

# The Million-Body Problem

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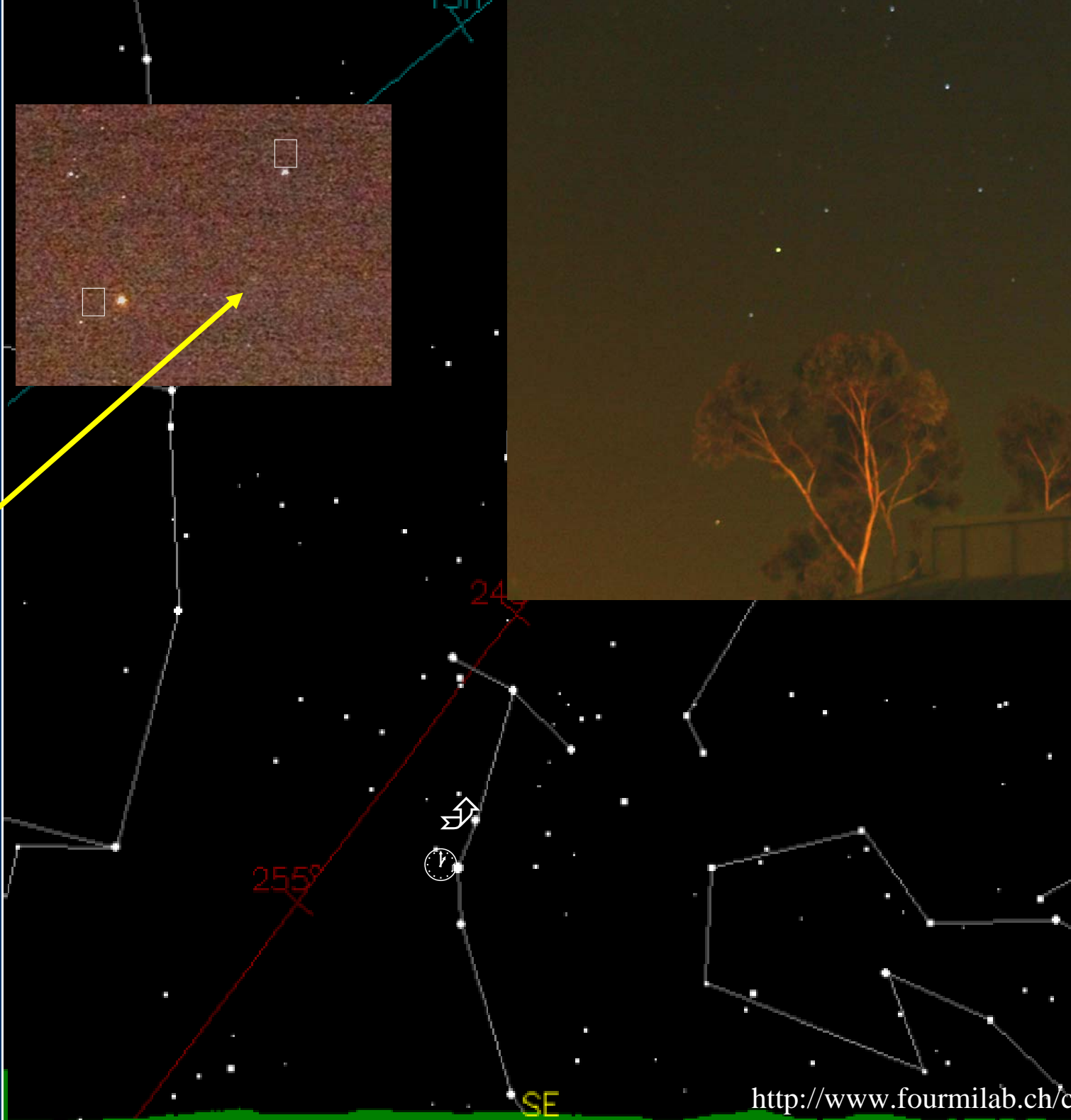


