Frustrated Magnetism and Quantum Spin Liquids, KITP, 2012

A phenomenological theory for the Z_2 spin liquid phase of S=1/2 Kagome Heisenberg Antiferromagnet

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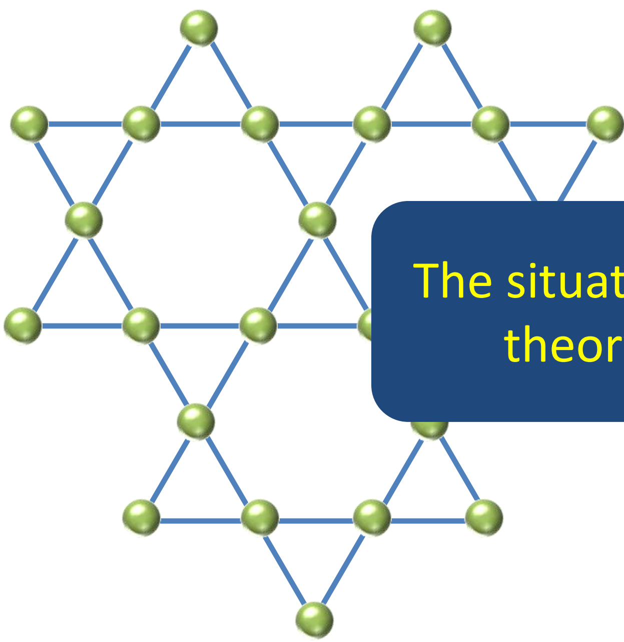
In collaboration with Oleg Tchernyshyov

Kagome HAF: The problem

$$H = \sum_{\langle ij \rangle} S_i \cdot S_j \quad (S = 1/2)$$



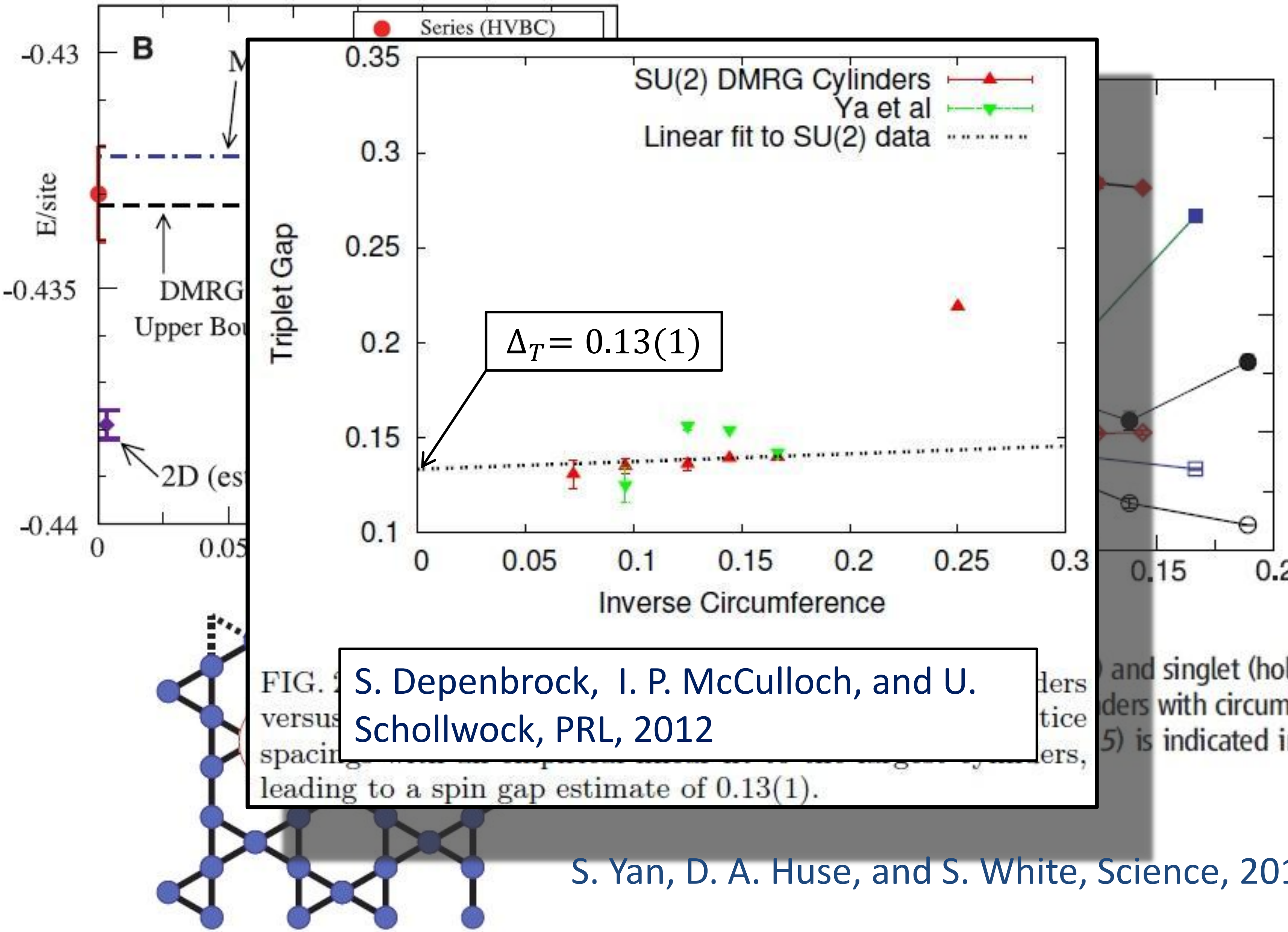
The "landscape" of competing T=0 phases:

- Valence bond crystals (Series expansion, QDM+ED, MERA)
- +VMC,
- "Chiral topological spin liquid" (SBMFT)
- "p6 symmetry" breaking state (CORE)
- "Striped spin liquid crystal" (SFMFT+VMC)

The situation is not hopeless comparing to string theory landscape problem (10^{500} vacua).

C. Zeng & V. Elser, PRB 1990; P. Nikolic & T. Senthil, PRB 2003; R. R. P. Singh & D. A. Huse, PRB 2007; Y. Ran et al, PRL 2007; D. Poilblanc et al, PRB 2010; G. Evenbly & G. Vidal, PRL 2010; S. Yan et al, Science 2011; S. Depenbrock et al, PRL 2012; L. Messio et al, PRL 2012; Y. Iqbal et al, arXiv: 1209.1928; B. Clark et al, arXiv: 1210.1585; S. Capponi, arXiv: 1210.5519…

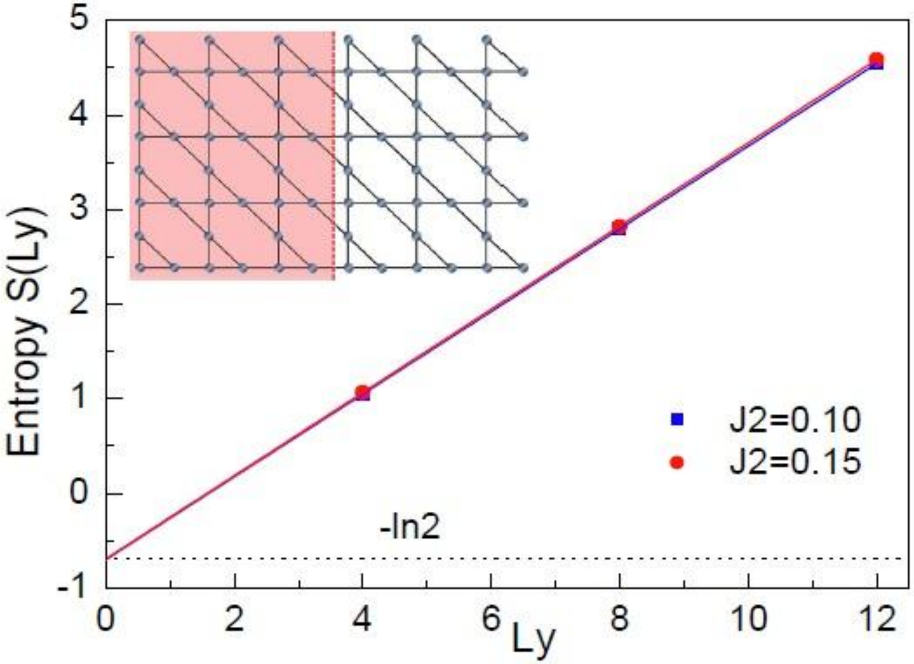
KHAF ground state according to DMRG



S. Depenbrock, I. P. McCulloch, and U. Schollwock, PRL, 2012

S. Yan, D. A. Huse, and S. White, Science, 2011

KHAF ground state according to DMRG



$$S(L_y) = aL_y - \gamma$$

$\gamma = 0.698\ (J_2 = 0.10)$

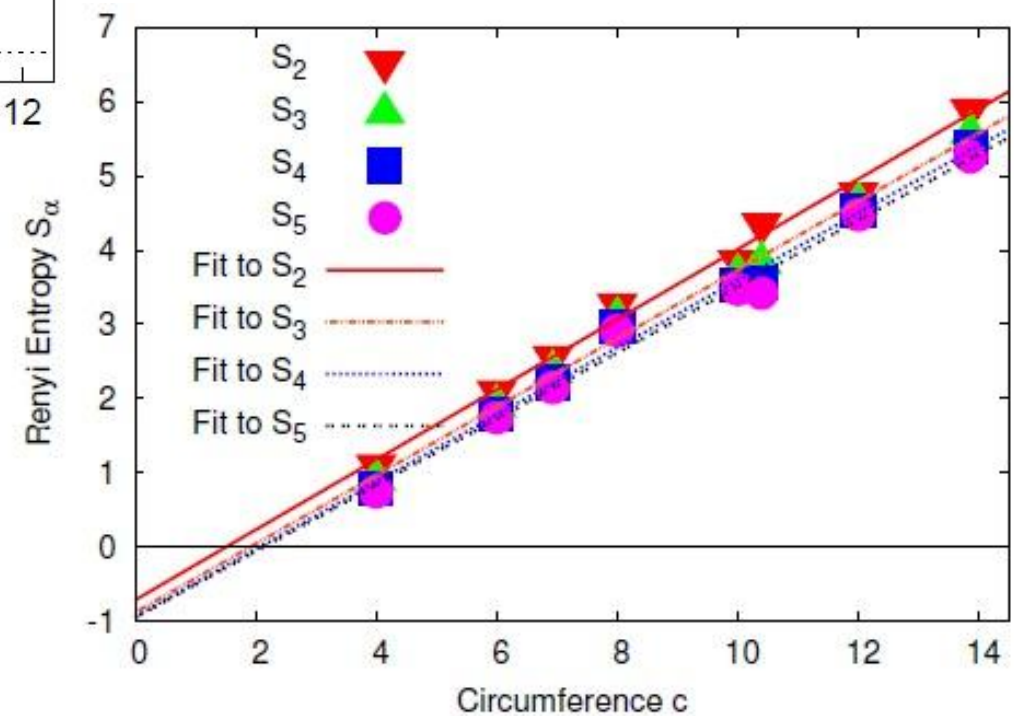
$\gamma = 0.694\ (J_2 = 0.15)$

Theory: $\gamma = \ln(2) = 0.693$

(alternatively, $\gamma = log_2(2) = 1$)

