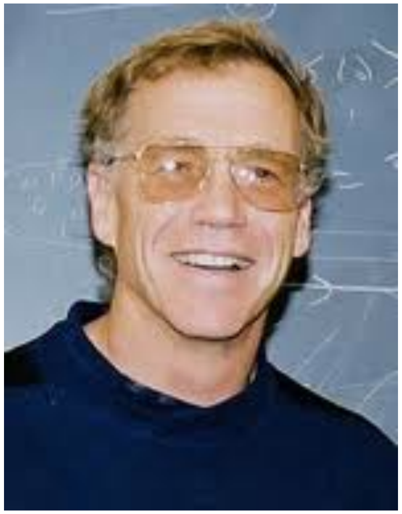
Happy Birthday Mike!



Non-Fermi Liquid (NFL) phases for itinerant electrons

60th Birthday Symposium for Michael Freedman
April 16, 2011

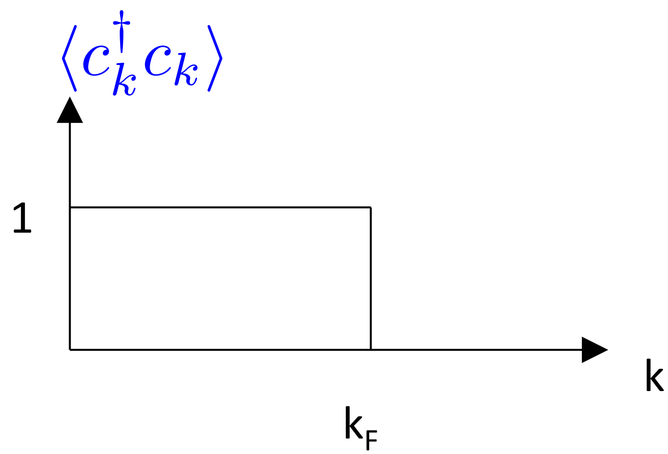
MPA Fisher with Hongchen Jiang, Matt Block, Ryan Mishmash,
Donna Sheng, Lesik Motrunich (in progress)

Goal: Construct and Analyze non-Fermi liquid phases of strongly interacting 2d itinerant electrons

- Wavefunctions for NFL – partons
- Example of a NFL: "D-wave Metal"
- Hamiltonian for the "D-wave Metal"?
- DMRG solution of Hamiltonian

2D Free Fermi Gas

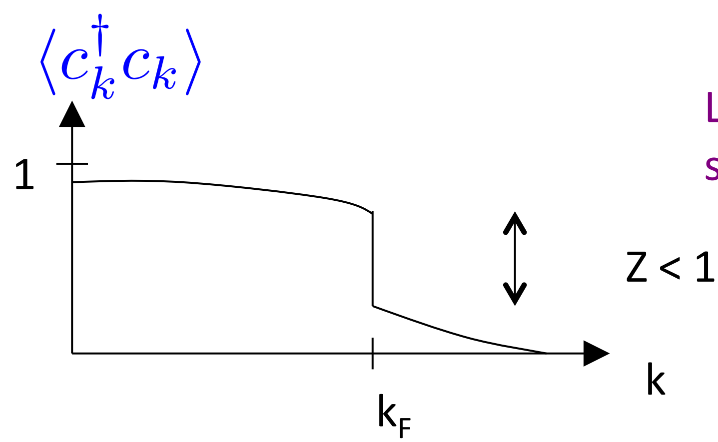
Momentum Distribution Function: $n_k = \langle c_k^\dagger c_k \rangle$



Volume of Fermi sea set by particle density

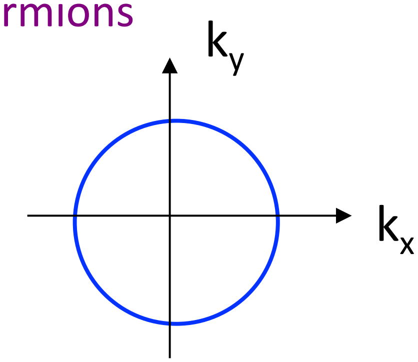
$$\rho = k_F^2 / 4\pi$$

2D Fermi Liquid Metal



Luttingers Thm: Volume inside Fermi surface still set by total density of fermions

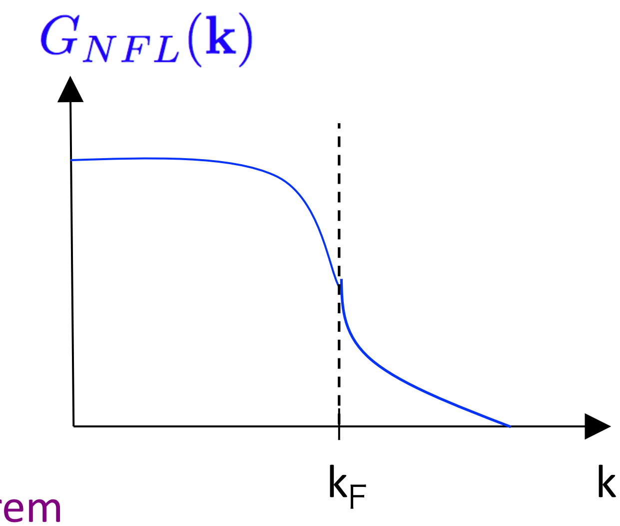
$$\rho = k_F^2 / 4\pi$$



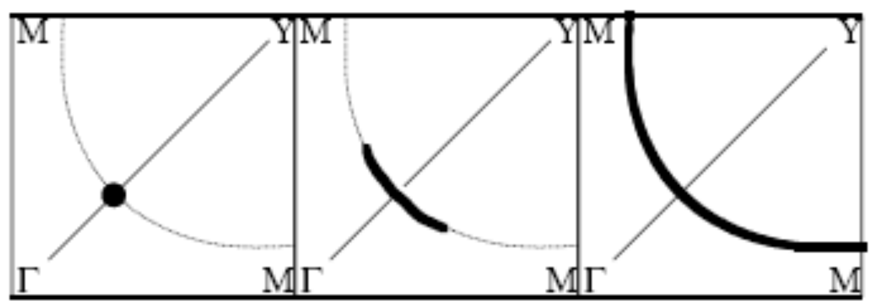
2D Non-Fermi Liquid Metal

Various possibilities:

1) A singular "Fermi surface" that satisfies Luttinger's theorem but without a jump discontinuity in momentum distribution function



2) A singular Fermi surface that violates Luttinger's theorem (eg. volume "x" rather than "1-x")



3) A singular "Fermi surface" with ``arc"

4.) Other….

Wavefunctions for 2d Fermions

Wf for **free Fermi gas**:
Slater determinant

$$\Psi_{FF}(\mathbf{r_{i\uparrow}}, \mathbf{r_{i\downarrow}}) = \det[\mathbf{e^{ik_i \cdot r_{j\uparrow}}}] \det[\mathbf{e^{ik_i \cdot r_{j\downarrow}}}]$$

Wf for an **interacting Fermi liquid:**

(a) Gutzwiller projection (no double occupancy)

$$\Psi_G = \mathcal{P}_G[\Psi_{FF}]$$

(b) Multiply by Jastrow factor

$$\Psi_{FL} = e^{-\sum_{i<j} u(\mathbf{R_i} - \mathbf{R_j})} \times \Psi_{FF} \qquad \{\mathbf{R_i}\} = \{\mathbf{r_{i\uparrow}}, \mathbf{r_{i\downarrow}}\}$$

Parton approach to NFL wavefunctions

Decompose electron:
spinless charge e boson,
s=1/2 neutral fermionic spinon

$$c_\sigma = b f_\sigma$$

Mean Field Theory

Treat "Spinons" and Bosons as Independent: $\mathcal{H} = \mathcal{H}_f + \mathcal{H}_b$

Wavefunctions $\psi_f(\mathbf{r_{i\uparrow}}, \mathbf{r_{i\downarrow}})$ $\psi_b(\mathbf{R_i})$

(enlarged Hilbert space - twice as many particles)

"Fix-up" Mean Field Theory

"Glue" together Fermion and Boson "partons"

$$\Psi \equiv \psi_f(\mathbf{r_{i\alpha}}) \times \psi_\mathbf{b}(\mathbf{R_i} \to \mathbf{r_{i\alpha}})$$

Project back into physical Hilbert space

Fermi and Non-Fermi Liquids?

Spinons in a filled Fermi sea $$\psi_f = \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\uparrow}}})]\det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\downarrow}}})]$$

Fermi Liquid: Bosons into Bose condensate $$\Psi_{FL} = \psi_f^{FF} \times \psi_b^{BEC}$$

$$\psi_b^{BEC} = e^{-\sum_{i<j} u(\mathbf{R_i}-\mathbf{R_j})} \qquad c_\sigma = \langle b \rangle f_\sigma \sim (const) f_\sigma$$

Non-Fermi Liquid: Bosons into *uncondensed* fluid - a "Bose metal"

$$\Psi_{NFL} = \psi_f^{FF} \times \psi_b^{BoseMetal}$$

NFL Metal: Product of Fermi sea and uncondensed Bose-Metal

Bose-metal: Fragment the charge sector

$$b = d_1 d_2 \qquad (c_\alpha = f_\alpha d_1 d_2)$$

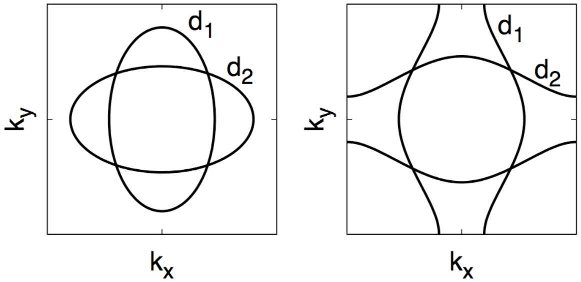
(all Fermionic decomposition of the electron)

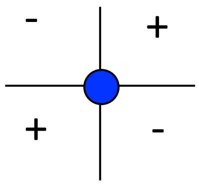
Gutzwiller wavefunction: $\psi_b = det_1 \times det_2$

Wavefunction for D-wave Bose-Metal (DBM)

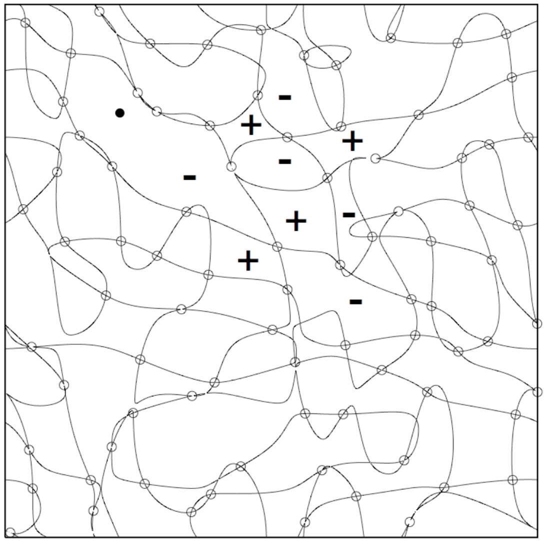
DBM: Product of 2 Fermi sea determinants, elongated in the x or y directions

$$\psi_{DBM} = det_x \times det_y$$





D_{xy} relative
2-particle correlations



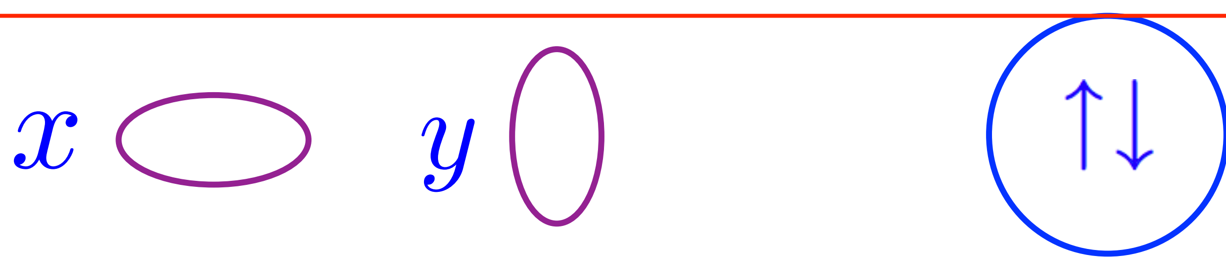
"D-Wave Metal"

Itinerant NFL phase of 2d electrons?

Parton construction $\qquad c^\dagger_\alpha(\mathbf{r}) = b^\dagger(\mathbf{r}) f^\dagger_\alpha(\mathbf{r}) = d^\dagger_x(\mathbf{r}) d^\dagger_y(\mathbf{r}) f^\dagger_\alpha(\mathbf{r})$

Wavefunction; Product of determinants $\qquad \{\vec{R}_i\} = \{\vec{r}_{i\uparrow}, \vec{r}_{j\downarrow}\}$

$$\Psi^{Metal}_{d_{xy}} = \det_x[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}] \cdot \det_y[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}] \times \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\uparrow}}}] \cdot \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\downarrow}}}]$$



Filled Fermi sea

Can use Variational Monte Carlo to extract equal time correlation functions from wf
But what about energetics???

Hamiltonian with D-wave Metal ground state???

