Quantum versus thermal fluctuations on the kagomé lattice

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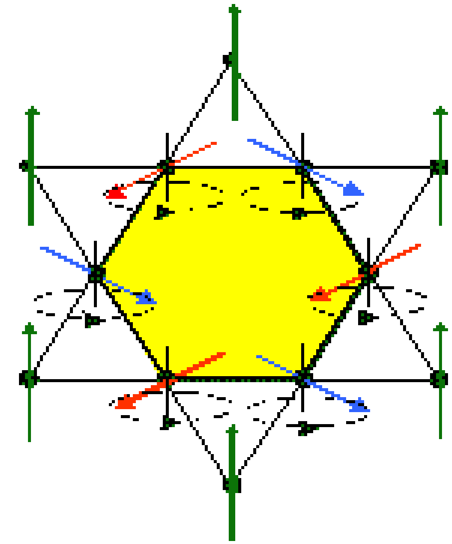
Classical Heisenberg Hamiltonian on the kagomé lattice (ħ=0)

$$H/J = \sum S_i . S_j$$
$$= \frac{1}{2} \sum_{\alpha} S_{\alpha}^2 + C$$
$$S_\alpha = (S_1 + S_2 + S_3)$$



- An infinite number of soft modes, an infinite T=0 degeneracy
- Order by disorder:

➢ Therm. fluct. select coplanar configurations below 10^{-2} J

J. Chalker, et al 92, Huse & Rutemberg 92 , Reimers & Berlinsky 93, Zhitomirsky (2008),

➢ Below 10^{-4} J thermal fluctuations favor √3√3 order

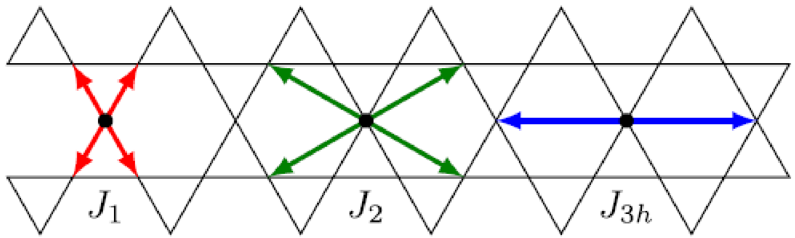
Hassan & Moessner (2012) KITP Conf (Exotic phases of frustrated magnets)

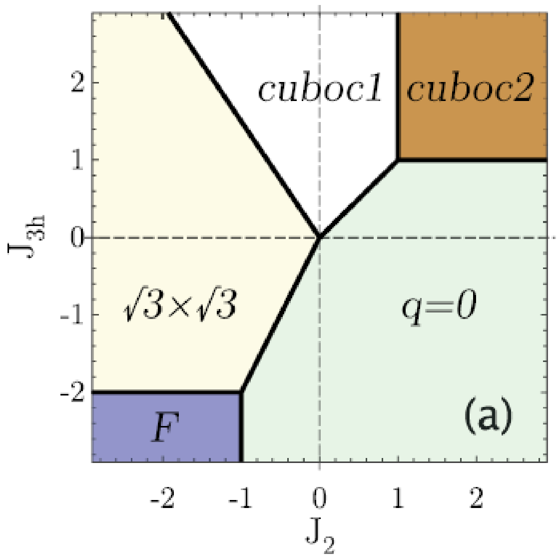
Thermal versus quantum fluctuations

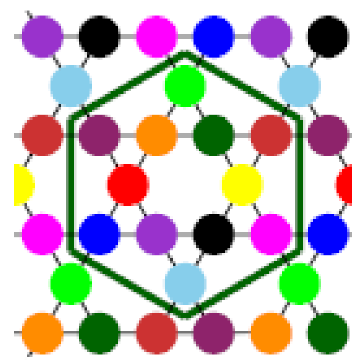
- Thermal fluctuations: Minimize F= U –TS
  - S propto the density of states near gs.
  - This dos is inv. prop to the stiffness of the state
  - Identical for all planar states in the harmonic approx.
  - No more the case in the anharmonic approximation the √3√3 order is selected
- Spin wave calculations and beyond
  - Essentially the same result: amongst planar states √3√3 order wins