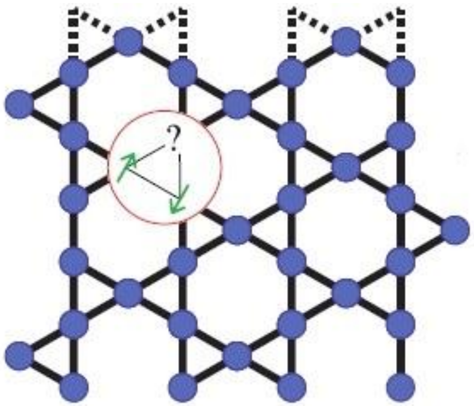
S. Depenbrock, I. P. McCulloch, and U. Schollwock, PRL, 2012

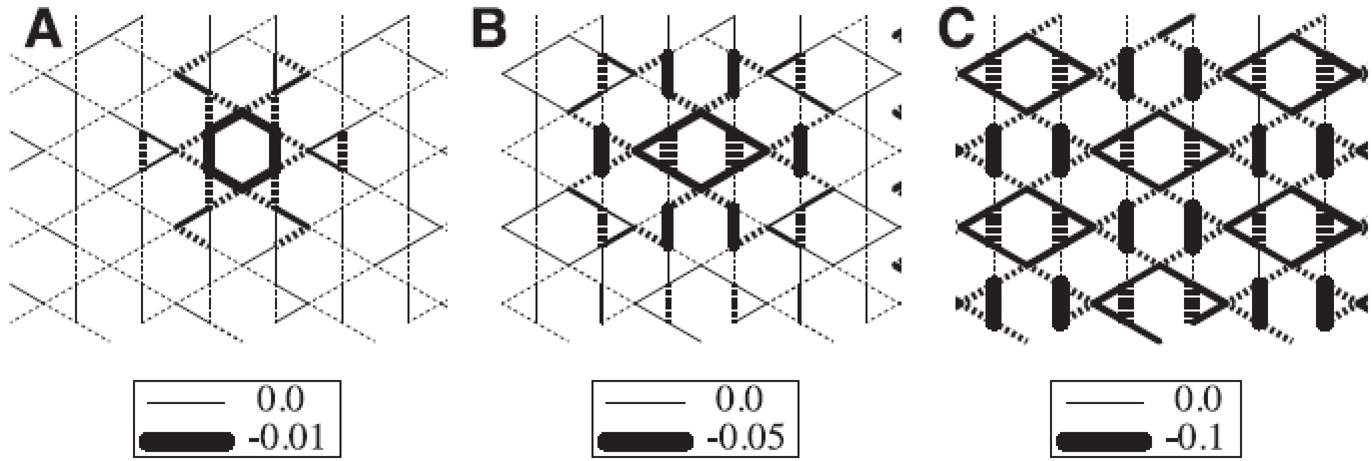
H. C. Jiang, Z. H. Wang, and L. Balents, arXiv: 1201.4289.



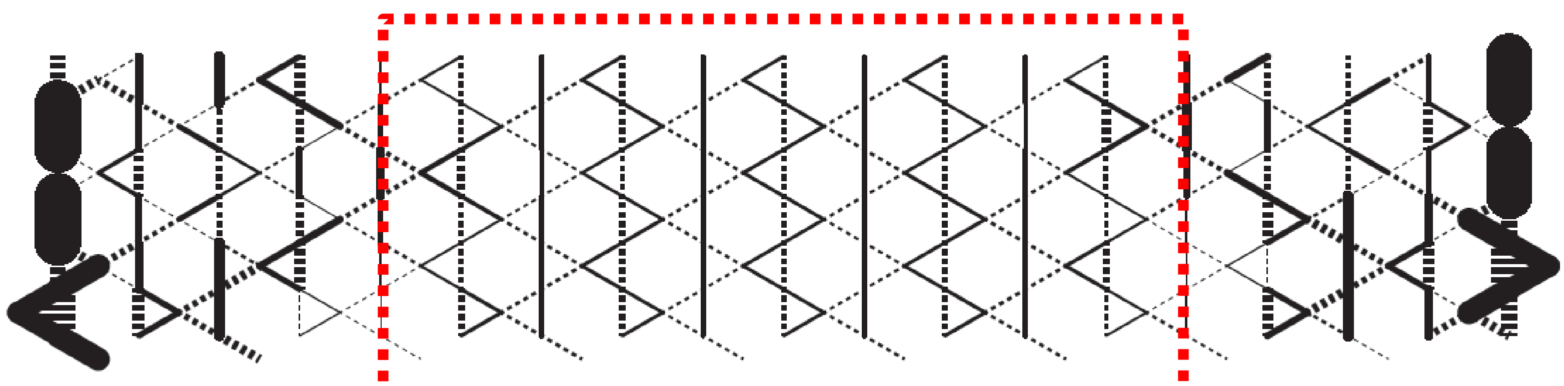
KHAF ground state according to DMRG

"Diamond Resonance"





"Even-Odd Effect" (c.f. J1-J2 Square HAF, H. C. Jiang et al, PRB 2012)



From S. White, KITP talk, 2010

A summary of DMRG findings

“Universal” properties of
ga
sp

- No discernible signature of SU(2) symmetry or translational symmetry breaking.
- A sizable spin triplet gap

Can we construct a phenomenological Hamiltonian capturing the DMRG phenomenology?

- Z_2 topological entanglement entropy.
- “Diamond resonance”.
- Even-odd effect and the valence bond density wave.

cf. H. C. Jiang, H. Yao, and L. Balents, PRB 2012.

" With four parameters I can fit an elephant, and with five I can make him wiggle his trunk.."

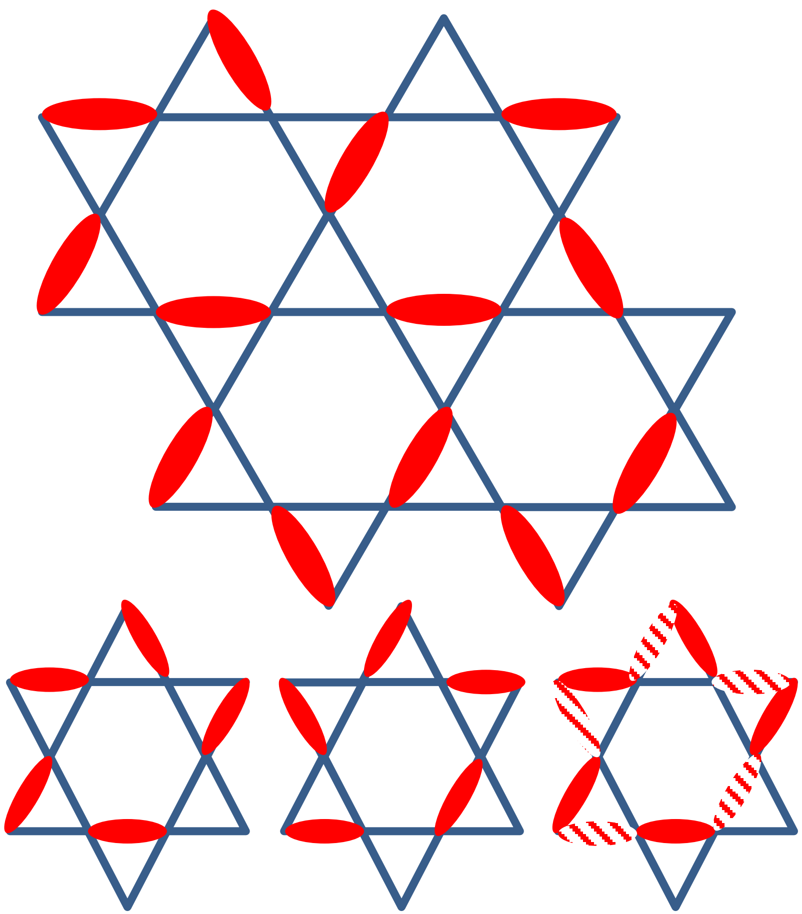
-- John von Neumann

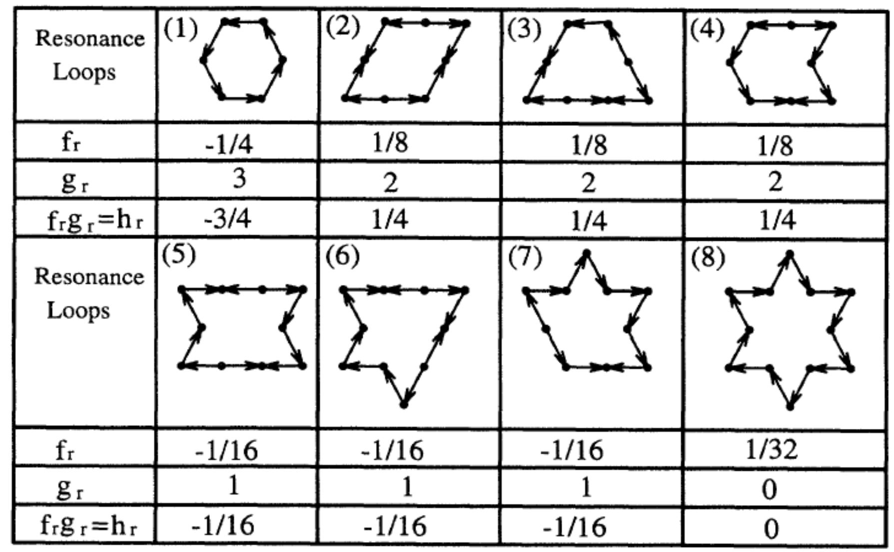
The goal: make the elephant wiggle his trunk with two parameters.

