Sankar Das Sarma, Univ. of Maryland April 15, 2011

Majorana Returns!

- **Majorana/Fermi (1937) : *"Real version" of Dirac Theory***
- **Majorana disappears (1938)**
- **Neutrino as Majorana (??): *Double beta decay (lepton number!)***
- **Neutralino of SUSY is a Majorana fermion**
- **Read and Green (2000): *5/2 FQHE = Chiral p+ip SC***
- **Kitaev (2001): *Majorana 1D chain***
- **Das Sarma, Freedman, Nayak (2005): *5/2 FQH 'Majorana' Qubit***
- **Das Sarma, Nayak, Tewari (2006):*$SrRuO_4$ Majorana (half-vortex)***
- **Fu, Kane (2008): *Topological Insulator + SC***
- **Sau, Lutchyn, Tewari, Das Sarma (2010): *SO/SM + SC + FMI***
- **Alicea (2010): *SO/SM + SC***
- **Lutchyn, Sau, Das Sarma (2010): *Majorana Nanowire***
- **Freedman, Kitaev: *Topological Quantum Computation (~1995---)***

Vortices in 2D spinless $(p_x + ip_y)$ Superconductor

CHIRAL, p-WAVE, SPINLESS; ZERO ENERGY MODE AT THE CORE

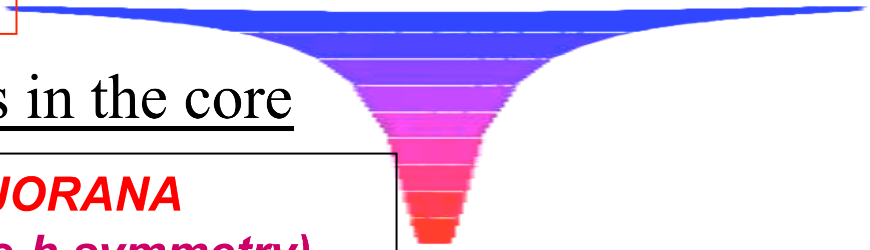
Order parameter phase rotates by $2\pi n$ around the core

Order parameter amplitude suppressed at the core

Majorana is an anyon

Not a regular fermion because it is zero-energy!

Bound states in vortex cores are e-h symmetric



Low energy normal bound states in the core

ZERO-ENERGY CORE STATE IS MAJORANA PROTECTED BY INDEX THEOREM (e-h symmetry)

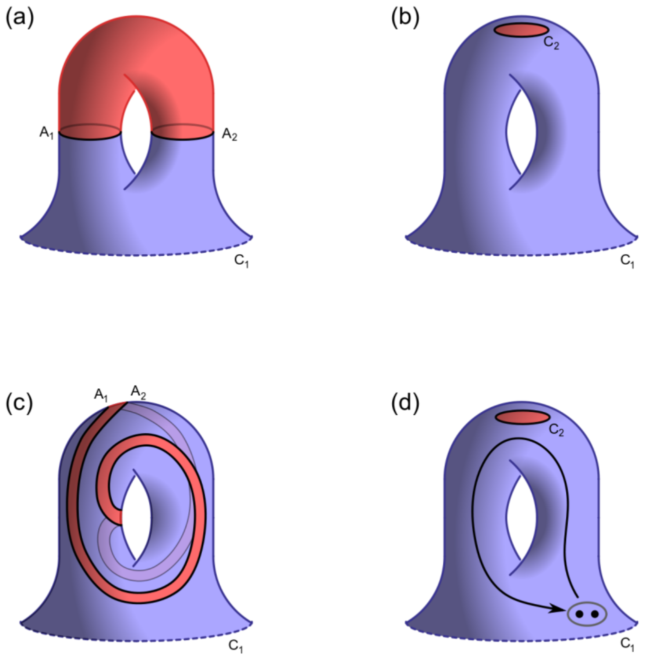
Missing operations in TQC using Majorana (Ising anyons)

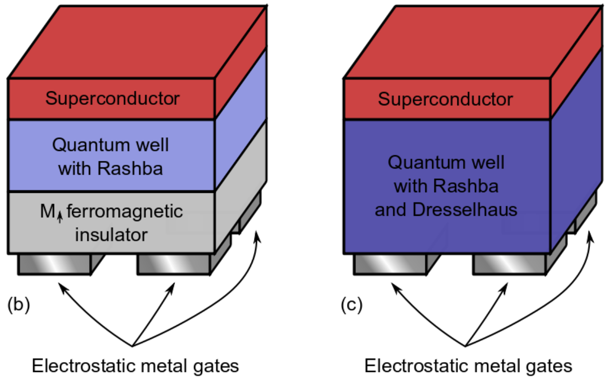
Nayak, Simon, Stern, Freedman, Das Sarma, Rev. Mod. Phys. (2008)

Ising anyons are *almost* universal: need $\vartheta = \pi/4$ phase gate and CNOT gate.

- CNOT gate can be implemented by measuring the total parity of two topological qubits
- One idea for $\vartheta = \pi/4$ phase gate is Dynamical Topology Change (DTC) (Bravyi, Kitaev 2000; Bonderson, Das Sarma, Freedman, Nayak 2010)

Hard to implement in FQHE systems, but perhaps feasible in other systems such as Superconductor/Semiconductor/Magnetic Insulator heterostructures



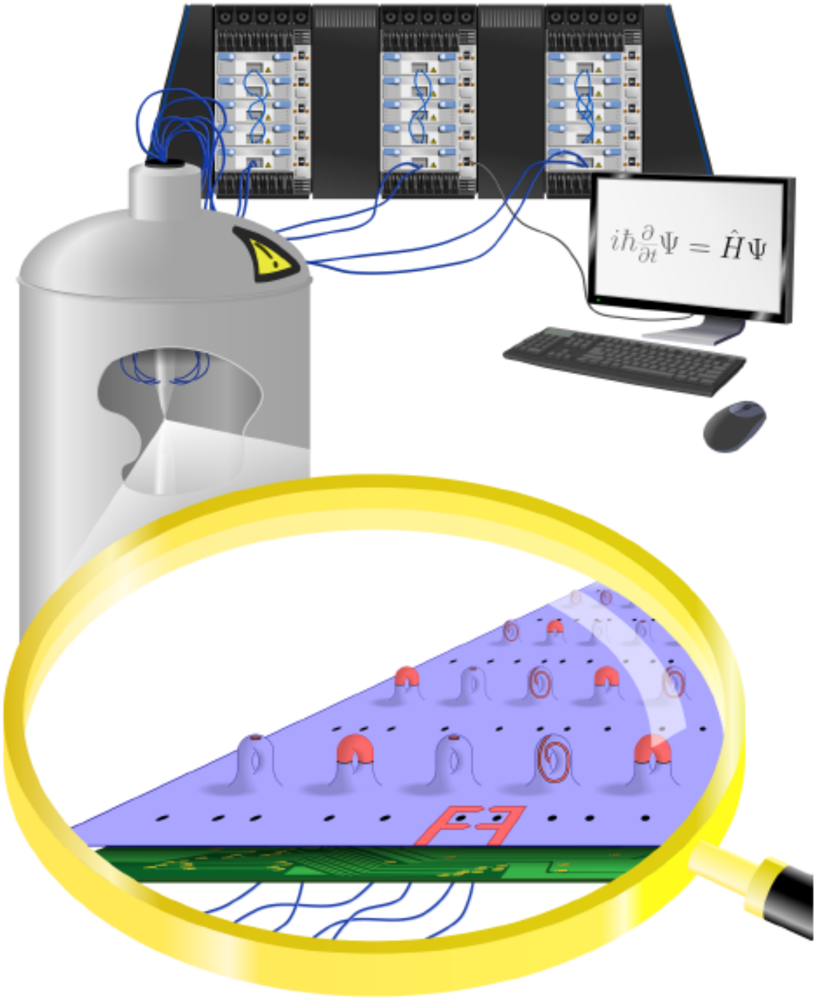


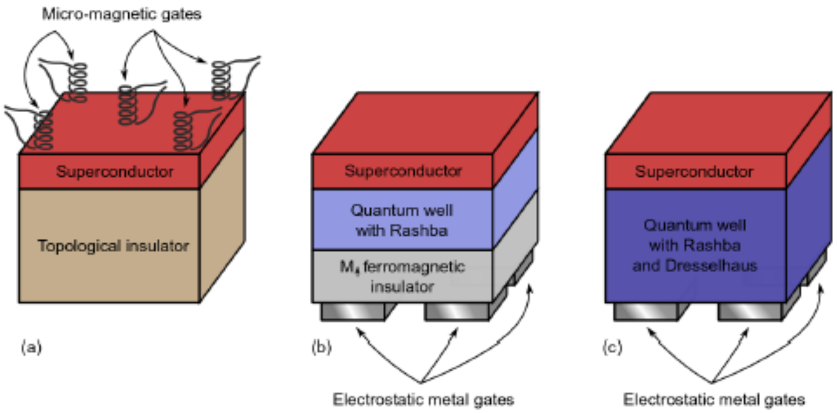
ISH (Ising Semiconductor Heterostructure)

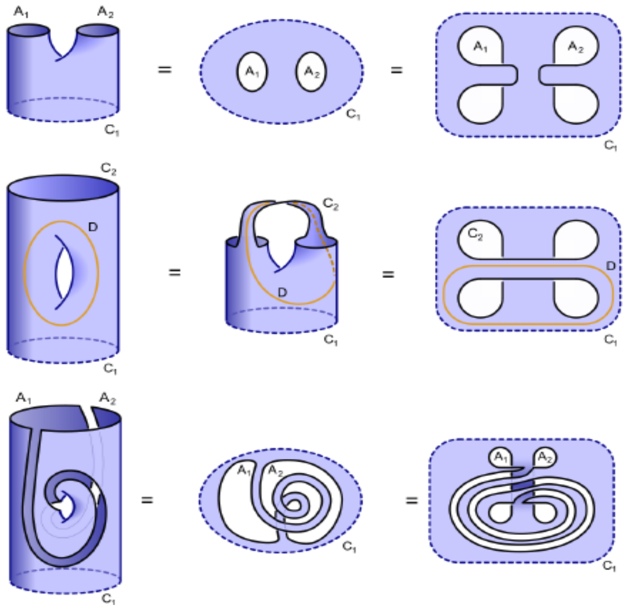
A Blueprint for a Topologically Fault-tolerant Quantum Computer

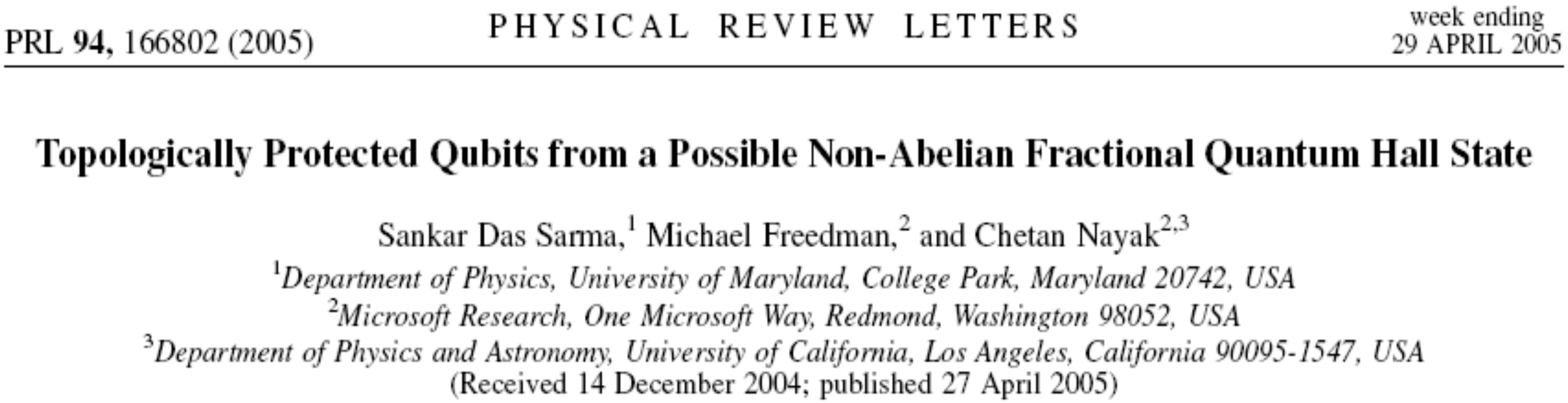
arXiv 2010

Parsa Bonderson,[1] Sankar Das Sarma,[1,2] Michael Freedman,[1] and Chetan Nayak[1,3]
[1]*Microsoft Research, Station Q, Elings Hall, University of California, Santa Barbara, CA 93106*
[2]*Department of Physics, University of Maryland, College Park, MD 20742*
[3]*Department of Physics, University of California, Santa Barbara, CA 93106*









