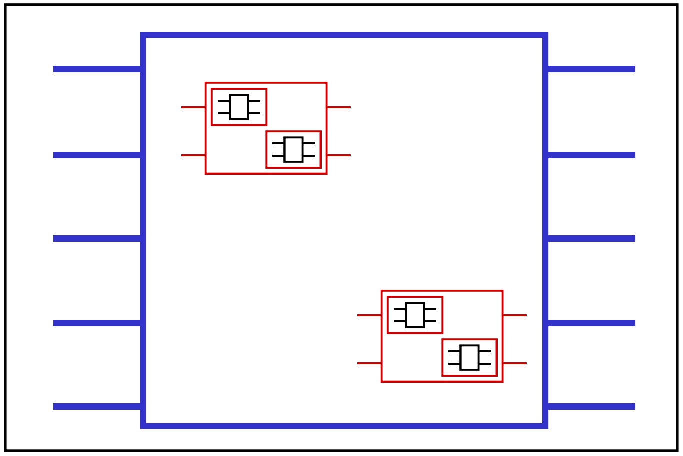
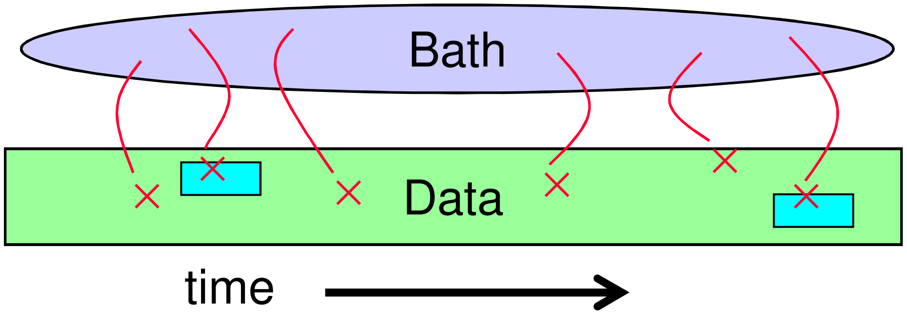
Topology and quantum error correction





John Preskill, Caltech
Freedman Symposium
15 April 2011

Freedman

AP+M Build 7th floor 7260

Muir College

Urey Hall

Wiley 5720

Talked to Freedman 2/4

25% Lornicor Patty

800-248-2683

ATM building

From hjkimble@cco.caltech.edu Mon Feb 2 11:36:33 1998
From: "H. Jeff Kimble" <hjkimble@cco.caltech.edu>
To: preskill@theory3.caltech.edu
Subject: M. Freedman
Date: Mon, 2 Feb 1998 11:36:22 -0800 (PST)

John,
Apart from a host of other issues to discuss concerning the STC, this email is to alert you that I just spoke with M. Freedman who will try to contact you concerning quantum computing stuff, including the STC. I emphasized that a discussion with you might let him know better if involvement with our center would be of interest to him and Microsoft. I also stressed our interest in contacts with him independent of any official Microsoft tie.
Regards,
Jeff
ps- If you want to jump first, his telephone at UCSD is 619-534-2647 and his email there is mfreedman@math.ucsd.edu, where I am not sure if the m and freedman are run together.

LaJolla Cove
UCSD

Radisson:

J. Wiley 619 534-~~21~~ 5?
-1178

www.
lajollacove.com

Accuracy?

Measure #P "sharp P"

coef in

$0 - 3^n$

degree n

n measurements would need n bits of accuracy

Oversampling because of variations

Bar Natan

Wed. Feb. 11

Go visit Freedman

Proc. Natl. Acad. Sci. USA
Vol. 95, pp. 98–101, January 1998
Computer Sciences

P/NP, and the quantum field computer

MICHAEL H. FREEDMAN

Microsoft Research 9N, 1 Microsoft Way, Redmond, WA 98052

Contributed by Michael H. Freedman

ABSTRACT The central problem in computer science is the conjecture that two complexity classes, *P* (polynomial time) and *NP* (nondeterministic polynomial time—roughly those decision problems for which a proposed solution can be checked in polynomial time), are distinct in the standard Turing model of computation: $P \neq NP$. As a generality, we propose that each physical theory supports computational models whose power is limited by the physical theory. It is well known that classical physics supports a multitude of implementation of the Turing machine. Non-Abelian topological quantum field theories exhibit the mathematical features necessary to support a model capable of solving all #*P* problems, a computationally intractable class, in polynomial time. Specifically, Witten [Witten, E. (1989) *Commun. Math. Phys.* 121, 351–391] has identified expectation values in a certain $SU(2)$-field theory with values of the Jones polynomial [Jones, V. (1985) *Bull. Am. Math. Soc.* 12, 103–111] that are #*P*-hard [Jaeger, F., Vertigen, D. & Welsh, D. (1990) *Math. Proc. Comb. Philos. Soc.* 108, 35–53]. This suggests that some physical system whose effective Lagrangian contains a non-Abelian topological term might be manipulated to serve as an analog computer capable of solving *NP* or even #*P*-hard problems in polynomial time. Defining such a system and addressing the accuracy issues inherent in preparation and measurement is a major unsolved problem.

and a tape. The head is capable of being in one of a finite number of "internal states" $\{q_i\}$ and can read and overwrite a symbol $\in \{S_j\}$ from a finite set of symbols and then shift one block left or right along the tape. It contains a finite internal program that directs its operations.

Consider a problem Q, with a yes/no answer, for which infinitely many instances exist, for example, the satisfiability of Boolean formulae. One asks: what is the fastest possible running time as a function of the size of the instance which a fixed program might achieve in correctly answering all of the instances of Q? One says that Q is in class P, if there is a program whose running time is bounded by a polynomial function of the number n of bits required to describe the instance I of Q on the Turing machine's tape. One says Q is in NP if there is an "existential" program operating on I plus a number of "guess bits" that correctly answer all instances I of Q in polynomial time. The existential program is deemed to say "yes," iff some setting of the guess bits returns a "yes" answer in poly-time. Clearly $P \subseteq NP$. It is easy to map NP into an apparently larger class of questions $\#P$ which ask of a given NP algorithm (with a fixed polynomial time cut-off), how many settings of the guess bits lead to "yes"?

The word "complete," following a class, is used to denote a problem $\bar{Q}$ within a class, which is maximally hard in the sense that any other problem in the class can be solved—again in poly-time—with an oracle giving, in a single clock cycle,

Fault-tolerant quantum computation by anyons

A. Yu. Kitaev

L.D.Landau Institute for Theoretical Physics,
117940, Kosygina St. 2

e-mail: kitaev@itp.ac.ru

Abstract

A two-dimensional quantum system with anyonic excitations can be considered as a quantum computer. Unitary transformations can be performed by moving the excitations around each other. Measurements can be performed by joining excitations in pairs and observing the result of fusion. Such computation is fault-tolerant by its physical nature.

A quantum computer can provide fast solution for certain computational problems (e.g. factoring and discrete logarithm [1]) which require exponential time on an ordinary computer. Physical realization of a quantum computer is a big challenge for scientists. One important problem is decoherence and systematic errors in unitary transformations which occur in real quantum systems. From the purely theoretical point of view, this problem has been solved due to Shor's discovery of fault-tolerant quantum computation [2], with subsequent improvements [3, 4, 5, 6]. An arbitrary quantum circuit can be simulated using imperfect gates, provided these gates are close to the ideal ones up to a constant precision δ. Unfortunately, the threshold value of δ is rather small[1]; it is very difficult to achieve this precision.