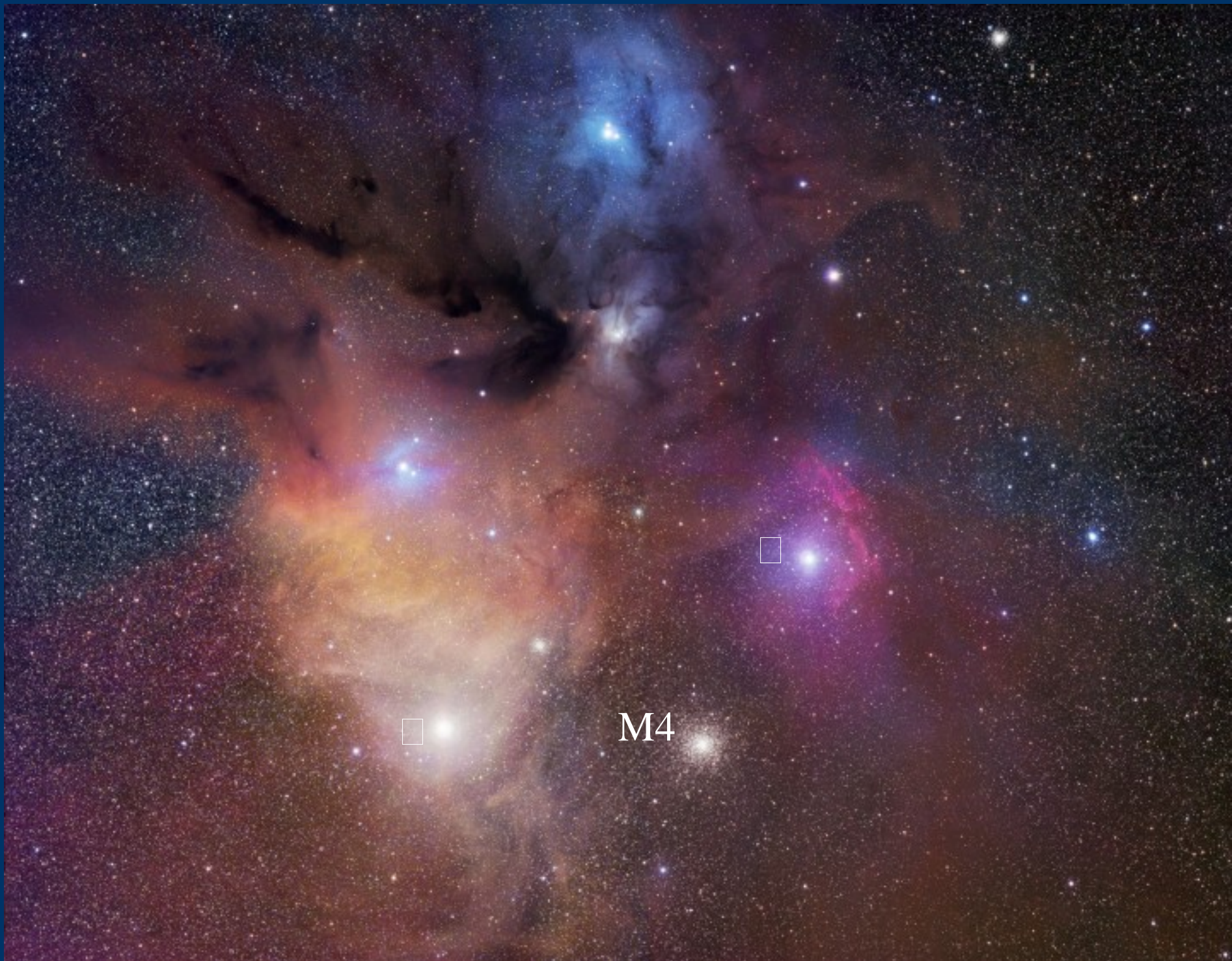
Santa Barbara +34

Edinburgh +56

KITP  
Chalk  
Talk

M4





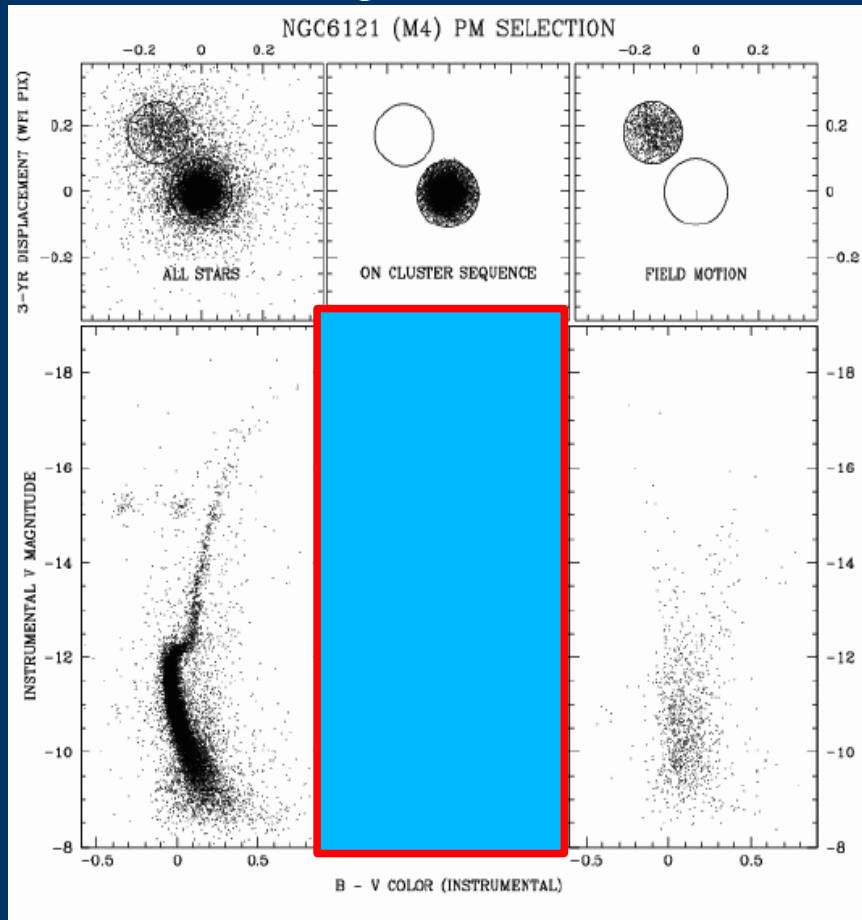
## Part 1: Why Globular Star Clusters Matter

