TOPOLOGICAL PROPERTIES OF $\mathbb{Z}_2$ SPIN LIQUIDS

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(3)


(4)

- Phys. Rev. B 83, 245134 (2011) [1,2,3]
- Phys. Rev. B 86, 014404 (2012) [1,2,4]
- arXiv:1203.4816 (in press) [1,2,4]

OUTLINE

- «Idealized» spin liquids (= NN RVB w.f.s) and topological sectors
- Tensor network methods - Tensor product representation (=PEPS) of RVB states
- Topological energy splitting for infinite cylinders
- Entanglement properties: entanglement entropy and more...

Exotic states of matter

* no broken symmetry
* no local order
* GS degeneracy depends on **topology** of space

→ Topological order   X. G. Wen

Beyond the "order parameter paradigm": correlations "missed" by two-point correlation functions can be detected by entanglement measures

Solid numerical evidence that a (gapped) spin liquid GS can be realized in simple frustrated SU(2) Heisenberg QAF

* Kagome lattice

S. Yan, D.A. Huse & S. White, Science 2011
S. Depenbrock, I.P. McCulloch & U. Schollwoeck, PRL 2012

H.C. Jiang, Z. Wang & L. Balents, arxiv:1205.4289

* J1-J2 on the square lattice

H.C. Jiang, H. Yao & L. Balents, PRB 2012
L. Wang, Z.-C. Gu, F. Verstraete & X.-G. Wen, arxiv:1112.3331

...but topological features not fully revealed

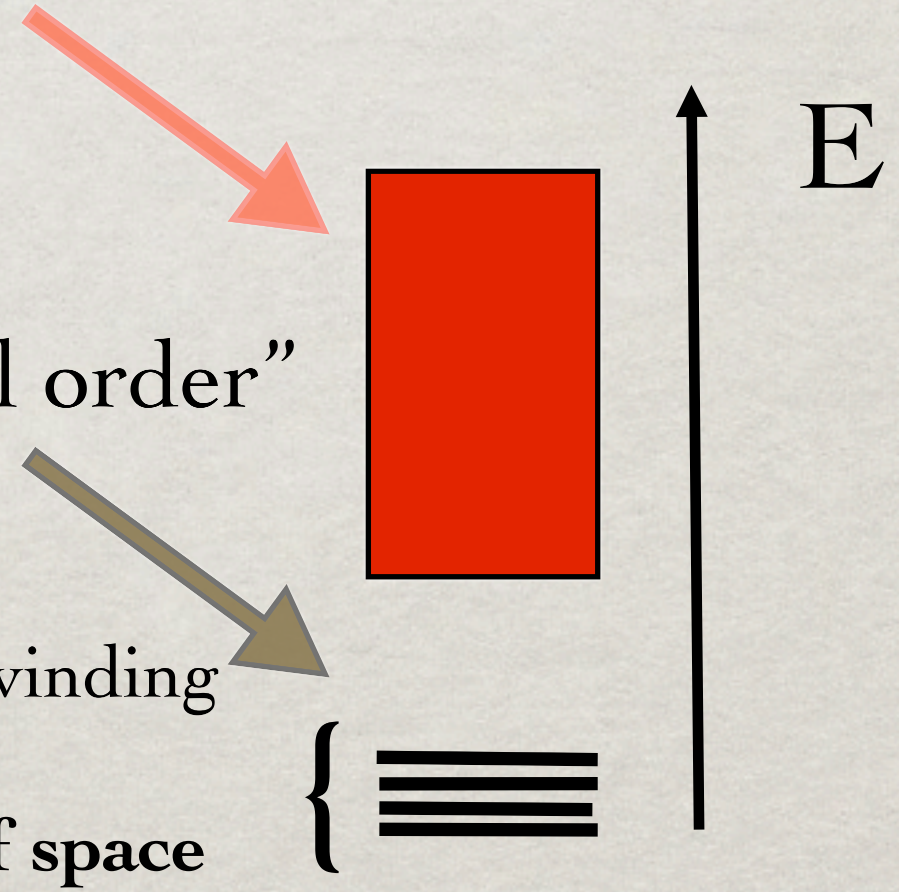
Topological spin liquids

$(\mathbb{Z}_2)$

Excitations are topological "visons"

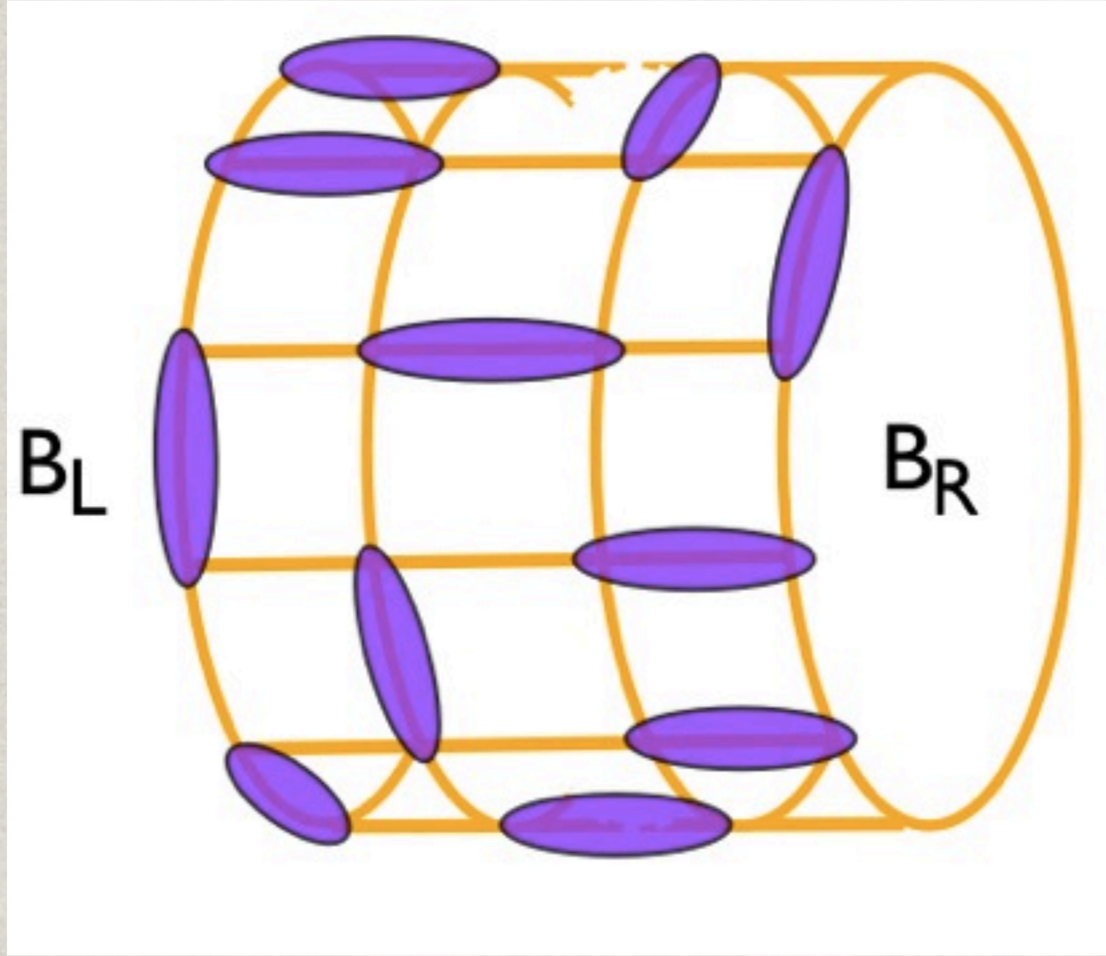
Degeneracy from "topological order"

GS have different "winding numbers"
but **same IRREP of space group**



degeneracy depends on topology:
g=4 on the torus
g=2 on the cylinder

Resonating Valence Bond states



RVB = equal-weight superposition of NN singlet coverings

P. Fazekas and P.W. Anderson
Philosophical Magazine **30**, 423-440 (1974)

Are such simple «idealized» spin liquid w.f.s

1) «good enough» to describe the $\mathbb{Z}_2$ spin liquids on t kagome and/or square lattice ?

2) useful to understand topological properties of such liquids (i.e. finite size scalings etc...) ?

=> feedback for numerics ?

Some properties of RVB wavefunctions

* Kagome lattice: short-range dimer-dimer correlations

Misguich et al., PRL 2002
Moessner & Sondhi, PRL 2001
Yang & Yao, arXiv 2012
+ our work

=> gaped spin liquid

* Square lattice: algebraic-dimer correlations

Rohksar & Kivelson PRL
Albuquerque & Alet, PRB 2010

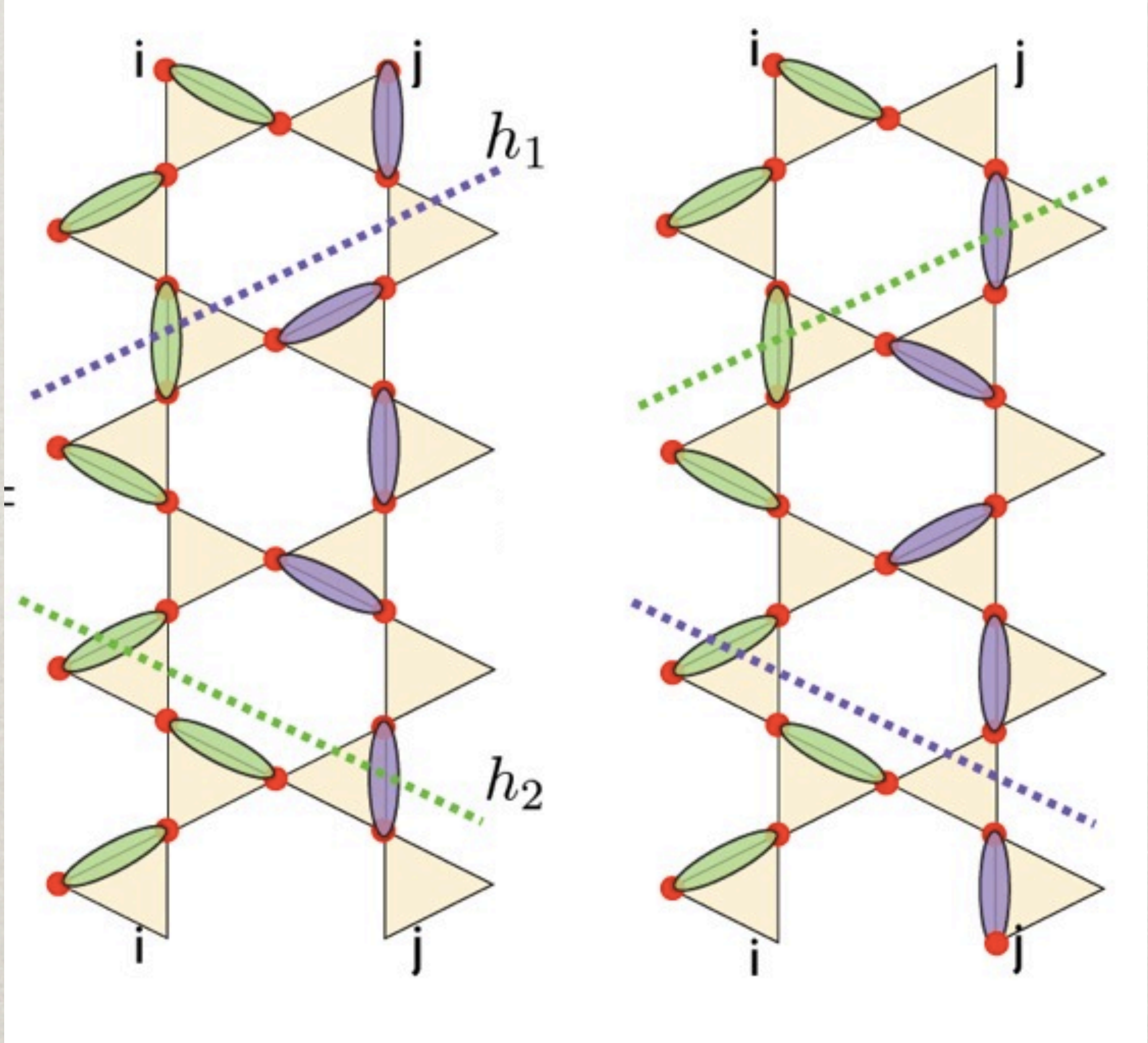
...but a gaped spin liquid can be constructed by allowing for longer range singlets bonds:

Ling Wang, DP, Z.-C. Gu, X.-G. Wen & F. Verstraete
in preparation

Disconnected topological sectors in the space of dimer lattice coverings

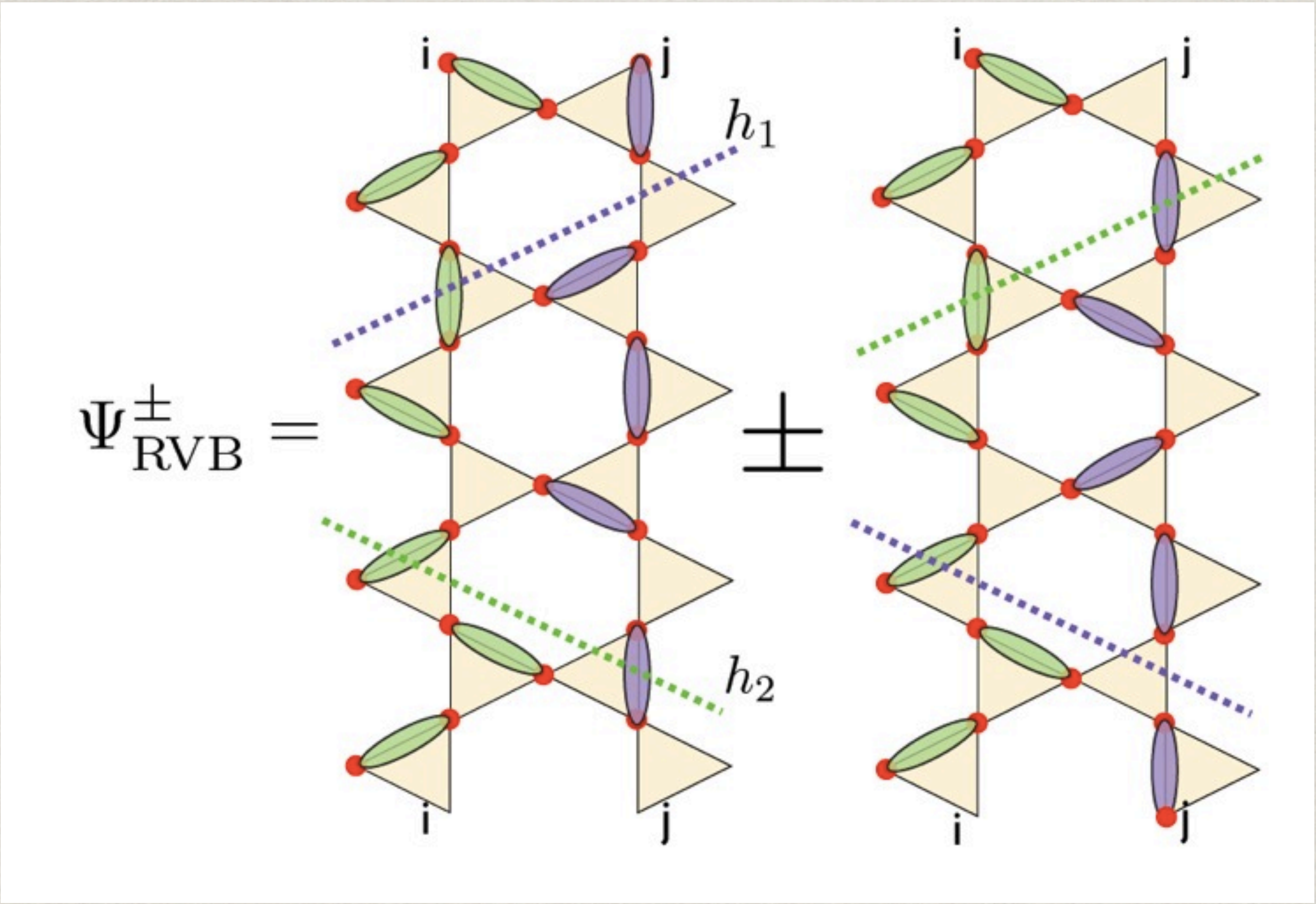
E.g. on a cylinder:

4×2 cylinders



conserved parity: $G_h = +1$ $G_h = -1$

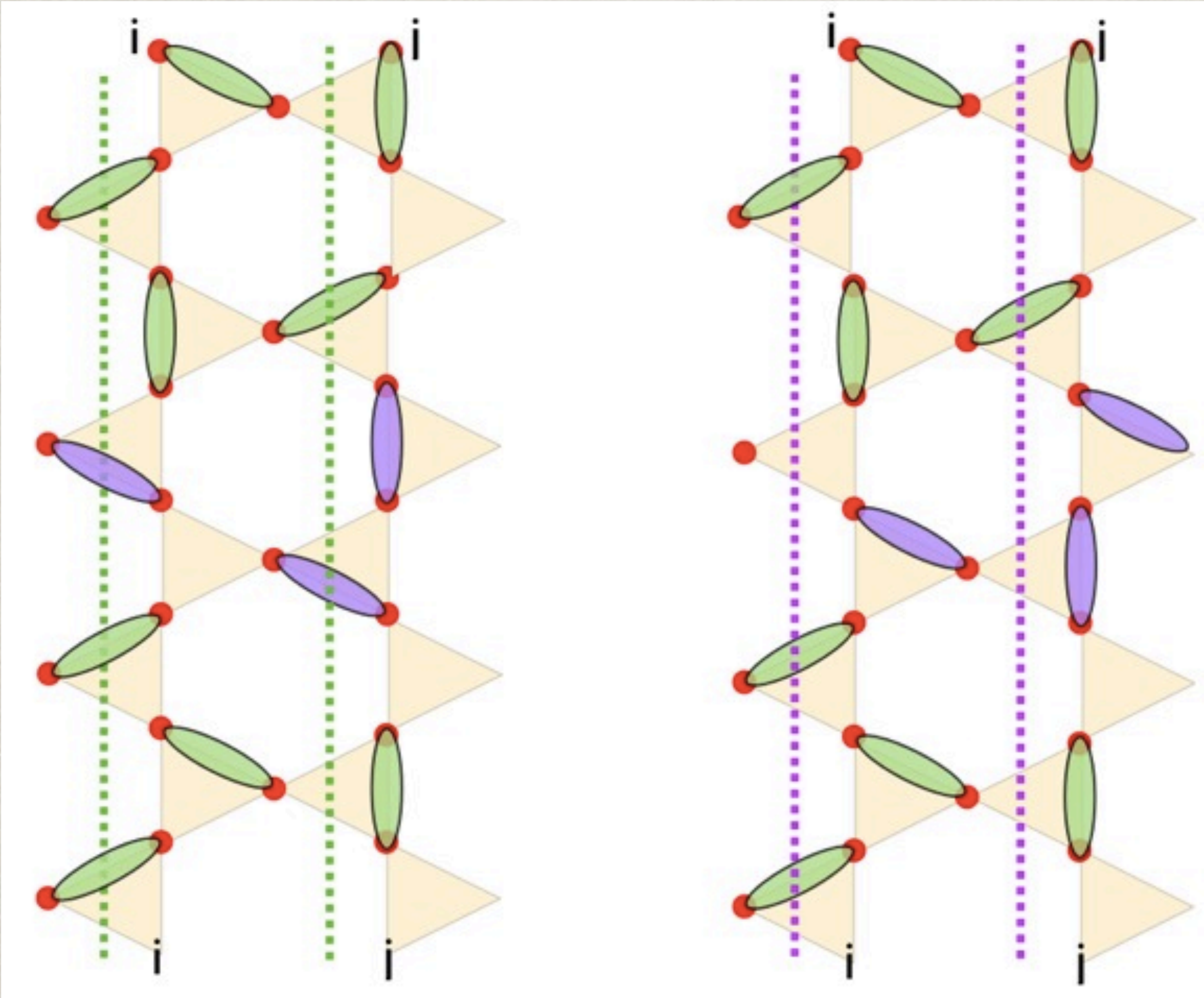
Eigenstates of a «Wilson loop» operator



$+$ no vison flux

$-$ $\mathbb{Z}_2$ vison flux

Fix the cylinder boundaries



«even»

$G_v = +1$

«odd»

$G_v = -1$

4 variational states

$\mathbb{Z}_2$ vison flux:

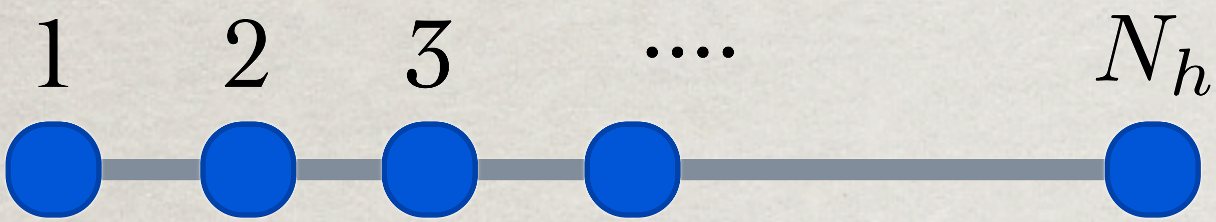
	0	1
even	$\Psi(+)_e$	$\Psi(-)_e$
odd	$\Psi(+)_o$	$\Psi(-)_o$

↑

cylinder boundaries

Tensor Network approaches

I. Cirac
F. Verstraete
G. Vidal



$$|\Psi\rangle = \sum_I c_I |i_1, i_2, ..., i_{N_h}\rangle \qquad i_k = -S, -S+1, \dots, S-1, S$$

Matrix Product States (1D) : $M^i_{\alpha_1,\alpha_2}$ $D \times D$ matrix

Totsuka and M. Suzuki.,
J. Phys.: Condens.Matter,7(1639), 1995.