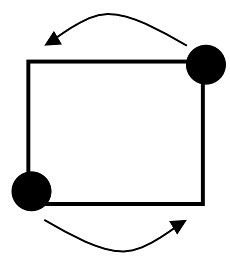
First: D-wave **Bose**-metal

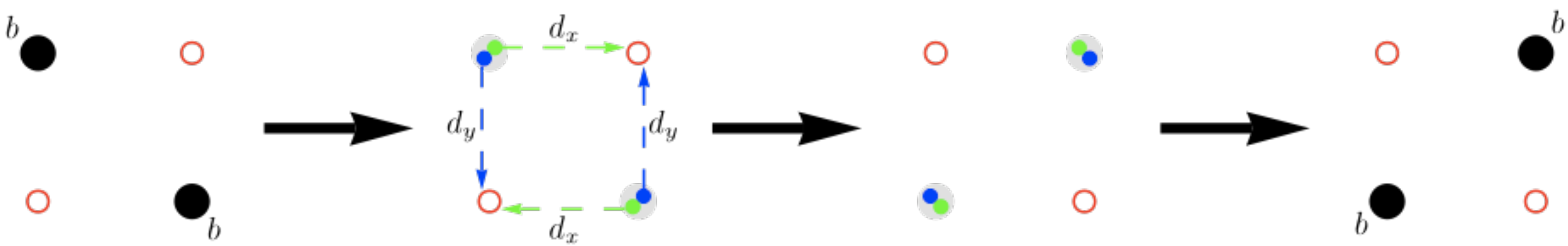
$$b = d_x d_y$$

U(1) gauge theory for d_x, d_y fermions
Strong coupling limit to obtain Boson Hamiltonian

$$\begin{aligned} H &= H_J + H_4 , \\ H_J &= -J \sum_{\mathbf{r};\, \hat{\mu}=\hat{\mathbf{x}},\hat{\mathbf{y}}} (b_{\mathbf{r}}^\dagger b_{\mathbf{r}+\hat{\mu}} + h.c.) , \\ H_4 &= K_4 \sum_{\mathbf{r}} (b_{\mathbf{r}}^\dagger b_{\mathbf{r}+\hat{\mathbf{x}}} b_{\mathbf{r}+\hat{\mathbf{x}}+\hat{\mathbf{y}}}^\dagger b_{\mathbf{r}+\hat{\mathbf{y}}} + h.c.) , \end{aligned}$$

"Ring exchange"





Hamiltonian for D-wave Metal?

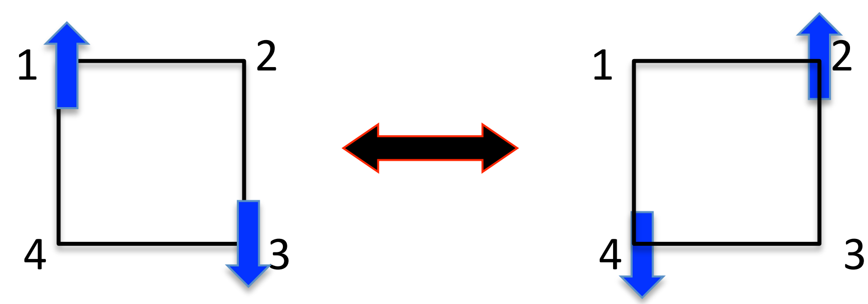
Strong coupling limit of parton gauge theory $c_\alpha = f_\alpha d_x d_y$

t-K "Ring" Hamiltonian (no double occupancy constraint)

$$\mathcal{H}_{tK} = -t \sum_{\langle ij \rangle} [c_{i\alpha}^\dagger c_{j\alpha} + h.c.] + K \sum_{\langle 1234 \rangle} [\mathcal{S}_{13}^\dagger \mathcal{S}_{24} + h.c.]$$

$$\mathcal{S}_{ij}^\dagger = \frac{1}{\sqrt{2}} [c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger - c_{i\downarrow}^\dagger c_{j\uparrow}^\dagger]$$

Electron singlet pair
"rotation" term



- Ring term will be generated when projecting Coulomb interaction into a single band model
- Ring term operates when two doped holes are nearby
- Ring term induces 2-particle singlet d-wave correlations (for K>0)

Phase diagram of electron t-K Hamiltonian?

$$\mathcal{H}_{tK} = -t \sum_{\langle ij \rangle} [c^{\dagger}_{i\alpha} c_{j\alpha} + h.c.] + K \sum_{\langle 1234 \rangle} [\mathcal{S}^{\dagger}_{13} \mathcal{S}_{24} + h.c.]$$

(Density and K/t)

Severe sign problem - intractable

Approach: Analyze t-K electron ring Hamiltonian on 2-leg ladder

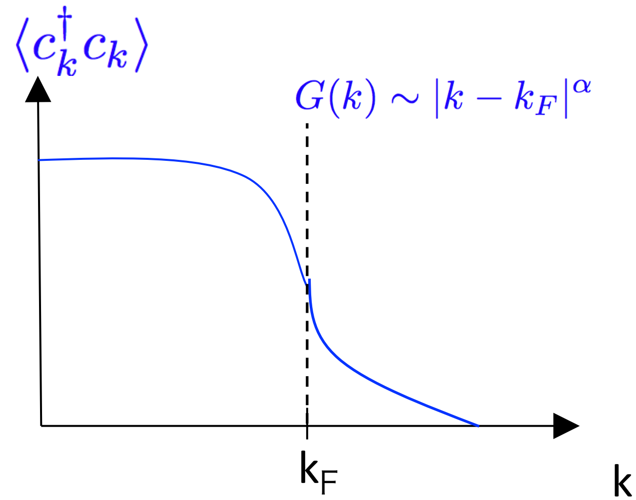
Possible to identify a NFL on a 2-leg ladder?

Interacting Fermions in 1d: A Luttinger liquid

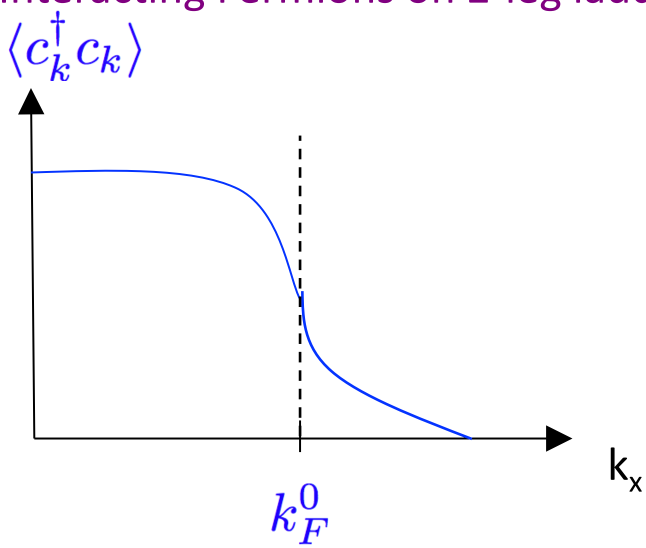
$$G(x) \sim \sin(k_F x)/x^{1+\alpha}$$

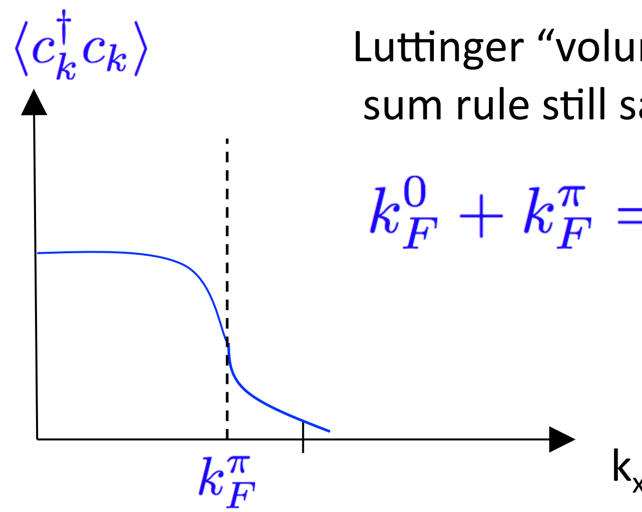
Luttinger liquid exponent: α

Momentum distribution function has (dominant) singularity at $k=k_F$ satisfying Luttinger sum rule



Interacting Fermions on 2-leg ladder: 2-bands





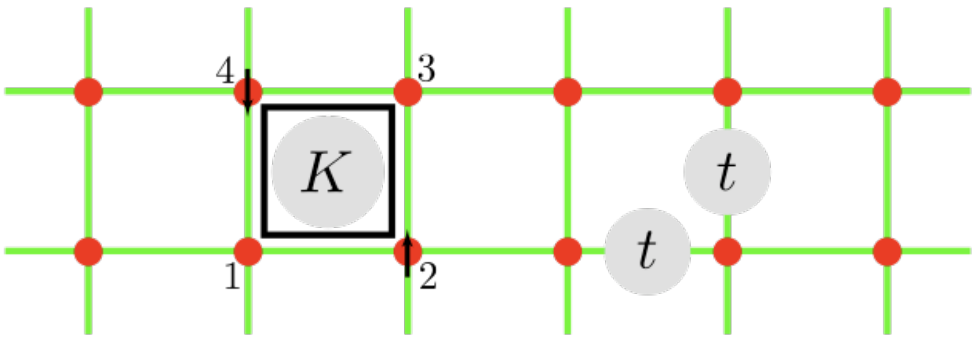
Luttinger "volume" sum rule still satisfied:

$$k_F^0 + k_F^\pi = \pi n_{1d}$$

Searching for a "non-Luttinger liquid" (ie. a Luttinger-liquid violating Luttinger's sum rule)

Electron t-K model on 2-leg ladder

$$\mathcal{H}_{tK} = -t \sum_{\langle ij \rangle} [c_{i\alpha}^{\dagger} c_{j\alpha} + h.c.] + K \sum_{\langle 1234 \rangle} [\mathcal{S}_{13}^{\dagger} \mathcal{S}_{24} + h.c.]$$

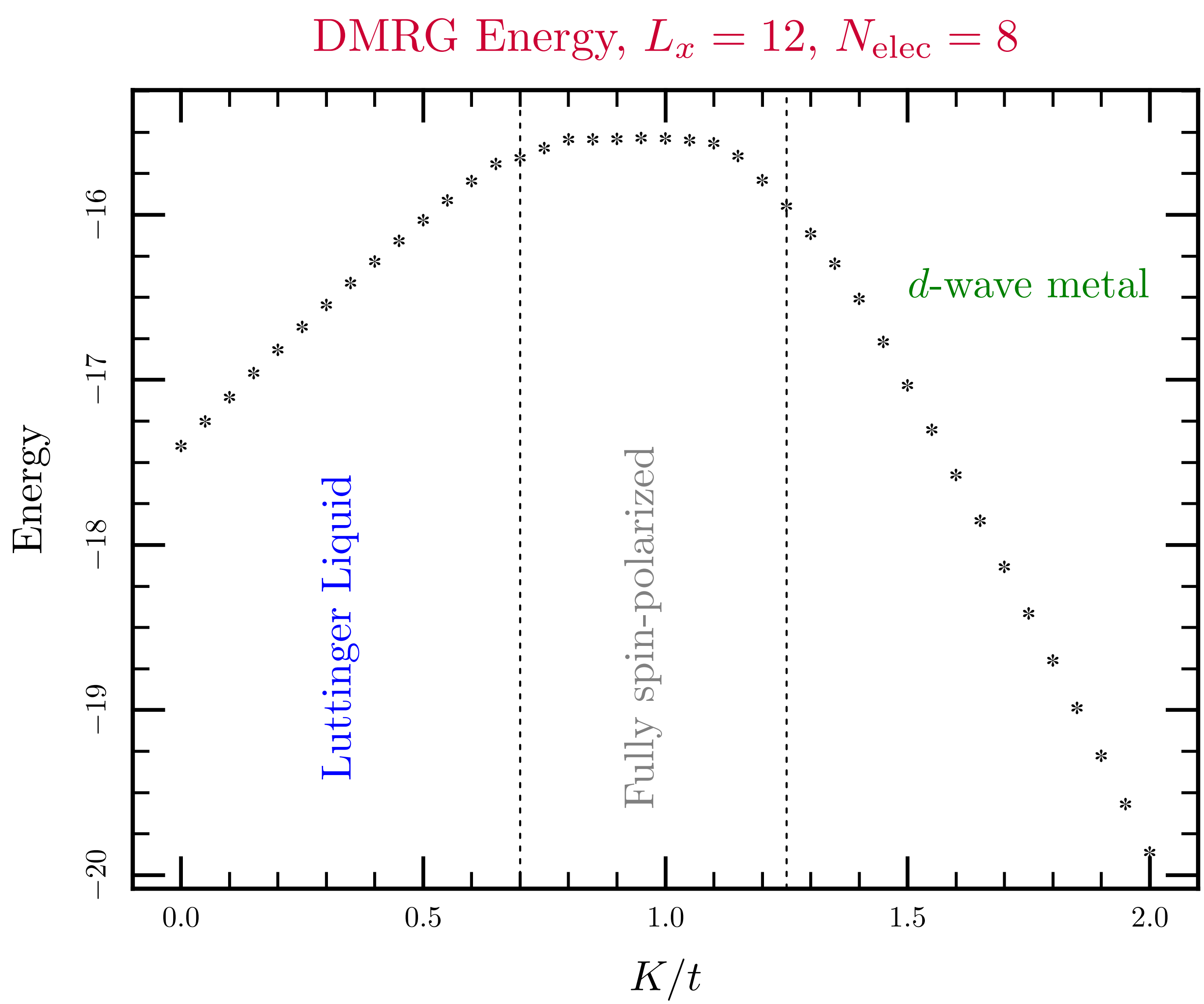


Two dimensionless parameters:
K/t and density n
(n=1/3 henceforth)
No double occupancy

Hongchen Jiang, Matt Block, Ryan Mishmash, Donna Sheng, Lesik Motrunich and MPAF (in progress)

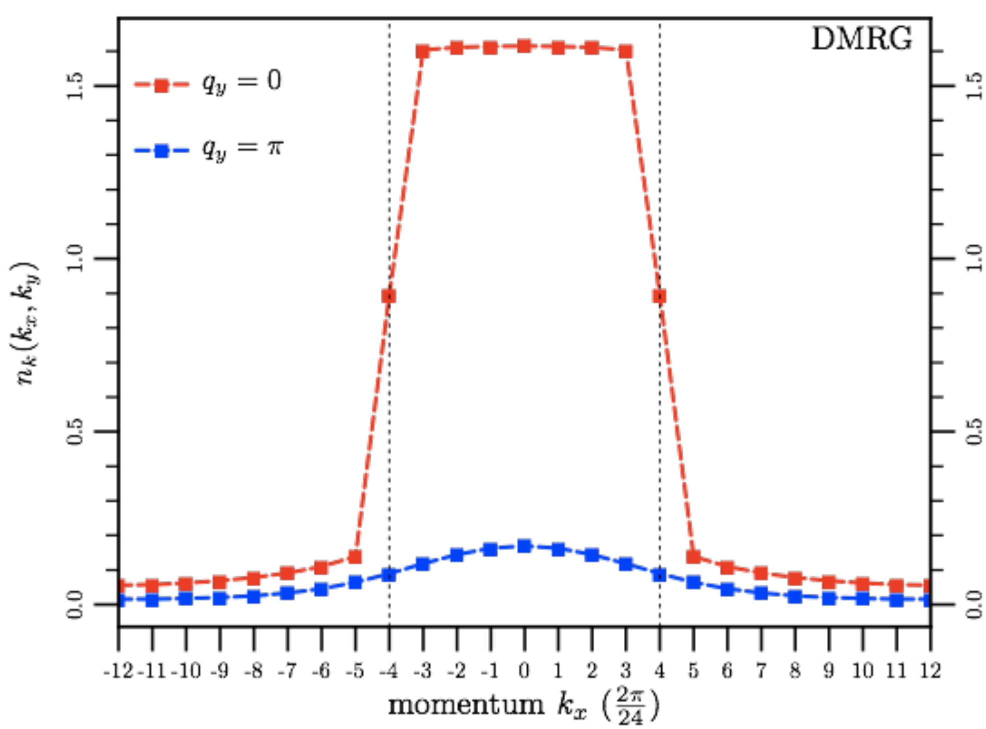
Attacked model using:
ED
DMRG
VMC
Bosonization of Quasi-1d U(1) gauge theory

