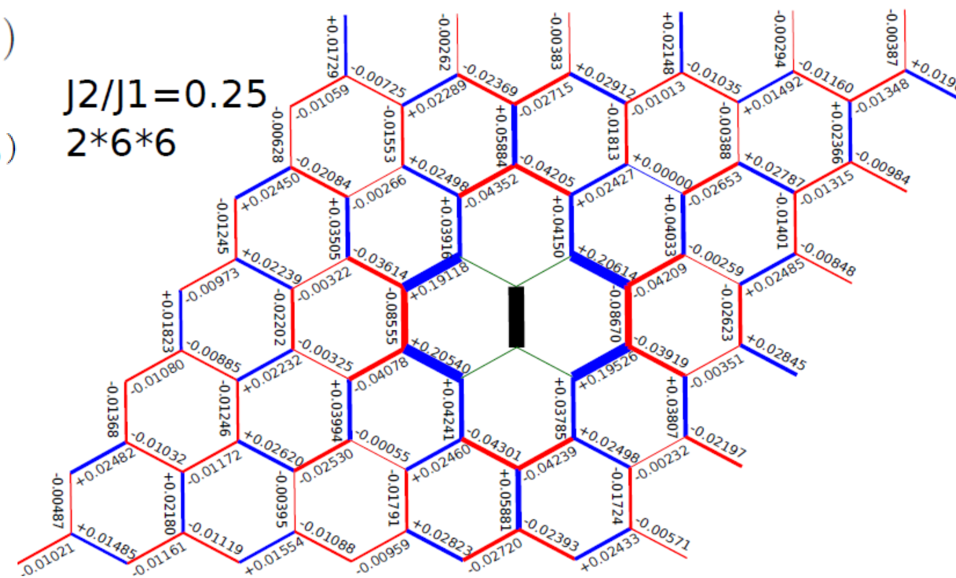


$$C_{(i,j),(k,l)} = 4(\langle(\mathbf{S}_i \cdot \mathbf{S}_j)(\mathbf{S}_k \cdot \mathbf{S}_l)\rangle - \langle\mathbf{S}_i \cdot \mathbf{S}_j\rangle\langle\mathbf{S}_k \cdot \mathbf{S}_l\rangle)$$

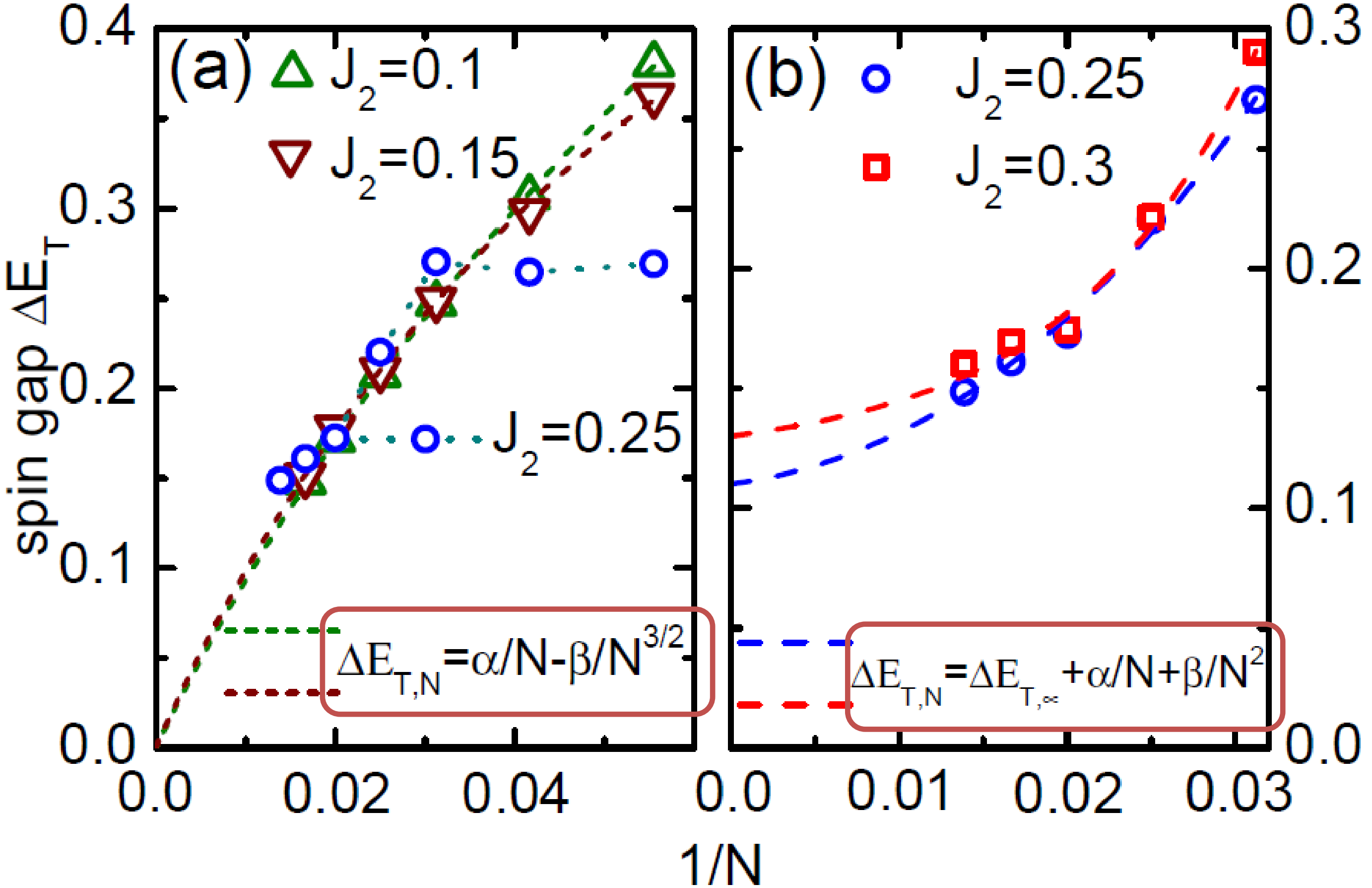
$$D_{aa}(\vec{q}) = \frac{1}{L_1 \times L_2} \sum_{(i,j),(k,l)} C_{(i,j),(k,l)} e^{i\vec{q}\cdot(\vec{r}_{(i,j)} - \vec{r}_{(k,l)})}$$

$$\mathbf{q} = (\tfrac{2}{3}\pi, \tfrac{4}{3}\pi) \text{ and } (\tfrac{4}{3}\pi, \tfrac{2}{3}\pi)$$

J2/J1=0.25
2*6*6



Spin gap:



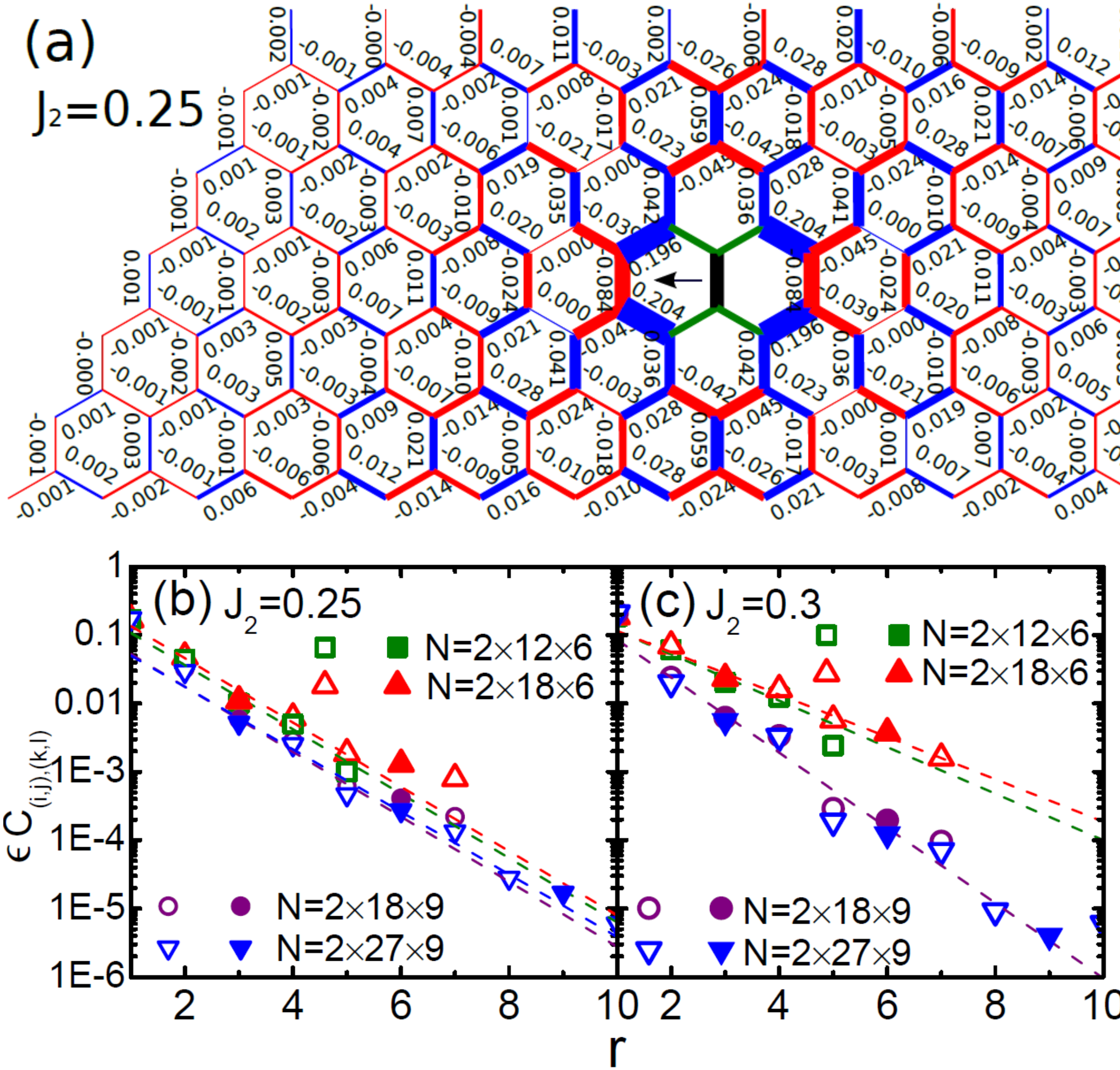
For J2=0.25, 0.3, N starts from N=2*4*4

N=2*3*3, 2*4*3, 2*4*4, 2*5*4, 2*5*5, 2*6*5, 2*6*6

Vanishing spin gap in Neel phase.

Nonzero spin gap in the intermediate phase.

Dimer correlation in zigzag cylinder:



N=2*18*6

$\epsilon = 1$ and -2 for blue and red bonds, respectively.

J_2=0.25: $\xi \simeq 0.89$

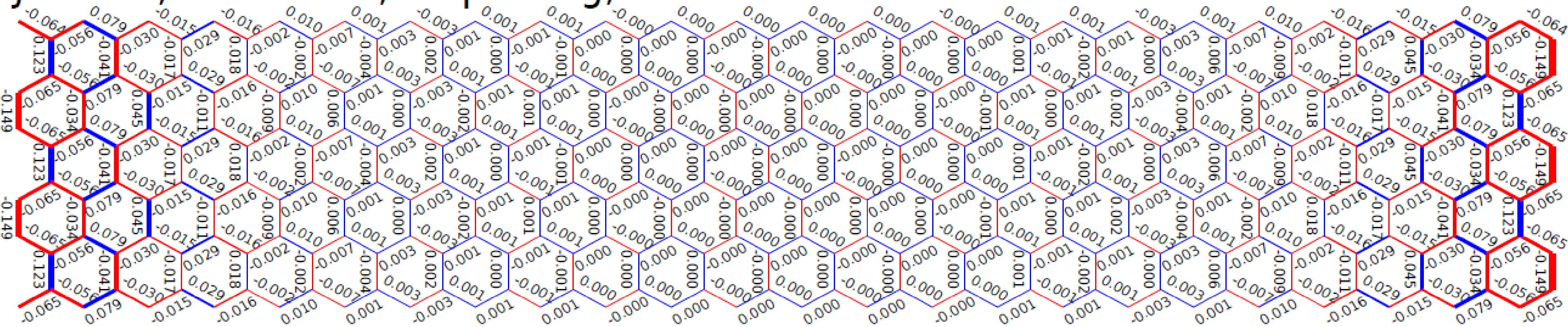
J_2=0.3: $\xi \simeq 1.4$ (L_2=6)

0.79 (L_2=9)

