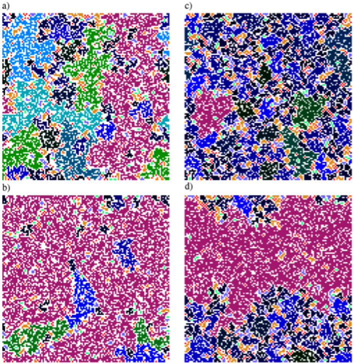
Extended magnetic degrees of freedom

mpipks

Roderich Moessner



PRL 107, 177202 (2011); 109, 096404 (2012); 109, 147204 (2012); PRB 85, 054425 (2012)

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Interesting directions in frustrated magnetism

Spin liquids and disorder

- disorder as fact of life
- disorder as diagnostics
- new disorder phases

'Itinerant frustration'

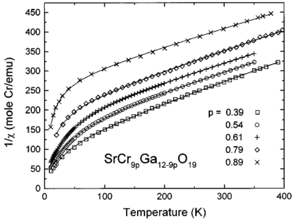
- AHE, Weyl metals, ...

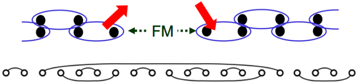
Material realisations

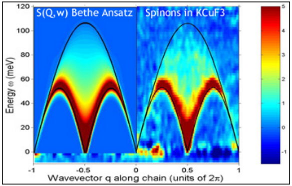
- long-ongoing search ...

Method+model development

- new phases and d.o.f.
- soluble models in $d \geq 2$
- 2d DMRG







How gauge fields can emerge (K)ITP activities for many years

High-energy d.o.f.: spins

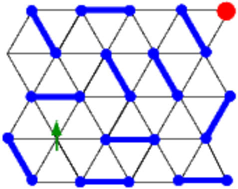
+ constraint due to interactions

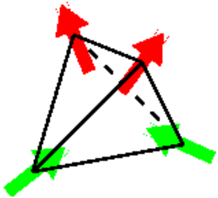
Origin of 'hardcore constraint'

- quantum dynamics of singlet formation
- local energetics: ice rule
- slave-particle treatment Max's tutorial

⇒ Low-energy d.o.f.: gauge field

- local (but can be redundant)
- fluxes/charges: can be 'physical' d.o.f
  - spin-charge separation; visons; ...
- often unstable





Genesis of new degrees of freedom

Fractionalisation

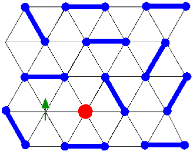
- electrons→holons+spinons
- electrons→Laughlin q.p.
- magnetic moments→ monopoles
- ions→fract. charge

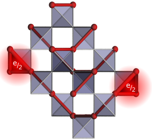
More prosaic: phase transitions

- phonons, magnons,...
- $D-1$ dimensional domain walls

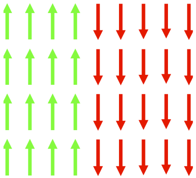
Other extended d.o.f.?

- polymers, cosmic strings, ...
- in magnetism?





Fulde etal.



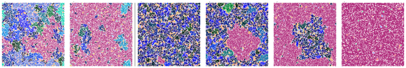
Outline: magnets as loop models

Loops and strings/worms in the Coulomb phase

- Ising and bionic Potts Coulomb phases
- Coupling to itinerant electrons
- Kasteleyn transition, simulations and log-corrections

RVB physics and the loop soup

- Neel order coexisting with dipolar bond correlations



Flat band ferromagnets

- as Pauli-correlated percolation

Loops and worms in the ice/six-vertex model

Corner-sharing square/tetrahedra

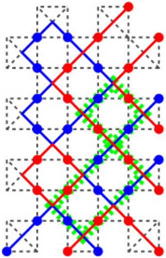
- Ising spins as basic d.o.f.

