

Image: Alison Martin

All but frustrating: When quantum materials meet topology

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Institute of Theoretical Physics
University of Frankfurt
Germany



KITP

*KITP, Chalk Talk
Tuesday, Sept. 17th 2019*

- **Topology**
- **Quantum Physics**
- **Topology meets Quantum Materials**

Topology

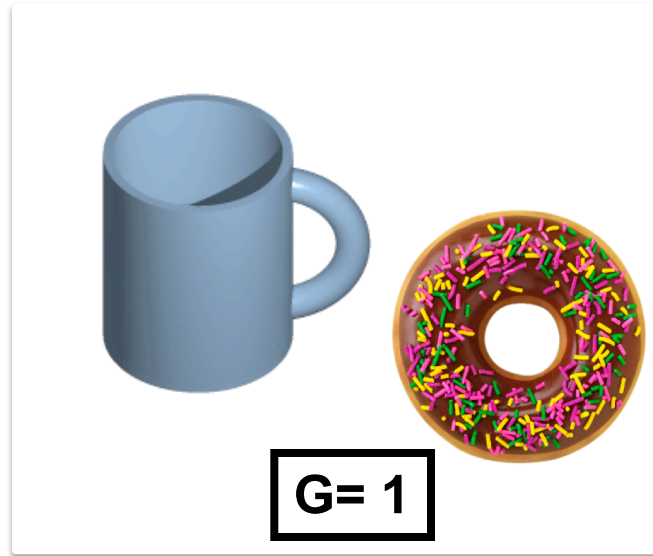
Greek: τόπος „place“ λόγος „study“

Studies the **properties** of a geometric object that are **preserved** under **continuous deformations** (stretching, twisting, crumpling)