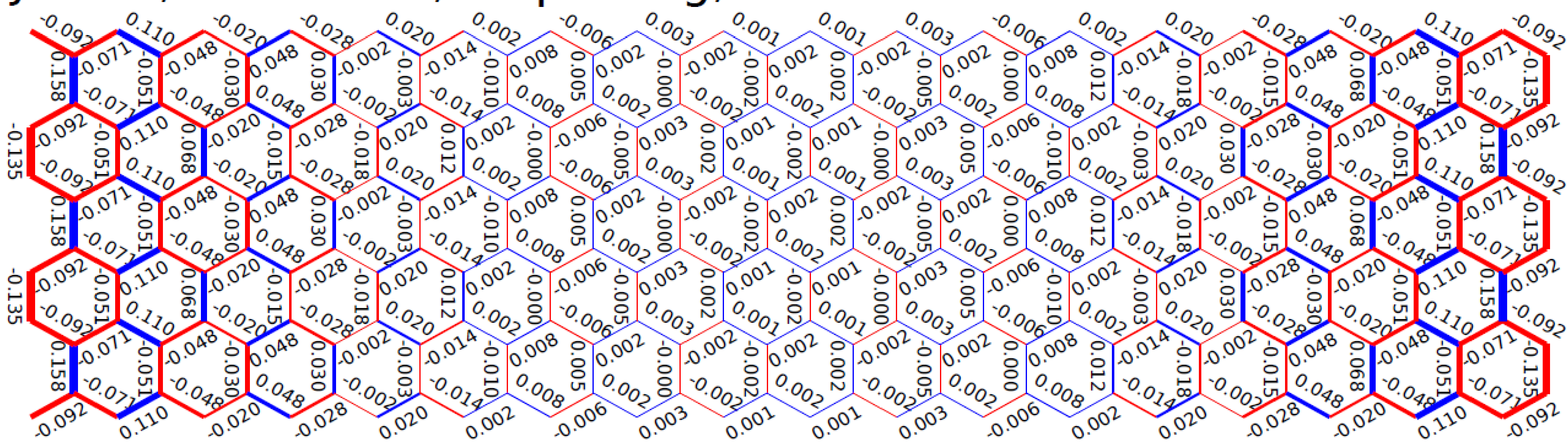
Dimer correlation in armchair cylinder:

Bond texture: $B_{i,j} \equiv \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle - e_\alpha$

J2=0.25, N=2*24*6, no pinning, bond texture

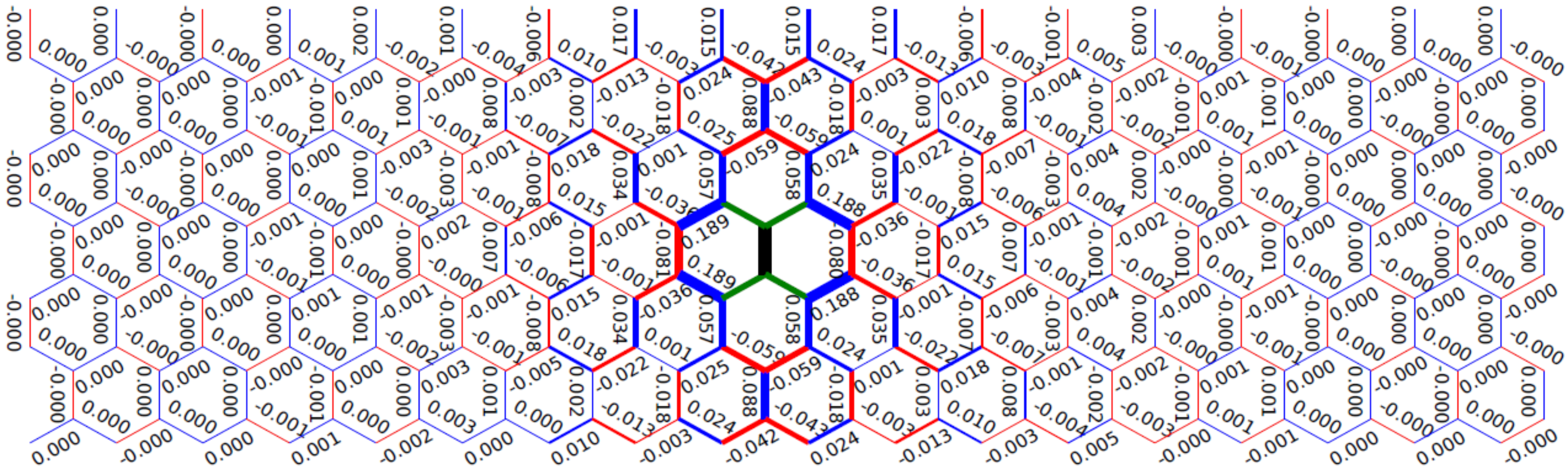


J2=0.3, N=2*18*6, no pinning, bond texture

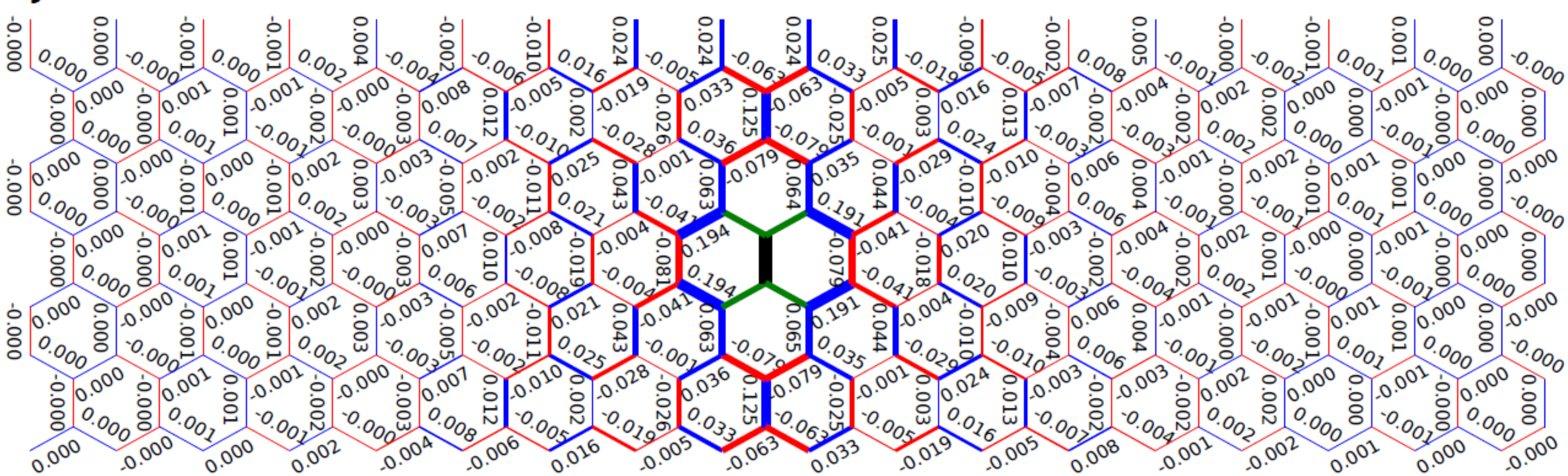


Dimer correlation in armchair cylinder:

J_2=0.25, N=2*18*6, dimer correlation

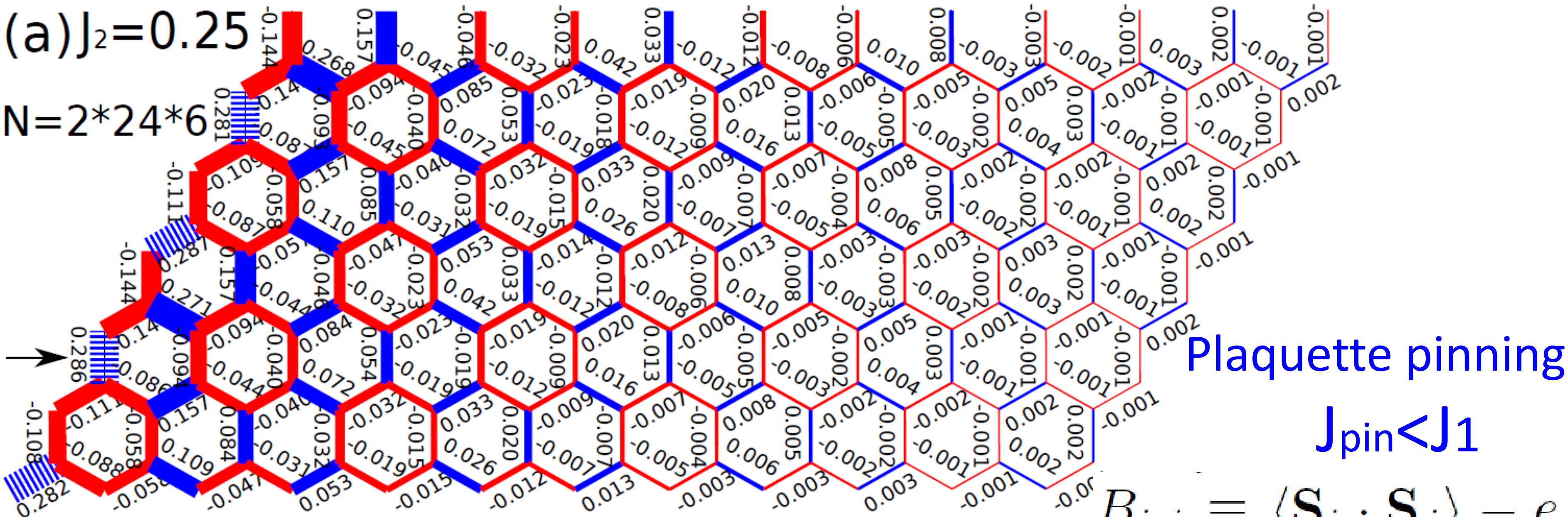


J_2=0.3, N=2*18*6, dimer correlation

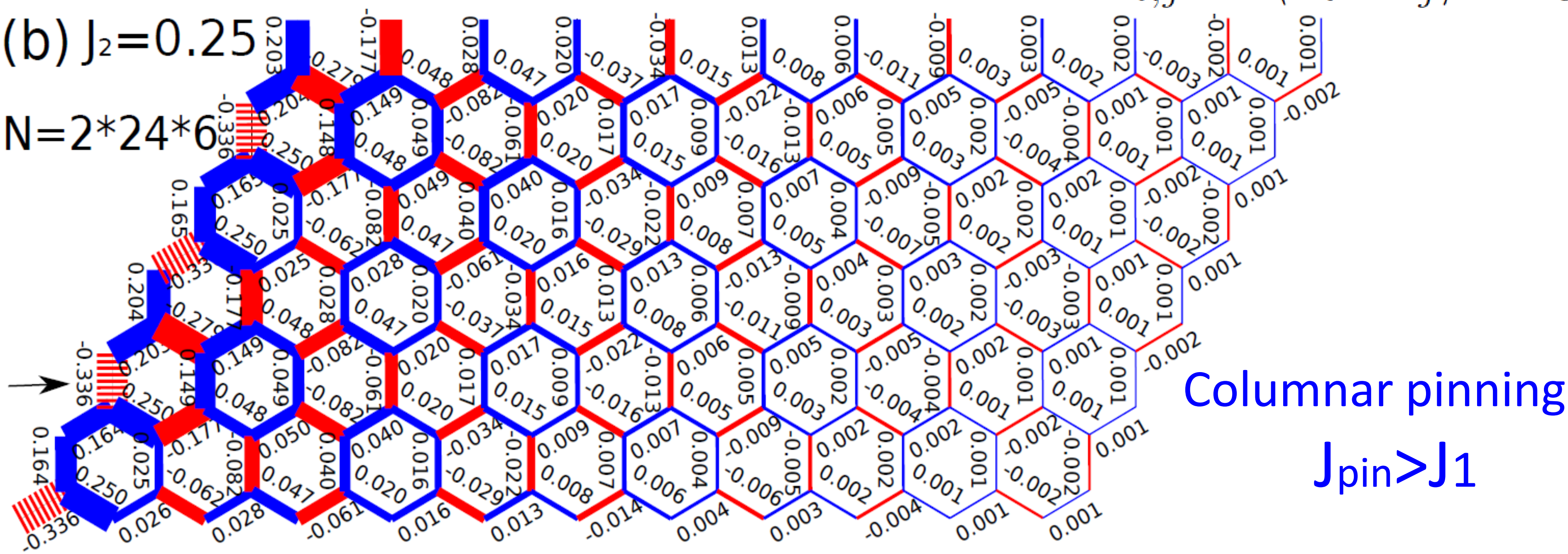


Dimer correlations decay a bit faster than the corresponding zigzag lattice

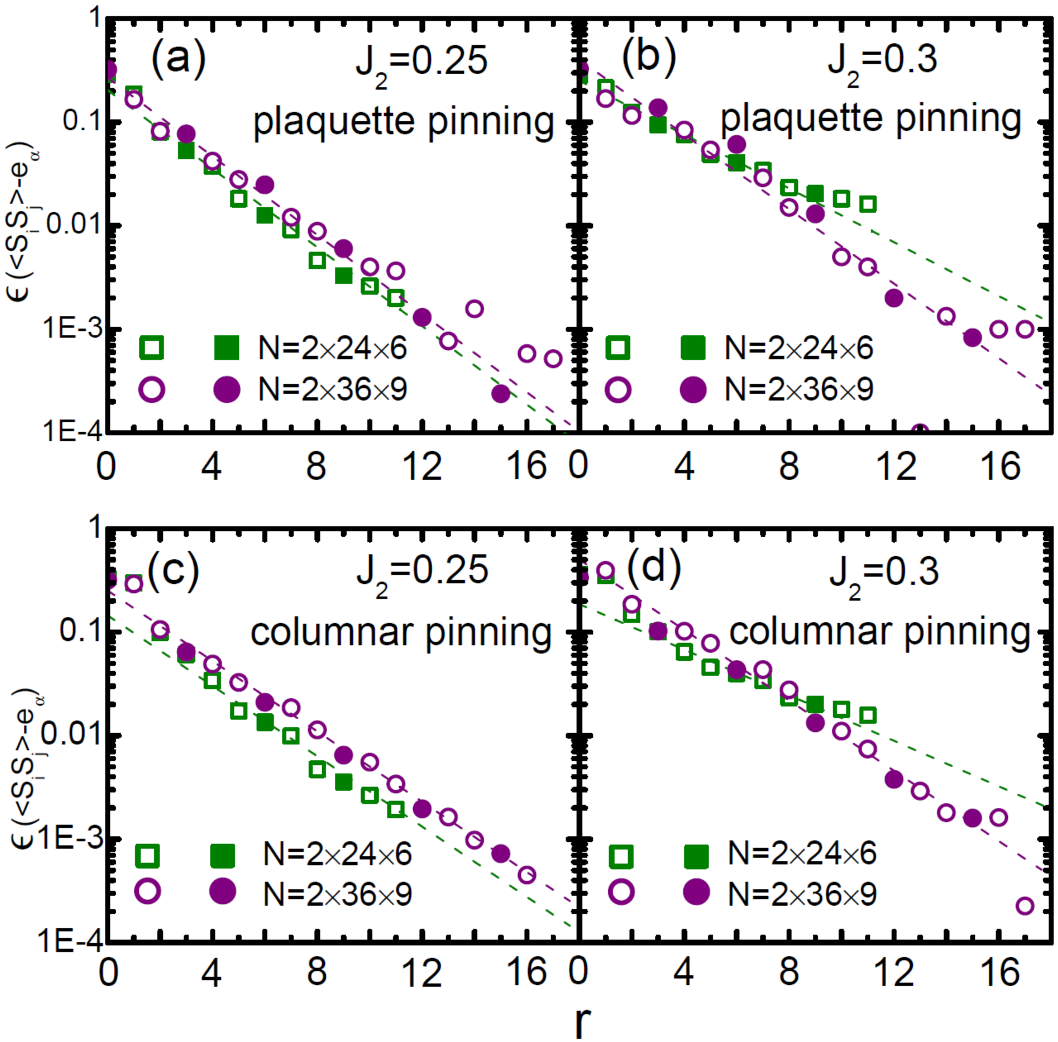
Response to VBS pertubation (zigzag):



$$B_{i,j} \equiv \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle - e_\alpha$$



Response to VBS pertubation (zigzag):



Jpin=0.5

J_2=0.25: $\xi_{\text{dec}} \simeq 2.3$