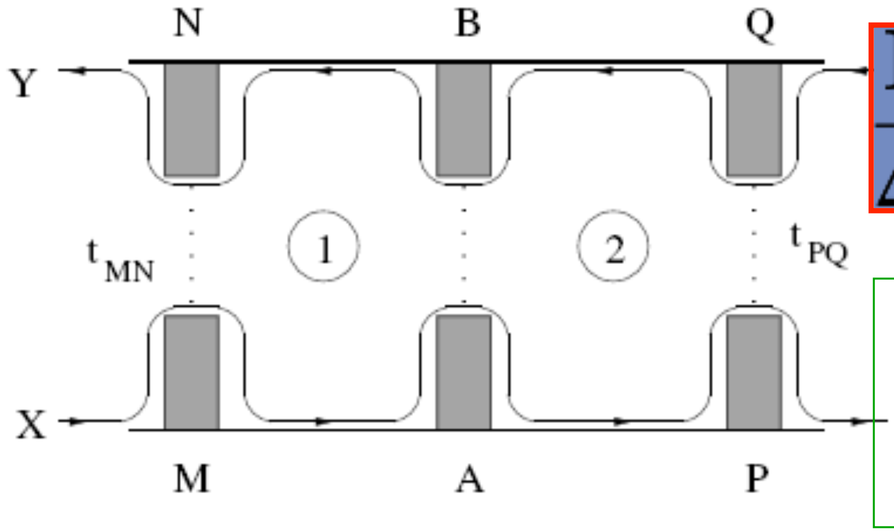
PRL **94,** 166802 (2005) PHYSICAL REVIEW LETTERS week ending 29 APRIL 2005

Topologically Protected Qubits from a Possible Non-Abelian Fractional Quantum Hall State

Sankar Das Sarma,[1] Michael Freedman,[2] and Chetan Nayak[2,3]

[1]*Department of Physics, University of Maryland, College Park, Maryland 20742, USA*
[2]*Microsoft Research, One Microsoft Way, Redmond, Washington 98052, USA*
[3]*Department of Physics and Astronomy, University of California, Los Angeles, California 90095-1547, USA*

(Received 14 December 2004; published 27 April 2005)



$$\frac{\Gamma}{\Delta} \sim \frac{T}{\Delta} e^{-\Delta/T} < 10^{-30}$$

Interference is the only definitive way of establishing non-Abelian-ness

The proposal for FQH topological qubit, quantum memory, and NOT gate using non-Abelian Ising $(SU_2)_2$ TQFT: **Fabry-Perot interferometery using the quasiparticle current paths along edges encircling anti-dots**

Das Sarma, Freedman, Nayak: Physics Today (2006)

Topology : A Global Property

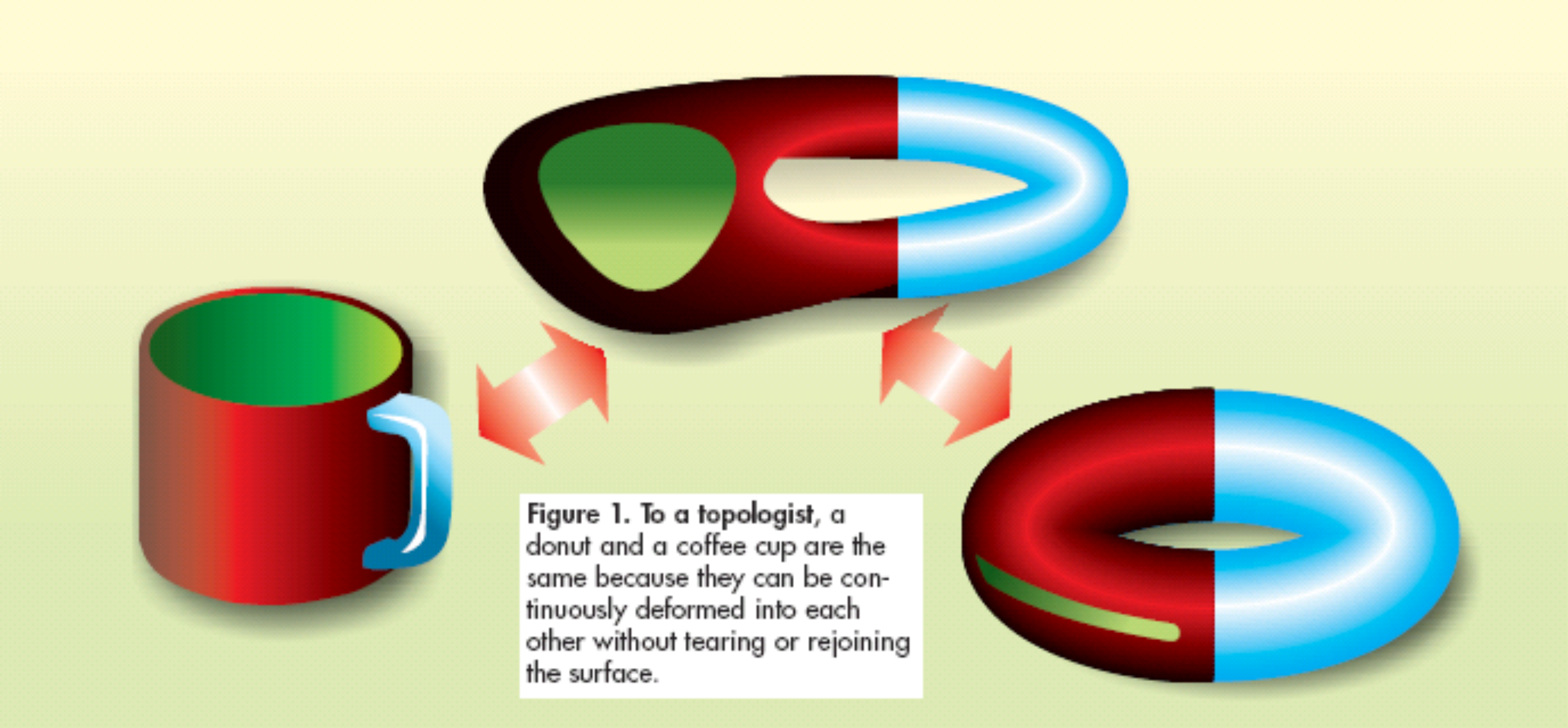


Figure 1. **To a topologist,** a donut and a coffee cup are the same because they can be continuously deformed into each other without tearing or rejoining the surface.

Look for a many-body quantum state sensitive only to the topology

Quasiparticles in $(\nu = 5/2)$ FQH system

Quasiparticles in vortex state of 2D p-wave superconductor

US007394092B2

(12) **United States Patent**
Freedman et al.

(10) **Patent No.: US 7,394,092 B2**
(45) **Date of Patent: Jul. 1, 2008**

(54) **QUASI-PARTICLE INTERFEROMETRY FOR LOGICAL GATES**

(75) Inventors: **Michael H. Freedman**, Redmond, WA (US); **Chetan V. Navak**, Santa Monica, CA (US); **Sankar Das Sarma**, Potomac, MD (US)

(73) Assignee: **Microsoft Corporation**, Redmond, WA (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

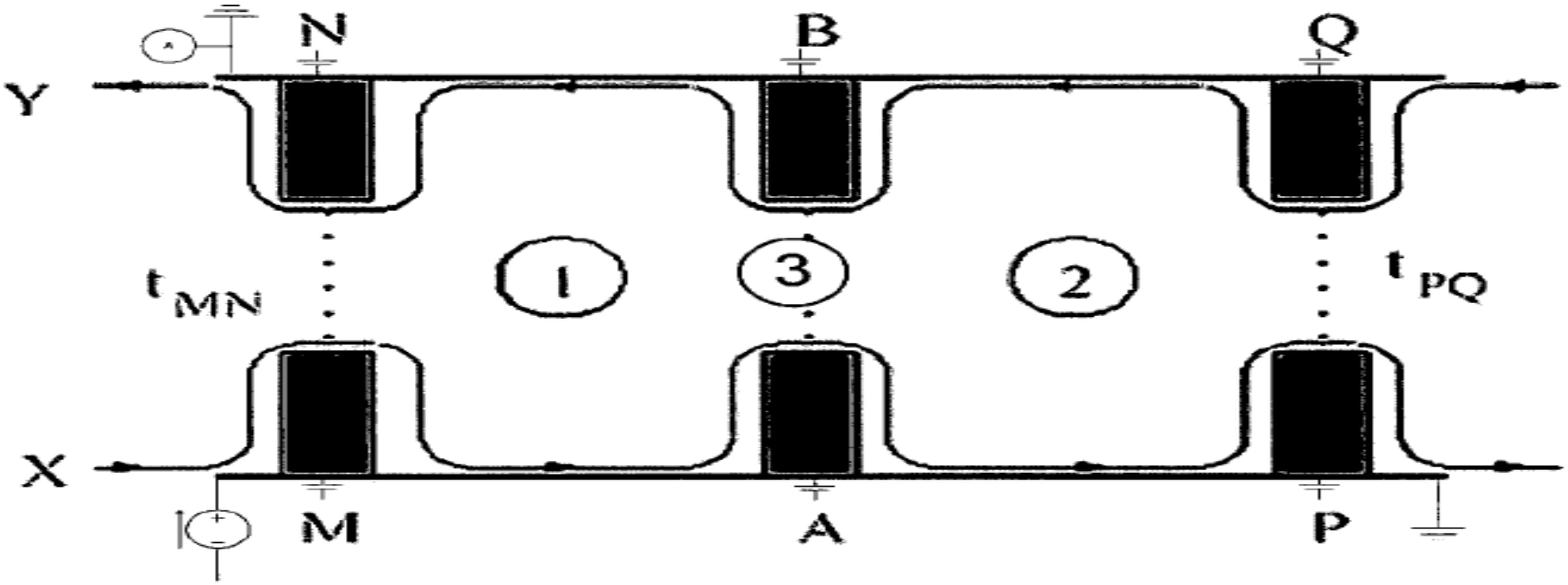
Freedman, M.H. et al., "A Modular Functor which is Universal for Quantum Computation", *Commun. Math. Phys.*, 2002, 227, 605-622.

Goldman, V.J. et al., "Resonant tunneling in the Quantum Hall Regime: Measurement of Fractional Charge", *Science*, 1995, 267, 1010-1012.

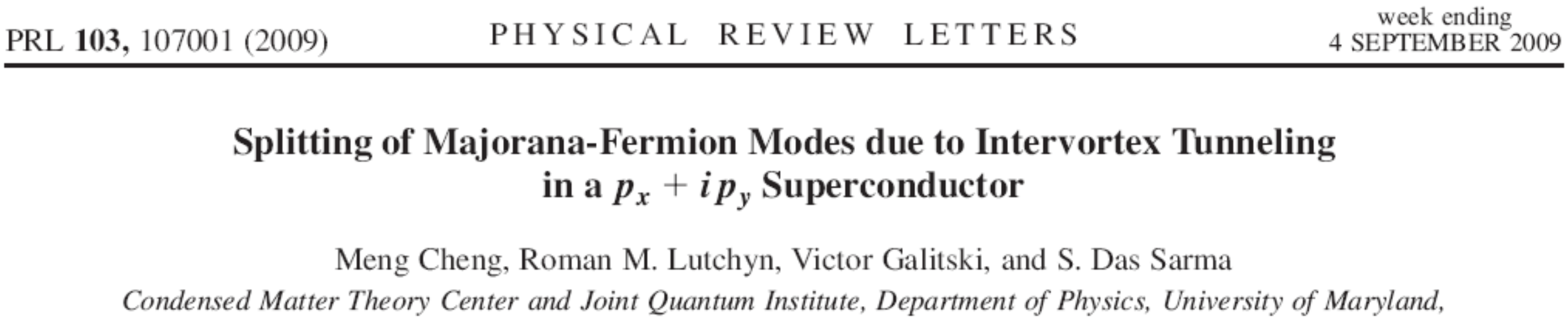
Greiter, M. et al., "Paired Hall State at Half Filling", *Physical Review Letters*, 1991, 66(24), 3205-3208.

Moore, G. et al., "Nonabelions in the Fractional Quantum Hall Effect", *Nuclear Physics*, 1991, B360, 362-396.

Nayak, C. et al., "2*n*-quasihole States Realize 2*n*-1-Dimensional Spinor Braiding Statistics in Paired Quantum Hall States", *Nuclear Physics*, 1996, B479, 529-553.



Qubit decoherence due to anyon tunneling: The limit to topological protection



PRL **103,** 107001 (2009) PHYSICAL REVIEW LETTERS week ending 4 SEPTEMBER 2009

Splitting of Majorana-Fermion Modes due to Intervortex Tunneling in a $p_x + ip_y$ Superconductor

Meng Cheng, Roman M. Lutchyn, Victor Galitski, and S. Das Sarma

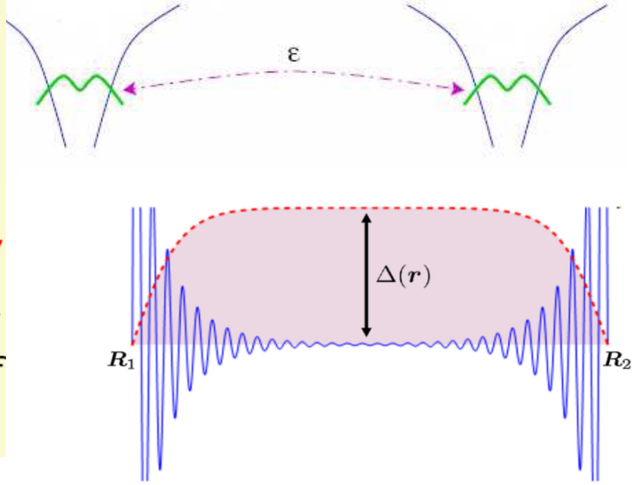
Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics, University of Maryland,

New results:

- Tunneling energy splitting calculated:

$$E_+ - E_- \approx \frac{\Delta_0}{\pi^{\frac{3}{2}}} \frac{\cos(k_F R + \frac{\pi}{4})}{\sqrt{k_F R}} e^{-\frac{R}{\xi}}$$

- New discovery is that the sign of the energy splitting oscillates! A behavior never seen before. It indicates that the additional effect of fluctuations is important for dephasing.