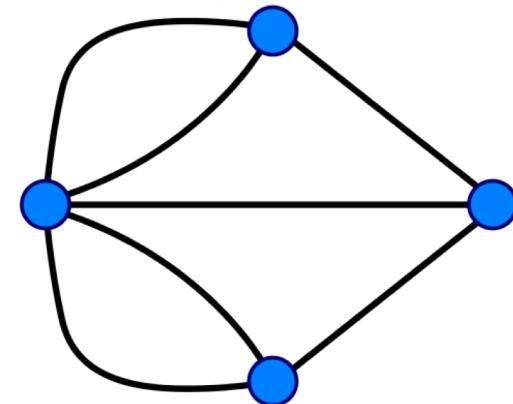
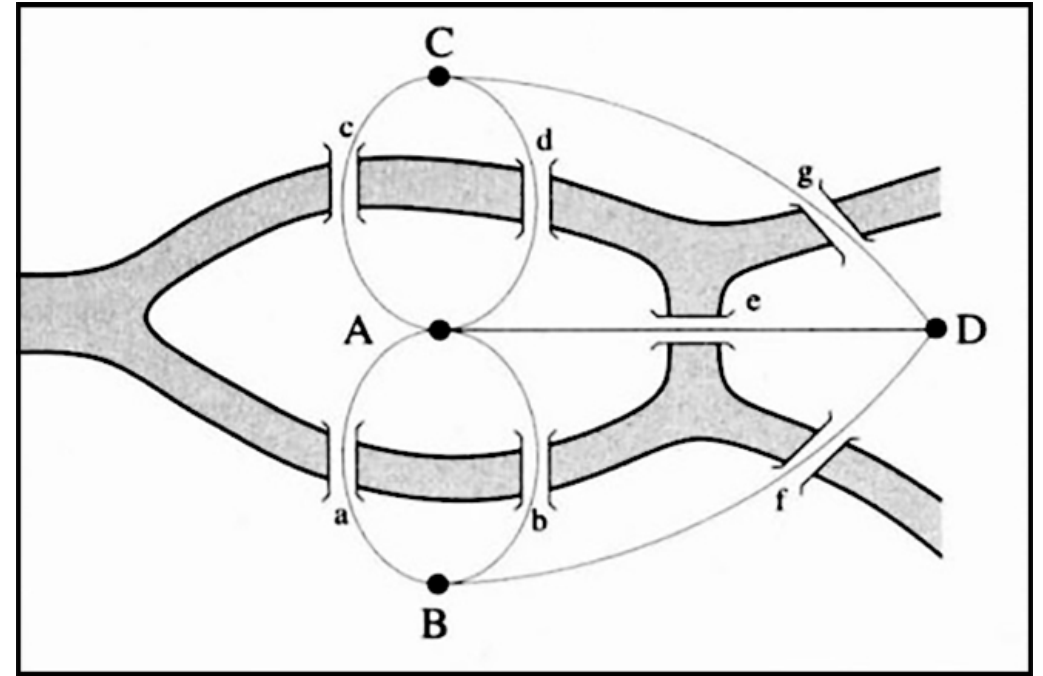
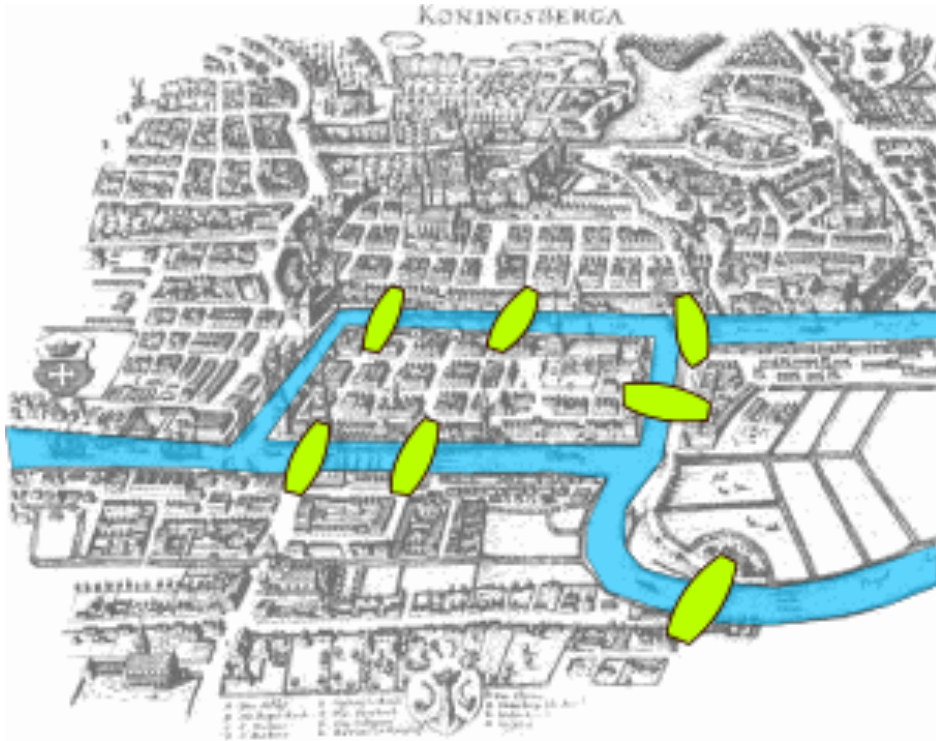
Topology

GENUS = number of holes ---- **Global** property



Studies the **properties** of a geometric object that are **preserved** under **continuous deformations** (stretching, twisting, crumpling)

Origins



1736: Leonhard Euler (**seven bridges of Königsberg**)

Is it possible to devise a walk through the city that would cross each of those bridges only once?

