Aspects of the spinon Fermi-surface state

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Kavli Institute for Theoretical Physics

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Plan:

• Introduction to the spinon Fermi-surface state of U(1) spin-liquid and some motivation

• Relation to

- Quantum Hall effect at $\nu = 1/2$

- Phase transitions in metals

• Theoretical status of the problem

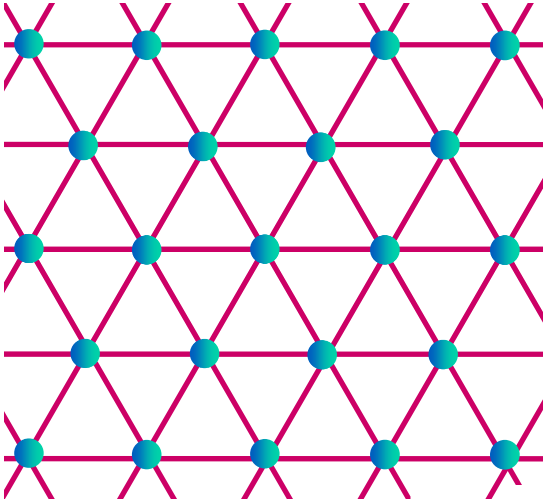
• BCS pairing of the spinon Fermi-surface state

- transition from a U(1) $\rightarrow$ Z_2 spin-liquid

Spin-liquids

- "Exotic" Mott insulators with no symmetry breaking.

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + \dots$$



- Fractionalize

$$S_i^a = \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta}^a f_{i\beta}, \qquad f_{i\alpha}^\dagger f_{i\alpha} = 1$$

- $f_\alpha \;-\; S = 1/2,\; q = 0$ fermionic quasiparticles (spinons)

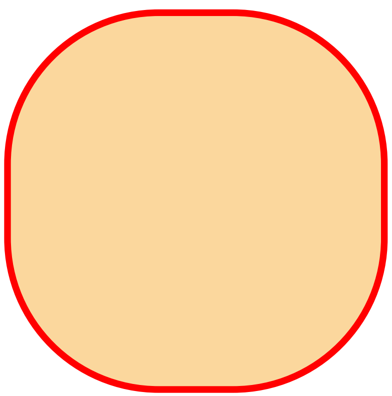
See e.g. Arun Paramekanti's talk
or review by P.A.Lee, N. Nagaosa and X.G. Wen, RMP (2006)

Spinon Fermi-surface

$$S_i^a = \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta}^a f_{i\beta}$$

$$H_{MF} = -\sum_{ij} (\chi_{ij} f_{i\alpha}^\dagger f_{j\alpha} + h.c.)$$

- Imagine χ_{ij} is such that the spinons form a Fermi-surface

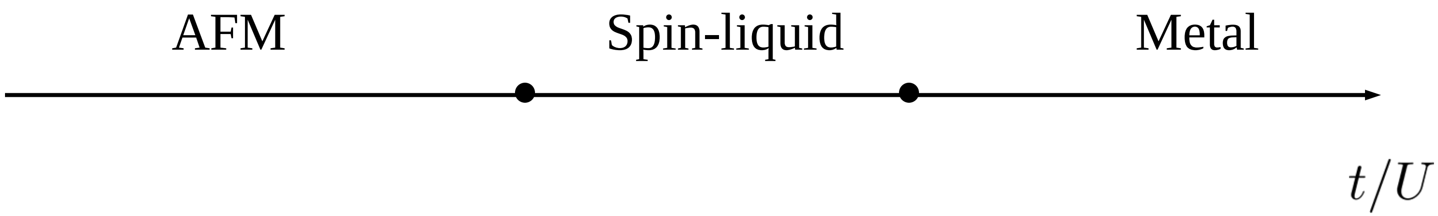


$$E_F^{\text{spinon}} \sim J$$

- lots of low energy spin excitations

Weak Mott Insulator

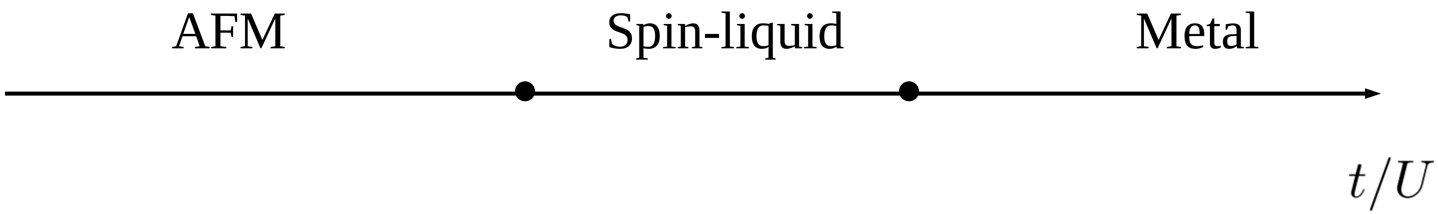
$$H = -t \sum_{\langle ij \rangle} c^\dagger_{i\alpha} c_{j\alpha} + \frac{U}{2} \sum_i n_i (n_i - 1)$$



- Evidence that this situation occurs on a triangular lattice

H. Morita, S. Watanabe and M. Imada (2002)
H.-Y. Yang, A. M. Lauchli, F. Mila, and K. P. Schmidt (2010)

Slave-boson theory



- A natural path to this phase diagram – slave boson (rotor) representation

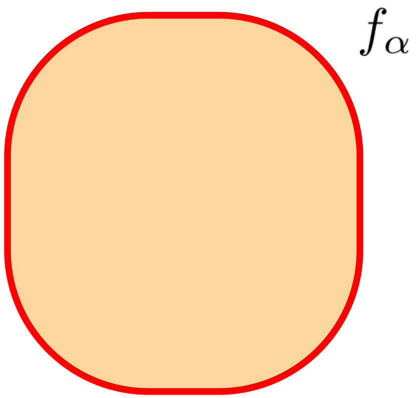
$$c_{i\alpha} = e^{i\varphi_i} f_{i\alpha} \qquad\qquad N_i^{\varphi} = f_{i\alpha}^{\dagger} f_{i\alpha}$$

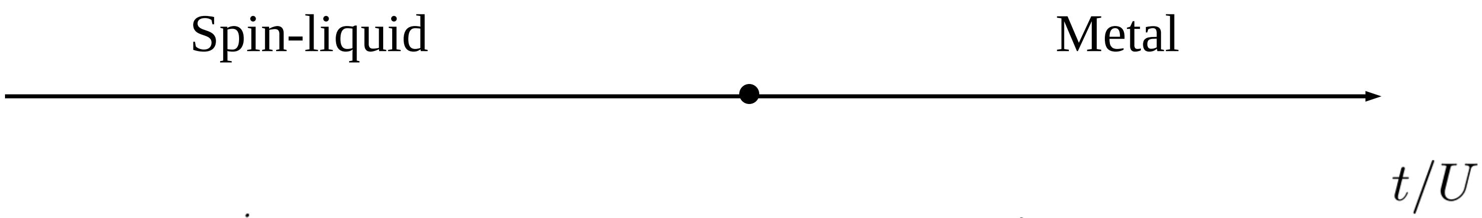
- $e^{i\varphi_i}$ - bosonic rotor variable $\quad e^{i\varphi_i}|N_i^{\varphi}\rangle = |N_i^{\varphi}+1\rangle \qquad q=1, \quad S=0$

  $f_{i\alpha}$ - fermionic spinon $\qquad q=0, \quad S=1/2$

Slave-boson theory

$c_{i\alpha} = e^{i\varphi_i} f_{i\alpha}$ $\qquad$ $N_i^\varphi = f_{i\alpha}^\dagger f_{i\alpha}$





$\langle e^{i\varphi_i} \rangle = 0$ $\qquad$ $\langle e^{i\varphi_i} \rangle \neq 0$

$e^{i\varphi}$ charge – gapped $\qquad$ $c_{i\alpha} \sim \langle e^{i\varphi_i} \rangle f_{i\alpha}$

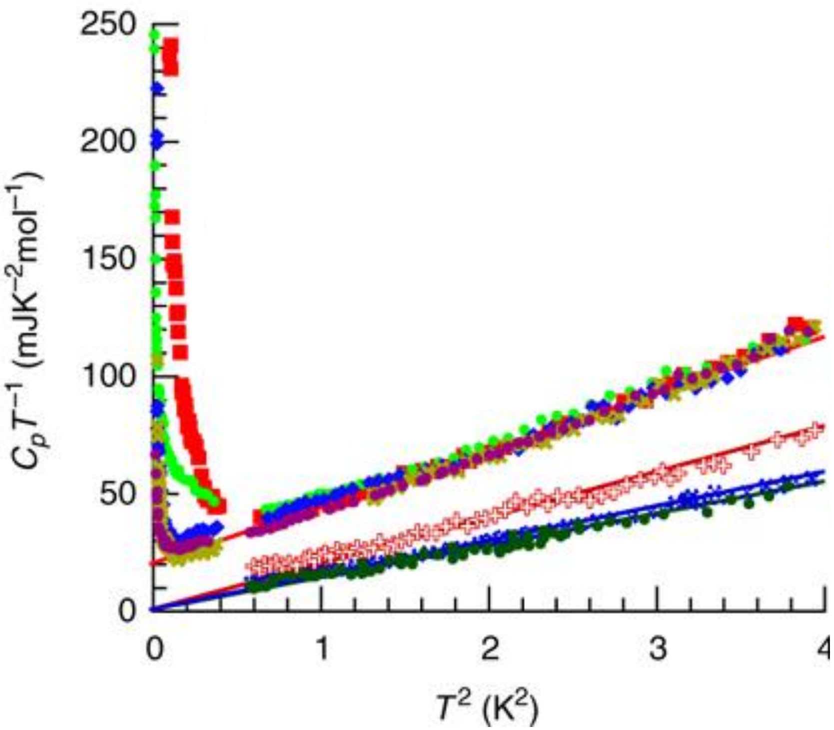
f_α spin - gapless

- Some numerical evidence for spinon FS in the spin liquid state on triangular lattice

H.-Y. Yang, A. M. Lauchli, F. Mila, and K. P. Schmidt (2010)

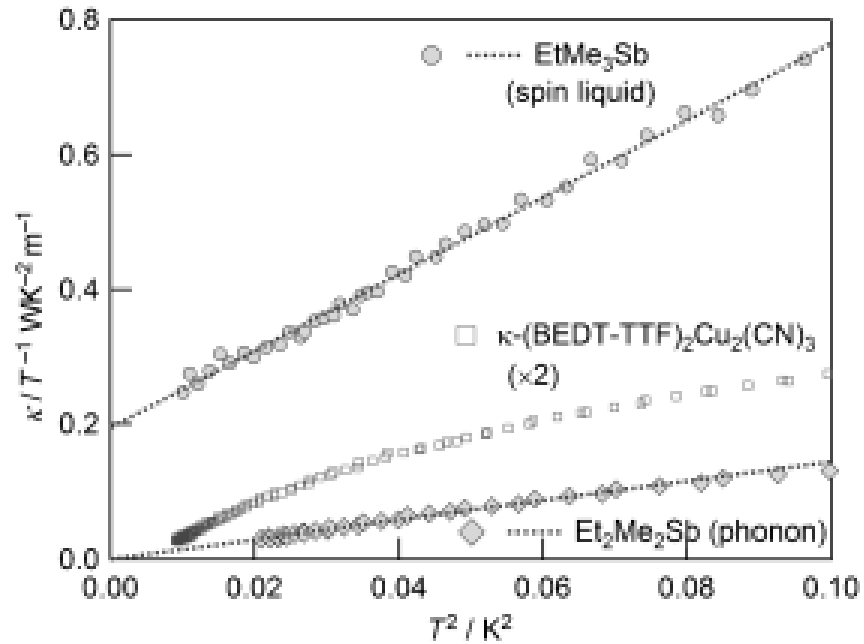
Material!!