Ground State energy: DMRG



K/t <0.7 Luttinger Liquid

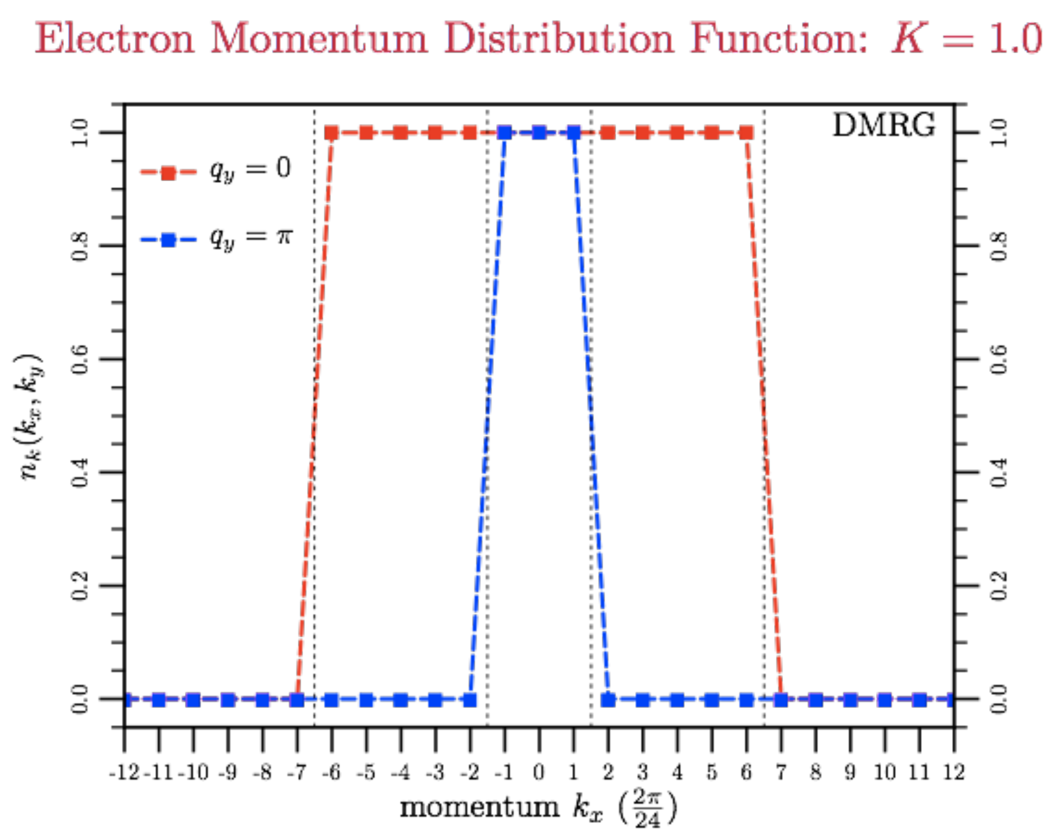
Electron Momentum Distribution Function: $K = 0.0$



Satisfies Luttinger's Theorem: the volume enclosed by the "Fermi surface" yields the particle density. (16 particles, singlet, 8 up and 8 down)

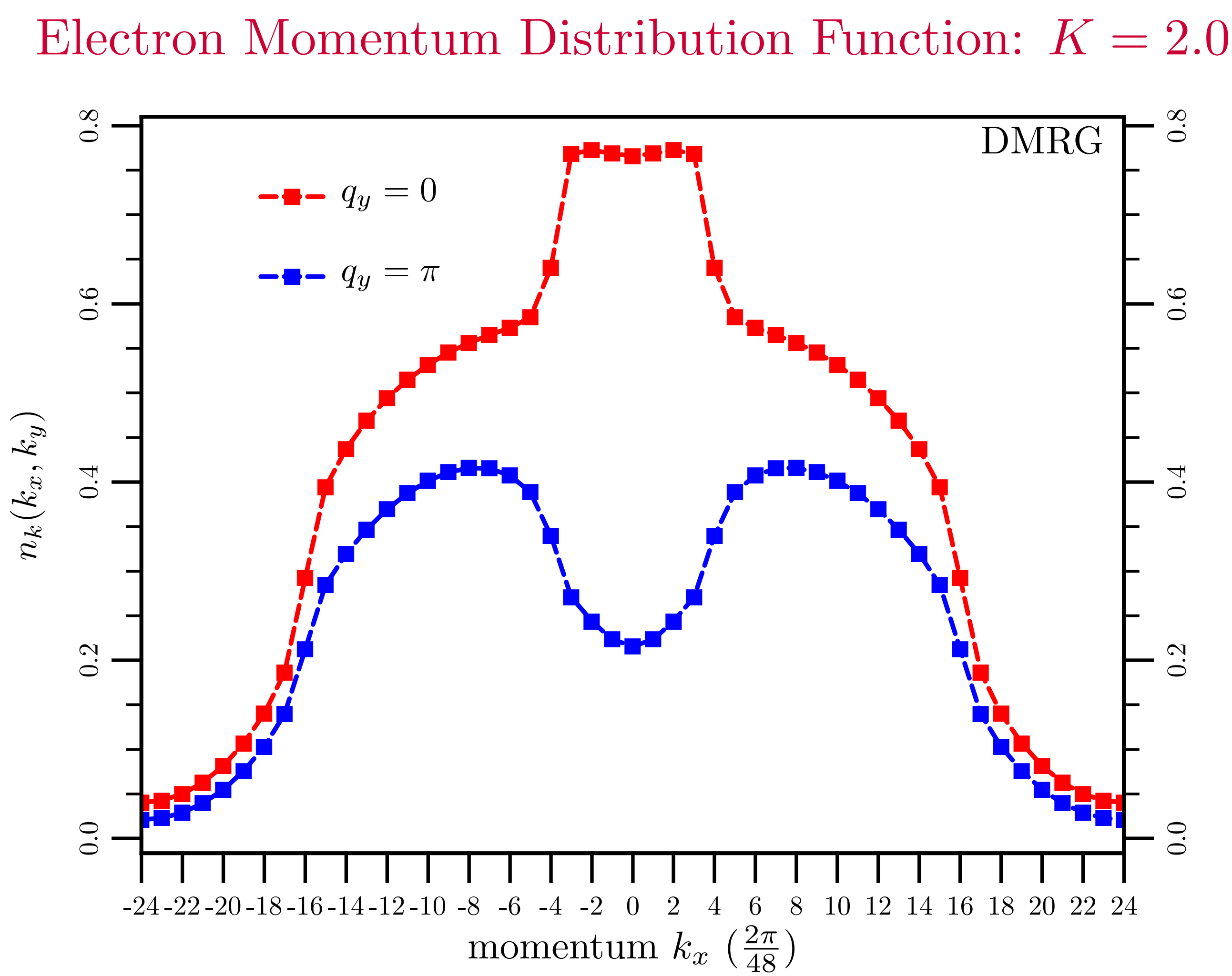
A canonical (single band) Luttinger liquid

0.7< K/t <1.25 : Spin Polarized

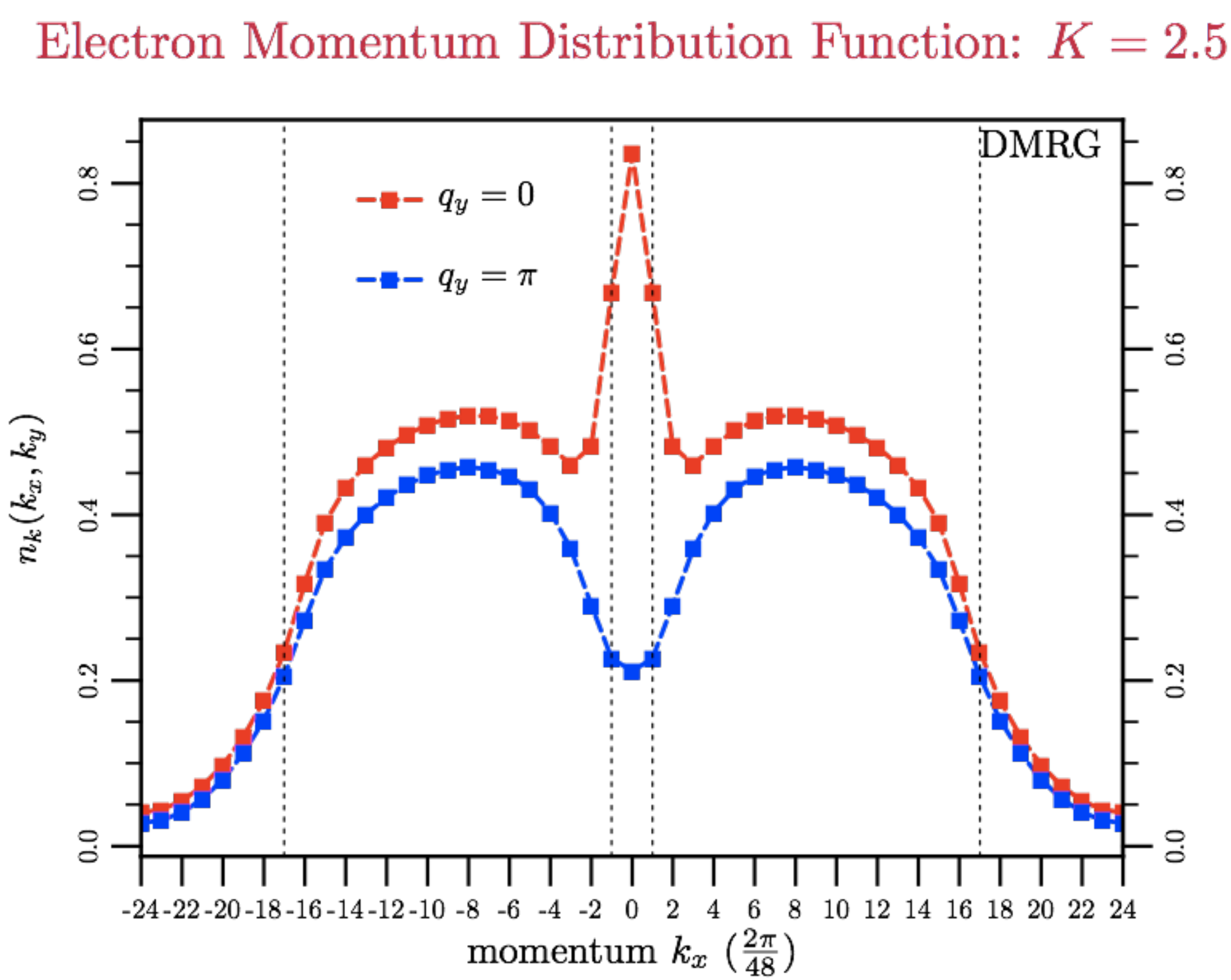


Non-interacting spin polarized Fermi sea is exact ground state here.
Luttinger theorem satisfied

K/t>1.25: Non-LL Phase



K/t > 1.25: Non-LL Phase



Non-monotonic momentum distribution function; No sign of Luttingers volume

Non-Luttinger-Liquid phase for K>1.25?

Electron momentum distribution function: Singular features,
but at momenta which do not satisfy Luttinger's volume theorem

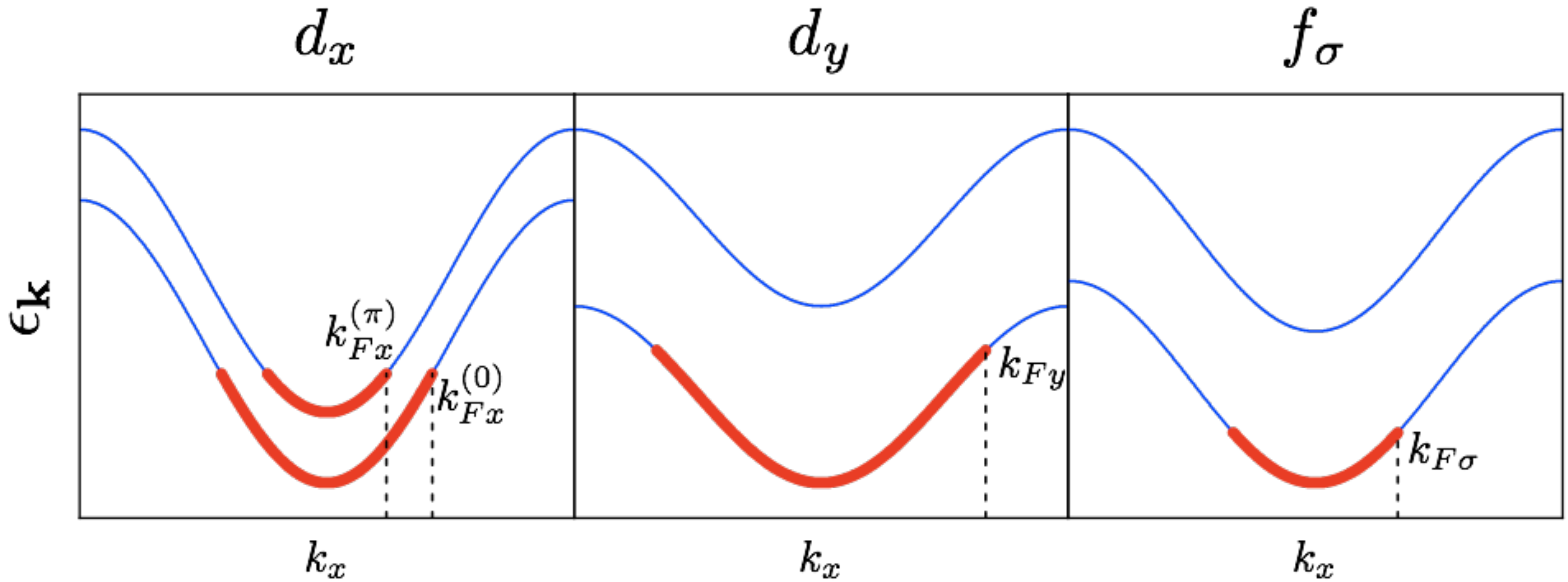
$$n_{k\sigma}(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \langle c^\dagger_{i\sigma} c_{j\sigma} \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$

Can we understand in terms of
D-wave Metal on 2-leg ladder??