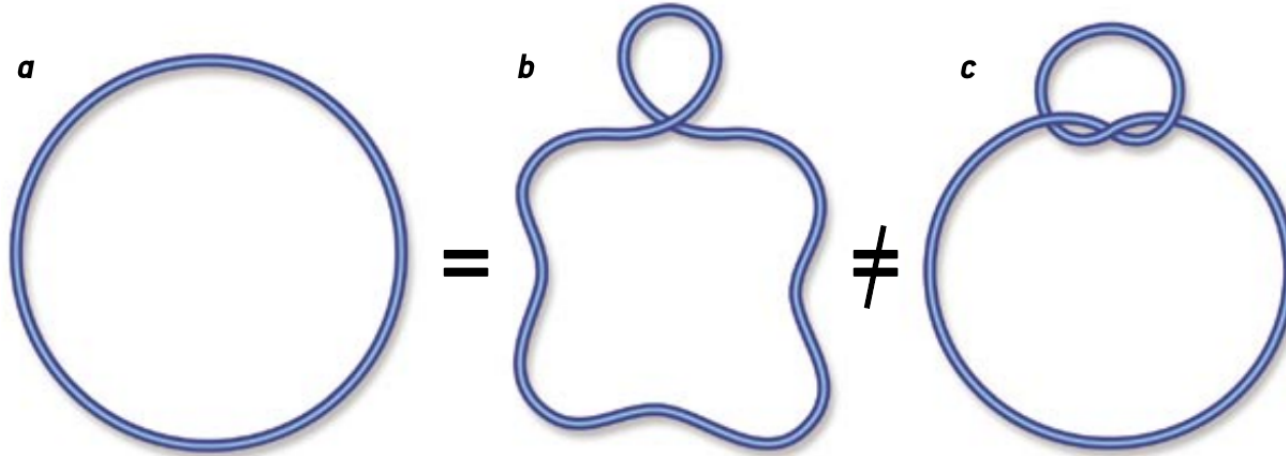
(graph theory)

connectivity

Knots

1771: Alexandre-Theophile Vandermonde

XIX century: Listing, Gauss, Tait

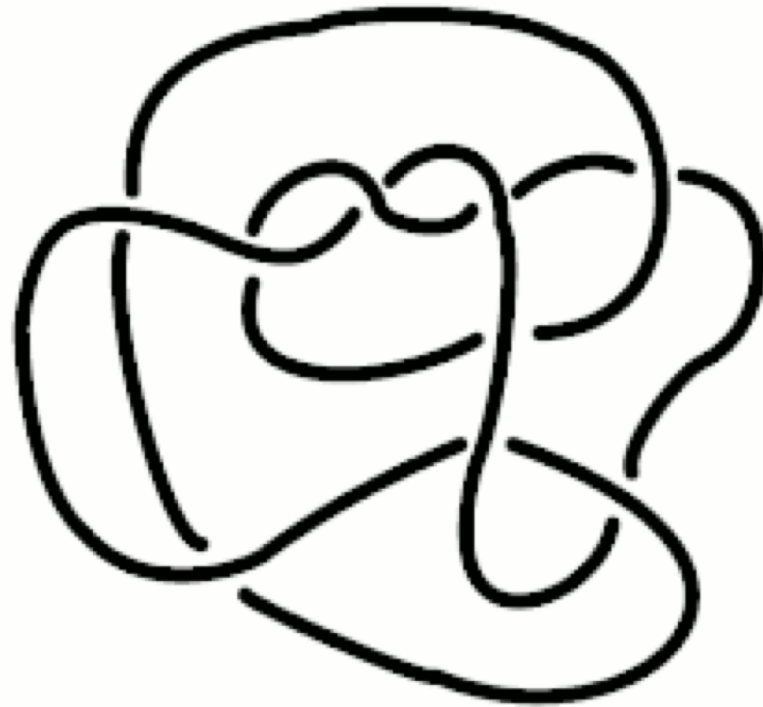


Celtic knot
Book of Kells
(Ireland) ~ 800 AD

(knot theory)

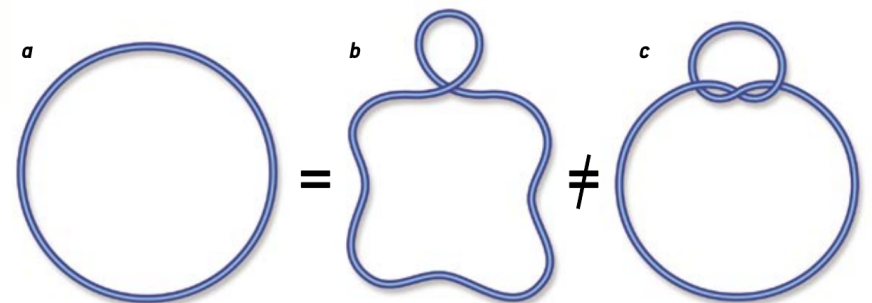
Two knots are equivalent if one can be transformed into the other via deformations in 3 dimensions