The globular  
star cluster M4



# The Colour-Magnitude Diagram

## Removing the non-members



Anderson et al, 2006, A&A, 454, 1029

## Determining the age from the theory of stellar evolution

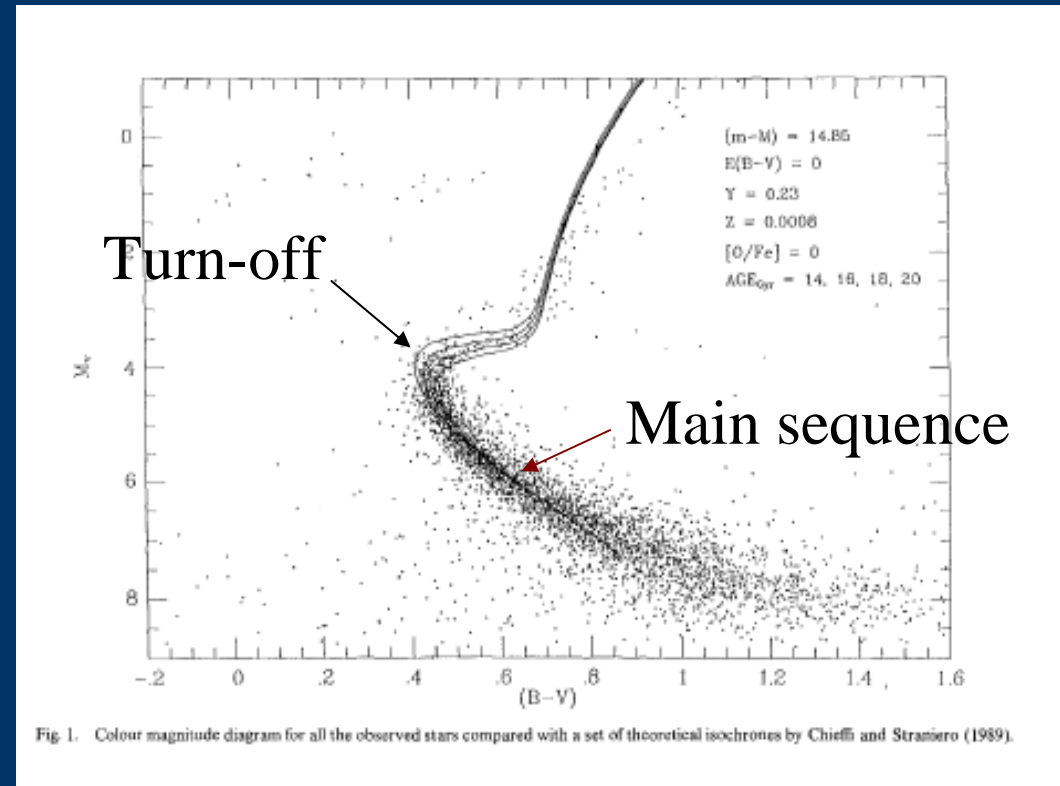


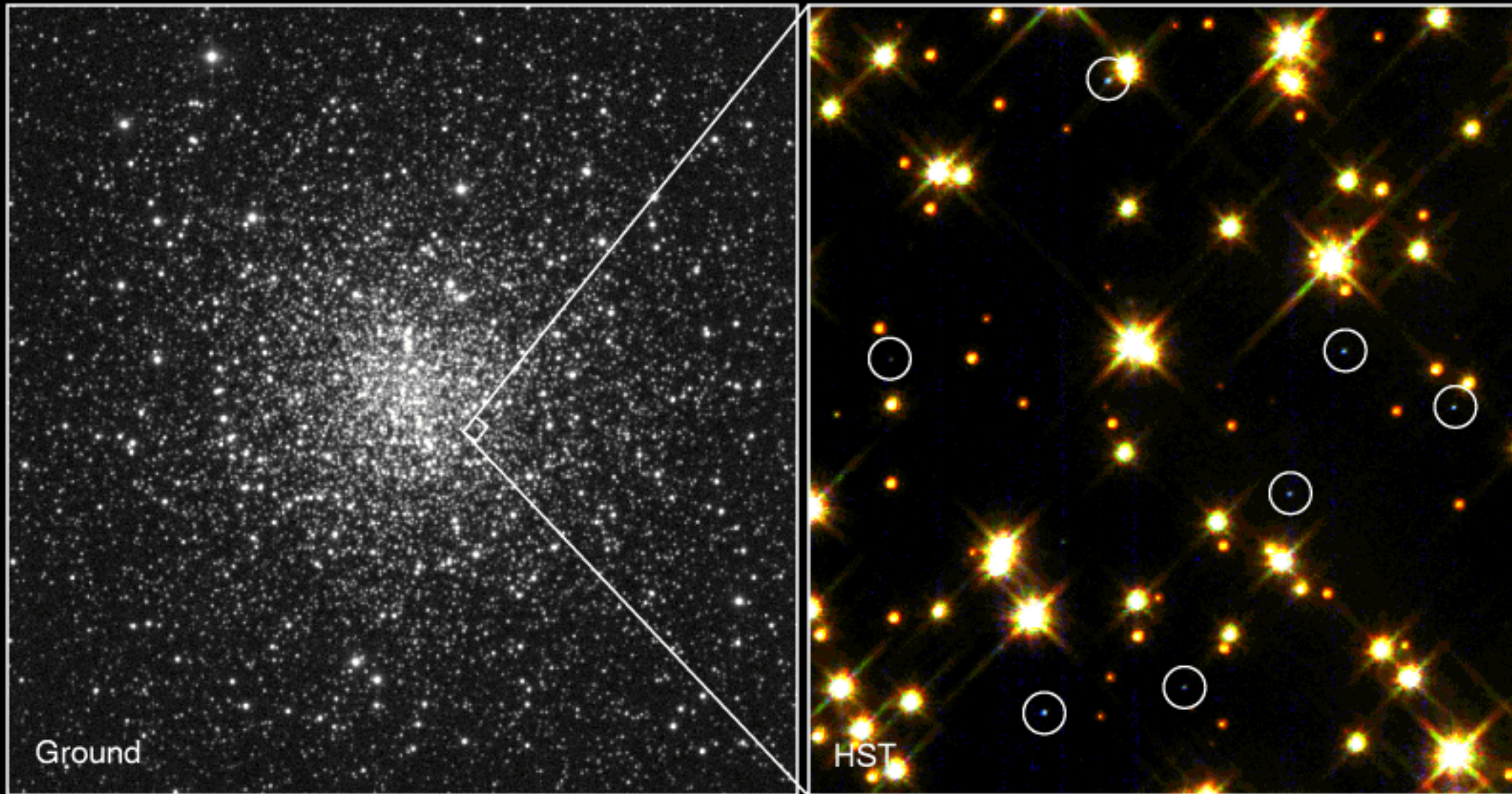
Fig. 1. Colour magnitude diagram for all the observed stars compared with a set of theoretical isochrones by Chieffi and Straniero (1989).

Paez et al, ApSpS,169,1990

The clusters are important for

- Stellar evolution
- Cosmology

White dwarfs:  
*the end-point of the evolution of low-mass stars*



**White Dwarf Stars in M4**

**HST · WFPC2**



# Blue stragglers in NGC 6397

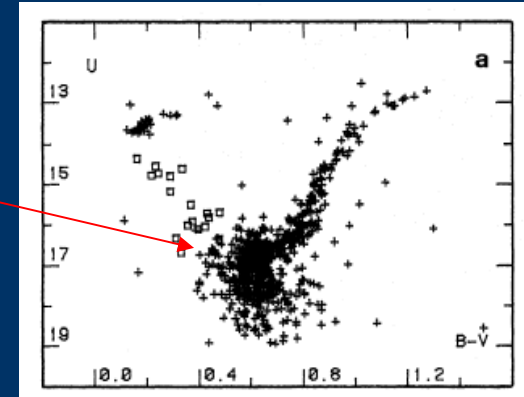
*young-looking massive stars formed by stellar collisions*

