Steve Kivelson **Apr 15, 2011** **Freedman Symposium**

Happy Nth Birthday Michael Freedman

“Exa[illegible]esults concerning the pha[illegible]iagram of the Hub[illegible] Model

S.Raghu, D.J. Scala[illegible] Liu, E. Berg

H. Yao, W-F. Tsai, [illegible]hli

G. Karakonstantakis, S. Whit[illegible] Cho

UCSB 2011

Exact:

Not approximate.

Involving precise statements that are either exactly right or simply wrong.

I will present evidence that they are right.

The Hubbard Model

- The paradigmatic model of strongly correlated electron systems

The Hubbard model is to correlated electron systems as the Ising model is to statistical mechanics

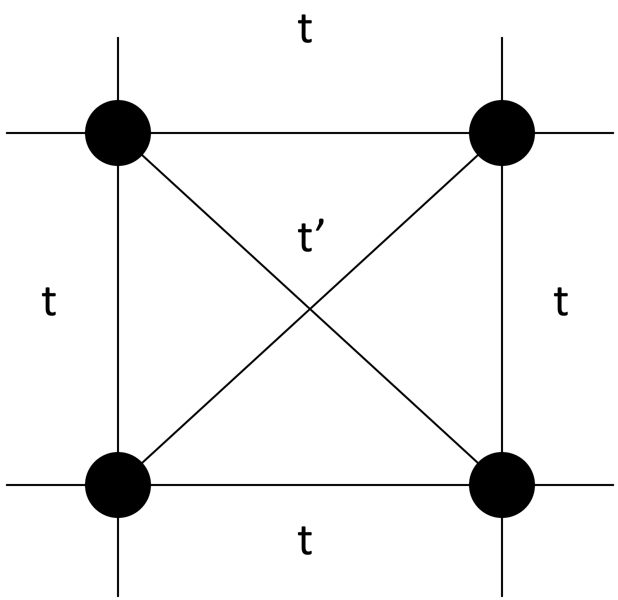
except that the phase diagram of the Ising model is well understood, while almost everything about the Hubbard model is subject to controversy.

The Hubbard Model

- The paradigmatic model of strongly correlated electron systems
- The most studied model of high temperature superconductivity
- The principle model of metallic ferromagnetism
- The fundamental model of quantum (insulating) antiferromagnetism

The Hubbard model

$$H = -\sum_{i,j,\sigma} t_{ij}\left[c^{\dagger}_{i,\sigma}c_{j,\sigma} + \mathrm{H.C.}\right] + U\sum_{j} c^{\dagger}_{j,\uparrow}c^{\dagger}_{j,\downarrow}c_{j,\downarrow}c_{j,\uparrow}$$



U > 0 Repulsive

Parameters:

U/t = "strength" of coupling

n = (1-x) = number of electrons per site

Dimensionality and lattice structure

t'/t = "band-structure"

In Someone's Theory of the Hubbard Model:

Ferromagnetic metal

Antiferromagnetic insulator

Antiferromagnetic metal

Fermi liquid

Spin 3/2 Fermi liquid

Charge density wave

Electronic Phase Separation

Unconventional Superconductivity:

d-wave, p-wave, f-wave, and s± superconductivity

Intertwined spin and charge density wave order:

"Stripes" and bubbles and checkerboards

Electron nematic and other electronic liquid crystalline phases

Spin liquid ground-states of various sorts:

Fully gapped Z_2, Nodal, pseudo-Fermi surface

Spatially oscillatory superconductivity:

Striped superconductor or PDW

Non-Fermi liquid or "hidden Fermi liquid" or RVB

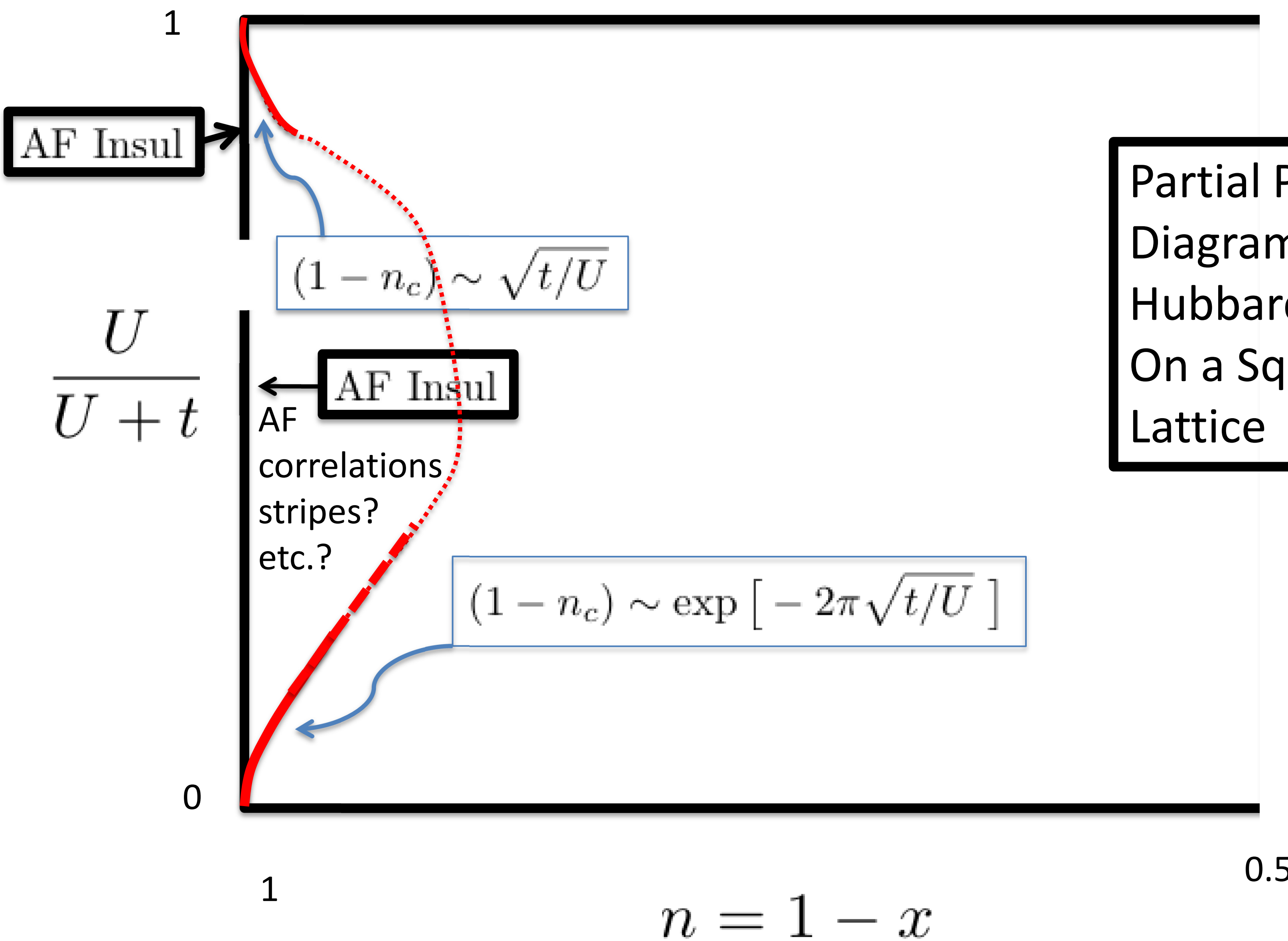
There is a large, and vocal community that holds
that superconductivity is impossible without attractive
(electron-phonon) interactions.

