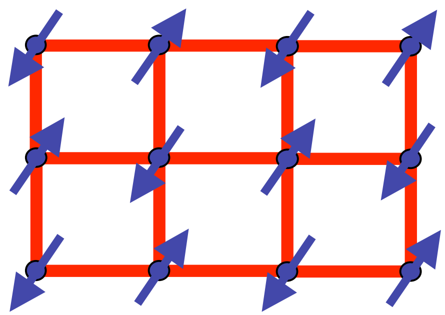
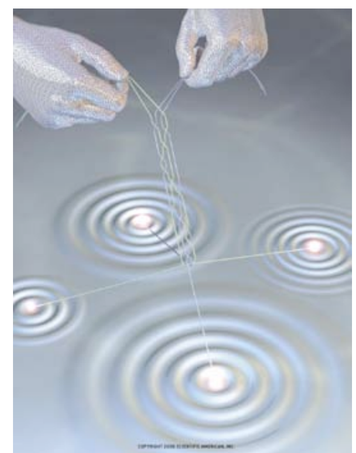
Quantum Crystals
Quantum Choreography
and Quantum Computing

Chalk Talk - KITP 4/12/2006

MPA Fisher





Crystals

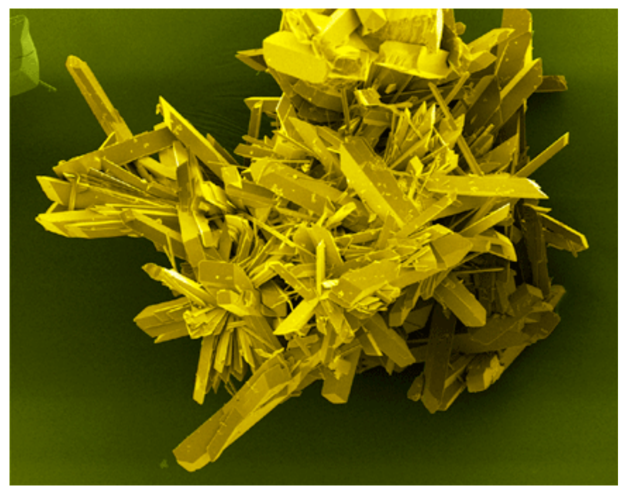
Quartz - SiO_2

Amethyst (purple)
Quartz (clear)
Citrine (yellow)

- In almost all rocks
- Principle constituent of glass
- Key component of computer chips
- Gem stones

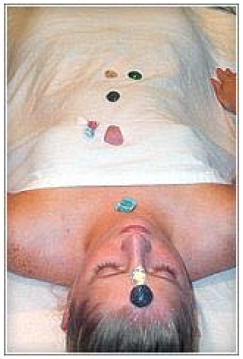
Crystals come in many shades

And in many shapes

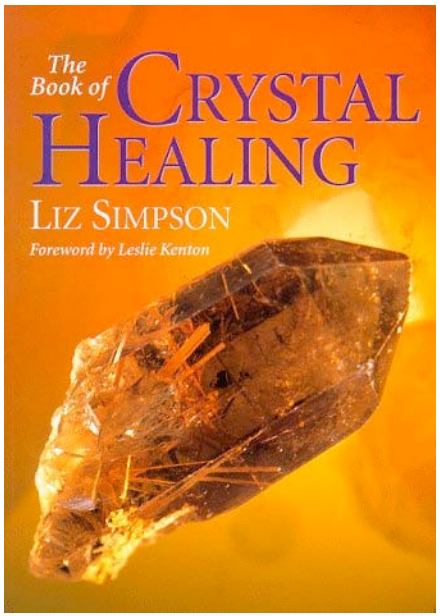


And with many “uses”:

Age old vanity, to new age voodoo



Crystals "enter" academia …



Advanced Crystal Healing

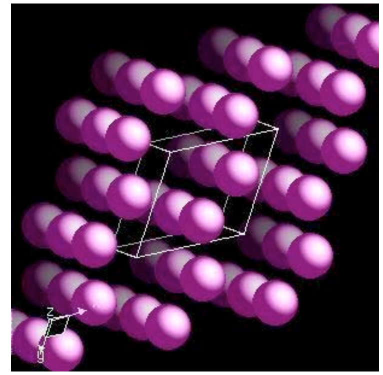
This is to certify that

Lyn E. Ayre

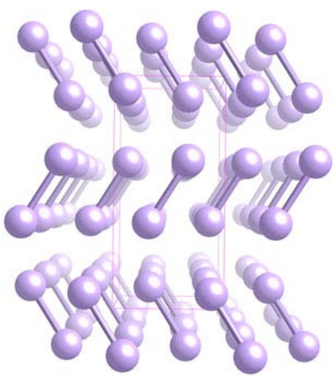
has successfully completed level 1 in Crystal Healing
on
August 31st, 2003

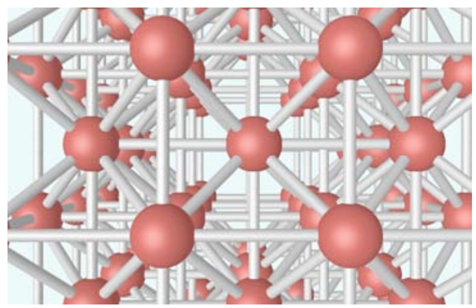
Monica Kiszel
Certified Crystal Healer, G.G., Herbalist, Reiki Level 1 & 2