Needless to say, *classical* computation can be also performed fault-tolerantly. However, it is rarely done in practice because classical gates are reliable enough. Why is it possible? Let us try to understand the easiest thing — why classical information can be stored reliably on a magnetic media. Magnetism arise from spins of individual atoms. Each spin is quite sensitive to thermal fluctuations. But the spins interact with each other and tend to be oriented in the same direction. If some spin flips to the opposite direction, the interaction forces it to flip back to the direction of other spins. This process is quite similar to the standard error correction procedure for the repetition code. We may say that errors are being corrected at the physical level. Can we propose something similar in the quantum case? Yes, but it is not so simple. First of all, we need a quantum code with local stabilizer operators.

I start with a class of stabilizer quantum codes associated with lattices on the torus and other 2-D surfaces [3, 8]. Qubits live on the edges of the lattice whereas the stabilizer operators correspond to the vertices and the faces. These operators can be put together to make up a

[1] Actually, the threshold is not known. Estimates vary from 1/300 [7] to 10^{-6} [4].

Simulation of topological field theories by quantum computers

Michael H. Freedman*, Alexei Kitaev†, and Zhenghan Wang‡

Abstract

Quantum computers will work by evolving a high tensor power of a small (e.g. two) dimensional Hilbert space by local gates, which can be implemented by applying a local Hamiltonian H for a time t. In contrast to this quantum engineering, the most abstract reaches of theoretical physics has spawned "topological models" having a finite dimensional internal state space with no natural tensor product structure and in which the evolution of the state is discrete, $H \equiv 0$. These are called topological quantum filed theories (TQFTs). These exotic physical systems are proved to be efficiently simulated on a quantum computer. The conclusion is two-fold:

1. TQFTs cannot be used to define a model of computation stronger than the usual quantum model "BQP."
2. TQFTs provide a radically different way of looking at quantum computation. The rich mathematical structure of TQFTs might suggest a new quantum algorithm.

A modular functor which is universal for quantum computation

Michael Freedman†, Michael Larsen‡, and Zhenghan Wang‡

† *Microsoft Research, One Microsoft Way, michaelf@microsoft.com*
‡ *Indiana Univ., larsen@math.indiana.edu and zhewang@indiana.edu*

Abstract

We show that the topological modular functor from Witten-Chern-Simons theory is universal for quantum computation in the sense a quantum circuit computation can be efficiently approximated by an intertwining action of a braid on the functor's state space. A computational model based on Chern-Simons theory at a fifth root of unity is defined and shown to be polynomially equivalent to the quantum circuit model. The chief technical advance: the density of the irreducible sectors of the Jones representation, have topological implications which will be considered elsewhere.

Truism:
the macroscopic world is classical.
the microscopic world is quantum.

Goal of QIS:
controllable quantum behavior in scalable systems

Why?

Classical systems cannot simulate quantum systems efficiently (a widely believed but unproven conjecture).

But to control quantum systems we must slay the dragon of decoherence …

Is this merely *really, really hard*?
Or is it *ridiculously* hard?

Delicious irony: macroscopic quantum systems can have intrinsic protection against decoherence!

Quantum Error Correction and Quantum Fault Tolerance

Shor

RAPID COMMUNICATIONS

PHYSICAL REVIEW A

ATOMIC, MOLECULAR, AND OPTICAL PHYSICS

THIRD SERIES, VOLUME 52, NUMBER 4 OCTOBER 1995

RAPID COMMUNICATIONS

The Rapid Communications section is intended for the accelerated publication of important new results. Since manuscripts submitted to this section are given priority treatment both in the editorial office and in production, authors should explain in their submittal letter why the work justifies this special handling. A Rapid Communication should be no longer than 4 printed pages and must be accompanied by an abstract. Page proofs are sent to authors.

Scheme for reducing decoherence in quantum computer memory

Peter W. Shor*

AT&T Bell Laboratories, Room 2D-149, 600 Mountain Avenue, Murray Hill, New Jersey 07974

(Received 17 May 1995)

Recently, it was realized that use of the properties of quantum mechanics might speed up certain computations dramatically. Interest has since been growing in the area of quantum computation. One of the main difficulties of quantum computation is that decoherence destroys the information in a superposition of states contained in a quantum computer, thus making long computations impossible. It is shown how to reduce the effects of decoherence for information stored in quantum memory, assuming that the decoherence process acts independently on each of the bits stored in memory. This involves the use of a quantum analog of error-correcting codes.

PACS number(s): 03.65.Bz, 89.70.+c

I. INTRODUCTION

Recently, interest has been growing in an area called *quantum computation*, which involves computers that use the ability of quantum systems to be in a superposition of many states. These computations can be modeled formally by defining a quantum Turing machine [1,5], which is able to be in the superposition of many states. Instead of considering the computer itself to be in a superposition of states, it is sufficient to assume that the contents of the memory cells are in a superposition of different states and that the computer presence of extra unextractable quantum information is a barrier to efficient simulation of a quantum computer on a classical computer.

It now appears that, at least theoretically, quantum computation may be much faster than classical computation for solving certain problems [5–7], including prime factorization. However, it is not yet clear whether quantum computers are feasible to build. One reason that quantum computers will be difficult, if not impossible, to build is *decoherence*. In the process of decoherence, some qubit or qubits of the computation become entangled with the environment, thus in ef-

Quantum error correction and topological order

A "logical qubit" is encoded using many "physical qubits." We want to protect the logical qubit, with orthonomal basis states $|0\rangle$ and $|1\rangle$, from a set of possible error operators $\{ E_a \}$.

For protection against bit flips:
$E_a |0\rangle \perp E_b |1\rangle$.