Employ parton construction, gauge theory and VMC $c_\alpha = f_\alpha d_x d_y$

The *d*-wave Metal on 2 Legs

$$c^\dagger_\sigma = d^\dagger_x d^\dagger_y f^\dagger_\sigma$$



$$k^{(0)}_{Fx} + k^{(\pi)}_{Fx} = k_{Fy} = 2k_{F\sigma} = 2\pi\rho$$

Gauge theory: Projects down to physical Hilbert space
Number of 1d modes = (number in MFT) – (gauge constraints) = (2+1+2)-(2) = 3
Central charge c=3, strongly entangled

Electron momentum distribution function

Mean Field Theory: electron momentum distribution, convolution of partons

$$n_c^{MFT}(k) = n_{d_x}(k) \otimes n_{d_y}(k) \otimes n_f(k) \qquad c_\sigma = d_x d_y f_\sigma$$

Gauge theory - certain wavevectors enhanced

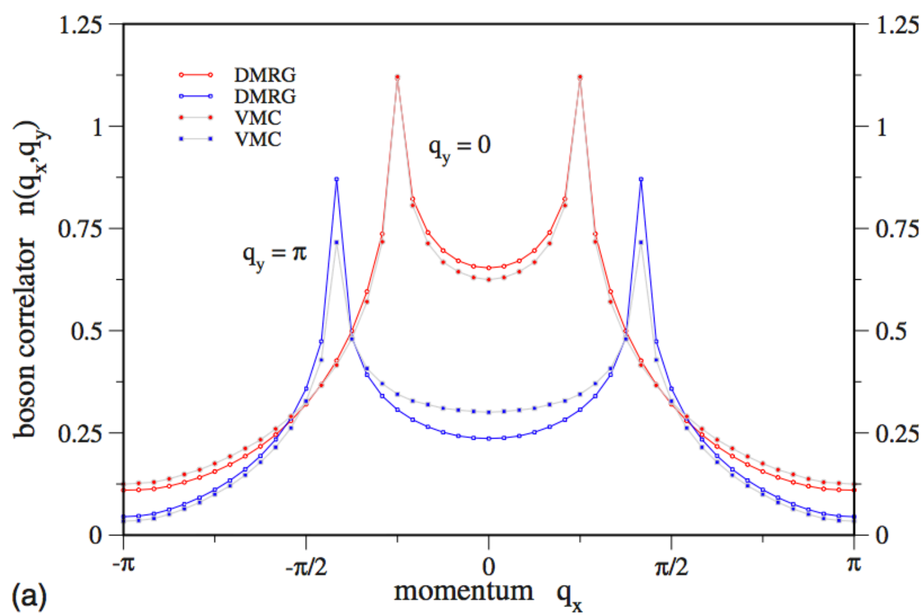
Illustrate with Boson ring model (MFT)

$$n_b^{MFT}(k) = n_{d_x}(k) \otimes n_{d_y}(k)$$

$$b = d_x d_y$$

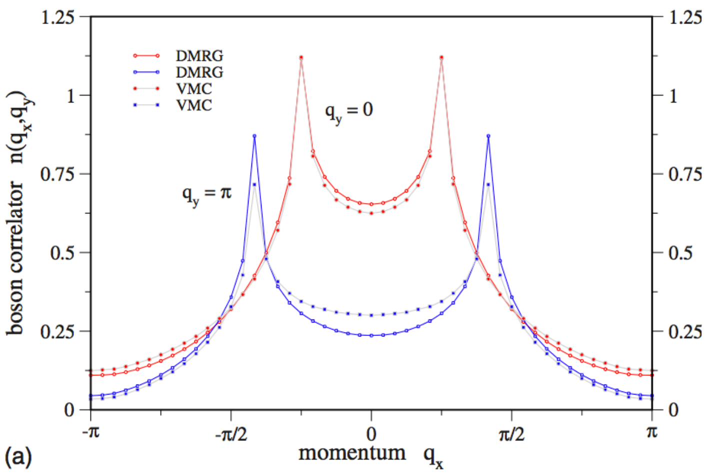
Very sharp peaks in the **exact** boson momentum distribution function! (from DMRG)

$$n_b(k)$$

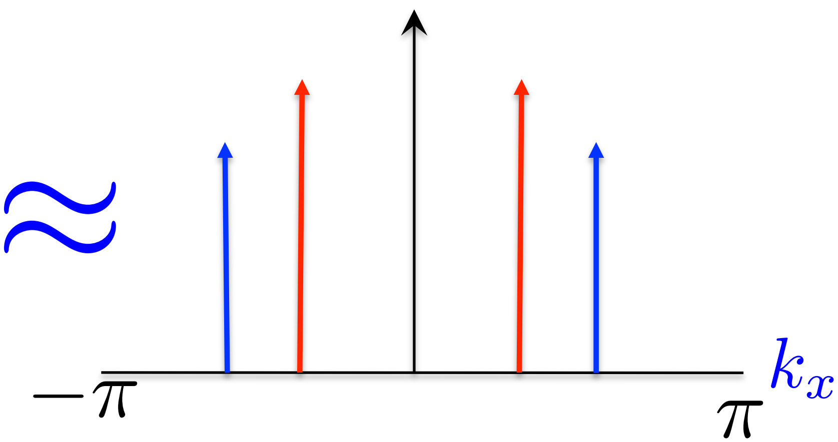


Momentum distribution function in the d-wave metal?

$$n_b(k) \qquad\qquad \tilde{n}_b(k)$$



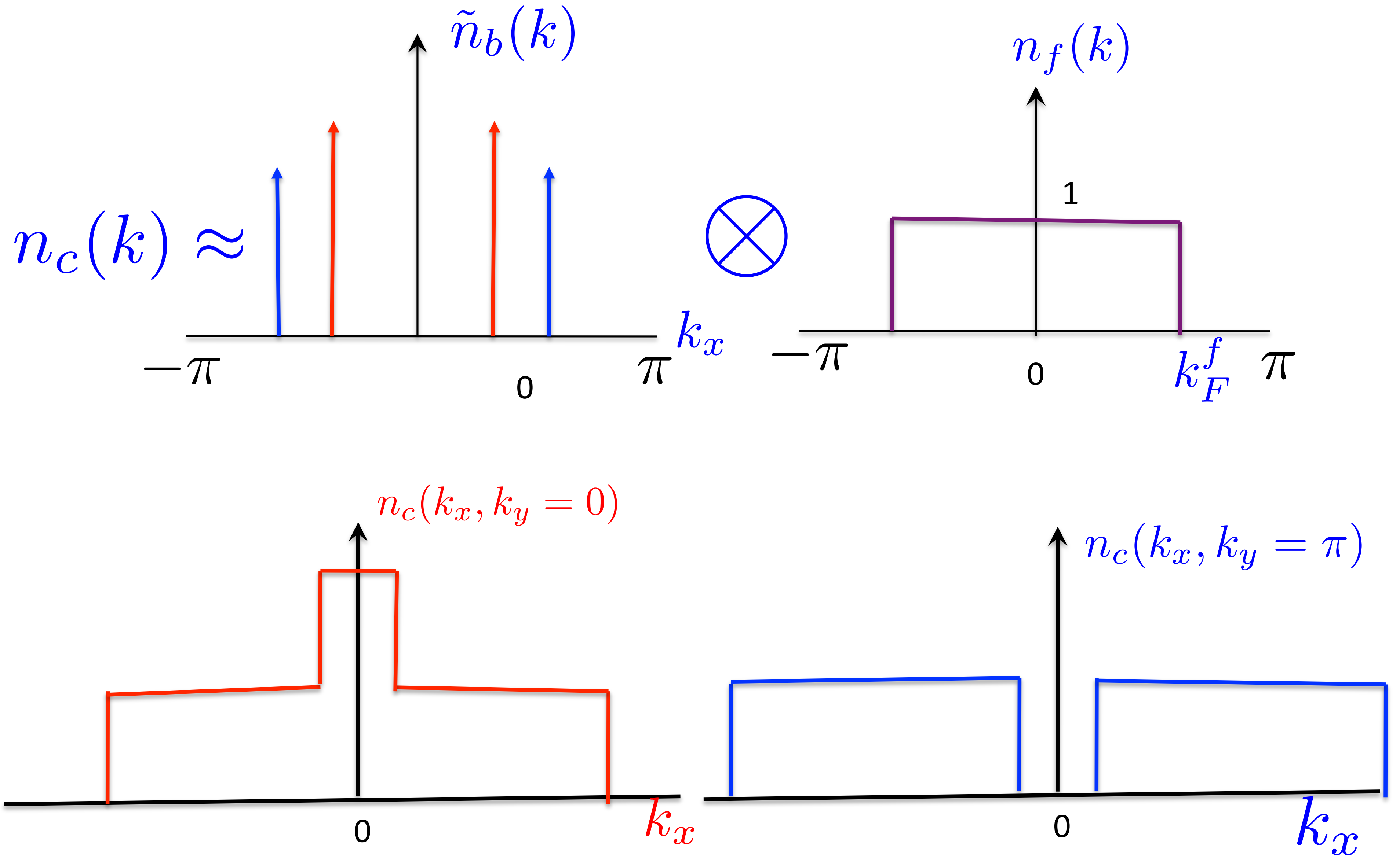
$$\approx$$



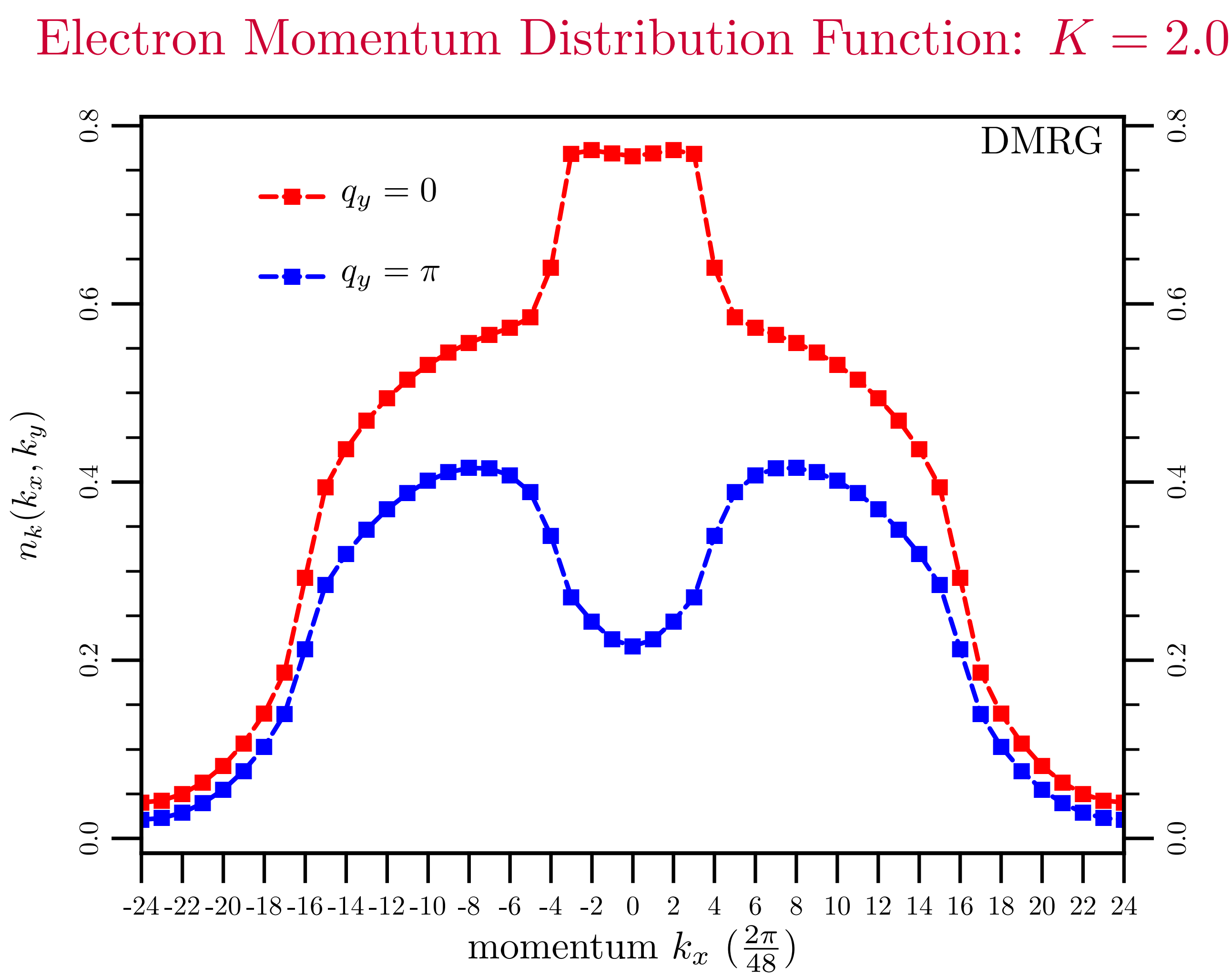
$$n_c(k) \overset{?}{\approx} \tilde{n}_b(k) \otimes n_f(k)$$

$n_f(k) = \Theta(K_F^f - |k|)$ (Free spinon sea)