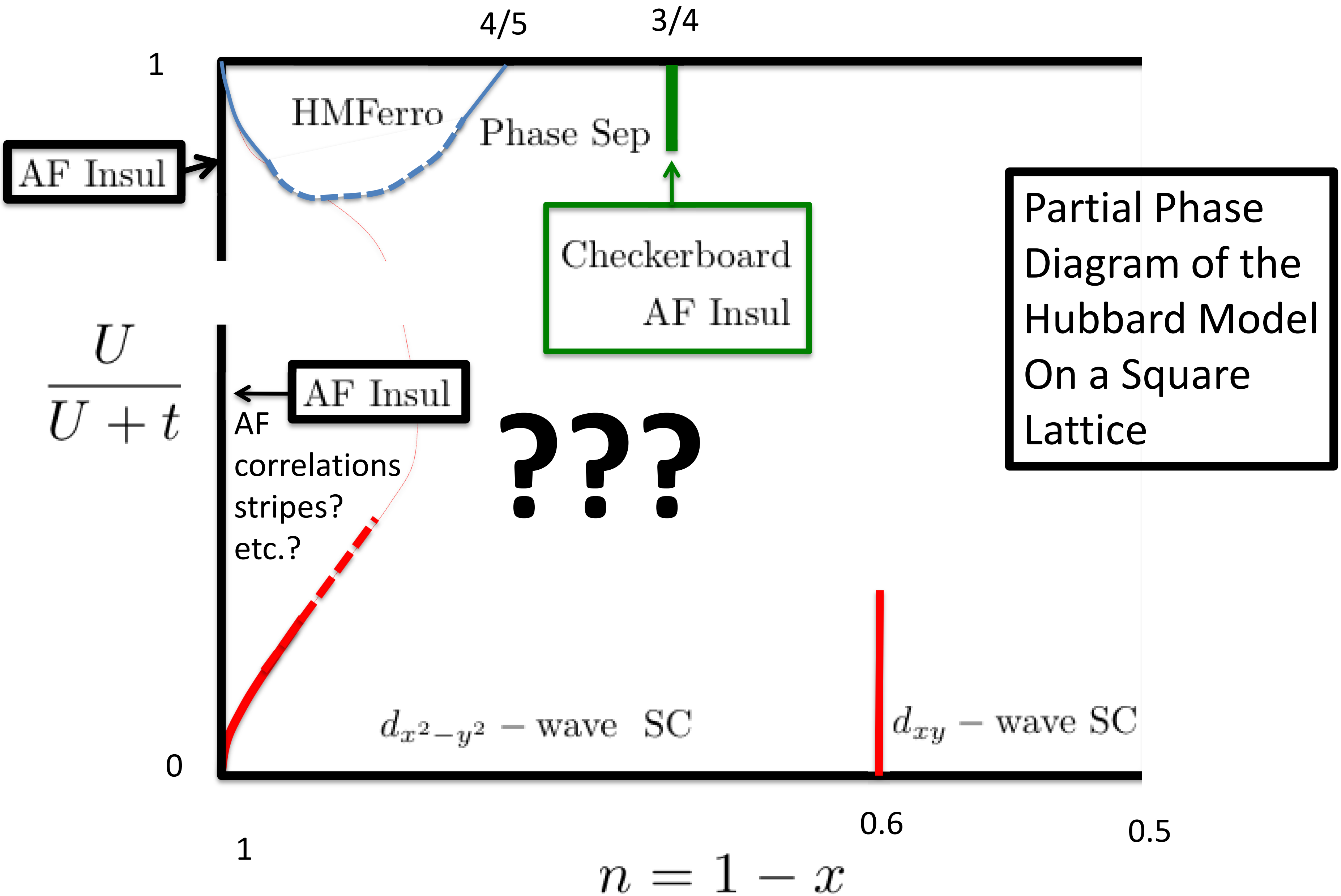
"No wonder American taxpayers are upset! They could well wonder what they are paying for. One way of looking at this is that we are relatively well off: Steve could have been selling subprime mortgages instead."

- *a very distinguished condensed matter theorist presenting a reasoned argument against superconductivity in the positive U Hubbard model*

However, they are wrong.

Partial Phase Diagram of the Hubbard Model On a Square Lattice




4/5
3/4
1
HMFerro
Phase Sep
AF Insul
Checkerboard AF Insul
Partial Phase Diagram of the Hubbard Model On a Square Lattice
$\frac{U}{U+t}$
AF Insul
AF correlations stripes? etc.?
???
$d_{x^2-y^2}$ − wave SC
d_{xy} − wave SC
0
1
0.6
0.5
$n = 1 - x$

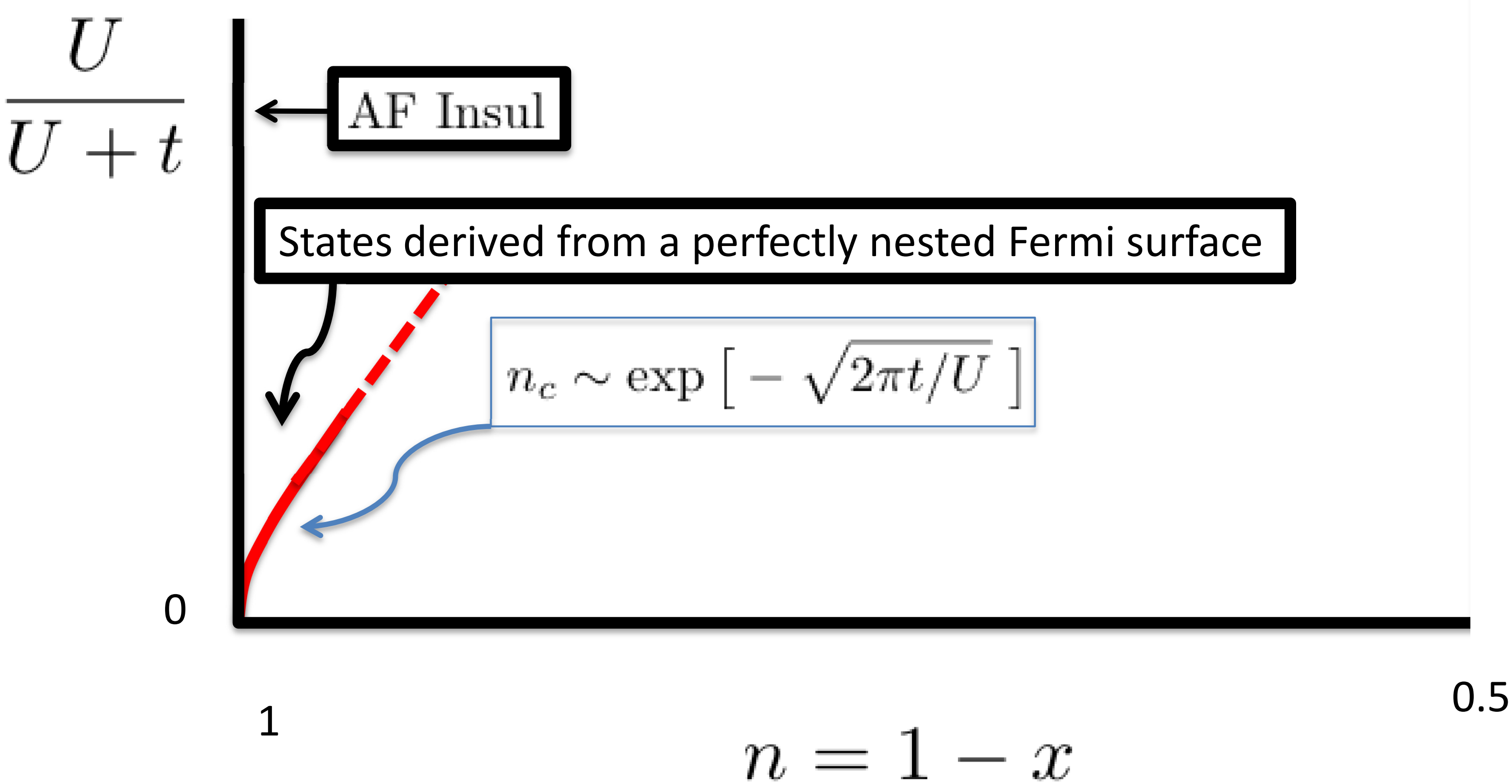
Small U/t limit of the Hubbard Model:

S. Raghu, SAK, and D. J. Scalapino - *PRB **81**, 224505 (2010)*

Draws on work of Kohn and Luttinger, Shankar and Polchinski, Chubukov and Kagan, and many others

1) The only infinitesimal instabilities of a Fermi gas are the superconducting (Cooper) instability, except under fine-tuned circumstances (e.g. conditions of perfect nesting).

Small U/t limit of the Hubbard Model:



Small U/t limit of the Hubbard Model:

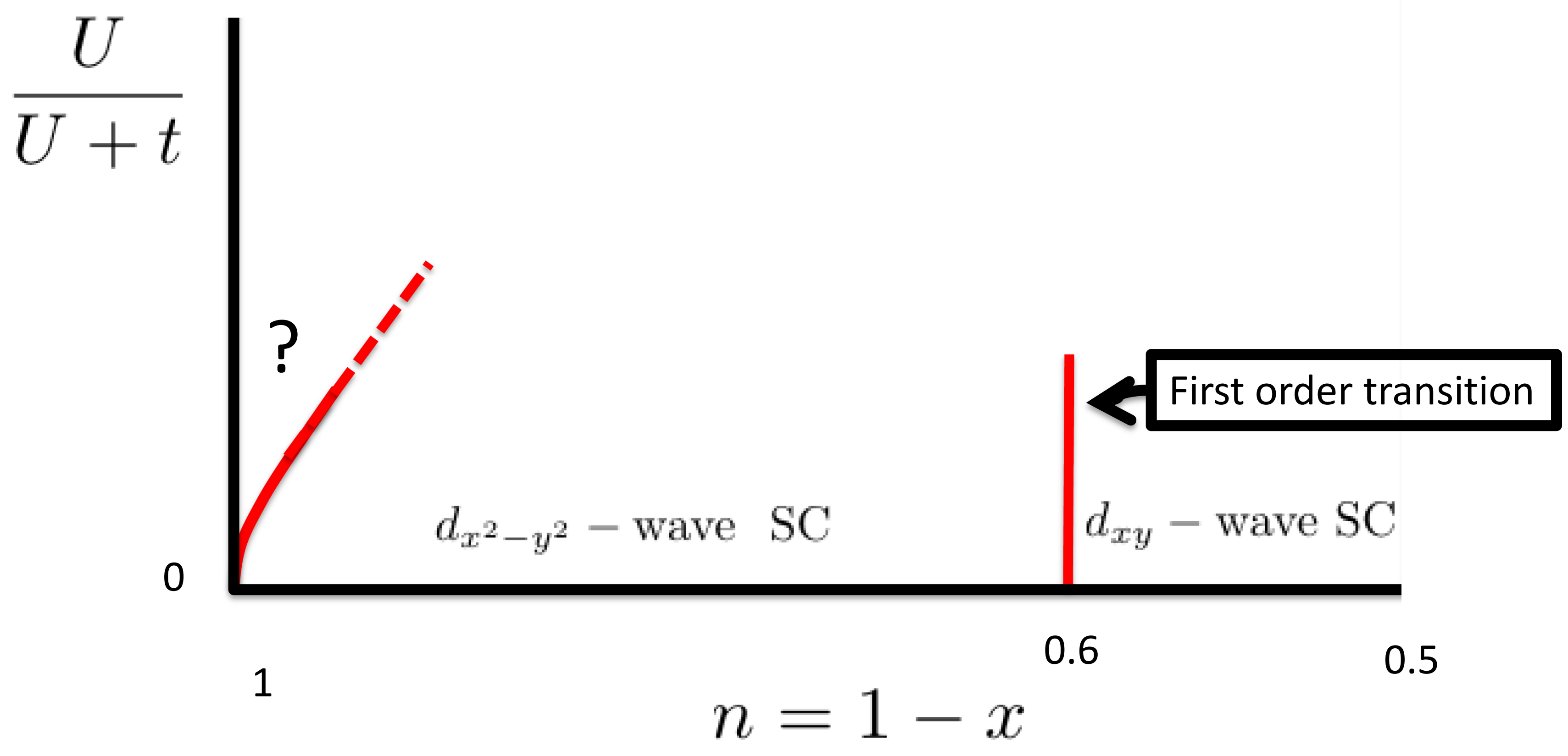
1) The only infinitesimal instabilities of a Fermi gas are the superconducting (Cooper) instability, except under fine-tuned circumstances (e.g. conditions of perfect nesting).

2) Effective attraction for unconventional pairing is generated by transient flows as irrelevant operators scale to 0.

"Unconventional superconductivity" means that the gap averaged over the Fermi surface vanishes (or approximately vanishes).

$$\left|\langle c^{\dagger}_{\vec{R},\uparrow} c^{\dagger}_{\vec{R},\downarrow}\rangle\right| \ll \mathrm{Max}\left\{ \ \left|\langle c^{\dagger}_{\vec{R},\sigma} c^{\dagger}_{\vec{R}',\sigma'}\rangle\right| \ \right\}$$

Small U/t limit of the Hubbard Model:



Small U/t limit of the Hubbard Model:

Generic "unconventional" superconducting ground-state with <u>low</u> T_c.

Asymptotic expansion:

$$T_c \sim 4t \ \exp\{-1/\lambda\}$$

$$\lambda = (U/t)^2\,[\mathbf{A} + \mathbf{B}(U/t) + \mathbf{C}(U/t)^2 + \ldots]$$

Can calculate **A** and **B** exactly from band-structure properties of non-interacting system

Can calculate symmetry and structure of the gap and the pair wave-function.

Small U/t limit of the Hubbard Model:

Generic "unconventional" superconducting ground-state with <u>low</u> T_c.

Asymptotic expansion:

$$T_c \sim 4t \;\exp\left\{ -1/\lambda \right\}$$

$$\lambda = (U/t)^2 \,[A + B(U/t) + C(U/t)^2 + \ldots]$$

Note that this is not a theory of a high superconducting T_c.

The normal state is, with high precision, a Fermi liquid.

There is no significant regime of superconducting fluctuations above T_c.

There are no competing orders.

There are no exotic phases or phase transitions.

However, if extrapolated to intermediate coupling …

Infinite U Hubbard model

New paper by Li, Yao, Berg, and Kivelson – arxiv:1103.3305

$$H = -\sum_{<i,j>,\sigma} \mathcal{P}[c^\dagger_{i,\sigma} c_{j,\sigma} + \mathrm{H.C.}]\mathcal{P}$$

$\mathcal{P}$ is the projection operator onto the space of no-double occupancy

$$n_i = \sum_\sigma c^\dagger_{i,\sigma} c_{i,\sigma} = 0 \quad \text{or} \quad 1 \qquad \text{for all } i$$

All there is the lattice structure and the mean electron density.

Infinite U Hubbard model

New paper by Li, Yao, Berg, and Kivelson – arxiv:1103.3305

We have studied this model by DMRG on 2, 3, 4, 5, and 6 leg ladders.

