

Quantum spin ice

Shigeki Onoda

Condensed Matter Theory Lab., RIKEN

SO-Tanaka, PRL **105**, 047201 (2010), PRB **83**, 094411 (2011).

SO, J. Phys.: Conf. Series **320**, 012065 (2011).

L.-J. Chang, SO, Y. Su et al. Nature Comm. 3:992 (2012).

S. Lee, SO, L. Balents, PRB, to be published (arXiv:1204.2262).

Some unpublished works.

Thanks to

Theory:

Y. Tanaka (RIKEN)	Superexchange int.
L. Balents, S. Lee (KITP, UCSB)	Gauge theory
Y.-J. Kao (Natl. Taiwan Univ.)	Fitting with neutron exp.

Experiments on Yb pyrochlore:

L.-J. Chang (Natl. Cheng Kung Univ.)	Neutron exp.
Y. Su (Julich Centre for Neutron Science)	Neutron exp.
Y. Yasui (Nagoya Univ),	Sample
K. Kakurai (JAEA)	Neutron exp.
M. R. Lees (Univ. of Warwick)	Specific heat
K.-D. Tsuei (NSSRC, Hsinchu)	EXAFS

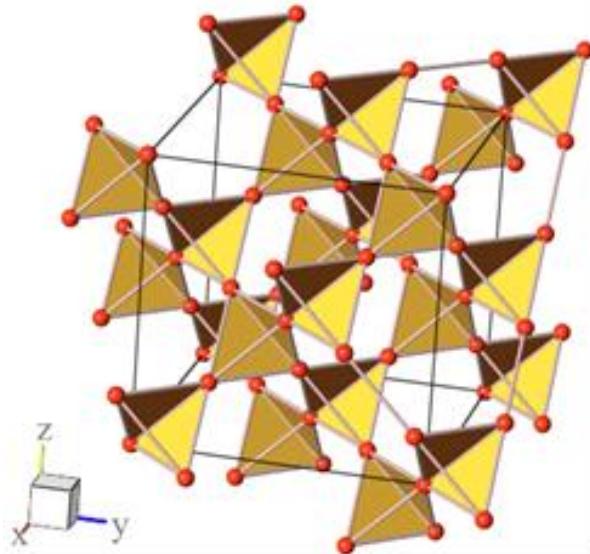
Outline

- Introduction
 - Spin ice: emergent magnetic monopoles
 - Simple quantum effects on monopoles: U(1) QSL
 - Indication of quantum spin ice in materials
- Generic quantum spin ice model
 - Nontrivial superexchange interaction between local doublets of f-electrons for Pr, Yb, (Nd, Er)
- Gauge theory
 - Analogous QED with gapless “photon”
 - A novel excitation
- Experiments on $\text{Yb}_2\text{Ti}_2\text{O}_7$
 - A first-order phase transition via the Higgs mechanism

Spin ice & emergent monopoles

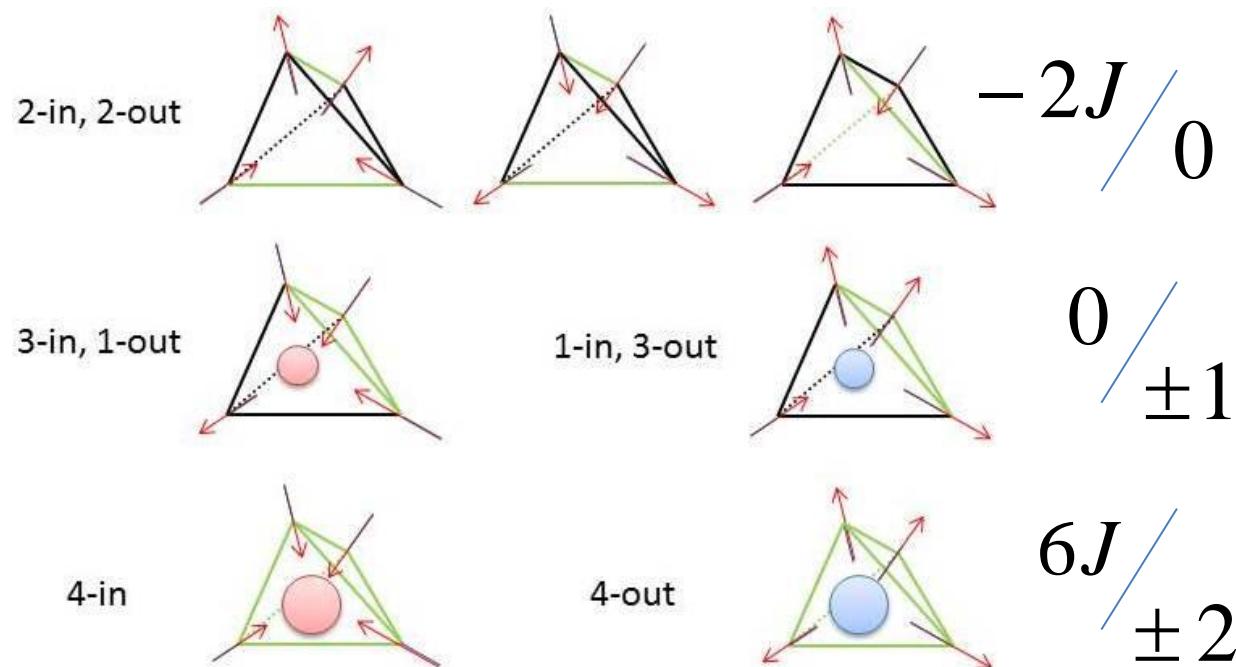
AF Ising model on a pyrochlore lattice

$$H = 4J \sum_{\langle r, r' \rangle}^{n.n.} S_r^z S_{r'}^z \quad S_r^z : \text{Ising } (S=1/2) \text{ spin}$$



Moessner-Sondhi

Energy/Monopole charge



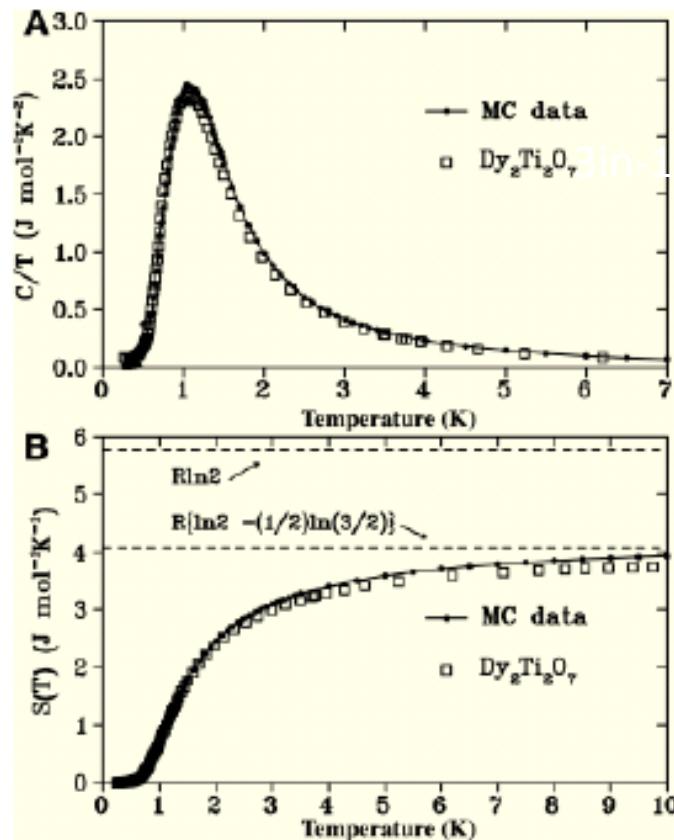
Classical Coulomb-phase physics: divergence-free $\nabla \cdot S^z = 0$

Experiments and numerics on dipolar spin ice

Harris, Ramirez, Bramwell, Sakakibara , Hiroi, Maeno, Gingras

$\text{Dy}_2\text{Ti}_2\text{O}_7$, $\text{Ho}_2\text{Ti}_2\text{O}_7$

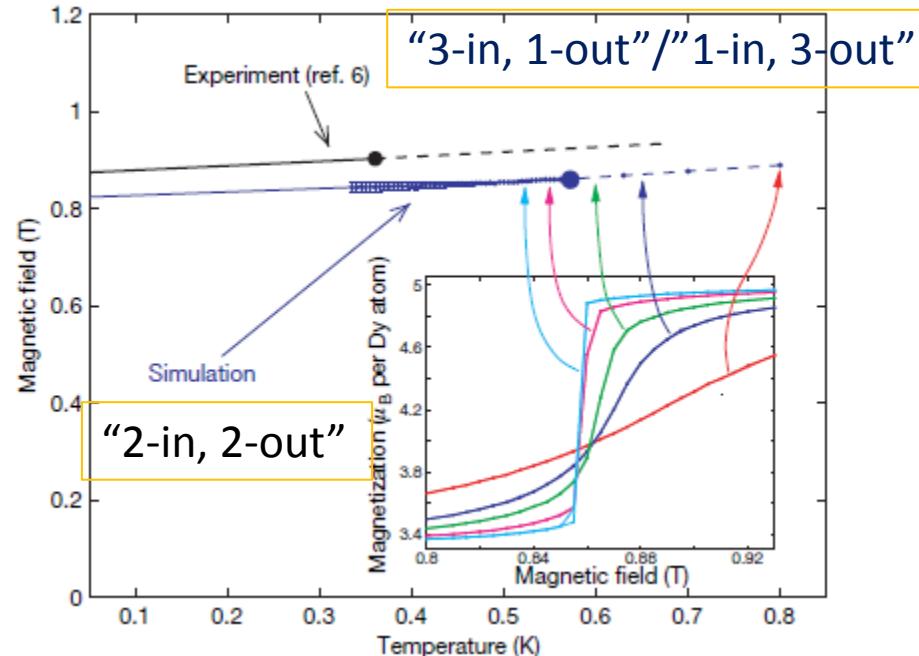
N.N. Ising coupling $J \sim 2.4 \text{ K}$



Gingras

Castelnovo-Moessner-Songhi, Nature 451, 42-45 (2008)

Metamagnetic transition under $H // (111)$
→ liquid-gas phase transition of monopoles



Exp., Sakakibara et al.,
Phys. Rev. Lett. 90, 207205 (2003).

Dipolar spin correlations

- O(N) Heisenberg antiferromagnet

S.V. Isakov, K. Gregor, R. Moessner, S. L. Sondhi,

Phys. Rev. Lett. 93, 167204 (2004).

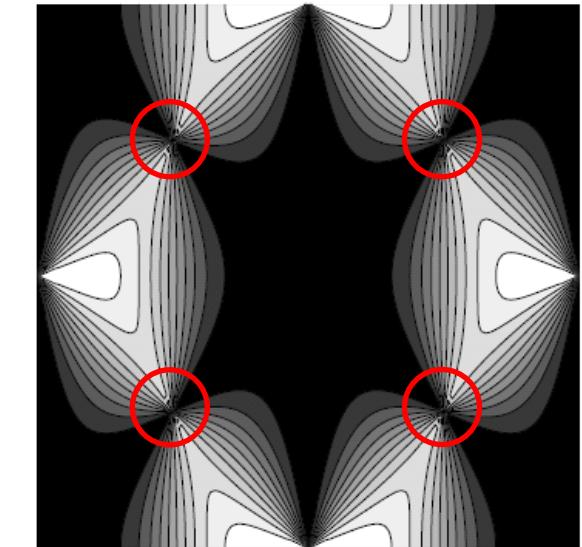
1/N expansion → dipolar spin correlations

$$\mathcal{L}(S, \lambda) = \sum_{i,j} \sum_{\alpha=1}^N \frac{1}{2} S_i^\alpha A_{ij} S_j^\alpha + i \frac{\lambda_i}{2} \delta_{i,j} (S_i^\alpha S_i^\alpha - N),$$

$$2 \begin{pmatrix} 1 & c_{xz} & c_{xy} & c_{yz} \\ c_{xz} & 1 & c_{\bar{yz}} & c_{\bar{xy}} \\ c_{xy} & c_{\bar{yz}} & 1 & c_{\bar{xz}} \\ c_{yz} & c_{\bar{xy}} & c_{\bar{xz}} & 1 \end{pmatrix}$$

$$c_{ab} = \cos\left(\frac{q_a + q_b}{4}\right) \text{ and } c_{\bar{ab}} = \cos\left(\frac{q_a - q_b}{4}\right)$$

Works well for N=1 (Ising) and infinity.