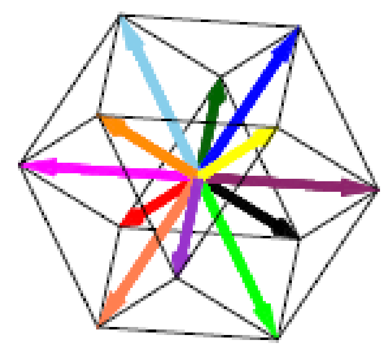
    (*Chubukov 92, Cepas 11*)
  - Not totally a surprise : the smaller zero point energy in these approaches, is directly related to the softness of excitations
- Are planar states the good departure point to look at quantum phases?

Classical instabilities on the kagome latt.





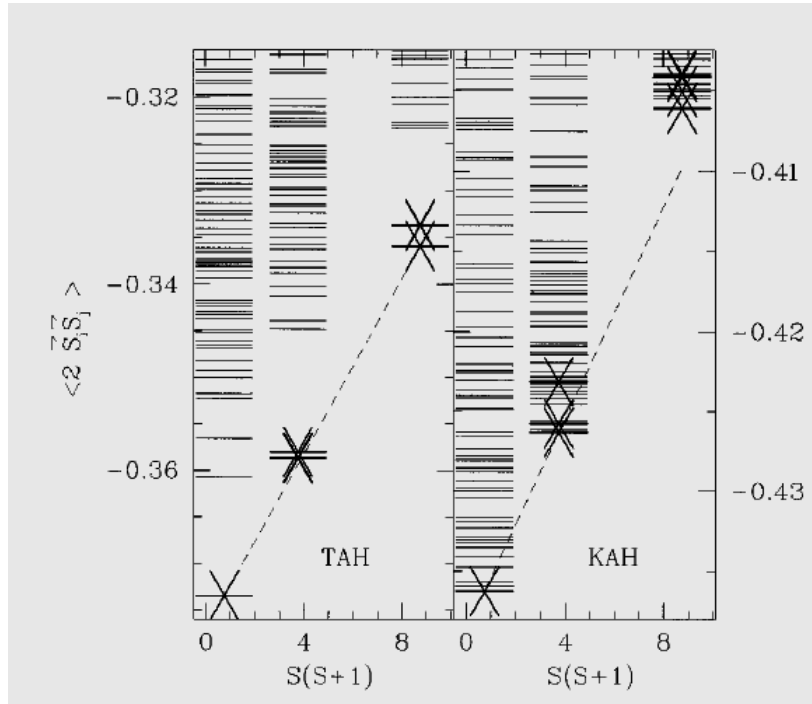




In a classical or semi classical approximation cuboc1 cannot compete with planar states. (This non planar LRO is stiffer than the coplanar configurations)

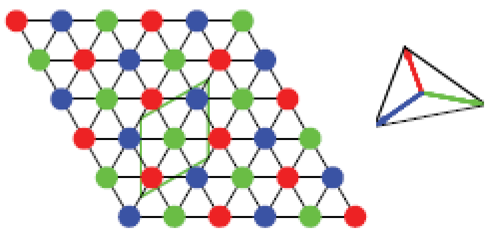
BUT what can we say of this SR arrangement from a quantum point of view?