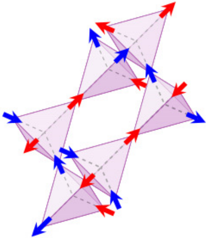
Each square/tetrahedral unit

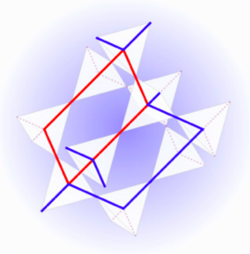
- two up/two down spins
- realises six-vertex model

Two red and two blue sites each

- worms = alternating red/blue
  - emergent gauge flux = spins!
- adjacent red (blue) spins form red (blue) loops
  - fully-packed two-color loop model Kondev+Henley







Statistics of loops

Algebraic length distribution

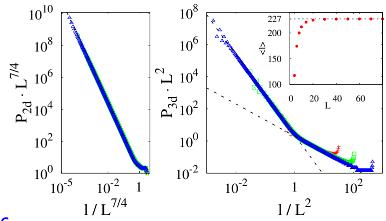
- but finite average loop length (24 Jacobsen vs. 227)
- 2d vs. 3d qualitatively different

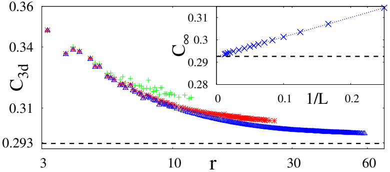
Specific properties of 3d

- two populations: (non-)winding
- finite fraction of sites on each of longest loop
- 6% of spins on non-w. loops

Different effective descriptions

- 2d critical percolation
- 3d Brownian motion
  - topological phase!

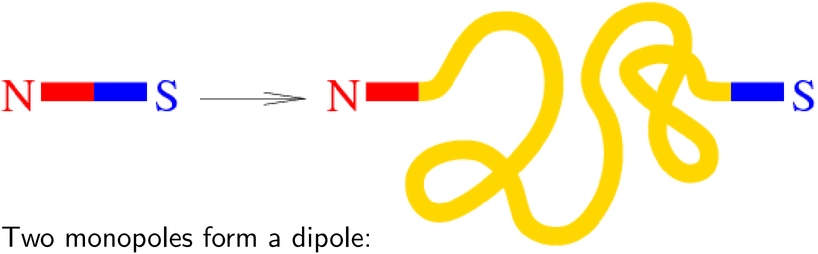




Intuitive picture for monopoles

Simplest picture does not work: disconnect monopoles

Next best thing: no string tension between monopoles:



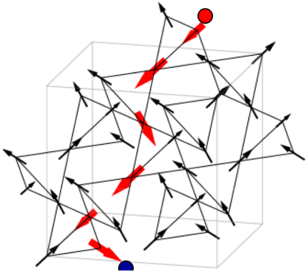
Two monopoles form a dipole:

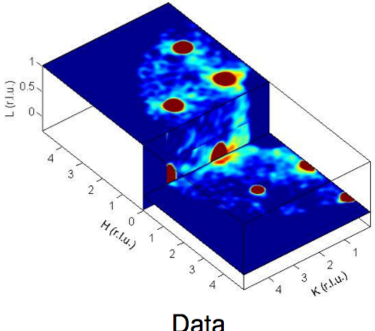
- connected by tensionless ‘Dirac string’
- Dirac string <u>is</u> observable

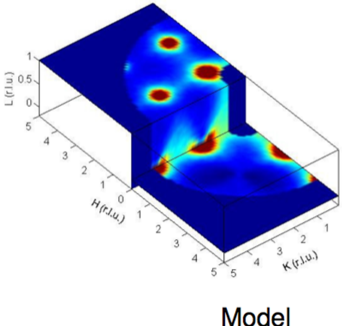
Signatures of loops – ‘Dirac strings’ HMI collaboration

Neutron scattering in spin ice

- saturate spins with field
- Kasteleyn transition
  - flipped spins occur in strings/loops
  - loops execute 'random walk'



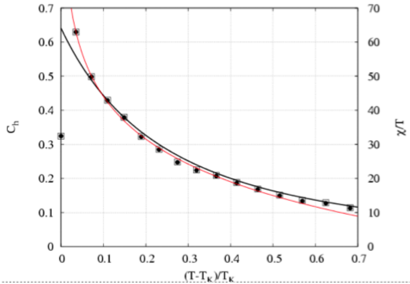




Use for numerical simulations Newman+Barkema; Gingras et al; Isakov et al; . . .