J_2=0.3: $\xi_{\text{dec}} \simeq 3.3$ (L_2=6)

ξ_{dec} ~2.4 (L_2=9)

Jpin=2.0

J_2=0.25: $\xi_{\text{dec}} \simeq 2.5$

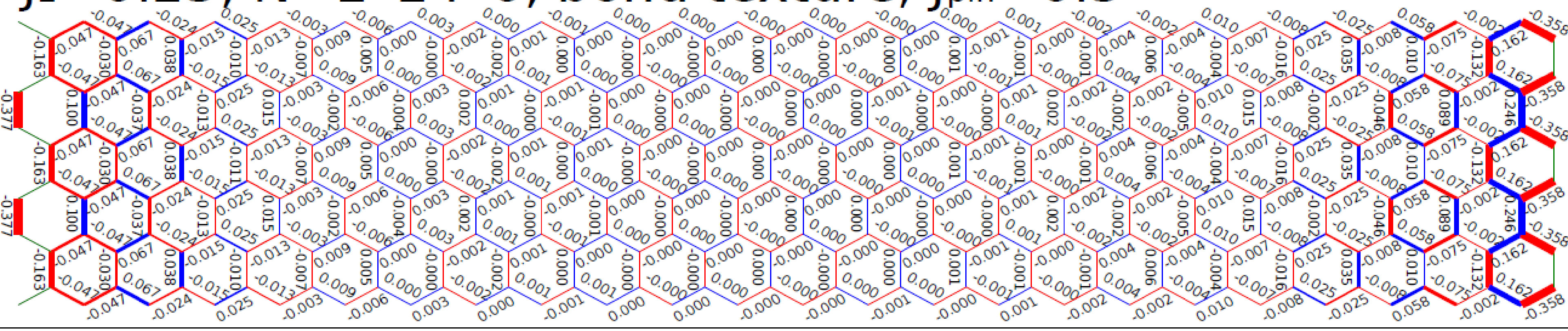
J_2=0.3: $\xi_{\text{dec}} \simeq 3.9$ (L_2=6)

ξ_{dec} ~2.5 (L_2=9)

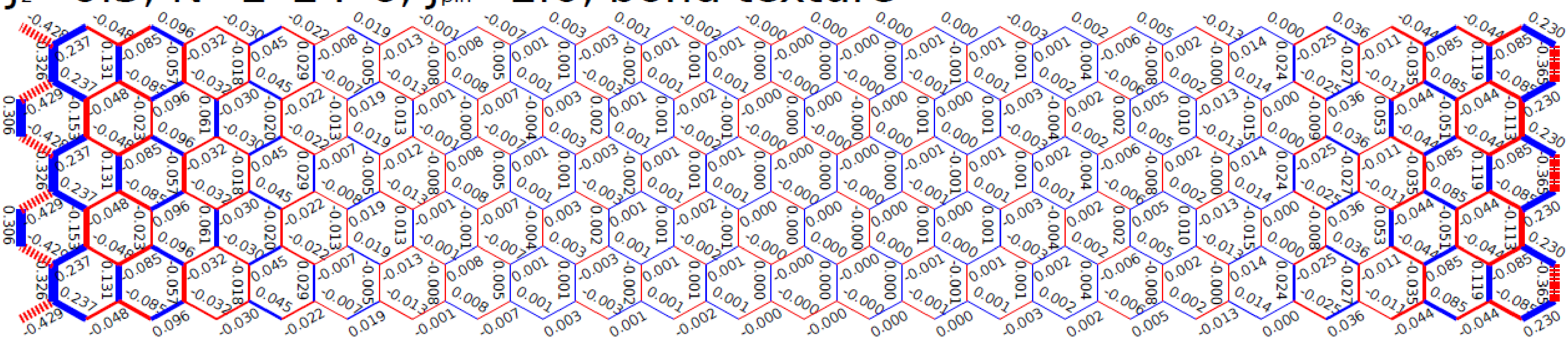
Decay faster in wider system

Response to VBS pertubation (armchair):

J_2=0.25, N=2*24*6, bond texture, J_{pin}=0.5



J_2=0.3, N=2*24*6, J_{pin}=2.0, bond texture



Almost uniform in the bulk of the lattice!