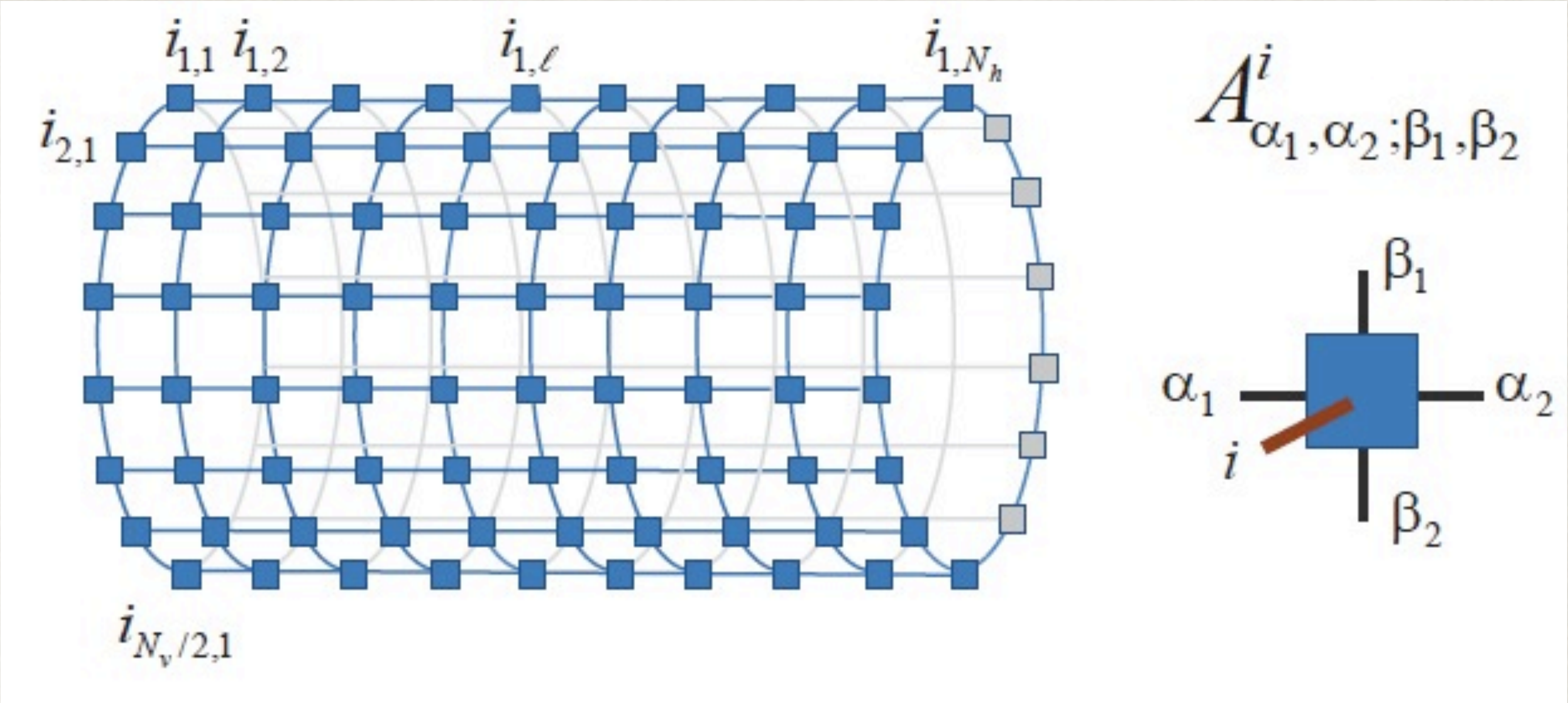
$$c_I = \sum_\alpha L^{i_1}_{\alpha_1} M^{i_2}_{\alpha_1\alpha_2} \dots M^{i_{N_h-1}}_{\alpha_{N_h-2}\alpha_{N_h-1}} R^{i_{N_h}}_{\alpha_{N_h-1}}$$

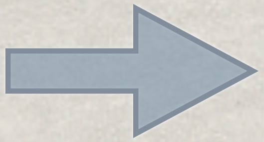
$$= L^{i_1} M^{i_2} \dots M^{i_{N_h-1}} R^{i_{N_h}}$$

Equivalent to DMRG ! Romer and Ostlund (PRL, 1995)

D ~ m parameter controling the DMRG truncation

Tensor Network for d=2 (and higher): Projected Entangled Paired States (PEPS)





"contract" product of tensors

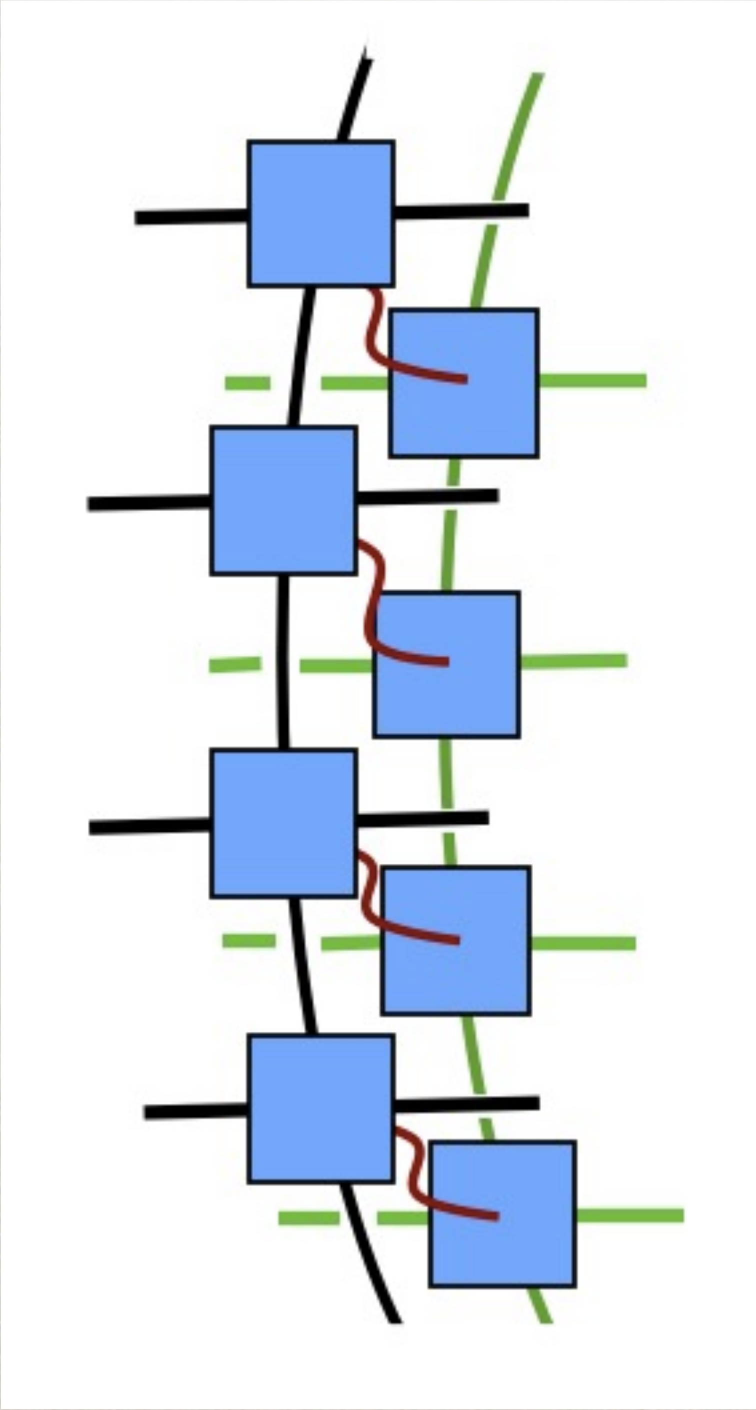
$$c_I = \sum_\Lambda L^{I_1}_{\Lambda_1} B^{I_2}_{\Lambda_1,\Lambda_2} \ldots B^{I_{N_h-1}}_{\Lambda_{N_h-2},\Lambda_{N_h-1}} R^{I_{N_h}}_{\Lambda_{N_h-1}}$$

$$B^{I_n}_{\Lambda_{n-1},\Lambda_n} = \mathrm{tr}\left[\prod_{k=1}^{N_v} \hat{A}^{i_{k,n}}_{\alpha_{k,n-1},\alpha_{k,n}}\right]$$

$$\Lambda_n = (\alpha_{1,n}, \alpha_{2,n}, \ldots, \alpha_{N_v,n})$$
$$I_n = (i_{1,n}, i_{2,n}, \ldots, i_{N_v,n})$$

build «double layer» tensor network by contracting physical variables

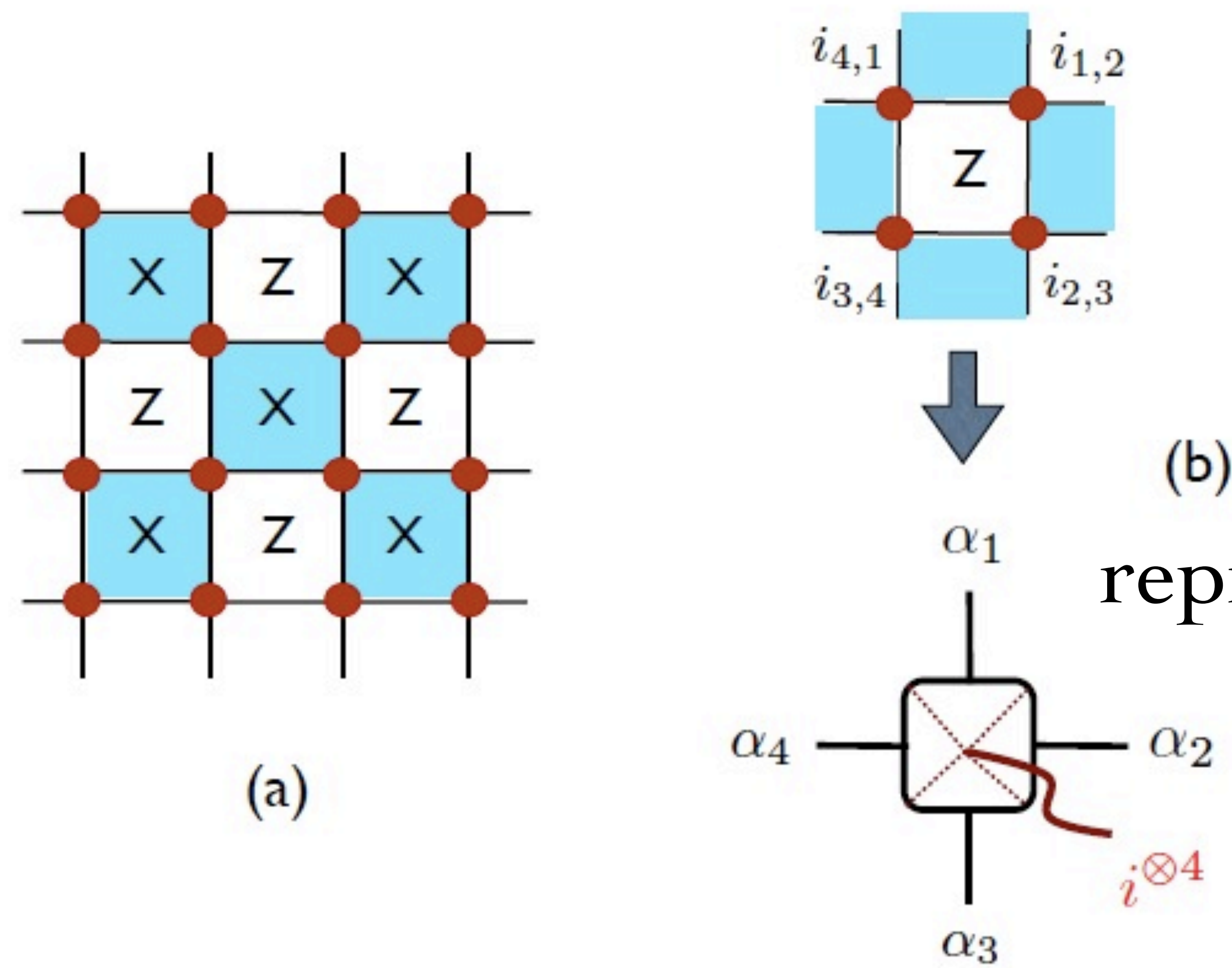
$$\langle \Psi | \Psi \rangle$$



«transfer matrix»