Enrico Fermi apparently said:
“There are many categories of scientists, people of second and third rank, who do their best, but do not go very far. There are also people of first class, who make great discoveries, which are of capital importance for the development of science. But then there are the geniuses, like Galileo and Newton. Well, Ettore was one of these. “

Ettore Majorana

born in Catania, Sicily, 1906
rose rapidly through academic ranks
friend and scientific collaborator of Fermi, Heisenberg etc
stream of high quality papers

1933 problems… gastritis, reclusive,
no publications for several years

1937 Fermi was allowed to write-up and submit under Majorana’s name his last and most profound paper which Majorana had derived some years before.

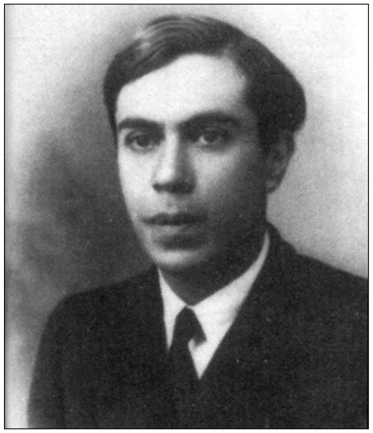
At Fermi’s urging, Majorana applied and got Chair in Naples (1938)

March 1938: trip to Palermo, arrived, boarded a ship straight back to Napoli
DISAPPEARED without a trace.

a) retired to monastery,
   to escape spiritual crisis and to embrace his deep Catholic faith
b) jumped overboard in suicide

Brief history of Majorana fermions

Question: Are equations for spin-1/2 particles necessarily complex ?



E. Majorana (1937)

Simple clever modification of the Dirac equation that involves ONLY REAL numbers

↓

Majorana fermion - electrically neutral particle which is its own antiparticle $\gamma = \gamma^\dagger$

Relevance:
particle physics (neutrinos)
(neutralinos)

Experimental status:
NOT observed

EMERGENT MAJORANA?

perspective

Majorana returns

F. Wilczek, Nature Physics'09

‘Unusual’ recent popularity of emergent Majorana modes in solids

Majorana returns

Frank Wilczek Nature Physics 2009 September

In his short career, Ettore Majorana made several profound contributions. One of them, his concept of ‘Majorana fermions’ — particles that are their own antiparticle — is finding ever wider relevance in modern physics.

Non-Abelian states of matter

Ady Stern[1] Nature 2010 March

Barbara Goss Levi Search and Discovery Physics Today 2011 March

The expanding search for Majorana particles

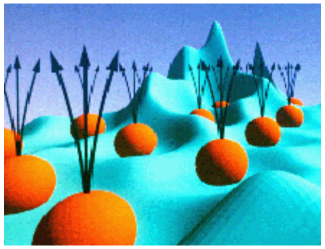
NEWS

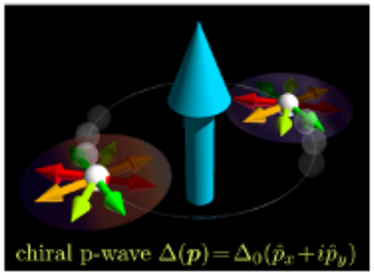
Robert F. Service Science 2011 April

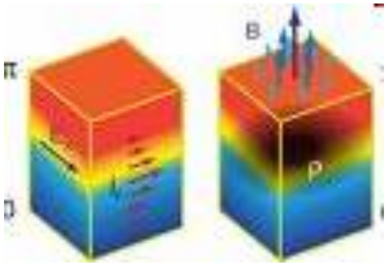
Search for Majorana Fermions Nearing Success at Last?

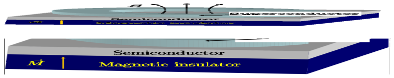
Researchers think they are on the verge of discovering weird new particles that borrow a trick from superconductors and could give a big boost to quantum computers

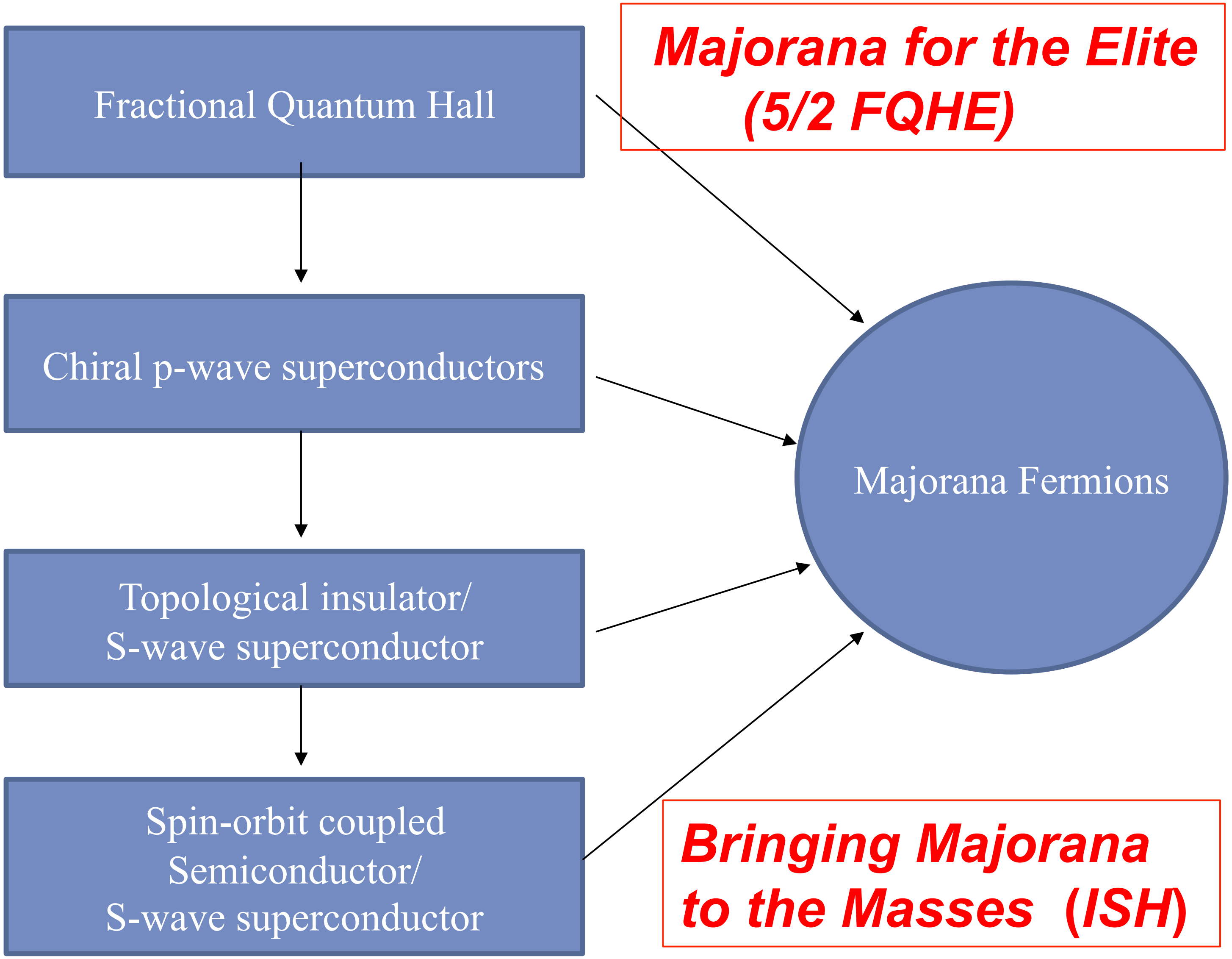
Non-abelian statistics and topological qubits
increasing simplicity











Fractional Quantum Hall

Chiral p-wave superconductors

Topological insulator/
S-wave superconductor

Spin-orbit coupled
Semiconductor/
S-wave superconductor

Majorana Fermions

Majorana for the Elite
(5/2 FQHE)

Bringing Majorana
to the Masses (ISH)

PRL **104,** 040502 (2010) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2010

Generic New Platform for Topological Quantum Computation Using Semiconductor Heterostructures

Jay D. Sau,[1] Roman M. Lutchyn,[1] Sumanta Tewari,[1,2] and S. Das Sarma[1]

PRL **105,** 077001 (2010) PHYSICAL REVIEW LETTERS week ending 13 AUGUST 2010

Majorana Fermions and a Topological Phase Transition in Semiconductor-Superconductor Heterostructures

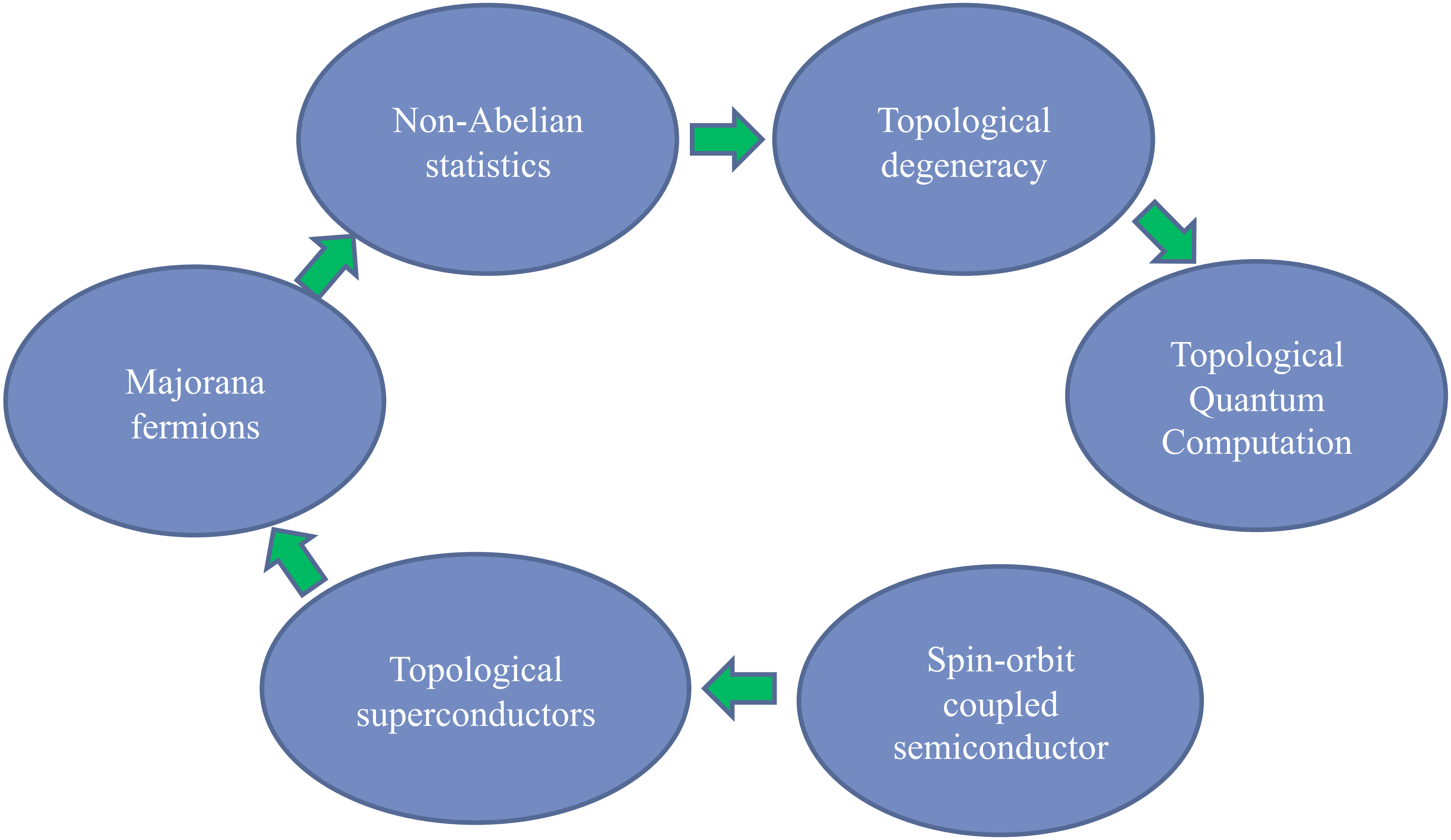
Roman M. Lutchyn, Jay D. Sau, and S. Das Sarma

PRL **106,** 127001 (2011) PHYSICAL REVIEW LETTERS week ending 25 MARCH 2011

Search for Majorana Fermions in Multiband Semiconducting Nanowires

Roman M. Lutchyn,[1,3] Tudor D. Stanescu,[1,2] and S. Das Sarma[1]