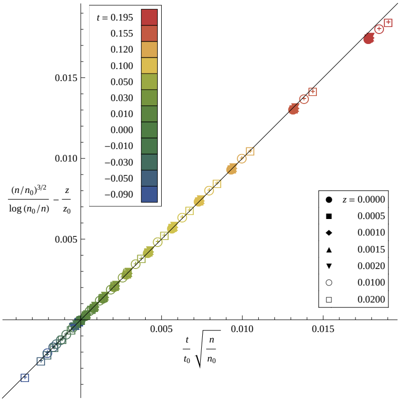
Algorithm flips worms – weighted by length of worm

- in $d = 3$, each MC move flips finite fraction of sample
- can simulate unconventional phase transition very accurately
  - log-corrections at upper critical dim. of Kasteleyn transition



$$\frac{t}{t_0}\left(\frac{n}{n_0}\right)^{1/2} = \frac{1}{\ln(n_0/n)}\left(\frac{n}{n_0}\right)^{3/2} - \frac{z}{z_0}$$

Powell, unpub (2012)



Itinerant electrons–double exchange

Consider (double-exchange) model: electrons hop only along loops

- zero-point energy favors short loops
- transition to short-loop phase cf Nahum+Chalker as density increases

  ⇒ L. Jaubert

Signature of loops – 'glassy order'

Heisenberg magnet (pyrochlore $CsNiCrF_6$)

- Cr/Ni coloured red/blue
- species ice rule Banks+Bramwell

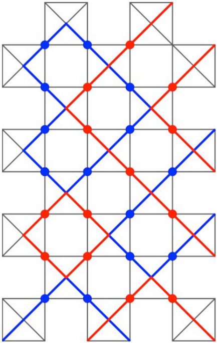
Cr-Cr and Ni-Ni exchange larger than Cr-Ni

- dimensional reduction: decoupled ordered 1D loops

Finite prob. to be on same loop

⇒ fractal magnetic LRO

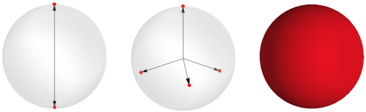
- fate of state under fluctuations?



Pyrochlore Potts afm: a bionic Coulomb liquid

From Ising to Heisenberg via Potts

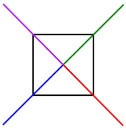
- Potts still discrete but spans 3D
- more components (e.g. from spin-orbital?)
- Potts ice rule gives 4-colouring model



(a) (b) (c)

Potts AFM: unit contain all colors

- 4-colouring model on square/diamond
- dipolar algebraic correlations



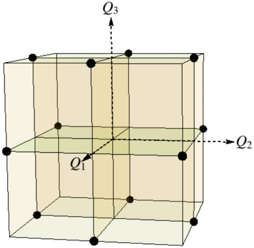
Effective field theory

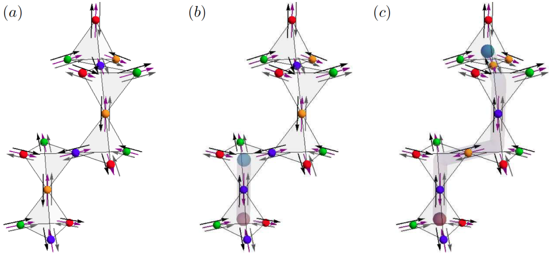
- Three independent emergent gauge fields cf. Chern+Wu

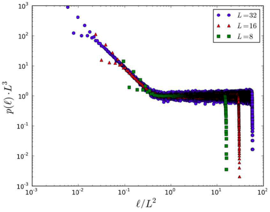
Bionic excitations

Defects charged under two gauge fields

- Potts nature of spins shows up in matter content
  - Heisenberg: charges independent, no longer quantised
- Corresponding worms/Dirac strings have same statistics as in spin ice







Néel and dipolar correlations in RVB

Resonating valence bond wavefunctions

- parent of superconducting state? PWA
- singlet-dominated phase

Encodes magnetic correlations

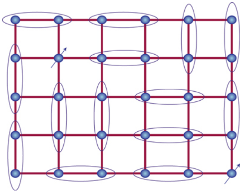
- on square lattice, long(short)-range RVB have (no) Néel order Liang etal

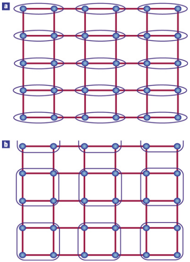
Nature of bond (energy) correlations?