A simple example: the Kitaev code model

H =

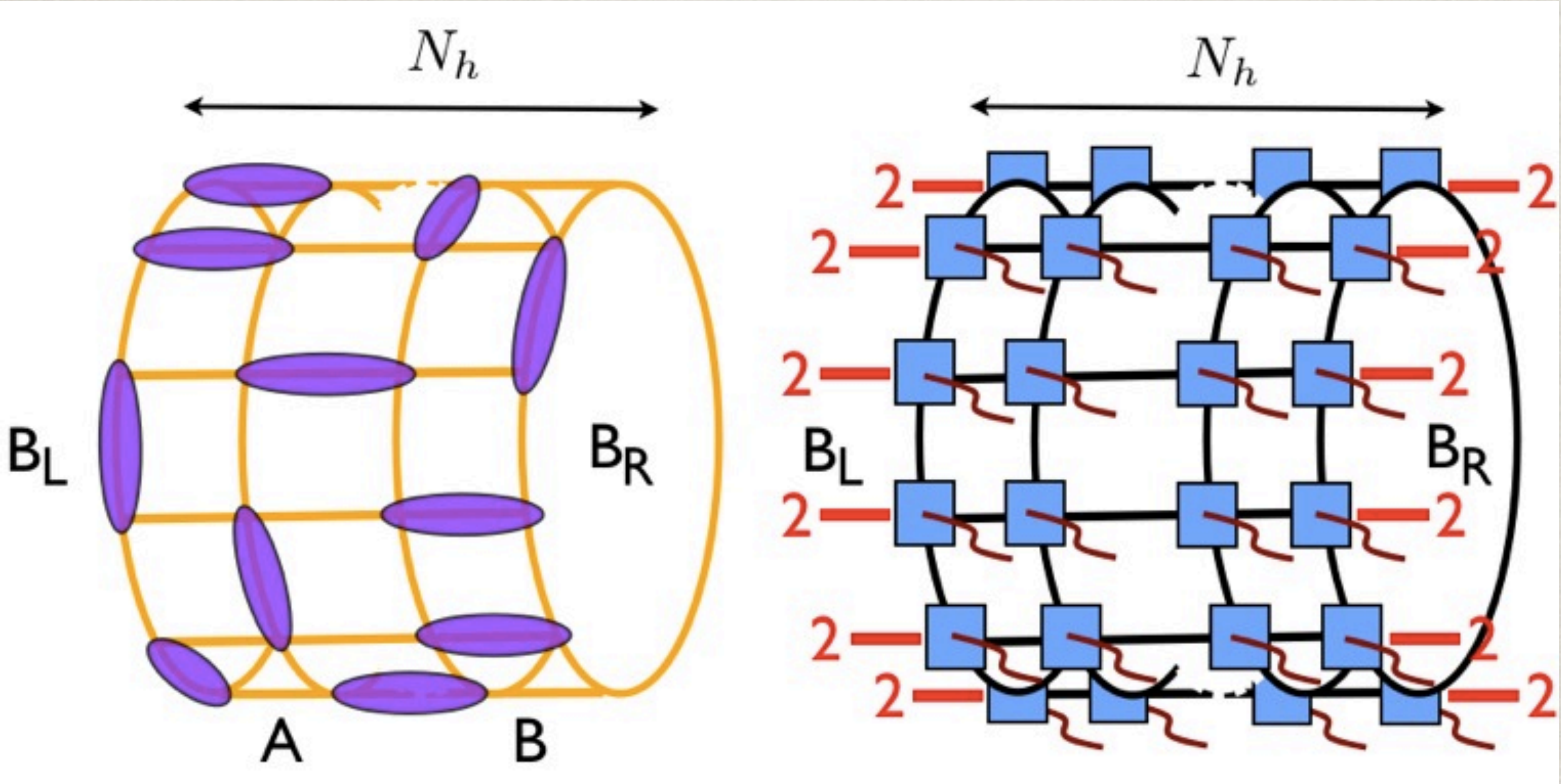


PEPS representation

D=2 !

$$A^{i_{1,2},i_{2,3},i_{3,4},i_{4,1}}_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} = \begin{cases} 1 \text{ if } i_{x,x+1} = \alpha_{x+1} - \alpha_x \mod 2 \ \forall x \\ 0 \text{ otherwise.} \end{cases}$$

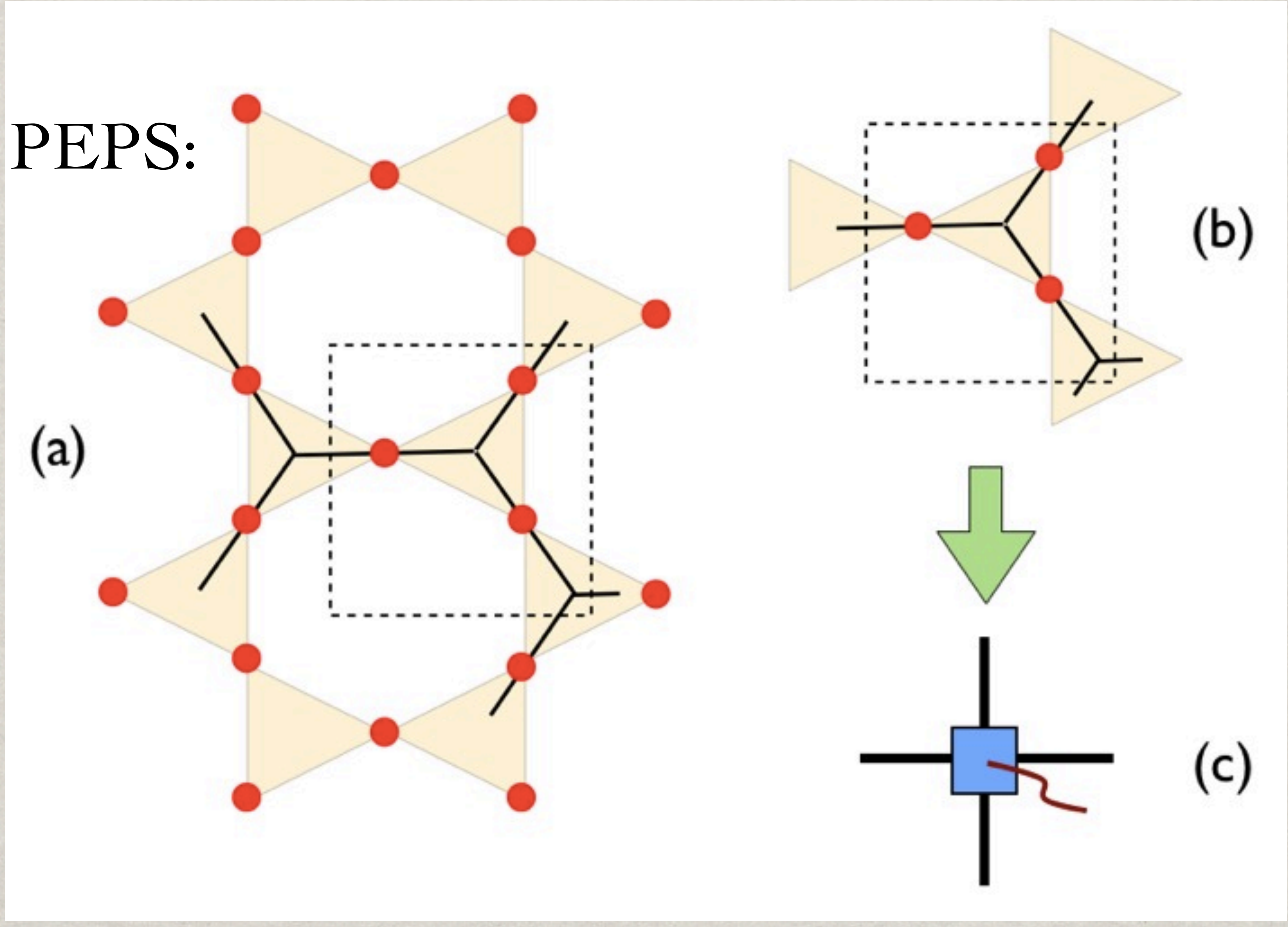
PEPS formulation of RVB



RVB → D=3 PEPS

F. Verstraete et al., 2006

PEPS RVB on the kagome lattice

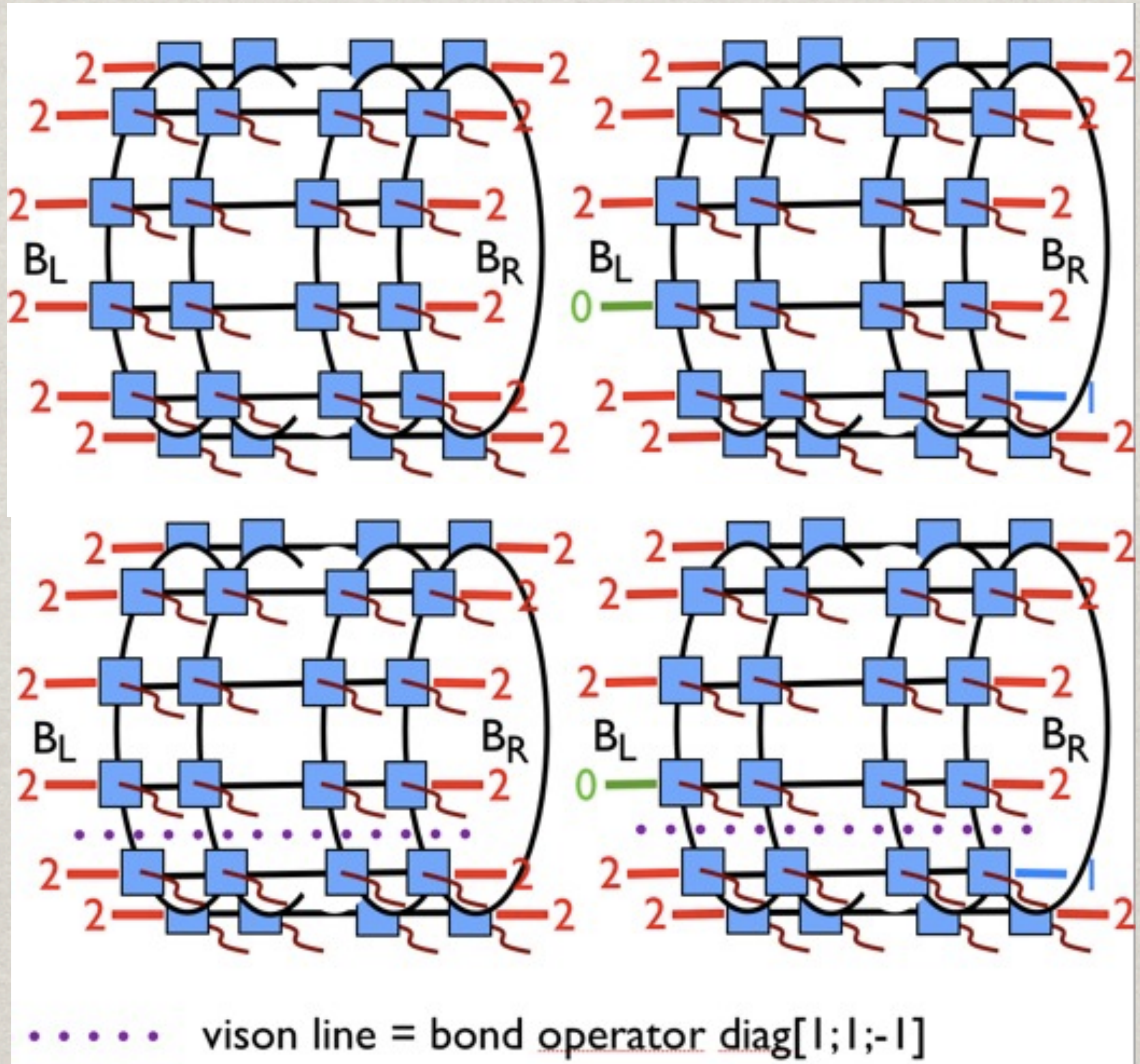


map on a square lattice
(but no reflection symmetry)