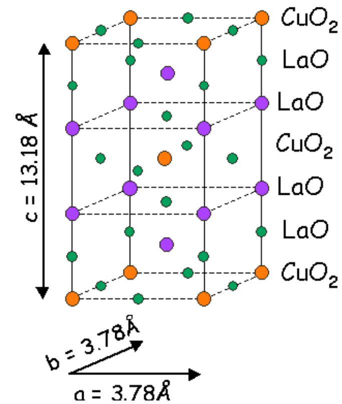
“Look” inside crystals



Periodic array of atoms







Many arrangements

Many different atoms

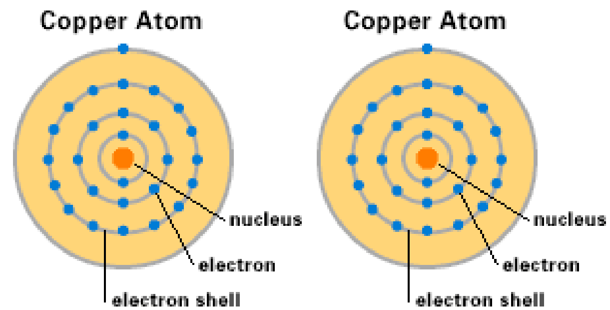
Many, many crystals

"Quantum Crystals"

Why "Quantum"??

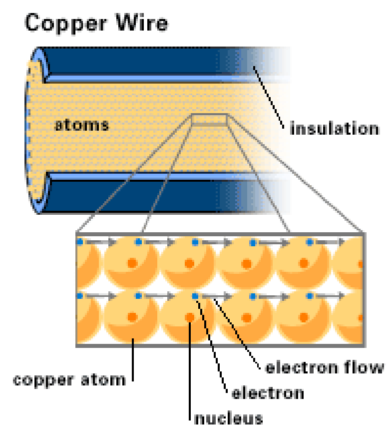
Atoms:

Tiny Nucleus

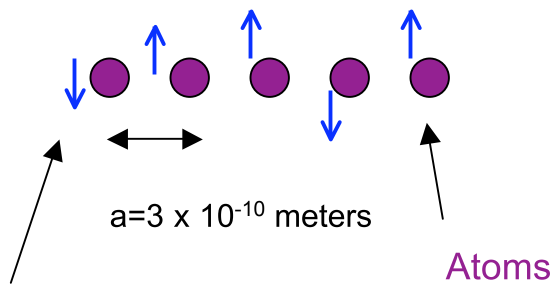


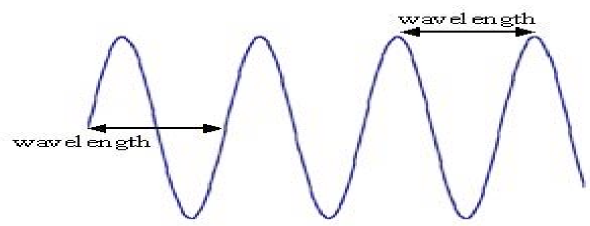
Outer shell electrons can often move from one atom to another

Electrons are so light that their motion thru crystal is always Quantum Mechanical!



Electrons are Particles AND Waves





Electron “particles” (with spin)

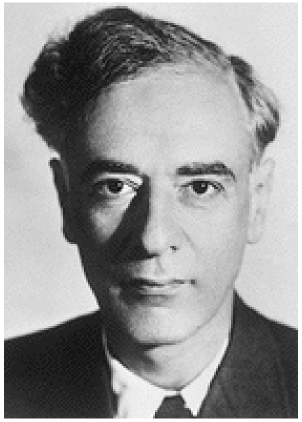
Electron “waves”

Electron wavelength much greater than spacing between atoms
- even at room temperature!

Inside Crystals:
Quantum “wave” mechanics of 10^{23} electron “particles”

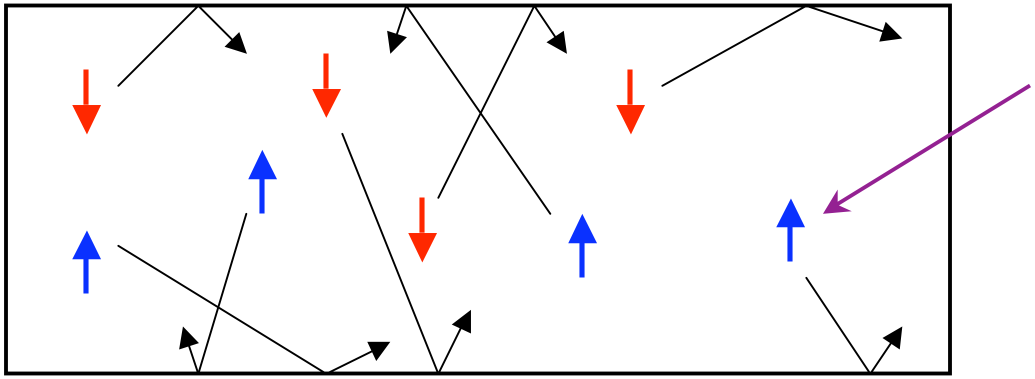
Landau's Theory of Quantum Crystals

- Explains why copper conducts and why quartz does not, and why silicon is a semiconductor and much, much more
- Enabled the computer revolution!



Lev Landau

Landau says - "Electrons do their own thing"

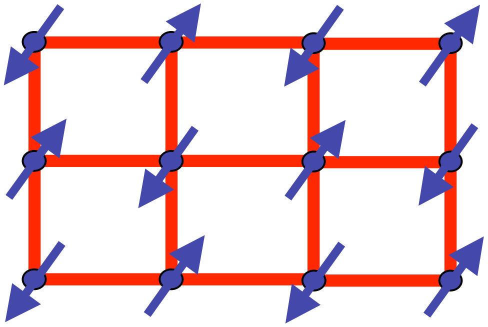


But sometimes they don't…

Many “Complex” Crystals are “bad actors”

Landau - “complex crystals should conduct - like copper.”
But because the electrons are too crowded they do NOT.