- proximity to valence-bond solid in 2D
- what happens on 3D cubic lattice?

Consider RVB wave function of n.n. dimer coverings, $|c\rangle$ Rokhsar+Kivelson

$$|\Psi\rangle = N_c^{-1/2} \sum_c |c\rangle$$





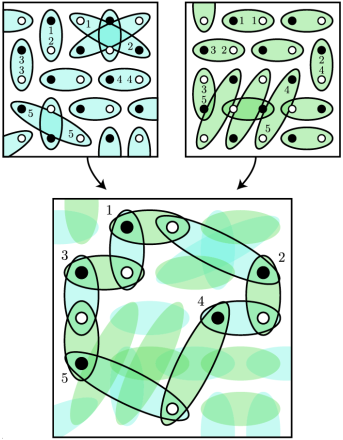
Sachdev

Correlations from RVB wavefunctions Sutherland; Beach, Sandvik

$\langle S_i \cdot S_j \rangle = N_c^{-1} \sum_{c,d} \langle d | S_i \cdot S_j | c \rangle$

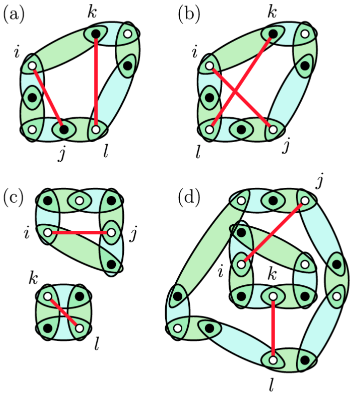
- contribution if i,j on same loop

⇒ properties of loop soup?



Bond correlators

- contributions more complex



Results for cubic n.n. RVB

Loop soup has two populations

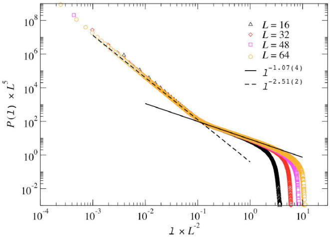
- long loops give rise to Neel order

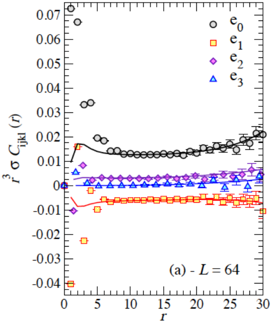
Bond correlators have algebraic dipolar form

- different power law from conventional Néel state

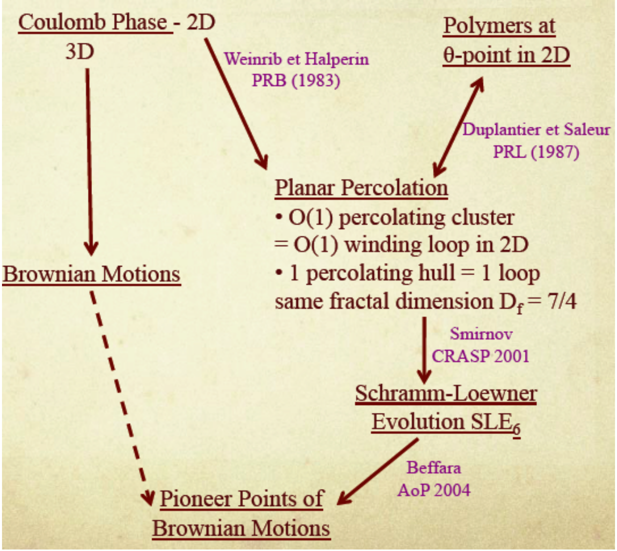
Field theory: two emergent gauge fields

- Néel order can disappear independently





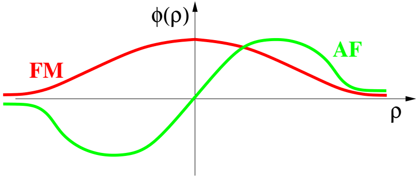
Connections to physical/mathematical problems