Majorana fermions, topological superconductivity, spin-orbit coupling…


Spin-orbit coupled semiconductor

Topological superconductors

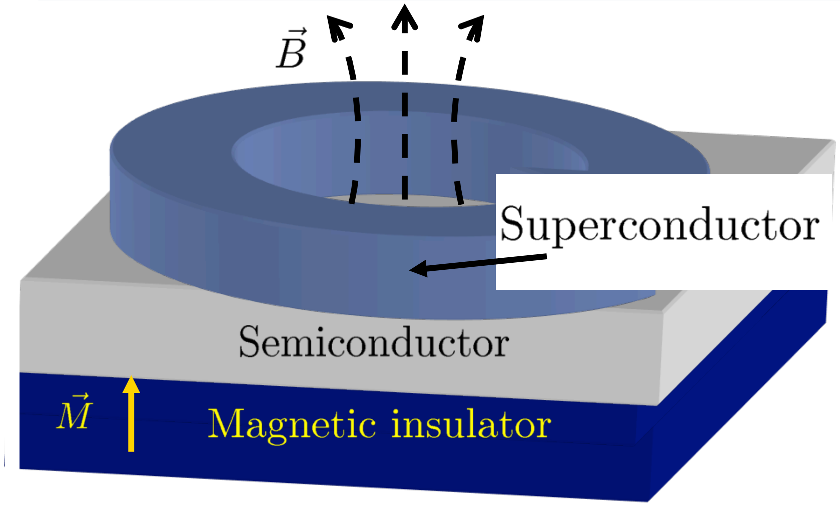
Majorana fermions

Non-Abelian statistics

Topological degeneracy

Topological Quantum Computation

Majorana bound states in SM/SC heterostructure



Bogoliubov-de-Gennes equations

$$H_{\mathrm{BdG}}\Psi = E\Psi$$

$$\Psi = (\psi_\uparrow, \psi_\downarrow, \psi_\downarrow^\dagger, -\psi_\uparrow^\dagger)^T$$

$$H_{\mathrm{BdG}} = \left(-\frac{\nabla^2}{2m} - \mu - i\alpha(\vec{\sigma}\times\vec{\nabla})\hat{z} + V_z\sigma_z\right)\tau_z + \Delta_0(r)(\tau_x\cos\theta + \tau_y\sin\theta)$$

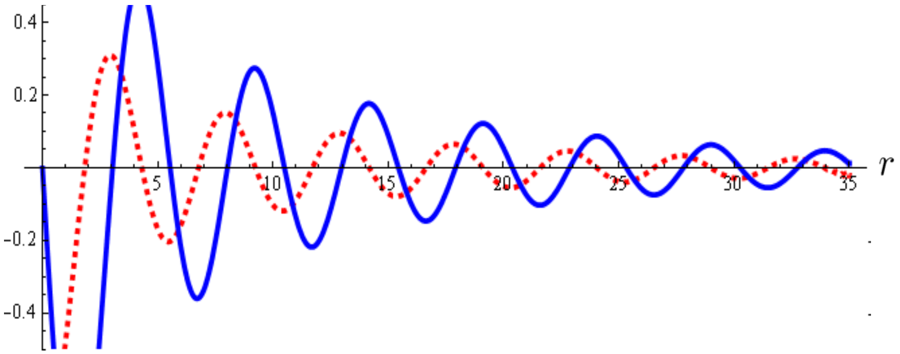
Non-degenerate zero-energy solution

exists when $\sqrt{\mu^2+\Delta_0^2} < |V_z|$

Majorana number $\mathcal{M}$

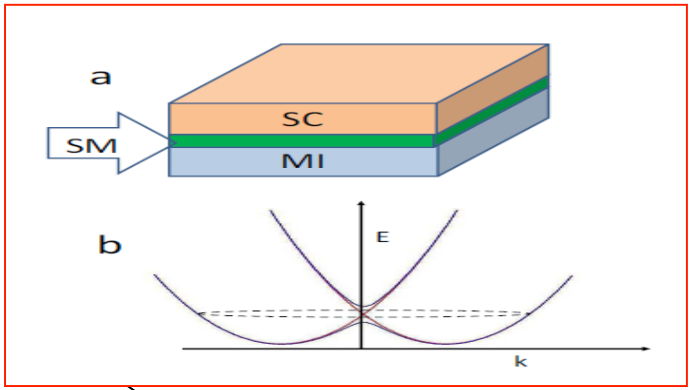
$$\mathcal{M} = e^{i\pi C_1} = \pm 1$$

artificially created
$h/2e$ vortex

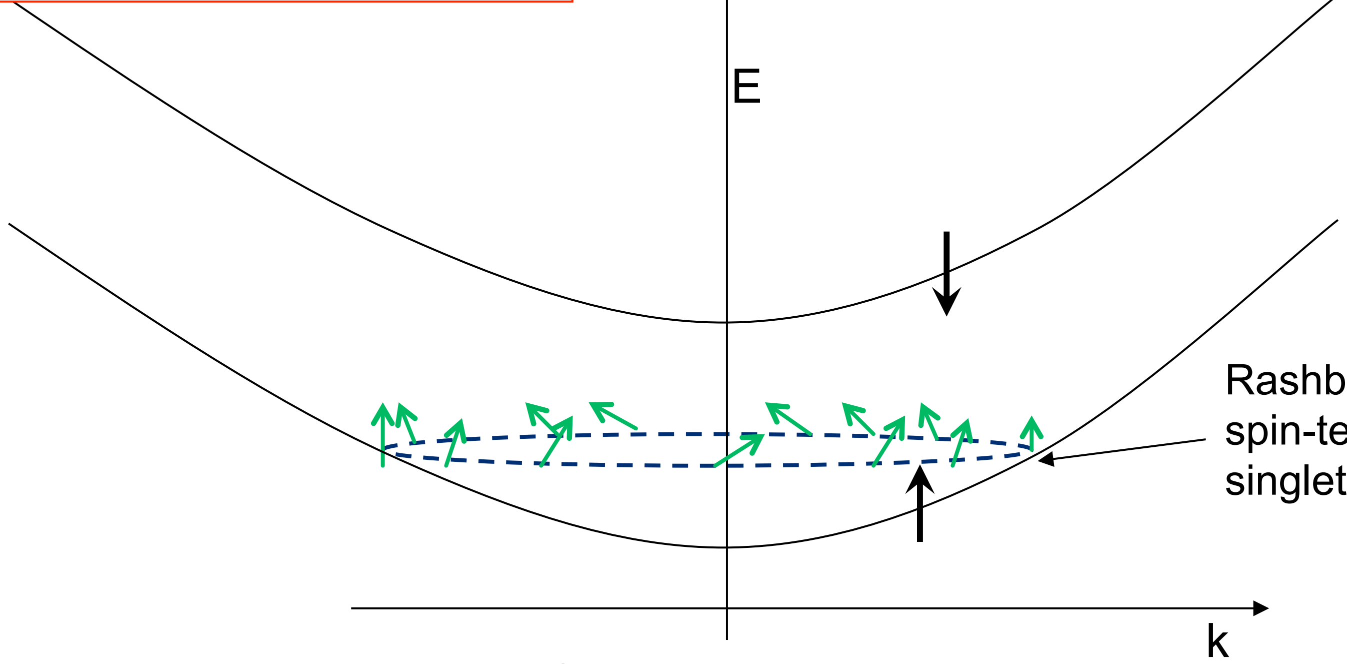


Sau, Lutchyn, Tewari, Das Sarma, PRL'10

Majorana at spin-orbit coupled semiconductor (Sm)-SC interfaces



Need single non-degenerate Fermi-surface
$H_{Sm} = k^2 + V_z \sigma_z$



Rashba induced chiral spin-texture allows singlet pairing

Band structure of spin-split spin-orbit coupled semiconductor

Rashba + Zeeman break inversion and time-reversal for chiral edge mode

Sau, Lutchyn, Tewari, Das Sarma PRL(2010)

Superconductors are natural hosts for Majorana

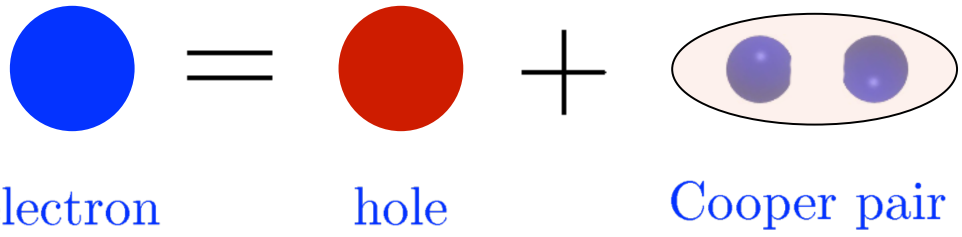
Bogoliubov quasiparticle $\gamma = u\psi + v\psi^\dagger$

$u = v^*$ → equal superposition of a particle and a hole

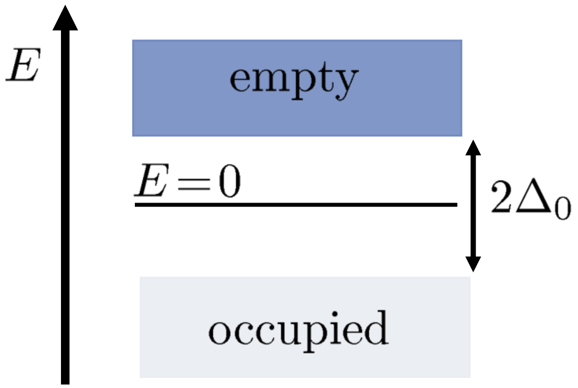
↓

Majorana fermion

$$\gamma = \gamma^\dagger$$



Look for ZERO energy states !



Bound states in vortices

Midgap states at the interfaces

Topological protection of zero-energy mode

Bogoliubov-de-Gennes equations

$$\begin{pmatrix} h_0 & \Delta \\ \Delta^\dagger & -h_0^T \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}$$

Particle-hole symmetry:

If $\begin{pmatrix} u \\ v \end{pmatrix}$ is a solution with E

If $\begin{pmatrix} v^* \\ u^* \end{pmatrix}$ is a solution with $-E$

For spinless fermions particle-hole symmetry guarantees Majorana mode at $E=0$

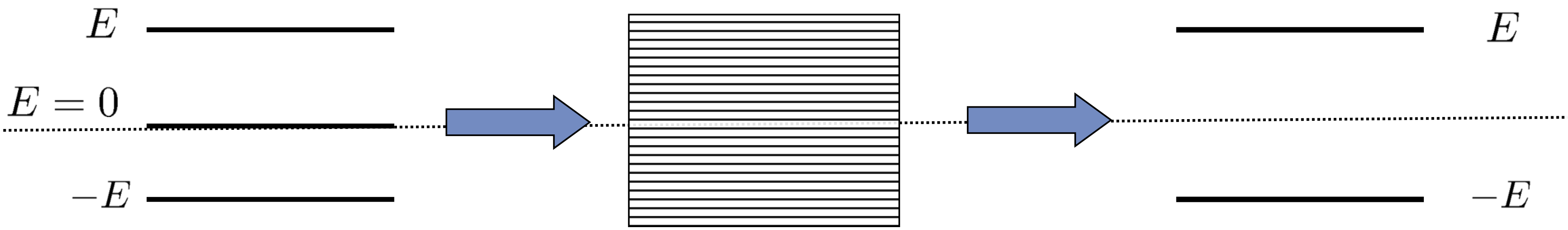
Two topological classes of BdG Hamiltonians

$2N+1$ solutions

Topological reconstruction of the spectrum requires closing of the bulk gap

Bound states in the vortex core

$2N$ solutions



Read and Green, PRB'00

Example: 2D chiral p-wave superconductors

Zero-energy states appear in chiral superfluids

He-3: Kopnin and Salomaa PRB’91;

Chiral superfluids/superconductors, Volovik (1999), Read & Green(2000)

Chiral p-wave superconductor SrRuO_4: Das Sarma, Nayak, Tewari (2006)

Chirality may originate from the order parameter or band structure

Caroli-de Gennes-Matricon bound states

s-wave SC

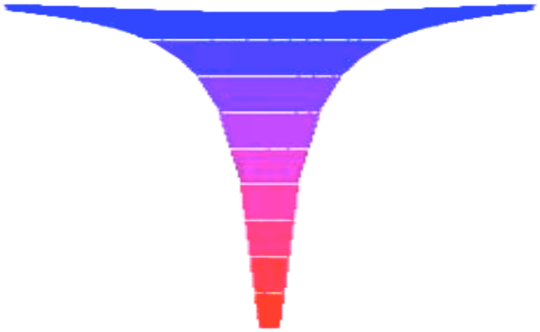
$$E_n = \omega_0 \left(n + \frac{1}{2}\right)$$

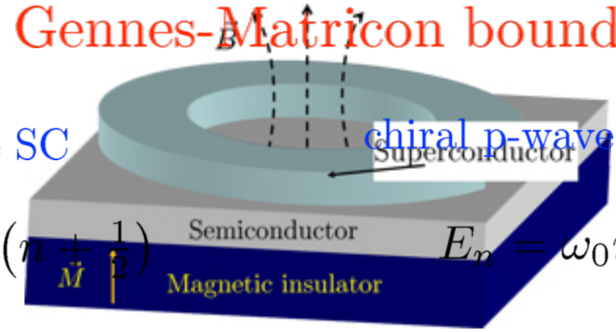
$$\Psi(r, \theta + 2\pi) = -\Psi(r, \theta)$$

chiral p-wave SC

$$E_n = \omega_0 n$$

$$\Psi(r, \theta + 2\pi) = \Psi(r, \theta)$$





Chiral superconductors:

- strontium ruthenate

Rice & Sigrist, 1995

Heterostructures:

- topological insulator/s-wave superconductor
- semiconductor/s-wave superconductor

... among others

Engineering spinless p+ip superconductor

Rather than looking for $p_x + ip_y$ SC in nature, we could try to engineer suitable Hamiltonians via proximity effect

Chirality has to come from the bandstructure

Strong spin-orbit interaction is necessary
to avoid fermion doubling

Ordinary S-wave SC
+
2D (or 1D) Semiconductor
with Strong SO Coupling

Superconducting heterostructures

2D: Majoranas "live" in vortices

1D: Majoranas "live" at the ends of wires

Sau, Lutchyn, Tewari, Das Sarma, PRL'10

1D Lutchyn, Sau, Das Sarma, PRL(2010)

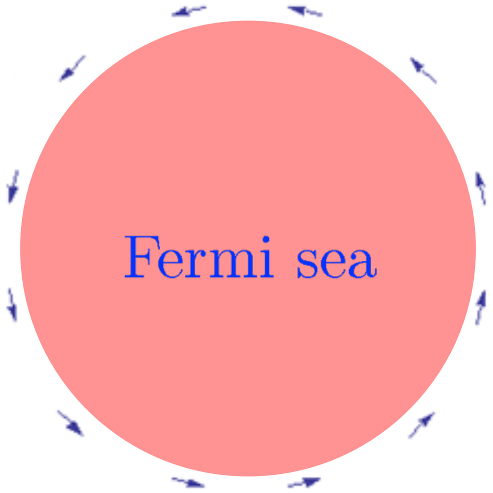
Sau et al. PRB (2010)
Sau, Tewari, Das Sarma Ann Phys'10

Q1D Lutchyn, Stanescu, Das Sarma, PRL'11

Semiconductor with spin-orbit interaction

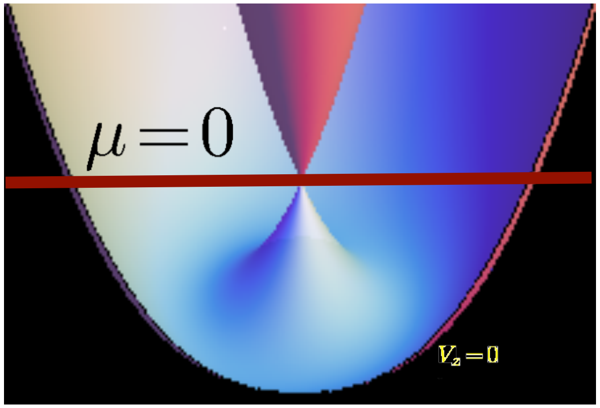
Semiconductor with Rashba interaction

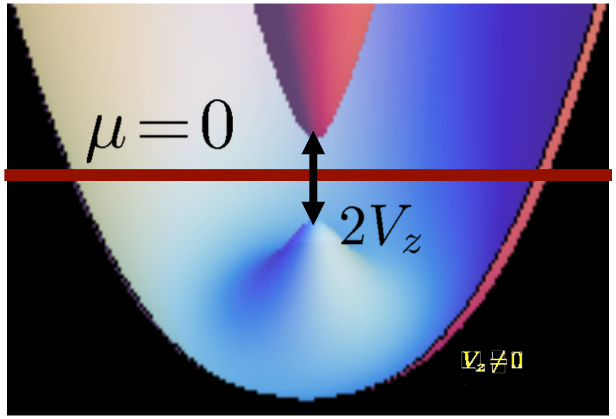
$$H_0 = \begin{pmatrix} \frac{p^2}{2m} - \mu & \alpha i(p_x - ip_y) \\ -\alpha i(p_x + ip_y) & \frac{p^2}{2m} - \mu \end{pmatrix}$$



spin orientation changes around Fermi surface

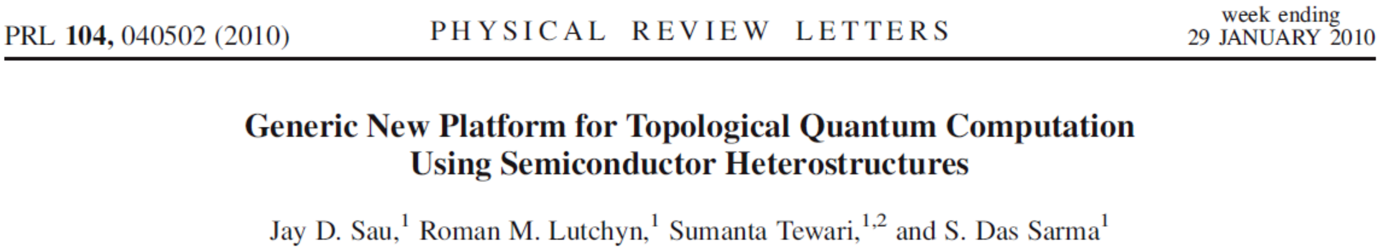
Sau, Lutchyn, Tewari, Das Sarma, PRL'10;





Single Fermi surface !

Practical route to spinless p+ip superconductivity


PRL **104,** 040502 (2010) PHYSICAL REVIEW LETTERS week ending 29 JANUARY 2010

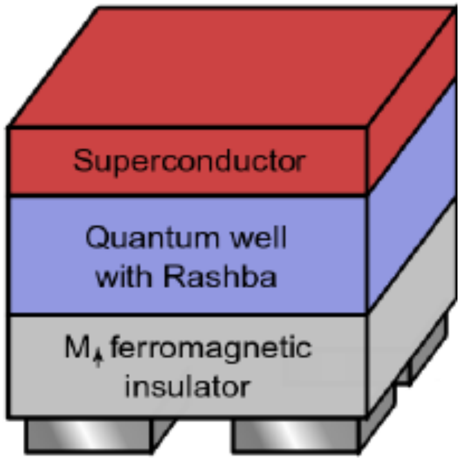
Generic New Platform for Topological Quantum Computation Using Semiconductor Heterostructures

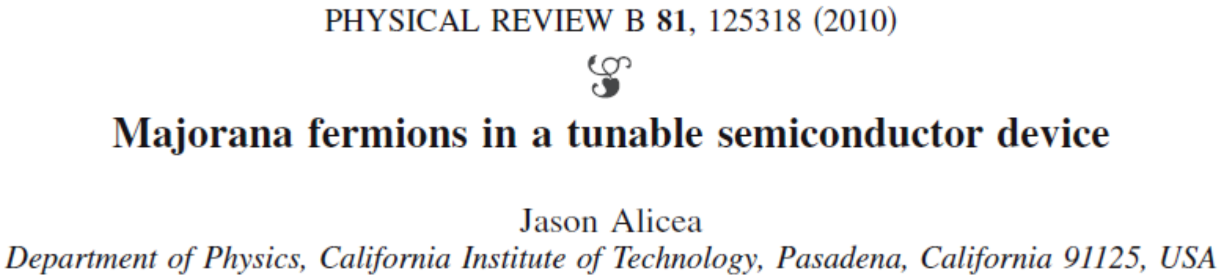
Jay D. Sau,[1] Roman M. Lutchyn,[1] Sumanta Tewari,[1,2] and S. Das Sarma[1]

Proximity-induced Δ_{ind}

Proximity-induced V_z

Challenge: creating two interfaces




PHYSICAL REVIEW B **81**, 125318 (2010)

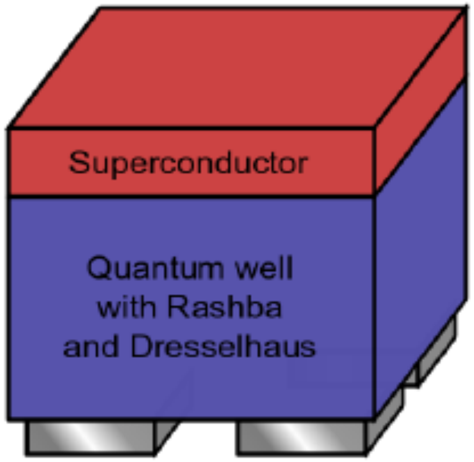
Majorana fermions in a tunable semiconductor device

Jason Alicea
Department of Physics, California Institute of Technology, Pasadena, California 91125, USA

Proximity-induced Δ_{ind}

In-plane magnetic field

Challenge: low electron density, effects of disorder



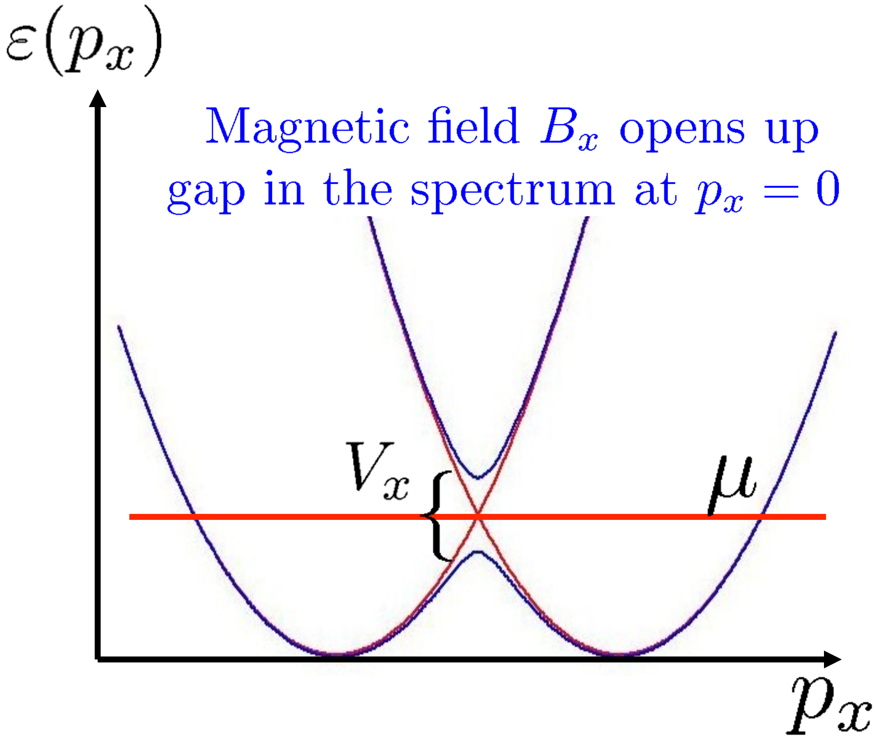
1D wires with spin-orbit: helical state

$$H_0=\int_{-L}^{L} dx\,\psi^\dagger_\sigma(x)\left(-\frac{\partial_x^2}{2m^*}-\mu+i\alpha\sigma_y\partial_x+V_x\sigma_x\right)_{\sigma\sigma'}\psi_{\sigma'}(x)$$

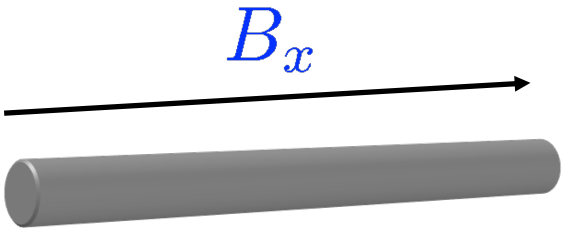
single channel nanowire

spin-orbit coupling

Zeeman splitting



Magnetic field B_x opens up gap in the spectrum at $p_x=0$



InAs, InSb nanowires

large spin-orbit ($\alpha \sim 0.1 eV\mathring{A}$)

large g-factor ($g \sim 10-50$)

good contacts with metals

→

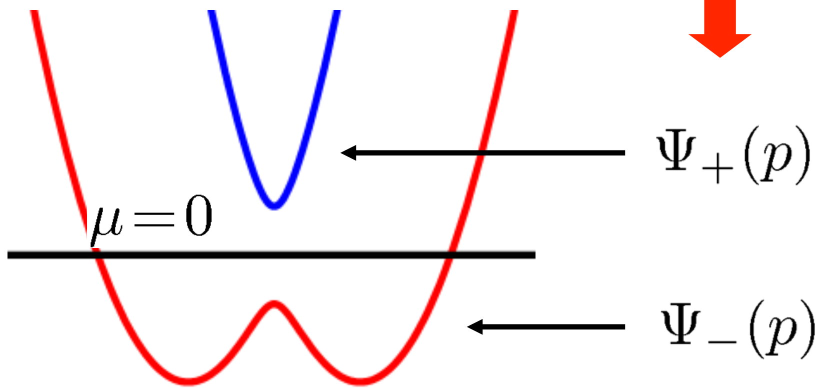
Majorana quantum wires

$$H_{\rm MW} = \int_{-L}^{L} dx \left[\psi^\dagger_\sigma \left(-\frac{\partial_x^2}{2m^*} - \mu + i\alpha\sigma_y\partial_x + V_x\sigma_x \right)_{\sigma\sigma'} \psi_{\sigma'} + \Delta_0^* \psi_\uparrow \psi_\downarrow + \Delta_0 \psi^\dagger_\downarrow \psi^\dagger_\uparrow \right]$$