Quantum Spin Liquid

Quantum Spin Liquid

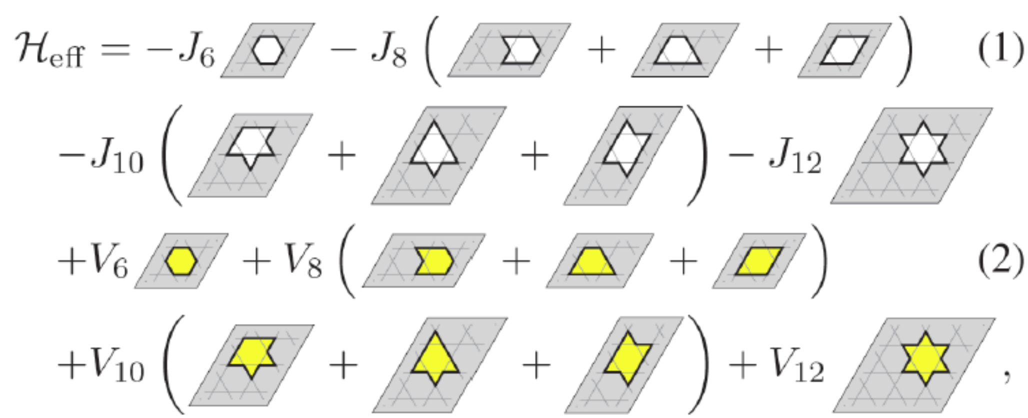
Constructing kagome quantum dimer model





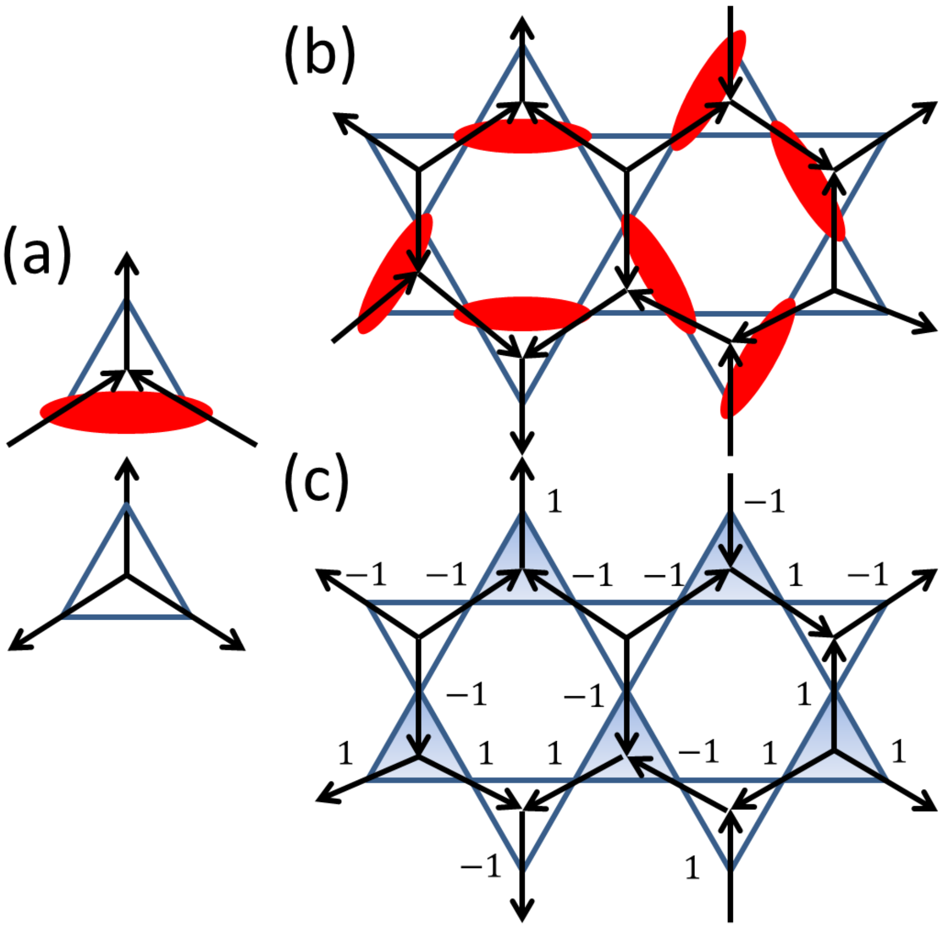
Resonance Loops	(1)	(2)	(3)	(4)
f_r	-1/4	1/8	1/8	1/8
g_r	3	2	2	2
$f_r g_r = h_r$	-3/4	1/4	1/4	1/4
Resonance Loops	(5)	(6)	(7)	(8)
f_r	-1/16	-1/16	-1/16	1/32
g_r	1	1	1	0
$f_r g_r = h_r$	-1/16	-1/16	-1/16	0

C. Zeng and V. Elser, PRB 1995.



D. Poilblanc, M. Mambrini and D. Schwandt, PRB 2010.

From kagome QDM to Z_2 gauge theory



Arrow rep: V. Elser and C. Zeng, PRB 1993.

Cf. P. Nikolic and T. Senthil, PRB 2003; Y. Huh, M. Punk and S. Sachdev, PRB 2011.

Map arrows to Z_2 electric fluxes:

$$\sigma^x_{ij} = \begin{cases} 1 & \text{pointing from A to B} \\ -1 & \text{pointing from B to A} \end{cases}$$

Constraint:

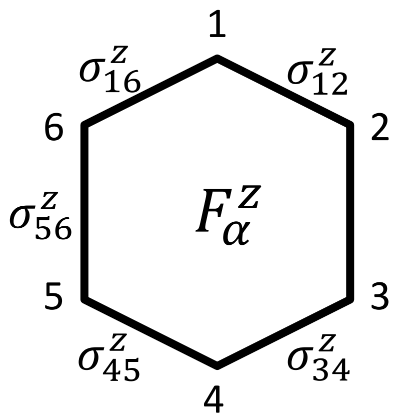
$$Q_i \equiv \sigma^x_{i1}\sigma^x_{i2}\sigma^x_{i3} = \begin{cases} 1 & i \in A \\ -1 & i \in B \end{cases}$$

Z_2 Gauss Law!

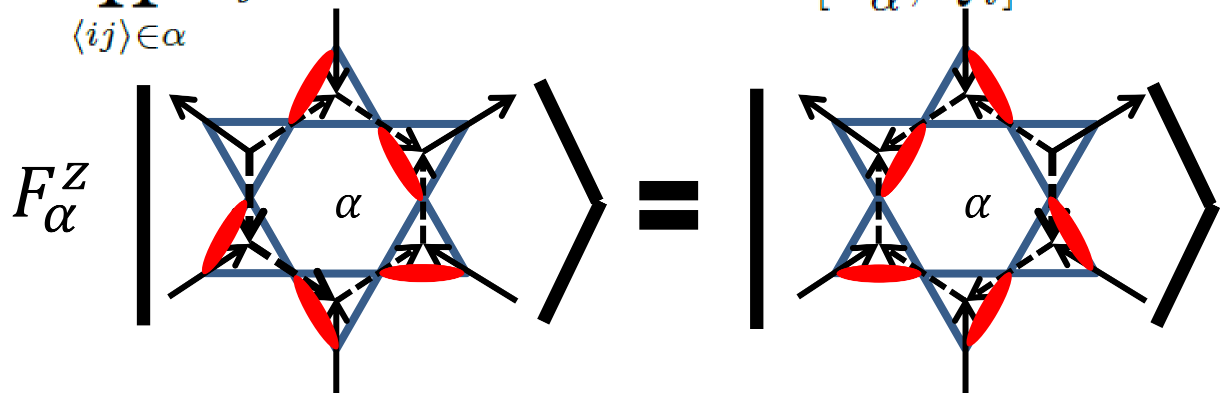
The Hilbert space of kagome QDM is exactly identical to the Hilbert space of honeycomb Z_2 gauge theory; Any kagome QDM Hamiltonian can be written as a Z_2 gauge theory Hamiltonian.