- Experimental candidate: organic triangular lattice insulator $EtMe_3Sb[Pd(dmit)_2]_2$



$$\frac{C}{T} \to const$$

S. Yamashita et. al., (2011)



$$\frac{\kappa}{T} \to const$$

M. Yamashita et. al., (2011)

Material!!

- Experimental candidate: organic triangular lattice insulator $EtMe_3Sb[Pd(dmit)_2]_2$
- Low energy properties like a Fermi-liquid (except an insulator)!

$$\chi_s \rightarrow const$$

$$\frac{C}{T} \rightarrow const$$

$$\frac{\kappa}{T} \rightarrow const$$

- But are these actually properties of the spinon FS state?

Effective theory of spinon Fermi-surface state

- $$S_i^a = \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta}^a f_{i\beta}, \qquad f_{i\alpha}^\dagger f_{i\alpha} = 1$$
- Invariant under a local symmetry $U(1): \quad f_i \to e^{i\lambda_i} f_i$
- Low energy description will involve a compact U(1) gauge field

$$a_\mu \to a_\mu + \partial_\mu \lambda$$

- Note: the spinon representation actually has a larger SU(2) symmetry

$$SU(2): \quad \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix} \to U_i \begin{pmatrix} f_{i\uparrow} \\ f_{i\downarrow}^\dagger \end{pmatrix}$$

However, the mean-field Hamiltonian $H_{MF} = -\sum_{ij} (\chi_{ij} f_{i\alpha}^\dagger f_{j\alpha} + h.c.)$

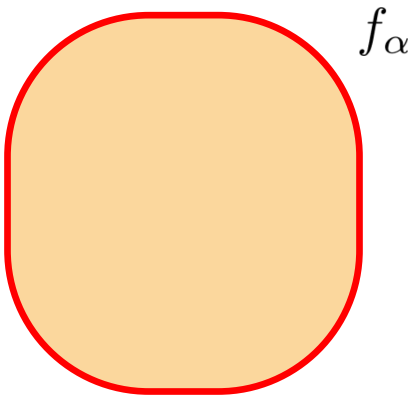
breaks $SU(2) \to U(1)$.

Effective theory of spinon Fermi-surface state

- Low energy theory involves gapless spinons near the Fermi-surface and gapless fluctuations of the U(1) gauge field.

- The coupling of gauge-fluctuations to spinons destroys sharp spinon quasiparticles.

- Non-Fermi-liquid (critical Fermi-surface) of spinons.



Plan:

- Introduction to the spinon Fermi-surface state of U(1) spin-liquid and some motivation
- Relation to
  - Quantum Hall effect at $\nu = 1/2$
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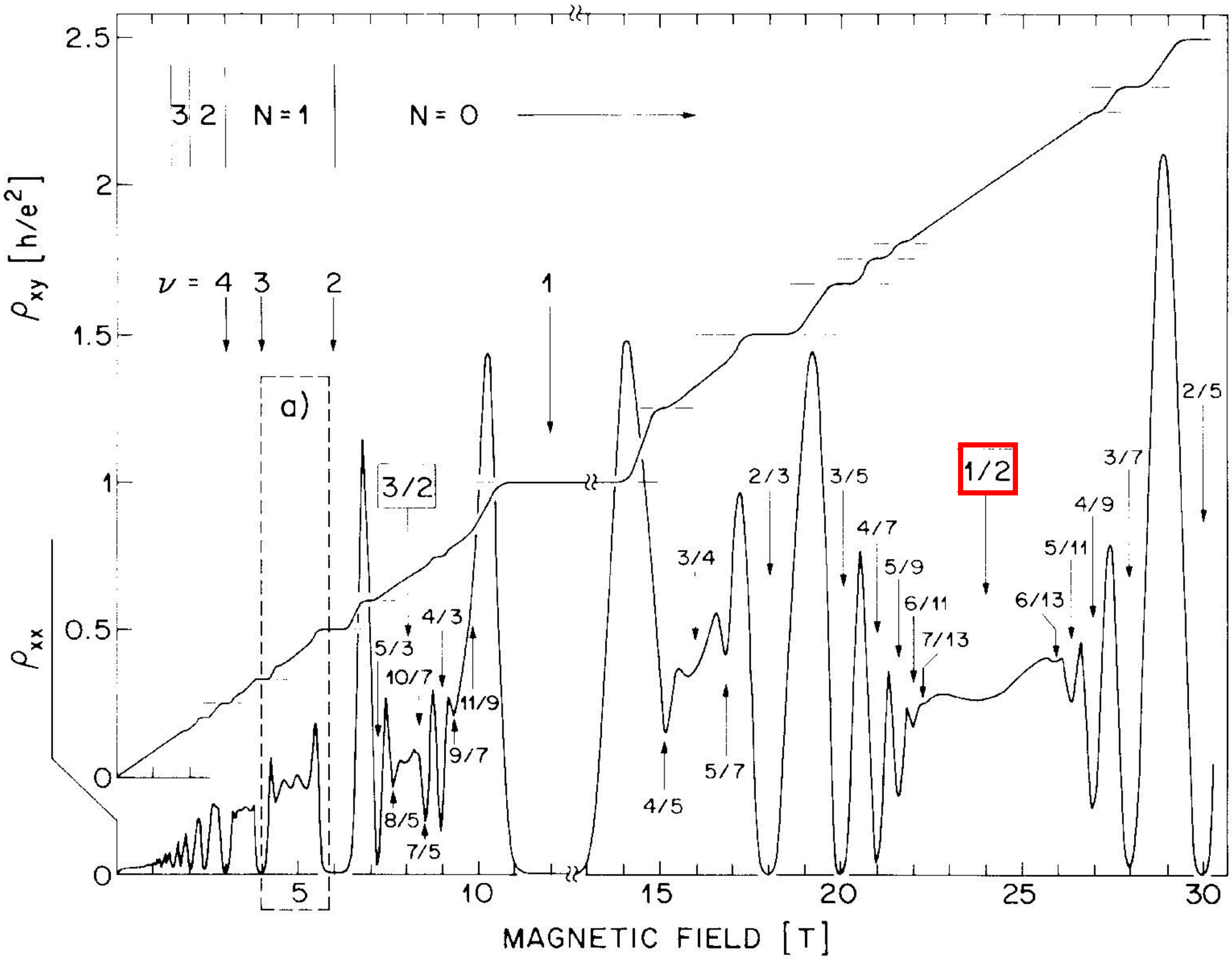
Half-filled Landau level

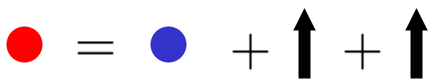


R. Willet et. al. (87)

Half-filled Landau level: Composite Fermion Liquid

B. I. Halperin, P. A. Lee, N. Read (1993)

- Form composite fermions by attaching two flux-quanta to each electron



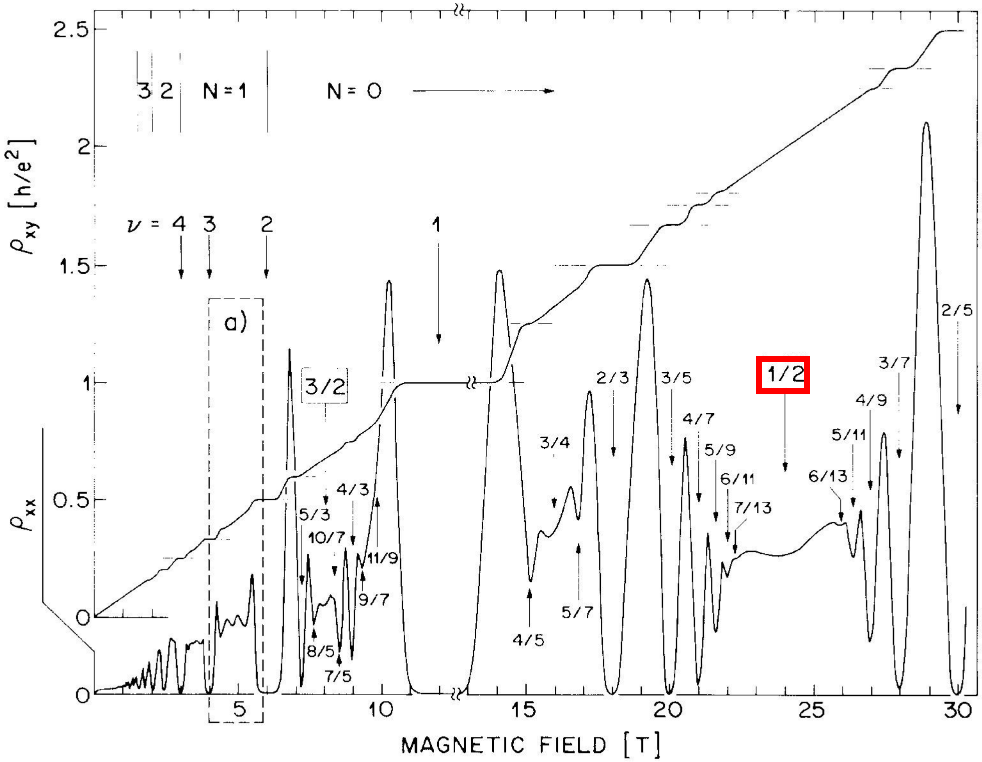
- Composite fermions see no net magnetic field and form a Fermi-surface
- A U(1) Chern-Simons gauge field is used to attach flux:

$$L = f^\dagger(\partial_\tau - i a_\tau) f + \frac{1}{2m} |(\nabla + i \frac{e}{c} \vec{A} - i \vec{a}) f|^2 + \frac{i\nu}{4\pi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$$

$$\nabla \times \vec{a} = 2(2\pi) f^\dagger f$$

- Fluctuations of a_μ make the Fermi-surface of the composite fermion critical.