Rashba spin-orbit+in-plane field

Proximity-induced superconductivity

Diagonalize H_0



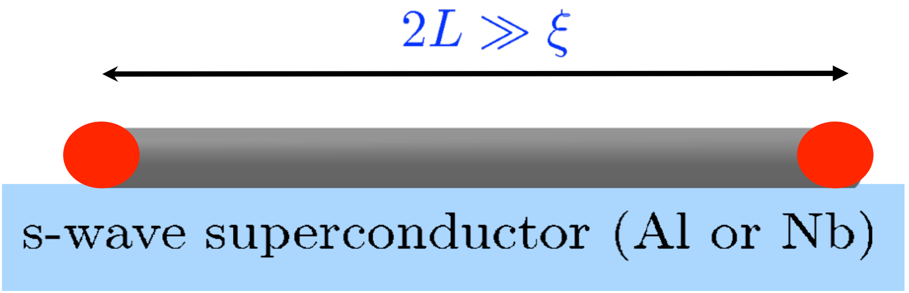
$$H_{\rm SC} = \begin{cases} \Delta_{--}(p)\Psi^\dagger_-(p)\Psi^\dagger_-(-p) \\ \cancel{\Delta_{+-}(p)\Psi^\dagger_+(p)\Psi^\dagger_-(-p)} \\ \Delta_{--}(p) \propto \Delta_0 p_x \\ \cancel{\Delta_{++}(p)\Psi^\dagger_+(p)\Psi^\dagger_+(-p)} \end{cases}$$

Lutchyn, Sau, Das Sarma PRL 2010

Drive topological phase transition by changing V_x or μ

Density of states across phase transition

Finite-size numerical studies $L_x = 10\mu\text{m}$

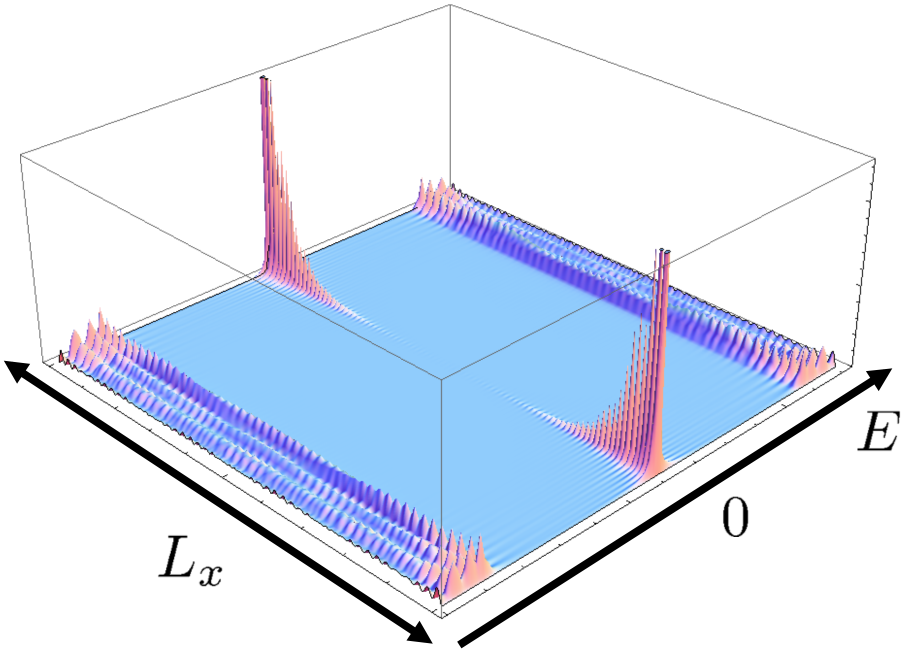


$2L \gg \xi$

s-wave superconductor (Al or Nb)

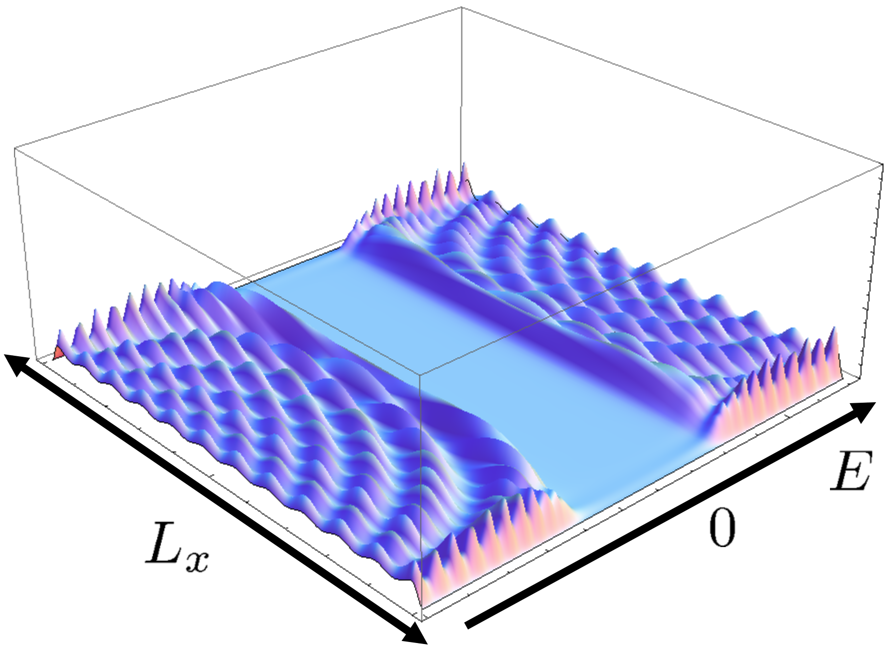
DoS in topologically non-trivial phase

$|V_x| > \sqrt{\mu^2 + \Delta_0^2}$



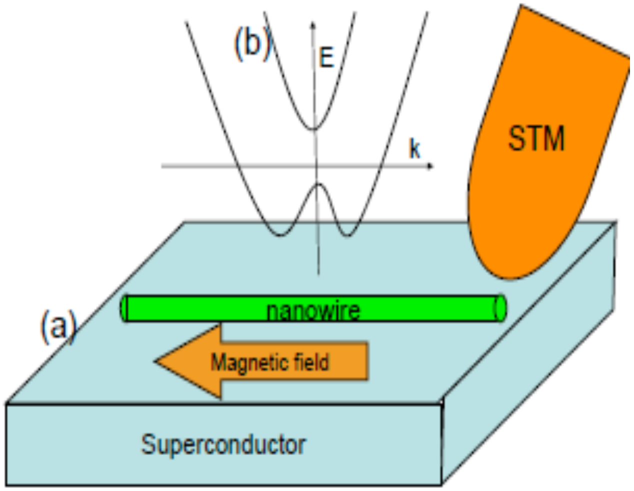
DoS in topologically trivial phase

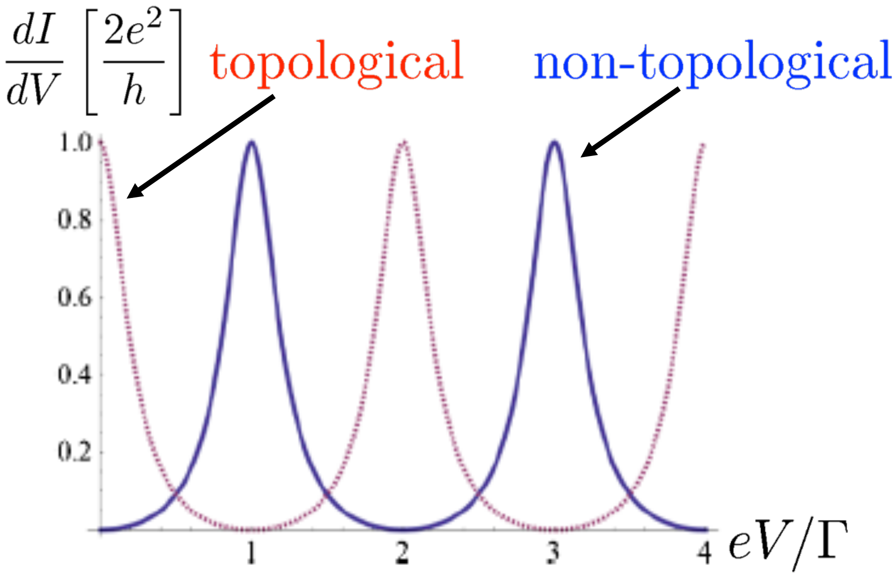
$|V_x| < \sqrt{\mu^2 + \Delta_0^2}$



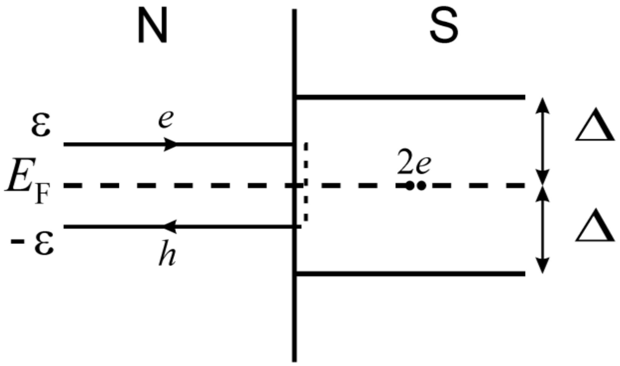
Tunneling experiments

- probing Majorana bound states using tunneling experiments





Resonant Andreev reflection



$T = 0$

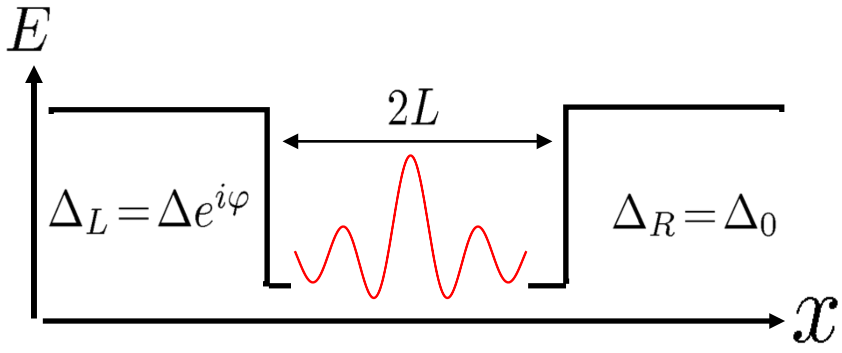
$$G = \frac{2e^2}{h} \qquad |V_x| > \sqrt{\mu^2 + \Delta_0^2}$$

$$G = 0 \qquad |V_x| < \sqrt{\mu^2 + \Delta_0^2}$$

Sau, Tewari, Lutchyn, Satansecu, Das Sarma, PRB’10

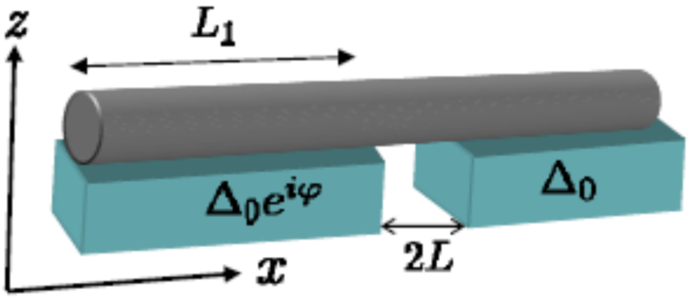
Fractional ac Josephson effect

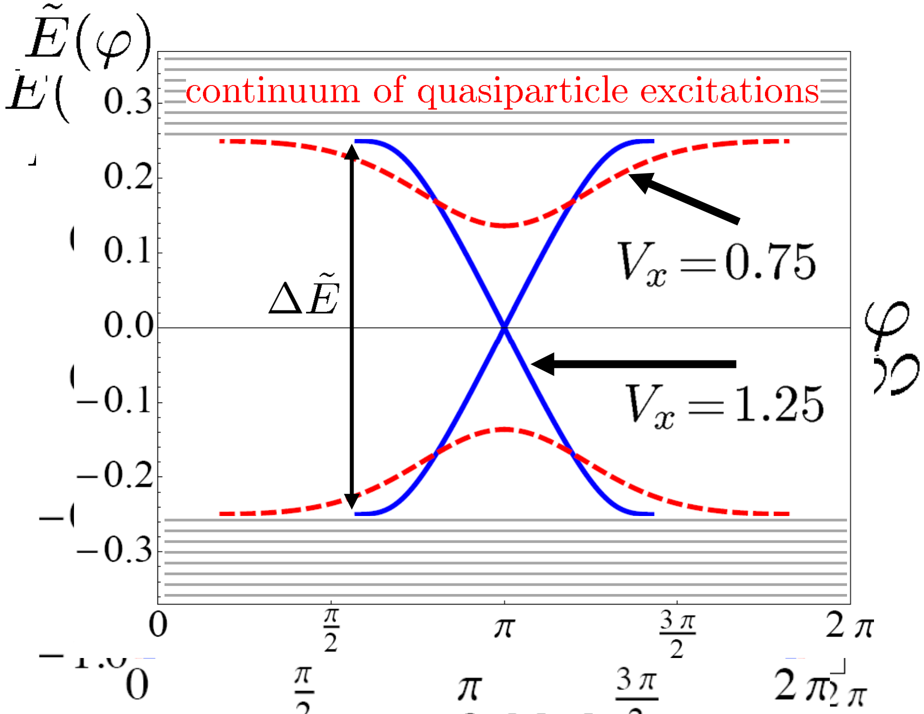
Andreev bound states



Short junction limit
($L \ll \xi$)