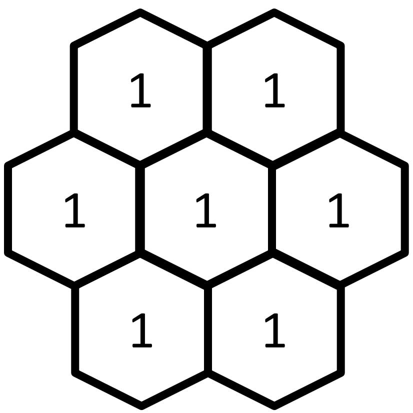
Cf. Poilblanc, Misguich, etc

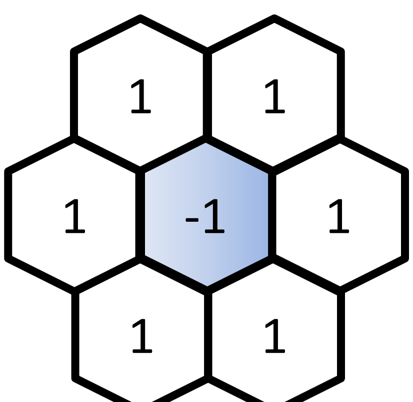
Misguich-Serban-Pasquier Hamiltonian* and Z_2 gauge theory



$$F^z_\alpha = \prod_{\langle ij\rangle \in \alpha} \sigma^z_{ij} = \sigma^z_{12}\sigma^z_{23}\sigma^z_{34}\sigma^z_{45}\sigma^z_{56}\sigma^z_{61} \qquad [F^z_\alpha, Q_i] = 0 \quad \forall \alpha,\ i,$$







Hamiltonian (h>0):

$$H = -h \sum_\alpha F^z_\alpha,$$

Ground State Obeys:

$$F^z_\alpha |G\rangle = |G\rangle; \quad \forall \alpha.$$

The ground state is the Rohksar-Kivelson state:

$$|G\rangle = \sum_{\text{arrow patterns } \alpha} |\alpha\rangle = \sum_{\text{dimer coverings } D} |D\rangle$$

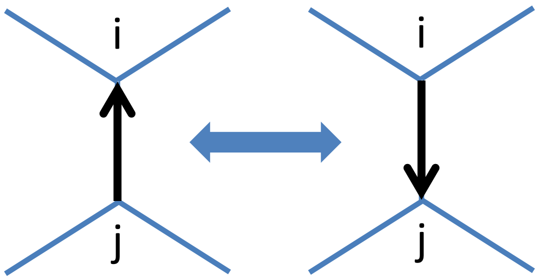
Extremely short-ranged correlations; Dispersionless visons.

*G. Misguich, D. Serban, and V. Pasquier, PRL 2002.

Building the phenomenological Hamiltonian

Perturbing the MSP Hamiltonian:

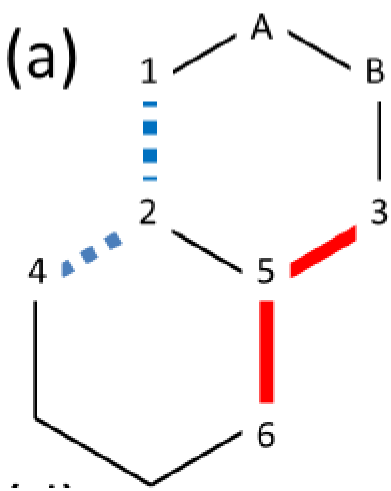
$$H = -h\sum_{\alpha} F^z_{\alpha} - \sum_{\langle ij \rangle} A^{(1)}_{ij}\sigma^x_{ij} - \sum_{\langle ij \rangle, \langle kl \rangle} A^{(2)}_{ij;kl}\sigma^x_{ij}\sigma^x_{kl}.$$



Inversion symmetry demands:

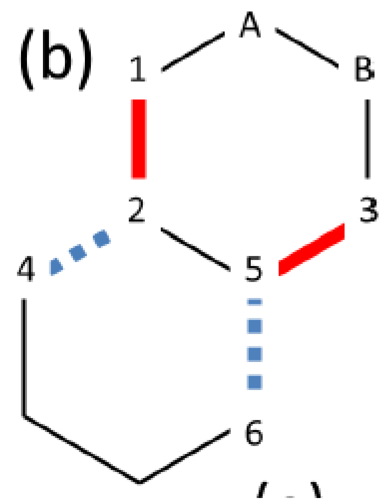
$$A^{(1)}_{ij} = 0,$$

N.N and 2nd N interactions vanish:

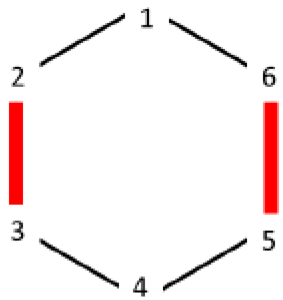


$$\sigma^x_{12}\sigma^x_{24} + \sigma^x_{35}\sigma^x_{56} = \sigma^x_{25} - \sigma^x_{25} = 0.$$

$$\sigma^x_{12}\sigma^x_{35} + \sigma^x_{24}\sigma^x_{56} = (\sigma^x_{24}\sigma^x_{25})(-\sigma^x_{56}\sigma^x_{25}) + \sigma^x_{24}\sigma^x_{56}$$
$$= -\sigma^x_{24}\sigma^x_{56} + \sigma^x_{24}\sigma^x_{56} = 0$$

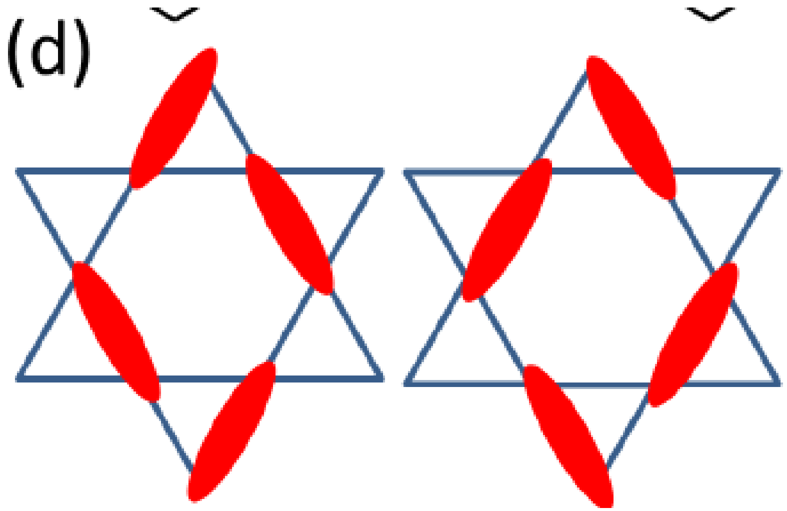


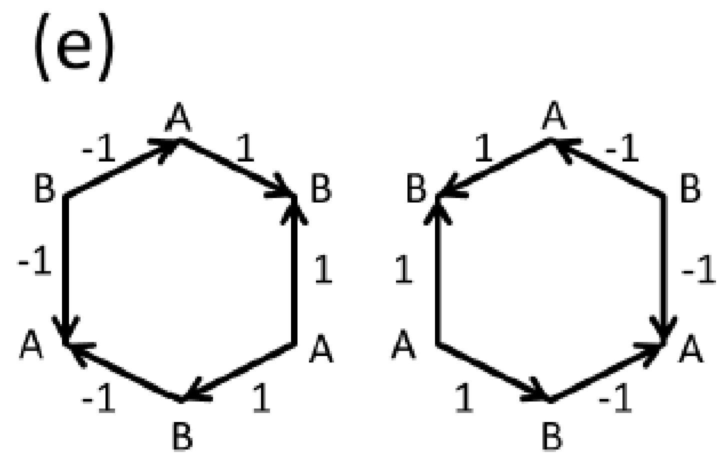
Building the phenomenological Hamiltonian



$$H = -h \sum_{\alpha} F_{\alpha}^{z} + K \sum_{3.n.} \sigma_{ij}^{x} \sigma_{kl}^{x},$$

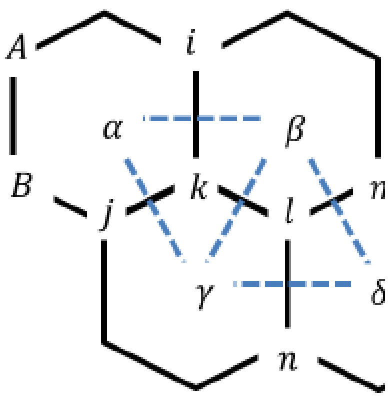
$(h > 0, K > 0 \; or < 0?)$

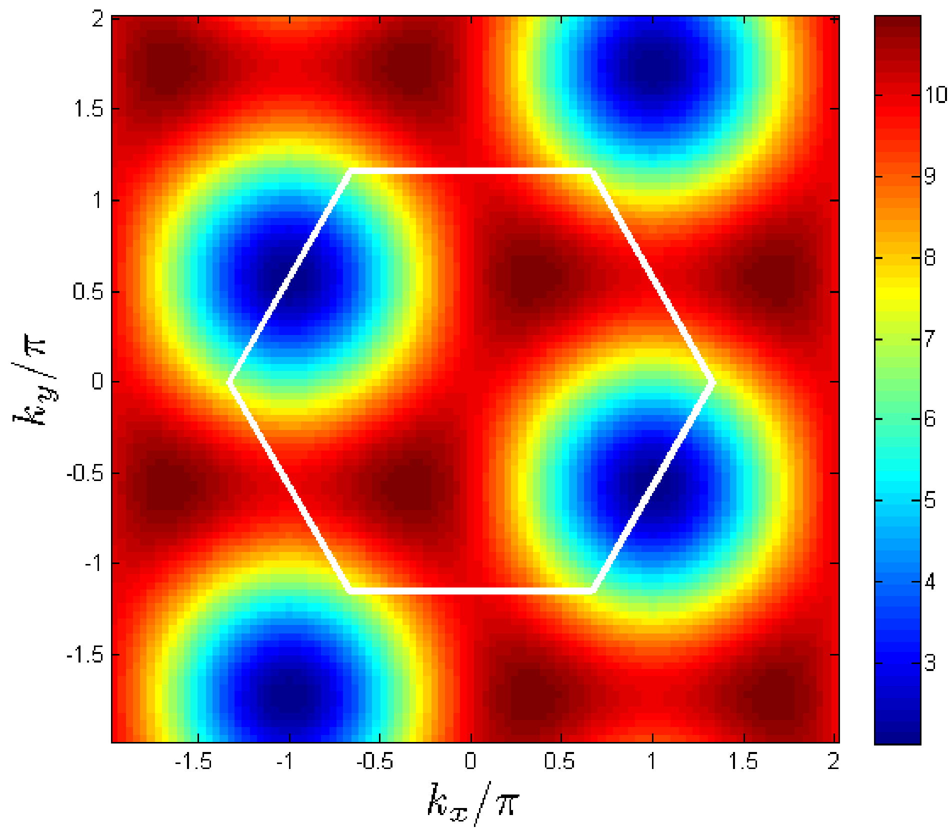




"Diamond resonance" → K>0

The dual Ising theory





onary:

nsverse field term

Ising coupling

stration condition

$+ H_D$

del

Cf. the four-component real field ϕ_i transforming under PSG $GL(2,\mathbb{Z}_3)$ in Huh, Punk & Sachdev, PRB 2011.