Each electron gets stuck



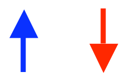
“Mott Insulators”

Sir Neville Mott

Electrons spins can
flop around ….
Quantum mechanically

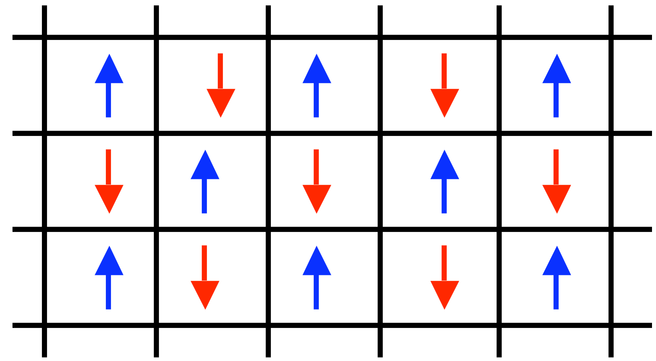
But how???

"Quantum Choreography"



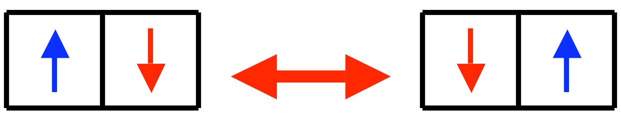
"males and females"

Electrons are -
- "homophobes"
- basically shy

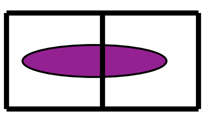


Antiferromagent
(boring)

Electrons like to dance

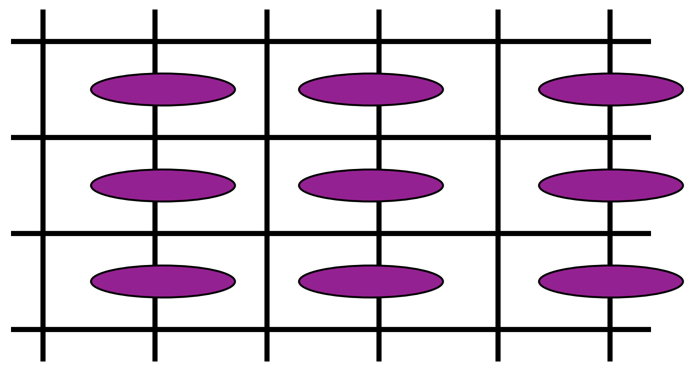


"Quantum Docey Doe"



$$= \frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$$

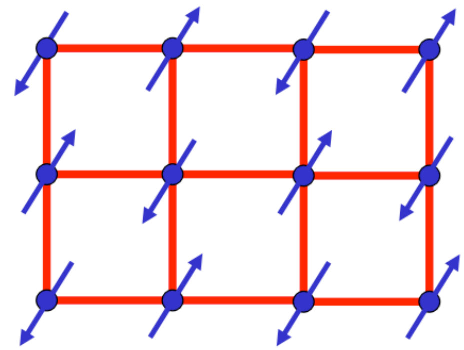
chemical (or "valence") bond



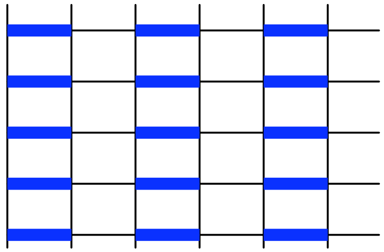
Valence Bond Crystal

"ORDER"

Antiferromagnetic ORDER



Valence Bond Crystal ORDER

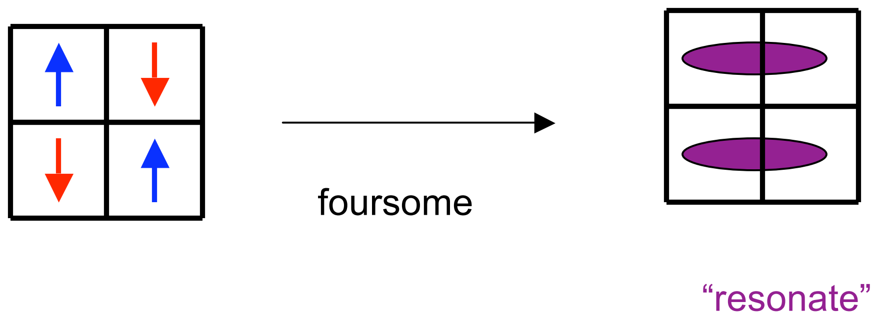


ORDER is Boring!

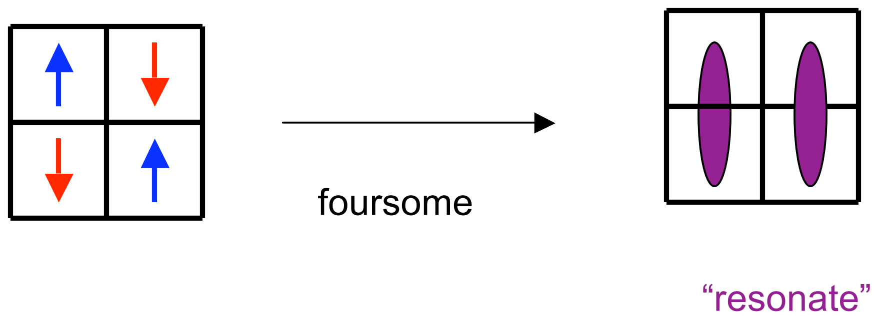

ORDERVILLE


Fisher's
GOLD &
JEWELRY
ROCKS & GEMS

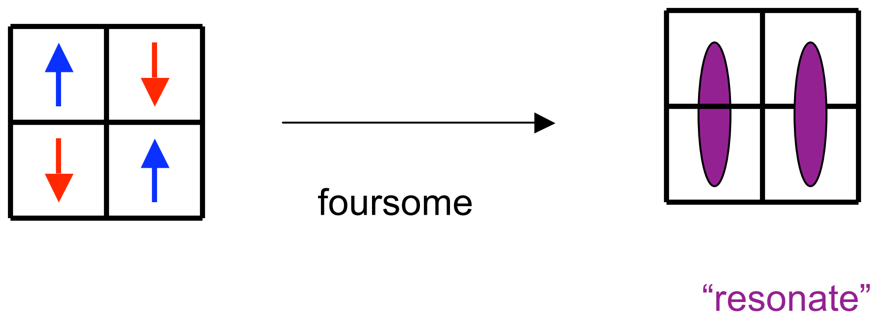
Electrons like to "swing"