Origin of pairwise magnetic interactions

Ferromagnetism

Pauli principle



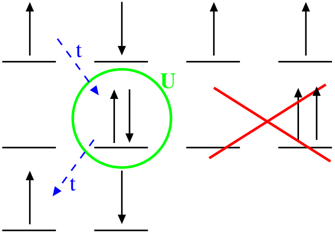
antisymmetric real space wavefctn:

$\Psi(r_1, r_2) = \Phi(R)\phi(\rho = r_1 - r_2)$

- suppresses Coulomb repulsion
- node: kinetic energy cost

Anderson superexchange

uncertainty principle



$$H = -t \sum c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

- $J \propto -|t|^2/U$
- kinetic energy gain

Removing kinetic energy: flat bands

How to get rid of kinetic energy?

- need constant kinetic energy (may be nonzero)

$\Rightarrow$ cf. degeneracy of Hund's rule

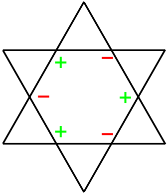
In lattice, need ∞ effective mass

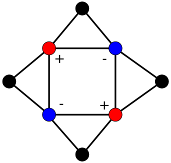
- $E(k) = (\hbar k)^2/2m^* \to 0$

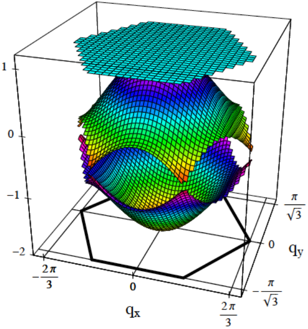
$\Rightarrow$ flat (dispersionless) band

$\Leftrightarrow$ 'local modes'

$\Rightarrow$ frustrated lattices







Flat-band ferromagnetism

Hubbard model:

$$H = -t \sum c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

- kinetic + interaction energy

Single electron states

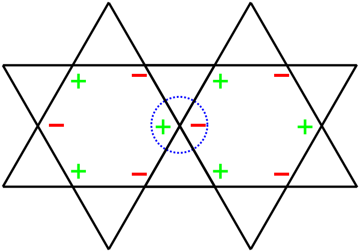
- kinetic energy constant in flat band

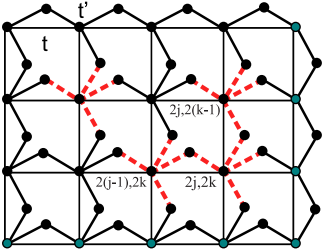
Many-body states

- on-site repulsion invisible to ferromagnetic state

Model: (decorated) Tasaki lattice

- simple orthogonal basis

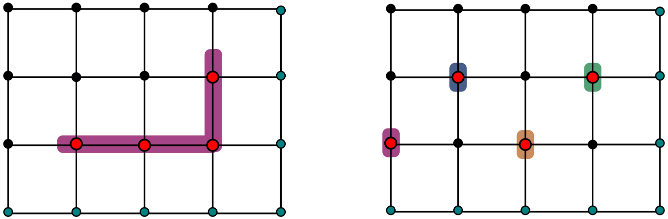




Quantum-statistical effective interaction

Electrons with overlapping wavefunctions have aligned spins

- cluster wavefunction $L+1$-fold degenerate



$L = 4,\ W = L + 1 = 5$ $\quad$ $L = 1,\ W = (1+1)^4 = 16$

Nature of statistical interaction

- geometric
- range-free but bounded by $\ln 2$ per particle
- multi-particle interaction: not two-body decomposable
- 'entropic interaction': finite weight ratios at $T = 0$