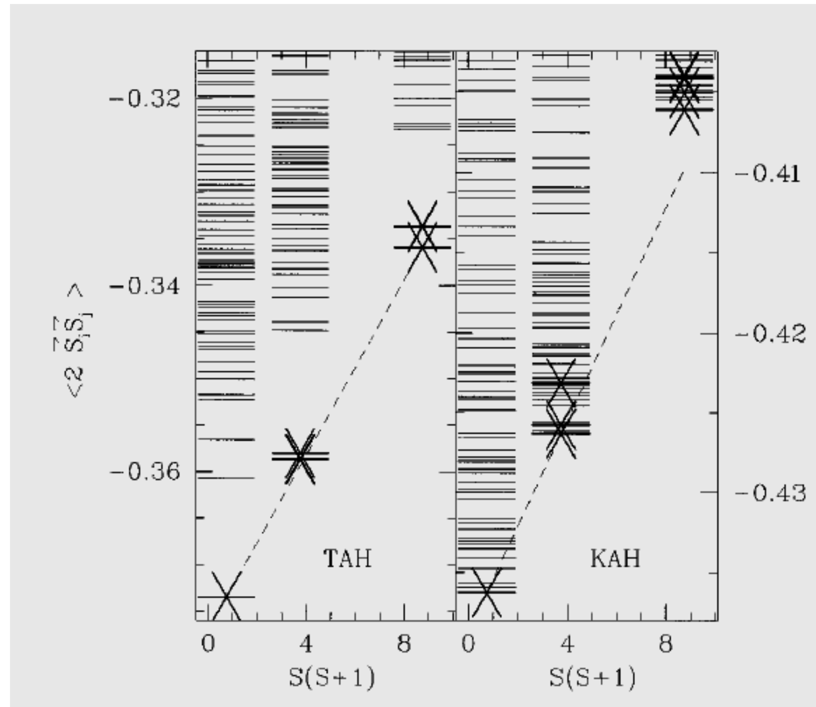
Could cuboc1 SRO be the clue to small size ED spectra?



Exact eigenstates of the Heisenberg model on triangular lattice (left N=28)
& on kagomé lattice (right N=27).
Note the much lower energy of the KAH

Could cuboc1 SRO be the clue to small size ED spectra?

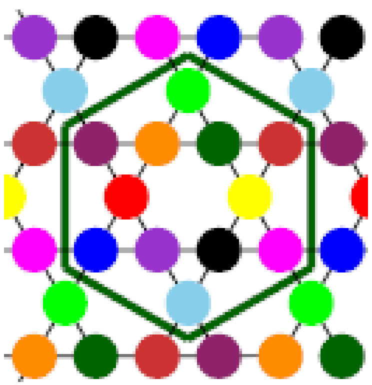


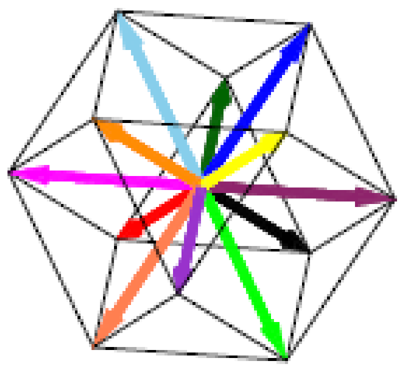


- 3 sublattice order

- This order gives a simple spectrum of low lying excitations
- First approx: the 3-sublattice rigid model with 3 spin directions at 120°
  -> 1 singlet, 3 triplets etc..
  Whatever the number of sites in each sublattices

Short range arrangements and their quantum description (continued)





Back of the envelope calculation!
•angular momentum addition of 12 spins ½ with AF correlations on first, second and third neighbors
-> 8 low energy singlets *(there are 7 low lying singlets in the exact spectrum below the first triplet)…*

•Does this short range order explains the 237 singlets below the spin gap of the exact spectrum of a N=36 sample? The sophisticated analysis of Lecheminant (PRB 52 1995) remains to be done!

A bet!

Short range quantum fluctuations absent from usual semi-classical approaches might contribute to the energetic stabilization of the spin-1/2 Heisenberg spin liquid on the kagomé lattice

A bosonic large N approach
Messio PRL 108, 2012

The Schwinger boson operator $\hat{b}^{\dagger}_{i\sigma}$ ($\sigma = \uparrow$ or $\downarrow$)

The hamiltonian $H = \sum_{\langle i,j \rangle_\alpha} J_\alpha \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j,$ is rewritten in the mean field approx. as

$$H_{\mathrm{MF}} = \sum_{\langle i,j \rangle_\alpha} J_\alpha (\mathcal{B}_{ij} \hat{B}^{\dagger}_{ij} - \mathcal{A}_{ij} \hat{A}^{\dagger}_{ij}) + \mathrm{H.c.} - \sum_i \lambda_i \hat{n}_i + \epsilon_0,$$