Half-filled Landau level



• Fractional QH states near $\nu = 1/2$ can be interpreted as Integer QH states of the composite fermion liquid.

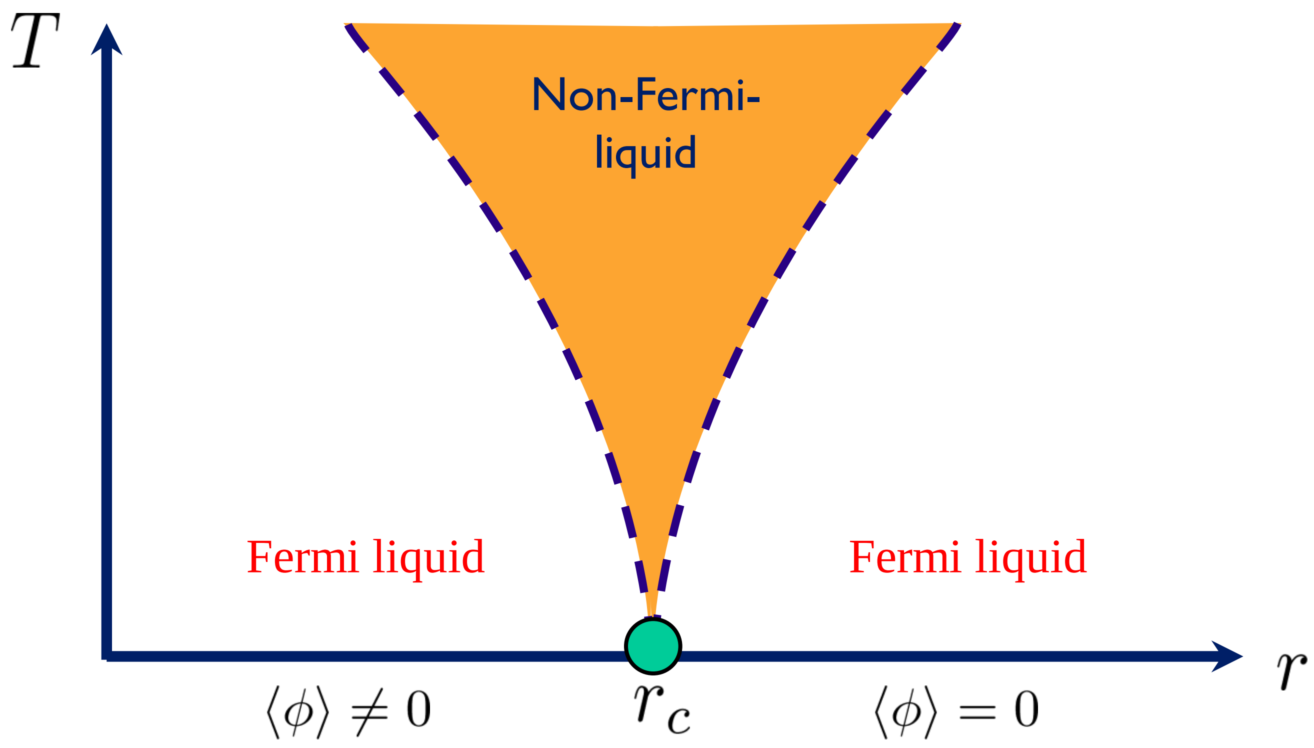
• Strong experimental evidence for emergence of a Fermi-surface at $\nu = 1/2$ (e.g. from surface-acoustic wave).

• Strong numerical evidence for emergence of Fermi-surface.

• Conflicting results regarding criticality of the Fermi-surface.

Phase transitions in metals

• Order parameter: ϕ



• heavy-fermion compounds, $Sr_3Ru_2O_7$, electron-doped cuprates, pnictides, organics, hole-doped cuprates

Critical Fermi surface states

- Spinon Fermi-surface state of Mott-insulators
  (Bose metals)

- Composite-fermion liquid of QHE system

- Phase transitions in metals

- All three states involve a Fermi surface interacting with a gapless boson
- Difficult problem, due in part to an absence of a full RG program.
- "Solved" in $N \to \infty$ limit in early 90's
  Declared open again after work of S. S. Lee (2009).

Plan:

- Introduction to the spinon Fermi-surface state of U(1) spin-liquid and some motivation
- Relation to
  - Quantum Hall effect at $\nu = 1/2$
  - Phase transitions in metals
- Theoretical status of the problem (focus on dimension d = 2)
- BCS pairing of the spinon Fermi-surface state
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Emergence of gauge field

$$S_i^a = \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta}^a f_{i\beta}, \qquad f_{i\alpha}^\dagger f_{i\alpha} = 1$$

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j = -\frac{J}{2} \sum_{\langle ij \rangle} (f_{i\alpha}^\dagger f_{j\alpha})(f_{j\beta}^\dagger f_{i\beta})$$

$$L = \sum_i f_{i\alpha}^\dagger \partial_\tau f_{i\alpha} - H - i \sum_i a_{\tau i} \left(f_{i\alpha}^\dagger f_{i\alpha} - 1 \right)$$

- a_τ - local Lagrange multiplier enforcing the single occupancy constraint.

Emergence of gauge field

$$H = J \sum_{ij} \vec{S}_i \cdot \vec{S}_j = -\frac{J}{2} \sum_{ij} (f^\dagger_{i\alpha} f_{j\alpha})(f^\dagger_{j\beta} f_{i\beta})$$

$$L = \sum_i f^\dagger_{i\alpha} \partial_\tau f_{i\alpha} - H - i \sum_i a_{\tau i} \left(f^\dagger_{i\alpha} f_{i\alpha} - 1 \right)$$

$$\begin{aligned} L &= \sum_i f^\dagger_{i\alpha} (\partial_\tau - i a_{\tau i}) f_{i\alpha} + i \sum_i a_{\tau i} \\ &+ \sum_{\langle ij \rangle} (\chi_{ij} f^\dagger_{i\alpha} f_{j\alpha} + h.c.) + \frac{2}{J} \sum_{\langle ij \rangle} |\chi_{ij}|^2 \end{aligned}$$

- $\chi_{ij}(\tau)$ - Hubbard-Stratonovitch field.

Emergence of gauge field

$$
\begin{aligned}
L &= \sum_i f^\dagger_{i\alpha}(\partial_\tau - i a_{\tau i}) f_{i\alpha} + i \sum_i a_{\tau i} \\
&+ \sum_{\langle ij \rangle} (\chi_{ij} f^\dagger_{i\alpha} f_{j\alpha} + h.c.) + \frac{2}{J} \sum_{\langle ij \rangle} |\chi_{ij}|^2
\end{aligned}
$$

$$\chi_{ij} = |\chi_{ij}| e^{i a_{ij}}$$

$$U(1):\ f_{i\alpha} \to e^{i\lambda_i(\tau)} f_{i\alpha}, \quad e^{i a_{ij}} \to e^{i\lambda_i} e^{i a_{ij}} e^{-i\lambda_j}, \quad a_{i\tau} \to a_{i\tau} + \partial_\tau \lambda_i$$

- Focus on slow fluctuations $a_{i\tau}, a_{ij}$. $\quad a_{ij} \sim \vec{a} \cdot (\vec{R}_i - \vec{R}_j)$

$$a_\mu = (a_\tau, \vec{a})$$

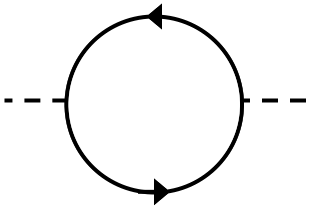
$$U(1):\ f \to e^{i\lambda(x)} f, \quad a_\mu \to a_\mu + \partial_\mu \lambda$$

Theory of the spinon Fermi-surface

- Theory of a gapless gauge field interacting with the spinon Fermi-surface

$$L = f_\alpha^\dagger(\partial_\tau - ia_\tau + \epsilon(-i\vec{\nabla} - \vec{a}))f_\alpha + \frac{1}{2e^2}(\nabla \times \vec{a})^2$$

- Integrate the fermions out at the RPA level (work in $\nabla \cdot \vec{a} = 0$ gauge):



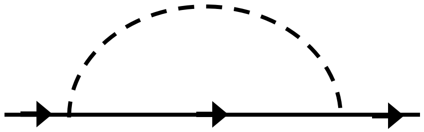
$$\delta S = \frac{1}{2} a_\mu \Pi_{\mu\nu} a_\nu$$

$$\Pi_{\tau\tau}(\omega = 0, \vec{q} \to 0) = N(0)$$ - Debye screened

$$\Pi_{ij}(\omega, \vec{q}) = \left(\gamma(\hat{q})\frac{|\omega|}{|\vec{q}|} + C\vec{q}^2\right)\left(\delta_{ij} - \frac{q_i q_j}{\vec{q}^2}\right)$$ - Landau damped

- Magnetic field fluctuations are very soft: $\omega \sim |\vec{q}|^z$ $\quad z = 3$

Feedback on the fermions

$\Sigma(\omega, \vec{k}) =$  $= -ic_f sgn(\omega)|\omega|^{2/3}$

$$G^{-1}(\omega, \vec{k}) = -\cancel{i\omega} - ic_f sgn(\omega)|\omega|^{2/3} + v_F(|k| - k_F)$$

- Non Fermi-liquid!

$$G(k, \omega) = \frac{Z}{i\omega - v_F(|k| - k_F)}$$

$$Z \to 0, \quad v_F \to 0$$

$$C_v \sim T^{2/3}$$

P. A. Lee, N. Nagaosa (1992); J. Polchinski (1993);
B. I. Halperin, P. A. Lee, N. Read (1993).

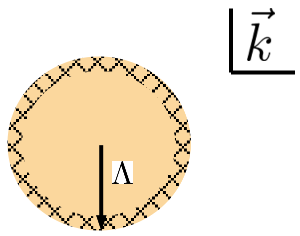
How to scale?