Ferromagnetic transition as percolation problem

Ferromagnetic transition: need infinite cluster

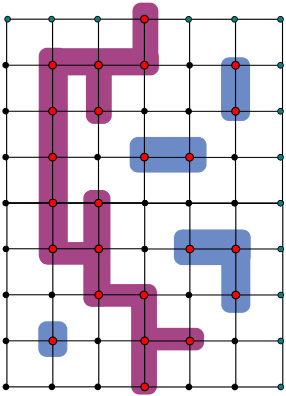
- otherwise no symmetry breaking

Compare percolation

- emergence of spanning cluster with $O(N)$ sites Mielke+Tasaki
- non-trivial weights: Pauli-correlated percolation

Complexity reduction

- quantum many-body problem $\rightarrow$ classical combinatorics

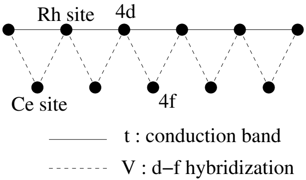


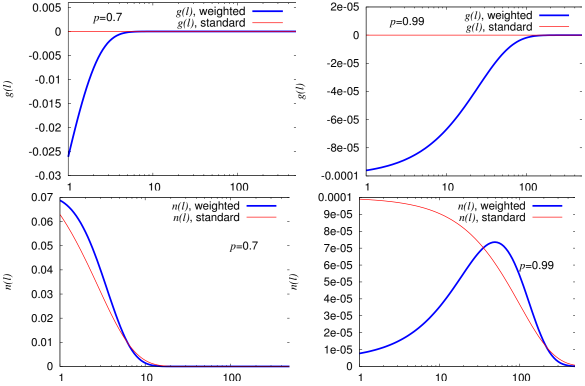
Exact solution in $d = 1$

Continuous transition at density $n = 1$

- new structure compared to standard percolation

Weird lattices exist in reality!





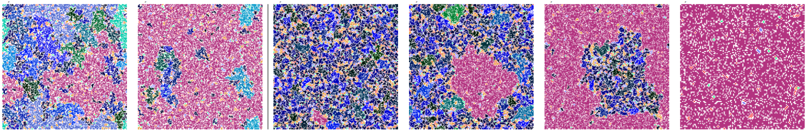
Higher dimension: efficient simulations

Simulations much harder than standard percolation

- but much easier than quantum ones

Percolative transition to ferromagnetism

- transition around $n \approx 0.65$
- first-order transition: density jumps with chemical potential
  - for canonical ensemble, phase separation is evident

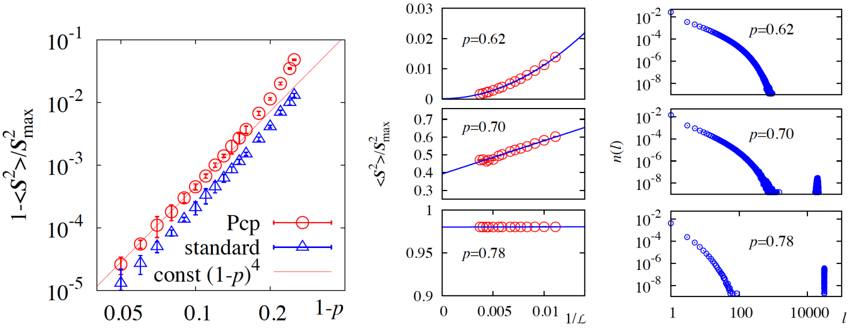


Moment size and cluster distribution

Maximal moment size only at $p = 1$ (full coverage)

- near full coverage, $\Delta S \propto (1-p)^4$
- below percolation: no moment
- magnetic structure of coexistence regime unkonwn!

Cluster-size distribution never algebraic (first order)



Summary: magnetic extended d.o.f.

One-dimensional strings/loops/worms

- Spin ice (loops/worms)
  - fractal order; Dirac strings in neutron scattering; efficient simulations; ...
- Potts ice
  - bionic charges and strings
- loops in RVB physics
  - long-range magnetic order independent of dipolar bond order

Percolative transition for flat bands

- Pauli correlated percolation with quantum statistical weight
  - first-order transition; unsaturated magnetism; large-scale simulations; coexistence properties?