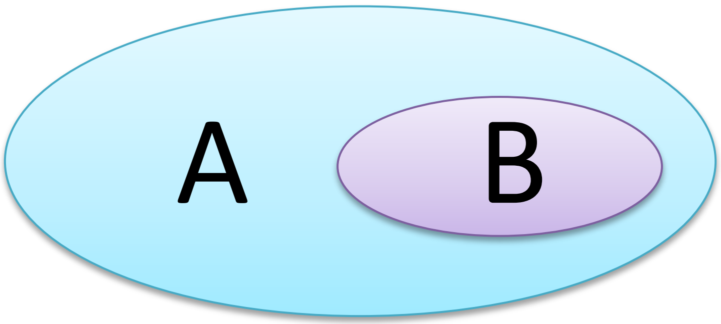
Entanglement entropy:

For a gapped phase

$E(L_B) = aL_B - \gamma$ ← universal constant

A. Kitaev and J. Preskill, PRL 96, 110404 (2006),

M. Levin and X.-G. Wen, PRL 96, 110405 (2006).



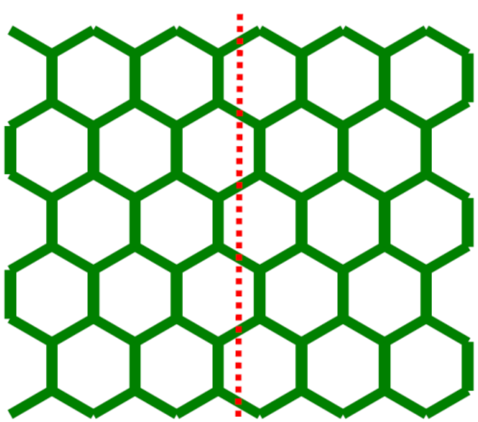
$$E = -\sum_i \lambda_i \ln \lambda_i$$

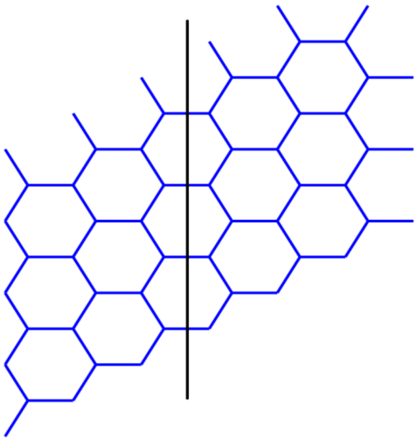
For a topological ordered phase, γ=ln(D).
D is the total quantum dimension.

DMRG in *cylinder* systems.

H. C. Jiang, Z. Wang, L. Balents, arxiv:1205. 4289.

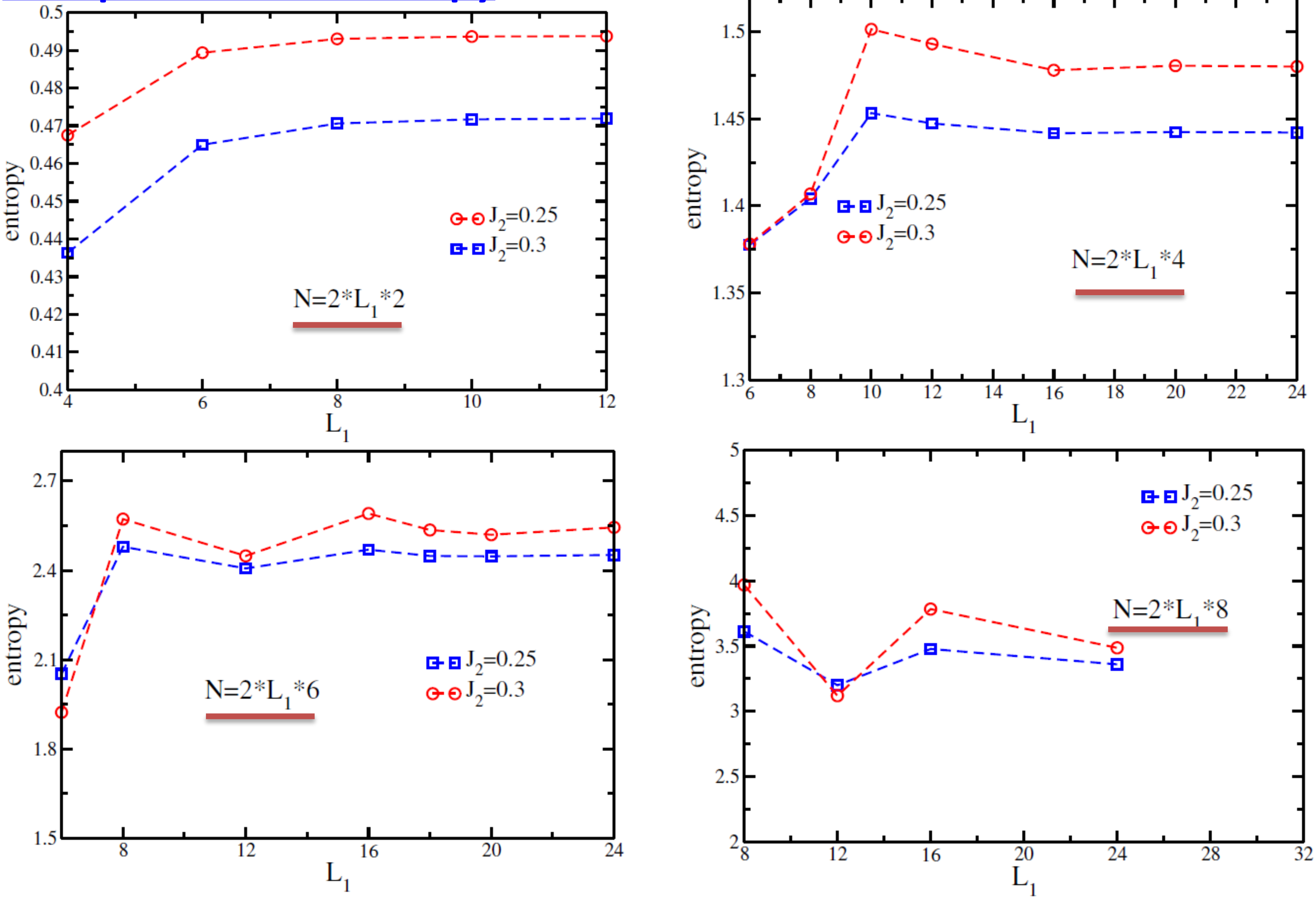
In large *L*x limit, DMRG gets the minimal entangled state with vertical cut in cylinder