Globular Cluster NGC 6397



Hubble  
Heritage

Turn-  
off

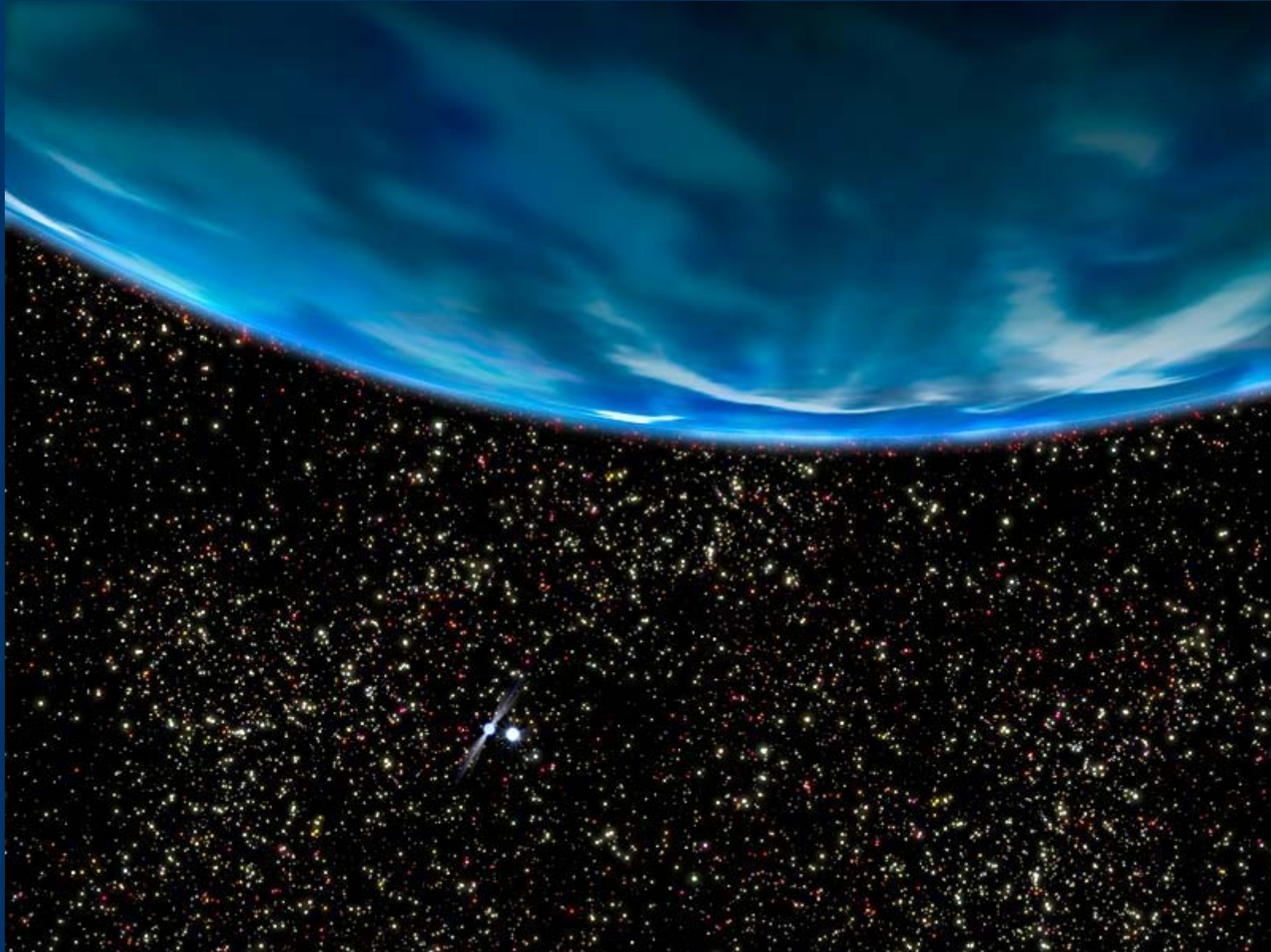


Lauzeral, 1992, A&A, 262, 63L

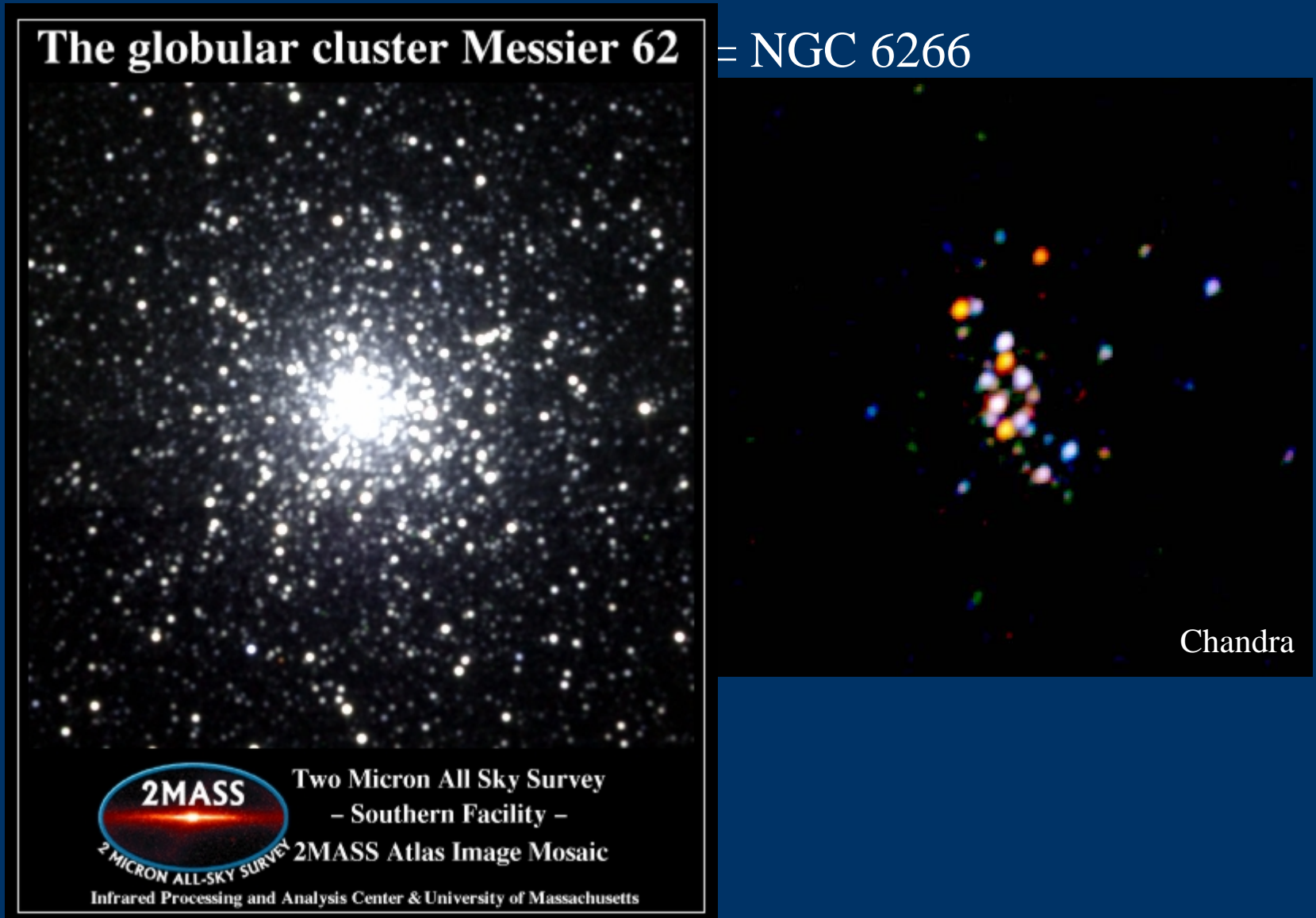
Movie credit: J. Lombardi

<http://webpub.allegheyeny.edu/employee/j/jalombar/movies/>

# The pulsar triple in M4: a planet in a globular cluster

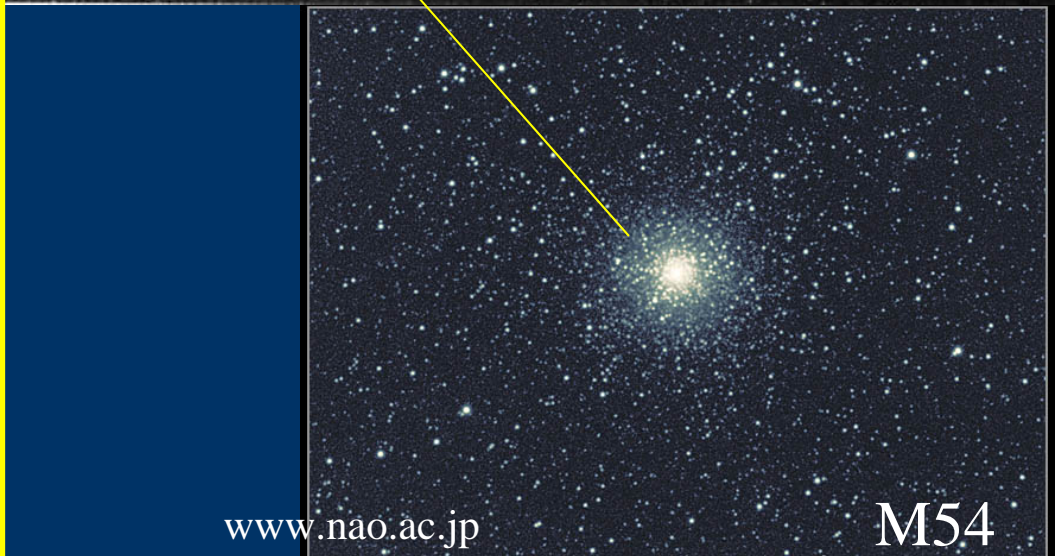
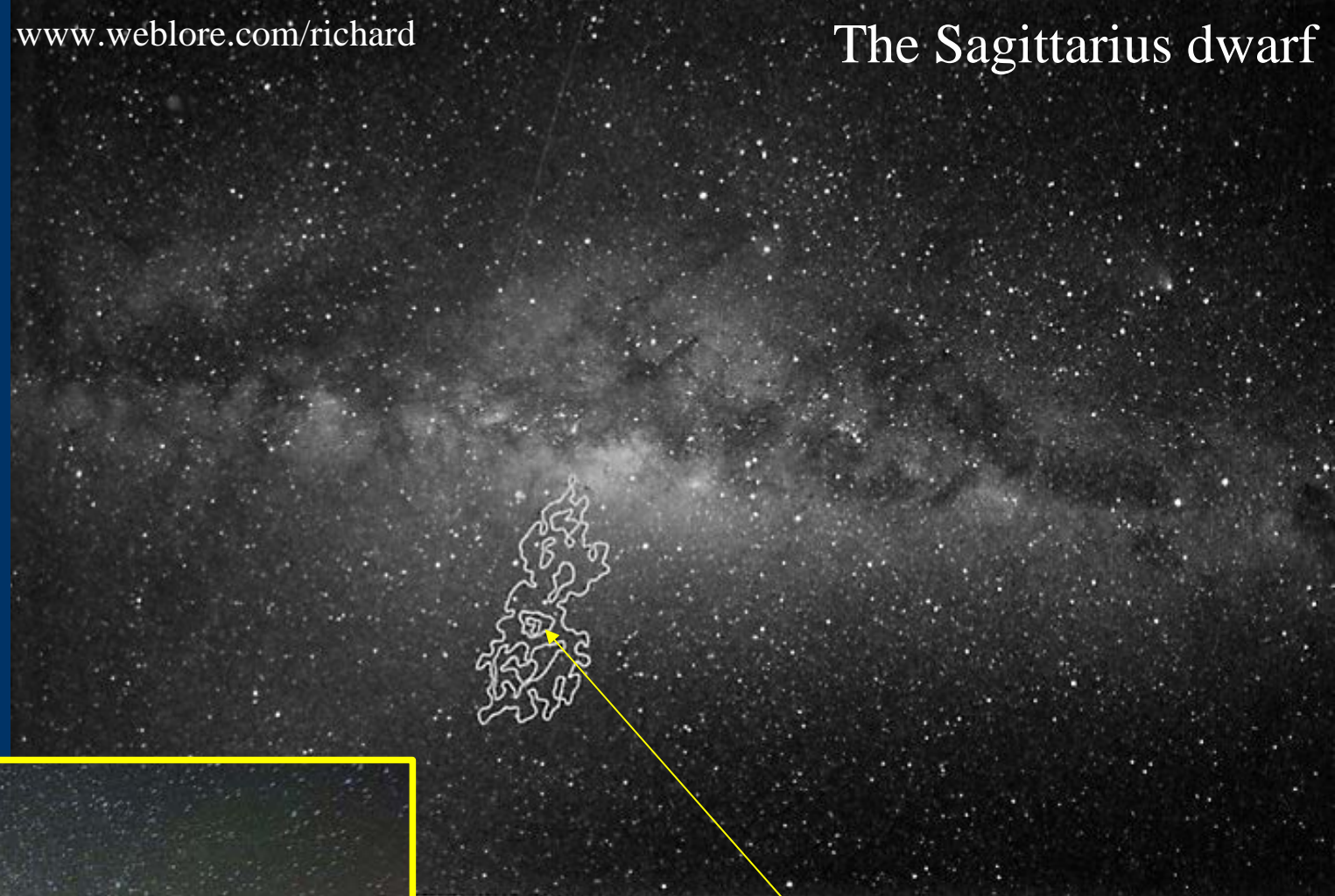