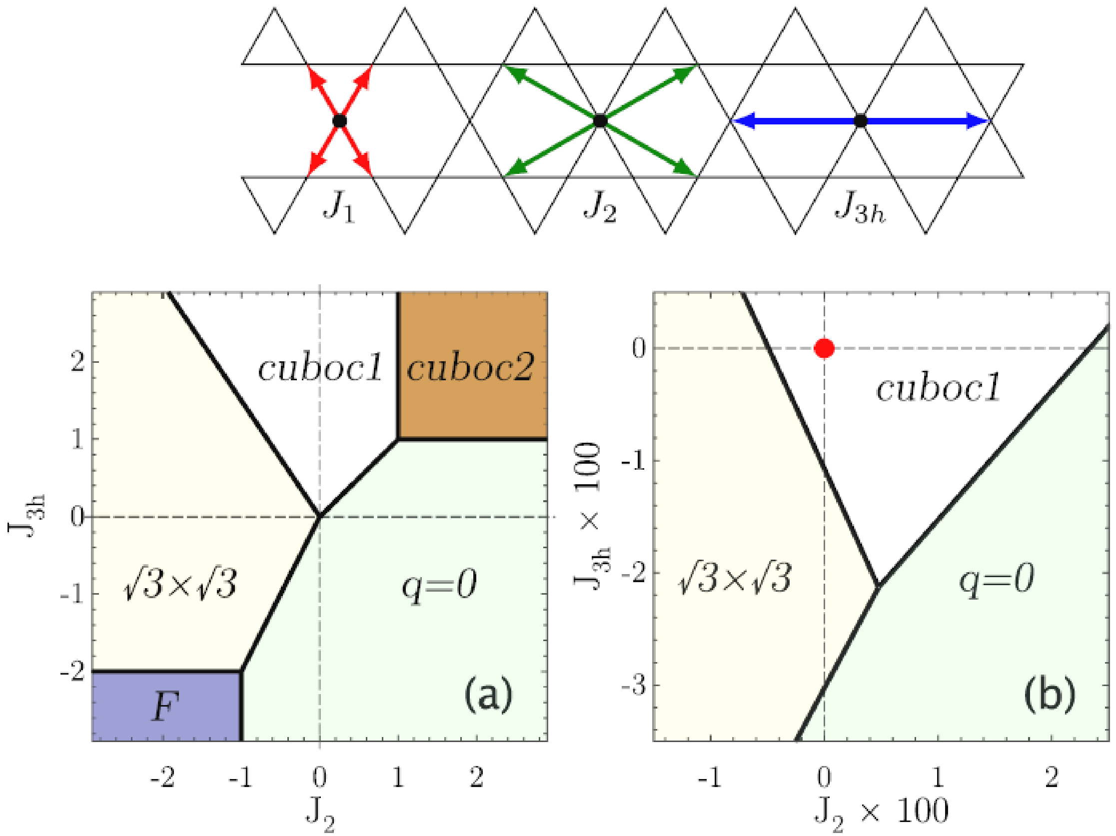
where

$$2\hat{A}_{ij} = \hat{b}_{i\uparrow}\hat{b}_{j\downarrow} - \hat{b}_{i\downarrow}\hat{b}_{j\uparrow} \text{ and } 2\hat{B}_{ij} = \hat{b}^{\dagger}_{i\uparrow}\hat{b}_{j\uparrow} + \hat{b}^{\dagger}_{i\downarrow}\hat{b}_{j\downarrow}.$$

•The use of the two bond fields is essential in frustrated magnets (Flint and Coleman 2009, Mezio et 2011)
• Only spin liquids ground-states have been searched for..
• Extension of the PSG analysis of F. Wang and A. Vishwanath (2006) to deal with putative T-reversal –symmetry-breaking phases