2-leg ladders up to length N=50

4-leg ladders up to length N=20

6-leg ladders up to length N=8

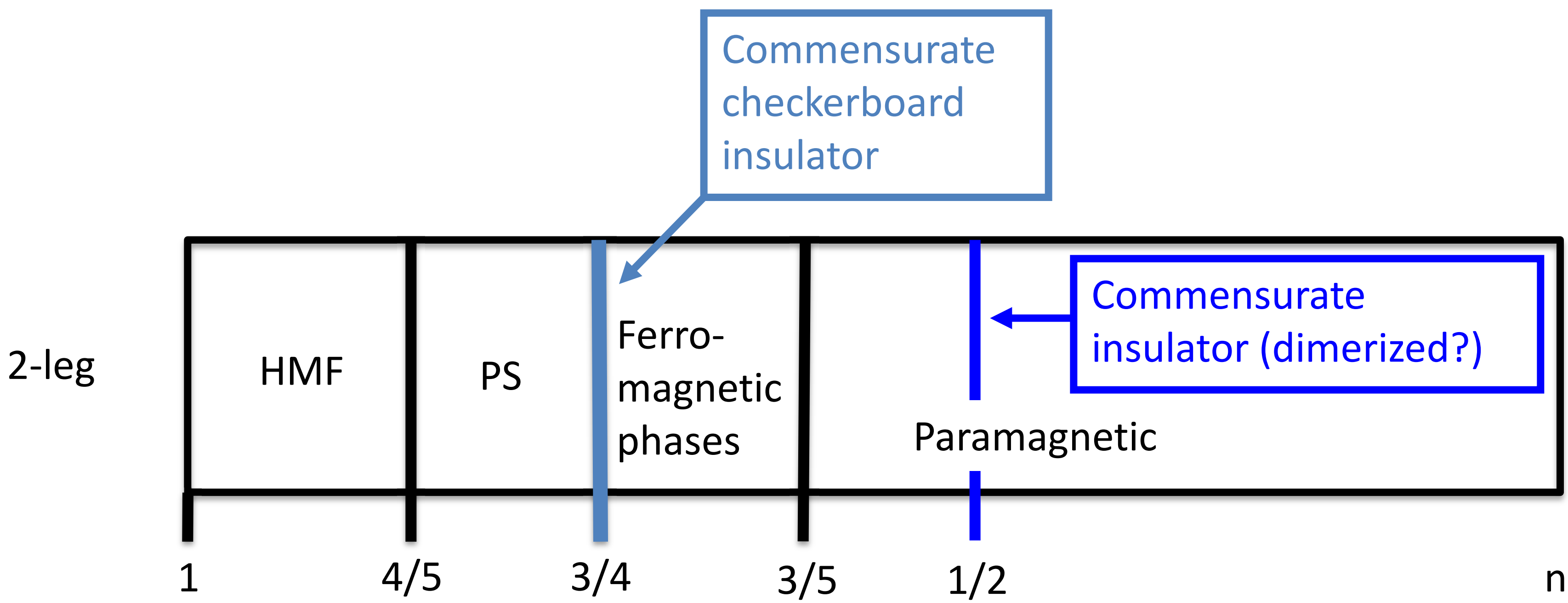
2-leg Infinite U Hubbard ladder

HMF= “Half metallic ferromagnet” = fully spin polarized Fermi liquid

PS = phase separation = regime of 2-phase coexistence

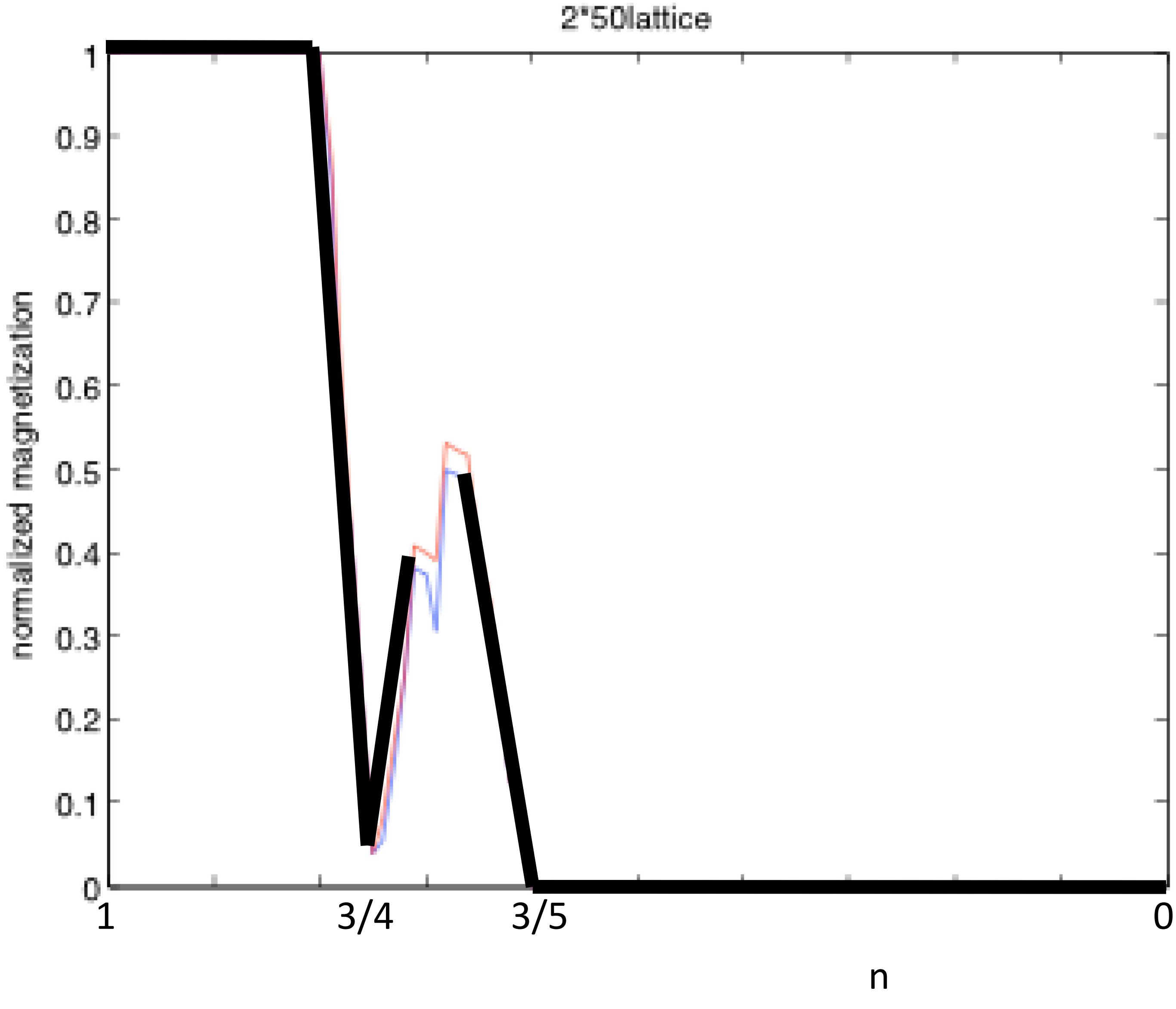
Ferromagnetic phases = partially spin polarized electron fluids

Paramagnetic = spin unpolarized = magnetization-density vanishes as N □□



$m = M/M_{max}$

2 leg ladder at infinite U



HMF & Paramagnetic Phases

lattice size	n_F
2×20	0.800
2×30	0.800
2×40	0.800
2×50	0.800
4×10	0.800
4×15	0.817
4×20	0.800
6×10	0.817
6×15	0.800

For all n such that

$1 > n \geq n_F$

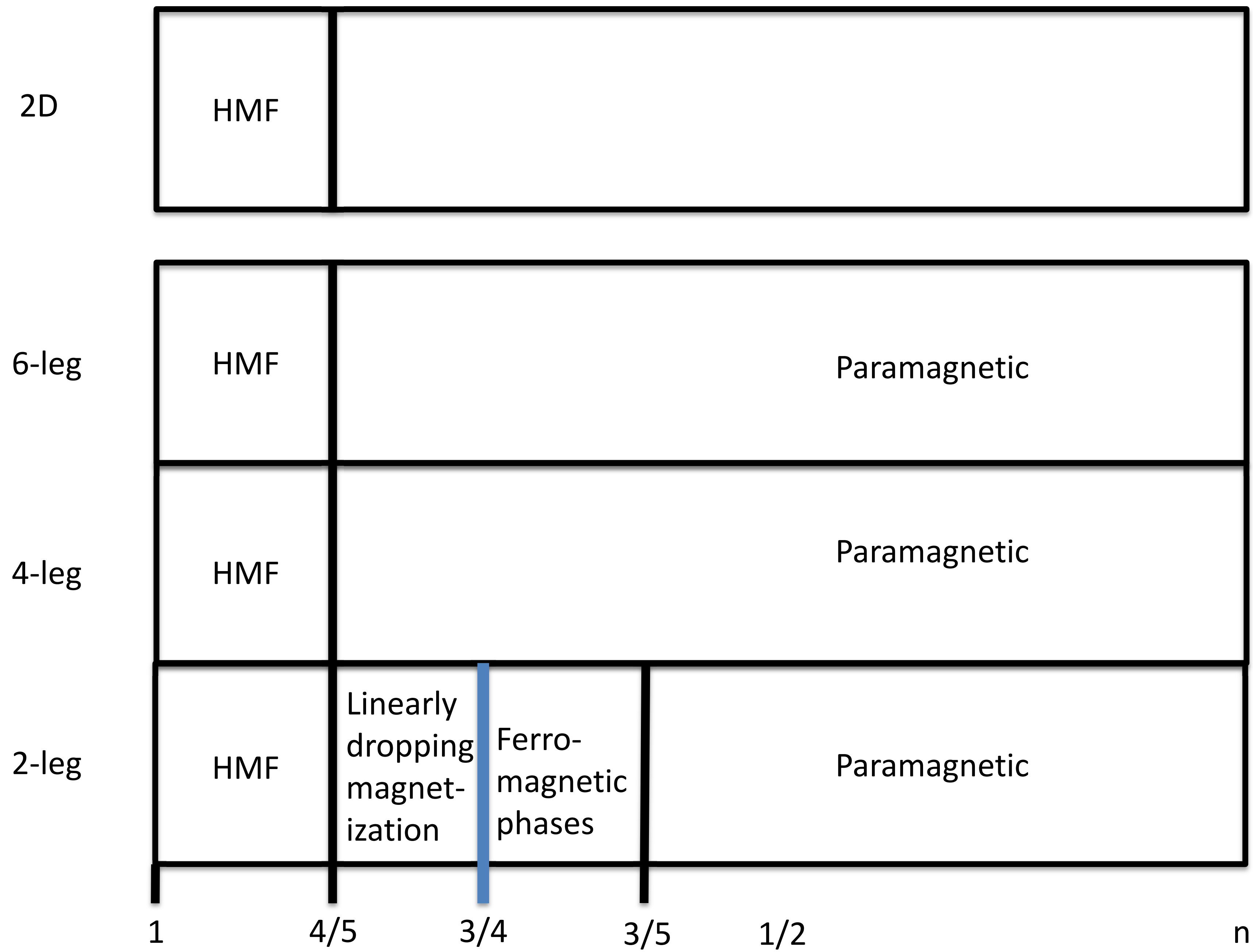
the groundstate is fully polarized.

For all n such that

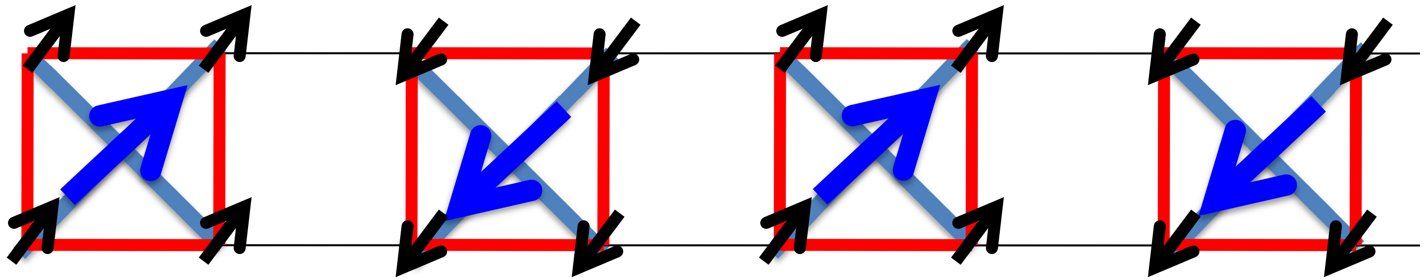
$n_F > n$

the groundstate is incompletely or unpolarized.