N=∞, 1

cf. pinch-point singularity

C. L. Henley, *Phys. Rev. B* 71 014424 (2005)

divergence-free condition of spin-ice rule

Experiments on dipolar spin ice: Morris et al., Fennell et al.

From classical to quantum spin ice

Assumptions:

- (i) A large amplitude of magnetic moments.

$$\hat{H}_{\text{D}} = \frac{\mu_0}{4\pi} \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \left[\frac{\hat{m}_{\mathbf{r}} \cdot \hat{m}_{\mathbf{r}'}}{(\Delta r)^3} - 3 \frac{(\hat{m}_{\mathbf{r}} \cdot \Delta \mathbf{r})(\Delta \mathbf{r} \cdot \hat{m}_{\mathbf{r}'})}{(\Delta r)^5} \right]$$

- (ii) Spins obey the classical statistics.

$$\hat{H}_{\text{Ising}} = -D_{\text{Ising}} \sum_{\mathbf{r}} (\mathbf{n}_{\mathbf{r}} \cdot \hat{\mathbf{J}}_{\mathbf{r}} / J)^2, \quad \rightarrow \text{This is taken to infinity!}$$

- (iii) Higher-order multipolar interactions are ignored.

$$\hat{H}_{\text{H}} = -3J_{\text{n.n.}} \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle}^{\text{n.n.}} \hat{\mathbf{J}}_{\mathbf{r}} \cdot \hat{\mathbf{J}}_{\mathbf{r}'} / J^2. \quad \leftarrow \begin{array}{l} \text{Ad-hoc Heisenberg exch. Int.} \\ \text{Not so simple} \end{array}$$

With a smaller moment amplitude and/or a larger D_{3d} crystal field, these assumptions do not hold in general.

$\text{Tb}_2\text{TM}_2\text{O}_7, \text{Pr}_2\text{TM}_2\text{O}_7, \text{Yb}_2\text{TM}_2\text{O}_7$ ($\text{TM}=\text{Ti}, \text{Zr}, \text{Sn}, \text{Hf}, \text{Ir}, \dots$)

Spin-flipping interactions: quantum mechanics!

Classical-to-quantum Coulomb-phase physics

- Classical case: particles obeying a Coulombic law

$$H_{cl} \approx \frac{1}{8\pi} \mathbf{E}^2 - \mu \psi^+ \psi + u \psi^+ \psi \psi^+ \psi$$

—————> Coulomb propagator

$$\nabla \cdot \mathbf{E} = g(\psi^+ \psi) \leftarrow \hat{S}^z = \mathbf{E} \cdot \mathbf{z}$$

ψ^+, ψ : **Spinon** operators creating and annihilating the gauge charge

- Quantum case: kinetic energy with gauge field

$$H_{qm} \approx \frac{1}{8\pi} (\mathbf{E}^2 + \mathbf{B}^2) + \frac{1}{2m} \psi^+ (-i\hbar \nabla + g\mathbf{A})^2 \psi - \mu \psi^+ \psi + u \psi^+ \psi \psi^+ \psi$$

$$\nabla \cdot \mathbf{E} = g(\psi^+ \psi) \leftarrow \hat{S}^z = \mathbf{E} \cdot \mathbf{z}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \leftarrow \hat{S}_r^\pm = \psi_{r\pm d}^+ e^{\pm i A_{r+d, r-d}} \psi_{r\mp d}$$

$$[A_{r+d, r-d}, E_{r+d, r-d}] = i$$

Abelian Higgs models:
Savary-Balents
(non-interacting spinons)
S.Lee-S.O.-Balents
(interacting spinons)

Weak quantum effects on spin-ice manifold

A simple quantum pseudospin-1/2 Hamiltonian

Hermelé-Fisher-Balents, PRB 69, 64404

$$\hat{H} = J_{n.n.} \sum_{\langle r,r' \rangle}^{n.n.} [g^{\parallel} \hat{\sigma}_r^z \hat{\sigma}_{r'}^z + g^{\perp} (\hat{\sigma}_r^x \hat{\sigma}_{r'}^x + \hat{\sigma}_r^y \hat{\sigma}_{r'}^y)]$$

1. Assume $J_{n.n.} > 0$, $g^{\parallel} > 0$.
2. Start from degenerate spin-ice ground states

3. 3rd-order perturbation in g^{\perp}

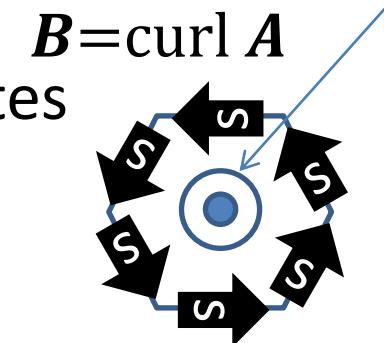
→ π-flux ($g^{\perp} > 0$) or 0-flux ($g^{\perp} < 0$)

→ emergent gauge fields!

→ deconfined bosonic spinons

The model is oversimplified for real materials, though...

Fictitious “magnetic” field



Fictitious QED

Ignore gapped spinon excitations first!

$$\mathcal{H}_p = \frac{U}{2} \sum_{\langle rr' \rangle} e_{rr'}^2 - K \sum_{\circlearrowleft} \cos \left(\sum_{rr' \in \circlearrowleft} a_{rr'} \right)$$

curl

→ Hamiltonian for electric field

Constraints for pseudospin-1/2 ($U \rightarrow \infty$)

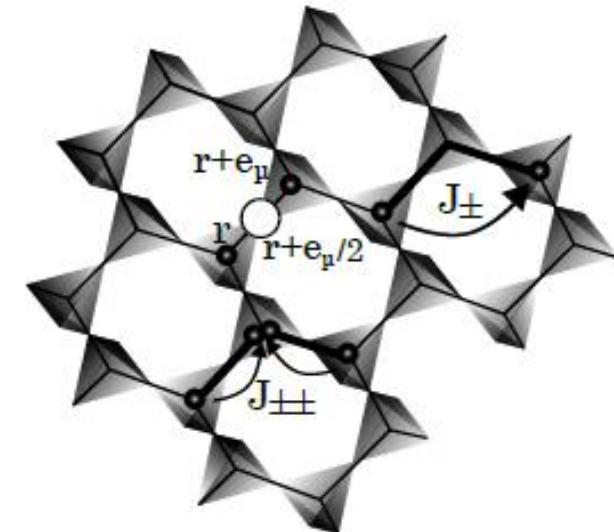
→ Hamiltonian for magnetic field penetrating hexagons

→ Deconfined Coulomb phase of 3+1D QED

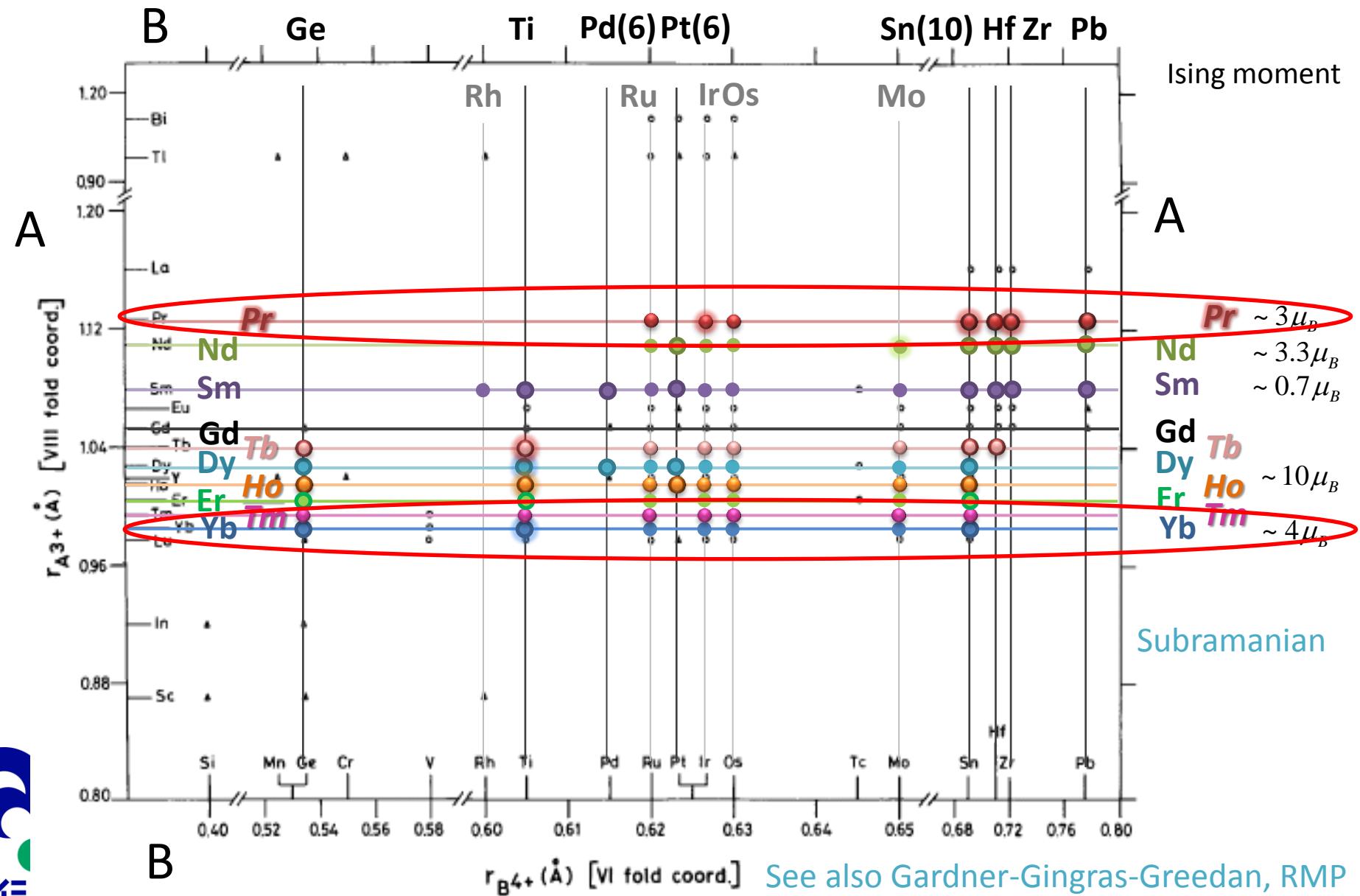
Linearly dispersive gapless “photons” excitations

What is “quantum spin ice”?