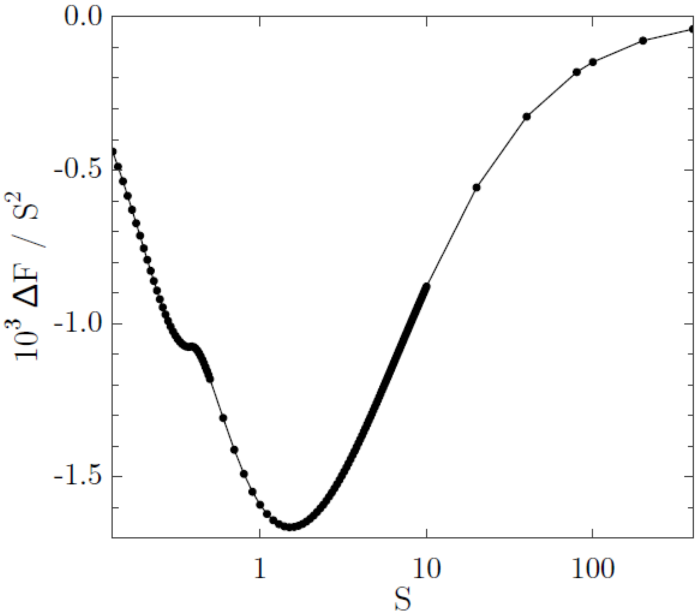
The bond fields are complex. The number of bosons by site is adjusted in average to 2S.

The large N limit phase diagram



Left: classical phase diagram Right: large N Sp(N)

SBMFT: Diff. in free energy between cuboc1 and $\sqrt{3}\sqrt{3}$ phase (T=0)



- Cuboc1 phase wins for any S at T=0
- $S_c \sim 0.38$
- For $\langle S^2 \rangle = \frac{3}{4}$ the cuboc1 phase is a gapped chiral spin liquid phase

Large N ground-state of the Heisenberg hamiltonian

- It is a chiral phase with cuboc1 long range order for large S
- For $S < S_c \sim 0.38$ the phase is gapped. It is a chiral SL
- has non zero fluxes around the hexagons

ordre	$\mathbb{B}(\triangle)$	$\mathbb{B}(\hexagon)$	$\mathbb{B}(\square)$	$\mathbb{A}(\hexagon)$	$\mathbb{A}(\square)$	$\psi^A(\hexagon)$	$\psi^A(\square)$
cuboc1	$-\frac{1}{8}$	$-\frac{1}{64}$	$-\frac{1}{256}$	$\frac{23-10\sqrt{2}i}{64}$	$-\frac{81}{256}$	$-\arctan\frac{10\sqrt{2}}{23}$	π

- Definition of loop operators and fluxes (next slide)

Loop operators and fluxes

Loop operators:

$< A_{12} A^{\dagger}_{23} A_{34} A^{\dagger}_{45} A_{56} A^{\dagger}_{61} >$ and $< B_{12} B_{23} B_{34} B_{45} B_{56} B_{61} >$

- physical observables (gauge invariant) related to permutation operators of spins
- The argument of these loop operators are the so-called fluxes.

Classical limit: fluxes are related to the solid angle described by the classical spins on the Bloch sphere .

The flux of B on a triangle is measured by the triple product of spins.

More complex for A fluxes:

Flux on hexagon = Arg $< A_{12} A^{\dagger}_{23} A_{34} A^{\dagger}_{45} A_{56} A^{\dagger}_{61} >$