Note that in the fully polarized sector of Hilbert space, the electrons are non-interacting.


2D
HMF
6-leg
HMF
Paramagnetic
4-leg
HMF
Paramagnetic
2-leg
HMF
Linearly dropping magnet-ization
Ferro-magnetic phases
Paramagnetic
1
4/5
3/4
3/5
1/2
n

Insulating phase at n=3/4 for two leg ladder with infinite U



$$K_{ij} = \sum_{\sigma} \langle [c^{\dagger}_{i,\sigma} c_{j,\sigma} + \mathrm{H.C.}] \rangle = \text{bond charge density}$$

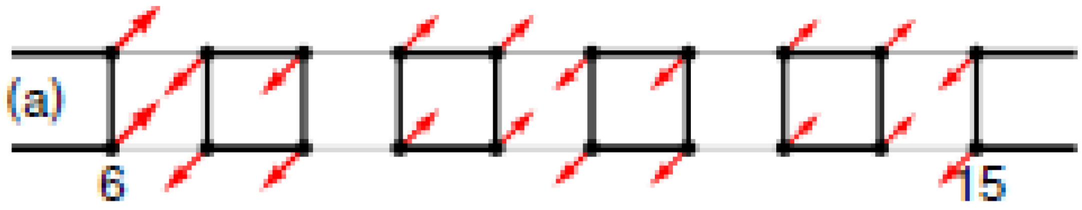
Breaks discrete symmetries

$$\vec{S}_j = \sum_{\sigma,\sigma'} \langle c^{\dagger}_{j,\sigma} \vec{\tau}_{\sigma,\sigma'} c_{j,\sigma'} \rangle = \text{spin density}$$

Continuous spin rotational symmetry so at most power law order

Equivalent to an effective spin 3/2 Heisenberg chain

Checkerboard AF Insulator at n=3/4



A Zeeman field of strength 1 has been applied to the lower left site, (0,0)

Bond density ranges from K_{ij}=0.3 to K_{ij} = 0.45

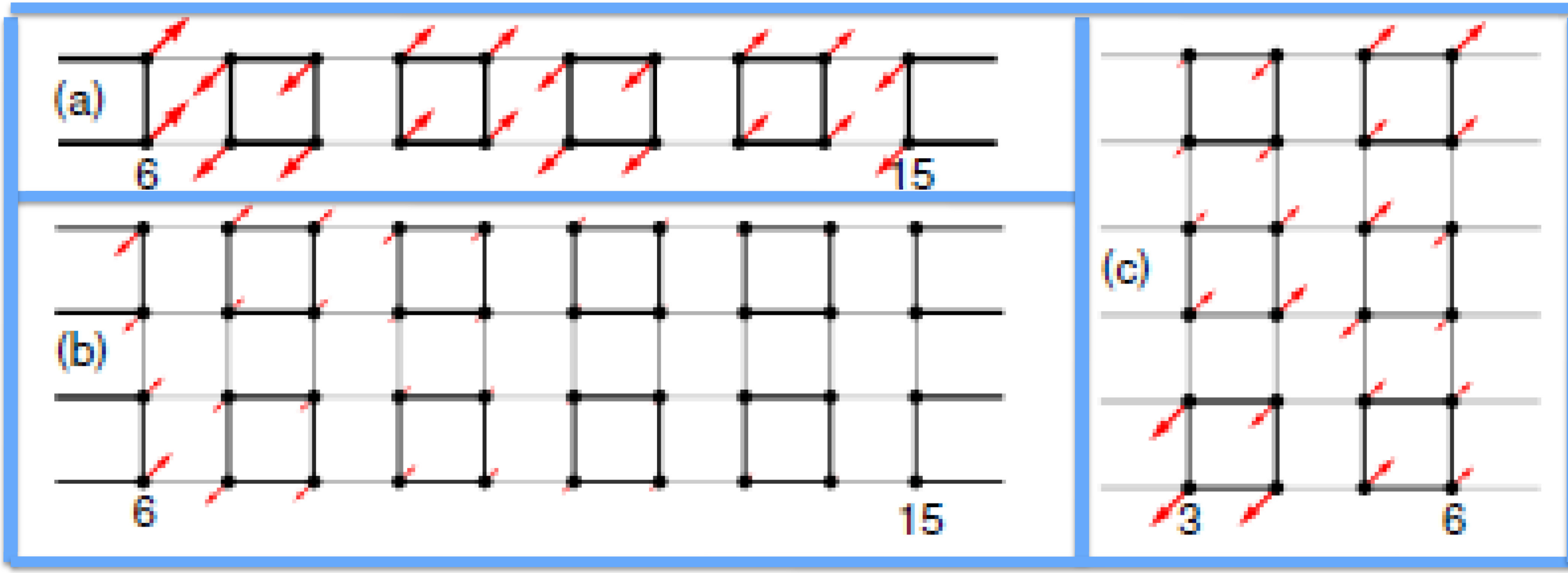
The magnitude of the spin density decays from 0.21 at the left to 0.19 at the right

Checkerboard AF Insulator with n=3/4

$$\Delta_c \equiv [E(N_{el}+2)+E(N_{el}-2)-2E(N_{el})]/2$$
$$\Delta_s \equiv [E(S=1;N_{el})-E(S=0;N_{el})]$$

	Δ_c	Δ_s
2x20 lattice	~ 0.286	~ 0.0003
2xN leg ladder N –› ∞	~ 0.245	< 0.0003

Checkerboard AF Insulator at n=3/4



2-leg ladder □——□ spin 3/2 chain with quasi LRO

4-leg ladder □——□ spin 3/2 2-leg ladder with SRO

6-leg ladder □——□ spin 3/2 3-leg ladder with quasi LRO

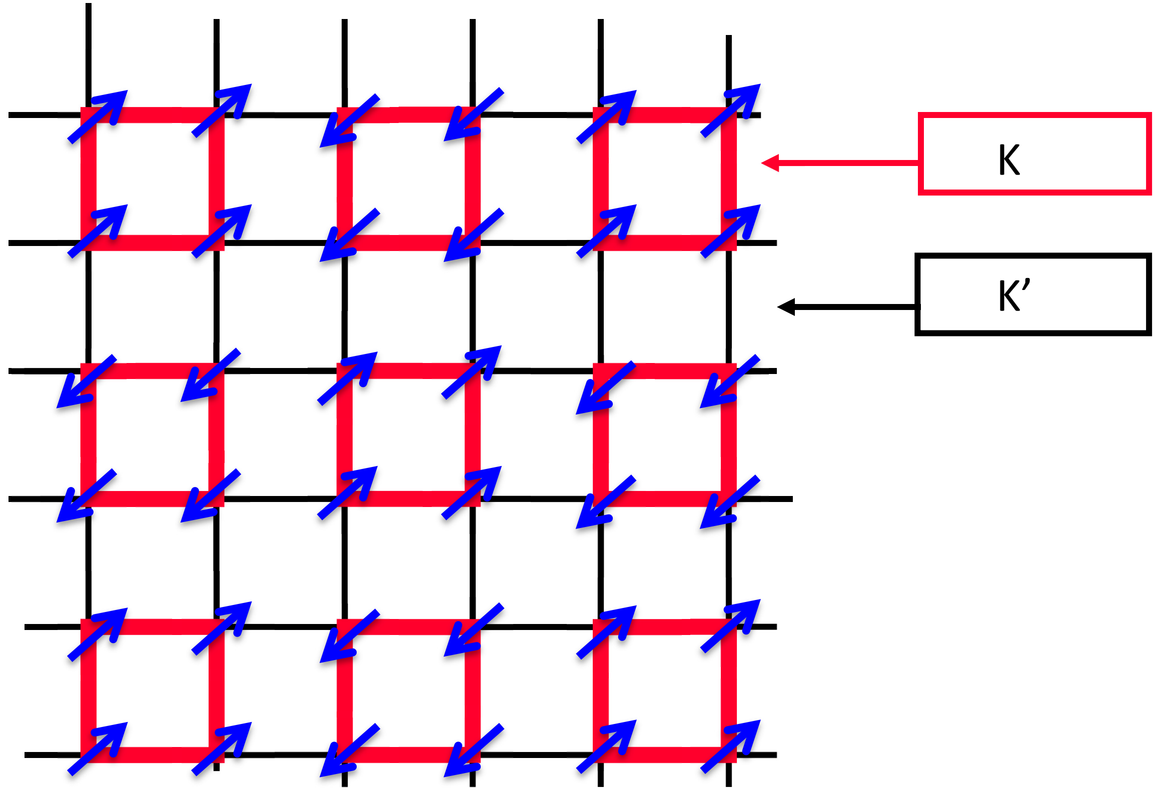
S. Chakravarty, Phys. Rev. Lett.77 , 4446 (1996)