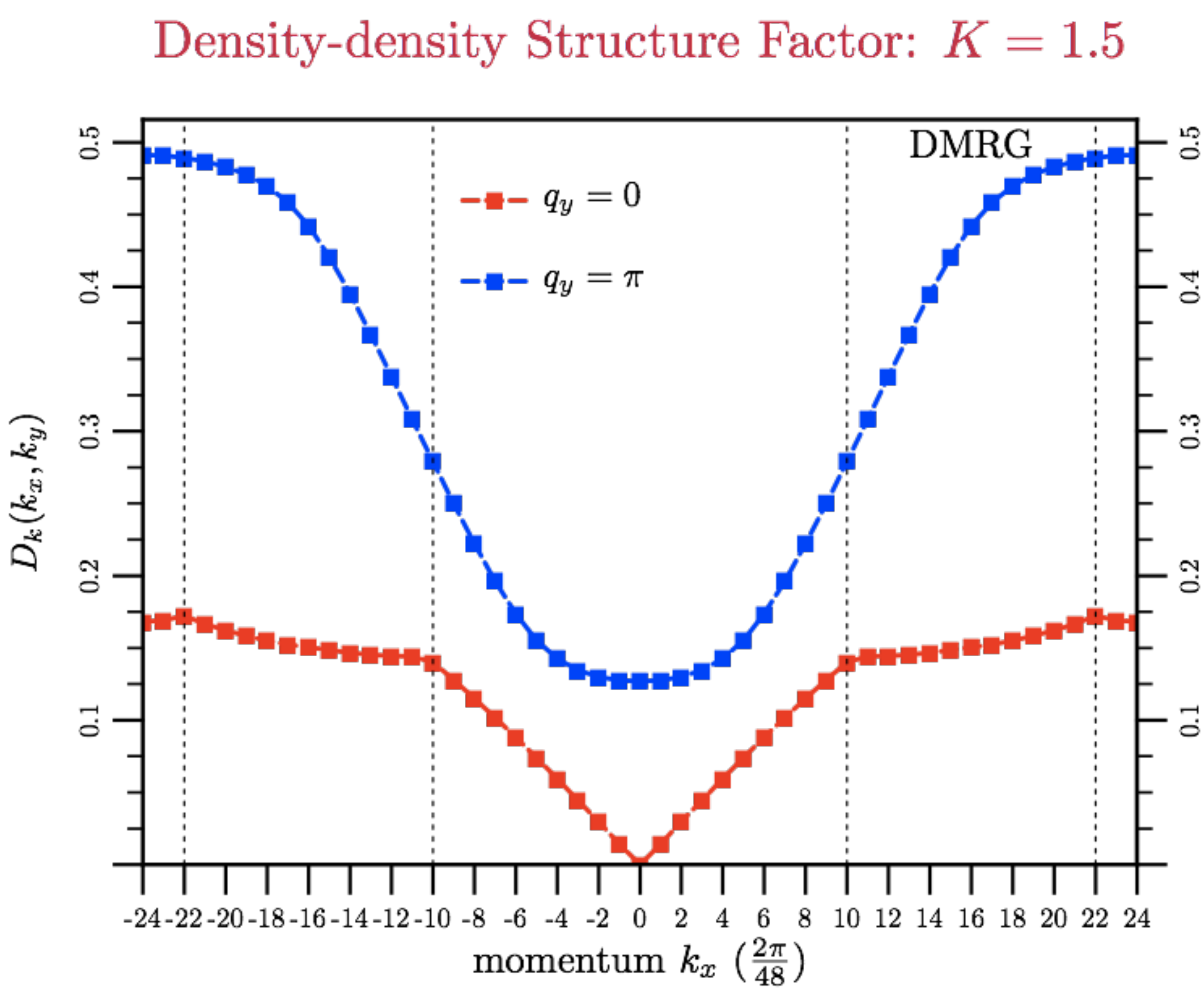
Convolution: c = b f



K/t>1.25: Non-LL Phase

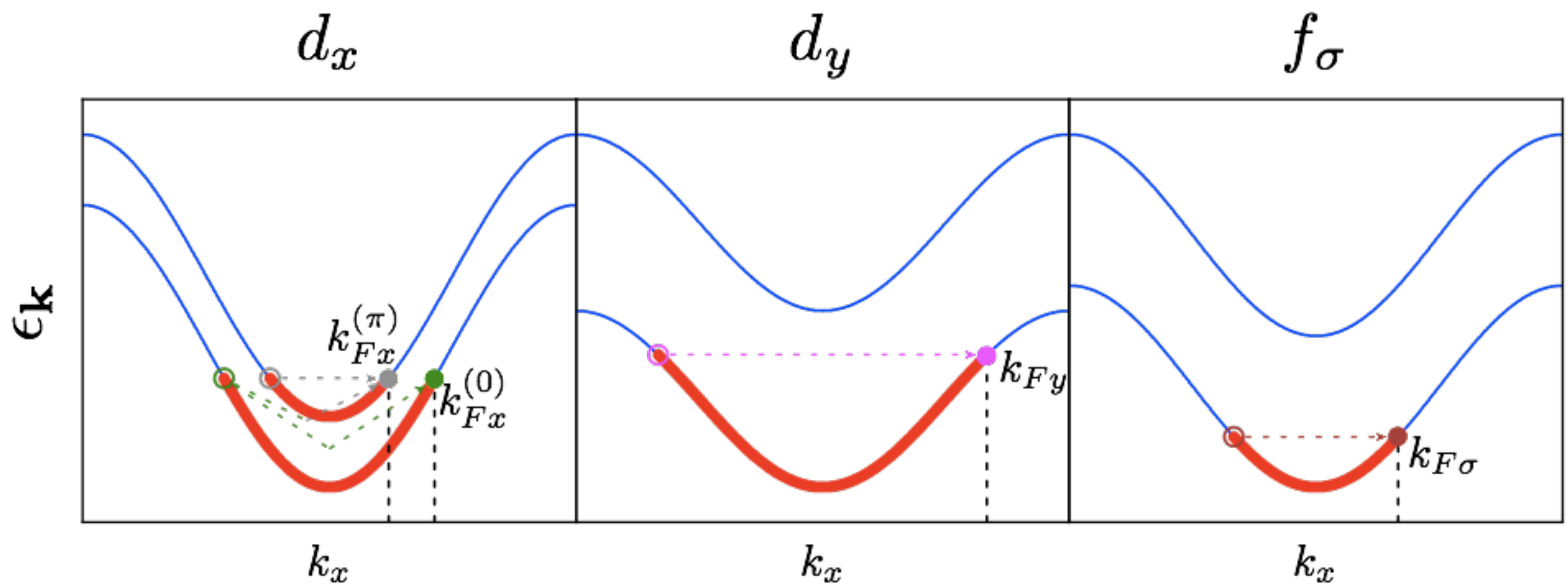


Density-density structure factor: DMRG



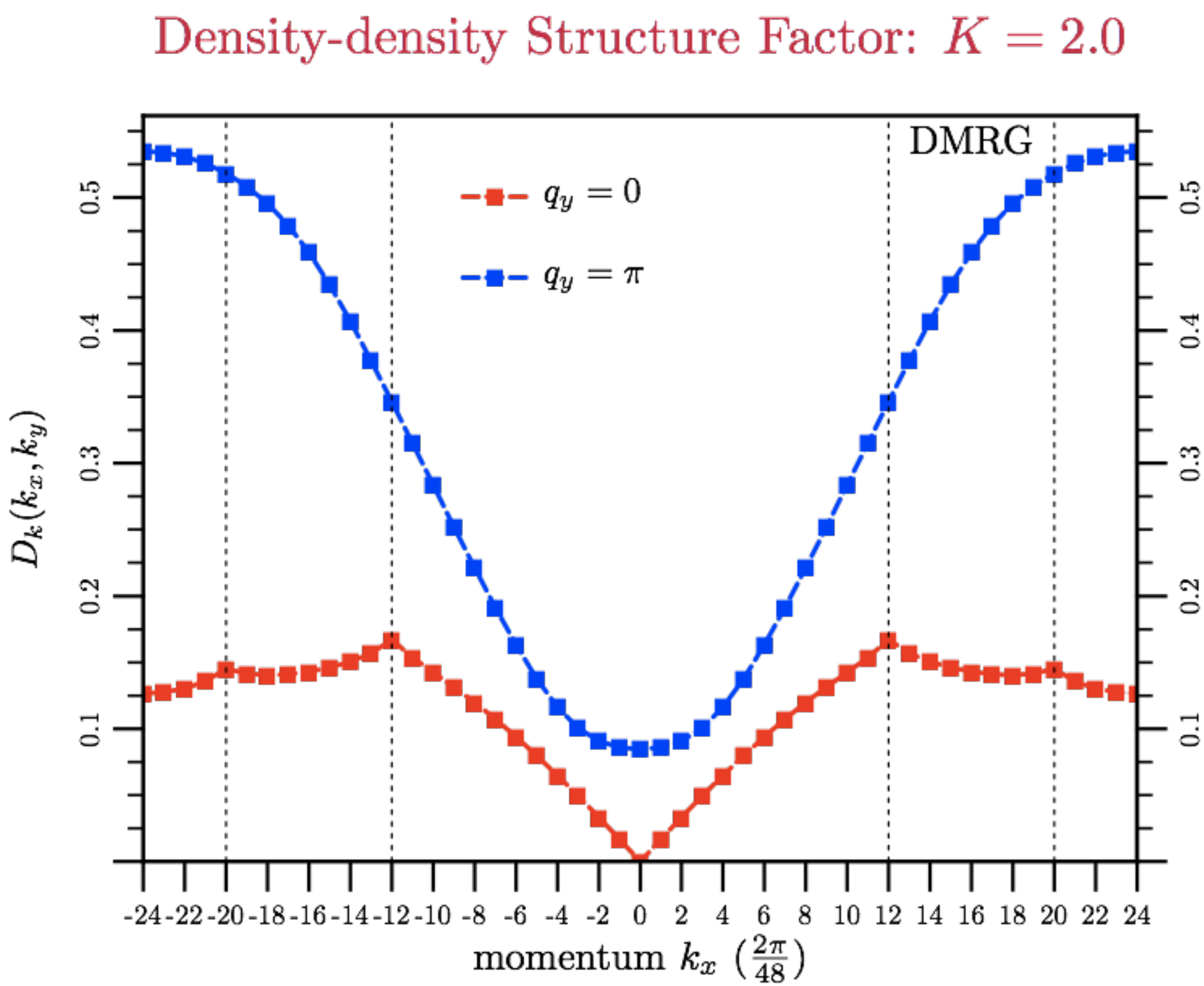
The *d*-wave Metal on 2 Legs

$$c_\sigma^\dagger = d_x^\dagger d_y^\dagger f_\sigma^\dagger$$

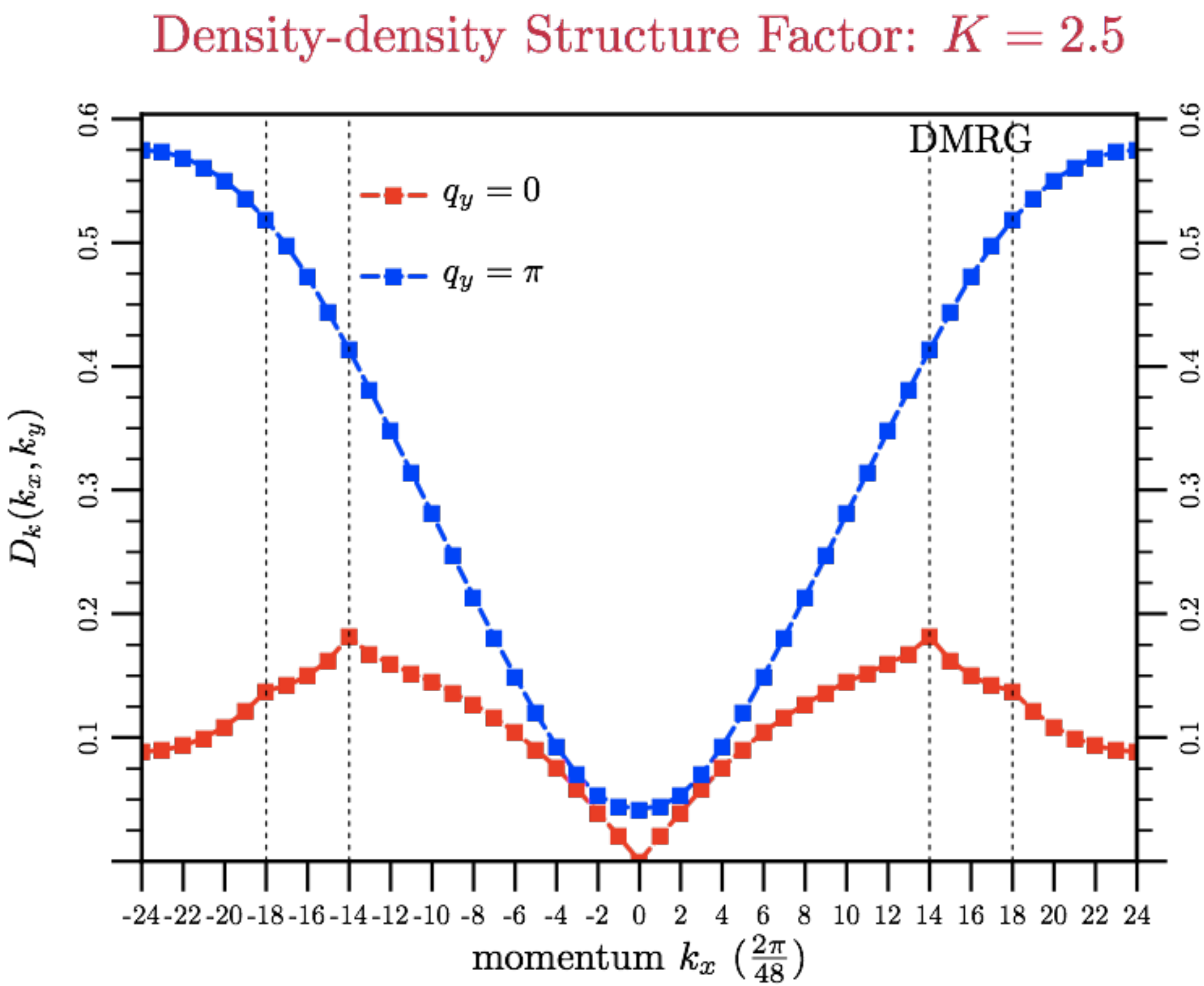


In D_k, enhanced singularities are predicted by the gauge theory at various "$2k_F$" wavevectors.