- **Deconfined spinons carrying monopole charge**
 - hosted by a frustrated N.N. Ising interaction as in classical spin ice
 - “Gapped” dressed quasiparticles, but not exactly classical monopole defects in spin ice
- **Perturbed by weak quantum-mechanical spin-flip interactions**
 - spinon hopping, spinon-spinon interaction
 - **Yb₂Ti₂O₇** → Higgs transition from a Coulomb liquid to a ferromagnet
Chang, SO et al., Nature Comm. 3:992 (2012)
 - **Pr₂Zr₂O₇** → U(1) spin liquid? [SO, unpublished]
- However, ... spinons are confined by strong Q.M. interaction or unfrustrated Ising interation → not a QSI
 - Er₂Ti₂O₇ ([Ross et al.](#)), Nd₂Zr₂O₇ ([Aldus et al.](#))



Candidate pyrochlore magnets $A_2B_2O_7$



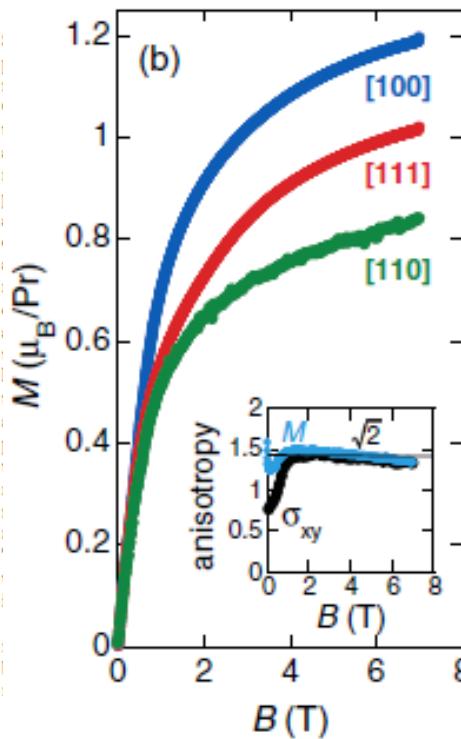
Pr2TM207

Chiral spin (Pr) state in $\text{Pr}_2\text{Ir}_2\text{O}_7$

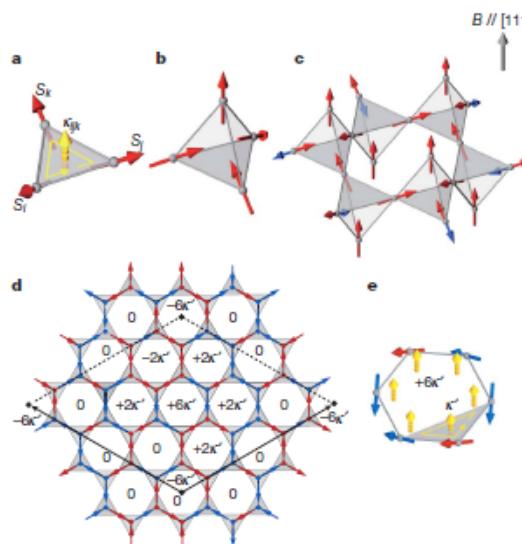
Machida, Nakatsuji, SO et al. Nature 463, 210 (2010)

Time-reversal symmetry breaking and spontaneous Hall effect without magnetic dipole order

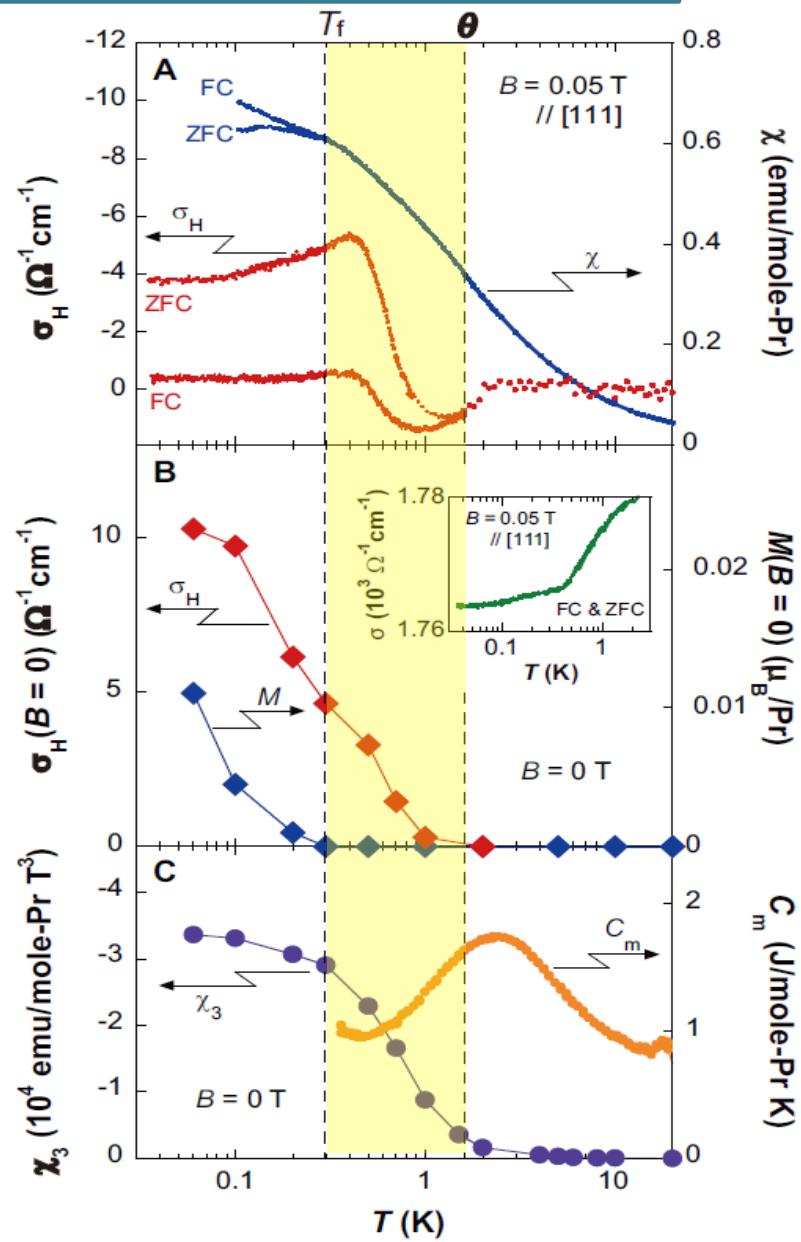
Yo Machida^{1†}, Satoru Nakatsuji¹, Shigeki Onoda², Takashi Tayama^{1†} & Toshiro Sakakibara¹



spins are essentially ordered. However, it is usually difficult to extract the scalar spin chirality κ_{ijk} reliably unless the spins are long-range-ordered⁴. In metallic magnets, on the other hand, a promising probe is available: the anomalous Hall effect (AHE)^{4,11}, which is the spontaneous Hall effect at zero applied magnetic field.



<111> Ising moments of Pr ions

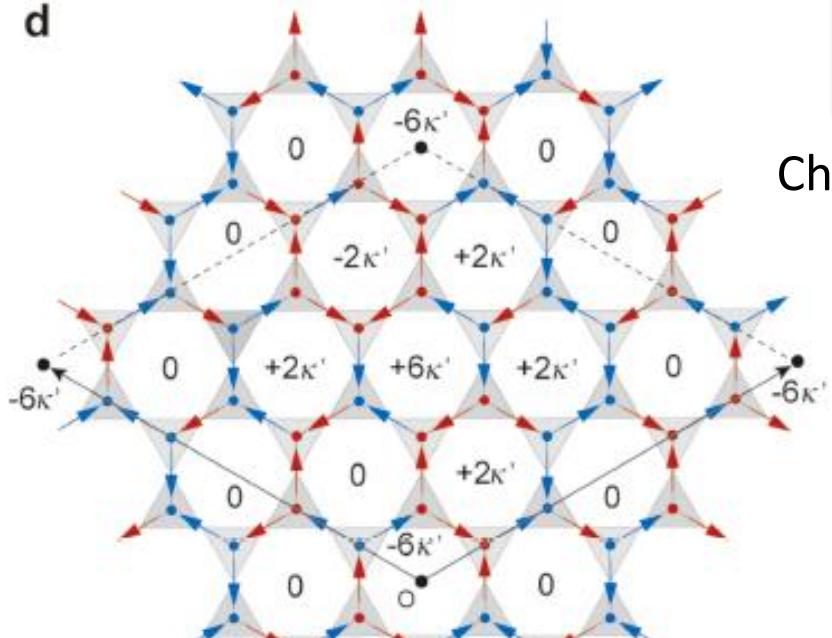


Nonzero uniform chirality with $M \sim 0$ at $H=0$: classical analysis (Pr)

Machida, Nakatsuji, SO et al. Nature 463, 210 (2010)

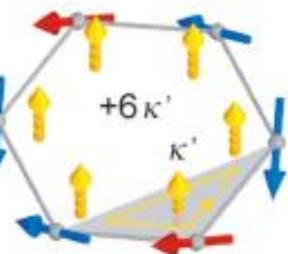
T-broken spin liquid at $T_f < T < \theta$ ($T_f \sim 0.3$ K, $\theta \sim 1.5$ K)

d



2I2O state having $M \sim 0$ but nonzero chirality and thus nonzero AHE

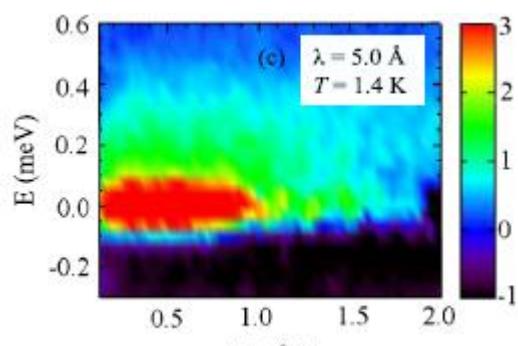
Chiral submanifold of the spin ice manifold



Nonvanishing uniform chirality summed over the hexagon

Quantum-mechanical superposition
However, classical spins are long-range ordered
of such classical chiral states, produced
or frozen, destabilizing a chiral spin-liquid state.
by dynamical generation of monopoles?

Pr₂Sn₂O₇ Zhou et al.

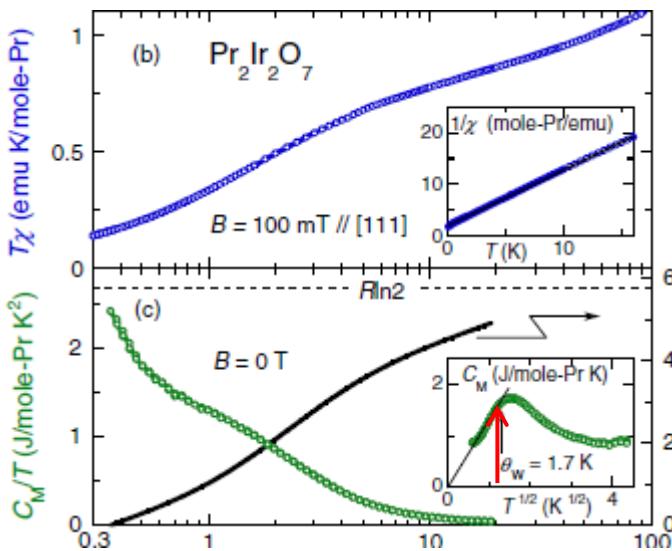


Quantum spin fluctuations

Similar low-T magnetism in $\text{Pr}_2\text{B}_2\text{O}_7$

B=Ir

Metallic



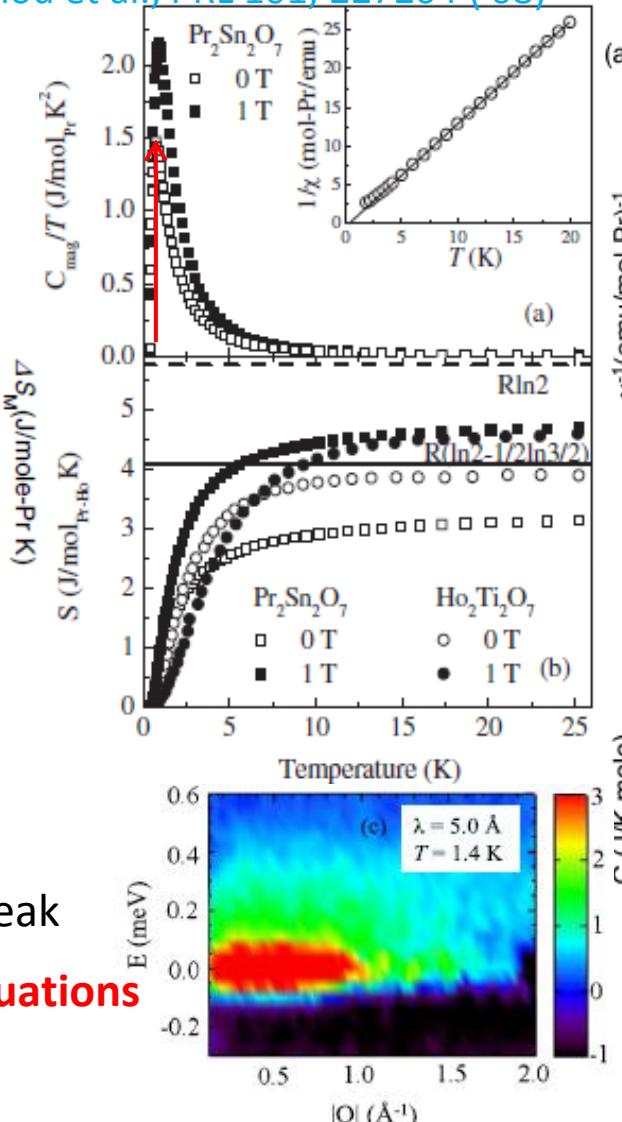
Nakatsuji et al. PRL 96, 087204 ('06)
c.f. Kondo effect at $\sim 20\text{-}40\text{ K}$