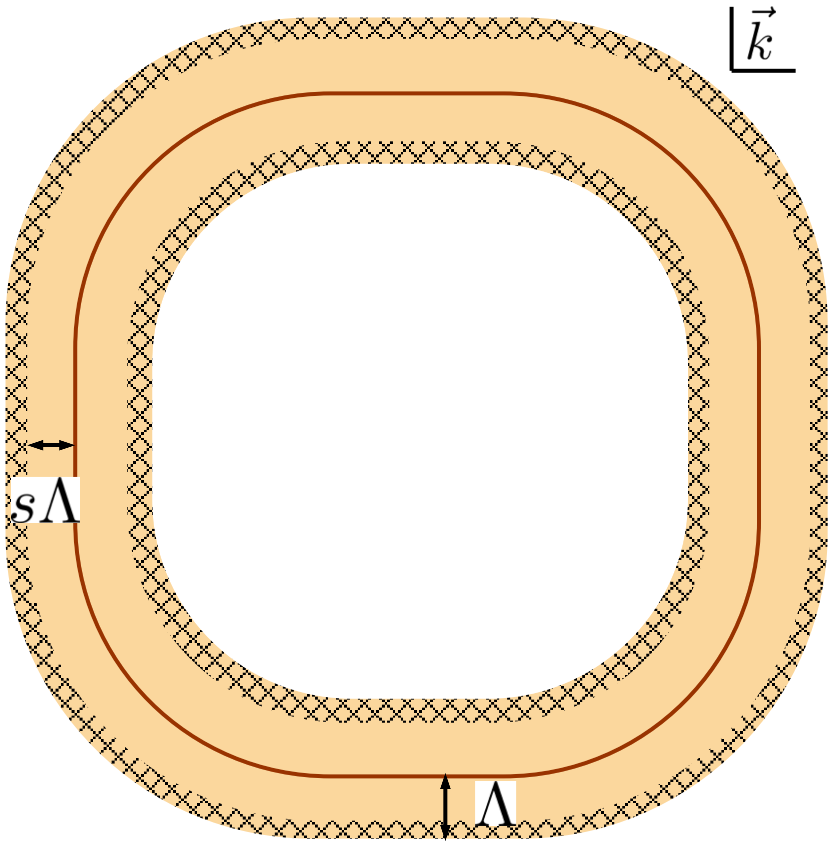
Gauge field: $\vec{a}(\vec{k}, \omega)$

$$\vec{k} \to s\vec{k}$$

Fermions: $\psi(k, \hat{k}, \omega)$

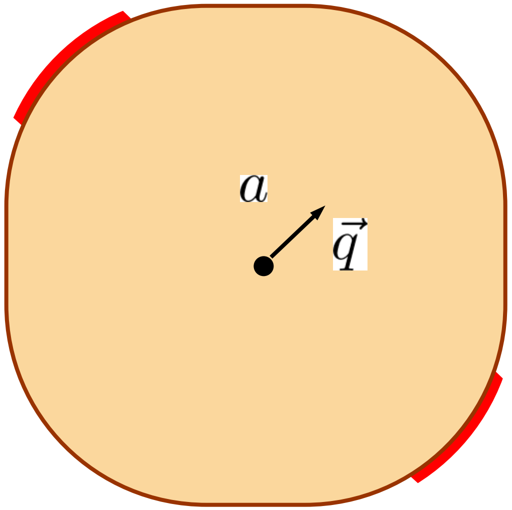
$$k \to sk, \ \ \hat{k} \to \hat{k}$$





Two-patch regime

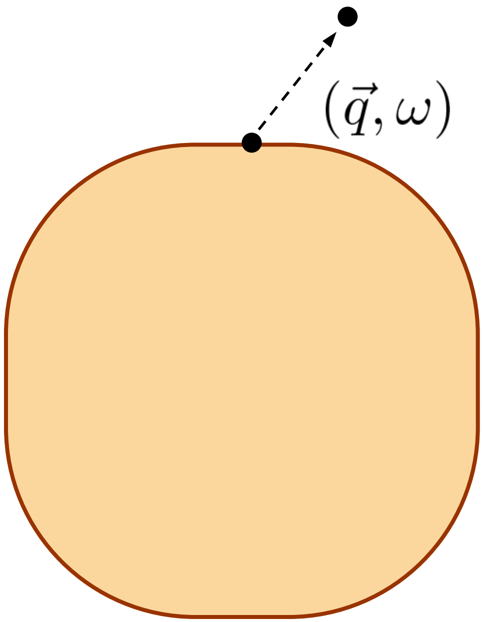
- Most singular kinematic regime: two-patch



J. Polchinski (1993); B. Altshuler, L. Ioffe, A. Millis (1994).

Why two-patch regime?

- Gauge fluctuations are very soft:



$$\omega \sim |\vec{q}|^3$$

$$\Delta\epsilon \sim v_F |\vec{q}| \sim \omega^{1/3}$$

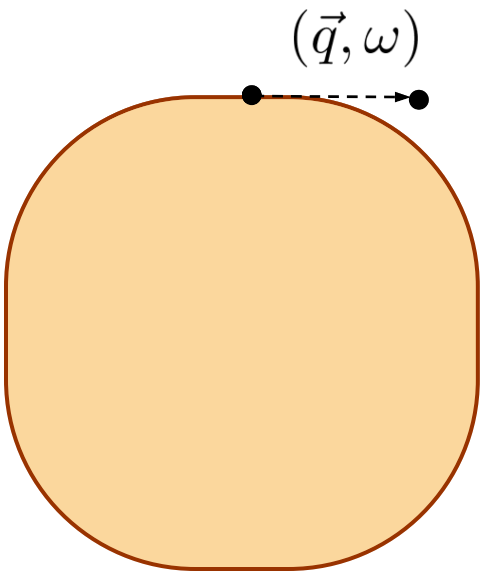
$$\Sigma(\omega) \sim \omega^{2/3}$$

- Cannot effectively absorb a Landau-damped bosonic mode.

B. Altshuler, L. Ioffe, A. Millis (1994).

Why two-patch regime?

- Gauge fluctuations are very soft:



$$\omega \sim |\vec{q}|^3$$

$$\Delta\epsilon \sim \frac{v_F |\vec{q}|^2}{2k_F} \sim \omega^{2/3}$$

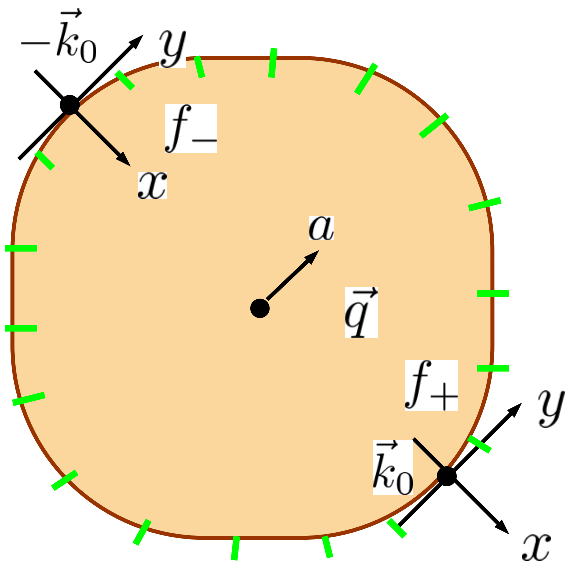
$$\Sigma(\omega) \sim \omega^{2/3}$$

- This “conspiracy” between fermions and bosons is special to 2+1 dimensions.

B. Altshuler, L. Ioffe, A. Millis (1994).

Two-patch theory

- For each $\hat{q}$ expand the fermion fields about two opposite points on the Fermi surface, $\vec{k}_0$ and $-\vec{k}_0$.



$$L_f = f_+^\dagger \left(\partial_\tau + v_F(-i\partial_x - \frac{\partial_y^2}{2K}) \right) f_+ + f_-^\dagger \left(\partial_\tau + v_F(i\partial_x - \frac{\partial_y^2}{2K}) \right) f_-$$

$$L_a = \frac{1}{2e^2}(\partial_y a)^2, \qquad a_i(\vec{q},\omega) = \epsilon_{ij}\frac{q_j}{|\vec{q}|}a(\vec{q},\omega)$$

$$L_{int} = v_F a(f_+^\dagger f_+ - f_-^\dagger f_-)$$

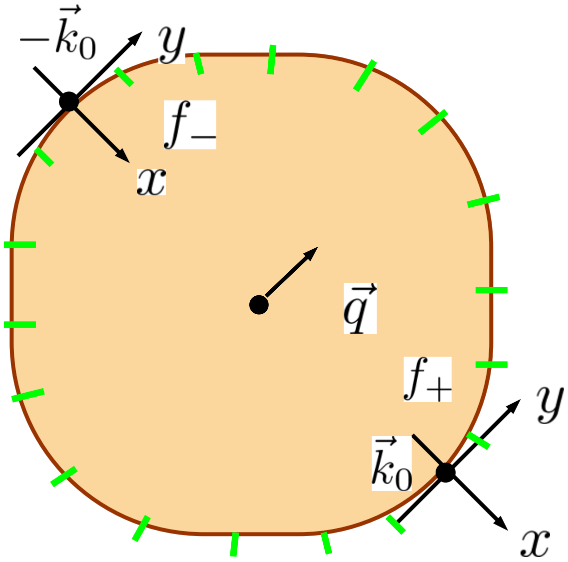
- Crucial to keep the Fermi-surface curvature radius K finite.

J. Polchinski (1993); S. S. Lee (2009)

Two-patch theory

$$S = \int d\tau dx dy \; L$$

- Have a large set of 2+1 dimensional theories labeled by points on the Fermi-surface ($\hat{q}$)



- Key assumption: can neglect coupling between patches.

Anisotropic scaling

$$L = \sum_s f_s^\dagger (\partial_\tau + v_F(-is\partial_x - \frac{\partial_y^2}{2K}))f_s + v_F a \sum_s s f_s^\dagger f_s + \frac{1}{2e^2}(\partial_y a)^2$$

- Fermion kinetic term dictates:

$$k_y \to s k_y, \quad k_x \to s^2 k_x$$

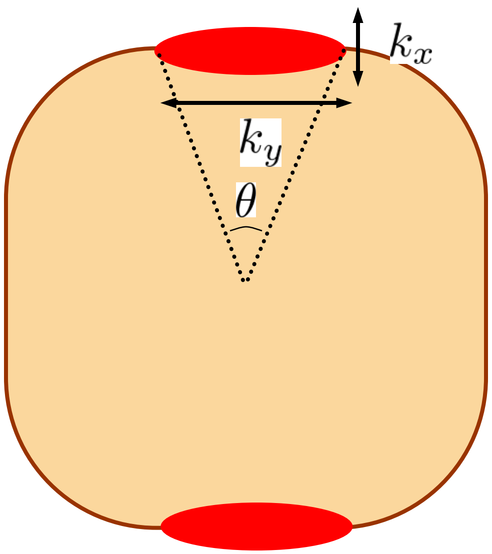
- A symmetry of the theory guarantees that this scaling is exact.