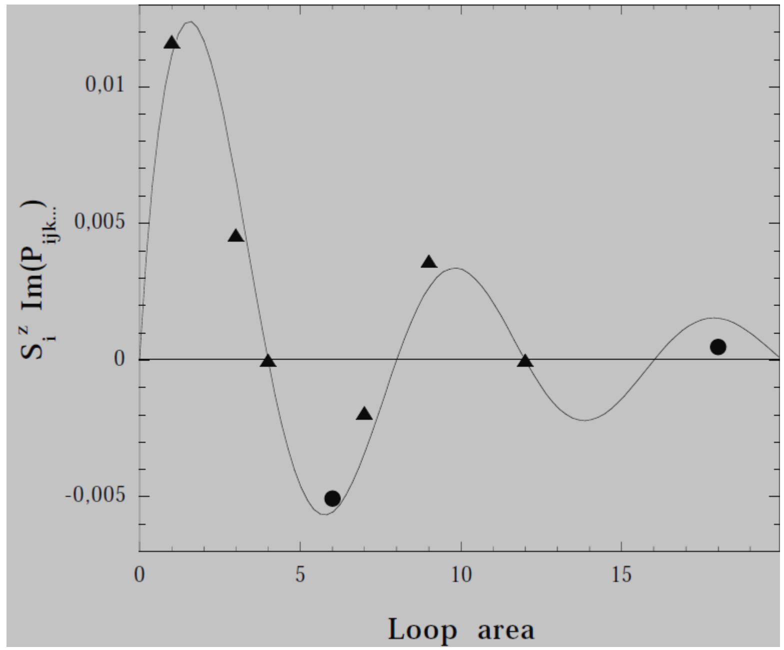
-> solid angle sustained by the spins: $\mathbf{S}_1$ **(-**$\mathbf{S}_2$**)** $\mathbf{S}_3$ **(-**$\mathbf{S}_4$**)** $\mathbf{S}_5$ **(-**$\mathbf{S}_6$ **)**

Fluxes could be measured in spin 1/2 approaches (ED, DMRG)

$$\widehat{P}_{(12..n)} = 2^n : \widehat{B}^{\dagger}_{12}\widehat{B}^{\dagger}_{23}...\widehat{B}^{\dagger}_{n1} :$$

$$2^{2n} : \widehat{A}^{\dagger}_{12}\widehat{A}_{23}\widehat{A}^{\dagger}_{34} \ldots \widehat{A}_{2n\,1} := \widehat{P}_{(12..2n)}(1 - \widehat{P}_{(23)})(1 - \widehat{P}_{(45)}) \ldots (1 - \widehat{P}_{(2n\,1)}).$$

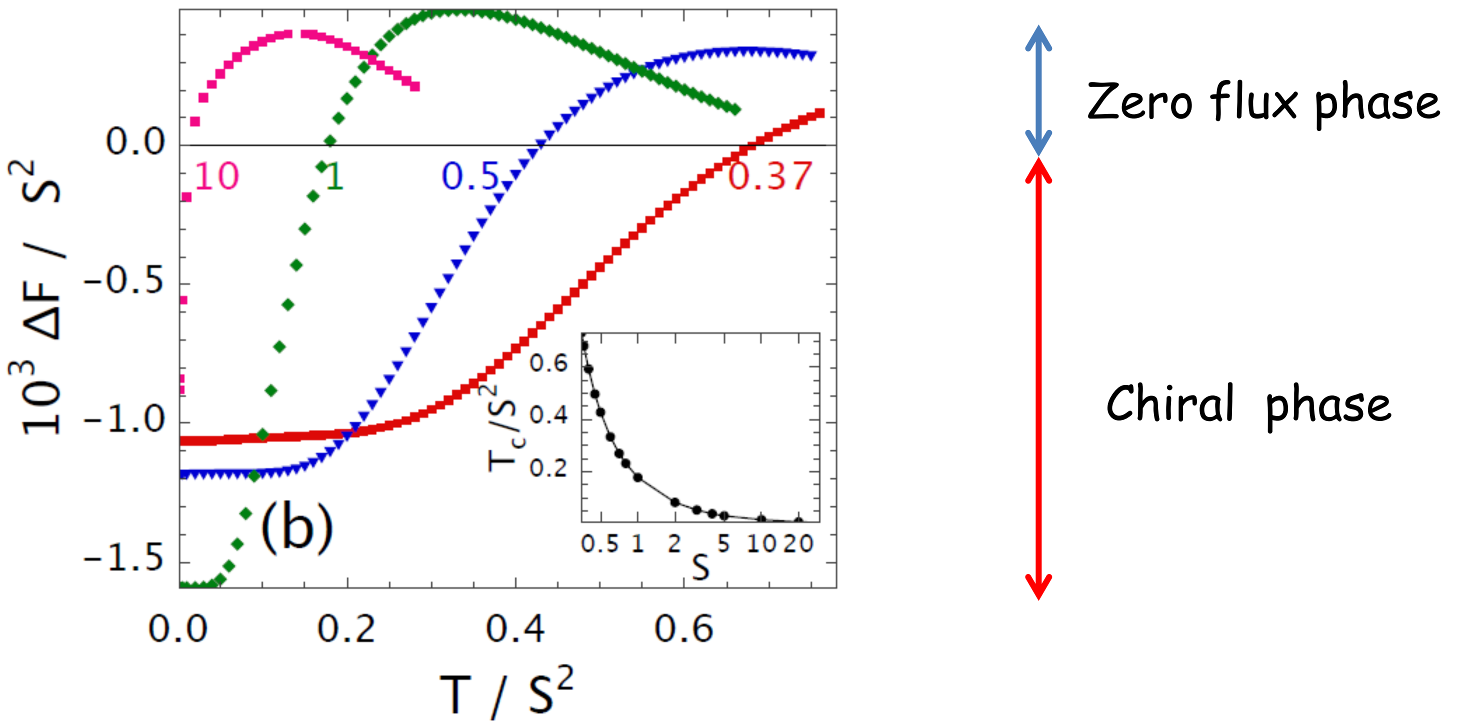
Im (B Loop operator) in a N=27 sample ground-state as a function of the length and area of various contours