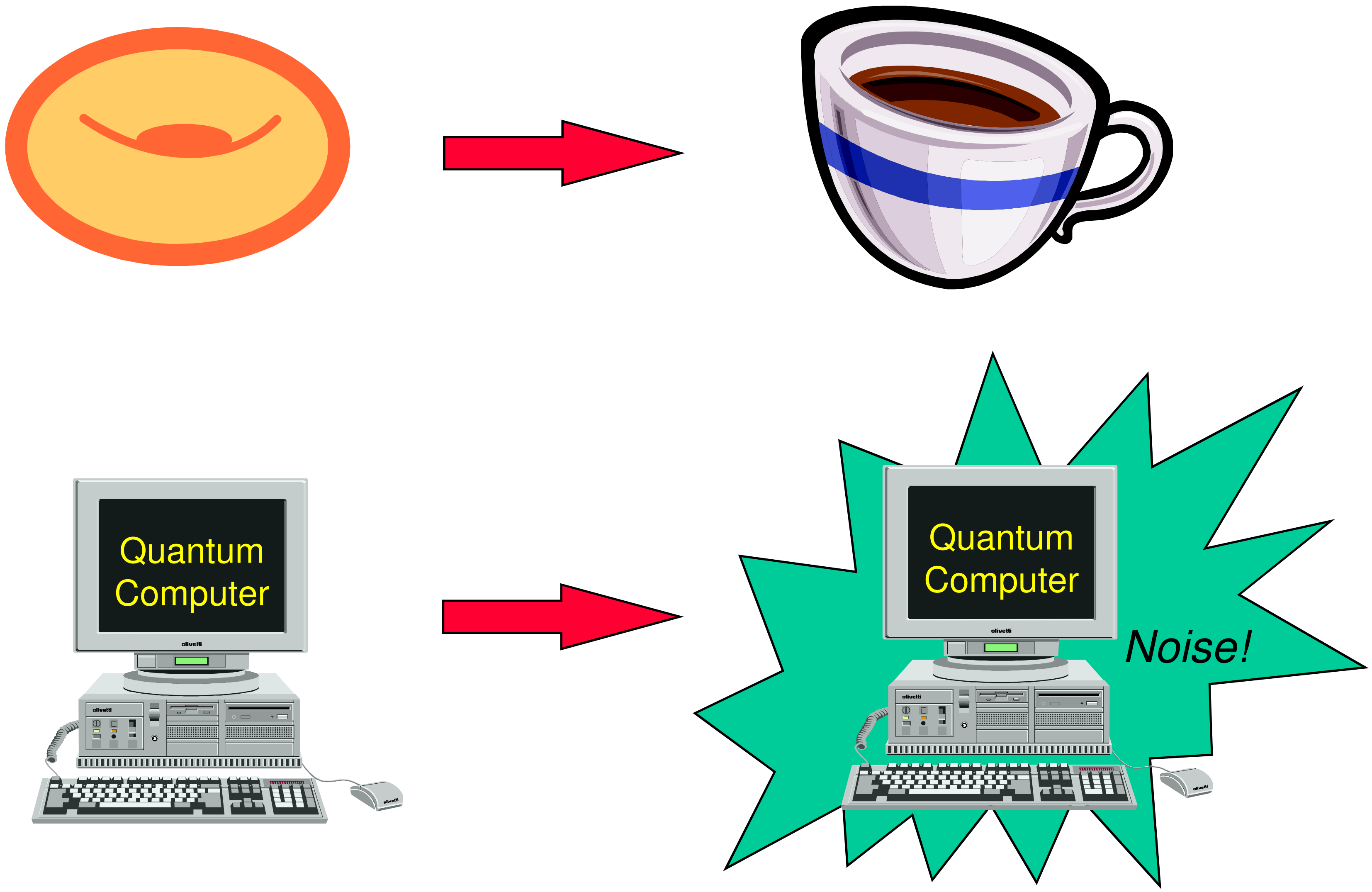
For protection against phase errors:
$E_a (|0\rangle + |1\rangle) \perp E_b (|0\rangle - |1\rangle)$.

In fact, these conditions suffice to ensure the existence of a recovery map.

It follows that
$\langle 0| E_b^\dagger E_a |0\rangle = \langle 1| E_b^\dagger E_a |1\rangle$.

Compare the definition of topological order: if V is a (quasi-)local operator and $|0\rangle$, $|1\rangle$ are ground states of a local Hamiltonian, then
$\langle 1| V |0\rangle = 0$, and $\langle 0| V |0\rangle = \langle 1| V |1\rangle$.
up to corrections exponentially small in the system size. (Ground states are locally indistinguishable.)

Topology



Quantum Error Correction and Quantum Fault Tolerance

Shor

Fault-Tolerant Quantum Computation

Peter W. Shor*
AT&T Research
Room 2D-149
600 Mountain Ave.
Murray Hill, NJ 07974, USA

Abstract

Recently, it was realized that use of the properties of quantum mechanics might speed up certain computations dramatically. Interest in quantum computation has since been growing. One of the main difficulties of realizing quantum computation is that decoherence tends to destroy the information in a superposition of states in a quantum computer, thus making long computations impossible. A futher difficulty is that inaccuracies in quantum state transformations throughout the computation accumulate, rendering the output of long computations unreliable. It was previously known that a quantum circuit with t gates could tolerate $O(1/t)$ amounts of inaccuracy and decoherence per gate. We show, for any quantum computation with t gates, how to build a polynomial size quantum circuit that can tolerate $O(1/\log^c t)$ amounts of inaccuracy and decoherence per gate, for some constant c. We do this by showing how to compute using quantum error-correcting codes. These codes were previously known to provide resistance to errors while storing and transmitting quantum data.

1 Introduction

It has recently been discovered that certain properties of quantum mechanics have a profound effect on abstract models of computation. More specifically, by using the superposition and the interference principles of quantum mechanics, one can devise a physics thought experiment giving a computing machine which is apparently more powerful than the standard Turing machine model of theoretical computer science. These quantum computers can use only poly-

The potentially most useful algorithms for quantum computers discovered so far include prime factorization and simulation of certain quantum mechanical systems.

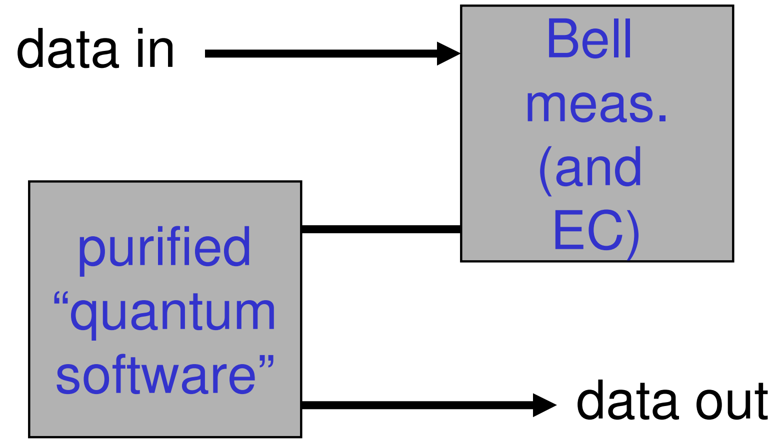
Given these theoretical results, a natural question is whether such computers could ever be built. Ingenious designs for such computers have recently been proposed [8], and currently several experiments are underway in attempts to build small working prototypes [14]. Even if small quantum computers can successfully be built, scaling these up to computers that are large enough to yield useful computations could present formidable difficulties.

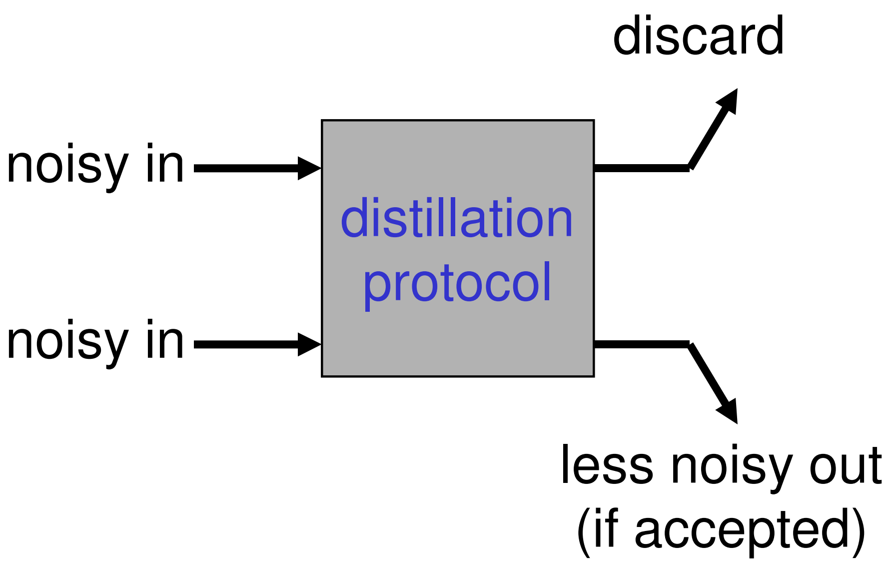
One of these difficulties is *decoherence* [12, 23, 9]. Quantum computation involves manipulating the quantum states of objects that are in coherent quantum superpositions. These superpositions, however, tend to be quite fragile and decay easily; this decay phenomenon is called decoherence. One way of thinking about decoherence is to consider the environment to be "measuring" the state of a quantum system. That is, if the environment interacts with the system in such a way that the effect on the environment depends on the state of the system, it will project the quantum system into an eigenstate of the interaction of the system and the environment [26].