Electrons like to “swing”



Electrons like to "swing"

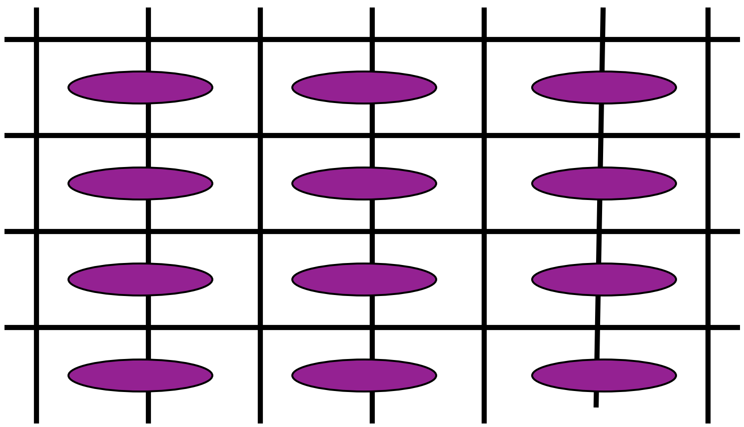


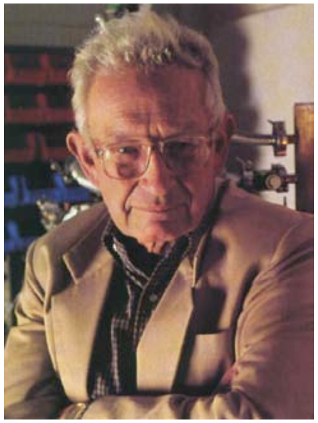
"Quantum Grand Right and Left"

“Quantum Grand Right and Left”

Resonating Valence Bond state (RVB)

PW Anderson





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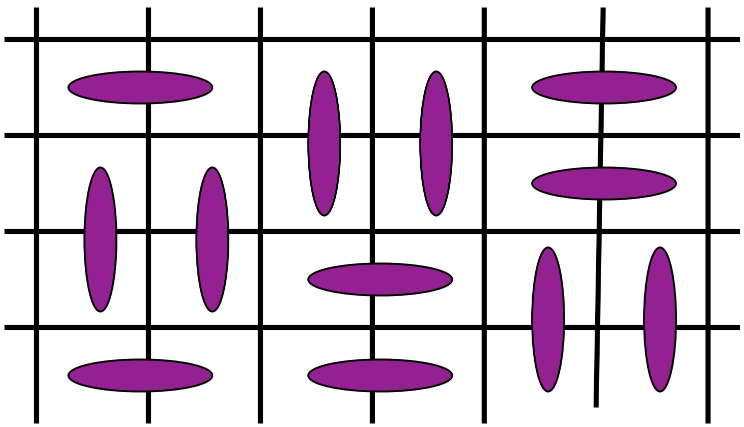

“Quantum Grand Right and Left”

Resonating Valence Bond state (RVB)

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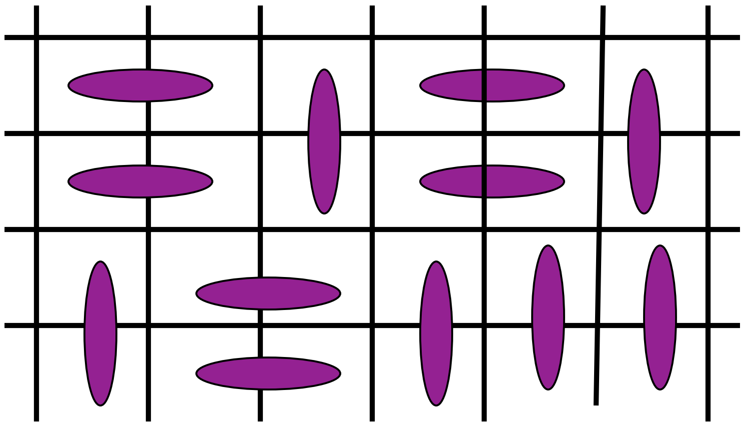
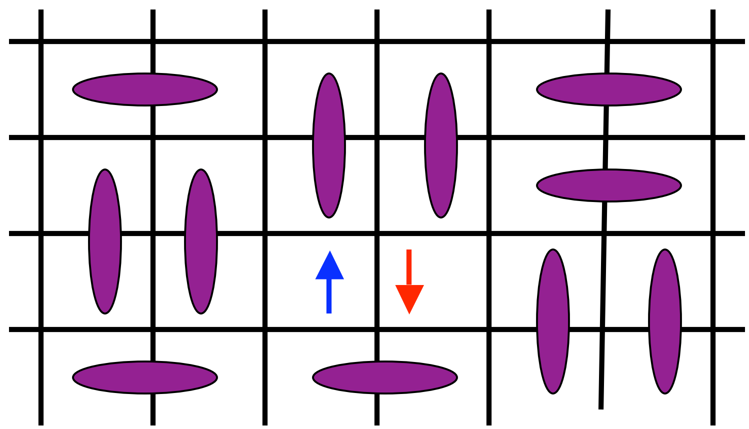
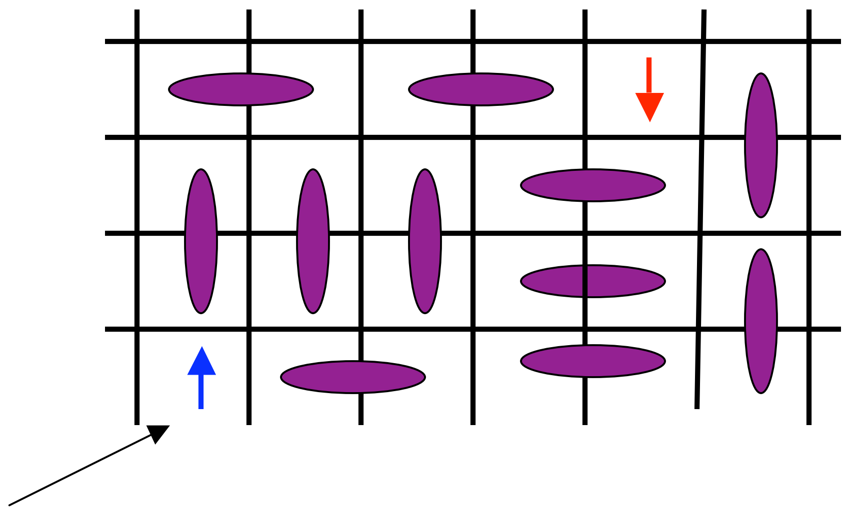