No magnetic Bragg peak

Quantum spin fluctuations
 $\sim 0.4\text{ meV}$

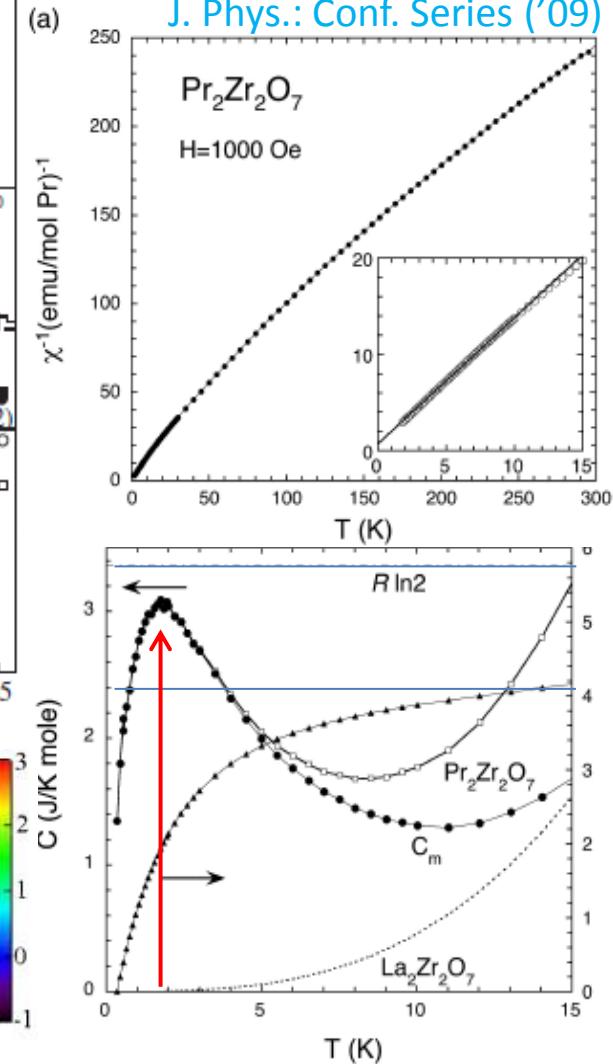
B=Sn (powder)

Zhou et al., PRL 101, 227204 ('08)



Insulating B=Zr

Matsuhira et al.,
J. Phys.: Conf. Series ('09)



$\text{Yb}_2\text{Ti}_2\text{O}_7$

First-order phase transition in Yb₂Ti₂O₇

Hodges et al. 2002

Mossbauer and muon spin relaxation spectroscopies:

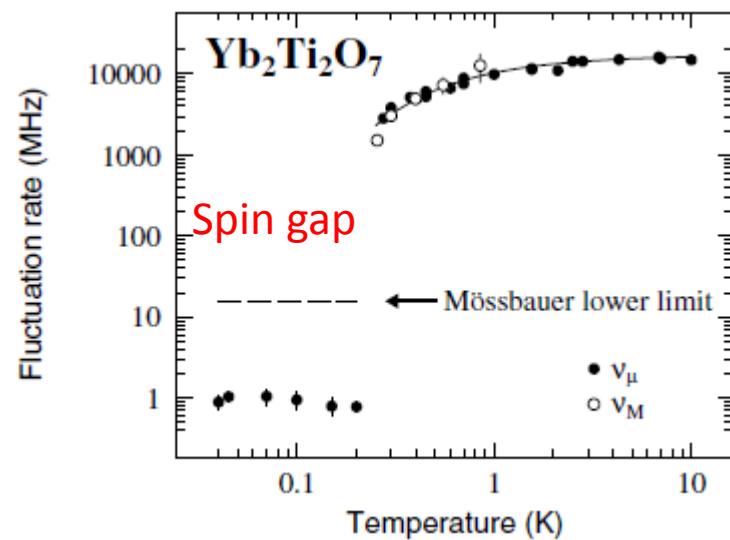
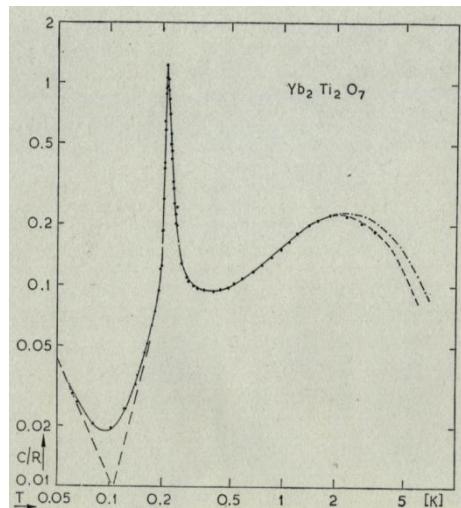
Local Yb ions → J_z=1/2 doublet

$$|\sigma\rangle_D = -\alpha\sigma|J_z = \frac{7}{2}\sigma\rangle + \beta|J_z = \frac{1}{2}\sigma\rangle + \gamma\sigma|J_z = -\frac{5}{2}\sigma\rangle$$

$$\alpha \approx 0.388, \beta \approx 0.889, \text{ and } \gamma \approx 0.242$$

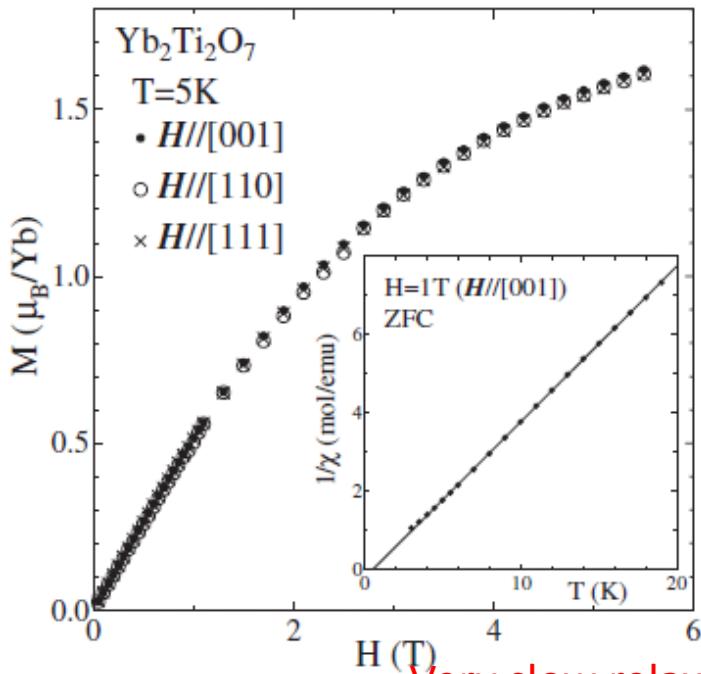
1st-order phase transition @ T ≈ 0.21 K

Blotte et al. 1969

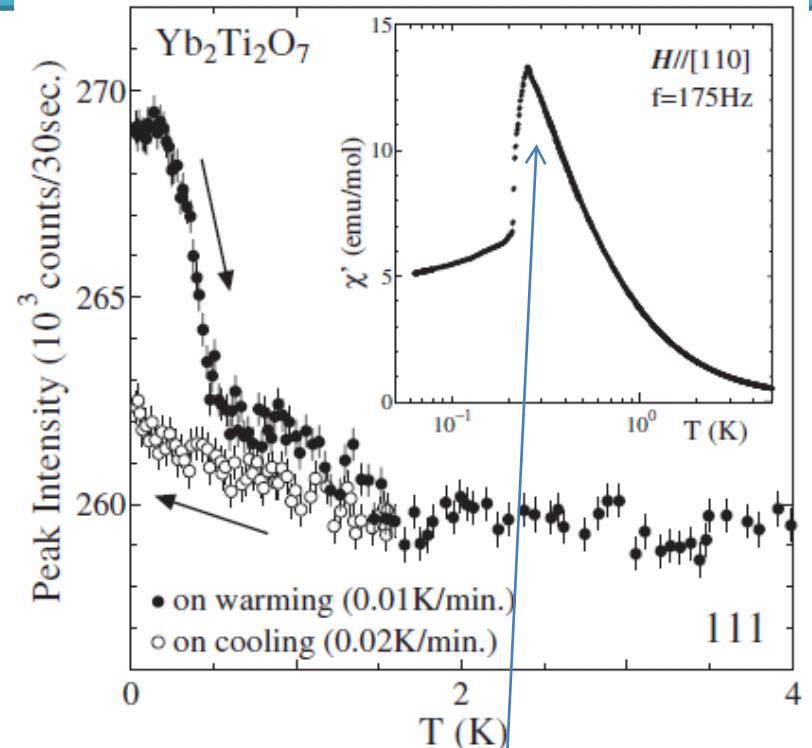


Evidence of the 1st-order phase ferromagnetic transition at ~ 0.21 K

Yasui et al. JPSJ (2003)



Very slow relaxation of magnetization
Magnetic Bragg peak evolves in 2 hours!