# X-rays from globular clusters



A globular cluster  
arriving with  
another galaxy

[www.areavoices.com/astrobob](http://www.areavoices.com/astrobob)



# The discoverer of the size of the galaxy – Harvard's Harlow Shapley



Three weeks later – Hollywood's shapely Harlow

## Part 1: Why Globular Star Clusters Matter – Summary

Evolution of stars

The age of the universe

Stellar collisions

Pulsars

Planets

X-ray sources

The formation of the Galaxy

The formation of stars

The shape of the Galaxy....

Part 2:

....Stellar dynamics –

the motions of stars under the forces acting between them

## *The Classical Gravitational N-Body Problem*

N point-masses (stars)

Inverse-square law gravity (long-range)

Newton's equations of motion

“

the “star

gas”

*An analogy – ideal gases*

N point masses (atoms, molecules)

Electrostatic interactions (“collisions”)

Newton's equations of motion

## Dynamic equilibrium – ideal gases

Example: ideal atmosphere

Rapid motion of atoms/molecules (few hundred m/s)

Gas at rest (on a good day)

Temperature gradient

## Dynamic equilibrium – star gases

Example: a globular cluster

Rapid motion of individual stars (few km/s)

No overall motion (expansion/contraction)

Stars move faster at centre than in the surrounding halo



# Computer simulation of a star cluster, showing dynamic equilibrium

[http://www.maths.ed.ac.uk/~heggie/orbits\\_13.mpg](http://www.maths.ed.ac.uk/~heggie/orbits_13.mpg)

What happens in the long run?

## Thermal energy – ideal gases

Atoms/molecules in motion

$$\text{Kinetic energy per particle} = \frac{1}{2}mv^2$$

Temperature is proportional to average value of  $v^2$

## Thermal energy – star gases

Stars in motion

$$\text{Kinetic energy per particle} = \frac{1}{2}mv^2$$

By analogy, temperature proportional to  $\langle v^2 \rangle$

## Heat conduction – ideal gases

In collisions, fast-moving particles tend to lose kinetic energy, slowly-moving particles tend to gain kinetic energy

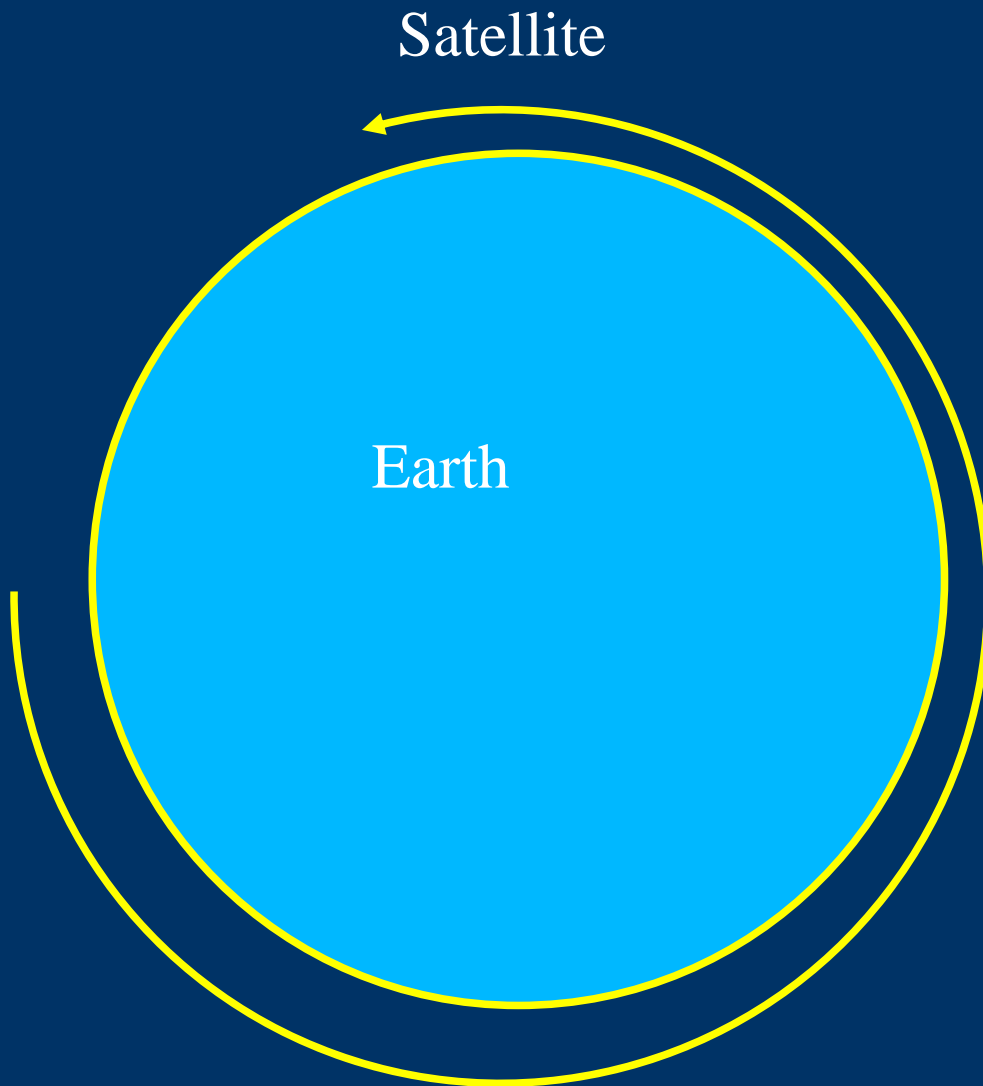
Heat (thermal energy) is conducted from hot regions to cold