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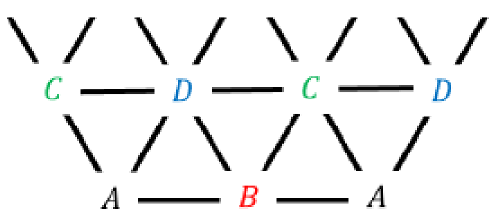
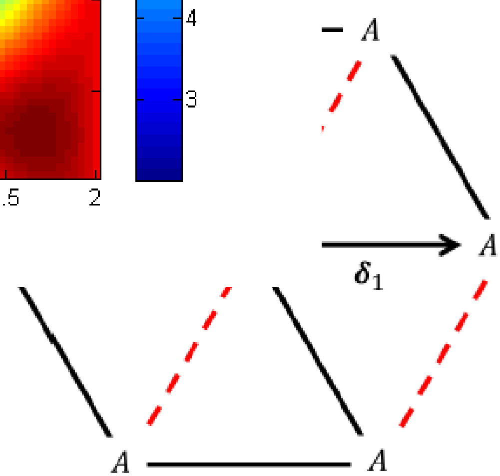
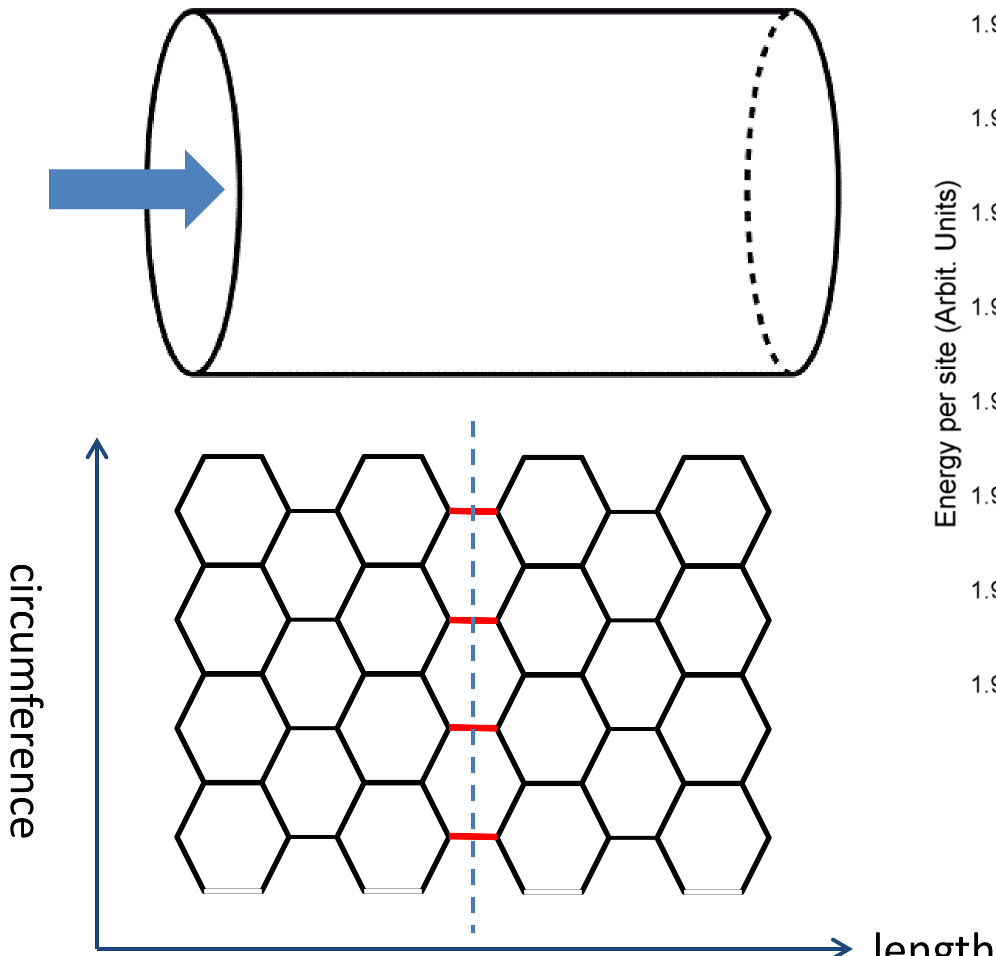


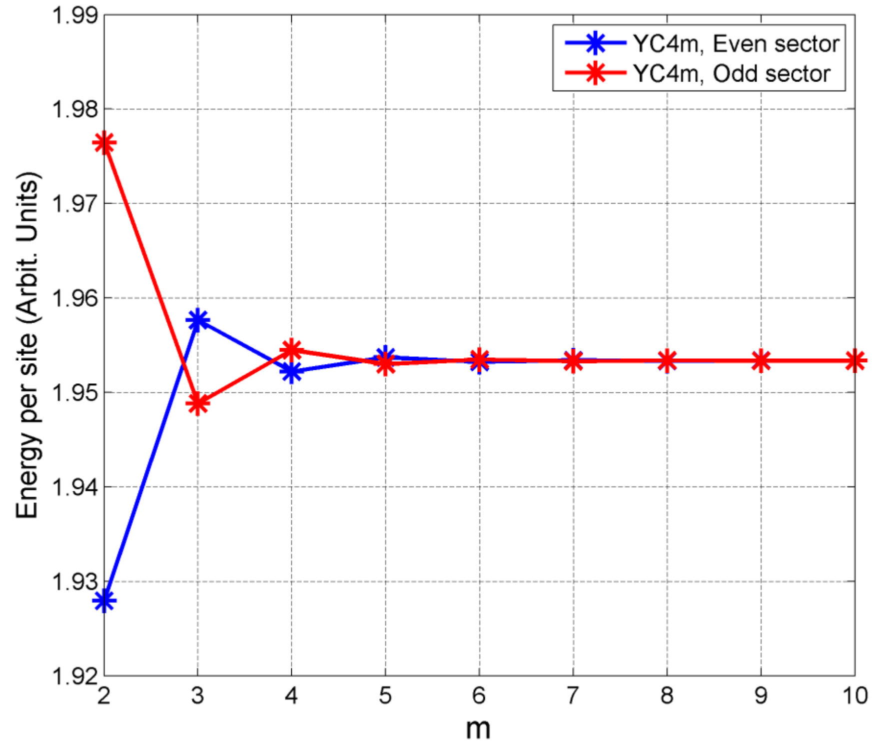


Does the phenomenological theory reproduce the numerics?

Lifted topological degeneracy on cylinder

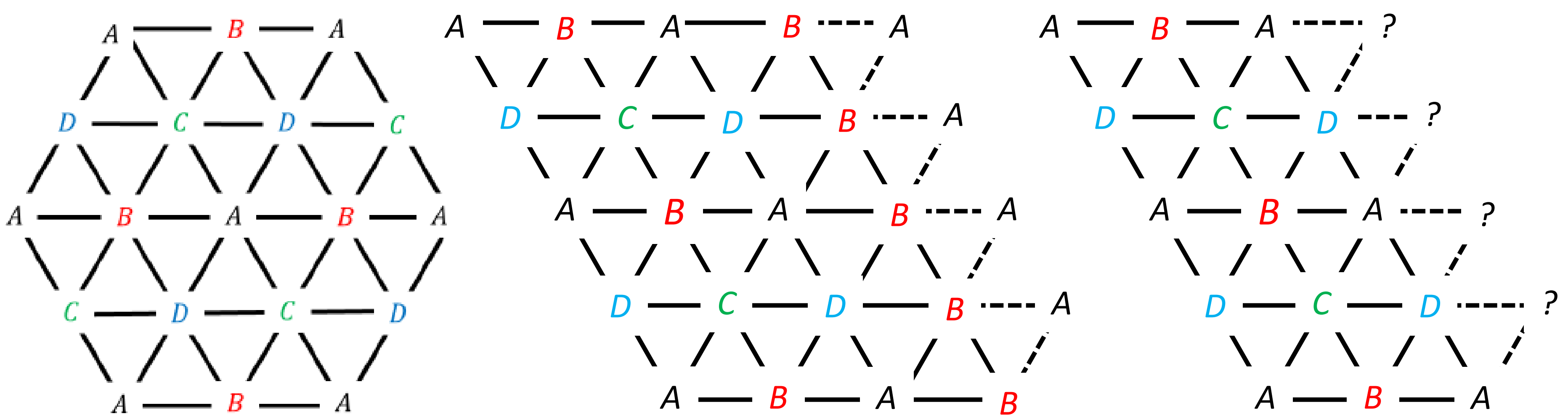
Threading electric flux = 1 (even) or -1(odd)