A second potential obstacle to building quantum computers is inaccuracy [12, 9]. Quantum computers are fundamentally analog-type devices; that is, the state of a quantum superposition depends on certain continuous parameters. For example, one of the common quantum gates used in quantum computations is a "rotation" of a quantum bit by an angle θ; when doing this transformation, there will naturally tend to be some inaccuracy in this angle θ. For the quantum

Gate teleportation and state distillation

In fault-tolerant schemes, a version of quantum teleportation is used to complete a universal set of protected quantum gates. Suitable “quantum software” is prepared and verified offline, then measurements are performed that transform the incoming data to outgoing data, with a “twist” (an encoded operation) determined by the software.





Reliable software is obtained from noisy software via a multi-round state distillation protocol. In each round (which uses CNOT gates and measurements), there are *n* noisy input copies of the software of which *n-1* copies are destroyed. The remaining output copy, if accepted, is less noisy than the input copies

Gottesman, Chuang; Knill; Bravyi, Kitaev

Scalability

Quantum Accuracy Threshold Theorem: Consider a quantum computer subject to quasi-independent noise with strength ε. There exists a constant $\varepsilon_0 > 0$ such that for a fixed $\varepsilon < \varepsilon_0$ and fixed $\delta > 0$, any circuit of size L can be simulated by a circuit of size L^* with accuracy greater than $1-\delta$, where, for some constant c,

$$L^* = O\left[L (\log L)^c \right]$$

Aharonov, Ben-Or
Kitaev
Laflamme, Knill, Zurek
Aliferis, Gottesman, Preskill
Reichardt

assuming:

parallelism, fresh qubits (*necessary* assumptions)

nonlocal gates, fast measurements, fast and accurate classical processing, no leakage (*convenient* assumptions).

"Practical" considerations:
Resource requirements, systems engineering issues

Matters of "principle":
Conditions on the noise model, what schemes are scalable, etc.

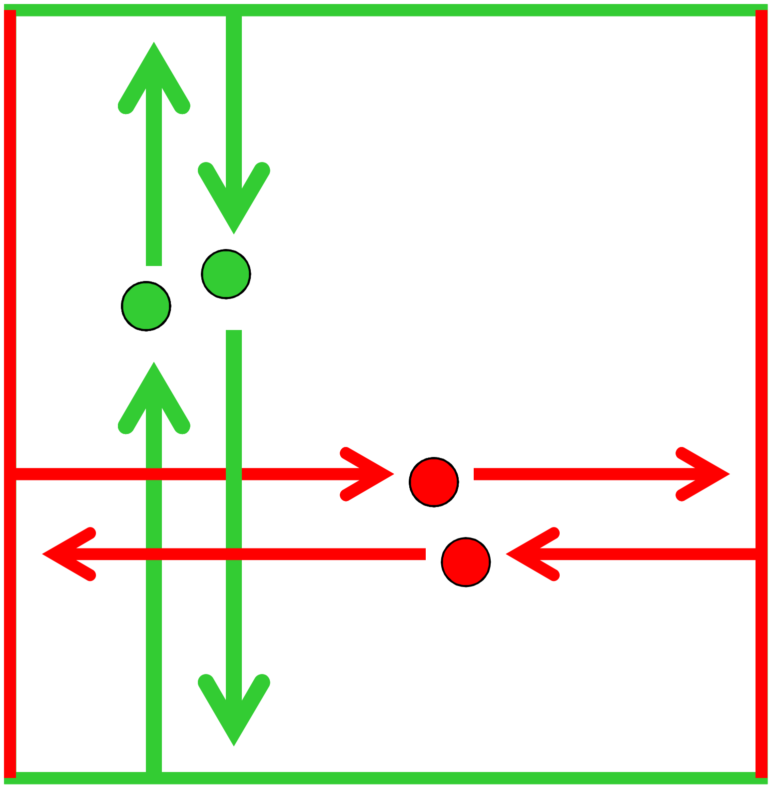
Better together: topology and quantum error correction

- Error correction and fault tolerance will be essential for operating large-scale quantum computers.
- The “standard” approach uses clever “software” to overcome the deficiencies of quantum hardware. It works in principle, if the hardware is not too noisy.
- The “physical” approach seeks quantum “hardware” that is intrinsically robust.
- The two approaches can be combined --- even robust quantum hardware will not be perfect, and the standard approach may still be needed to perform long computations reliably.
- Topology informs both approaches, by suggesting new schemes for constructing robust hardware, and by inspiring new kinds of software.

Three themes

- Topological codes
- Protected devices
- Self correction

Topological Degeneracy

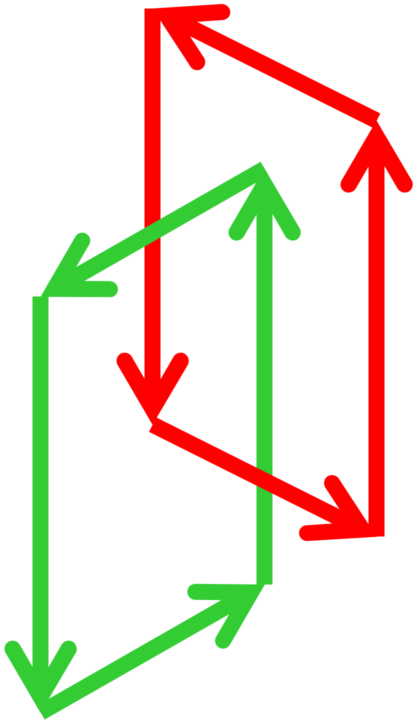


A two-dimensional system (with a mass gap) that supports quasiparticle excitations with nontrivial Aharonov-Bohm interactions has a ground state degeneracy that depends on the topology of the surface.

Example: two defects (green and red) with a Z_2 Aharonov Bohm phase. Green defects can be singly produced or annihilated at a green boundary, red defects can be singly produced or annihilated at at red boundary.

Two operators (R and G) both preserve the ground state, and obey a nontrivial commutation relation: $R^{-1}G^{-1}RG = -1$.
This algebra has no one-dimensional representations, hence the ground state is (two-fold) degenerate.

Topological Degeneracy



A two-dimensional system (with a mass gap) that supports quasiparticle excitations with nontrivial Aharonov-Bohm interactions has a ground state degeneracy that depends on the topology of the surface.

Example: two defects (green and red) with a Z_2 Aharonov Bohm phase. Green defects can be singly produced or annihilated at a green boundary, red defects can be singly produced or annihilated at at red boundary.

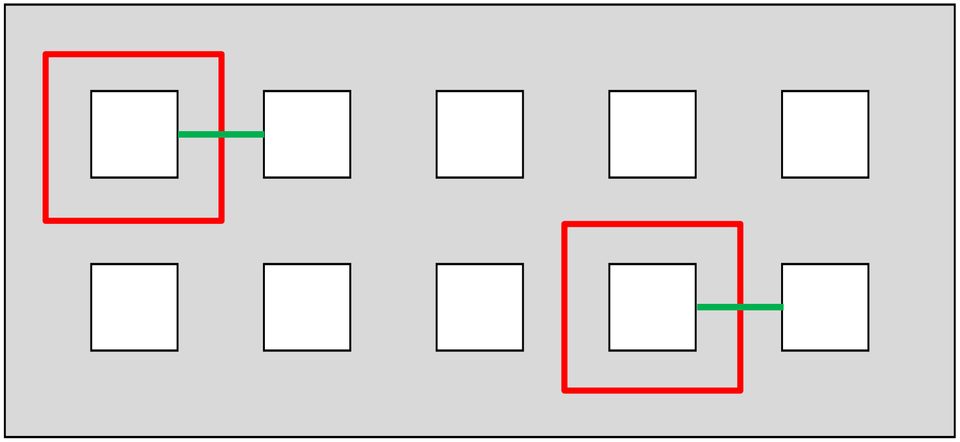
Two operators (R and G) both preserve the ground state, and obey a nontrivial commutation relation: $R^{-1}G^{-1}RG = -1$.
This algebra has no one-dimensional representations, hence the ground state is (two-fold) degenerate.