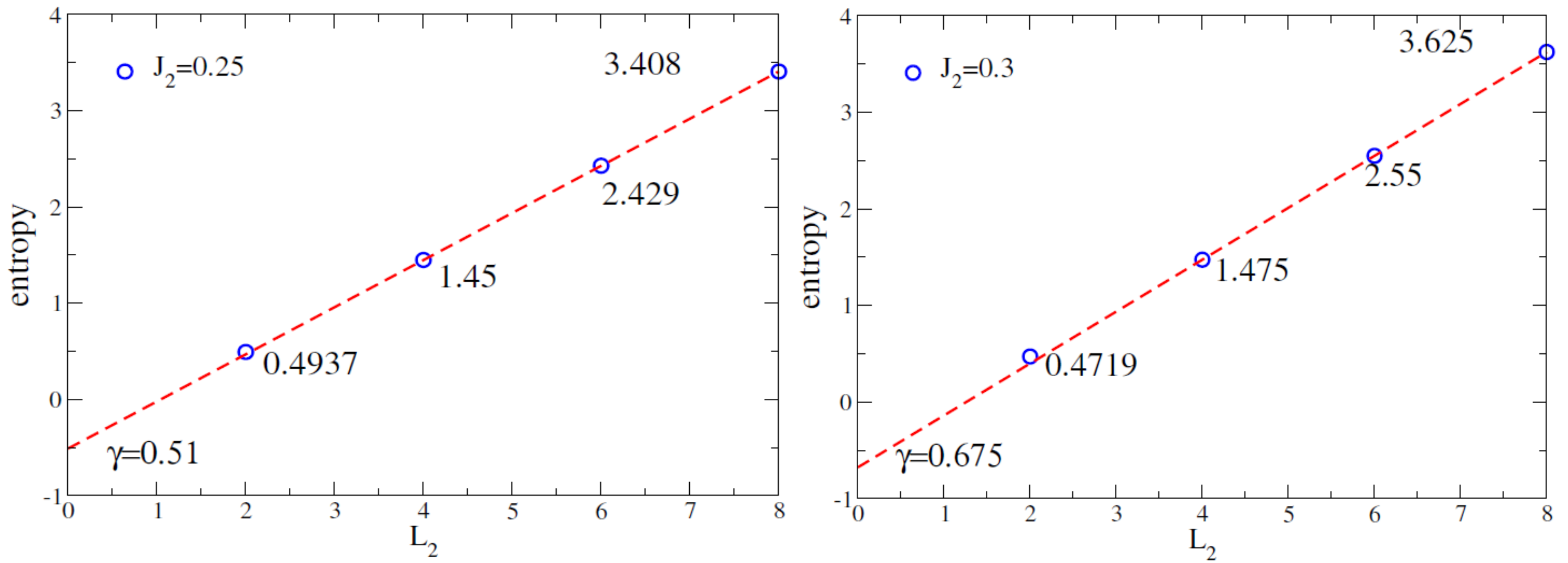


Entanglement Entropy (in armchair cut):

L1 dependence of entropy



Entanglement Entropy (in armchair cut):



J2=0.25, γ=0.51

J2=0.3, γ=0.675

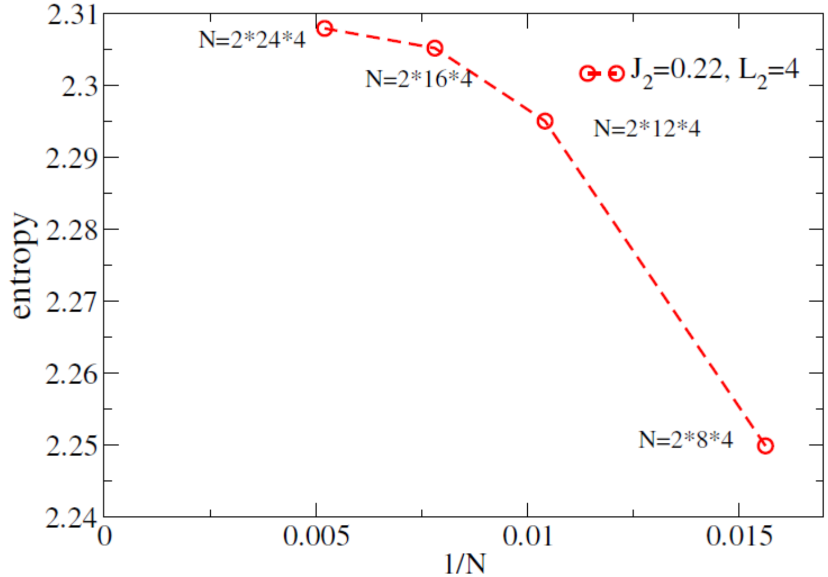
a. **Topological ordered phase**

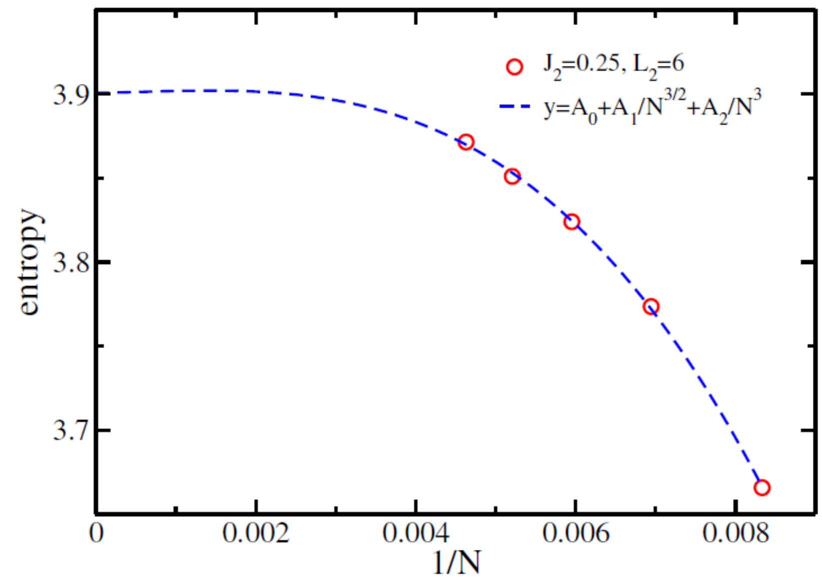
b. **γ is close to the value ln2 of Z_2 spin liquid**

Entanglement Entropy (in zigzag cut):

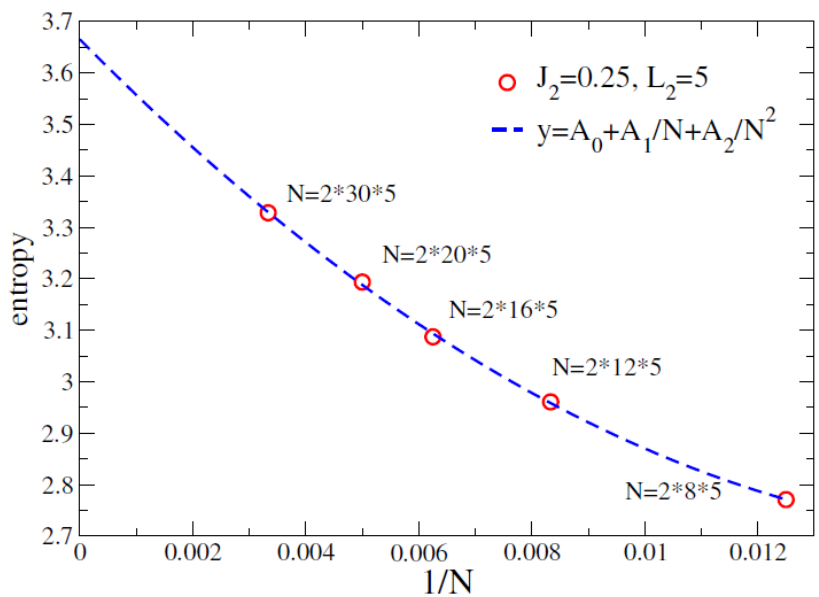
1/N dependence of entropy for give *L2*

L2=even





L2=5



The fittings of entropy in large L1 limit by L2=4, 6 lead to the TEE consistent with the armchair results.