“Spin Liquid”

Breakups - break valence bonds



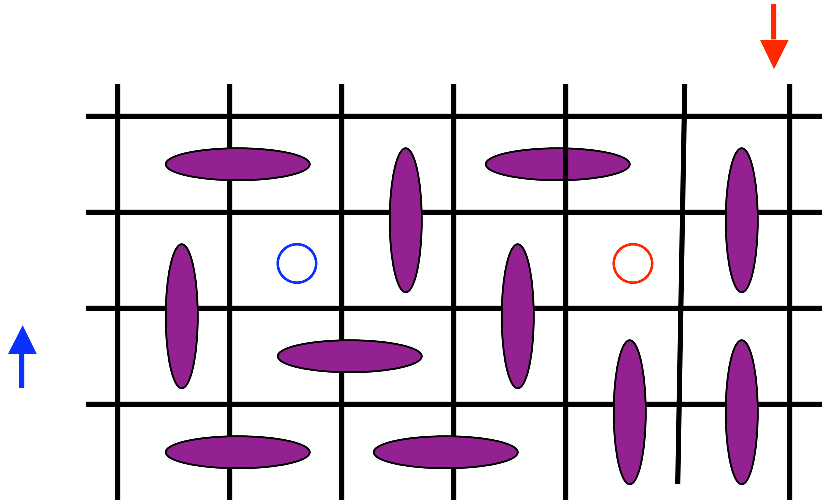
Separation



Loners - "Spinons"   Spin of electron but not charge

Electron "Fractionalization": "Spin-Charge separation"

Dejected and ejected: Dope in holes



“Holons” Charge of electron but not spin

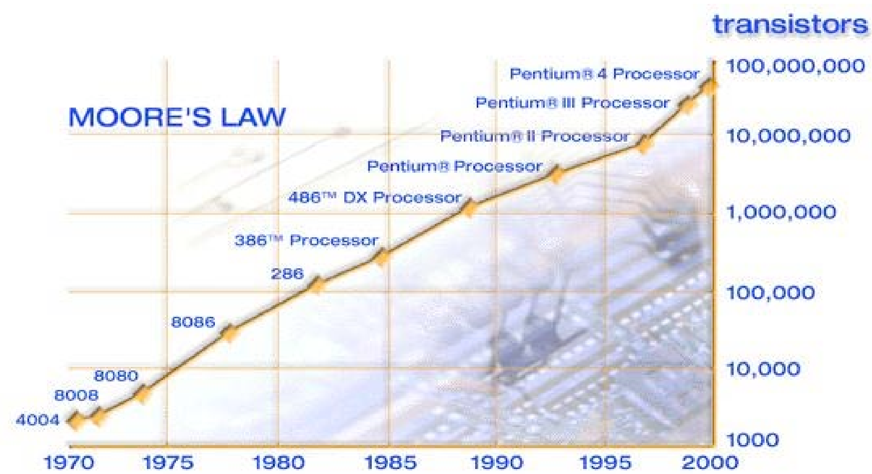
Electron “Fractionalization”: “Spin-Charge separation”

“Quantum Computers”

Can spin liquids be “useful” (make money)?

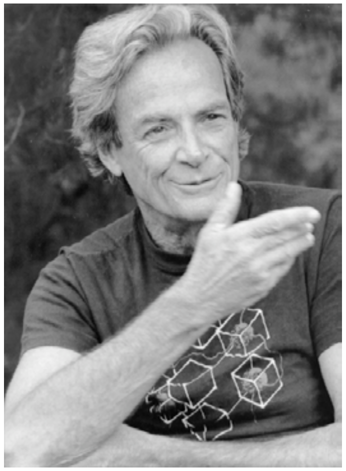
Motivation:

As the components of computers get smaller,
we are approaching the limit in which quantum effects become important.



Is this a problem? … or an opportunity?