Toward “architecture”…

Early work on quantum fault tolerance ignored geometry. It was assumed that a quantum gate could act on any pair of qubits, with a gate fidelity independent of the distance between qubits.

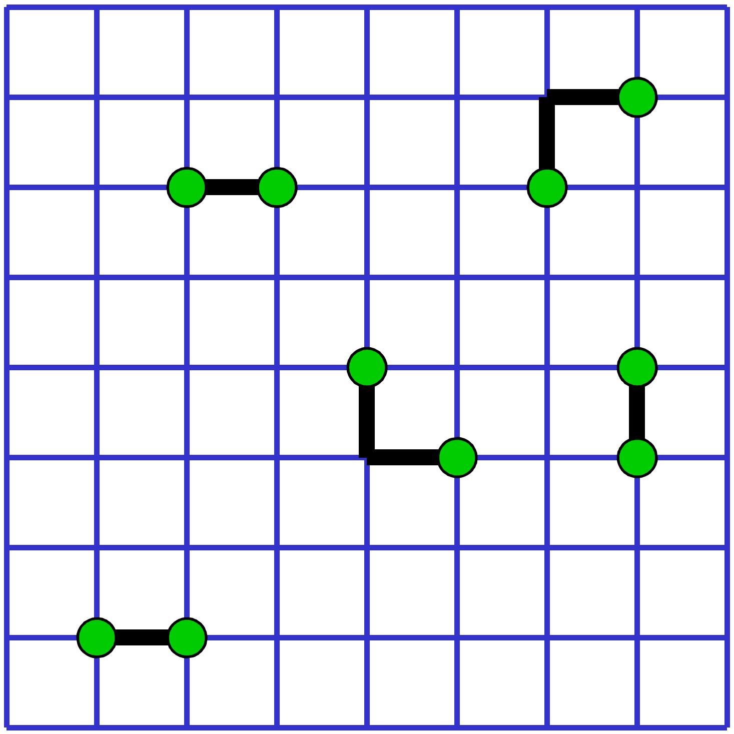
More realistically, later work considered two-dimensional layouts with local gates. (Qubits individually addressed using wires that extend into the third dimension.)

In a 2D layout, it is natural to use topological codes on a puctured plane, with qubits encoded using Z_2 “electric” (or “magnetic”) charges placed in the holes.



Logical (“string”) operations are realized by carrying a magnetic charge (red) around an electric hole, or by moving an electric charge (green) from one hole to another.

Error Recovery



Z-type errors create / annihilate pairs of electric charges, or move an electric charge to a neighboring site. X-type errors create / annihilate pairs of magnetic charges or move magnetic charges. If the error rate is small, the error chain segments are typically short, and the particle positions are strongly correlated. Once the particle positions are known, it is “easy” to guess how to bring particles together and annihilate them without a logical error. Measurements of particle positions are sometimes wrong, but we can repeat measurements to make our guess reliable.

There is an “accuracy threshold”

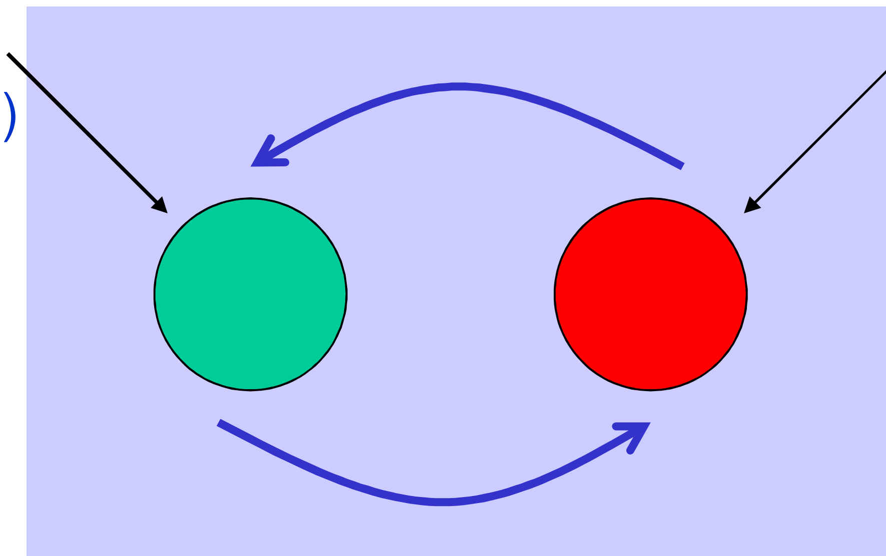
… if we assume accurate and instantaneous (poly-time) classical processing. The probability of a logical error decays exponentially with system size.

Dennis, Kitaev, Landahl, Preskill (2002).

Local fault tolerance with 2D topological codes

Qubits are arranged on a two-dimensional lattice with holes in it. Protected qubits are encoded (in either of two complementary bases) by placing "electric" charges inside "primal" holes or "magnetic" charges inside "dual" holes. The quantum information is well protected if the holes are large and far apart.

electric ("primal") hole



magnetic ("dual") hole

A controlled-NOT gate can be executed by "braiding the holes" which is achieved by a sequence of local gates or measurements.

The protected gates and error syndrome extraction can be done with local two-qubit gates or measurements. Numerical studies indicate an upper bound on the threshold for independent depolarizing noise:

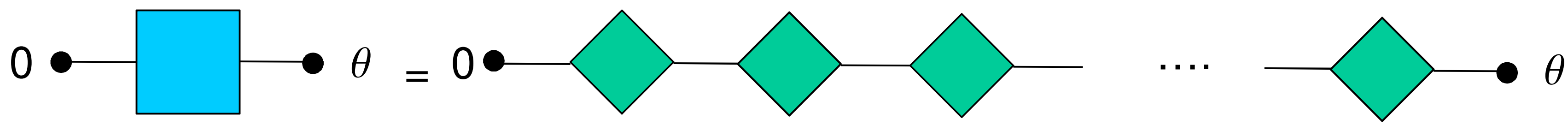
$\varepsilon_0 \sim 7.5 \times 10^{-3}$

Raussendorf, Harrington, Goyal (2007)
Dennis, Kitaev, Landahl, Preskill (2002)

Protected superconducting qubit

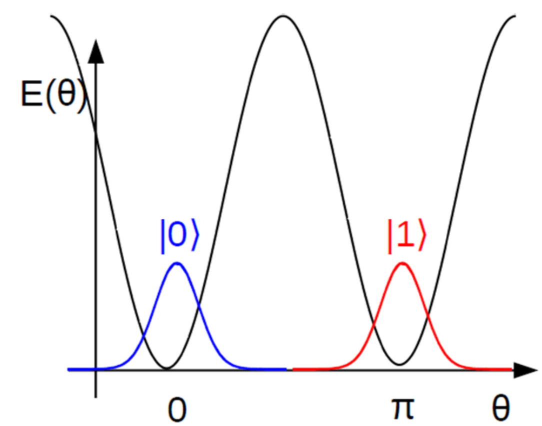
Ioffe et al.
Kitaev

One way to make a robust superconducting (0-Pi) qubit is to build a long chain of devices. Each individual device favors a phase change of 0 or π across its leads. The phase difference between the two ends of the chain can likewise be either 0 or π but with large local phase fluctuations along the chain.