Plaquette Checkerboard AF Insulator

$$\Delta_c \equiv [E(N_{el}+2) + E(N_{el}-2) - 2E(N_{el})]/2$$
$$\Delta_s \equiv [E(S=1; N_{el}) - E(S=0; N_{el})]$$

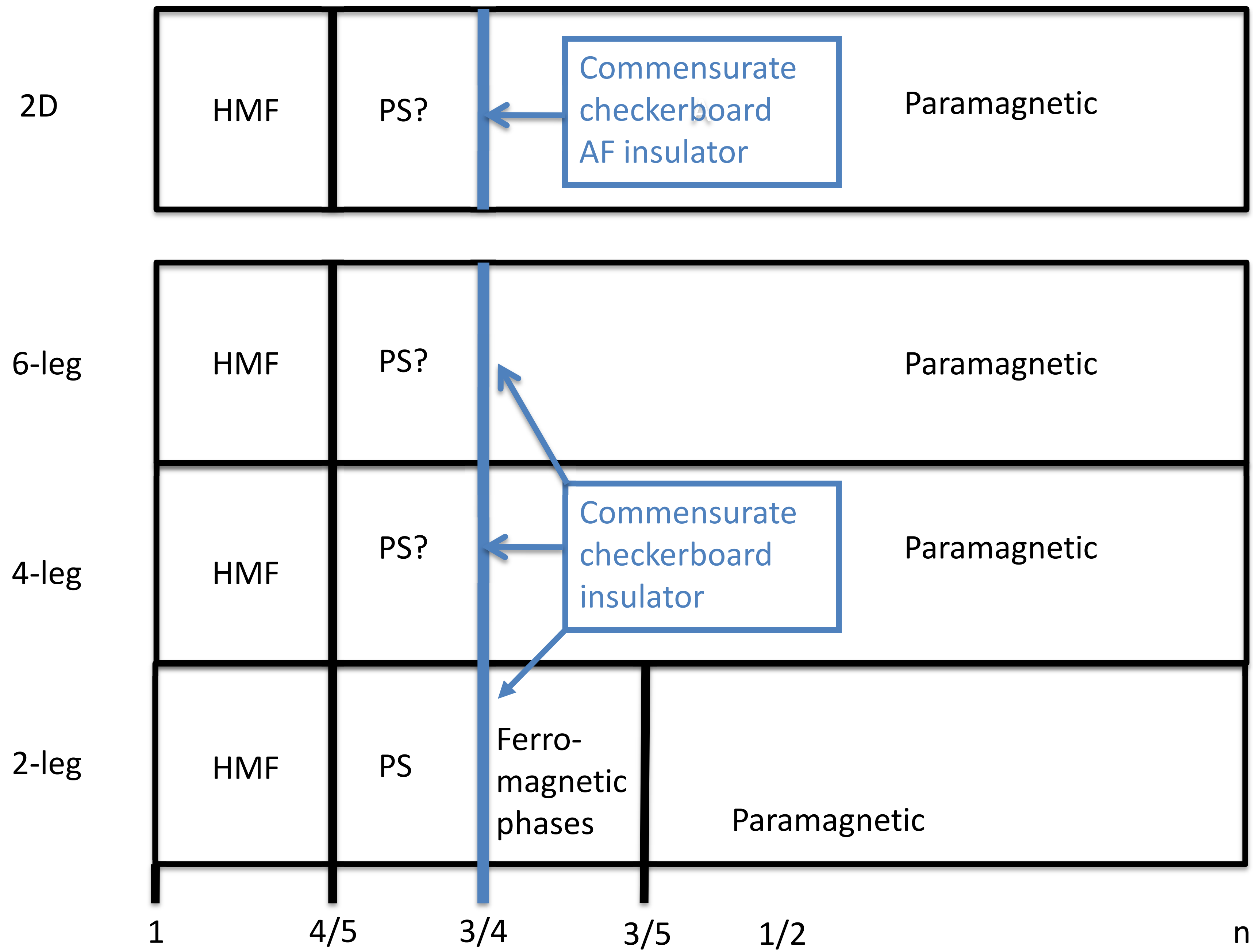
	Δ_c	Δ_s
2x20 lattice	~ 0.286t	~ 0.0003t
4x20 lattice	~ 0.23t	~ 0.008t
2xN leg ladder N –› ∞	~ 0.245t	< 0.0003t

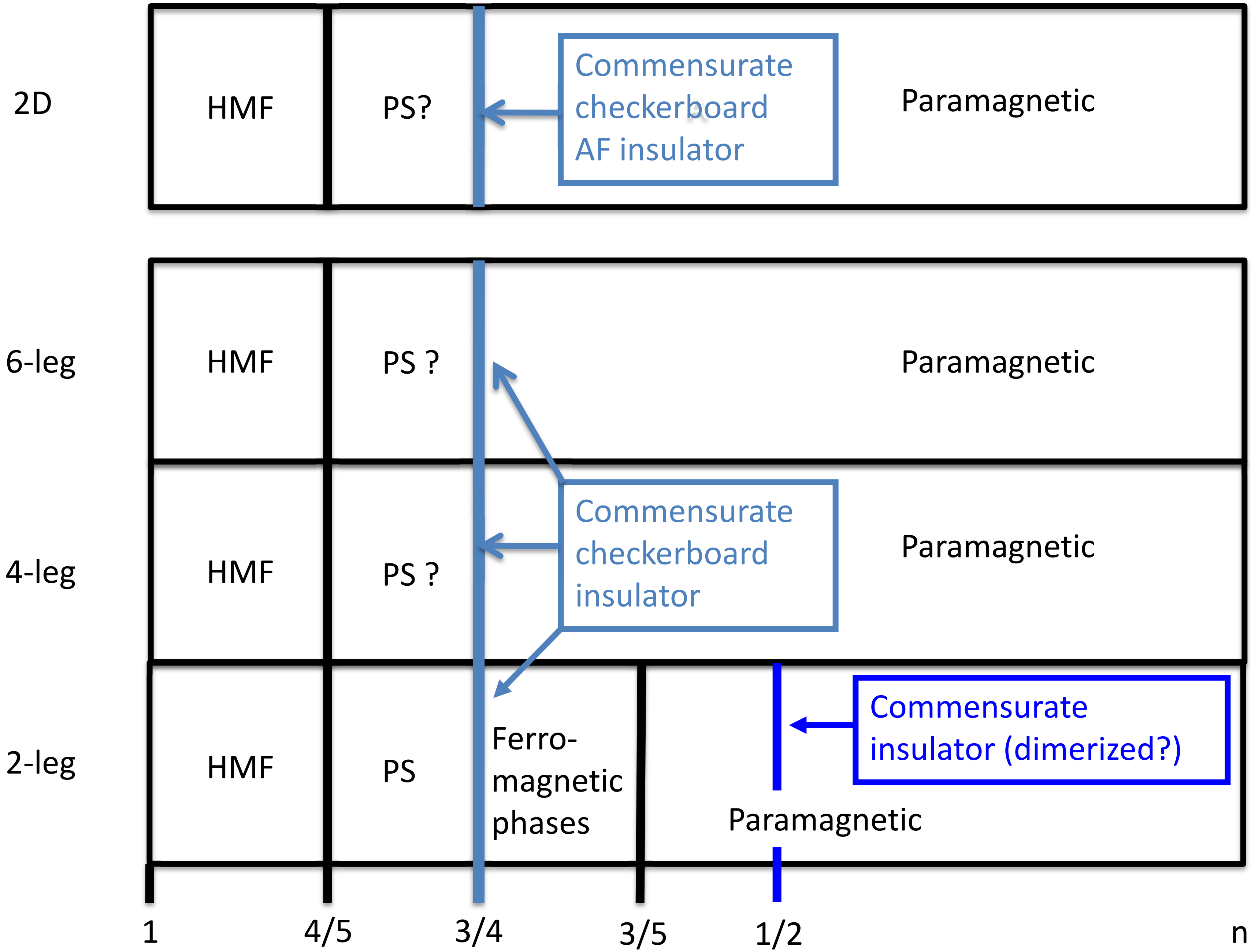
Checkerboard antiferromagnetic phase of the infinite U Hubbard model with n=3/4