Evolution of Peak Locations



Evolution of Peak Locations



Variational Monte Carlo (VMC)

D-wave Metal: Product of Slater determinants

$$\Psi_{d_{xy}}^{Metal} = \det_x[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}] \cdot \det_y[e^{i\mathbf{K_i}\cdot\mathbf{R_j}}] \times \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\uparrow}}}] \cdot \det[e^{i\mathbf{k_i}\cdot\mathbf{r_{j\downarrow}}}]$$

Variational Parameters:

Distribution of d_x partons between
bonding/anti-bonding bands (f-spinons and d_y partons
only in bonding band)

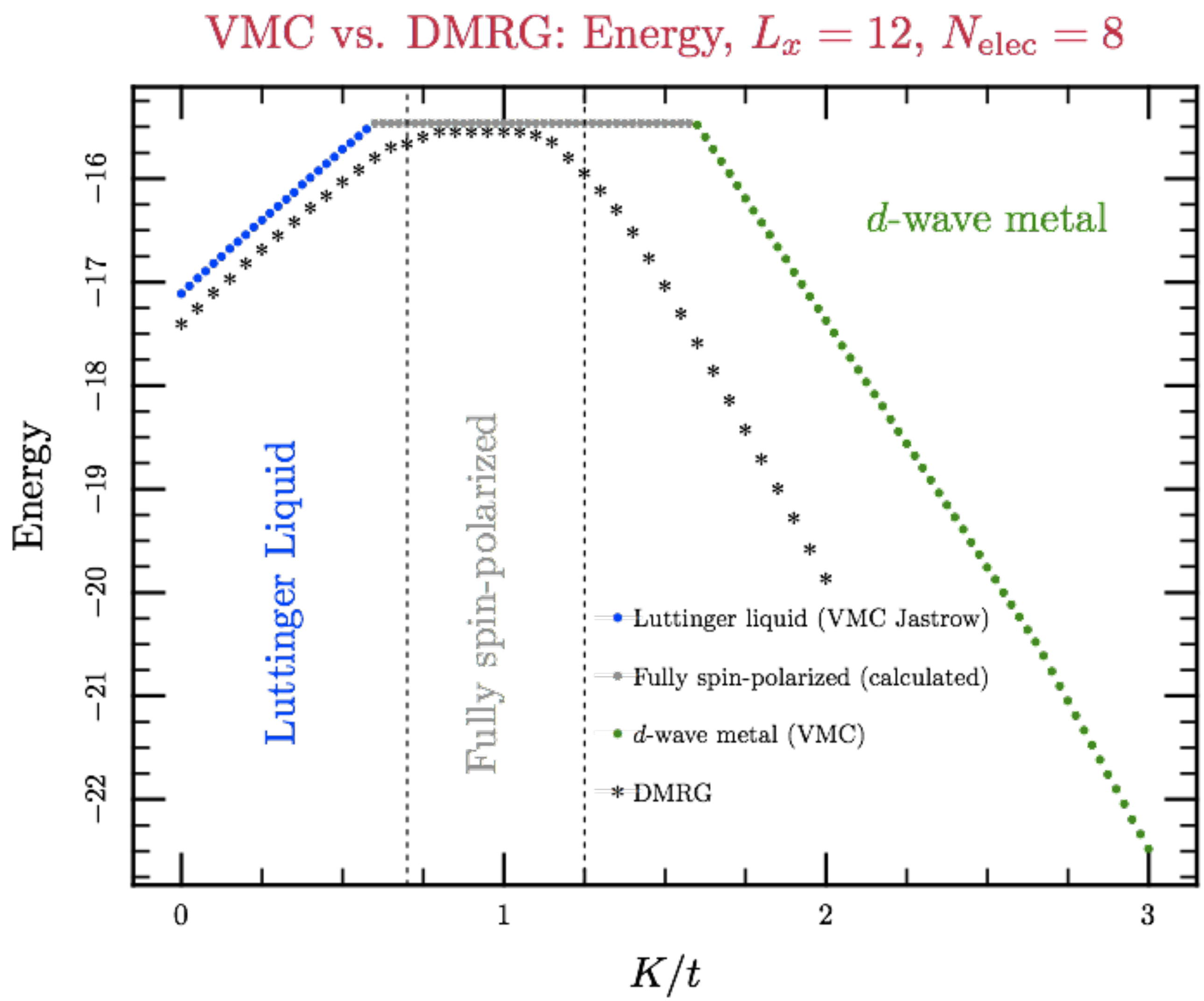
2 parameters to tune the Luttinger exponents

$$det_x = |det_x| sgn(det_x) \rightarrow |det_x|^{\gamma_x} sgn(det_x)$$

$$det_y = |det_y| sgn(det_y) \rightarrow |det_y|^{\gamma_y} sgn(det_y)$$

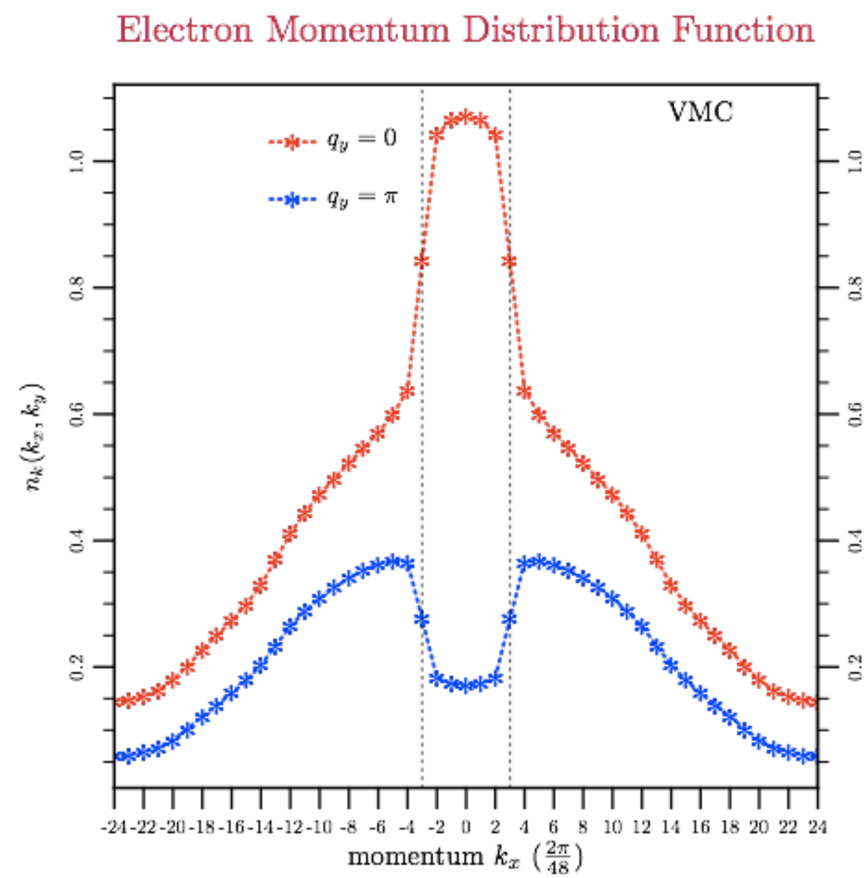
(Luttinger liquid phase: Jastrow factor multiplying filled Fermi sea)