## Heat conduction – star gases

In two-body encounters, fast-moving stars tend to lose kinetic energy, slowly-moving stars tend to gain it

Heat (kinetic energy) is conducted from hot regions to cold

# The paradoxical behaviour of motion under gravity – an example



As a satellite experiences friction from the Earth's atmosphere, it sinks into a lower orbit, and moves faster. Friction speeds things up.

## Specific heat – ideal gases

Addition of heat increases temperature

## Specific heat – star gases

Addition of heat *decreases* temperature:

Addition of heat (kinetic energy) makes stars move faster

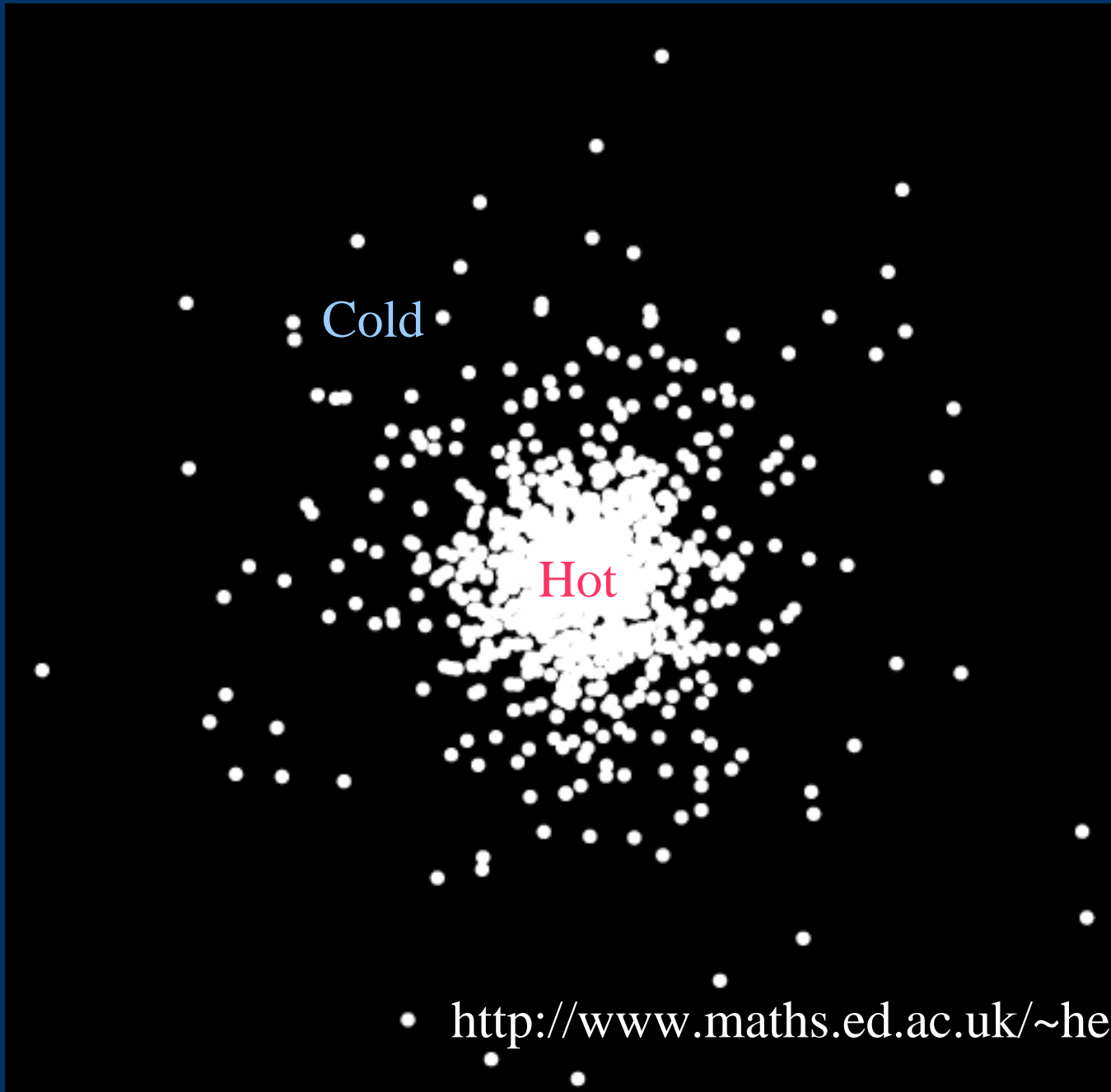
They move out from the cluster centre, losing kinetic energy

In fact they lose more kinetic energy than they gained initially

Star gases have *negative specific heat*:

if you add thermal energy (heat), they cool down.

## The Evolution of Star Clusters



Heat flows from the  
centre to the halo

The centre loses heat and  
gets hotter (and denser)  
the halo gains heat  
and gets cooler

The temperature difference  
has increased; the flow  
of heat increases

This is called  
*“gravothermal  
instability”*, or  
*“core collapse”*

*Movie*

• [http://www.maths.ed.ac.uk/~heggie/plots\\_13.mpg](http://www.maths.ed.ac.uk/~heggie/plots_13.mpg)

## Core collapse

A very slow process – billions of years

Central density becomes infinite (in theory)

The missing ingredient – binary stars  
(the *molecules* of the star gas)