$$< S_i^z \, \mathrm{Im}(P_{ijk\cdots lm}) >= C(N) \exp(-\frac{L}{\xi(N)}) \sin(\frac{\pi A}{4})$$

B. Bernu, P. Sindzingre (unpublished results 97)

Competition of thermal and quantum fluctuations



Large thermal fluctuations destabilize the cuboc1 phase

The characteristic temperature $T_c\ /S^2$ -> 0 when S -> ∞

Summary

•In the large Sp(N) limit the ground-state of the Heisenberg model is chiral

•It is a gapped Chiral Spin Liquid for physically reasonable 'spin parameters'

•Thermal fluctuations expel the flux of the chiral phase at non zero temperature (T = 0.1 for S=1/2)

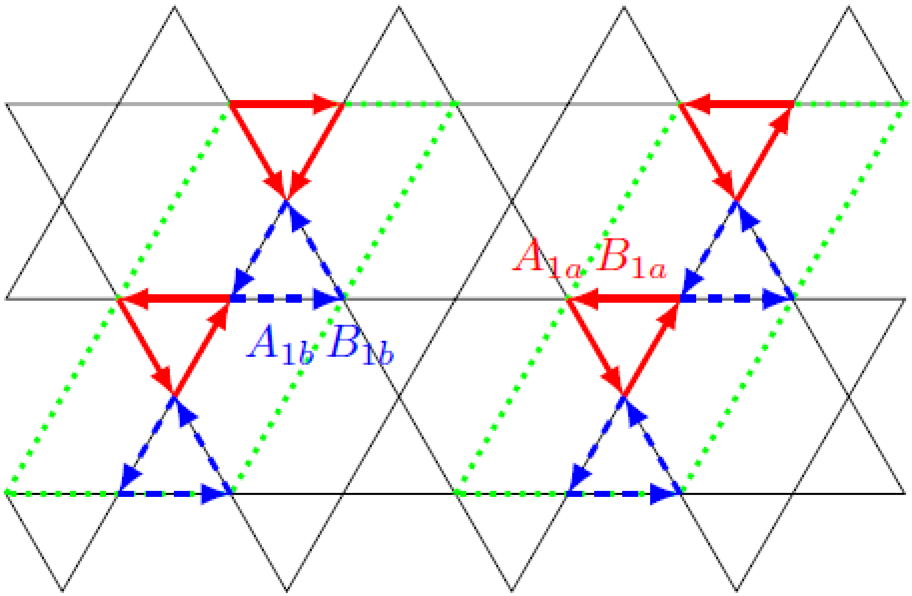
•The classical limit has $\sqrt{3}\sqrt{3}$ order for infinitesimal T