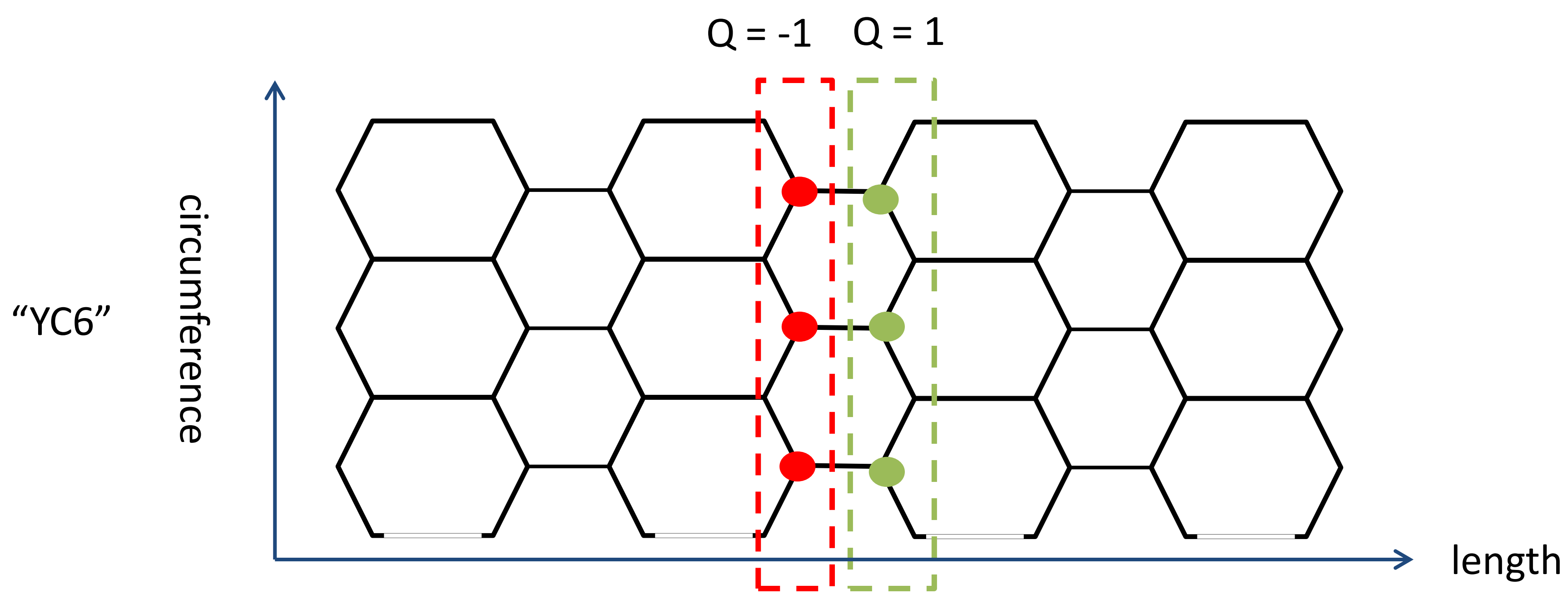




Even-odd effect

Infinite lattice: Unfrustrating periodic bc: Frustrating periodic bc:

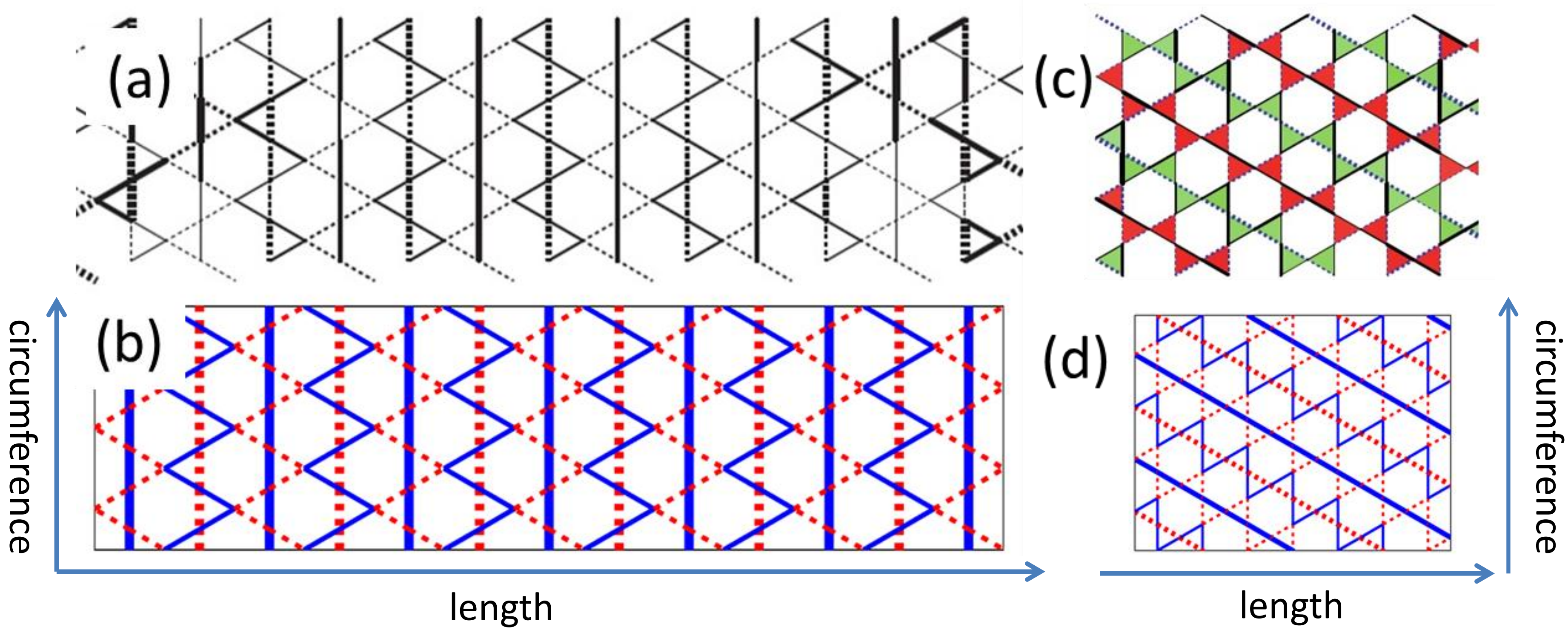




Even-odd effect

YC6 cylinder (top: DMRG; bottom: theory)

YC9-2 cylinder

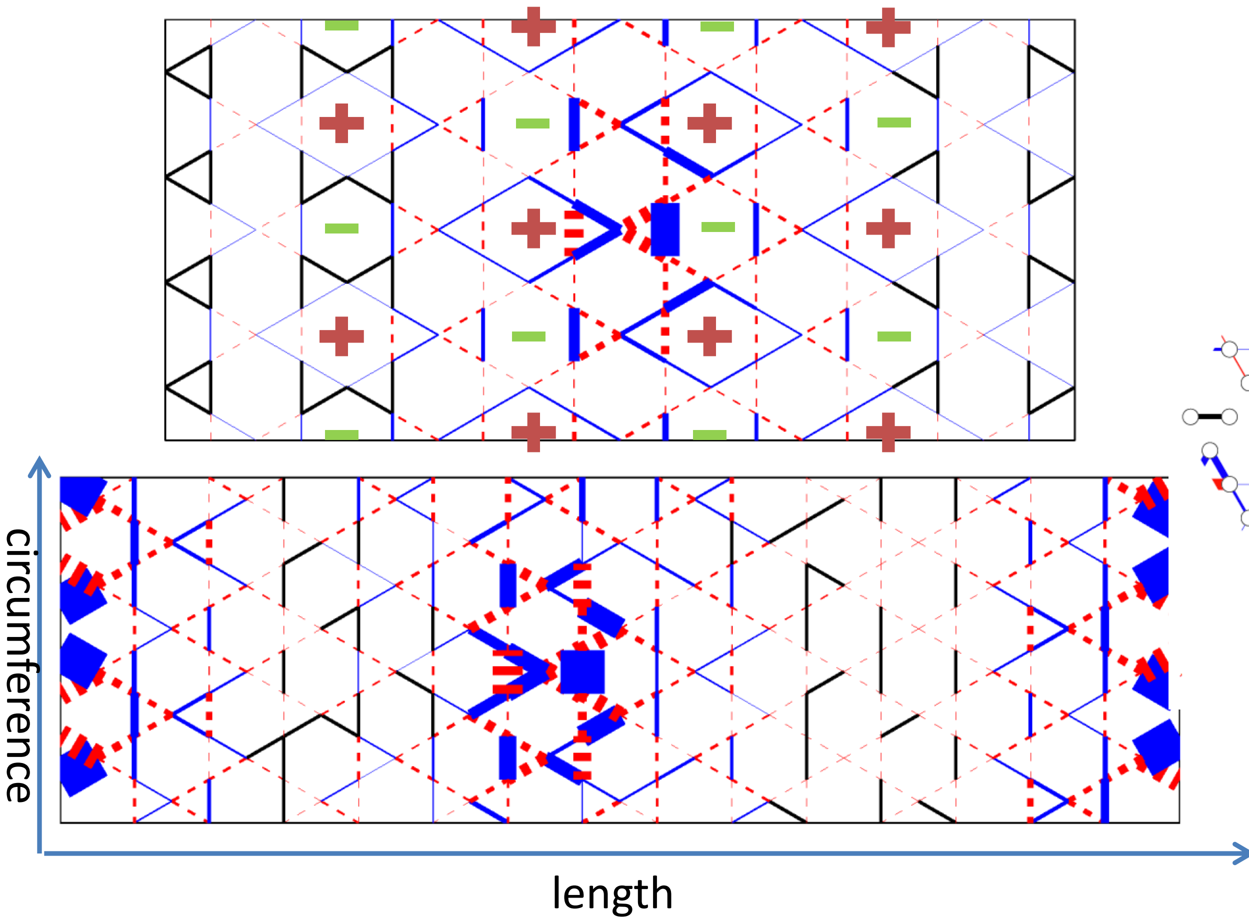


S. White, unpublished, 2010

S. Yan, D. A. Huse, and S. White, Science 2011.

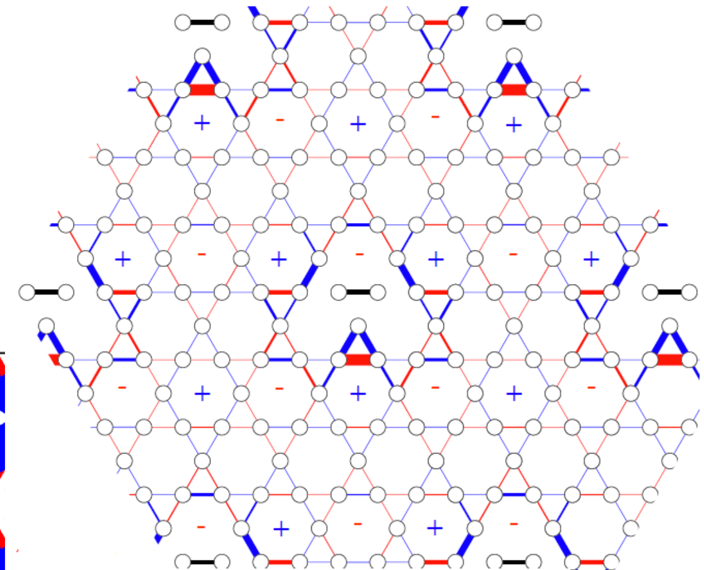
Dimer-dimer correlation

YC8 cylinder: theory (top) and DMRG (bottom)