particle-hole symmetry protects
true level crossing at $\varphi = \pi$



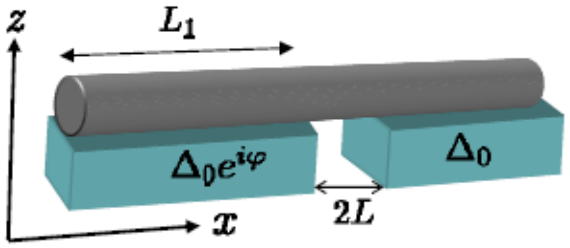


Magnetic field driven
topological phase transition

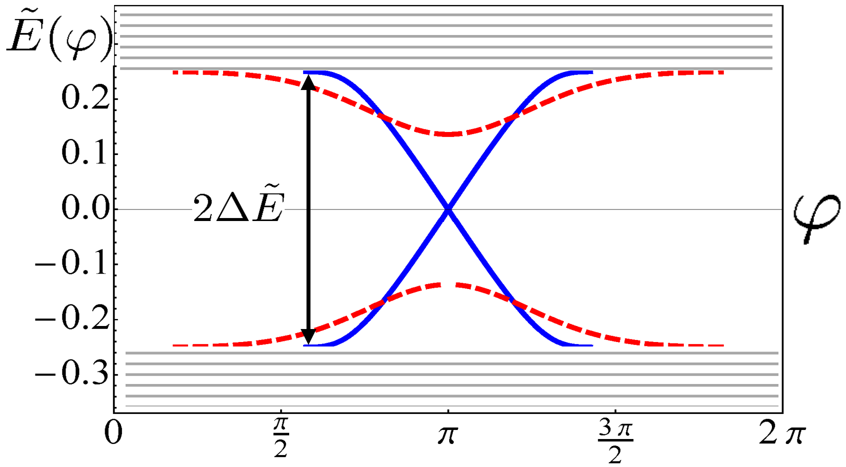
Lutchyn, Sau, Das Sarma, PRL'10

Topologically non-trivial phase: $V_x > \sqrt{\mu^2 + \Delta_0^2}$

Josephson current through heterostructure

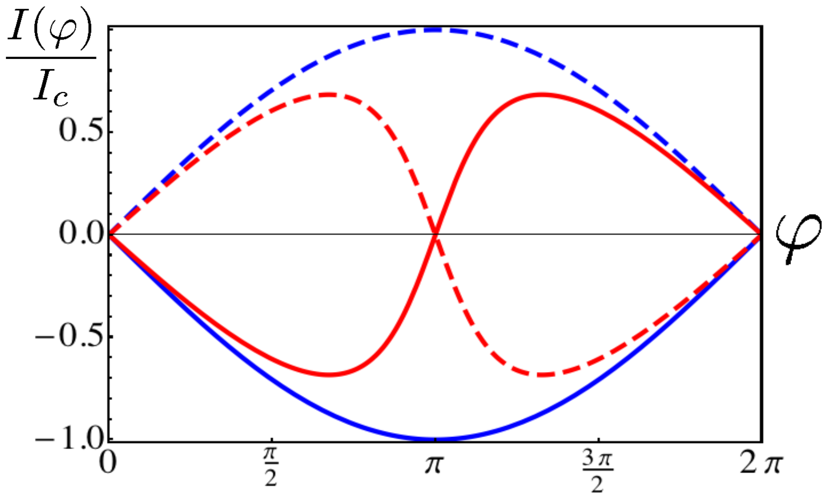


Lutchyn, Sau, Das Sarma, PRL'10



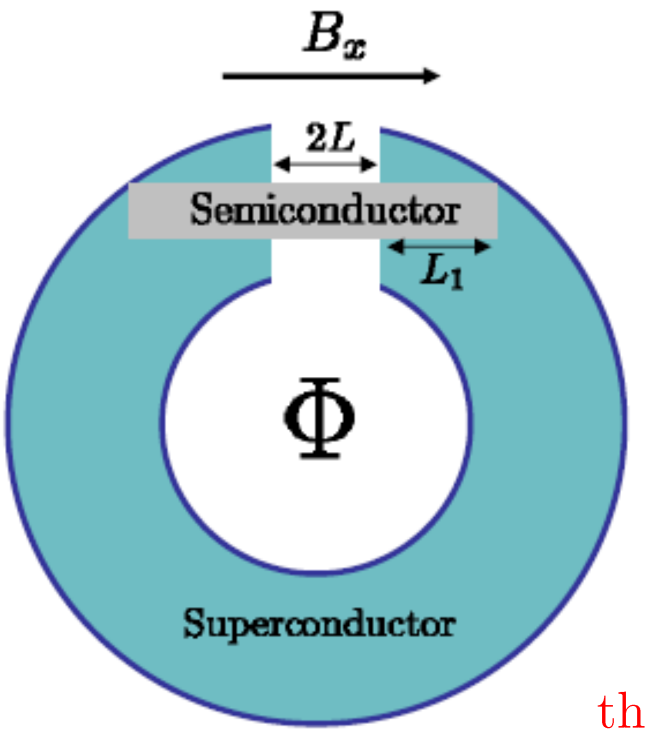
Josephson current through heterostructure

$$I_\pm(\varphi) \propto \frac{\partial E_\pm(\varphi)}{\partial \varphi}$$



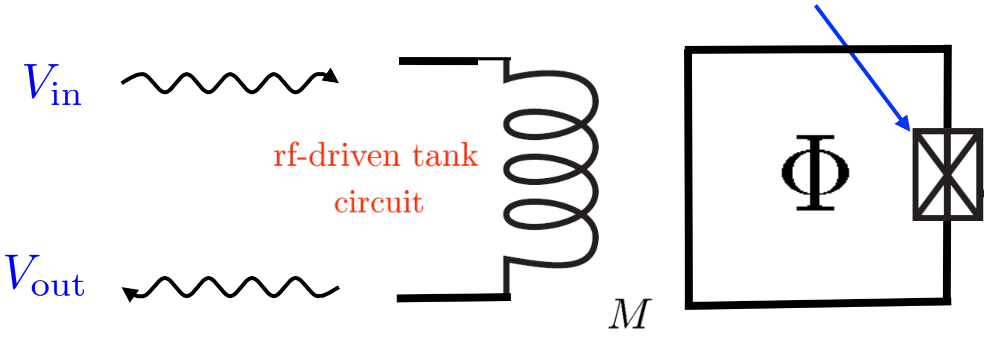
Fractional ac Josephson effect is a robust signature of topological SC

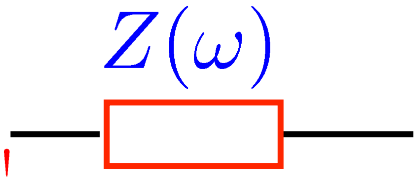
Experimental proposal: nanowire embedded into SQUID



Measure V_{out}
in time domain

SC/SM/SC
heterostructure





$Z(\omega)$ is a function of
the inductance of the SQUID !

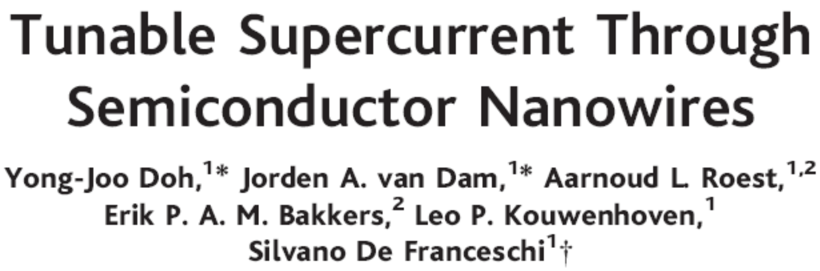
Measurement of Josephson inductance

$I_c \sim 10\text{nA}$

$L_J^{\text{min}} \sim 10 - 100\text{nH}$

Lutchyn, Sau, Das Sarma, PRL'10

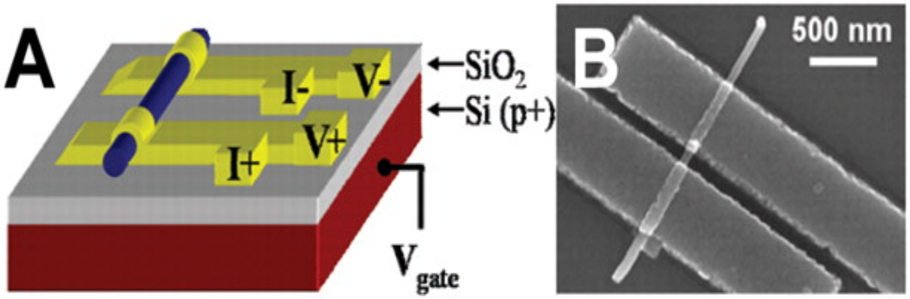
Experimental considerations



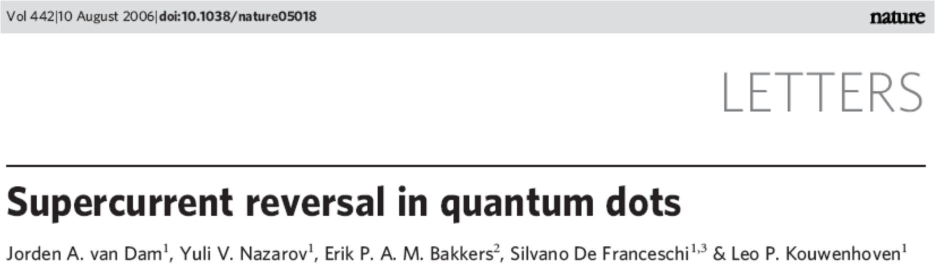
Tunable Supercurrent Through Semiconductor Nanowires

Yong-Joo Doh,[1]* Jorden A. van Dam,[1]* Aarnoud L. Roest,[1,2]
Erik P. A. M. Bakkers,[2] Leo P. Kouwenhoven,[1]
Silvano De Franceschi[1]†

Science **309**, 272 (2005)



Al/InAs/Al heterostructure



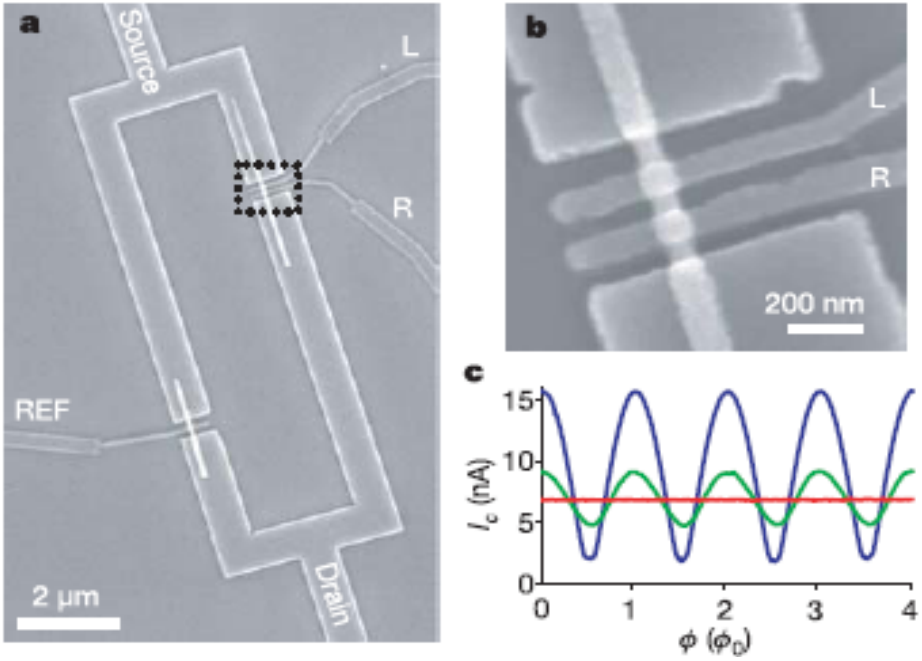
Vol 442|10 August 2006|**doi:10.1038/nature05018** **nature**

LETTERS

Supercurrent reversal in quantum dots

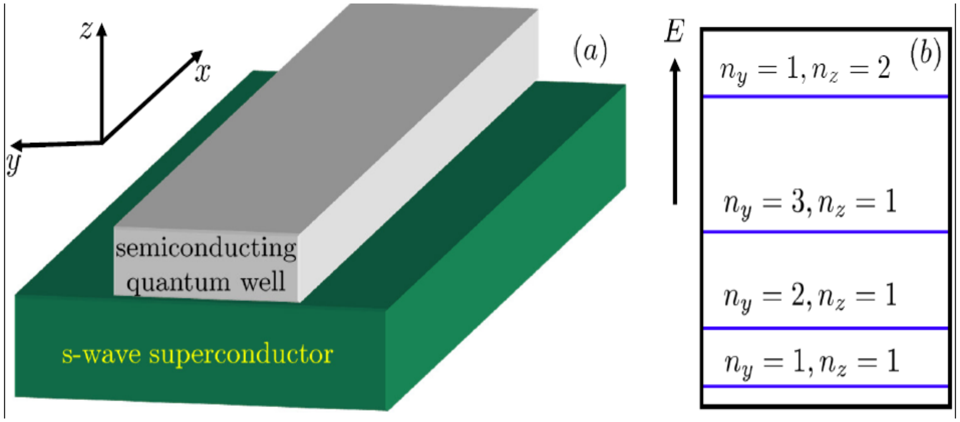
Jorden A. van Dam[1], Yuli V. Nazarov[1], Erik P. A. M. Bakkers[2], Silvano De Franceschi[1,3] & Leo P. Kouwenhoven[1]

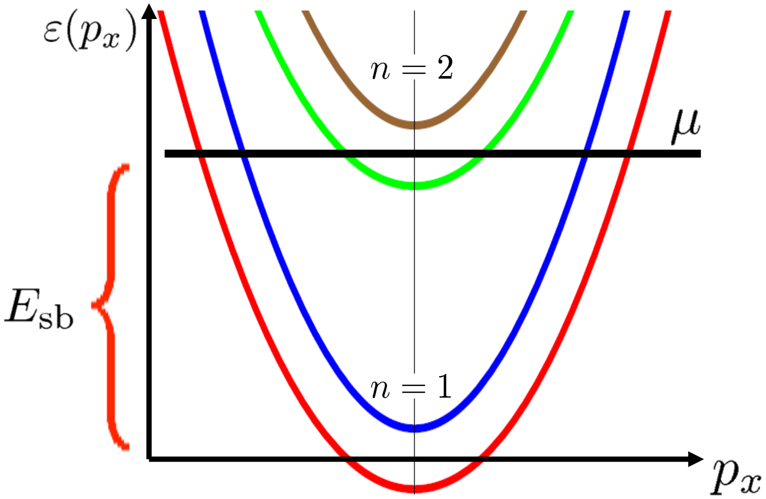
Nature 442, 667 (2006)



Experimental efforts: Delft, Harvard, McGill, UCSB, Weizmann ...