The two basis states of the qubit are distinguished by a global property of the chain --- both look the same locally. For long chain, the breaking of the degeneracy of the two states due to a generic local perturbation occurs in a high order of perturbation theory and is strongly suppressed.

The barrier is high enough to suppress bit flips, and the stable degeneracy suppresses phase errors. Protection arises because the encoding of quantum information is highly nonlocal, and splitting of degeneracy scales exponentially with (square root of) size of the device.



$$\boxed{E \approx f(2\theta) + O\left(\exp\left(-c\sqrt{\text{size}}\right)\right)}$$

Protected superconducting qubit

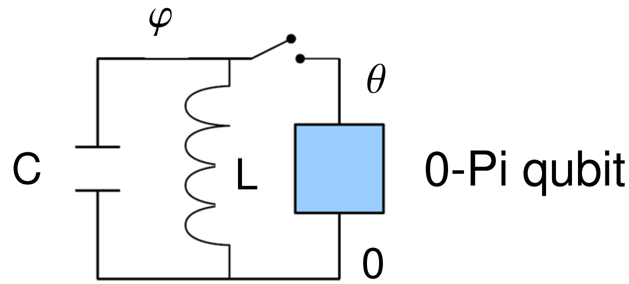
Kitaev,
Brooks,
Preskill

Some gates are also protected: we can execute

$$\boxed{\exp\left(i\frac{\pi}{4}Z\right) \text{ and } \exp\left(i\frac{\pi}{4}Z_1Z_2\right)}$$

single-qubit or two-qubit phase gate

with exponential precision. This is achieved by coupling a qubit or a pair of qubits to a "superinductor" with large phase fluctuations:

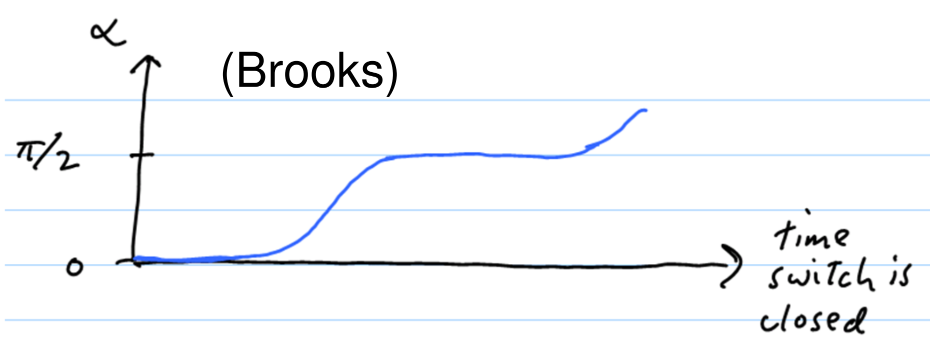


$$\sqrt{L/C} \gg$$

$$\hbar/(2e)^2 \approx 1k\Omega$$

To execute the gate, we (1) close the switch, (2) keep it closed for awhile, (3) open the switch. This procedure alters the relative phase of the two basis states of the qubit:

$$\left(a|0\rangle + b|1\rangle\right) \otimes |\text{init}\rangle \rightarrow \left(a|0\rangle + be^{-i\alpha}|1\rangle\right) \otimes |\text{final}\rangle$$

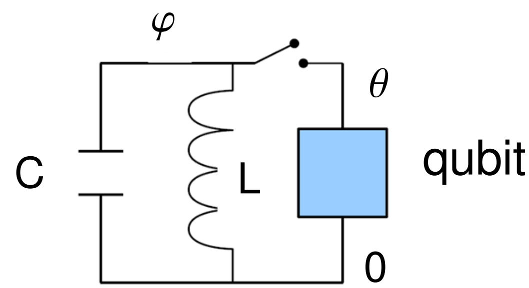


The relative phase induced by the gate "locks" at π/2. For $\sqrt{L/C} \approx 40$ Gate error < 10^{-4} is achieved for timing error of a few percent. Why?

Protected phase gate

Kitaev, Brooks, Preskill

$$\boxed{\exp\left(i\frac{\pi}{4}Z\right)}$$

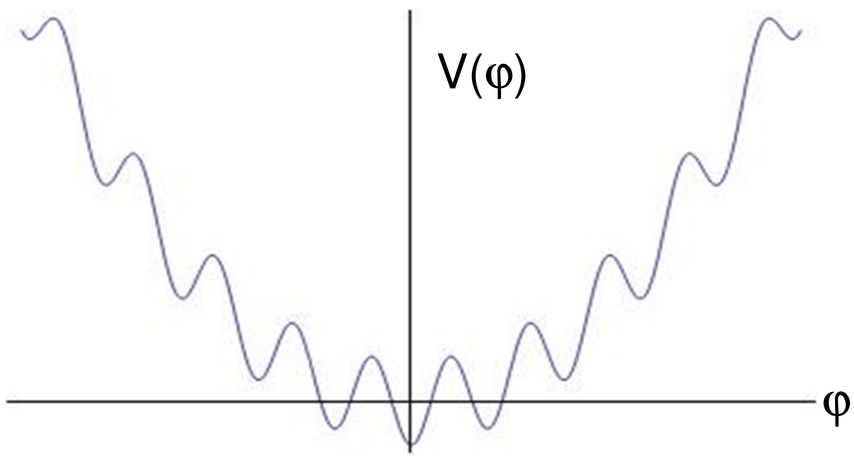


$$\sqrt{L/C} \gg \hbar/(2e)^2 \approx 1k\Omega$$

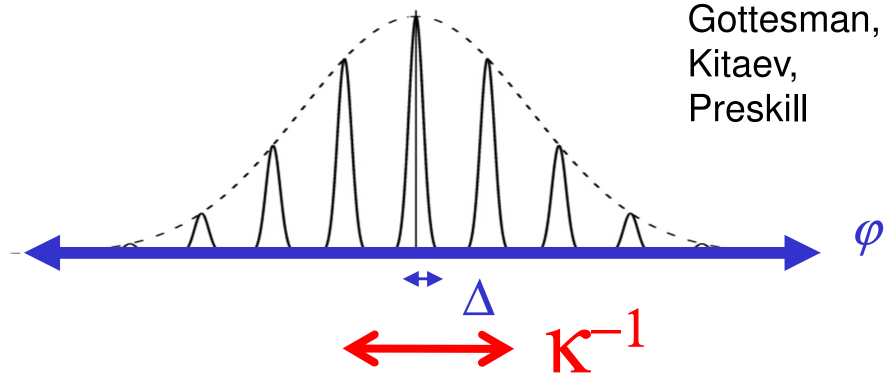
Switch is really a tunable Josephson junction:

$$H = \frac{Q^2}{2C} + \frac{\varphi^2}{2L} - J(t)\cos(\varphi - \theta)$$

Under suitable adiabaticity conditions, closing the switch transforms a broad oscillator state (e.g. the ground state) into a grid state (approximate codeword).