## Interactions between single stars



One star gains energy, the other loses

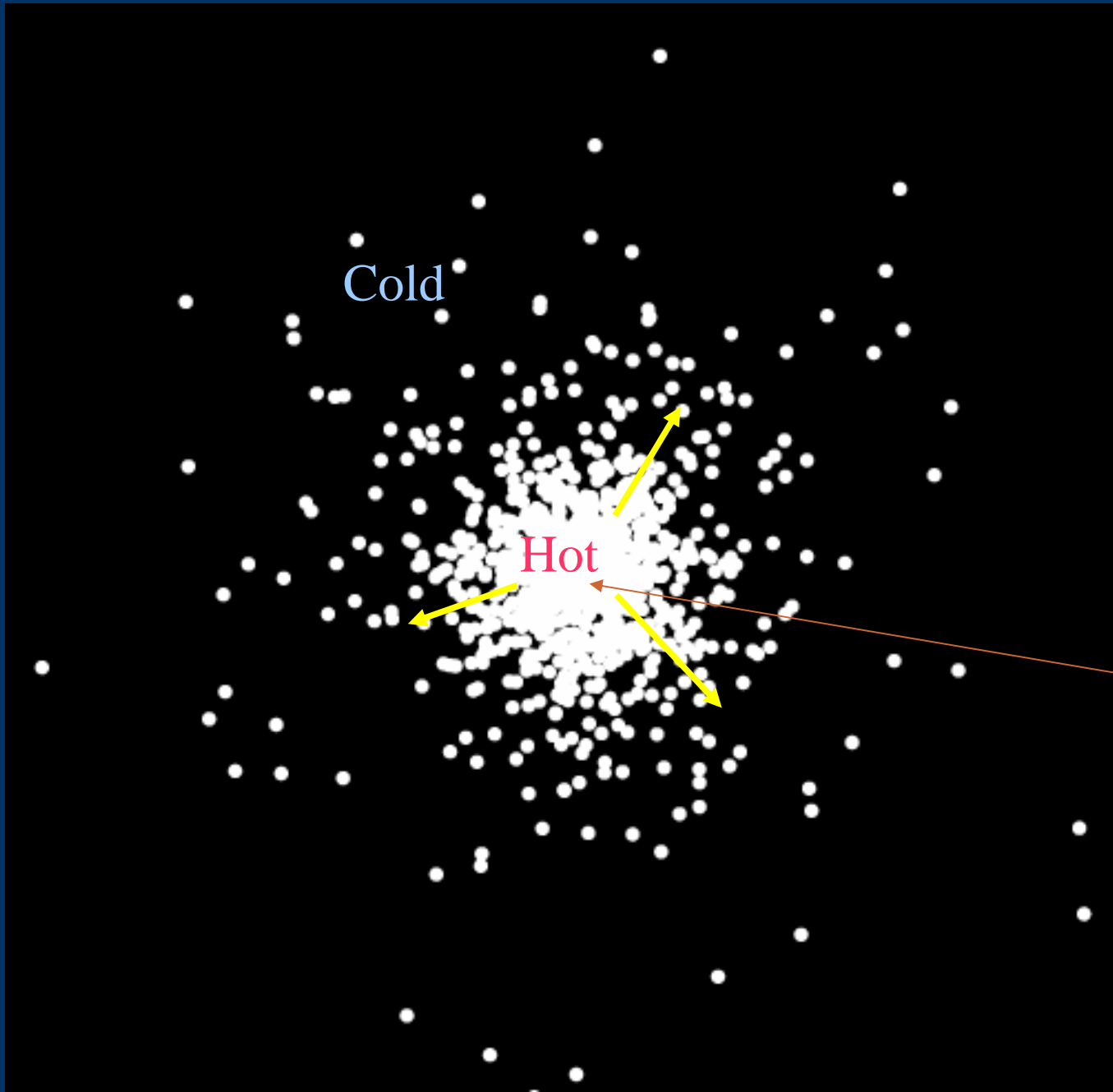


# Interaction with a binary



Both objects gain energy

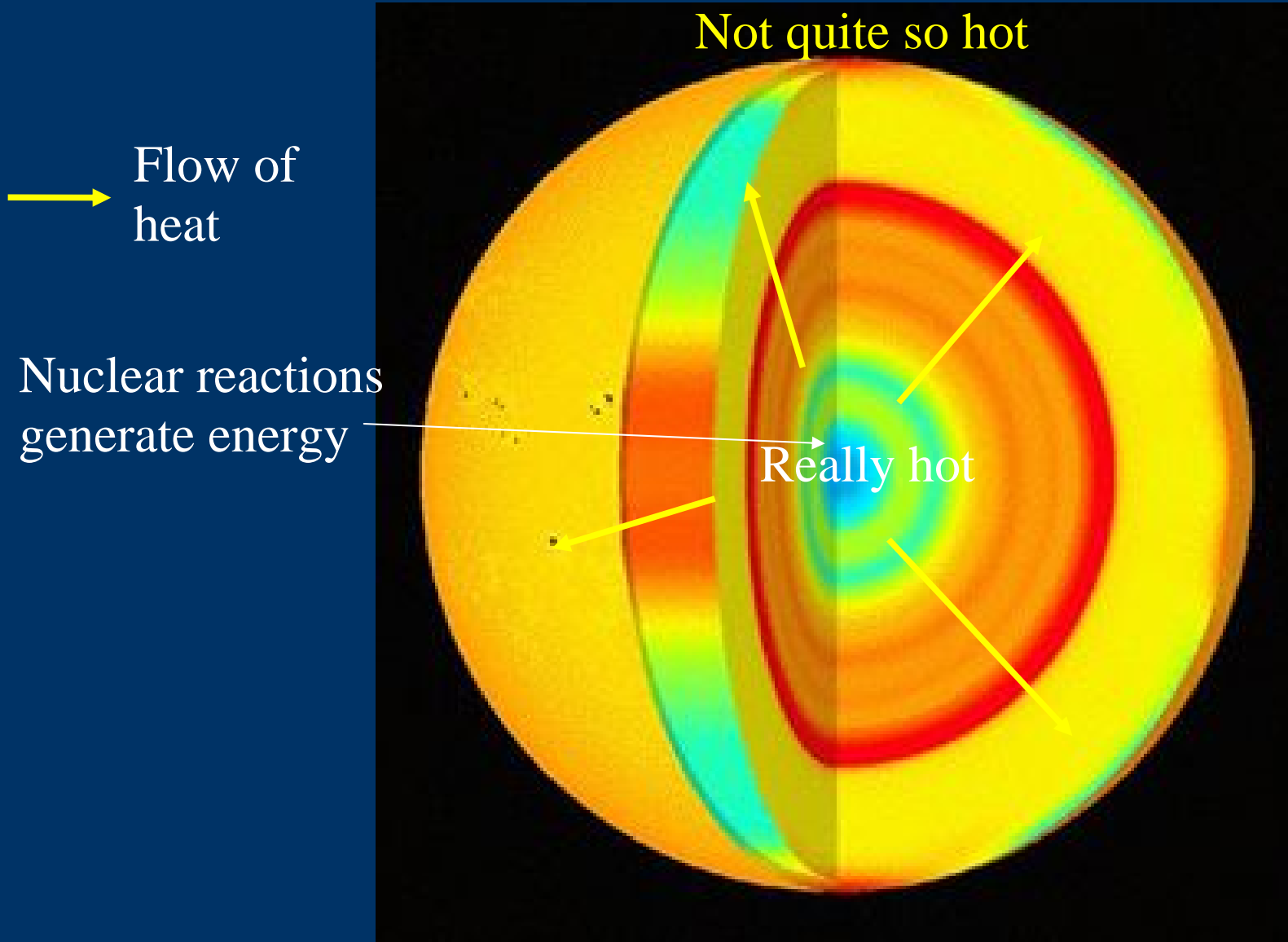
# A post-collapse cluster



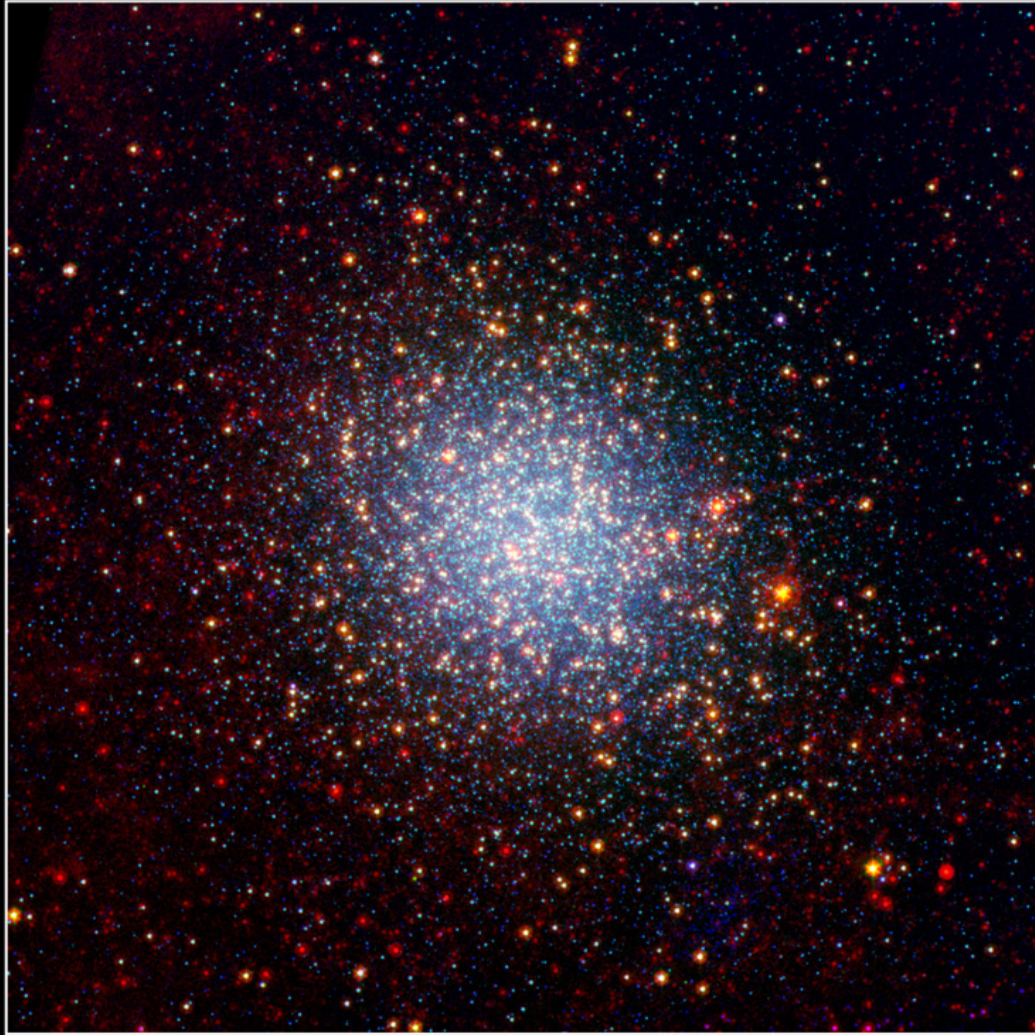
→ Flow of heat

Energy created by binaries in a dense region at the centre

# A star like the sun



# Omega Centauri – a star cluster with an uncollapsed core



Globular Cluster Omega Centauri