Construct 4 variational GS

$\Psi(+)_e$

$\Psi(+)_o$

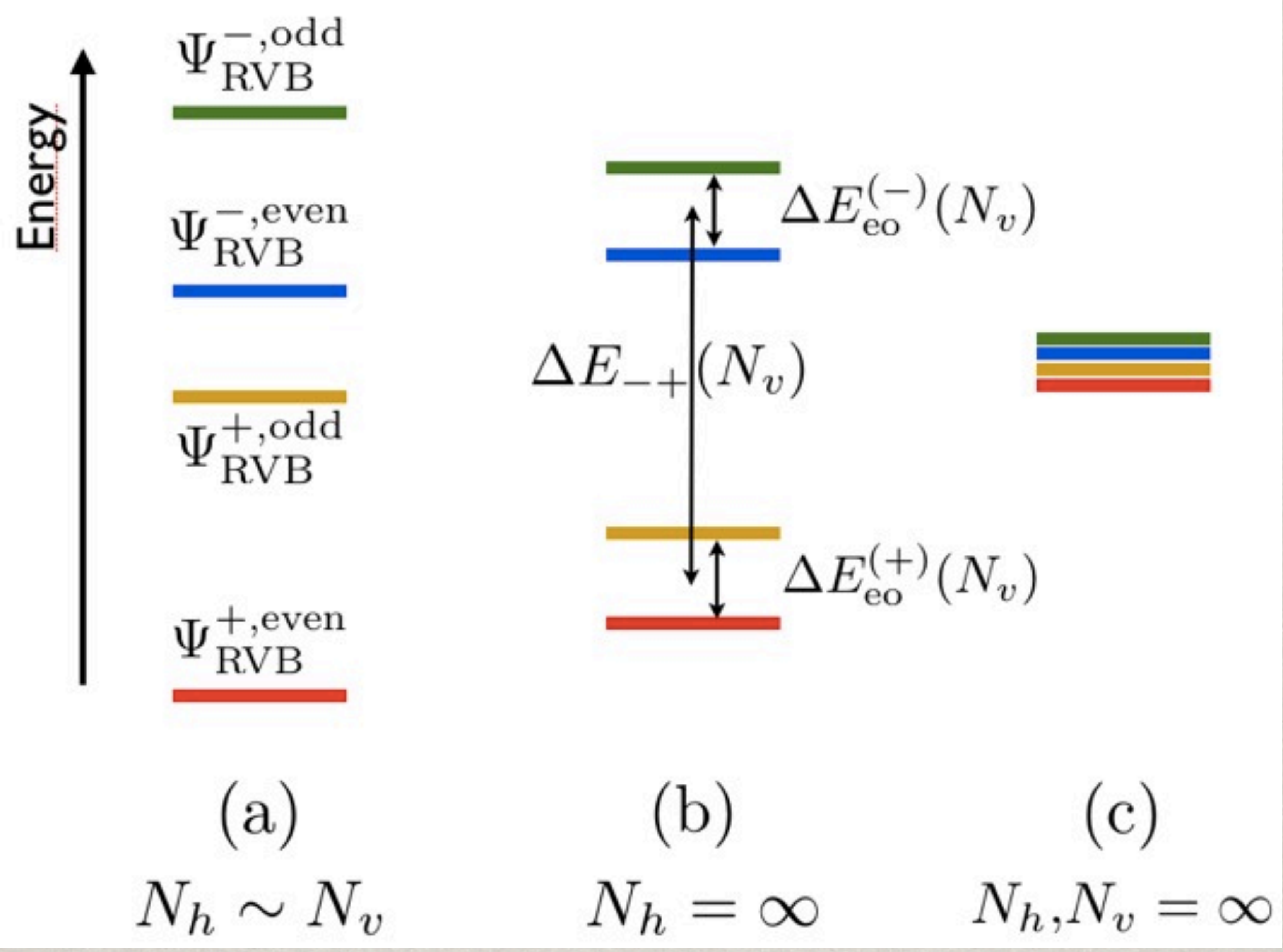


$\Psi(-)_e$

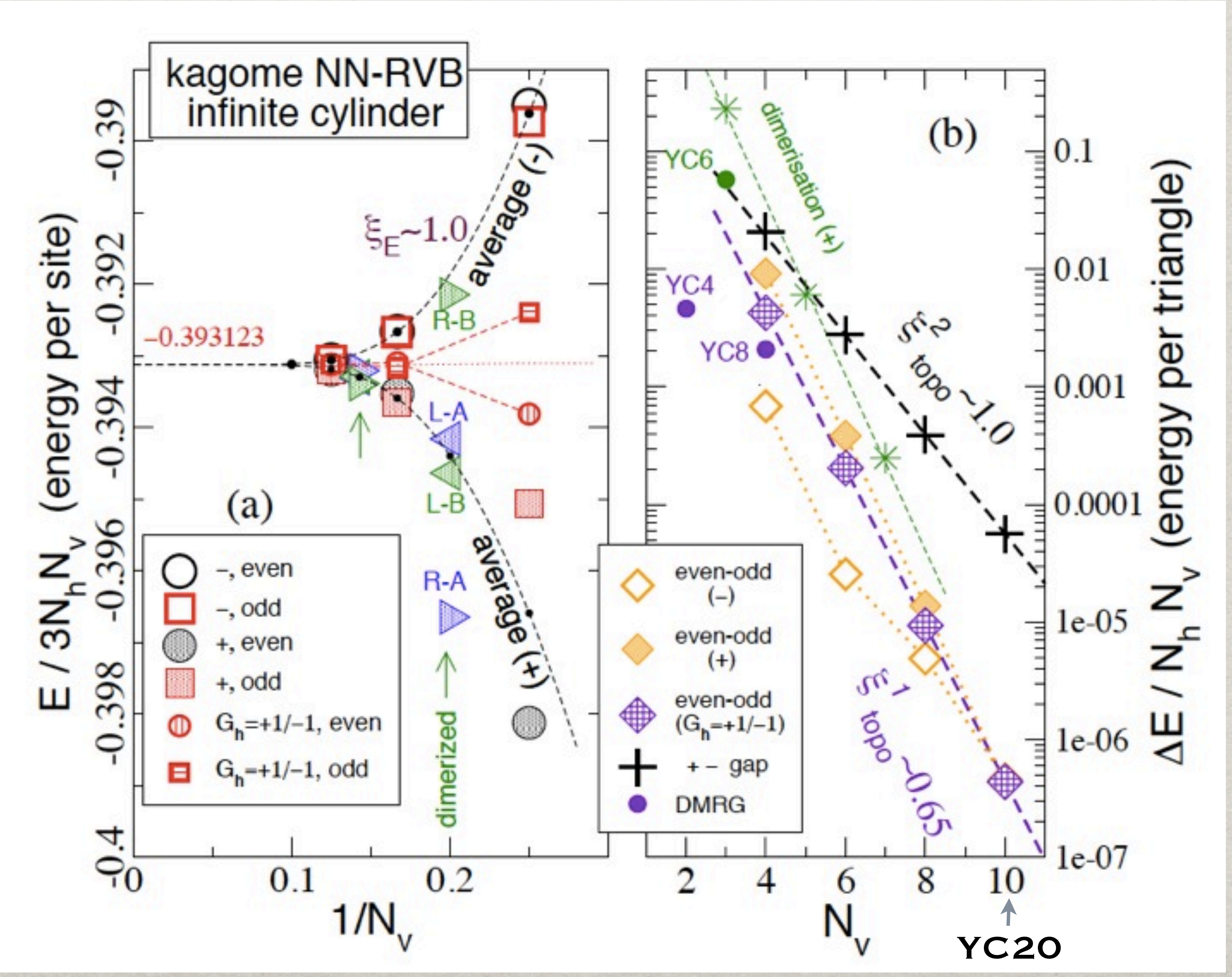
$\Psi(-)_o$

→ orthogonal in the limit of infinite cylinders $(N_h = \infty)$

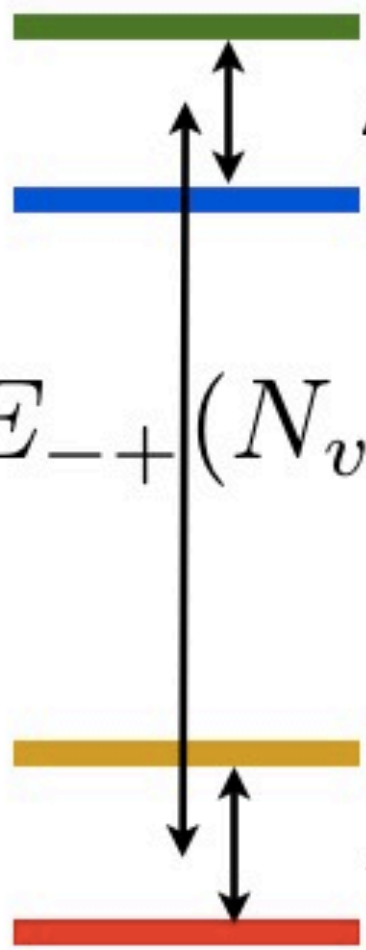
Topological energy scaling for Heisenberg kagome QAF (schematic)



Numerical data $(N_h = \infty)$



Topological energy scaling: quantitative fits



$$\Delta E_{\mathrm{eo}}^{(-)}(N_v) \simeq 1.95\, N_v N_h \exp\left(-1.54\, N_v\right)$$

$$\Delta E_{-+}(N_v) \simeq 1.06\, N_v N_h \exp\left(-0.99\, N_v\right)$$

$$\Delta E_{\mathrm{eo}}^{(+)}(N_v) \simeq 1.95\, N_v N_h \exp\left(-1.54\, N_v\right)$$

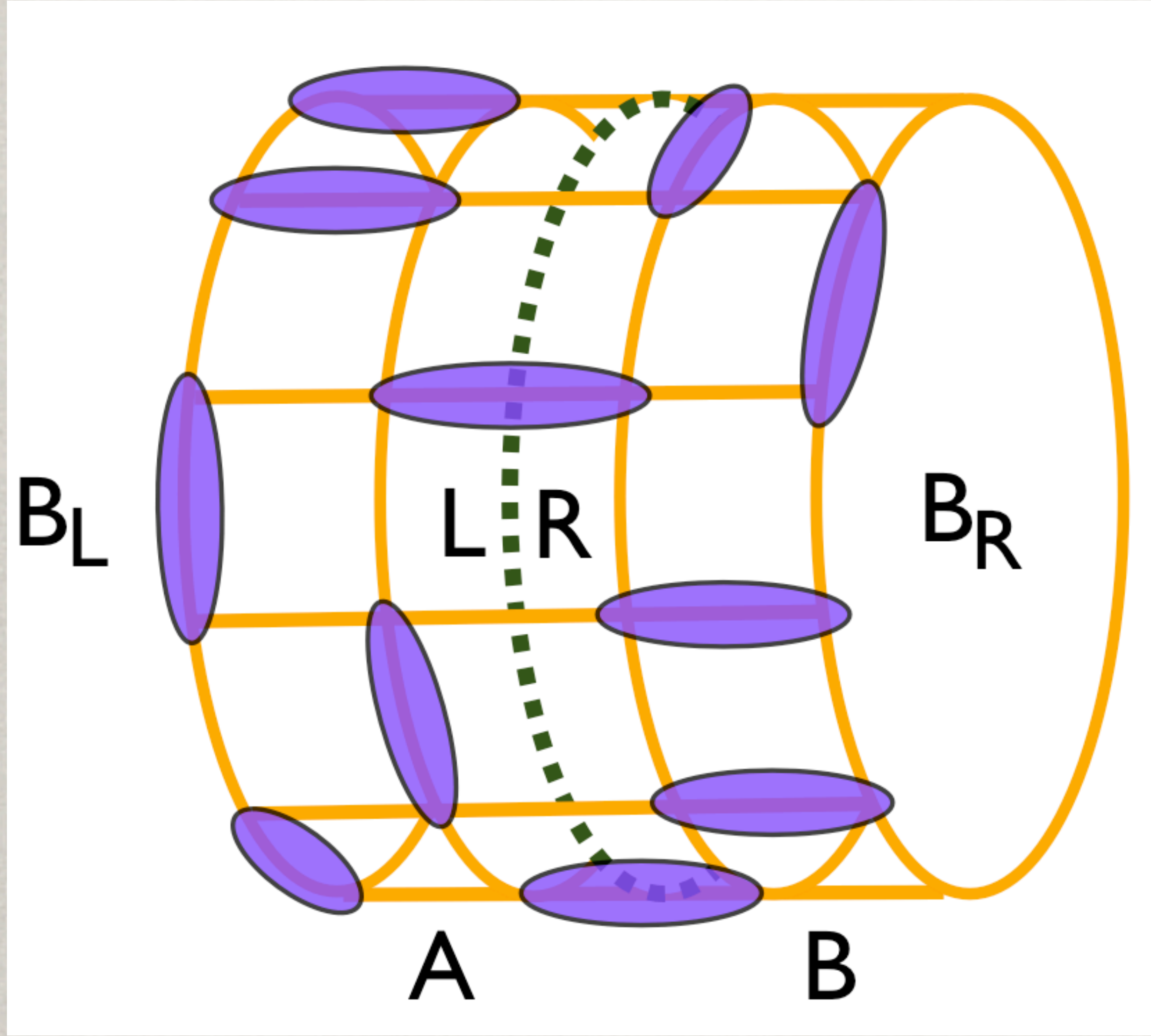
... of practical use in DMRG calculations ?