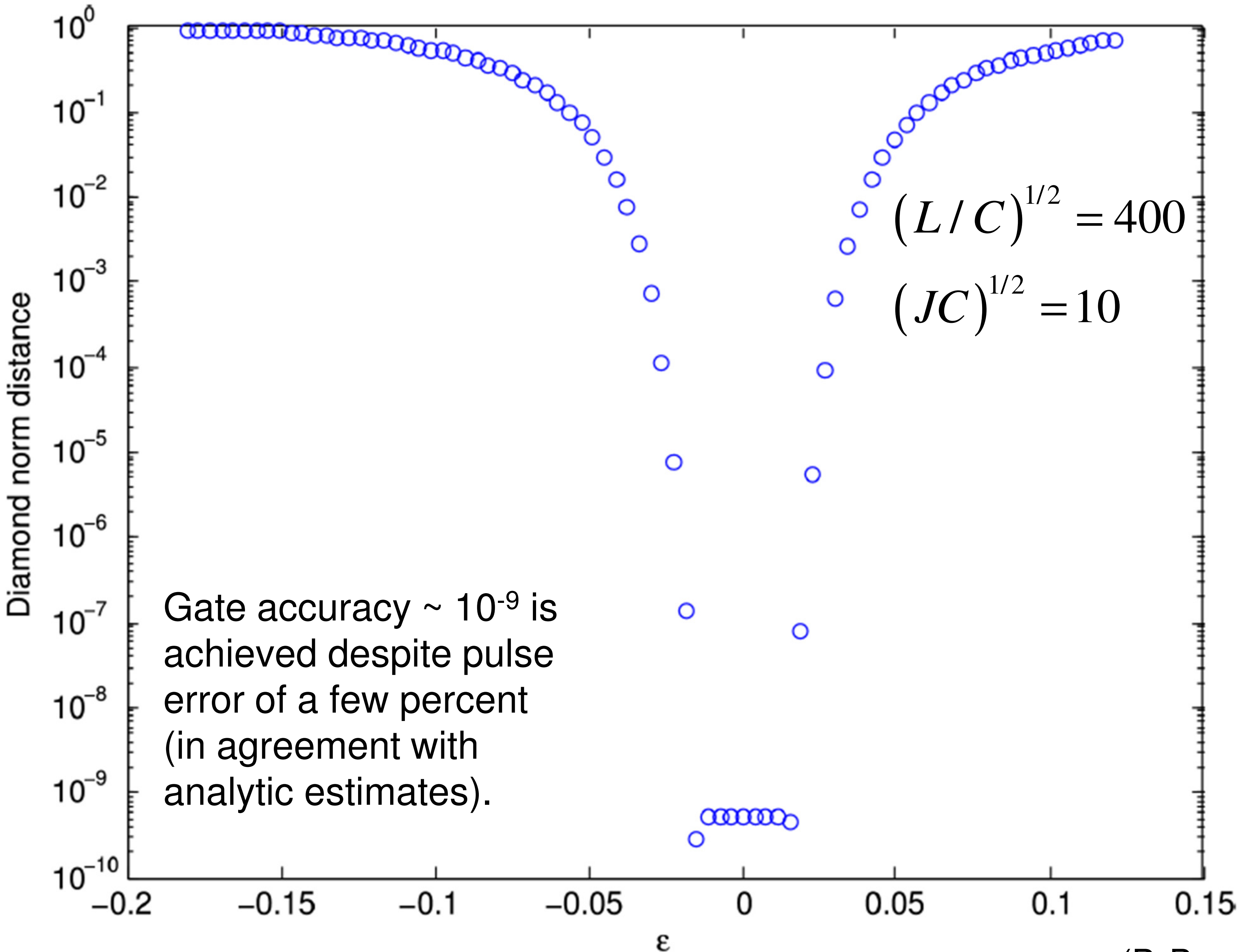


Gottesman, Kitaev, Preskill



Peaks are at even or odd multiples of π depending on whether θ is 0 or π, i.e. on whether qubit is 0 or 1. Inner width squared is $(JC)^{-1/2}$ and outer width is $(L/C)^{1/2}$

$$\omega_J^{-1} = \sqrt{C/J} \ll \text{switching time} \ll \omega^{-1} = \sqrt{LC} \gg 1$$

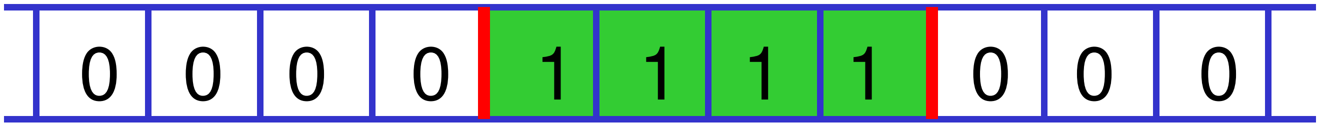


10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
Diamond norm distance
−0.2
−0.15
−0.1
−0.05
0
0.05
0.1
0.15
ε
$(L/C)^{1/2} = 400$
$(JC)^{1/2} = 10$
Gate accuracy ~ 10^{-9} is achieved despite pulse error of a few percent (in agreement with analytic estimates).

(P. Brooks)

Self Correcting Quantum Memory?

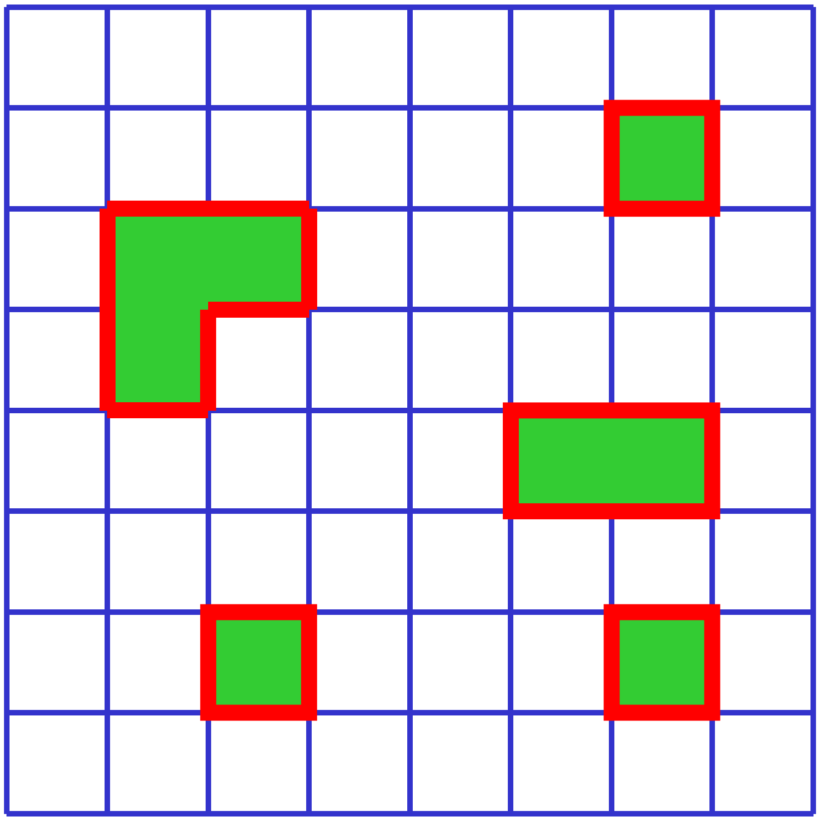
Example: 1D Ising model (repetition code)



When a connected (one-dimensional) droplet of flipped spins arises due to a thermal fluctuation, only the (zero-dimensional) boundary of the droplet contributes to the energy; thus the energy cost is independent of the size of the droplet.