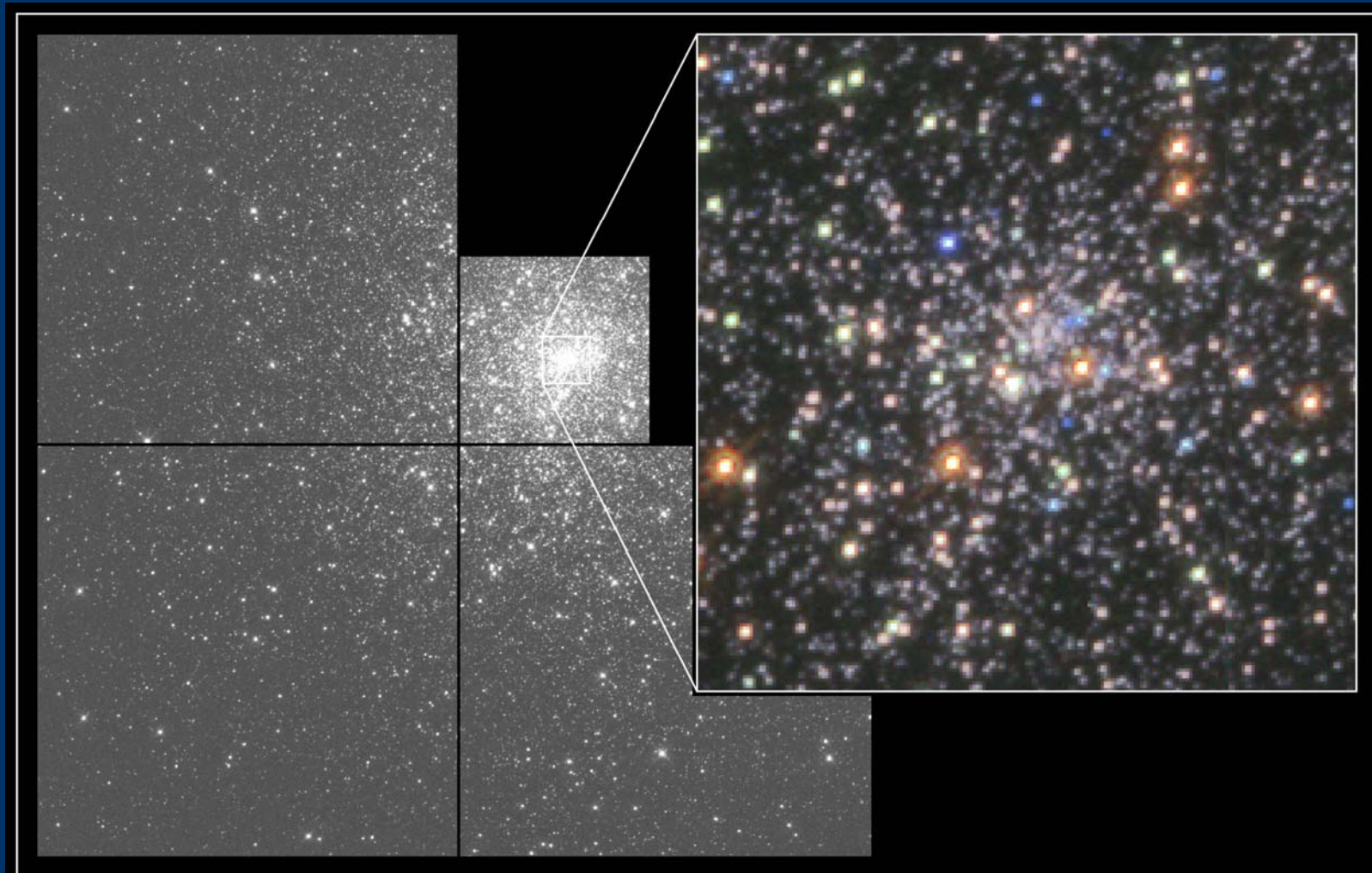
NASA / JPL-Caltech / M. Boyer (Univ. of Minnesota)

Spitzer Space Telescope  
IRAC • MIPS  
ssc2008-07a



HST

# M15 – a star cluster with a collapsed core



## Globular Cluster M15

Hubble Space Telescope • Wide Field Planetary Camera 2



For what could be more beautiful than the heavens, which contain all beautiful things?  
*Copernicus*