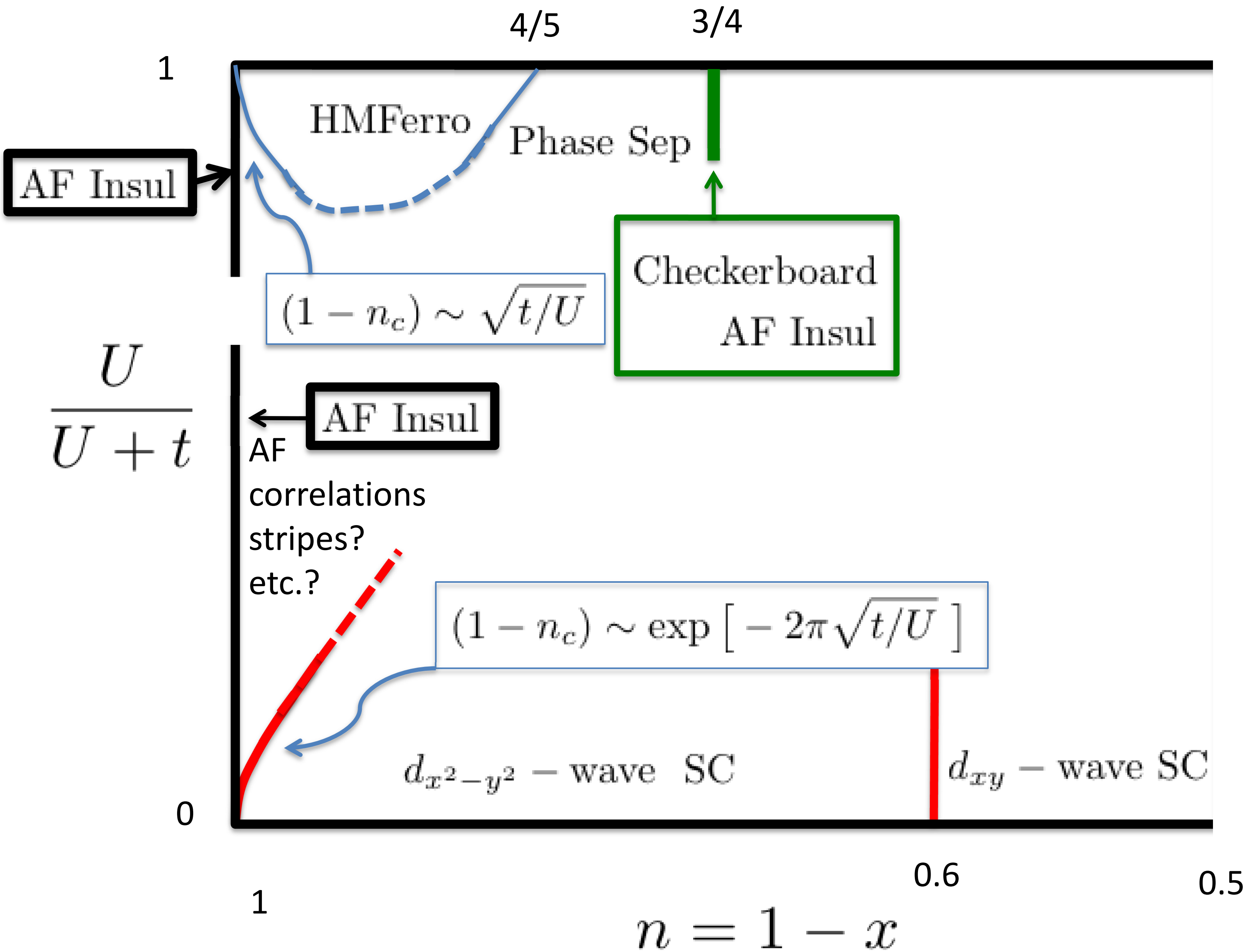


$$K = \sum_{\sigma} \langle [c^{\dagger}_{\vec{0},\sigma} c_{\hat{x},\sigma} + \mathrm{H.C.}] \rangle \qquad K' = \sum_{\sigma} \langle [c^{\dagger}_{\hat{x},\sigma} c_{2\hat{x},\sigma} + \mathrm{H.C.}] \rangle$$

Purely kinetic energy driven insulating density wave order!

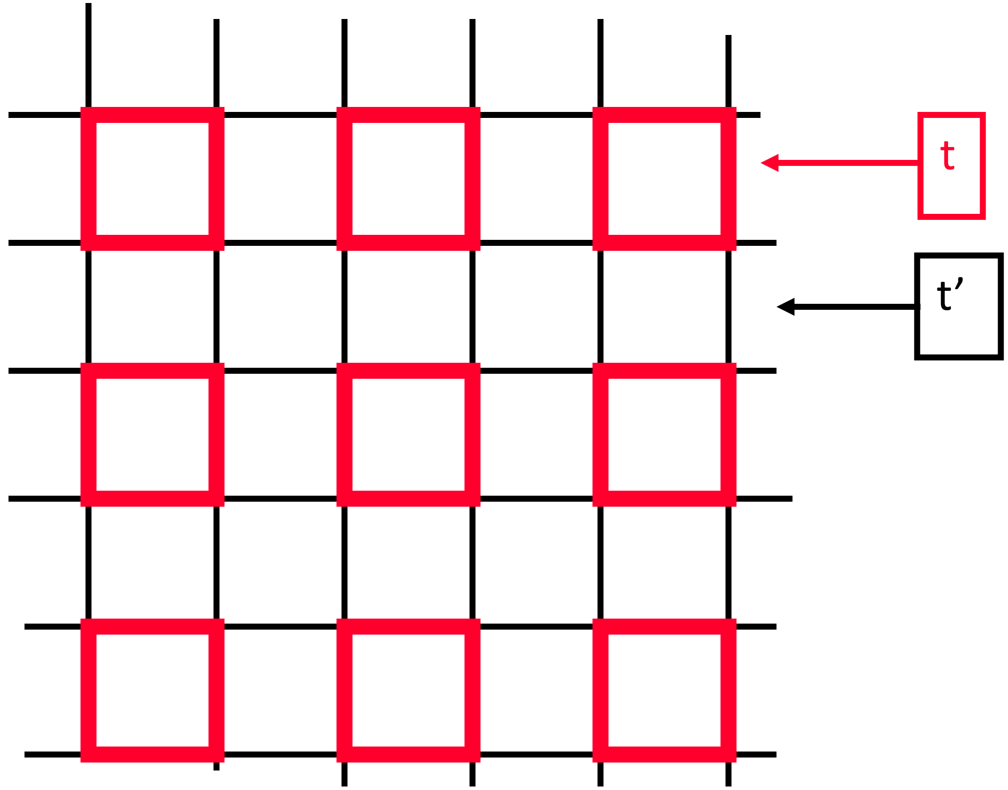

2D
HMF
PS?
Commensurate checkerboard AF insulator
Paramagnetic
6-leg
HMF
PS?
Paramagnetic
4-leg
HMF
PS?
Commensurate checkerboard insulator
Paramagnetic
2-leg
HMF
PS
Ferro-magnetic phases
Paramagnetic
1
4/5
3/4
3/5
1/2
n


2D
HMF
PS?
Commensurate checkerboard AF insulator
Paramagnetic
6-leg
HMF
PS ?
Paramagnetic
4-leg
HMF
PS ?
Commensurate checkerboard insulator
Paramagnetic
2-leg
HMF
PS
Ferro-magnetic phases
Commensurate insulator (dimerized?)
Paramagnetic
1
4/5
3/4
3/5
1/2
n


4/5
3/4
1
HMFerro
Phase Sep
AF Insul
Checkerboard
AF Insul
$(1-n_c) \sim \sqrt{t/U}$
$\frac{U}{U+t}$
AF Insul
AF
correlations
stripes?
etc.?
$(1-n_c) \sim \exp\left[-2\pi\sqrt{t/U}\,\right]$
$d_{x^2-y^2}$ – wave SC
d_{xy} – wave SC
0
1
0.6
0.5
$n = 1 - x$