J. Polchinski (1993); B. Altshuler, L. Ioffe, A. Millis (1994).

Two-patch scaling

<u>Critical Fermi surface</u>

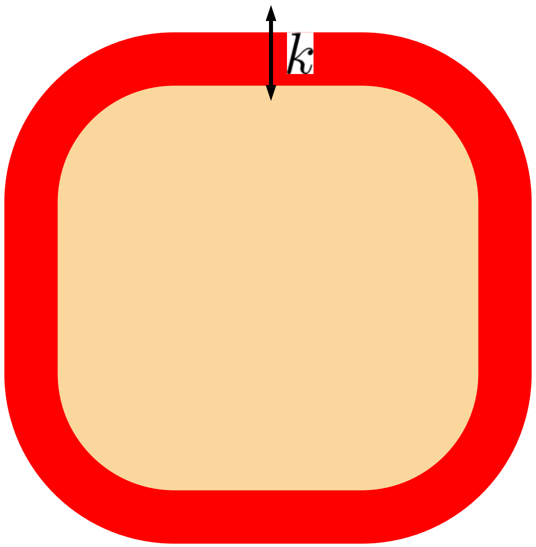
$$k_y \to s k_y, \quad k_x \to s^2 k_x$$



$$\theta \to s\theta$$

<u>Fermi-liquid</u>

$$k \to sk$$



θ does not flow

J. Polchinski (1993); B. Altshuler, L. Ioffe, A. Millis (1994), S. S. Lee (2008).

Scaling properties

$$L = \sum_s f_s^\dagger (\partial_\tau + v_F(-is\partial_x - \frac{\partial_y^2}{2K}))f_s + v_F a \sum_s s f_s^\dagger f_s + \frac{1}{2e^2}(\partial_y a)^2$$

- Reproduces RPA at one loop
- Theory strongly coupled
- Symmetries constrain the RG properties severely
- Only two anomalous dimensions

η_f - fermion anomalous dimension

z - dynamical critical exponent

Scaling forms: gauge field

- $$D^{-1}(\omega, \vec{q}) \sim |\vec{q}|^{z-1} f\left(\frac{|\omega|}{|\vec{q}|^z}\right)$$

- Simple Landau-damped frequency dependence consistent with scaling form

$$D^{-1}(\omega, \vec{q}) - D^{-1}(\omega = 0, \vec{q}) \sim \frac{|\omega|}{|\vec{q}|}$$

- Static behaviour

$$D^{-1}(\omega = 0, \vec{q}) \sim |\vec{q}|^{z-1}$$

Scaling forms: fermions

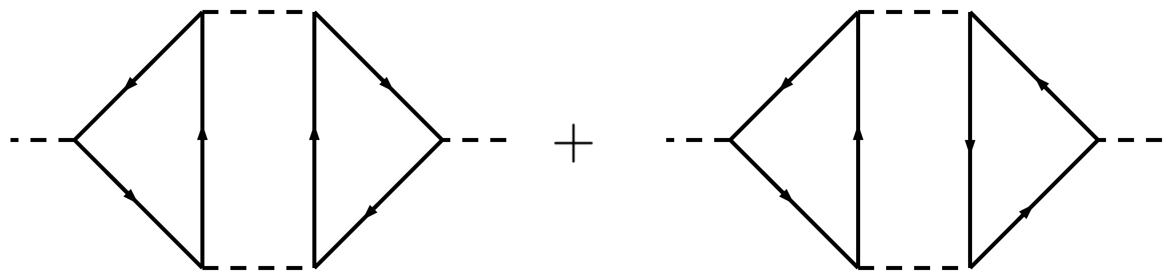
- $$G^{-1}(\omega, \vec{k}) \sim k^{1-\eta_f/2} g\left(\frac{|\omega|}{|k|^{z/2}}\right), \quad k = k_x + \frac{k_y^2}{2K}$$
- "Fermionic dynamical exponent" is half the "bosonic dynamical exponent"
- Static behaviour: $G^{-1}(0, \vec{k}) \sim k^{1-\eta_f/2}$
- Dynamic behaviour: $G^{-1}(\omega, 0) \sim \omega^{(2-\eta_f)/z}$
- ``Tunneling density of states": $N(\omega) \sim \omega^{\eta_f/z}$

M.M and S. Sachdev (2010)

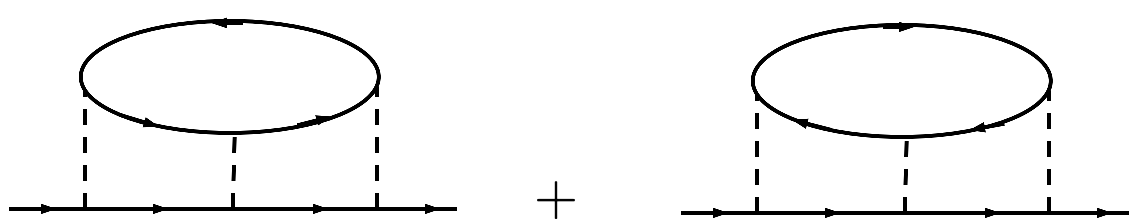
Problem

- No expansion parameter. Theory strongly coupled.
- Direct large-N expansion fails. *S. S. Lee (2009)*
- A more sophisticated genus expansion also fails. Unknown if $N \to \infty$ limit exists.
- Uncontrolled three loop calculations *M.M. and S. Sachdev (2010)*

$$\delta^3 \Pi(\omega = 0, q_y) =$$



$$\delta^3 \Sigma(\omega = 0, \vec{k}) =$$



Problem

- Uncontrolled three loop calculations give

$$z = 3$$

$$\begin{aligned} \eta_f &= -0.06824, \quad N = 2 \\ \eta_f &= -0.10619, \quad N = \infty \end{aligned}$$

$$C_v \sim T^{2/z}$$

M.M. and S. Sachdev (2010)

- Contrary to the previous belief that no qualitatively new physics beyond one loop.

How to control the expansion?

$$S_a = \frac{1}{2e^2}\int d^2x d\tau (\nabla a)^2 \rightarrow \frac{1}{2e^2}\int \frac{d\omega d^2\vec{q}}{(2\pi)^3}|\vec{q}|^{1+\epsilon}|a(\vec{q},\omega)|^2$$

- Long-range interaction in the half-filled Landau level

$$S_{int} = \frac{1}{2}\int d^2x d^2x' d\tau (f^\dagger f)(\vec{x},\tau)\frac{1}{|\vec{x}-\vec{x}'|^{1+\epsilon}}(f^\dagger f)(\vec{x}',\tau)$$

$$\nabla \times \vec{a} = 2(2\pi) f^\dagger f$$

- $\epsilon = 0$ - Coulomb interaction

  $\epsilon = 1$ - contact interaction

B. I. Halperin, P. A. Lee, N. Read (1993); C. Nayak and F. Wilczek (1994)

How to control the expansion?

$$L = \sum_s f_s^\dagger(\partial_\tau + v_F(-is\partial_x - \frac{\partial_y^2}{2K}))f_s + v_F a \sum_s s f_s^\dagger f_s + \frac{N}{2e^2} q_y^{1+\epsilon} a^2$$

- Perform more conventional scaling dictated by the fermion kinetic term:

$$\omega \to s^2\omega,\ k_x \to s^2 k_x,\ k_y \to s k_y$$

- Fermion-gauge field interactions are at tree level:

$\epsilon < 0$ - irrelevant

$\epsilon > 0$ - relevant

$\epsilon = 0$ - marginal

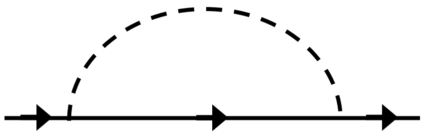
- Theory described by a single dimensionless coupling constant:

$$\alpha = \frac{e^2 v_F \Lambda_y^{-\epsilon}}{(2\pi)^2} \qquad \frac{d\alpha}{dl} = \frac{\epsilon}{2}\alpha$$

ε-expansion

- Quantum corrections to scaling

$$\Sigma(\omega, \vec{k}) =$$



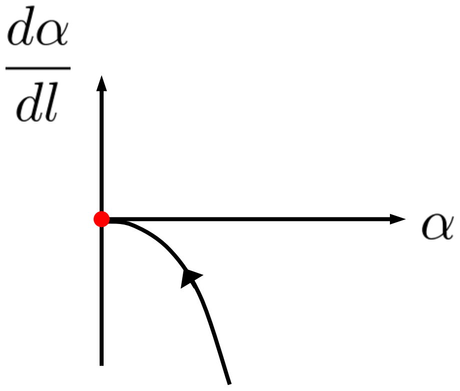
$$= -i\frac{\alpha}{N}\omega \log \frac{\Lambda_\omega}{|\omega|}$$

$$\frac{d\alpha}{dl} = \frac{\epsilon}{2}\alpha - \frac{\alpha^2}{N}$$

C. Nayak and F. Wilczek (1994)

ε-expansion

- $\epsilon = 0$ $$\frac{d\alpha}{dl} = -\frac{\alpha^2}{N}$$



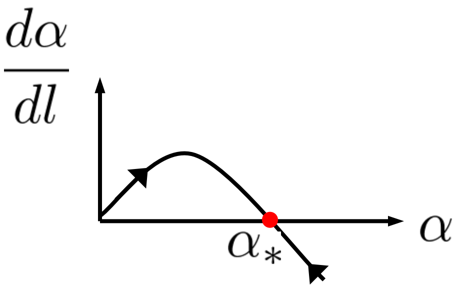
$$\Sigma(\omega) = -i\frac{\alpha}{N}\omega \log \frac{\Lambda_\omega}{|\omega|}$$

- interactions marginally irrelevant.
"Marginal Fermi-liquid."