S. White, private communications, 2011.

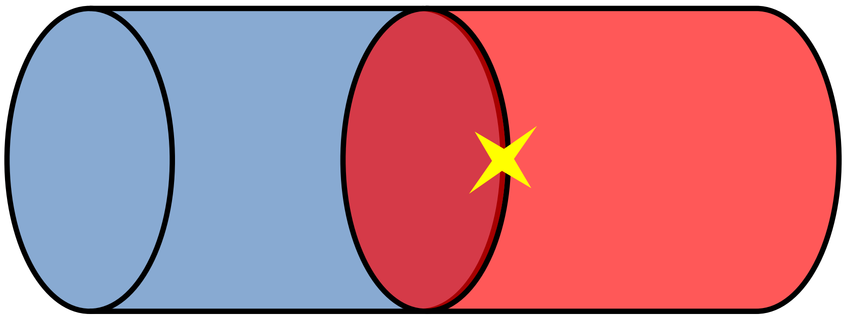
N=48 cluster Lanczos

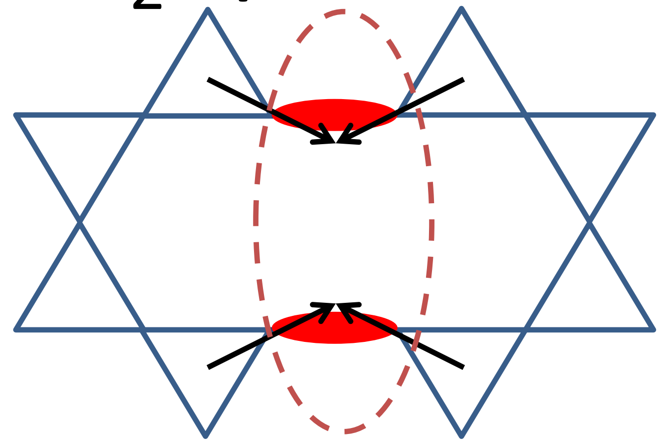


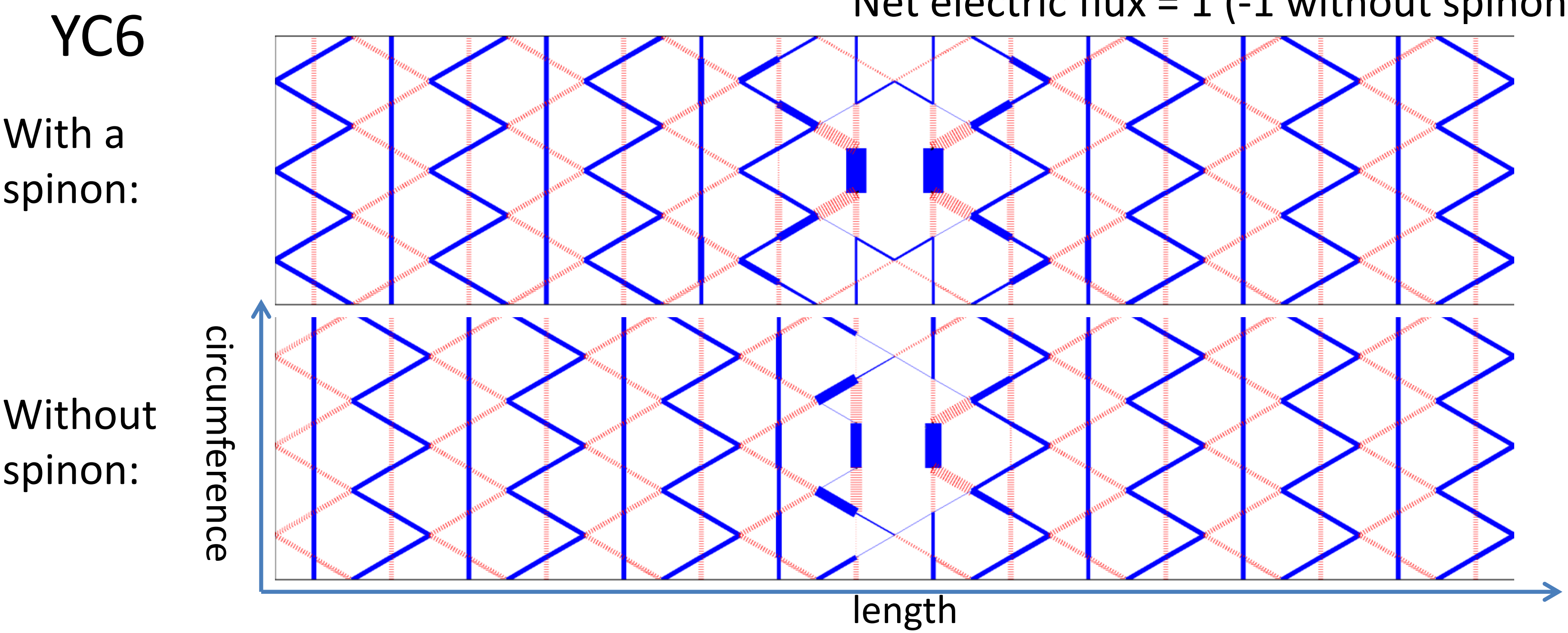
A. Läuchli, unpublished, 2012

Vacancy as a static Z_2 spinon

A spinon is a domain-wall of topological sectors

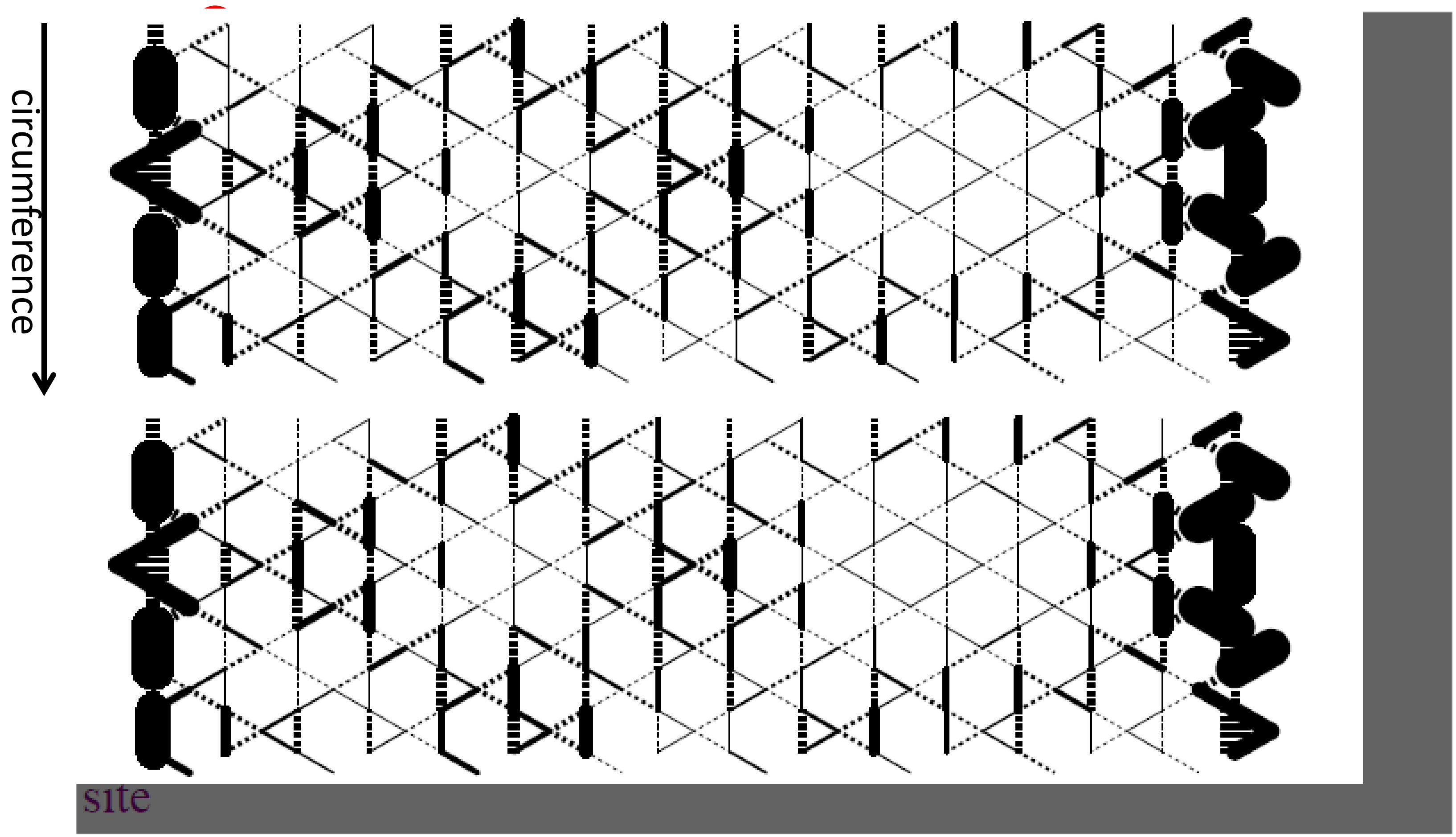






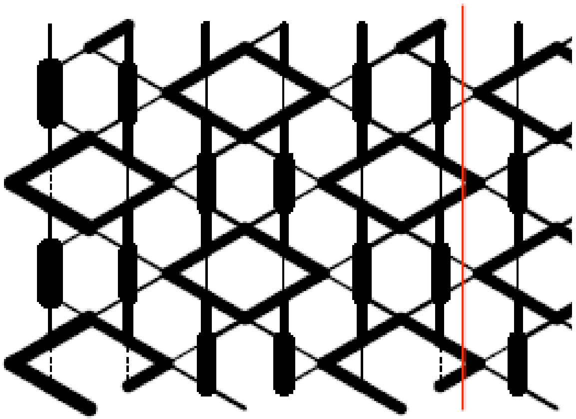
S. Dommange, M. Mambrini, B. Normand, and F. Mila, PRB 2003.