400-sites YC12: $\Delta E_{-+} \sim 0.35$

600-sites YC16: $\Delta E_{-+} \sim 0.07$

900-sites YC20: $\Delta E_{-+} \sim 0.014$ ⟵ below S=0 and S=1 gaps !

Entanglement properties



Reduced density matrix

$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

$$S_{\mathrm{entanglement}} = -\mathrm{Tr}\{\rho_A \ln \rho_A\}$$

(Von Neumann)

$$S_{\mathrm{entanglement}} \propto \xi L^{d-1}$$ "area" law

d=2: $\propto L$ (perimeter)

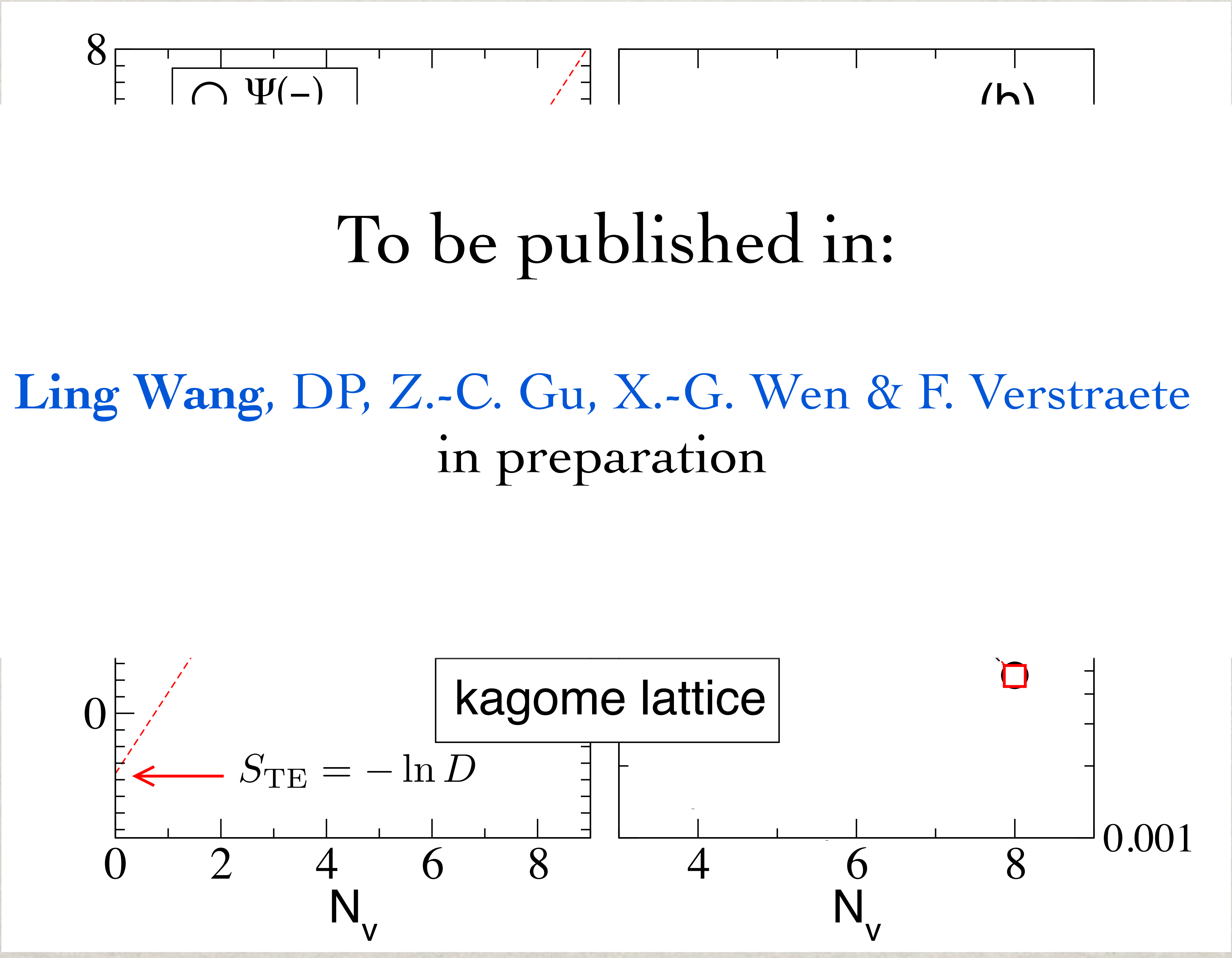
subleading correction to area law:

$$S_{\mathrm{TE}} = -\ln D$$

Kitaev & Preskill, 2006

Levin & Wen, 2006

Entanglement entropy of infinite cylinders vs cylinder perimeter



To be published in:

Ling Wang, DP, Z.-C. Gu, X.-G. Wen & F. Verstraete
in preparation

$$S_{\mathrm{VN}} = -\ln D + aN_v + b\exp\left(-cN_v\right)$$

OUTLOOK

* Boundary Hamiltonian and entanglement spectrum

$\rho_A = \exp(-H_b)$ → «edge states»

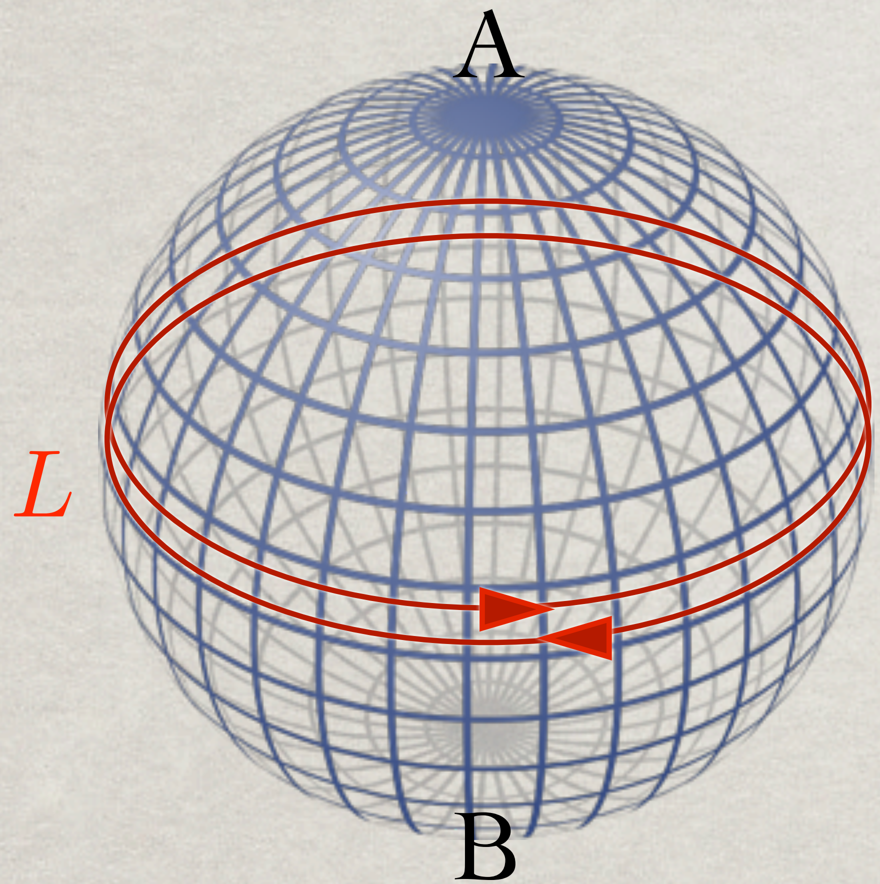
Haldane & Li, PRL 2008

* Interpolation between dimer state & SU(2) RVB

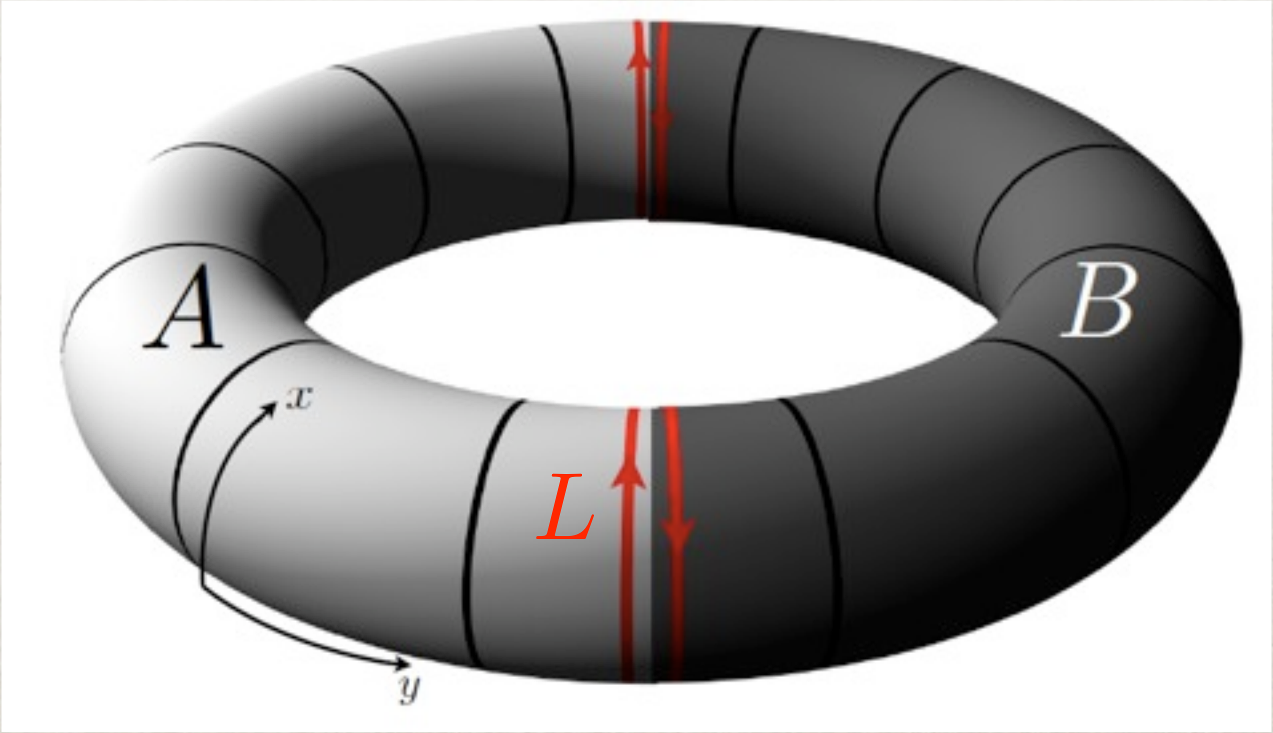
* Refinement of PEPS RVB is easy:
  - single vison energy (gap)
  - LR valence bonds
  - chiral RVB
  - spacial modulations => crystal

Extra material:

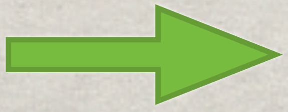
Topological FQH systems



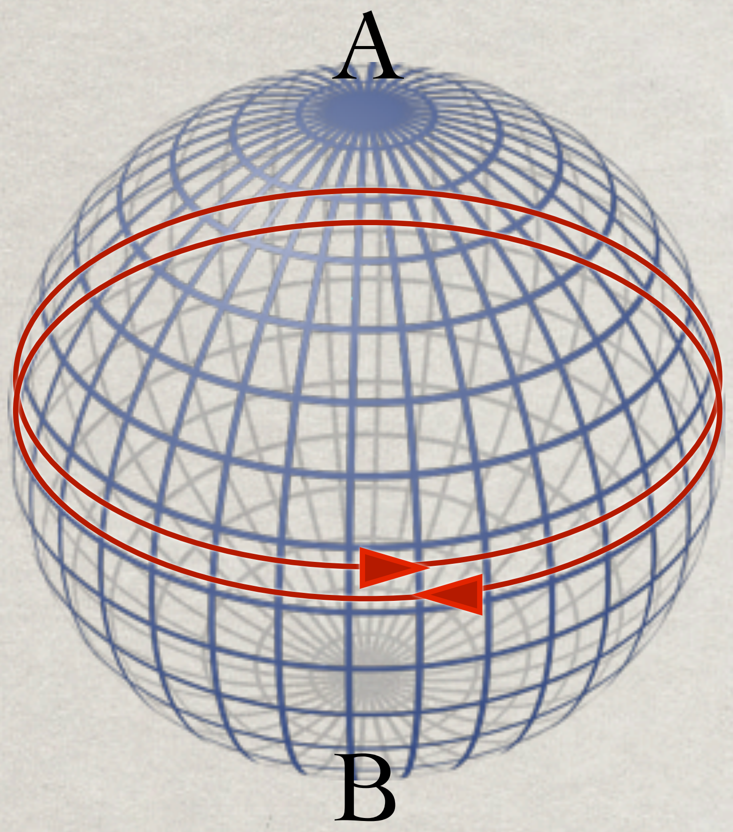
Li & Haldane
PRL 2008



Lauchli et al., 2009



Edge states ... connected to some entanglement features between A and B



Li & Haldane, 2008

Regnault, Bernevig & Haldane, 2009

"Haldane" Conjecture:

Precise correspondence between the entanglement spectrum of a FQH system partitioned into two sub-systems linked by some "edge" and the true sub-system spectrum

Questions:

- More generally, can the ES always be connected to the true edge spectrum ? Qi, Katsura and Ludwig, PRL 2012
- How do the ES/boundary Hamiltonian reflect bulk properties ?

Rewrite ρ_A as thermal density matrix

$$\rho_A = \sum_i \lambda_i^2 \, |i\rangle_A \langle i|_A$$

rewrite the weights as: $\lambda_i = \exp\left(-\xi_i/2\right)$

Entanglement spectrum : $\{\xi_i\}$

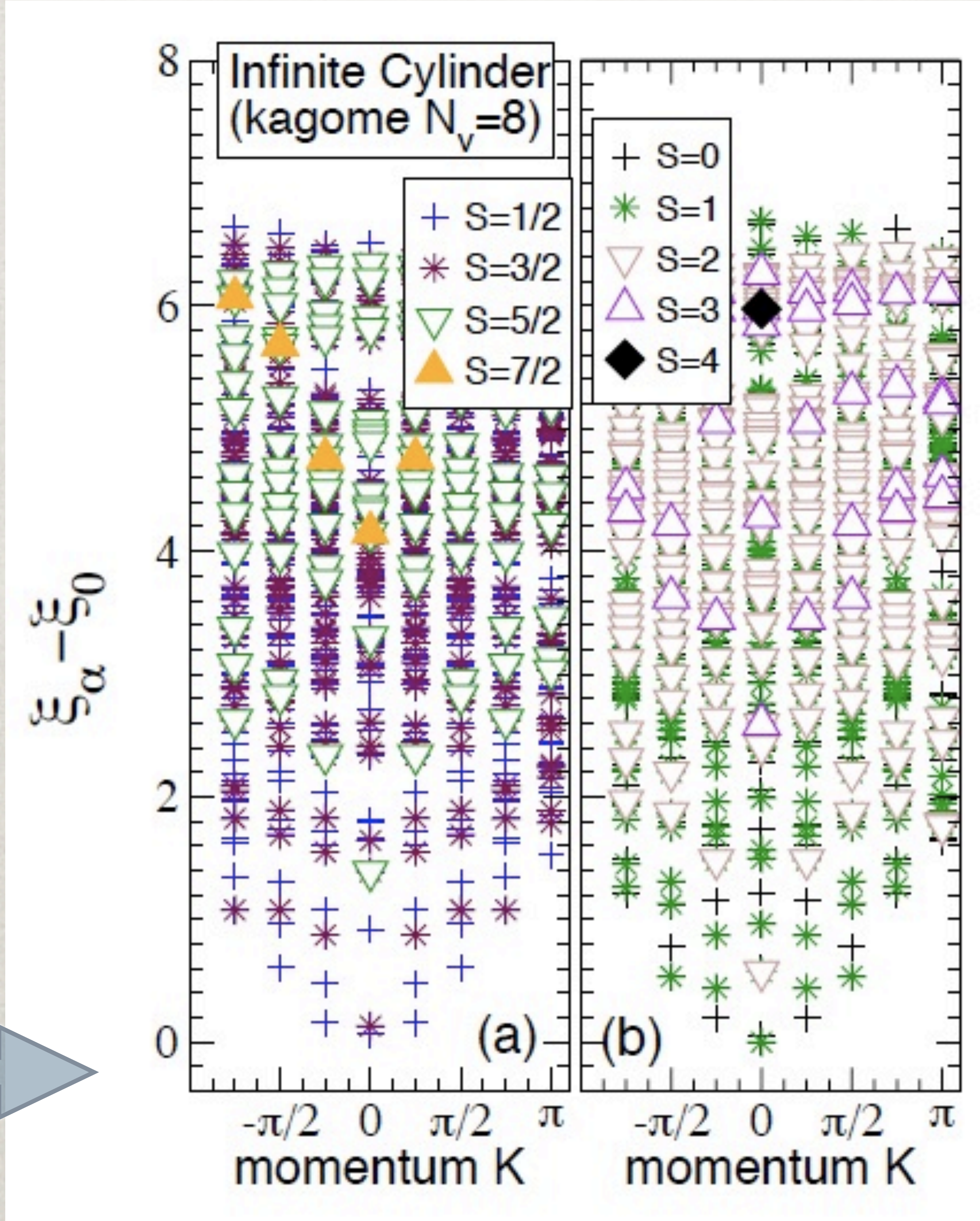
$$\rho_A = \exp\left(-\hat{\xi}\right)$$

Boundary Hamiltonien

Entanglement spectrum

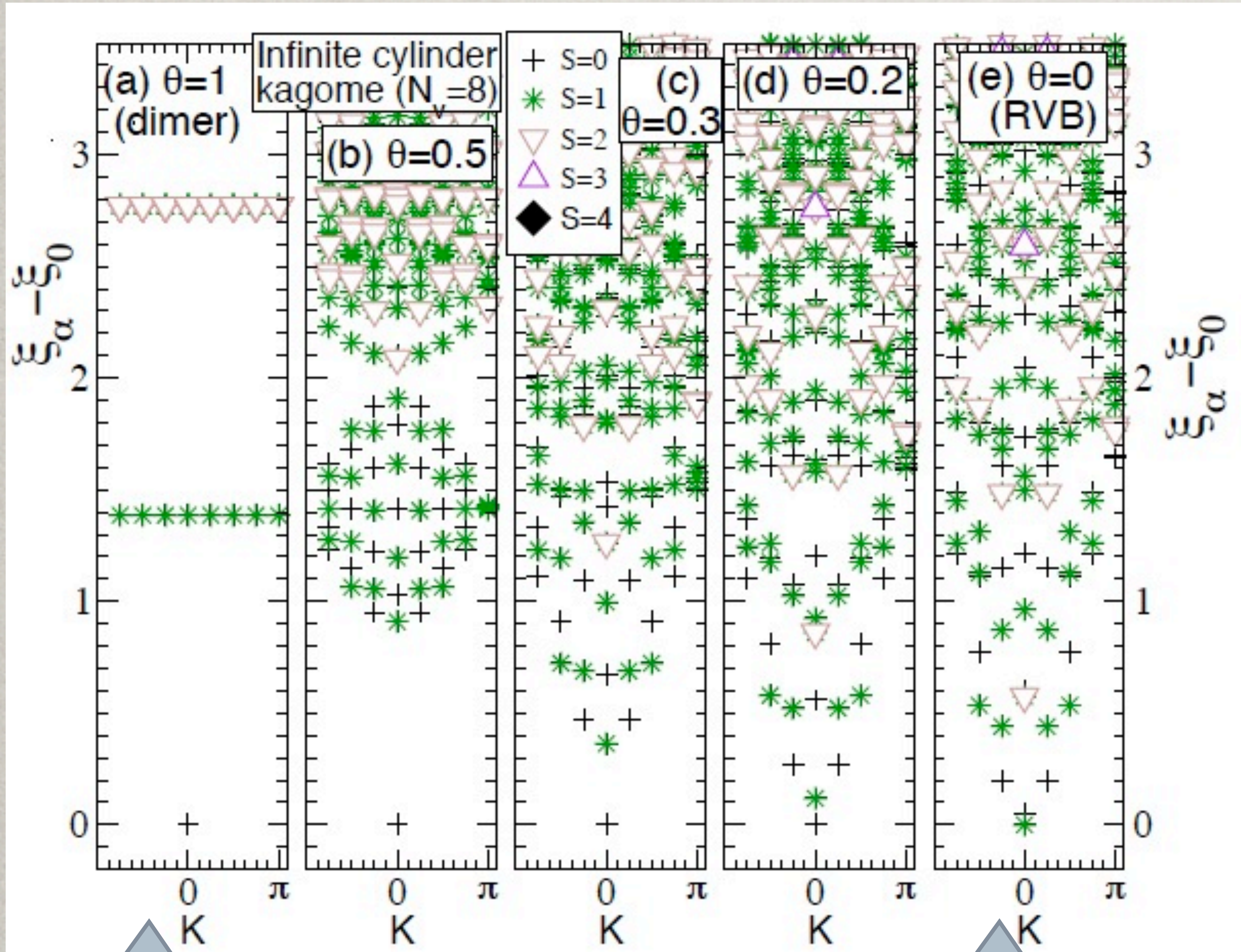
should reflect the **physical** edge spectrum