C. Nayak and F. Wilczek (1994)

ε-expansion

- $\epsilon > 0$ $$\frac{d\alpha}{dl} = \frac{\epsilon}{2}\alpha - \frac{\alpha^2}{N}$$



- new fixed-point: $\alpha_* = \frac{N\epsilon}{2}$

$$\Sigma(\omega) \sim -i|\omega|^{1-\epsilon/2}\text{sgn}(\omega)$$

C. Nayak and F. Wilczek (1994)

Beyond one-loop

- Two expansions have been suggested: $\alpha = \frac{e^2 v_F \Lambda_y^{-\epsilon}}{(2\pi)^2}$ $\alpha_* \approx \frac{N\epsilon}{2}$

  - $\epsilon \to 0, \;\; N$ - fixed, "perturbative" expansion

    C. Nayak and F. Wilczek (1994)

    potential difficulty: $D(\omega, q_y) = \frac{e^2}{N} \frac{1}{\frac{e^2 k_F}{2\pi}\frac{|\omega|}{|q_y|} + |q_y|^{1+\epsilon}}$

    systematic counting of powers of α needs to be done!

  - $\epsilon \to 0, \;\; N \to \infty, \;\; N\epsilon$ - fixed, $1/N$ - expansion.

    D. Mross, J. McGreevy, H. Liu and T. Senthil (2010)

    cannot be applied to the $\epsilon = 0$ case.

Response functions

- Same as before,

$$D^{-1}(\omega, \vec{q}) \sim |\vec{q}|^{z-1} f\left(\frac{|\omega|}{|\vec{q}|^z}\right)$$

$$G^{-1}(\omega, \vec{k}) \sim k^{1-\eta_f/2} g\left(\frac{|\omega|}{|k|^{z/2}}\right), \quad k = |\vec{k}| - k_F$$

- Except non-locality of the action constrains

$$z = 2 + \epsilon$$

- $\eta_f \sim \frac{1}{N^2} X(\epsilon N) \neq 0$ at three loop order.

D. Mross, J. McGreevy, H. Liu and T. Senthil (2010)

Open questions

- Prove (disprove) that $z = 3$ in physical theory $C_v \sim T^{2/z}$

  - for dMIT $C_v \sim T - ?$

- Systematic theory of forward-scattering channel

  $\chi_s \sim const$ C.P.Nave, S.S.Lee and P.A.Lee (2006)

  - holds for dMIT

- Transport properties

  $\kappa/T \sim T^{-2/3}$ C.P.Nave and P.A.Lee (2007)

  - for dMIT $\kappa/T \sim const$

- Monopole effects

  - monopoles irrelevant (confined)

L.B.Ioffe and A.I.Larkin (1989), S.S.Lee (2008)

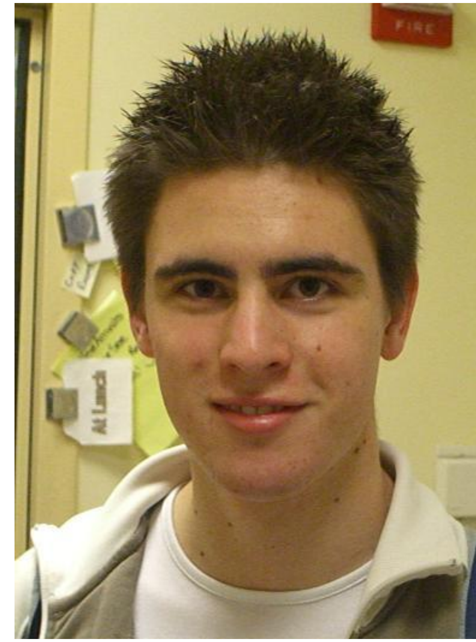
Plan:

- Introduction to the spinon Fermi-surface state of U(1) spin-liquid and some motivation
- Relation to
  - Quantum Hall effect at $\nu = 1/2$
  - Phase transitions in metals
- Theoretical status of the problem
- BCS pairing of the spinon Fermi-surface state
  - transition from a U(1) $\rightarrow$ Z_2 spin-liquid

Collaborators

Subir Sachdev
(Harvard)

Senthil Todadri
(MIT)



David Mross
(MIT)

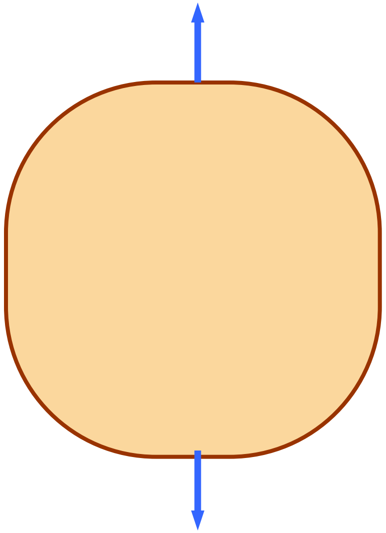
Pairing of critical Fermi surfaces

- A regular Fermi-liquid is unstable to arbitrarily weak attraction in the BCS channel.

- How about the spinon Fermi-surface?

Pairing instability of spinon Fermi-surface

- Magnetic fluctuations mediate a long-range repulsion

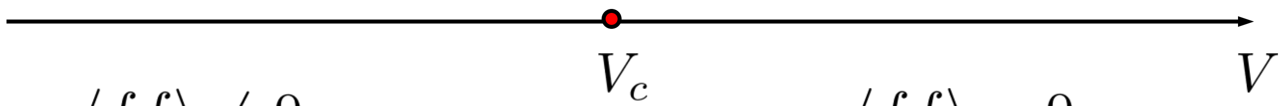


- Can a short range attractive interaction V compete with this?
- What is the critical interaction strength?

Pairing in a spin-liquid

$\mathbb{Z}_2$ - spin-liquid

U(1) spin-liquid
spinon Fermi surface



$\langle f f \rangle \neq 0$

$\langle f f \rangle = 0$

Excitations:

gapped spinons
gapped Abrikosov vortices
(visons)

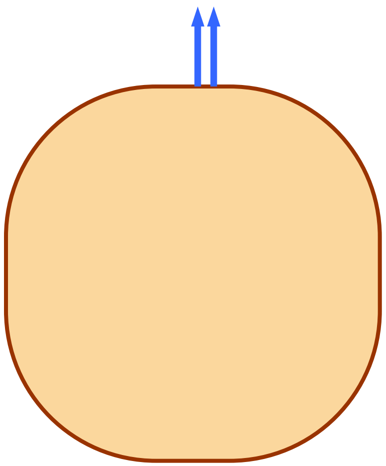
gapless spinons f_α
gapless gauge (magnetic field)
fluctuations

- Previous theoretical work claims a fluctuation induced first order transition

M. U. Ubbens and P. A. Lee (1994)

Amperean pairing

- Attraction for spinons on same side of the Fermi-surface



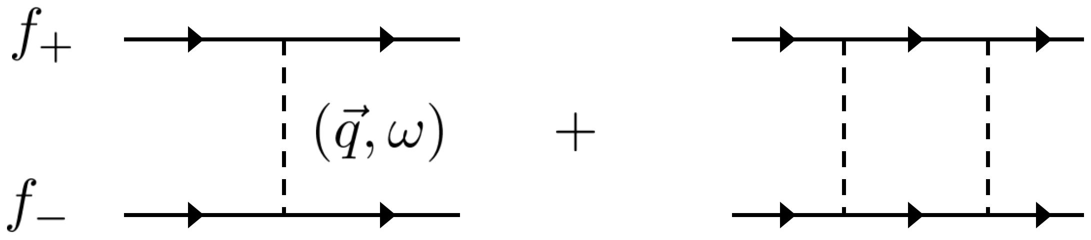
- Sympathetic to LOFF-like pairing $\langle f f(\vec{r}) \rangle \sim e^{i\vec{K}\cdot\vec{r}}\Delta$ $|\vec{K}| \sim 2k_F$

- Not discussing this possibility. For $\epsilon \ll 1$, system is “far” from this instability.

Sung-Sik Lee, Patrick A. Lee, T. Senthil (2007)

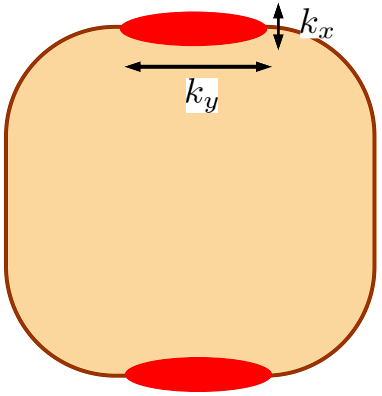
Pairing singularities

- Scattering amplitude in the BCS channel at $\epsilon = 0$

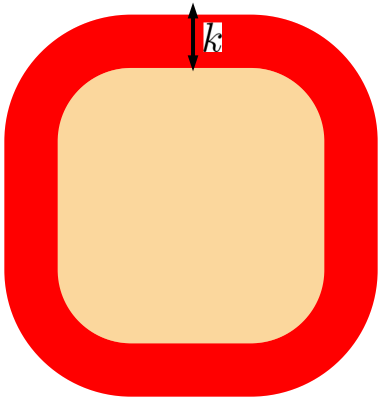


- In the regime $\omega \ll q_y^2$, the one loop diagram is enhanced by $-\frac{\alpha}{N} \log^2 \frac{q_y^2}{\omega}$
- $\delta L = g f^\dagger f^\dagger f f$