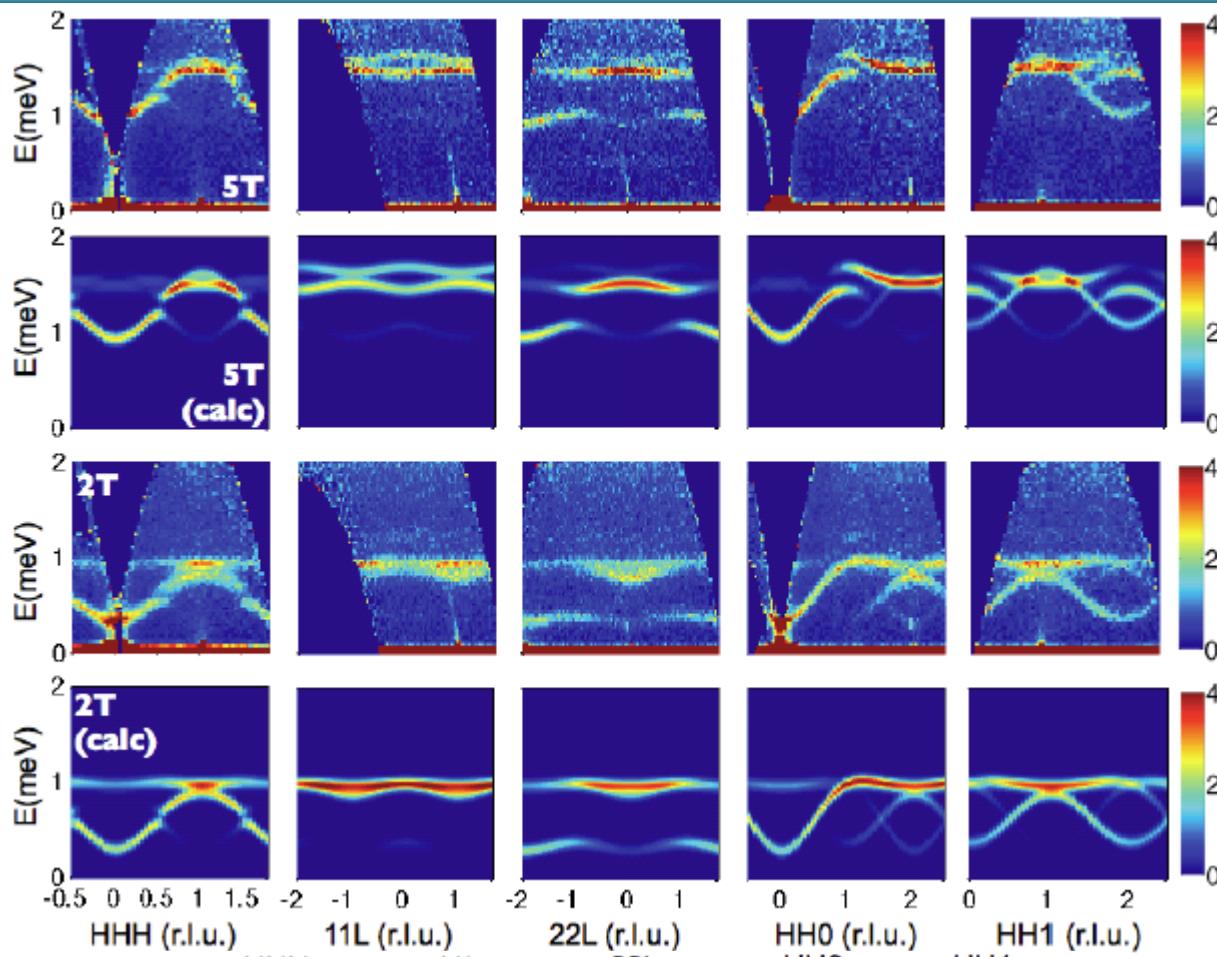


Spin excitations
are gapped.

Related anomaly in the specific heat [Blote et al. 1969]

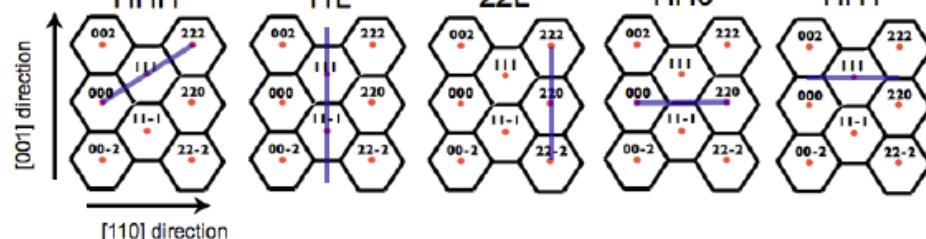
c.f. Sample dependence: the best available sample shows FM, while others does not.
Hodges et al, Thompson et al, Gardner et al, Ross et al

Spin-wave analysis of the high-field state



Ross et al.

$$\begin{aligned}g^{\perp} &\sim -0.6 \\g^q &\sim -0.6 \\g^K &\sim -1.6\end{aligned}$$



c.f.

Thompson-...-Gingras
Applegate-...-Gingras

Specific examples: Derivation of realistic superexchange int.

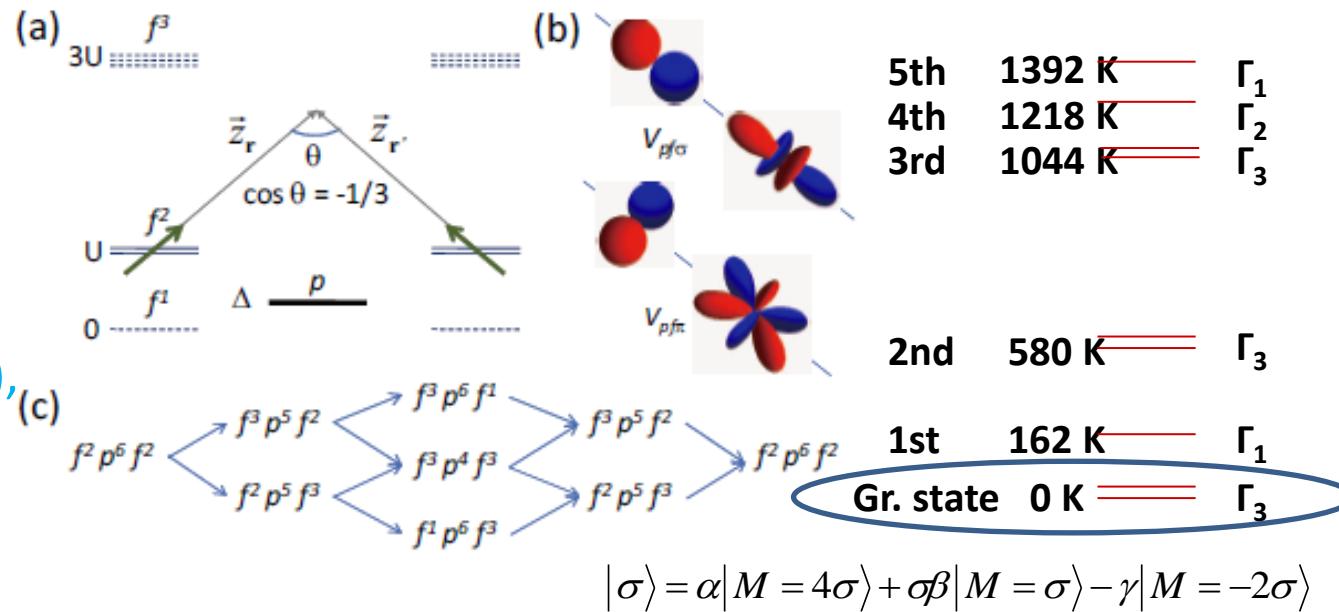
Anderson's superexchange int. → Project onto the gr. doublets

- $\text{Pr}_2\text{TM}_2\text{O}_7$
non-Kramers doublet
(integer-spins)

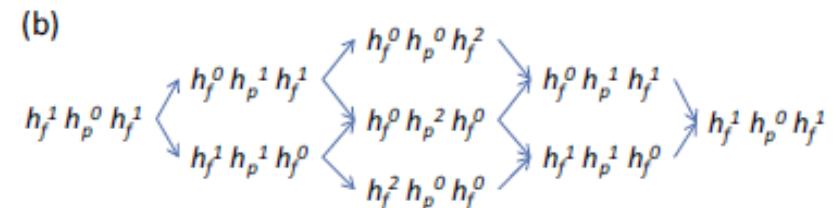
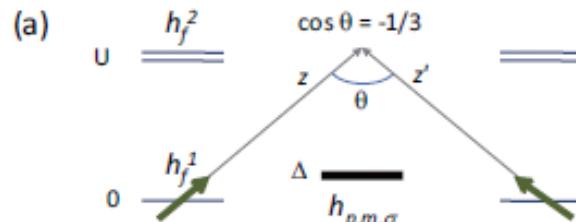
SO-Tanaka,
PRL **105**, 047201 (2010),
PRB **83**, 094411 (2011).

- $\text{Yb}_2\text{TM}_2\text{O}_7$

Kramers doublet (half-integer spins)
SO, J. Phys.: Conf. Series **320**, 012065 (2011)



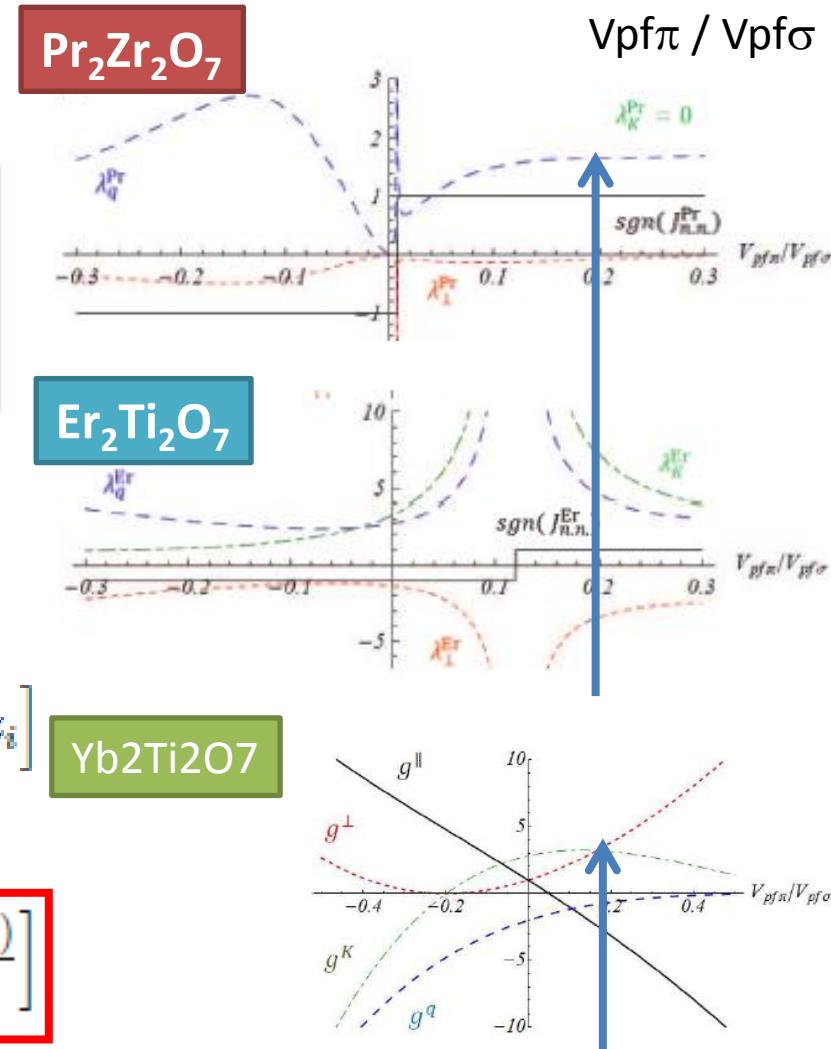
$$|\sigma\rangle_D = -\alpha\sigma|J_z = \frac{7}{2}\sigma\rangle + \beta|J_z = \frac{1}{2}\sigma\rangle + \gamma\sigma|J_z = -\frac{5}{2}\sigma\rangle$$



Effective pseudospin-1/2 model

Anisotropic superexchange interaction
[SO-Tanaka (2009, 2010), SO (2011)]

$$\hat{H}_{\text{SE}}^R = \frac{|J_{n.n.}^R|}{2} \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle}^{\text{n.n.}} \left[\text{sgn}(J_{n.n.}^R) \hat{S}_{\mathbf{r}}^z \hat{S}_{\mathbf{r}'}^z + \lambda_{\perp}^R \hat{S}_{\mathbf{r}}^+ \hat{S}_{\mathbf{r}'}^- \right. \\ \left. + \lambda_q^R e^{2i\phi_{\mathbf{r}, \mathbf{r}'}} \hat{S}_{\mathbf{r}}^+ \hat{S}_{\mathbf{r}'}^+ + \lambda_K^R e^{i\phi_{\mathbf{r}, \mathbf{r}'}} (\hat{S}_{\mathbf{r}}^z \hat{S}_{\mathbf{r}'}^+ + \hat{S}_{\mathbf{r}}^+ \hat{S}_{\mathbf{r}'}^z) \right] + h.c.$$



Magnetic moment

$$\hat{\mathbf{m}}_{\mathbf{r}}^R = g_J \mu_B \hat{\mathbf{J}}_{\mathbf{r}}^R = \mu_B \left[g_{\perp}^R \left(\hat{S}_{\mathbf{r}}^x \mathbf{x}_i + \hat{S}_{\mathbf{r}}^y \mathbf{y}_i \right) + g_{\parallel}^R \hat{S}_{\mathbf{r}}^z \mathbf{z}_i \right]$$

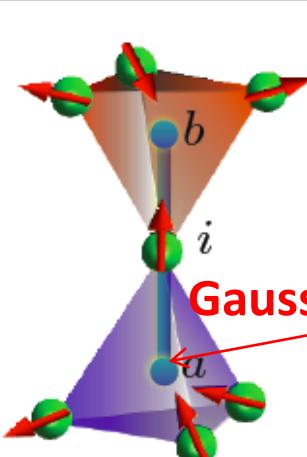
Yb2Ti2O7

Magnetic dipole interaction

$$\hat{H}_{\text{D}}^R = \frac{\mu_0}{4\pi} \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \left[\frac{\hat{\mathbf{m}}_{\mathbf{r}}^R \cdot \hat{\mathbf{m}}_{\mathbf{r}'}^R}{(\Delta r)^3} - 3 \frac{(\hat{\mathbf{m}}_{\mathbf{r}}^R \cdot \Delta \mathbf{r})(\Delta \mathbf{r} \cdot \hat{\mathbf{m}}_{\mathbf{r}'}^R)}{(\Delta r)^5} \right]$$

Best fit to
neutron-scattering exp.