A computer which operates "coherently" on a quantum system can be much, much, much more powerful than ANY imaginable conventional computer



Richard Feynamn (1981)

Massive quantum parallelism: Computing simultaneously in many "parallel" universes, interfering and measuring in one

Upside of Quantum Computing - Power

Prime Factorization: Most secure encryption method

World record: 200 digit number factorized after 170 CPU years on a Pentium (running at a 10^9 cycles/sec - 10^{19} cycles in total)

In 1994 Peter Shor developed an algorithm for prime factorization on a Quantum computer

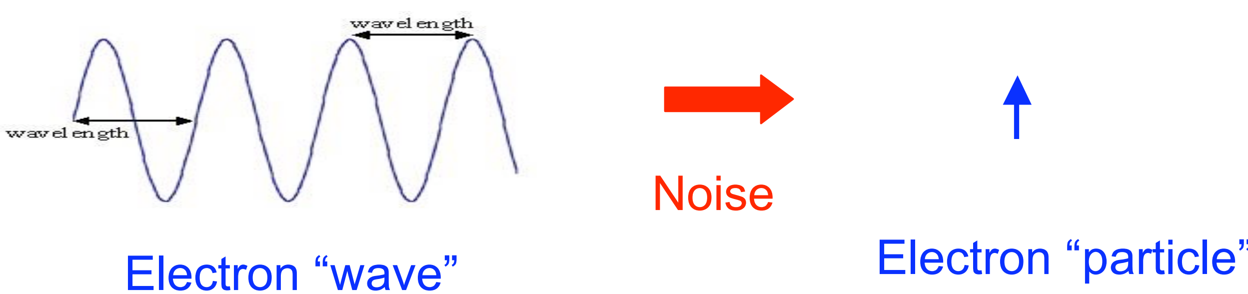
A quantum computer could readily (prime) factorize a 300 digit number!

Peter Shor (1994)

Downside of Quantum Comp. - "Decoherence"

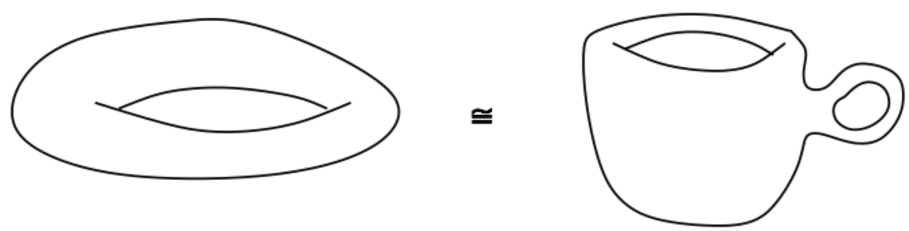
Quantum wave function is very, very "fragile"

Even tiny error (noise) can destroy the delicate quantum superposition (ie. "decohere") ruining the calculation

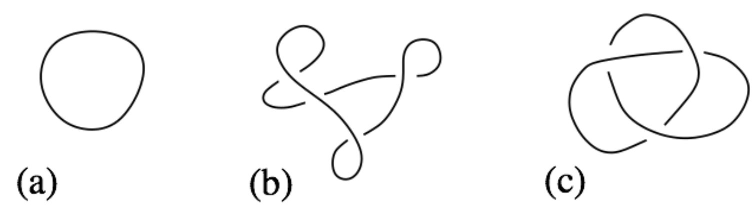


A trick: Exploit Topology

Topology: a branch of mathematics concerned with those properties of geometric configurations which are unaltered by local deformations.



Knots and Braiding



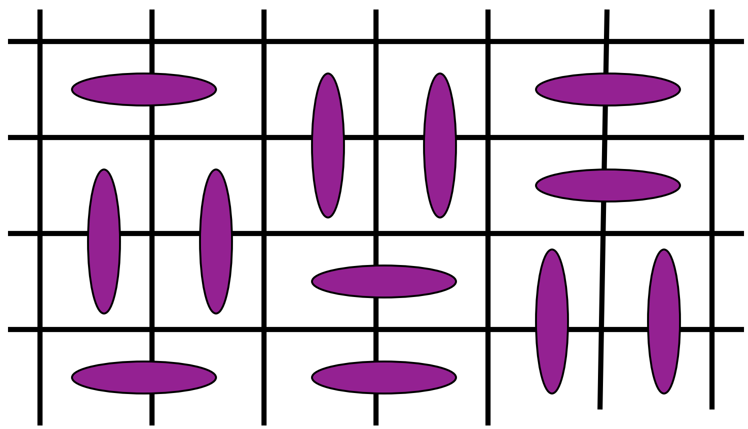
The first two loops can be deformed into each other, but the third cannot. It is a non-trivial *knot*.

Idea: Use "Quantum" Knots!

"A quantum system having particles with "topological" character would be automatically protected against errors caused by local disturbances"

Alexei Kitaev (1995)

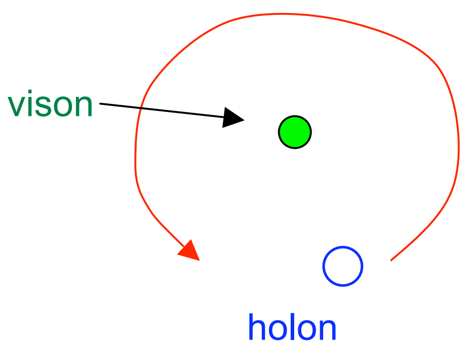
Resonating Valence Bond:
Simplest quantum state
with "topological" particles!



“Topological” Particles in RVB State

- Spinons and holons (with “electric charge”)
- “Visons” (with “magnetic charge”)

Quantum Braid



Braid holon
around vison

$$|holon\rangle \rightarrow -|holon\rangle$$

Minus sign

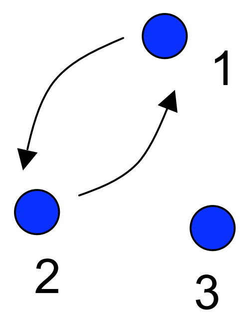
Created a Quantum Knot

More exotic topological states

Quantum states supporting particles with

“Non Abelian particles”

Braid 1 and 2



Multiple braids:

$$(1 \leftrightarrow 2, 2 \leftrightarrow 3) \neq (2 \leftrightarrow 3, 1 \leftrightarrow 2)$$