DMRG and VMC comparison:

Sublattice Pairing State

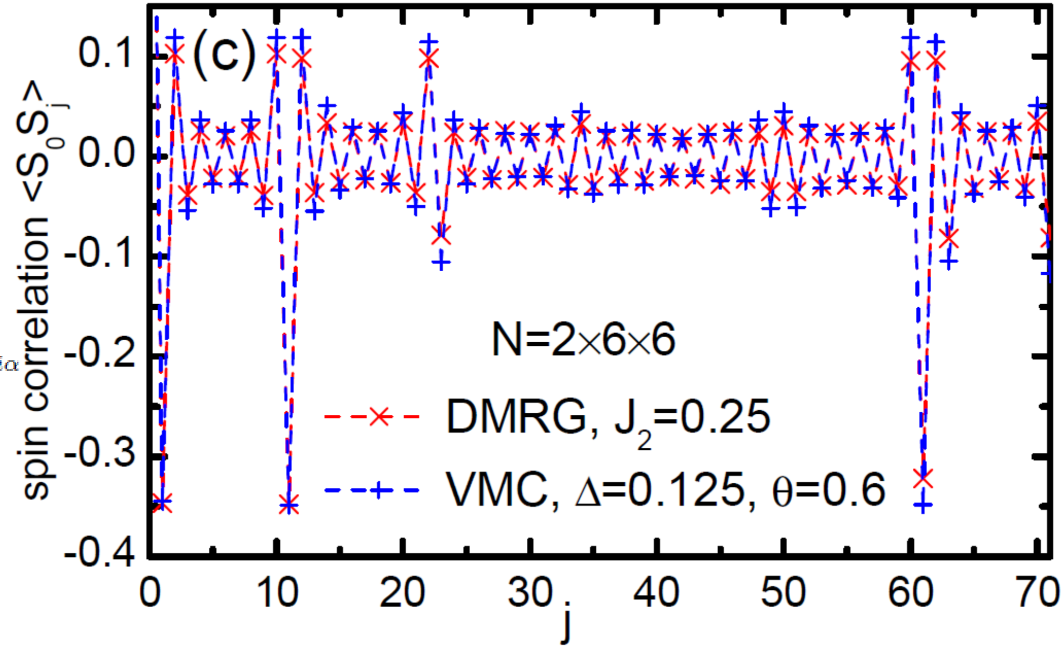
Y. M. Lu, et. al. PRB 84, 024420 (2011);
B. K. Clark, et. al. PRL 107, 087204 (2011).

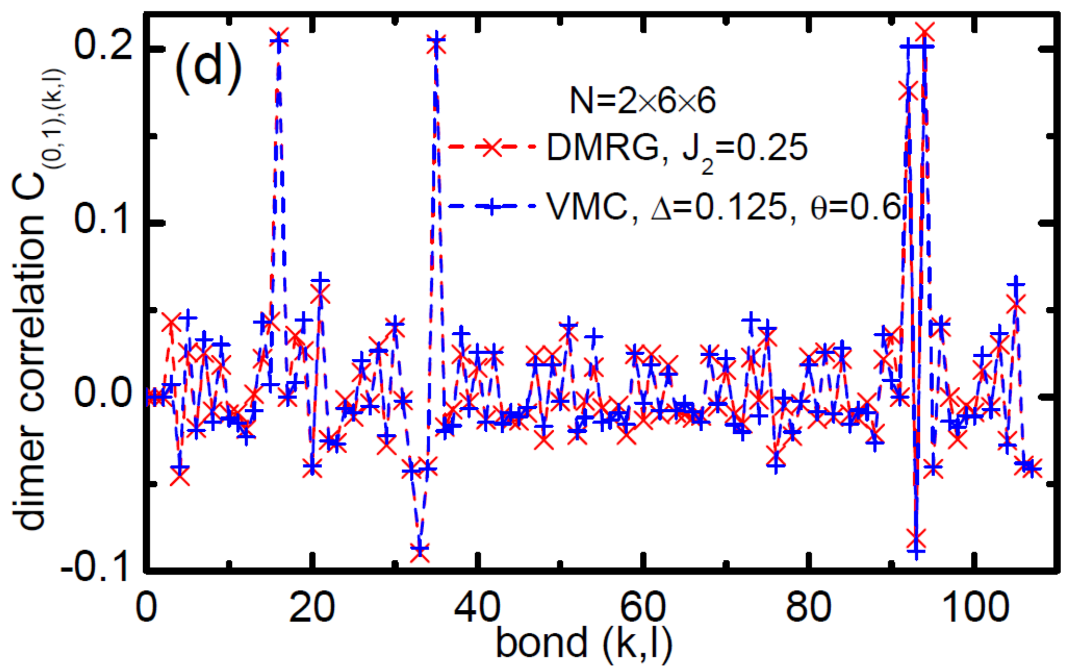
$$H_{\mathrm{mf}} = -\sum_{ij} t_{ij} f_{i\alpha}^{\dagger} f_{j\alpha} + \sum_{ij} \left(\Delta_{ij} f_{i\uparrow}^{\dagger} f_{j\downarrow}^{\dagger} + \mathrm{H.c.} \right) - \sum_{i} \mu_i f_{i\alpha}^{\dagger} f_{i\alpha}$$

The SPS variation energy is not very sensitive to θ for J2≥0.2. U(1) RVB (θ=0) has a close energy to the Z2 spin liquid.

Z_2 spin liquid for this VMC wave function

The agreement between the DMRG and VMC is striking!





Summary:

- The intermediate phase of this honeycomb J1-J2 model is identified as a gapped spin liquid with vanishing magnetic and dimer orders.

- The topological entanglement entropy and the DMRG-VMC comparisons support the Z_2 spin liquid for this phase.

- The model has a Neel, a Z_2 spin liquid and a staggered dimer phase with increasing J_2.

Thank you for your attention!