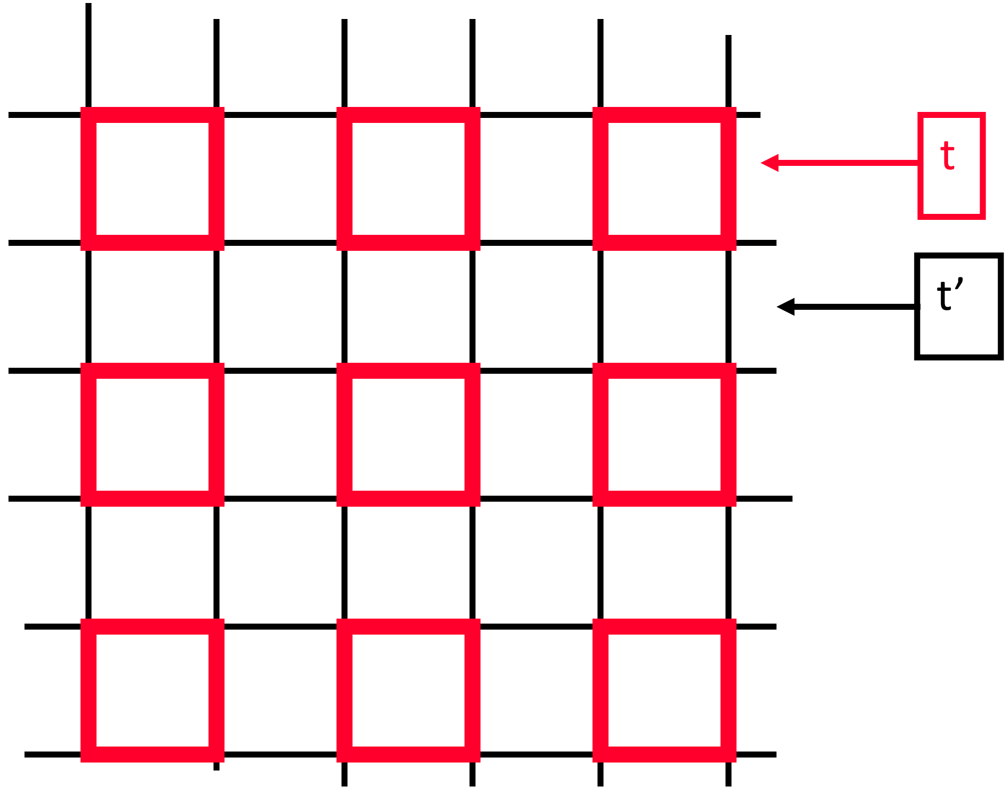
Phase diagram of checkerboard Hubbard model
– a route to the physics of intermediate coupling, U/8t ~ 1

"Homogeneous" (t'/t=1) to "highly inhomogeneous" (t'/t << 1)



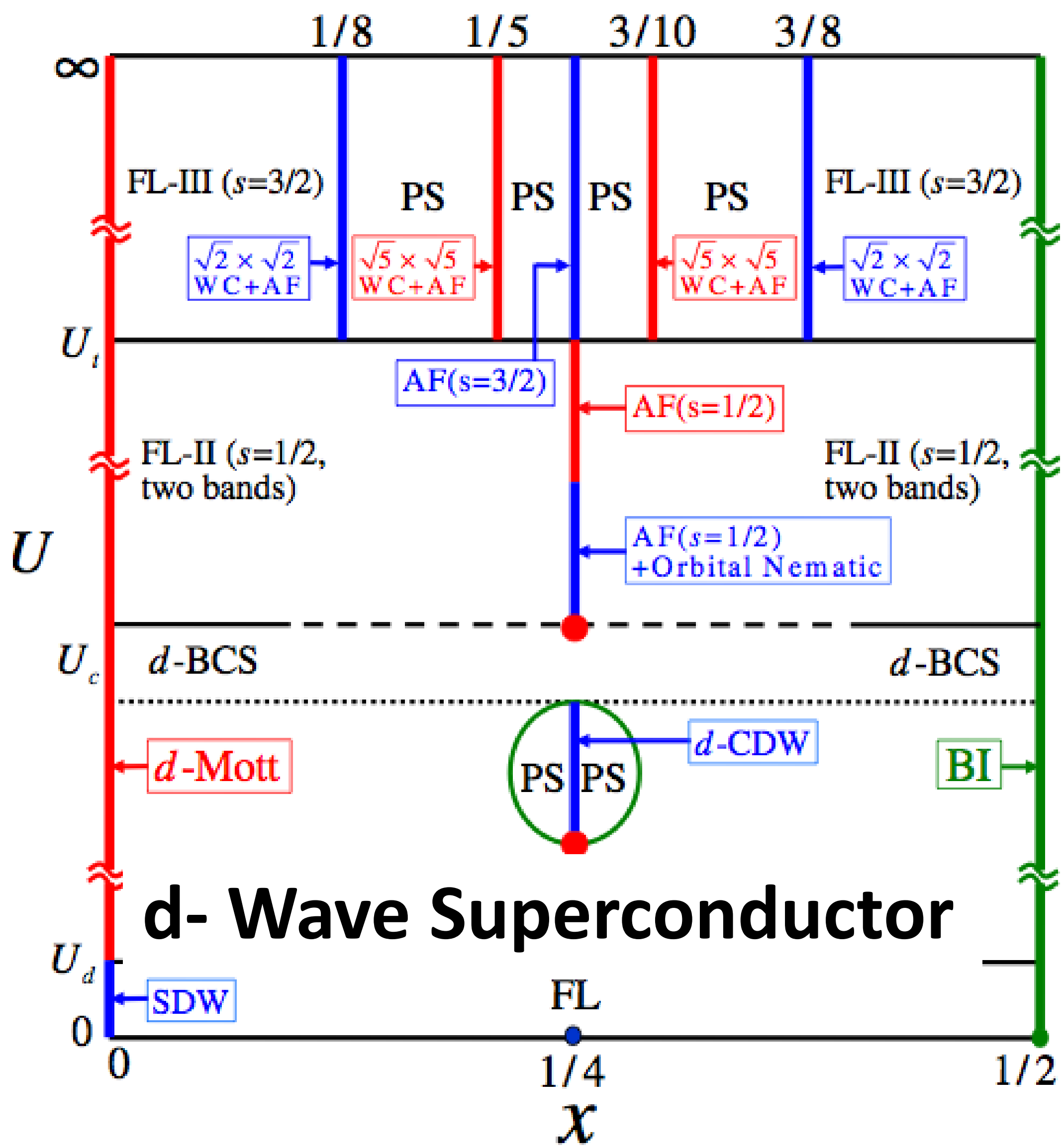
Phase diagram of checkerboard Hubbard model – a route to the physics of intermediate coupling, $U/8t \sim 1$

Asymptotic solution exists for arbitrary U/t in the limit $t'/t << 1$. *H.Yao et al, PRB 2007*



Myriad phases including d-wave superconducting, spin 3/2 Fermi liquid, electron nematic, AFI, ...

Phase diagram at T=0 for checkerboard model with t' << t.



Physics stories

- There are unconventional superconducting phases in the Hubbard model with purely repulsive interactions.
- For intermediate to strong interactions, there are myriad other ordered phases which “compete” or “intertwine” with superconductivity.
- For large enough U there is a half metallic ferromagnetic phase. (There probably are also partially polarized metallic phases.)

Open Questions

- Is superconductivity sufficiently strong, and T_c sufficiently high, that this plausibly captures (in essence) the mechanism of high temperature superconductivity?
- How robust is the superconductivity to inclusion of longer-range Coulomb repulsion and pair-breaking effects of phonons?
- What are the optimal conditions for high temperature superconductivity? How can one make high T_c higher?

Stay tuned

Stay tuned

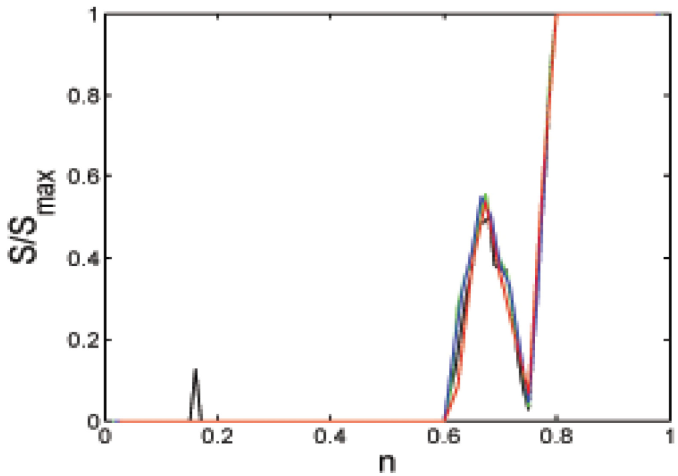
Phase Separation

Evidence:

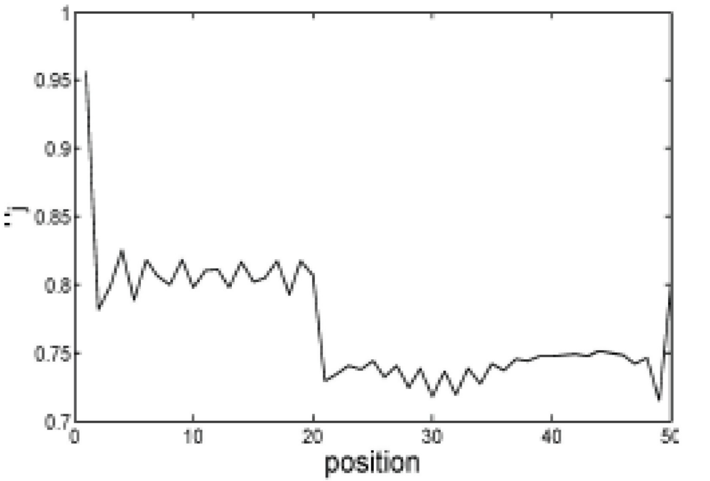
1 linear S dependence

2 linear energy dependence

3 potential induced phase separation



(a)



(b)