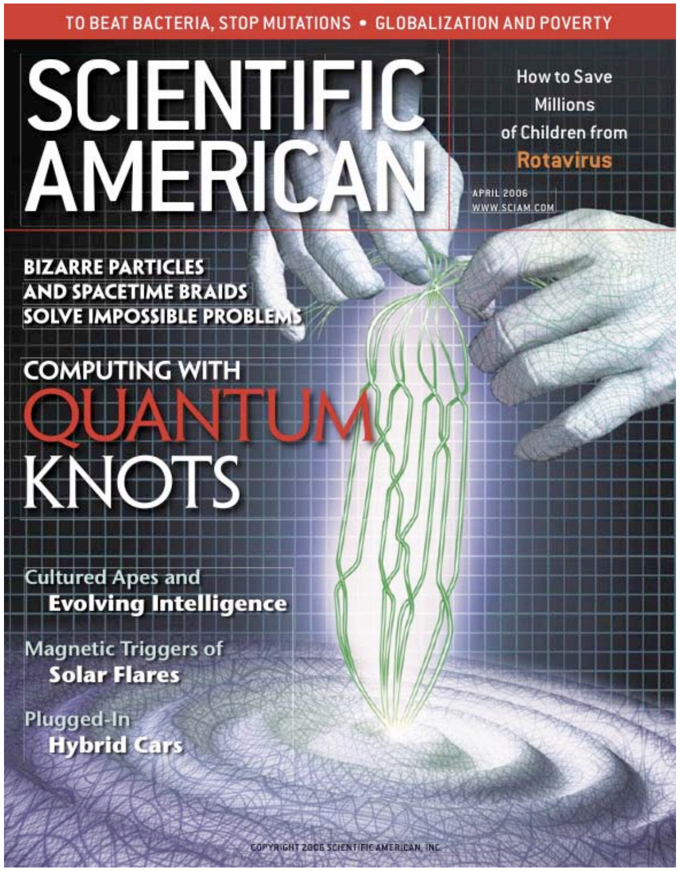
Order of braids matters!

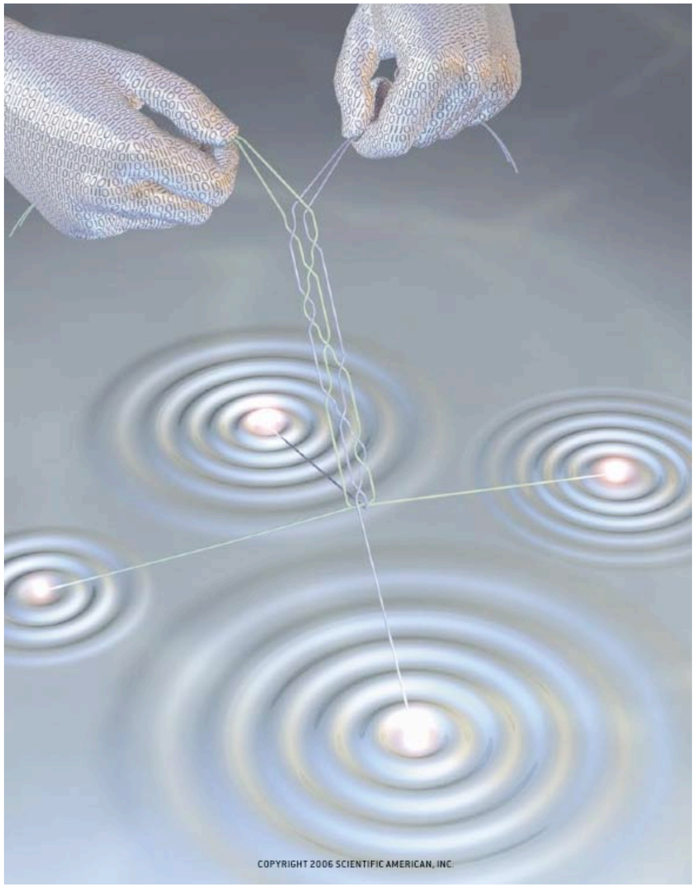
“Topological Quantum Computing”

Bill Gates enters the “hardware” game!

Station Q at UCSB (Michael Freedman)

“Braid” non Abelian particles - decoherence free quantum computing





- Quantum Crystals

  10^{23} electrons “particles” with quantum “wave” motion

- Quantum Choreography

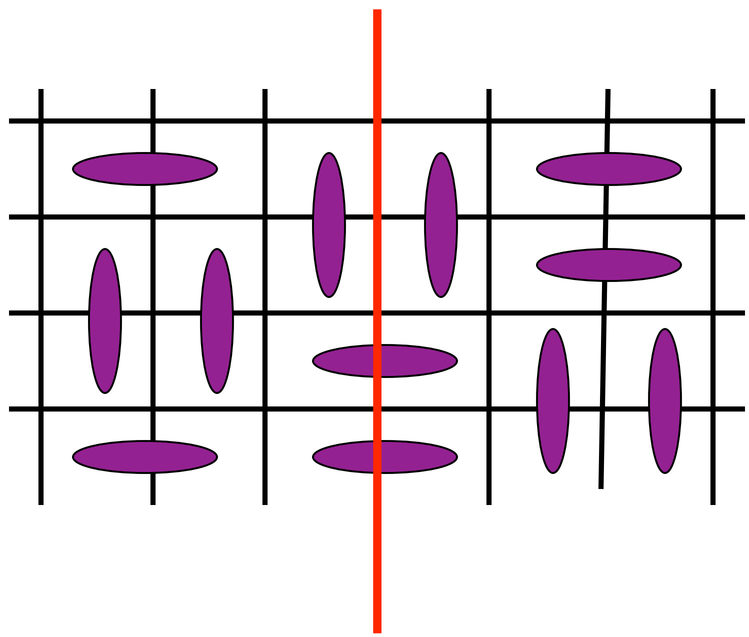
  Attempt to ascertain the fundamental rules underlying the quantum behavior of electrons in crystals

- Quantum Computing (Topological)

  Attempt to use quantum states with “topological” choreography to perform decoherence free quantum computing