Ground State energy: DMRG vs VMC



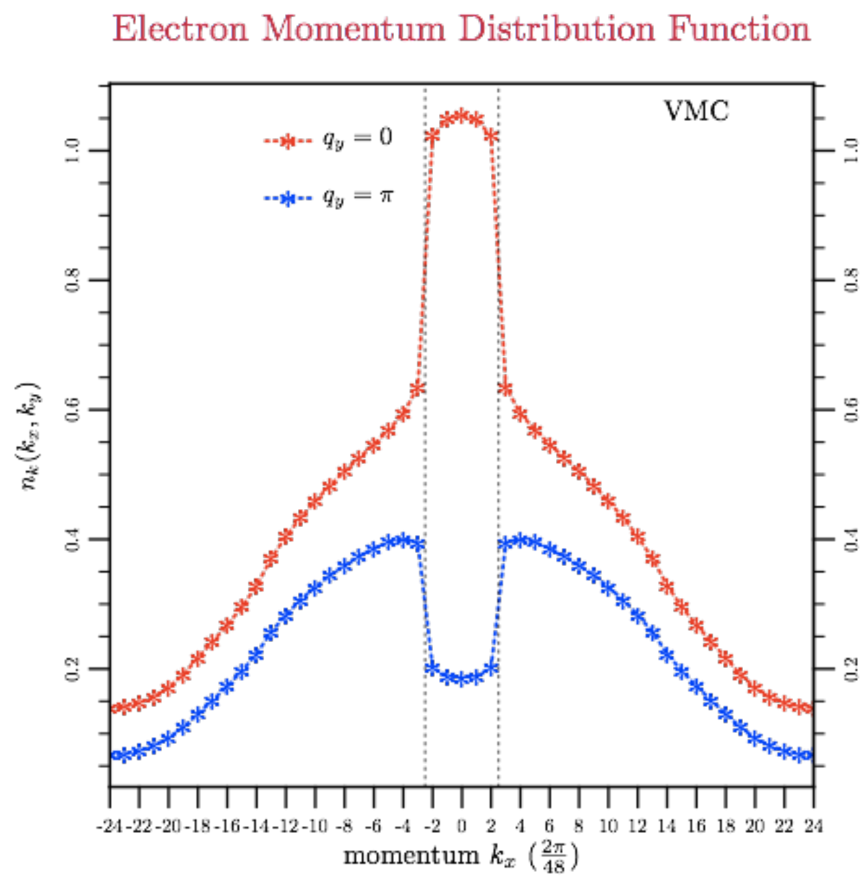
Evolution of VMC States

$$d_x : N_0 = 22, N_\pi = 10$$



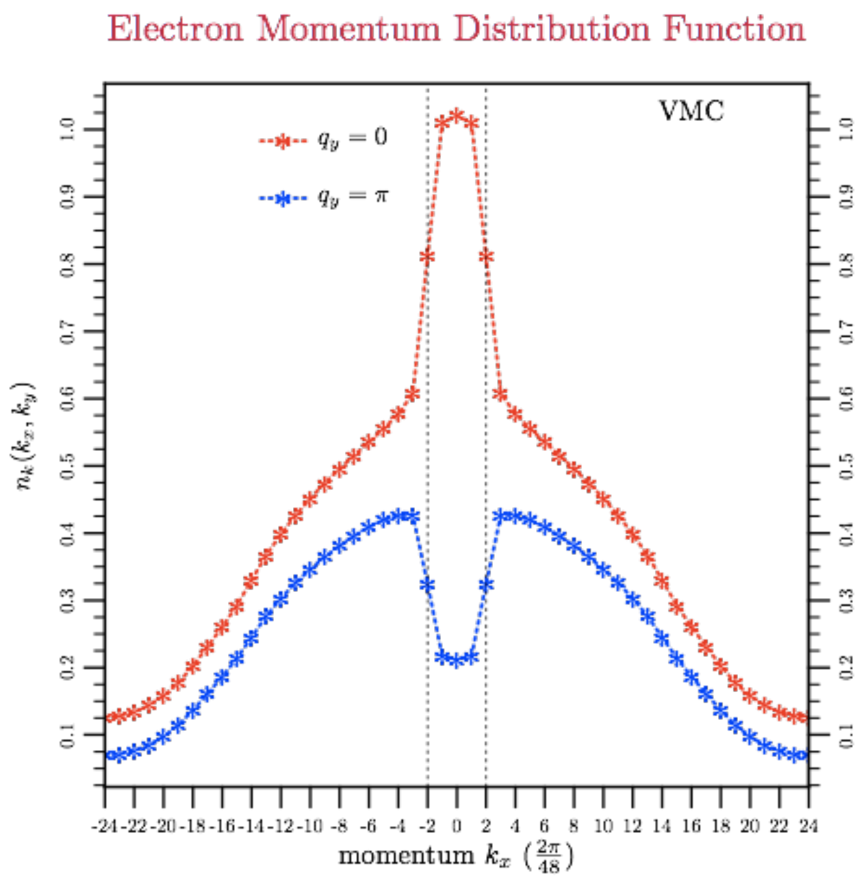
Evolution of VMC States

$$d_x : N_0 = 21, N_\pi = 11$$



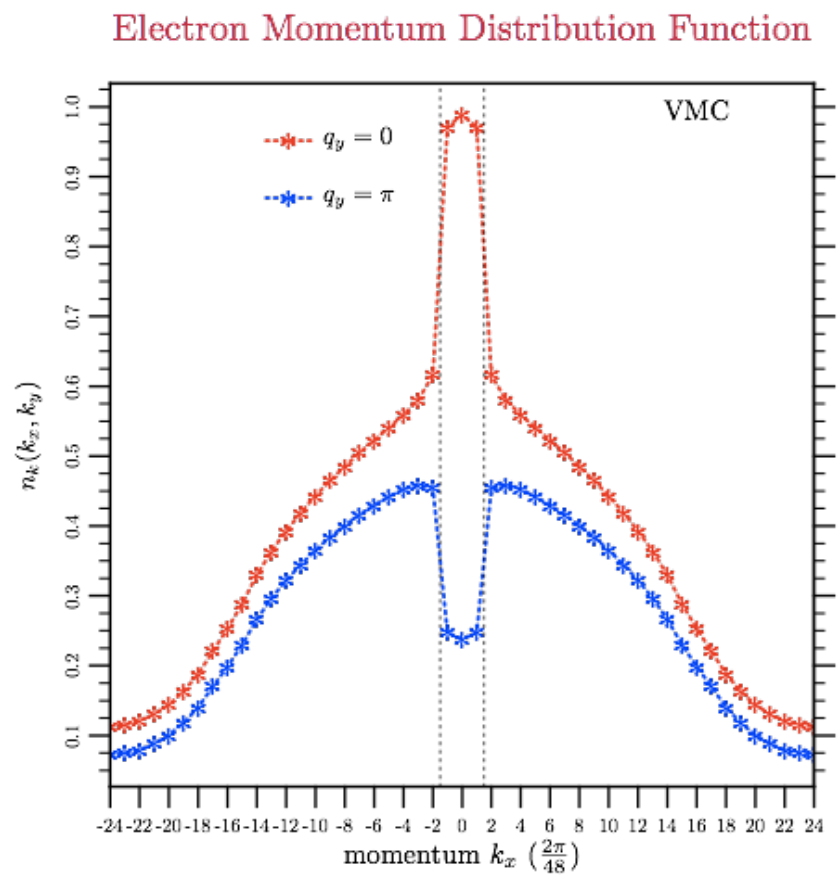
Evolution of VMC States

$$d_x : N_0 = 20, N_\pi = 12$$



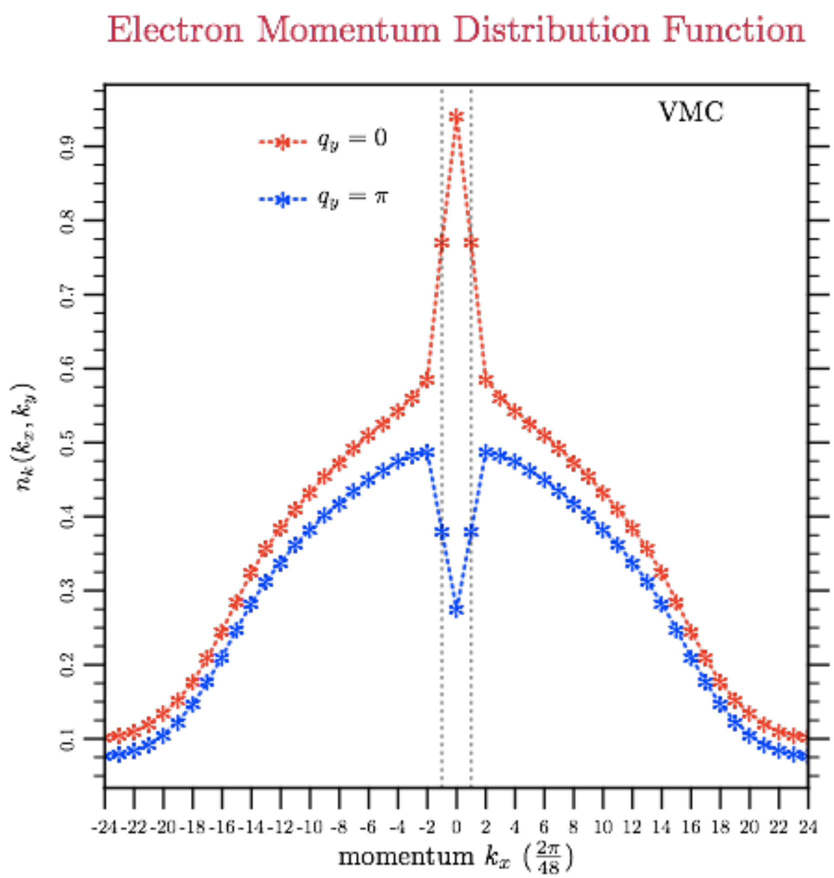
Evolution of VMC States

$$d_x : N_0 = 19, N_\pi = 13$$



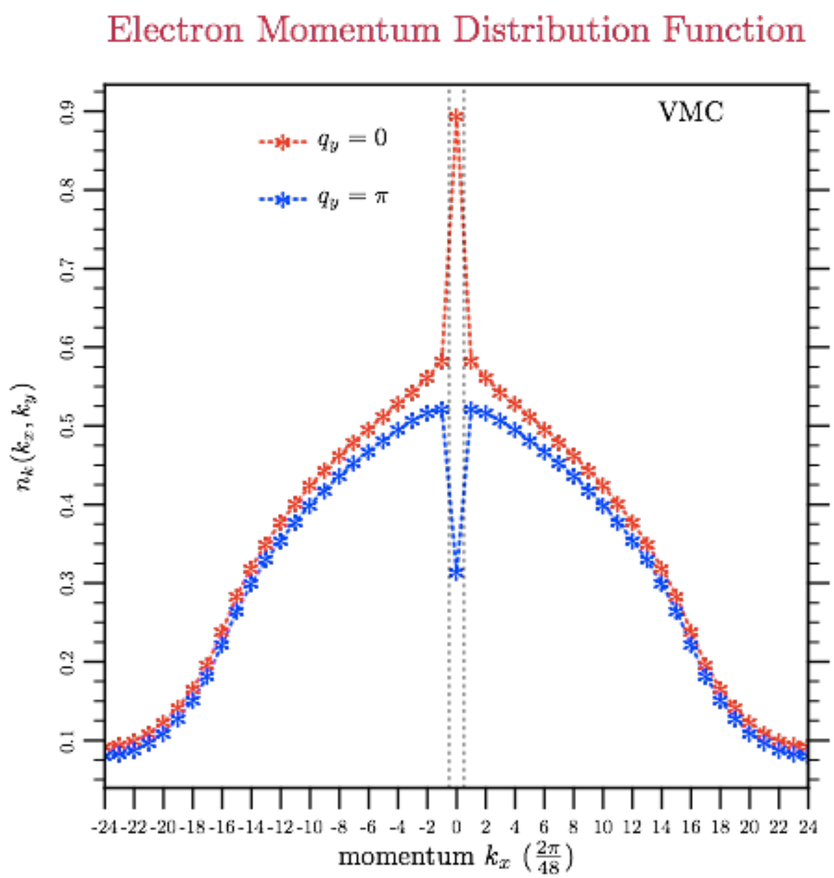
Evolution of VMC States

$$d_x : N_0 = 18, N_\pi = 14$$



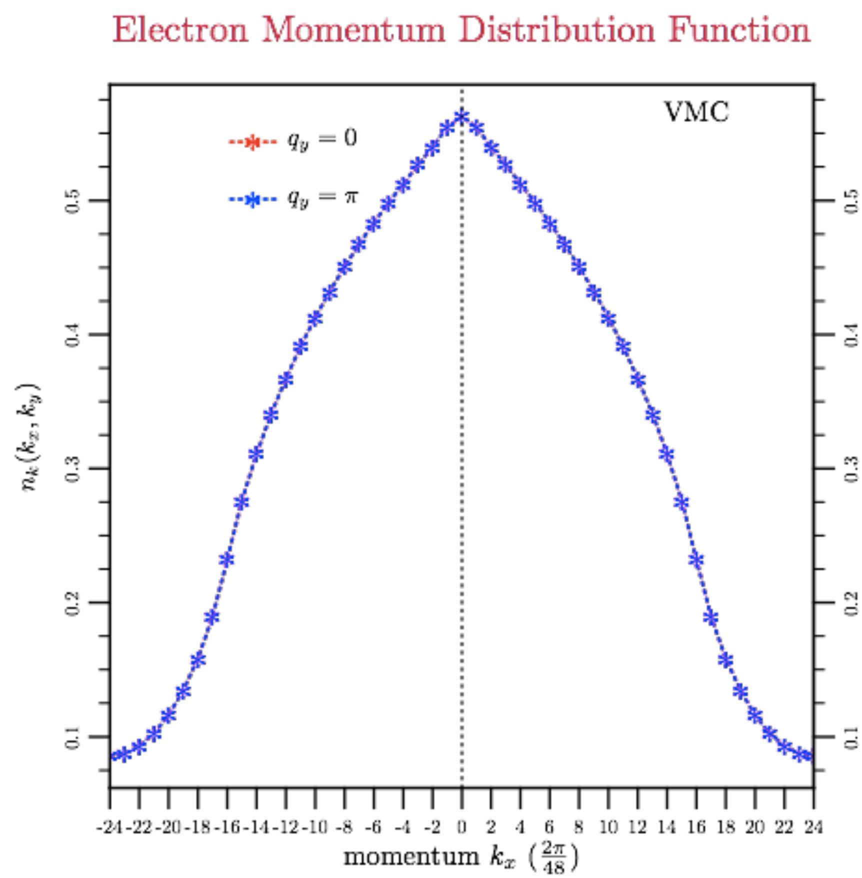
Evolution of VMC States

$$d_x : N_0 = 17, N_\pi = 15$$

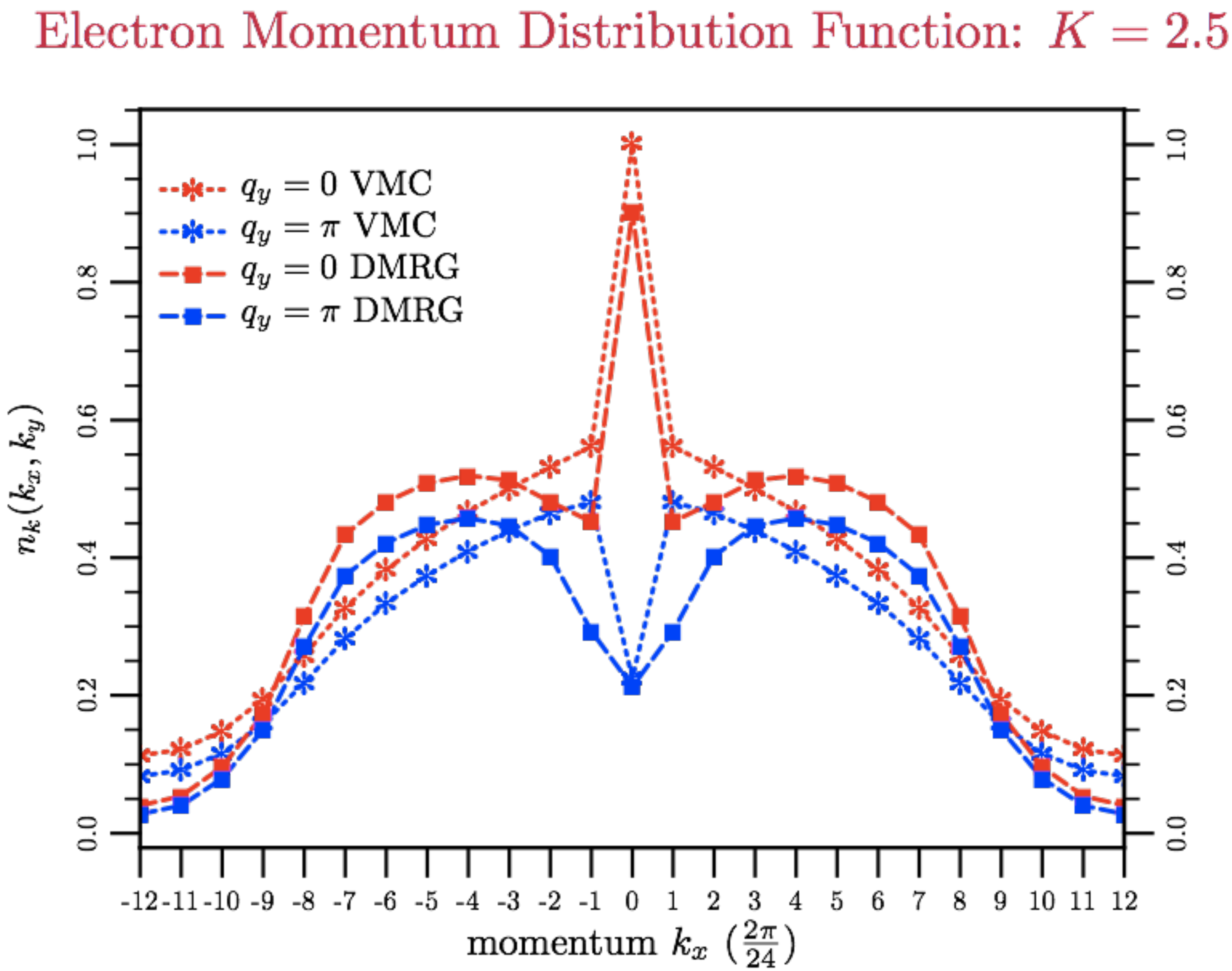


Evolution of VMC States

$$d_x : N_0 = 16, N_\pi = 16$$

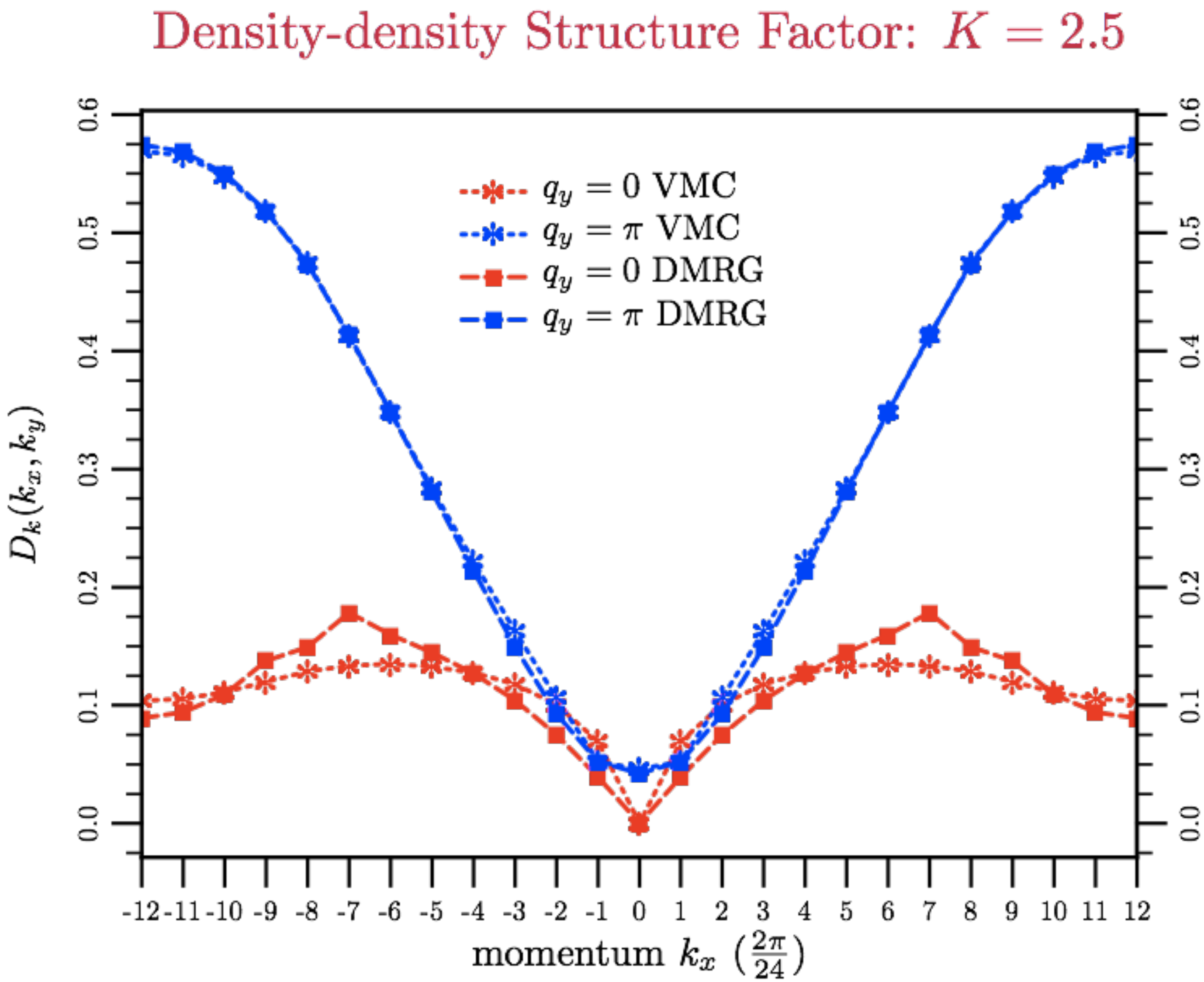


VMC vs. DMRG



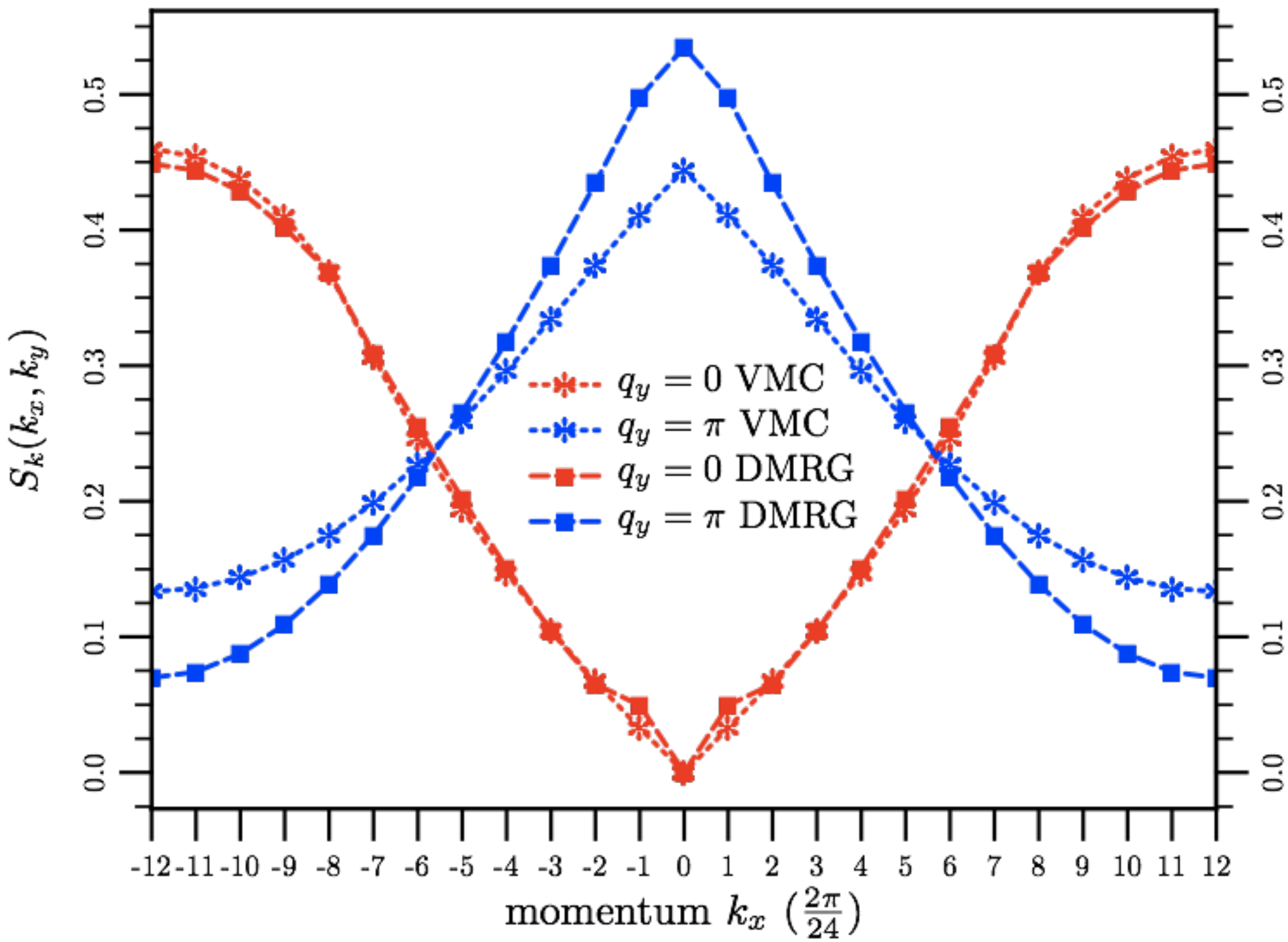
Electron Momentum Distribution Function: $K = 2.5$
$q_y = 0$ VMC
$q_y = \pi$ VMC
$q_y = 0$ DMRG
$q_y = \pi$ DMRG
$n_k(k_x, k_y)$
momentum k_x $(\frac{2\pi}{24})$
0.0
0.2
0.4
0.6
0.8
1.0
-12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12

VMC vs. DMRG


Density-density Structure Factor: $K = 2.5$
$q_y = 0$ VMC
$q_y = \pi$ VMC
$q_y = 0$ DMRG
$q_y = \pi$ DMRG
$D_k(k_x, k_y)$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
-12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12
momentum k_x $(\frac{2\pi}{24})$

VMC vs. DMRG

Spin-spin Structure Factor: $K = 2.5$



Conclusions

- NFL wf for a "D-wave Metal" and a candidate "ring" Hamiltonian
- Ring Hamiltonian on 2-leg ladder has a phase which is a "non-Luttinger liquid"
- Comparison between DMRG and VMC on a parton wf established that this non-LL is a ladder descendant of the 2d D-wave Metal.
- The D-wave Metal is very strongly entangled

Open Issues

- D-wave Metal on 2-leg ladder; Dynamics, other filling factors
- Multi-leg ladders towards 2d
- VMC energetics on 2d ring Hamiltonian (FL, D-wave BCS, D-wave Metal,…)
- Other wfs/Hamiltonians for 2d NFL phases??

Correlators and Structure Factors

Electron Momentum Distribution Function:

$$n_{k\sigma}(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \langle c_{i\sigma}^{\dagger} c_{j\sigma} \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \qquad n_k = n_{k\uparrow} + n_{k\downarrow}$$

Density-density Structure Factor:

$$D_k(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \left[\langle \rho(\mathbf{r}_i)\, \rho(\mathbf{r}_j) \rangle - \langle \rho(\mathbf{r}_i) \rangle \langle \rho(\mathbf{r}_j) \rangle \right] e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$

$$\rho(\mathbf{r}_i) = c_{i\uparrow}^{\dagger} c_{i\uparrow} + c_{i\downarrow}^{\dagger} c_{i\downarrow}$$

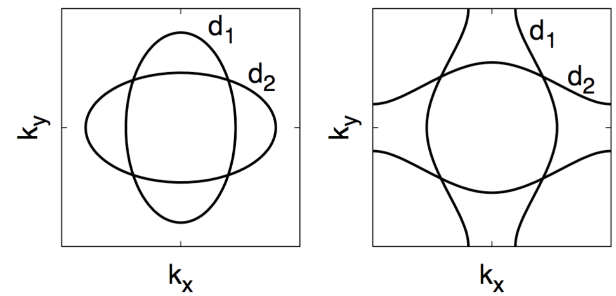
Spin-spin Structure Factor:

$$S_k(\mathbf{k}) = \frac{1}{L_x L_y} \sum_{i,j} \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle e^{i\mathbf{k}\cdot(\mathbf{r}_i - \mathbf{r}_j)}$$

Bose Surfaces in D-wave Bose-Metal

Mean Field Green's functions factorize:

$$G_b^{MF}(\mathbf{r},\tau) = G_{d_1}^{MF}(\mathbf{r},\tau) G_{d_2}^{MF}(\mathbf{r},\tau)/\bar{\rho}$$



$$G_{d_\alpha}^{MF}(\mathbf{r}) \approx \frac{1}{2^{1/2}\pi^{3/2}} \frac{\cos(\mathbf{k}_{F_\alpha}\cdot\mathbf{r} - 3\pi/4)}{c_\alpha^{1/2}|\mathbf{r}|^{3/2}}$$

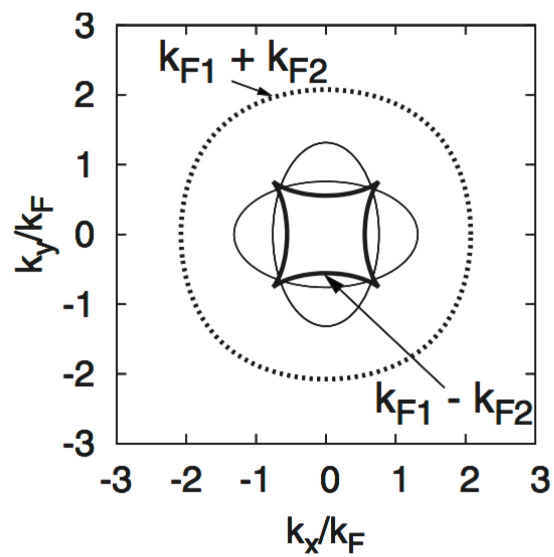
$$(\partial\epsilon_\alpha/\partial\mathbf{k})_{\mathbf{k}_{F\alpha}(\hat{\mathbf{r}})} = (const)\hat{\mathbf{r}}$$

Momentum distribution function:

$$n_b(\mathbf{k}) = \int G_b(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} d\mathbf{r}$$

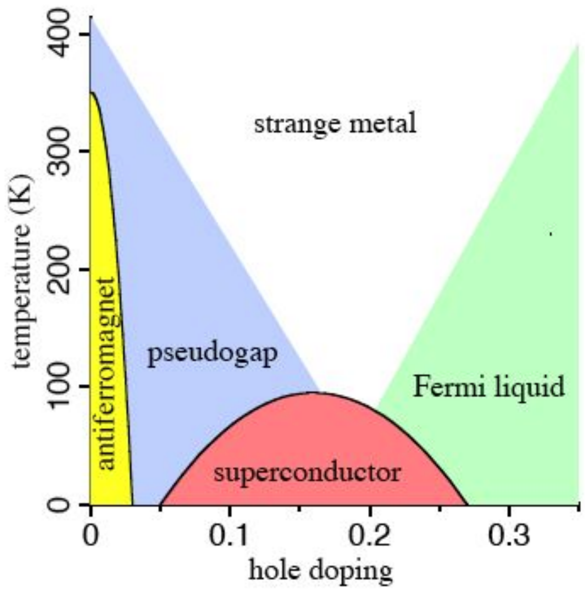
Two singular lines in momentum space, Bose surfaces:

$$\mathbf{k}_{F_1}(\hat{\mathbf{r}}) \pm \mathbf{k}_{F_2}(\hat{\mathbf{r}})$$



Motivation for Non-Fermi-Liquid Metal: "Abnormal" state of High T_c Superconductors

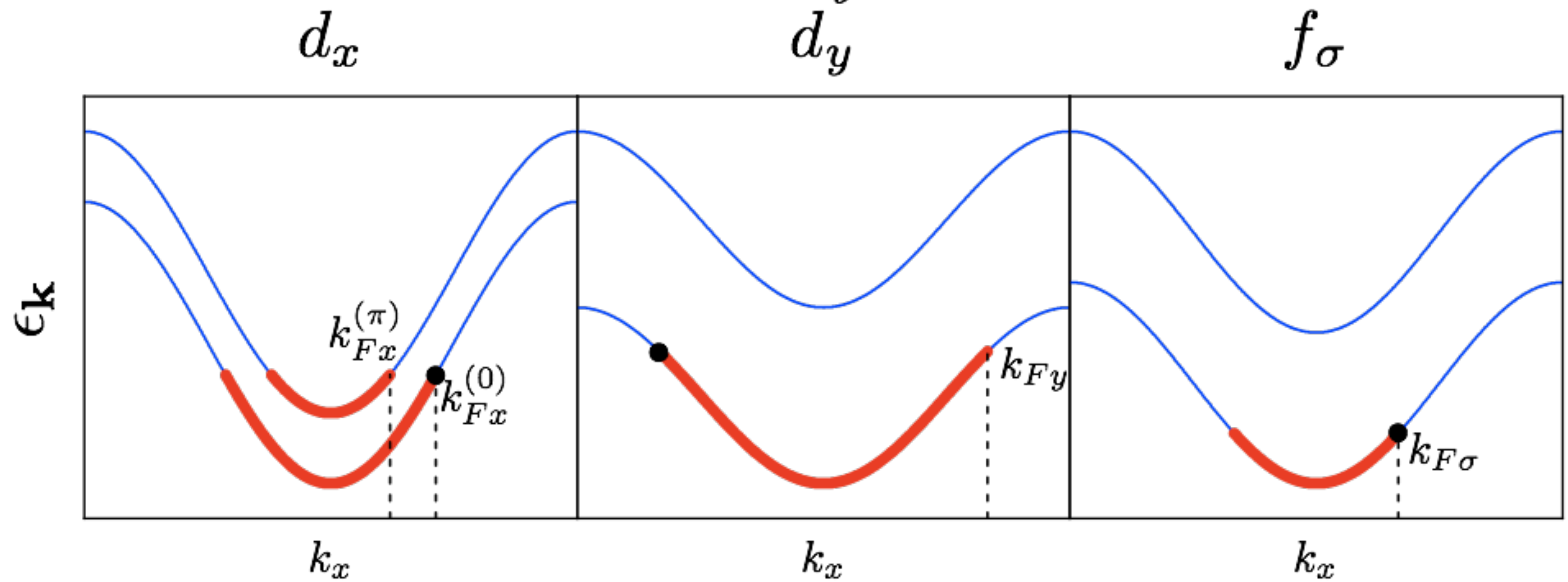
Phase Diagram



Strange metal: "Fermi surface" but quasiparticles are not "sharp"
Spectral function measured with ARPES suggests Z=0

The *d*-wave Metal on 2 Legs

$$c_\sigma^\dagger = d_x^\dagger d_y^\dagger f_\sigma^\dagger$$



In n_k, an enhanced singularity is predicted by the gauge theory at $k_{Fx}^{(k_y)} - k_{Fy} + k_{F\sigma}$