•Short range characteristics of cuboc1 spin-spin correlations could (probably) explain the long-standing mystery of ED spectra

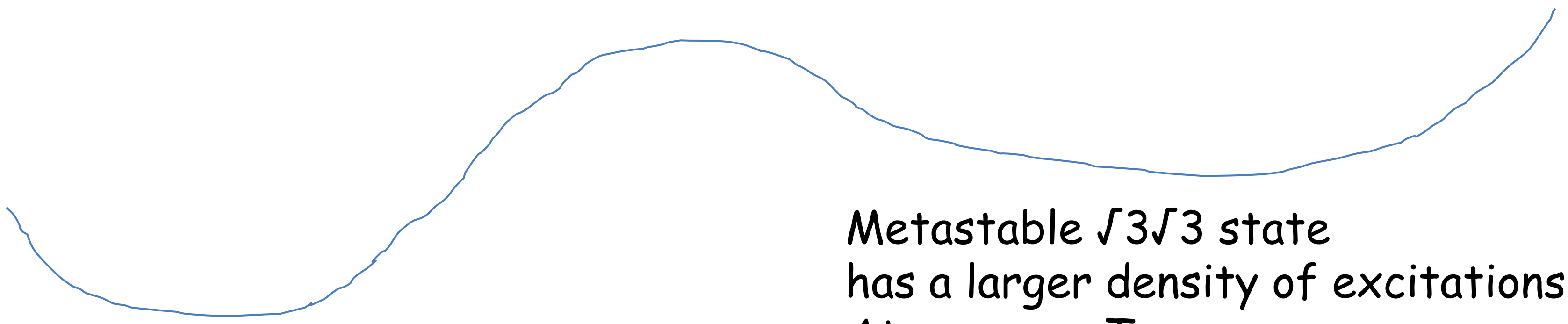
• The spontaneous chirality symmetry breaking if it exists would show up in the correlations of the sign of triple products of 2nd neighbor spins on hexagons and other more complex correlations

•Are short range quantum fluctuations described in this talk effective enough to counter zero flux phases in the SU(2) model at T=0?

Nearest neighbor cuboc1 Ansatz



Intuitive descrition of the free energy landscape of solutions in phase space in the large Sp(N) limit

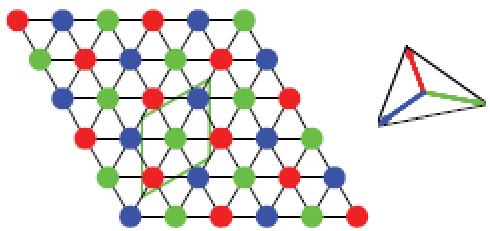


Metastable √3√3 state
has a larger density of excitations
At non zero T

Cuboc1 absolute g.s

Detour by the J1-J2 model on the triangular lattice

Lechemeinant PRB 52 (1995)



- J2/J1 < 1/8
  - 3 sublattice order
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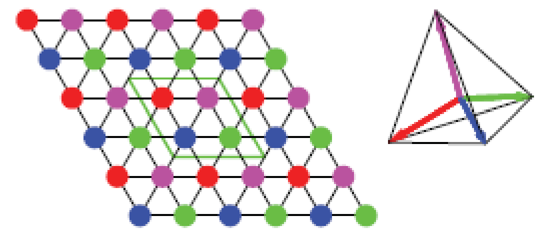

- J2/J1 >1/8
  - Classical degeneracy between 4-sublattice ord. and a striped collinear phase
  - The 4-sublattice rigid model with 4 spin directions gives

  2 low lying singlets for 4 sites

  5 low lying singlets for 16 sites

  8 low lying singlets for 28 sites

The large number of low lying singlets in the triplet gap is directly related to the number of sublattices

See precise analysis in
Lecheminant PRB 52 (1995)