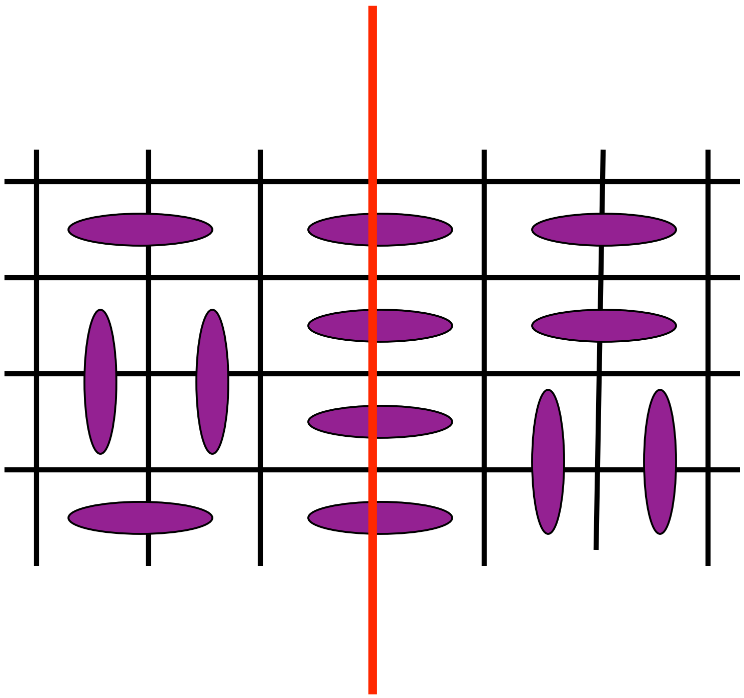
Recall the "cartoon" of the RVB state

Global property unaffected by local dynamics



Red line intersects an even number of bonds

Global property unaffected by local dynamics



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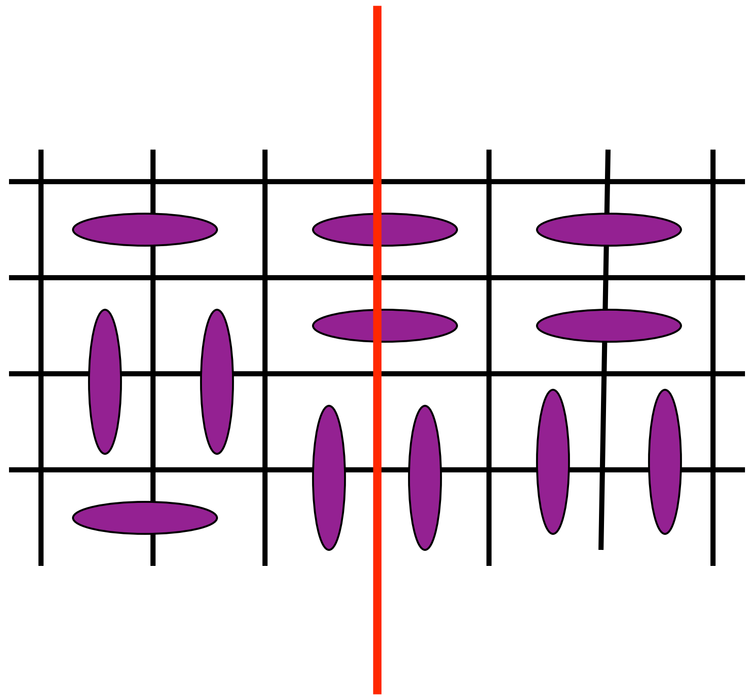
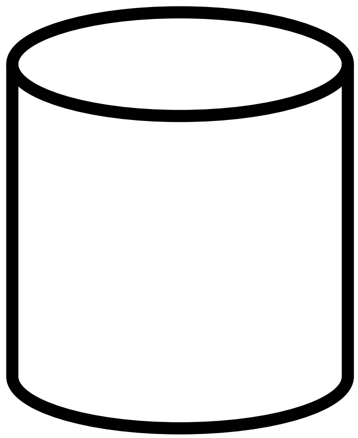


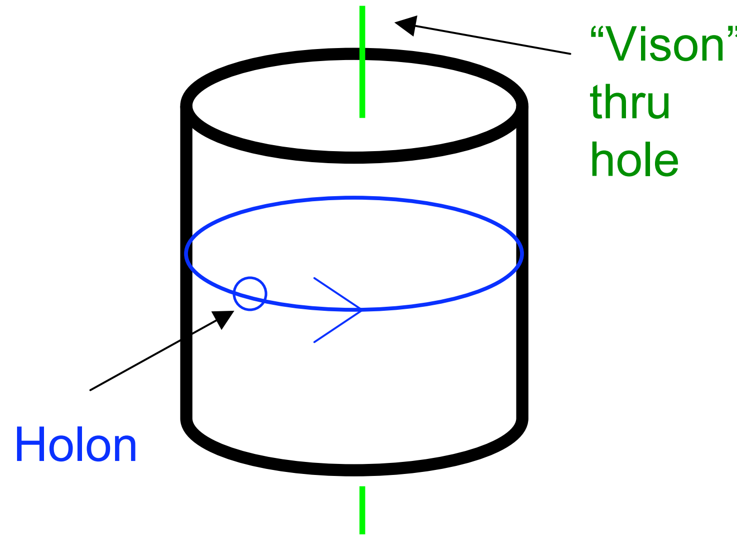
Red line intersects an even number of bonds

Two quantum states: $|even\rangle, \ |odd\rangle$

How is RVB Topological?

Two states on a cylinder, a surface with non-trivial topology





$$|1\rangle = |even\rangle + |odd\rangle$$

$$|2\rangle = |even\rangle - |odd\rangle$$

Take holon around cylinder - wave function changes sign

$$|2\rangle \rightarrow -|2\rangle$$