irrelevant in two-patch theory



marginal in Fermi-liquid theory

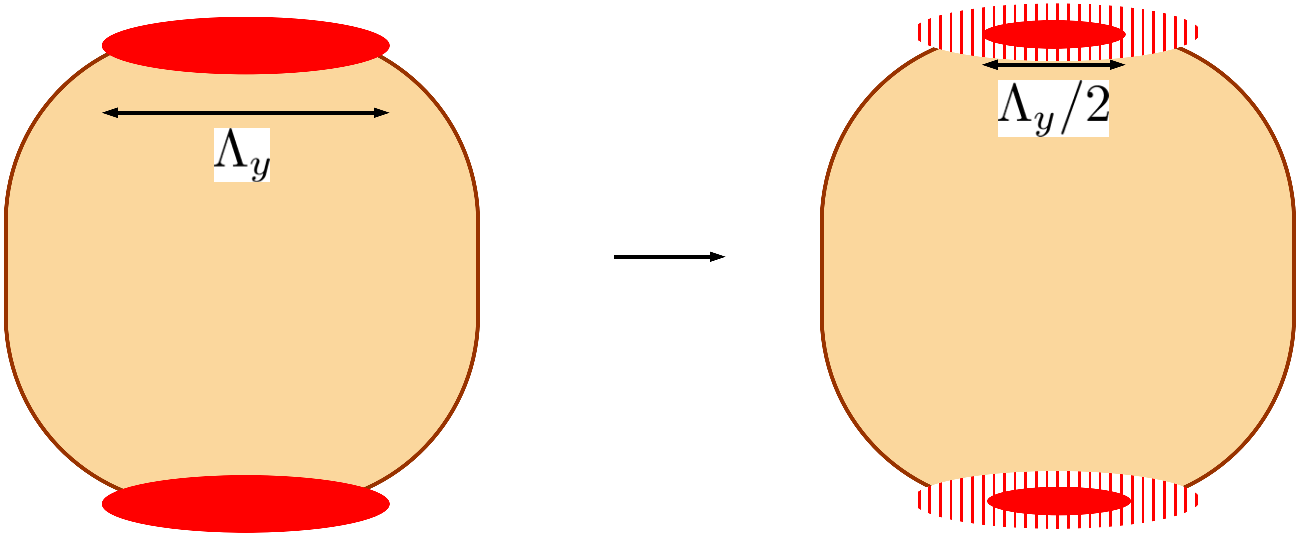


Conceptual difficulties with two-patch RG

- $\delta L = g f_+^\dagger f_-^\dagger f_+ f_-$

$$\sim g\left(1 \pm \frac{\alpha}{N} \log^2 \frac{\Lambda}{\omega}\right)$$

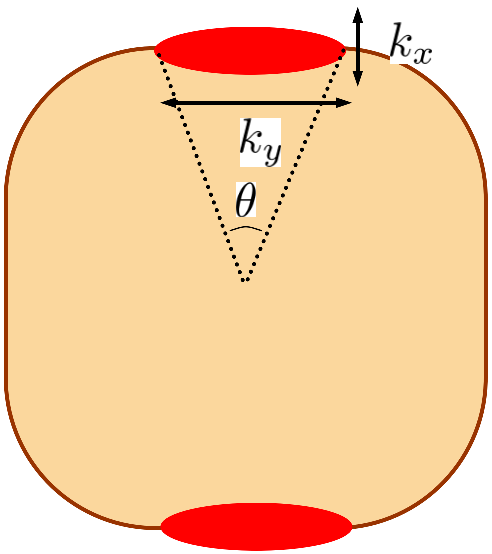
- Low-energy states on the Fermi-surface cannot be integrated out



Conceptual difficulties with two-patch RG

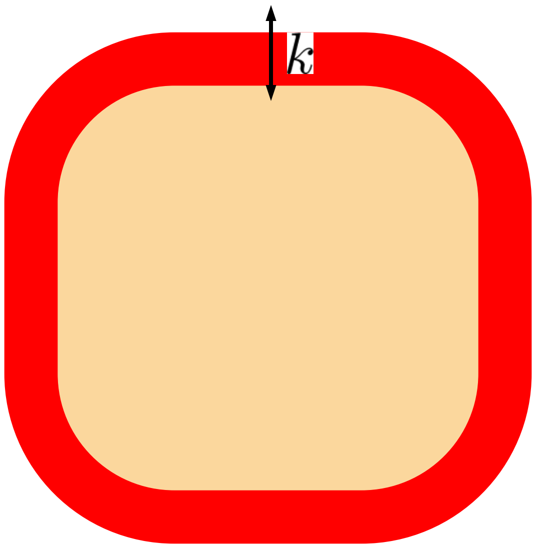
- Treatment of the pairing instability requires a marriage of two RG's:

$$k_y \to s k_y, \quad k_x \to s^2 k_x$$



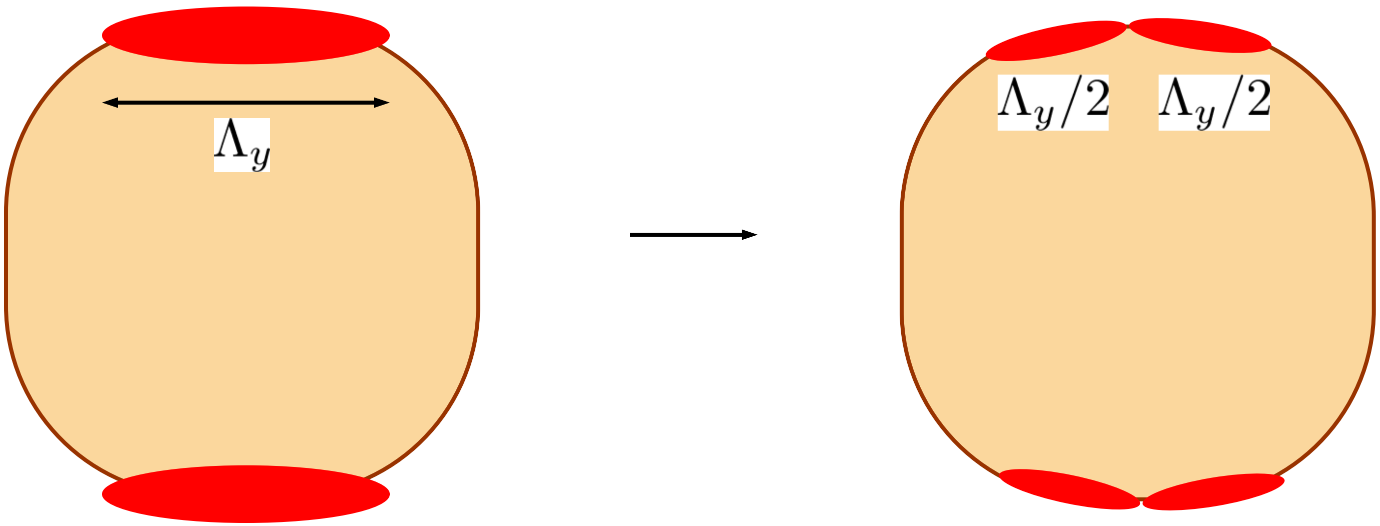
$$\theta \to s\theta$$

$$k \to sk$$



θ does not flow

Son's RG procedure



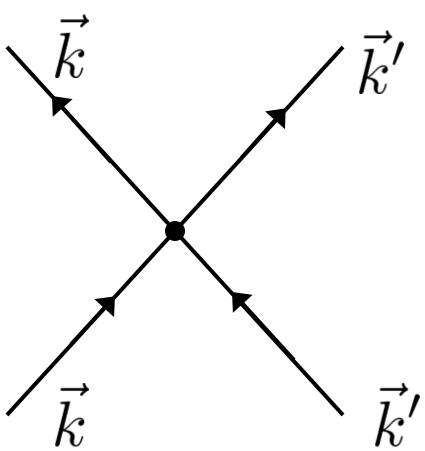
- Keep interpatch couplings!

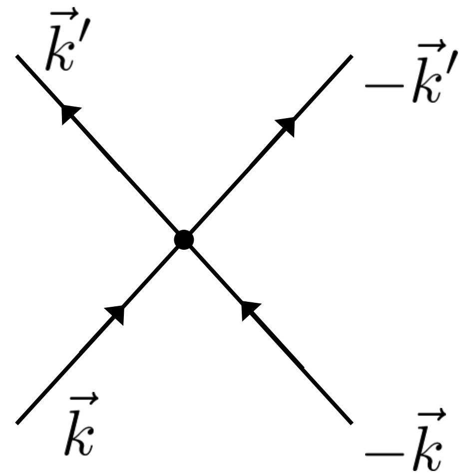
D. T. Son, Phys. Rev. D 59, 094019 (1999).

Perturbations

$$S_4 = -\frac{1}{4}\int \prod_{i=1}^{4} \frac{d^3 k_i}{(2\pi)^3} U_{\alpha\beta;\gamma\delta}(\hat{k}_1, \hat{k}_2; \hat{k}_3, \hat{k}_4)\psi^\dagger_\alpha(k_1)\psi^\dagger_\beta(k_2)\psi_\gamma(k_3)\psi_\delta(k_4)$$
$$\times(2\pi)^3\delta^3(k_1 + k_2 - k_3 - k_4)$$

- Only two types of momentum conserving processes keep fermions on the FS

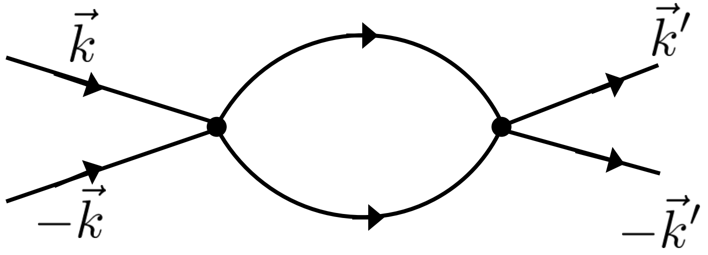




Forward-scattering $F^{s,a}(\vec{k}', \vec{k})$

BCS scattering $V^{s,a}(\vec{k}', \vec{k})$

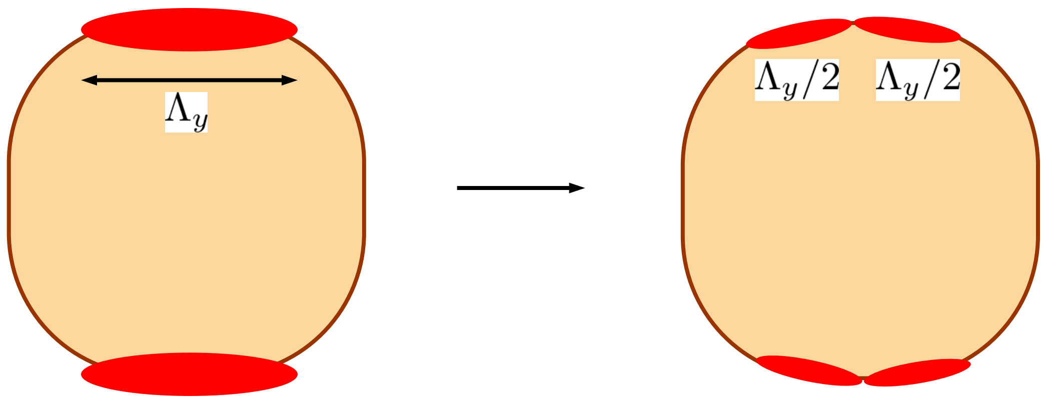
Fermi-liquid RG



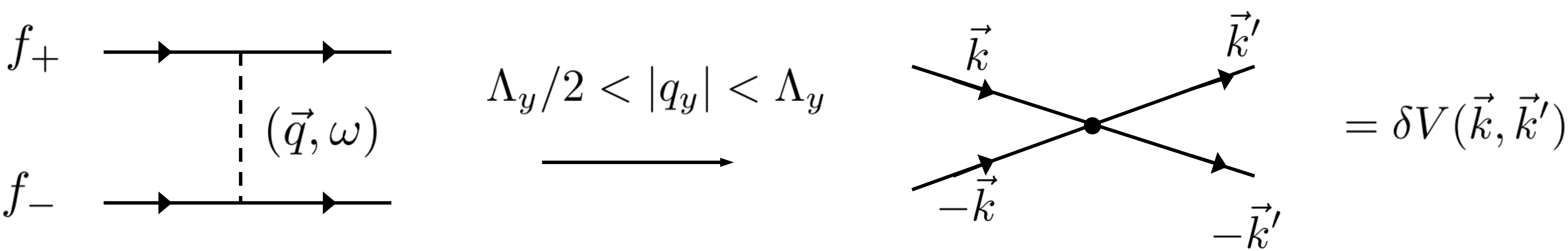
$$V_m^{s,a} = \int \frac{d\theta}{2\pi} V^{s,a}(\theta) e^{-im\theta}$$

$$\frac{dV_m^{s,a}}{d\ell} = -(V_m^{s,a})^2$$

Son's RG



- Generation of inter-patch couplings:



- Generates an RG flow: $$\frac{dV_m^{s,a}}{d\ell} = \frac{1}{N}\alpha$$

Combined RG

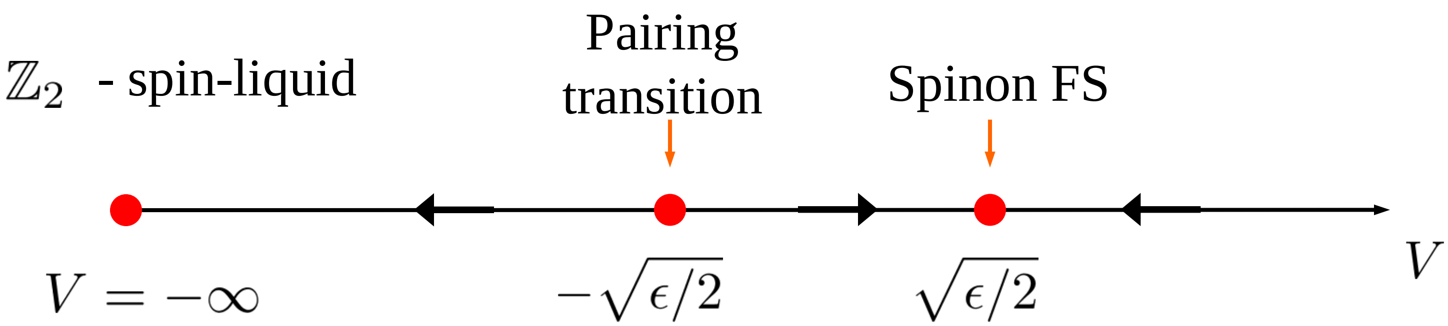
$$\frac{dV_m^{s,a}}{d\ell} = \frac{1}{N}\alpha - (V_m^{s,a})^2$$

$$\frac{d\alpha}{d\ell} = \frac{\epsilon}{2}\alpha - \frac{1}{N}\alpha^2$$

- (from intra-patch theory)

Pairing of the spinon Fermi-surface

$$\frac{dV_m^{s,a}}{d\ell} = \frac{1}{N}\alpha - (V_m^{s,a})^2 \qquad \frac{d\alpha}{d\ell} = \frac{\epsilon}{2}\alpha - \frac{1}{N}\alpha^2 \qquad \alpha_* = \frac{N\epsilon}{2}$$

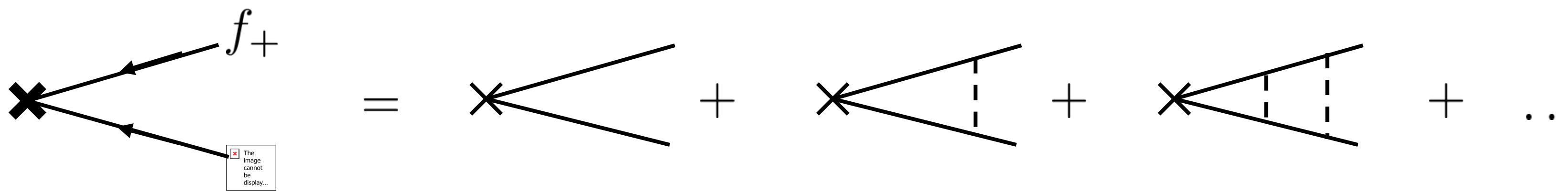


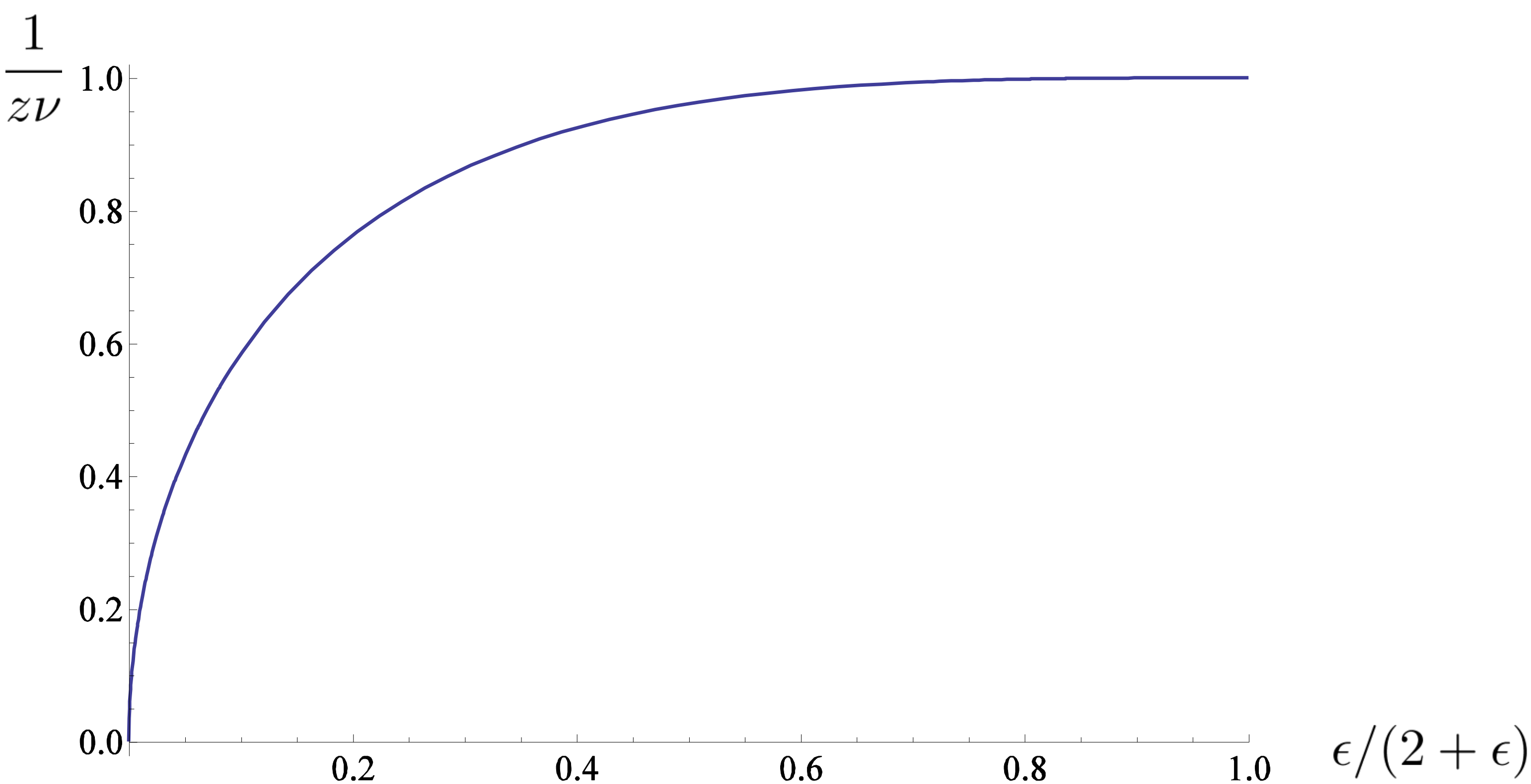
- Need a finite attraction to drive the pairing instability.
- Gap onsets as $\Delta = (V_c - V)^{z\nu}, \quad (z\nu)^{-1} = \sqrt{2\epsilon}$

M.M., D. Mross, S. Sachdev, T. Senthil, forthcoming

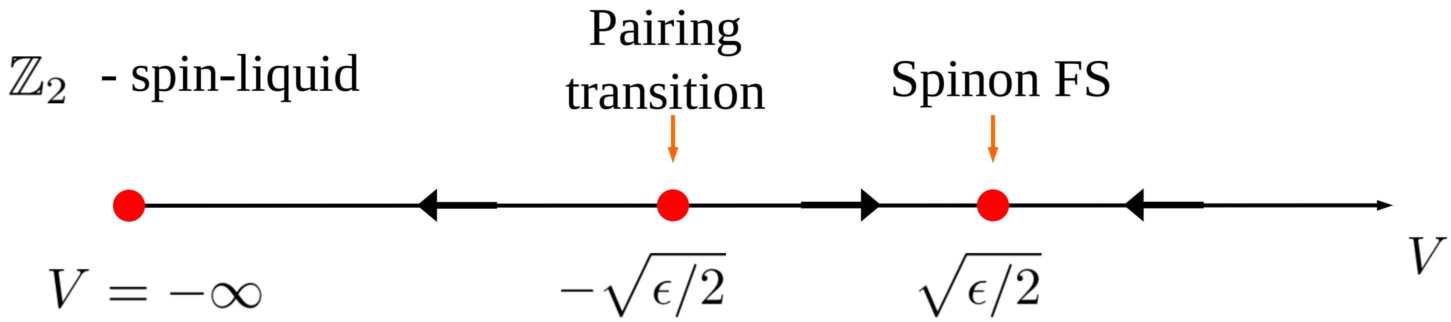
Eliashberg approximation

- All results for $\epsilon \ll 1$ can be reproduced by summing rainbow graphs in the Eliashberg approximation.





Properties of the paired state near transition



- is the "superconductor" type I or type II close to the transiton?

Likely type I: $\xi \sim \lambda^2 \gg \lambda$

- Motrunich effect $H = -\lambda h_{ext}(\partial_x a_y - \partial_y a_x)$ O.I.Motrunich (2006)

- Abrikosov lattice of visons unstable.

- how does the vortex mass (vison gap) vanish at the pairing transition?

$E_v \gg \Delta_f$

Conclusion

- Progress (and new challenges) in understanding spinon Fermi-surface state and its relatives.
- First theory of pairing transition out of the spinon Fermi-surface state.
- Lots of open questions

Thank you!