Interacting U(1) Higgs model: QED with charged bosonic spinons



Gauss' law

$$S_i^z = \eta_a E_{ab}$$

$$S_i^+ = \Phi_a^\dagger e^{iA_{ab}} \Phi_b$$

$$Q_a = (\text{div} E)_a$$

$$[A_{ab}, E_{ab}] = i$$

$$[\Phi_a, Q_a] = \Phi_a$$

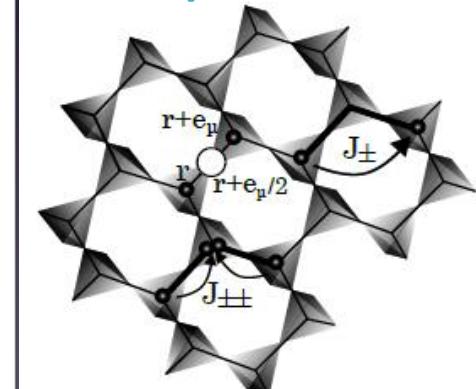
$$\eta_a = \pm 1 [a \in A(B)]$$

$$\Phi_a = e^{-i\varphi_a}$$

$$\Phi_a^\dagger \Phi_a = 1$$

**Monopolar spinons
(Higgs bosons)**
Increasing/decreasing the charge

S. Lee, S.O., L. Balents
PRB, in press



c.f. Savary-Balents

$$H_{QED} = \frac{J_{zz}}{2} \sum_{\mathbf{r}} Q_{\mathbf{r}}^2 - J_{\pm} \sum_{\mathbf{r}} \sum_{\mu \neq \nu} \Phi_{\mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\mu}}^\dagger \Phi_{\mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\nu}} s_{\mathbf{r}, \mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\mu}}^{-\eta_{\mathbf{r}}} s_{\mathbf{r}, \mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\nu}}^{+\eta_{\mathbf{r}}}$$

Starting from U(1) spin liquid
with deconfined spinons

$$+ \frac{J_{\pm\pm}}{2} \sum_{\mathbf{r}} \sum_{\mu \neq \nu} (\gamma_{\mu\nu}^{-2\eta_{\mathbf{r}}} \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\mu}} \Phi_{\mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\nu}} s_{\mathbf{r}, \mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\mu}}^{\eta_{\mathbf{r}}} s_{\mathbf{r}, \mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\nu}}^{\eta_{\mathbf{r}}} + h.c.)$$

$$+ q \sum_{\mathbf{r}} \sum_{\mu \neq \nu} (\gamma_{\mu\nu}^{-\eta_{\mathbf{r}}} \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\mu}} s_{\mathbf{r}, \mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\nu}}^{\eta_{\mathbf{r}}} + h.c.)$$

$$+ K \sum_{\mathbf{r}} \sum_{\mu \neq \nu} s_{\mathbf{r}, \mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\mu}}^z (\gamma_{\mu\nu}^{-\eta_{\mathbf{r}}} \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\nu}} s_{\mathbf{r}, \mathbf{r} + \eta_{\mathbf{r}} \mathbf{e}_{\nu}}^{\eta_{\mathbf{r}}} + h.c.) + \text{const..}$$

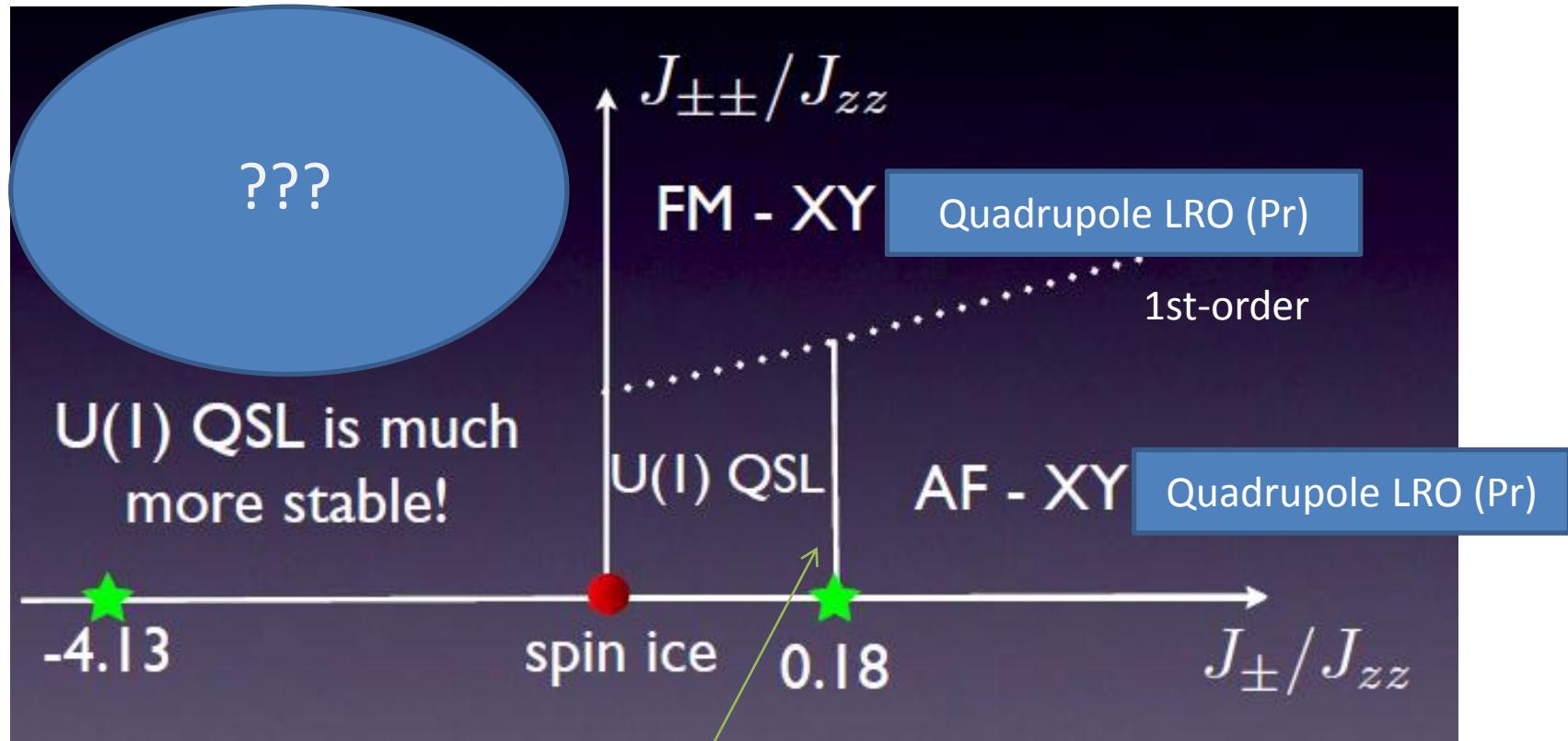
Non-Kramers doublets (integer spins) (Pr)

Classification of mean-field phases

Let's study the case of integer spins: non-Kramers doublets (Pr)

	$\langle s_{\mathbf{r}, \mathbf{r} \pm \mathbf{e}_\mu}^z \rangle$	$\langle s_{\mathbf{r}, \mathbf{r} \pm \mathbf{e}_\mu}^\pm \rangle$	$\langle \Phi_{\mathbf{r}} \rangle$	$\langle \Phi_{\mathbf{r}} \Phi_{\mathbf{r}} \rangle$	$\langle \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r} \pm \mathbf{e}_\mu} \rangle$
Ising order (confined)	$\neq 0$	0	0	0	0
QSL					
U(1)	0	$\neq 0$	0	0	0
Z ₂	0	$\neq 0$	0	$\neq 0$	0
(charge-2 Higgs)					
XY order					
U(1)	0	$\neq 0$	0	0	$\neq 0$
Classical	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$
(confined Higgs)					

Mean-field phase diagram in the case of non-Kramers doublets (Pr)



S. Lee, SO, L. Balents

Weakly first-order Higgs transition

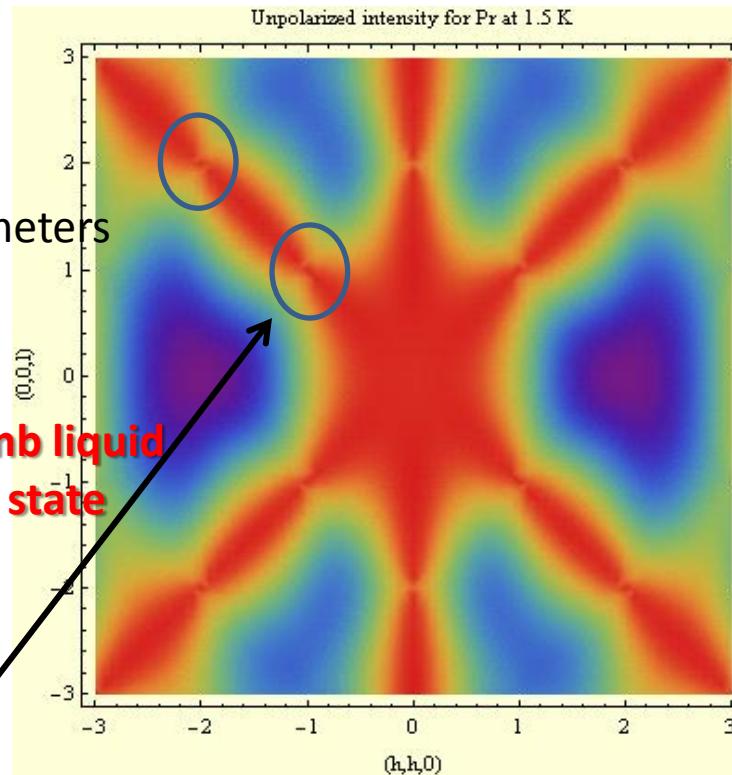
Is $\text{Pr}_2\text{Zr}_2\text{O}_7$ a U(1) QSL?

$$\frac{S(\vec{q})}{M_0^2} = \frac{1}{N} \sum_{r,r'} \sum_{i,j} \left(\delta_{i,j} - \frac{q_i q_j}{|\vec{q}|^2} \right) n_r^i n_{r'}^j \left\langle \sigma_r^z \sigma_{r'}^z \right\rangle_{\text{ave}} e^{i\vec{q} \cdot (r-r')}$$