Therefore, thermal fluctuations disorder the spins at any nonzero termperature. A one-dimensional ferromagnet is not a robust (classical) memory.

2D Ising model (repetition code)



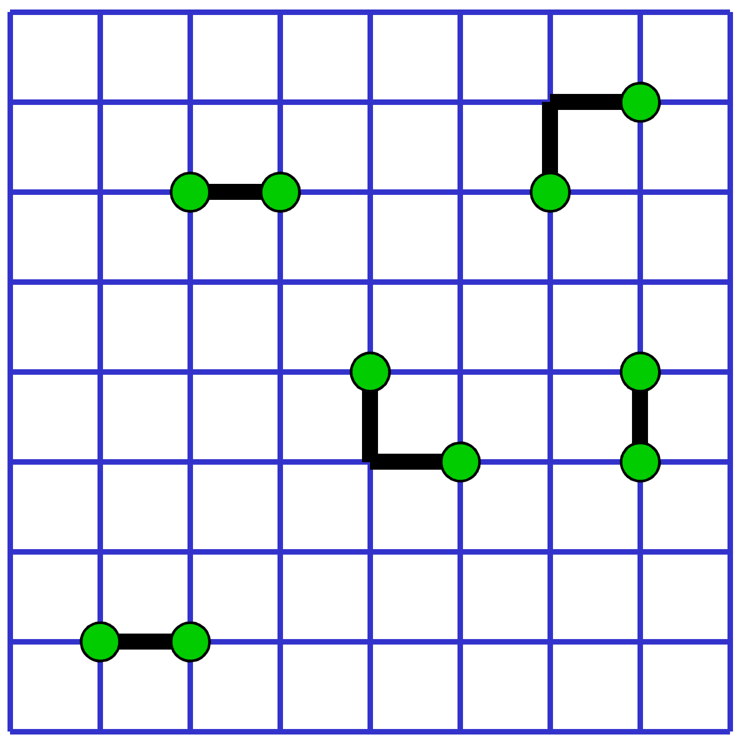
This memory is a repetition code, but with redundant (hence robust) parity checks.

Again, droplets of flipped spins arise as thermal fluctuations. But now the energy cost of a (two-dimensional) droplet is proportional to the length of its (one-dimensional) boundary.

Therefore, droplets with linear size L are suppressed at sufficiently low nonzero temperature T by the Boltzmann factor $\exp(-L / T)$, and are rare.

The storage time for classical information becomes exponentially long when the block size is large. (Actual storage media, which are robust at room temperature, rely on this physical principle.)

Topological Code

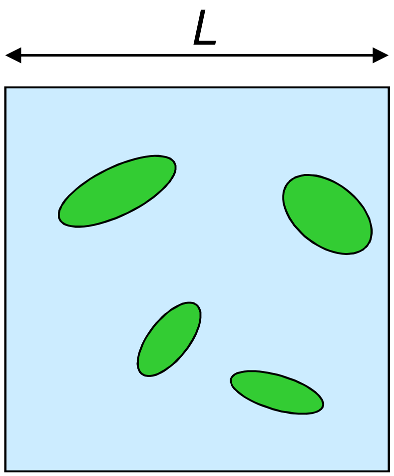


A topological medium in 2D is similar to the 1D Ising model: pairs of anyons are produced by thermal fluctuations at a rate that does not depend on the system size. These anyons can then diffuse apart without any additional energy cost. When anyons diffuse a distance comparable to the distance between pairs, logical errors arise.

Therefore, thermal fluctuations disorder the system at any nonzero termperature. A two-dimensional topological medium is not a robust quantum memory.

Topological order at finite temperature

Dennis et al. 2002
Alicki et al. 2010



In a 4D topological code, the energy cost of a 2D droplet of flipped qubits is proportional to the length of its 1D boundary.

To cause encoded errors, Droplets of linear size L, which could cause encoded errors, are suppressed at sufficiently low nonzero temperature T by the Boltzman factor $\exp(-L / T)$, and are rare.

Question: Is “finite-temperature topological order” possible in 2D or 3D?

In 2D, if the Hamiltonian is a sum of commuting local terms, there are logical string operators, hence no self correction (Bravyi & Terhal 2009, Poulin et al. 2010, Kay & Colbeck 2008, Haah & Preskill 2010).

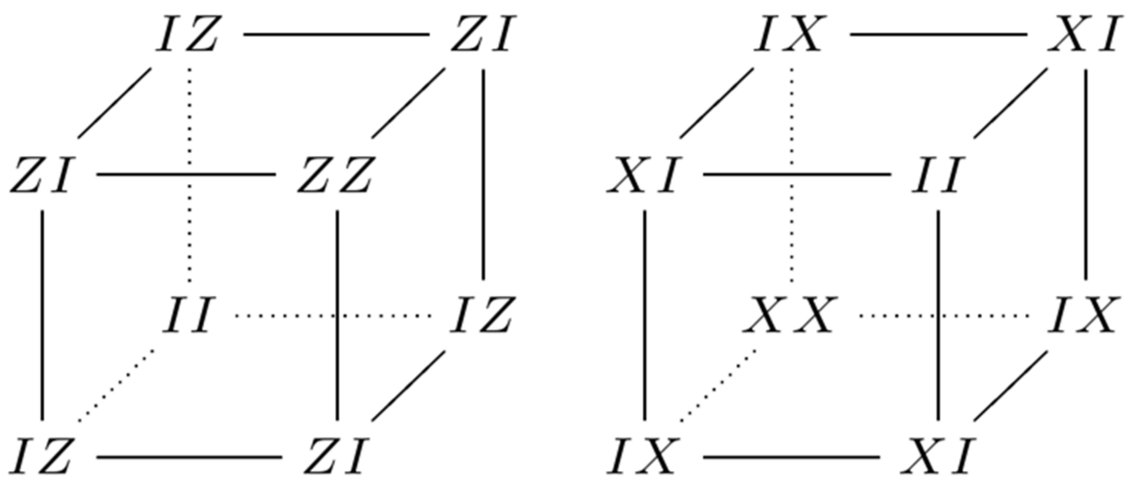
Long-range interactions? (Hamma et al. 2009, Chesi et al 2010.)
Localization? (Wooten & Pachos 2011, Tsomokos et al. 2011.)

In the 3D toric code, we can choose to have *point* defects at the boundary of 1D bit-flip error chains and *string* defects at the boundary of 2D phase-error droplets, or the other way around. (Robust classical memory.)

The same is true for any translationally-invariant local stabilizer code in 3D, provided the degeneracy is independent of system size (Yoshida 2011).

Haah's code

Haah 2011, Haah & Bravyi 2011



A local stabilizer code with two qubits per site on a simple cubic lattice.

Two stabilizer generators on each cube.

No logical string operators.

Code distance grows faster than linearly with linear system size L.

The barrier height for a logical error is O(log L).

Equilibrates slowly when cooled from high to low temperature (glass).

For weak noise, annealing corrects errors with high success probability.

Topology and quantum fault tolerance

Operating a large-scale quantum computer will be a grand scientific and engineering achievement.

Judicious application of the principles of fault-tolerant quantum computing will be the key to making it happen.

Fascinating connections with statistical physics, quantum many-body theory, device physics, and decoherence make the study of quantum fault tolerance highly rewarding.

Topological principles suggest new constructions for quantum error-correcting codes and new ways to realize robust quantum hardware.

We are making progress (in both experiment and theory) toward showing that fault-tolerant quantum computing can work effectively against realistic noise.

But we've got a long way to go.

Happy
Birthday!