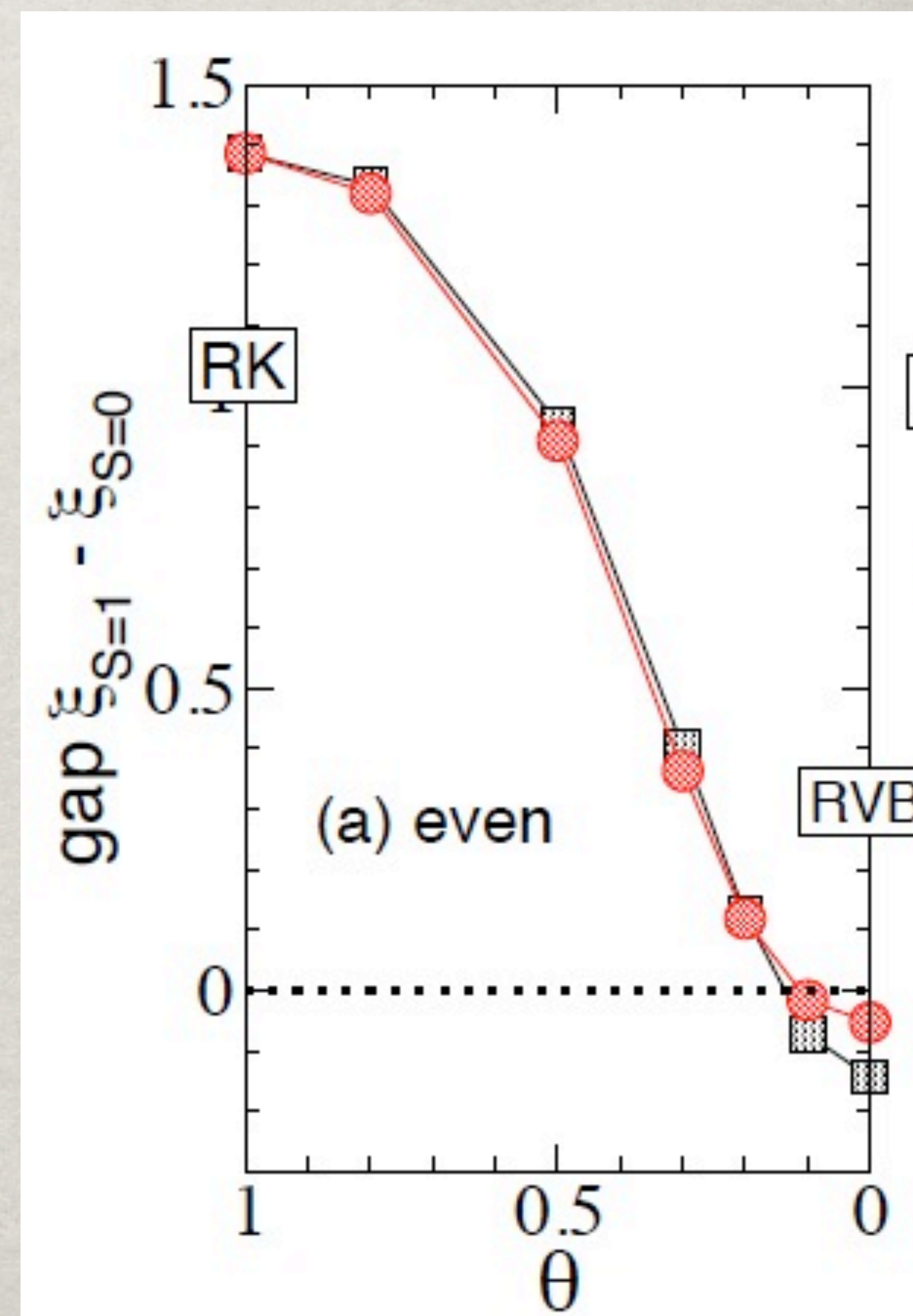
gapless ES !



Interpolation between orthogonal dimer and RVB states

ES spectrum reflects the edge spectrum



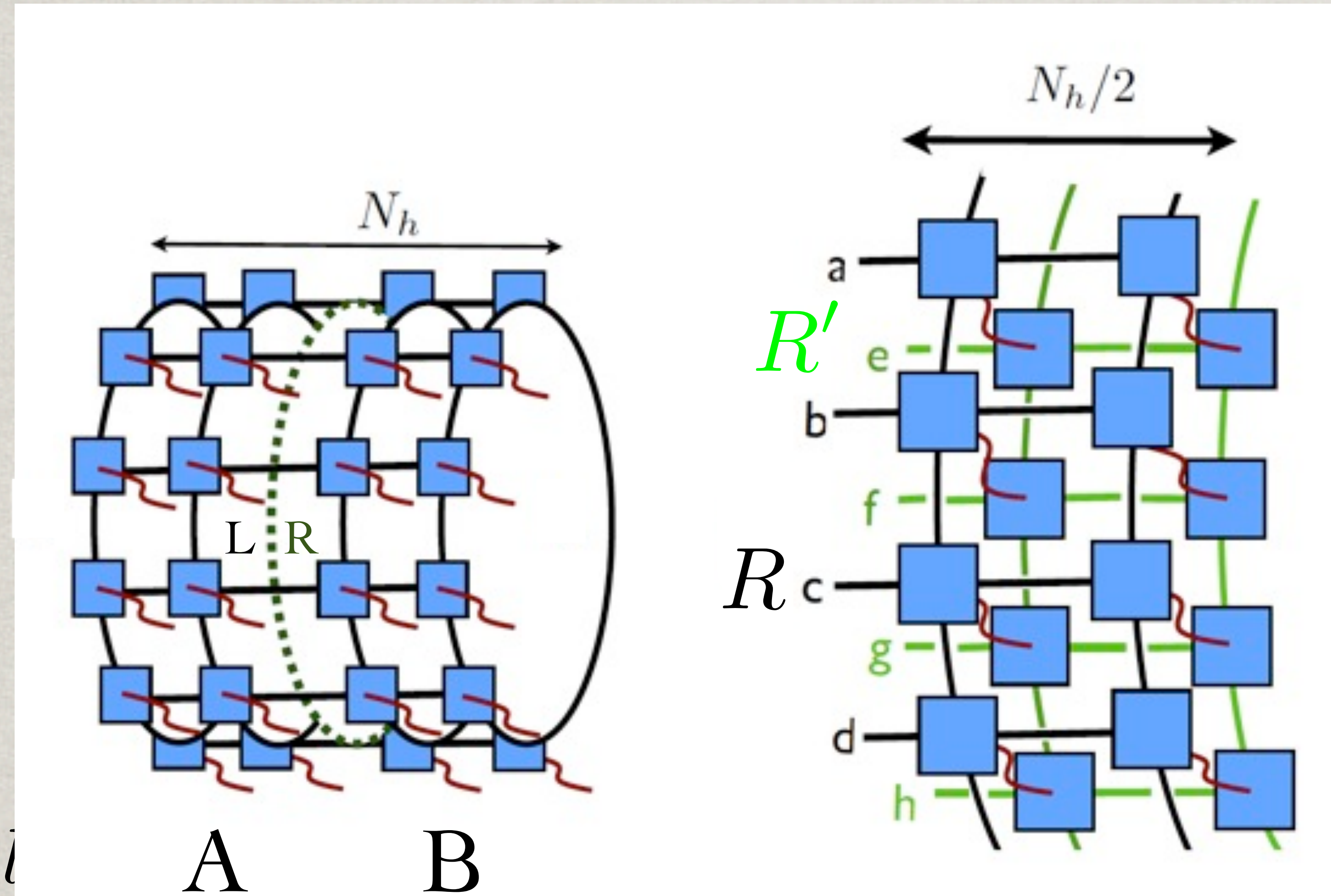


can be mapped onto Kitaev's toric code

gapless ES !

Holographic framework

ρ_A



σ_b^2

"lives" on the boundary: natural identification of EMERGING degrees of freedom

Basic formula: $\rho_A = U \sigma_b^2 U^\dagger$

isometry: maps 2D onto 1D

$$\sigma_b^2 = \exp(-H_b)$$

Consequence: expect area law !

Structure of the Boundary Hamiltonian

Topological sectors translate into
CONSERVATION LAWS of "transfer matrix"
[i.e. on (parity) of # of "2" on each row]

$$\sigma_b^2 = \exp\left(-\tilde{H}_b\right)$$

$$\tilde{H}_b = H_1 + \beta_\infty(\mathbf{1}^{\otimes N_v} - \mathcal{P})$$

$$\beta_\infty \to \infty$$

supported by the non-zero eigenvalue sector of the RDM

$$H_1 = H_{\text{local}}\mathcal{P}$$

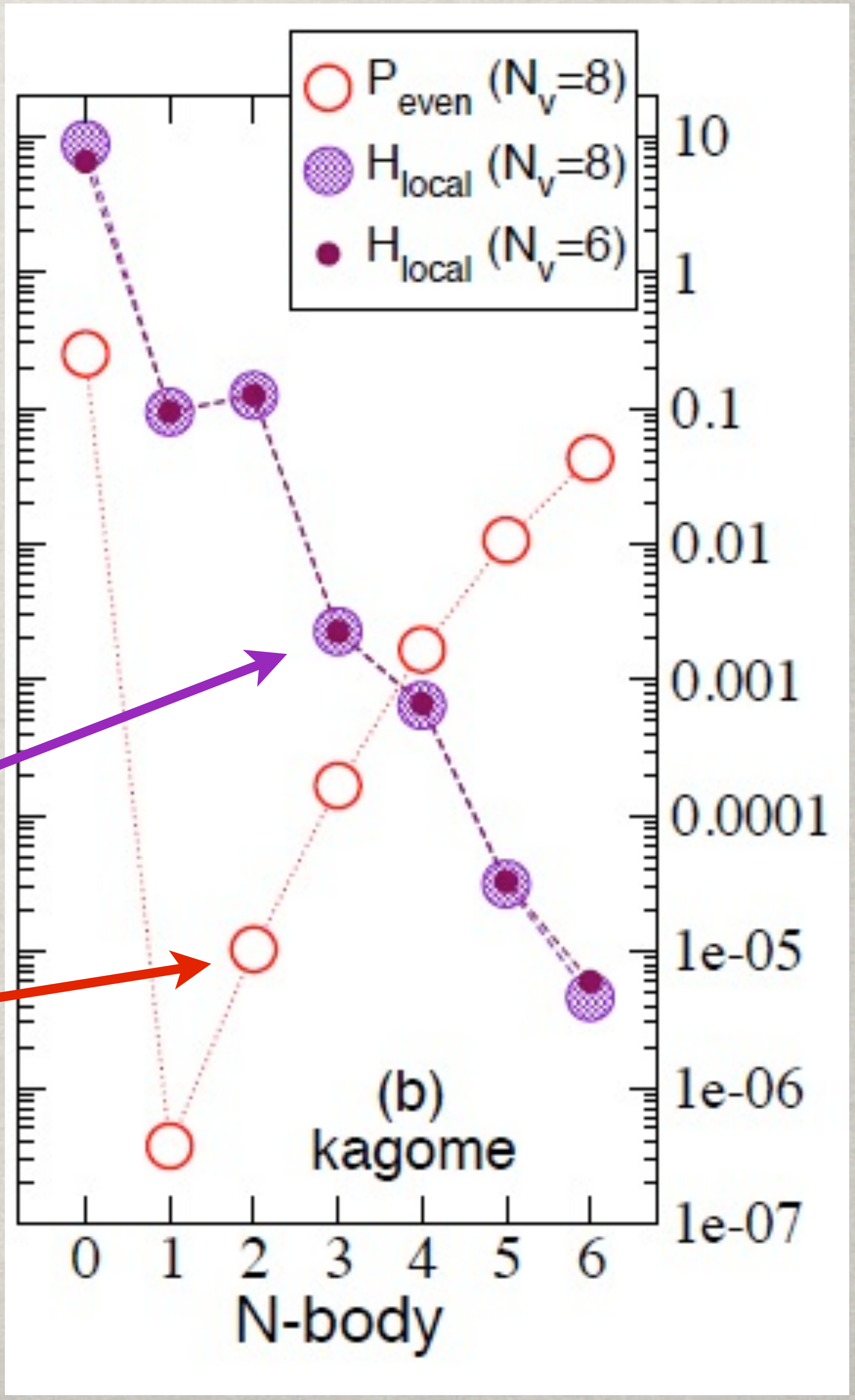
projector characterizing the sectors

Non-locality of Boundary Hamiltonian (acting on the edge)

local operator (on the edge):
$D \times D$ matrix $\Rightarrow$ basis of D^2 operators

$$\mathcal{O}_{\text{edge}} = c_0 N_v + \sum_{\lambda,i} c_\lambda \hat{x}^i_\lambda + \sum_{\lambda,\mu,r,i} d_{\lambda\mu}(r)\, \hat{x}^i_\lambda \hat{x}^{i+r}_\mu + \sum_{\lambda,\mu,\nu,r,r',i} e_{\lambda\mu\nu}(r,r')\, \hat{x}^i_\lambda \hat{x}^{i+r}_\mu \hat{x}^{i+r'}_\nu + \cdots ,$$

$$H_1 = H_{\text{local}} \mathcal{P}$$



H_{local} is an extended t-J model !

Topological entropy

of states on the edge contributing to even sector:

$$\# = 3^{N_v}/2$$

$$S_{\mathrm{VN}} \sim -\ln \#$$

$$S_{\mathrm{VN}} = S_0 + AN_v$$