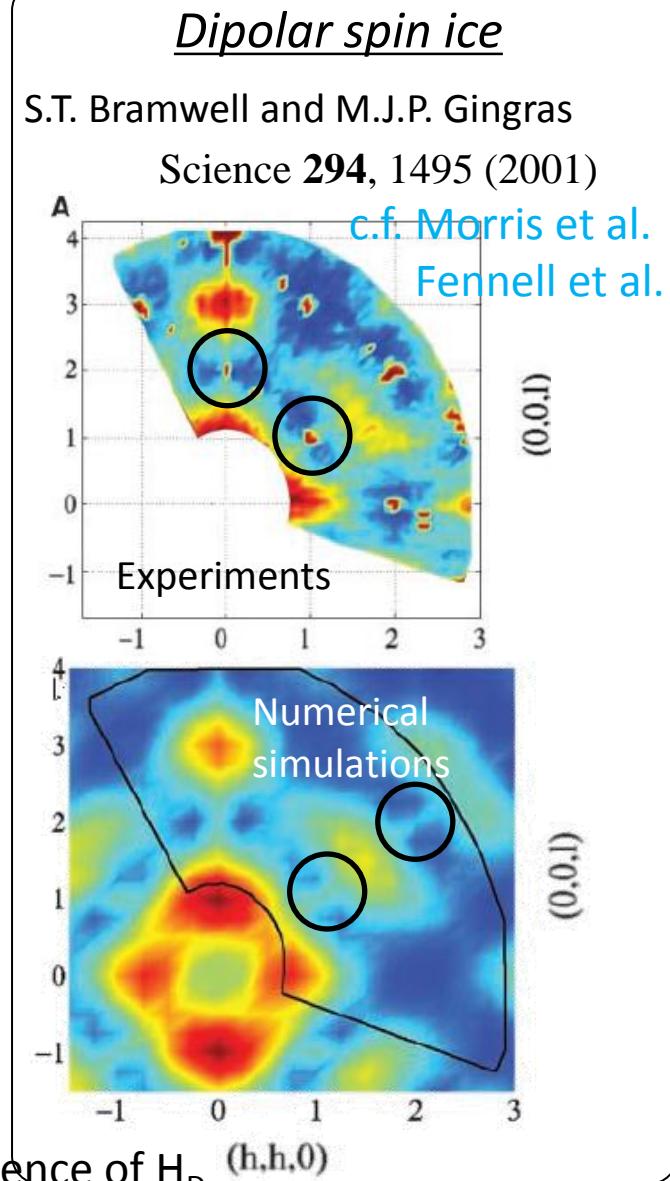
$$\vec{q} = \frac{2\pi}{a} (hhl), \quad M_0 = g_J \mu_B (4\alpha^2 + \beta^2 - 2\gamma^2)$$

(1/N) expansion

For exchange parameters
for $\text{Pr}_2\text{Zr}_2\text{O}_7$



**Pinch point singularity is broadened
by a dynamical violation of the ice rule**



Higgs transition

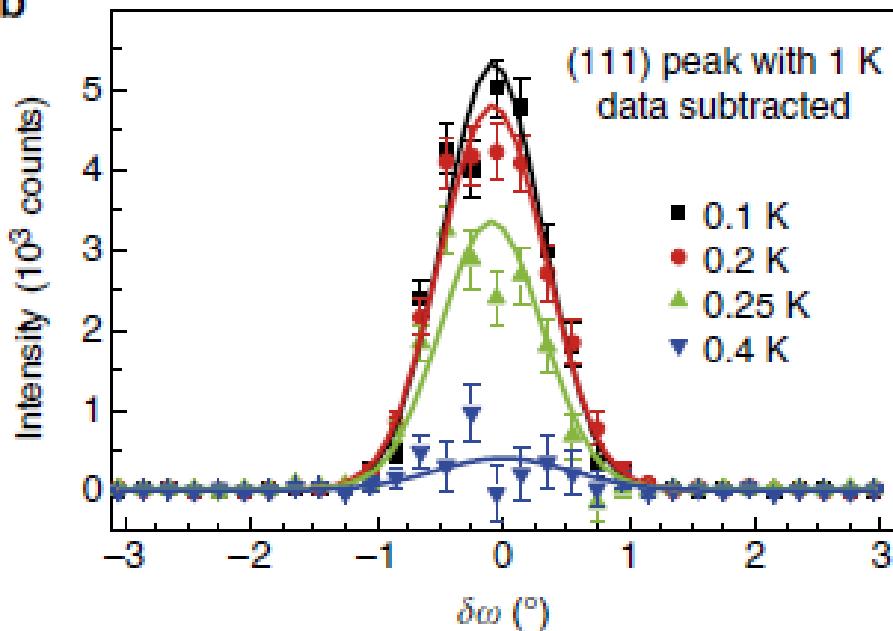
- Superconductivity:
 - Bose condensation of electrically charged particles
 - Described as a Higgs transition
 - Higgs mechanism:
 - fixing a U(1) phase → broken U(1) gauge structure
 - Nambu-Goldstone mode is absorbed into plasmons
 - Bose condensation of magnetically charged particles
 - Possible in quantum spin ice!
 - Analogous superconducting transition in magnetic monopoles
 - Higgs mechanism: gapped spin excitations!
- T>0 case: Low T_c → could be described with a Higgs transition!
Smoothly connected to conventional transitions with increasing T_c.

Evidence of first-order ferromagnetic transition

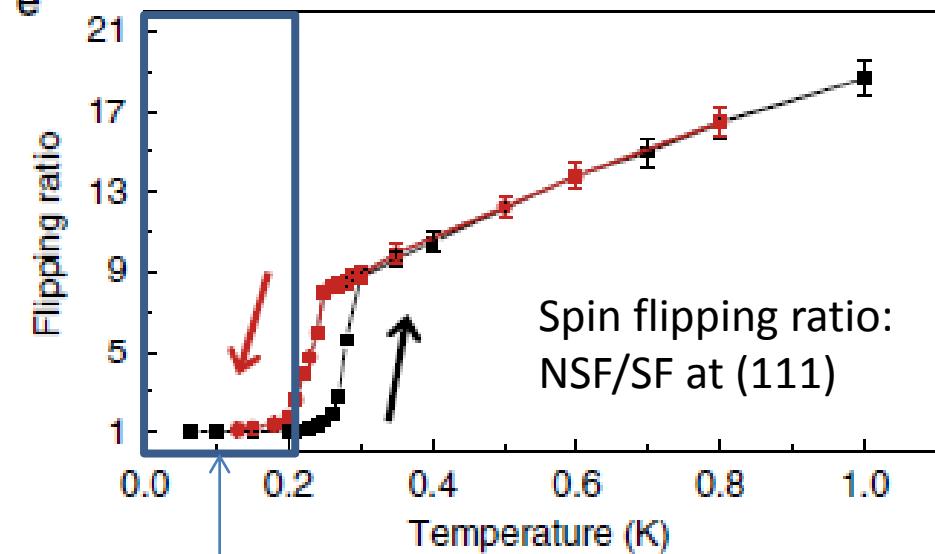
Polarized neutron-scattering intensity

Neutron-spin flipping ratio
showing thermal hysteresis

b

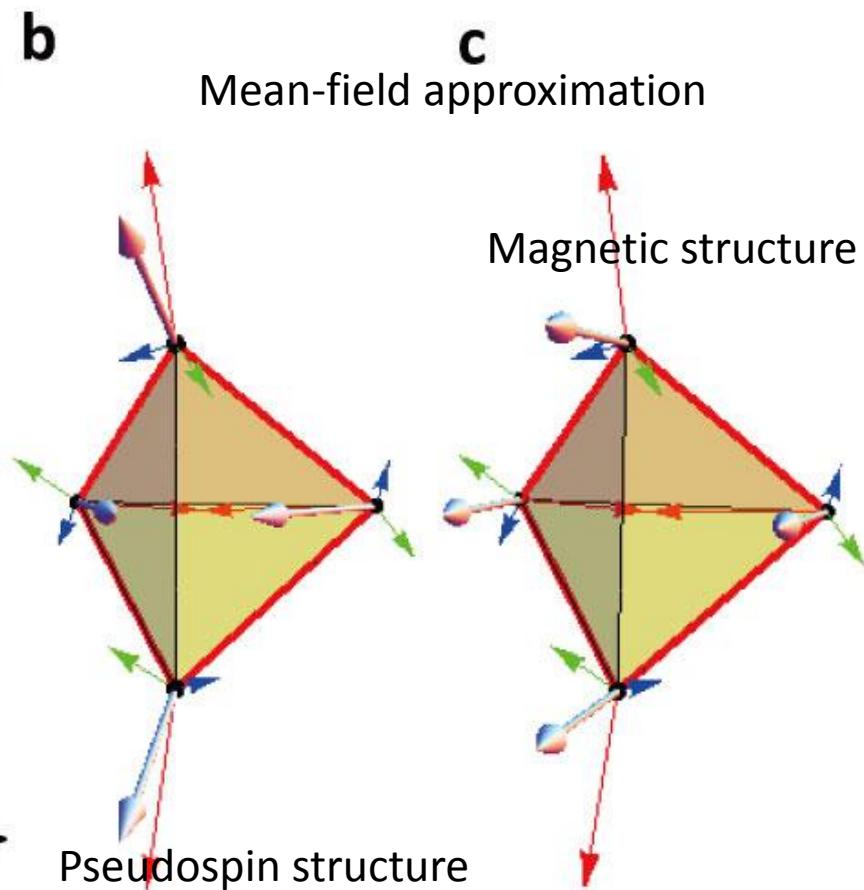
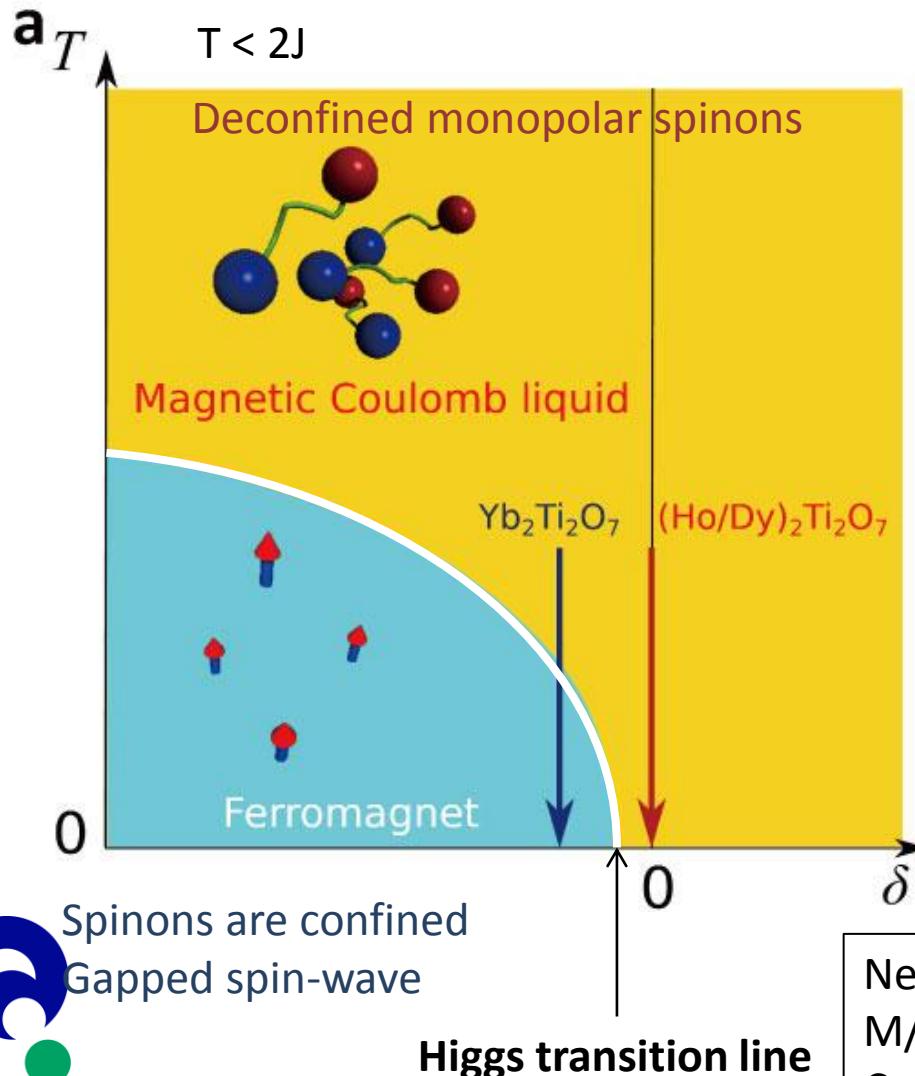


e



Full depolarization of neutron spins below 0.21 K
→ Macroscopic ferromagnetic domain