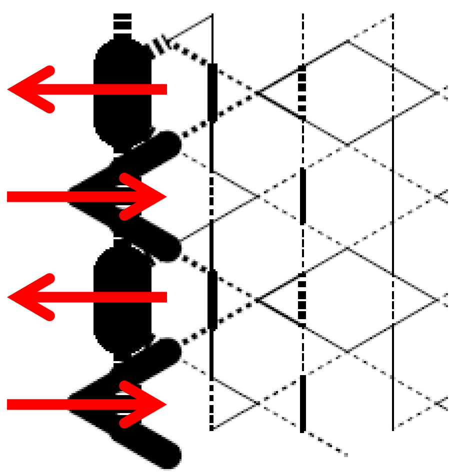
Mystery of the open boundary conditions



From S. R. White, KITP talk, 2010.

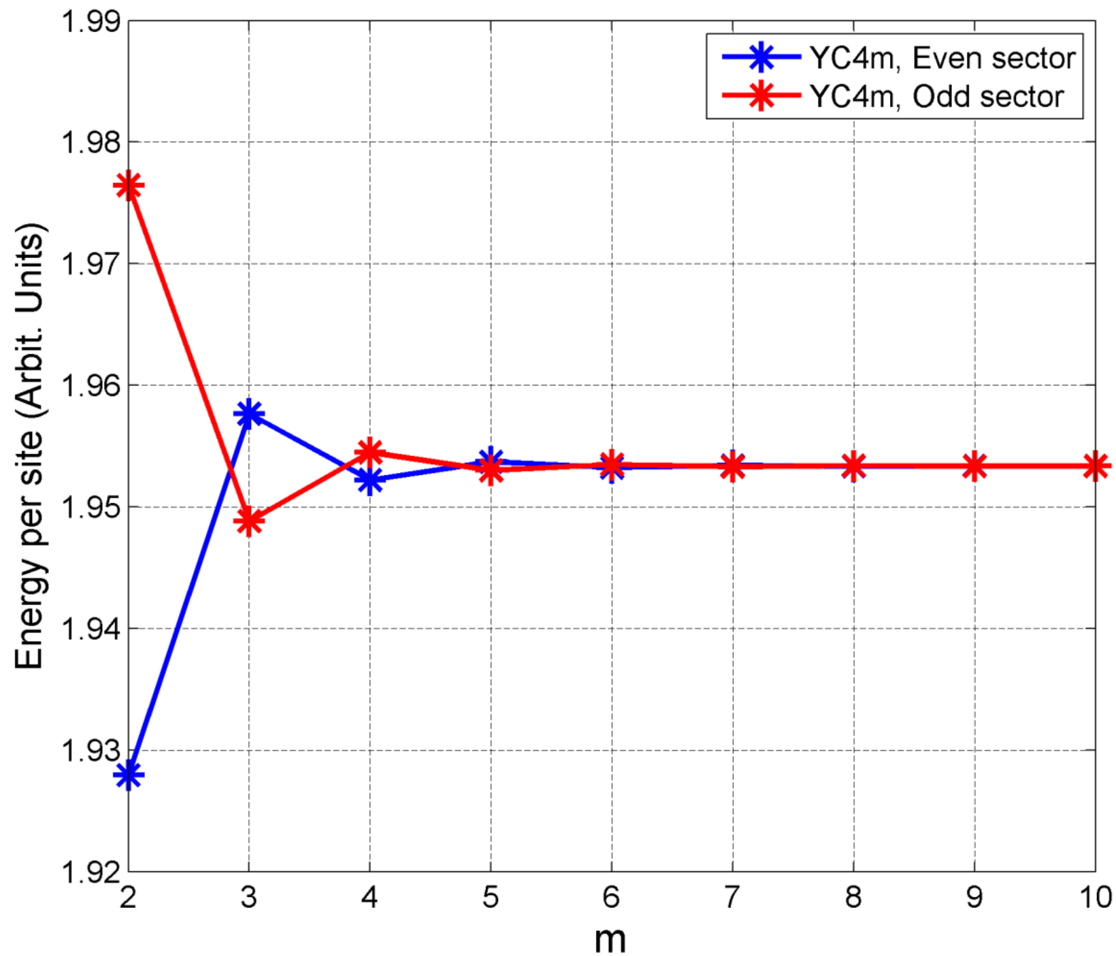
“Edge sector” and “bulk sector”





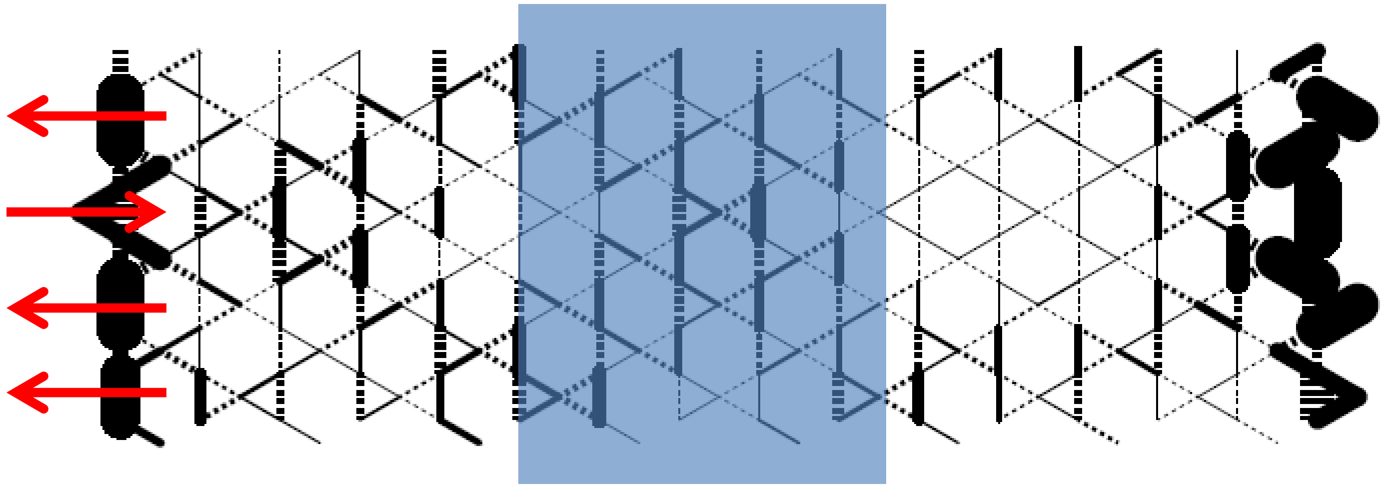
$Q_{edge} = 1$

Bulk energetics



Mismatching open boundaries and edge spinons

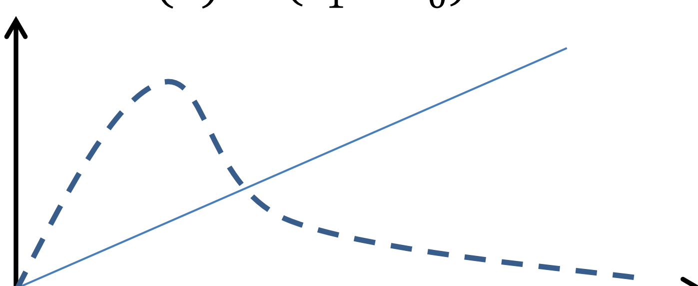
$Q_{edge} = -1$ $Q_{bulk} = 1$



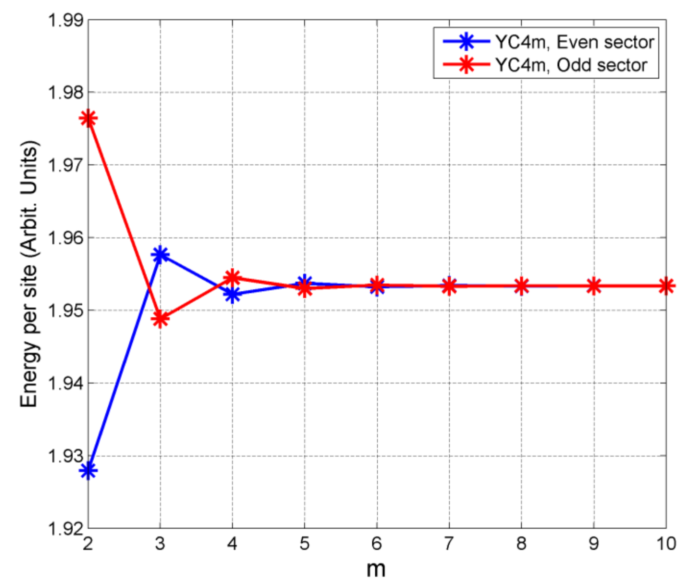
Spinon(s) bound at the edge?

$$V(x) = (\epsilon_1 - \epsilon_0)x$$

V



x



Summary

- A phenomenological quantum dimer model.
- "Diamond resonance" naturally incorporated; even-odd effect explained.
- Vacancy spinons and edge spinons ("Z_2 screening").
- Beyond singlet dynamics?