Multi-band semiconductor nanowires





Weak coupling analysis $\Delta \to 0$

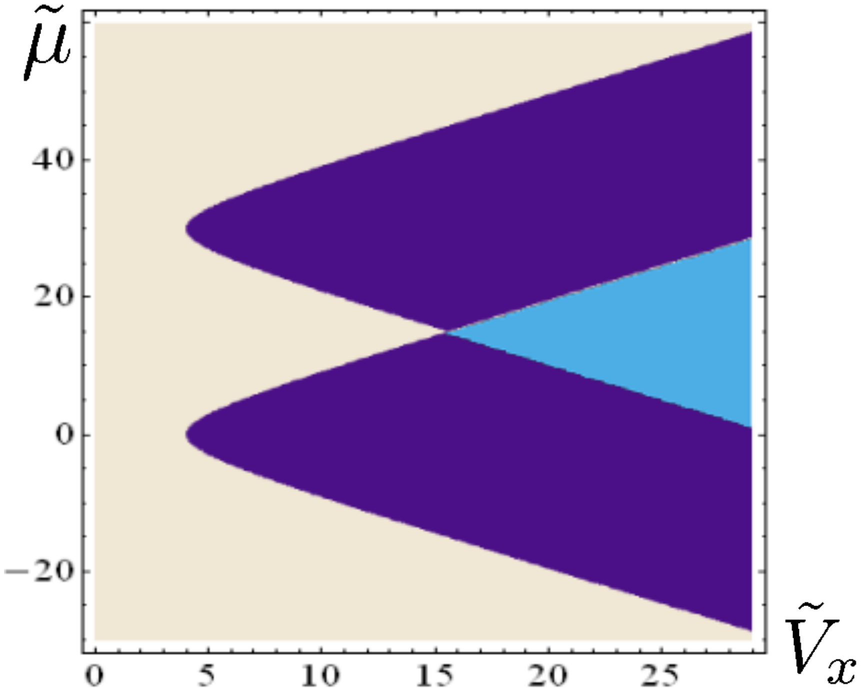
$\mathcal{M} = (-1)^{\nu(0)-\nu(\Lambda)}$ Kitaev, arXiv'00

Topological phase exists when

Second band $|V_x| > \sqrt{(\mu - E_{\text{sb}})^2 + \Delta_0^2}$

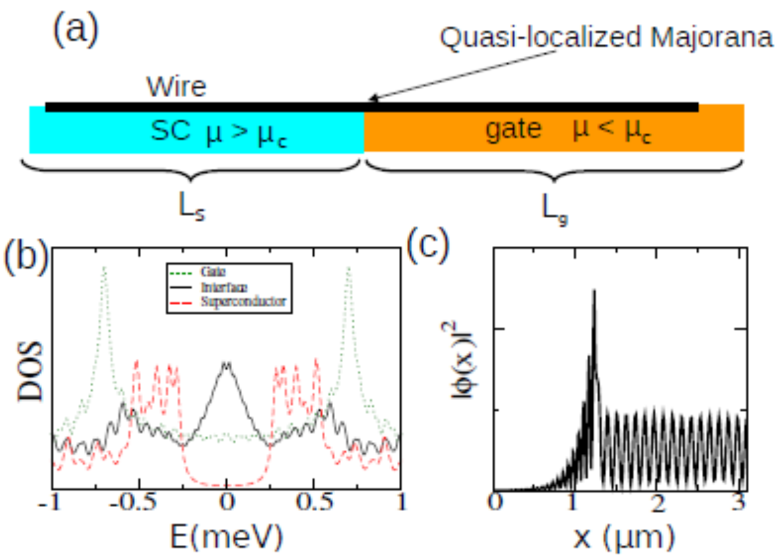
First band $|V_x| > \sqrt{\mu^2 + \Delta_0^2}$

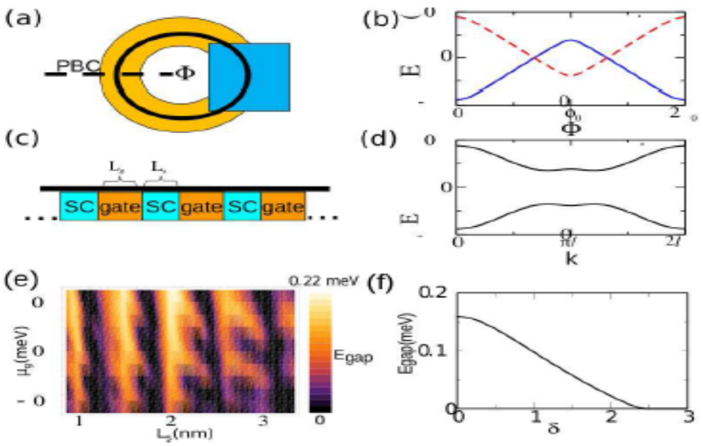
Lutchyn, Stanescu, Das Sarma, arXiv'10



arXiv: 1103.2770 Topological periodic superconductor-nanowire structures

Jay D. Sau[1], Chien Hung Lin[1], Hoi-Yin Hui[1], and S. Das Sarma[1]
[1] Condensed Matter Theory Center and Joint Quantum Institute, Department of Physics,
University of Maryland, College Park, Maryland 20742-4111, USA





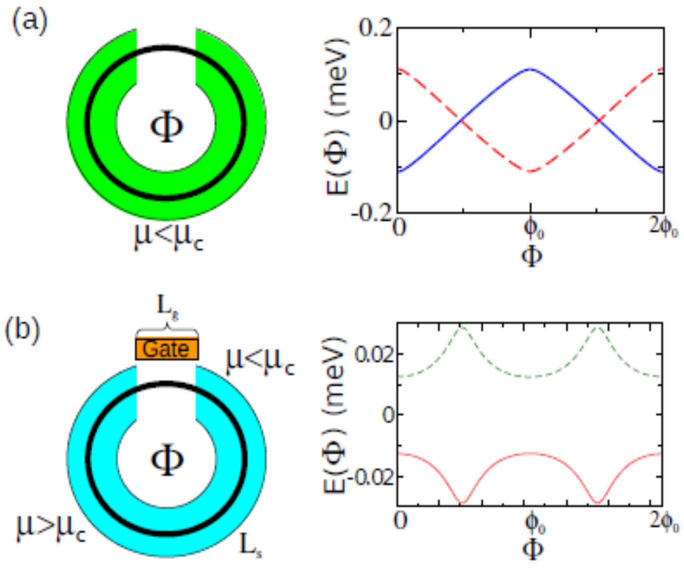


FIG. 2: (a) Junction in a ring topological superconductor structure with chemical potential control over entire structure (such that $\mu \sim 0$ meV $< \mu_c$) shows a fractional Josephson effect. Flux dependence of ABS energies and the corresponding Josephson current in junction show $2\Phi_0$ periodicity. (b) Experimental adaptations modify geometry so that $\mu_s > \mu_c$ in superconductor (length $L_s = 1.5\,\mu m$) with gate-induced chemical potential control only in junction ($\mu_g < \mu_c$) (length $L_g = 600$ nm). ABS spectrum show a conventional Josephson effect in this case despite tunneling signature of MFs in Fig. 1.

Super-current in a semiconducting nanowire

Jay Sau and Sankar Das Sarma, unpublished

$$J(x) = \int dx' \psi^{\dagger}(x') \frac{\delta H_0(A)}{\delta A(x)} \psi(x').$$

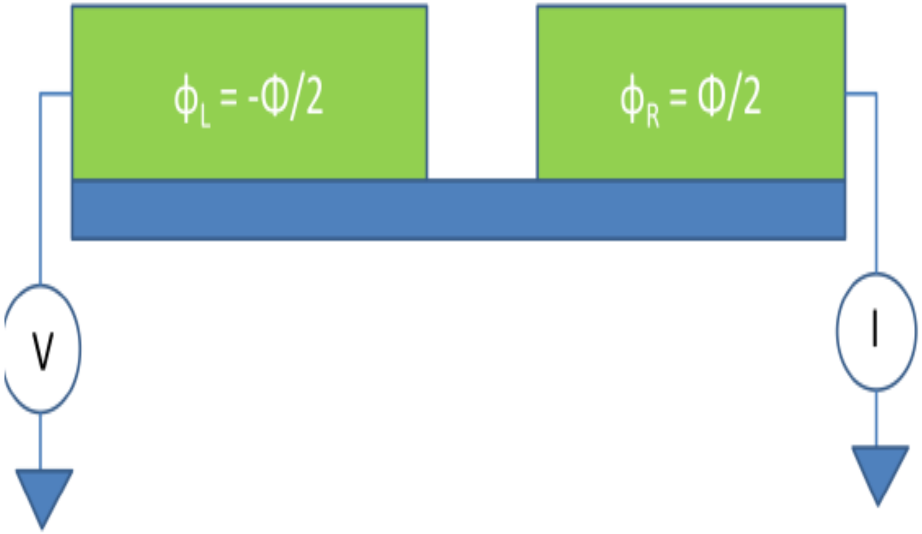
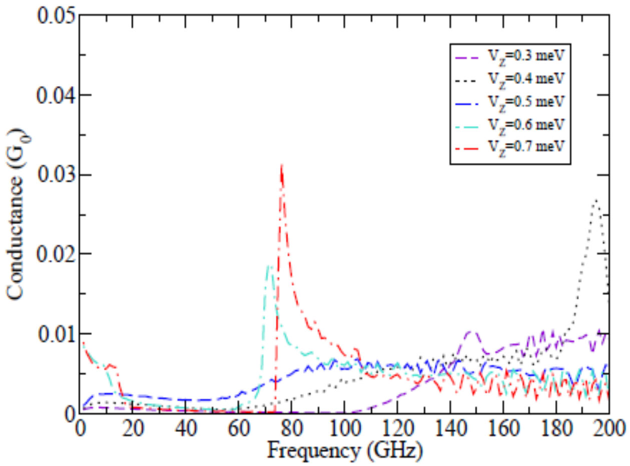
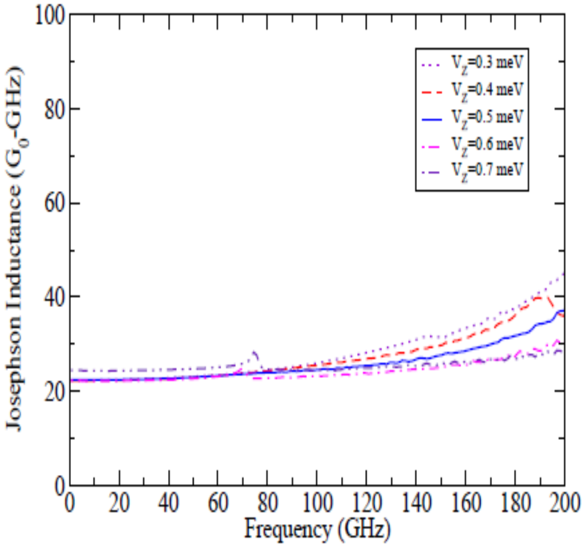


Figure 1: Nanowire geometry for measuring phase transition. All quantities $V(\omega)$,$I(\omega)$ and $\Phi(\omega)$ are frequency-dependent. The josephson phase $\Phi(\omega) = -\frac{i}{\omega}V(\omega)$.





The search for the Majorana 'fermion' may finally be coming to an end

The Majorana mode may soon be observed in table-top experiments as an emergent zero-energy mode in solid state semiconductor-superconductor sandwich structures

'Majorana' may return after a 75-year hiatus – thanks to Michael Freedman

Happy Birthday Michael