Phase diagram and the hypothetical magnetic structure

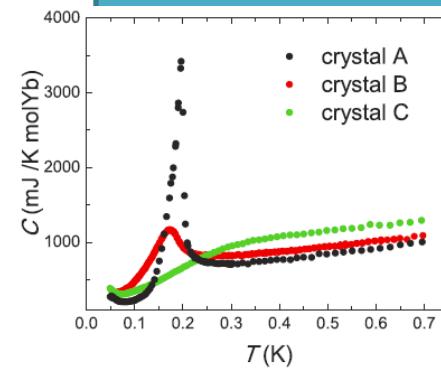


Nearly collinear ferromagnet (~1 degree canting)
 $M/[100]$
Consistent with the magnetic structure analysis

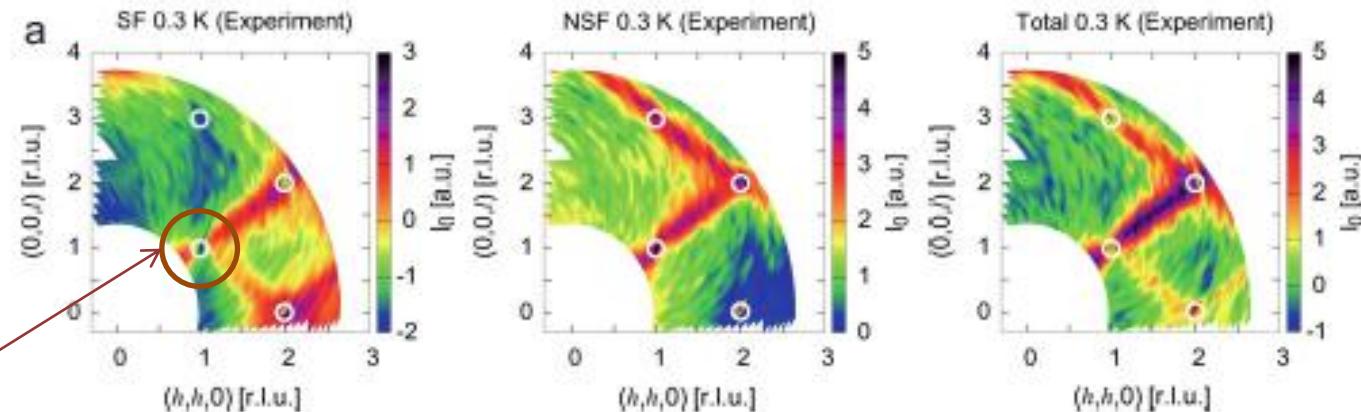
Summary

- Effective quantum pseudospin-1/2 models for $(\text{Pr,Yb})_2TM_2\text{O}_7$
 - Anisotropic superexchange interaction
 - ferromagnetic Ising coupling → start from spin ice
 - Magnetic monopole charges ($\nabla \cdot M \neq 0$) carried by spinons!
 - Emergent gapless U(1) spin liquid (Fictitious QED)
 - **Higgs transitions** to classical gapped ferromagnets
 - superconductivity of magnetic monopoles
 - **Neutron-scattering experiments on high-quality single crystal $\text{Yb}_2\text{Ti}_2\text{O}_7$**
 1. deconfined bosonic spinons carrying monopole charge in the high-temperature phase
 2. Confined spinons to form classical ferromagnetism in the low-temperature phase
- **$\text{Pr}_2\text{Zr}_2\text{O}_7$:**
 - U(1) quantum spin ?
 - Remnants of pinch-point singularity

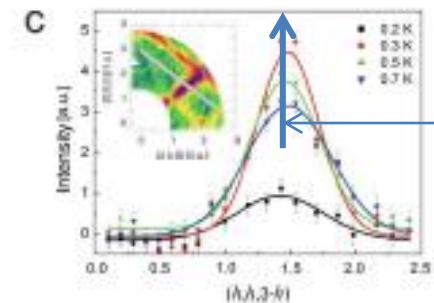
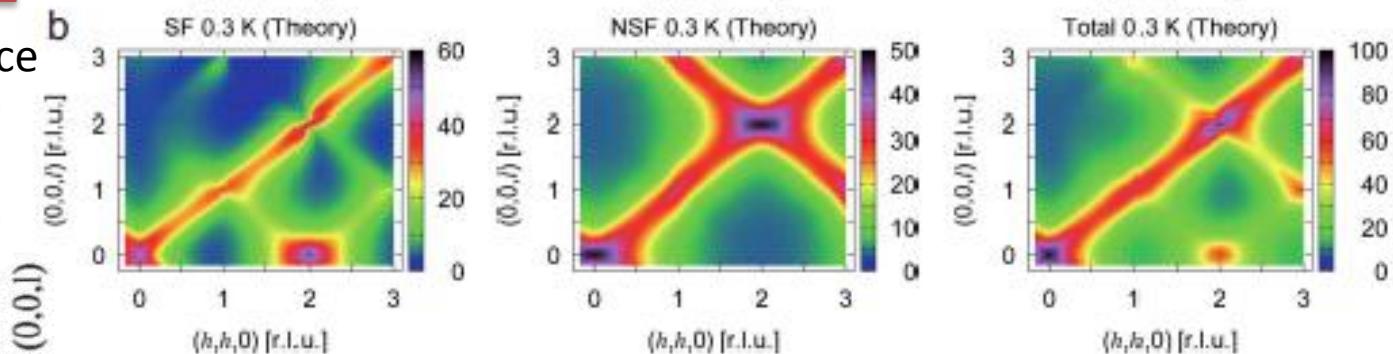
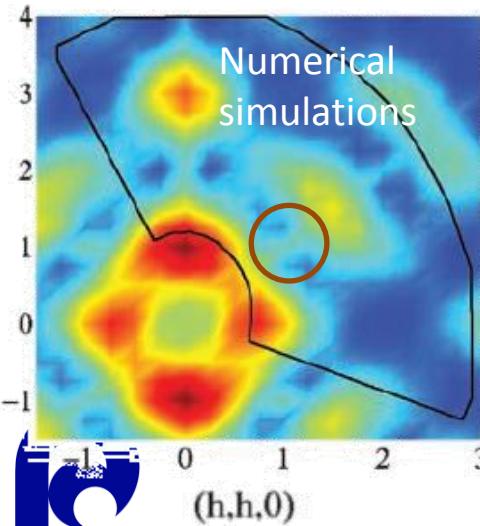
Fitting polarized neutron scattering results within the RPA



a remnant of
pinch-point singularity



c.f. classical dipolar spin ice



Anisotropic nature around (111)
grows with decreasing T!

Indication of Coulomb phase

$$g^{\perp} \sim -0.8$$

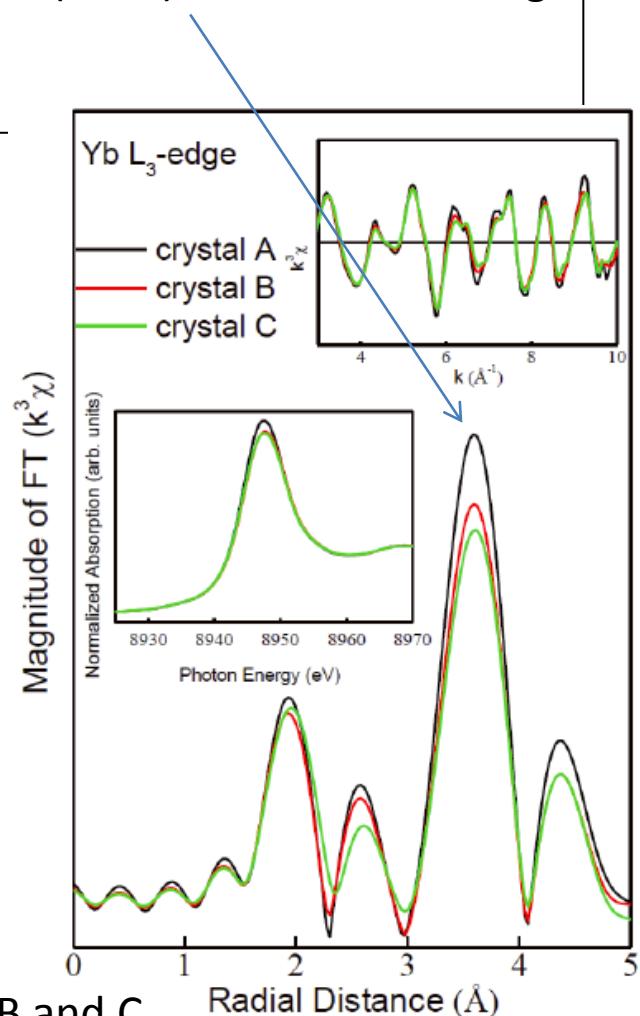
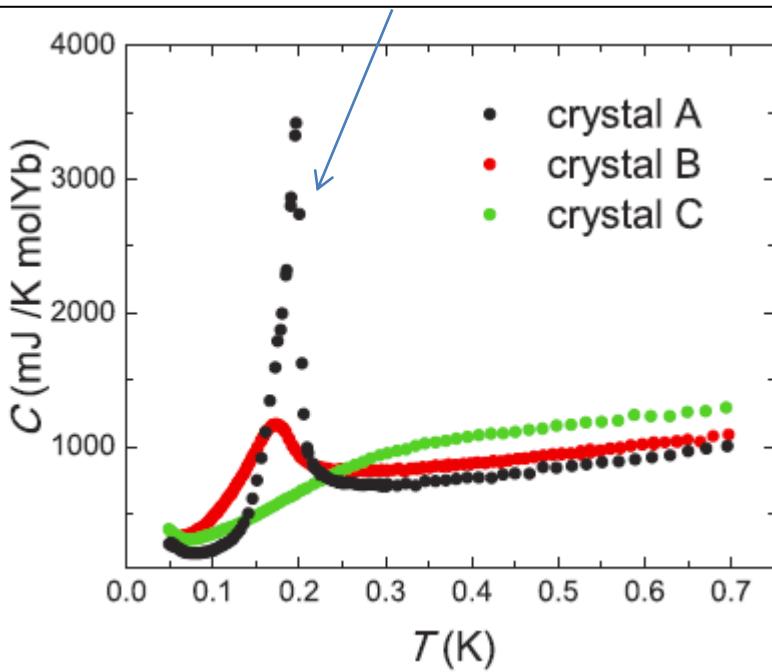
$$g^q \sim -0.6$$

$$g^K \sim -1.2$$

Close to results by Ross et al.

Ferromagnetic ground state is intrinsic! Dependence on different single crystals

The sample studied for unpolarized (2003) and polarized (2011) neutron scattering
Ferromagnet
Comparable to the result on powder samples by Blote



A bit sharper distribution of bond length
for single crystals showing sharper anomaly
in the specific heat

It turned out that Yb ions are more deficient in B and C.