Knots



https://en.wikipedia.org/wiki/Knot_theory

(knot theory)

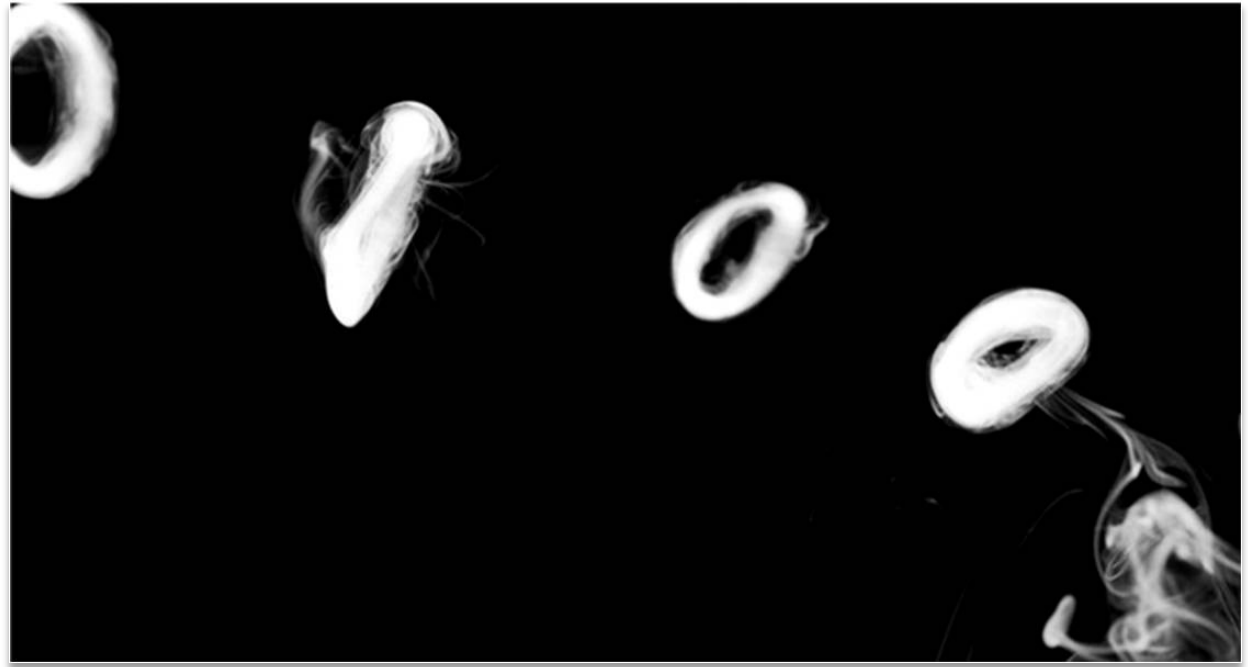
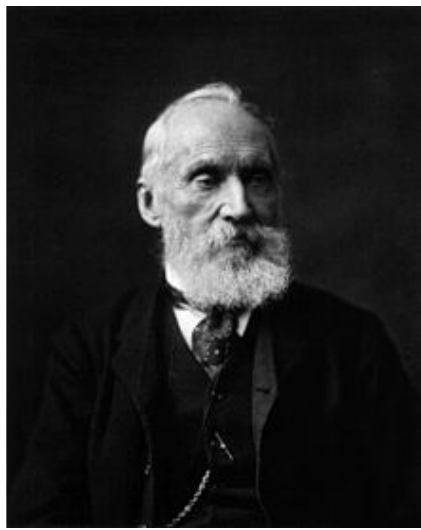


First encounters: topology and physics

XIX century:
Peter Guthrie Tait



William Thomson
(Lord Kelvin)



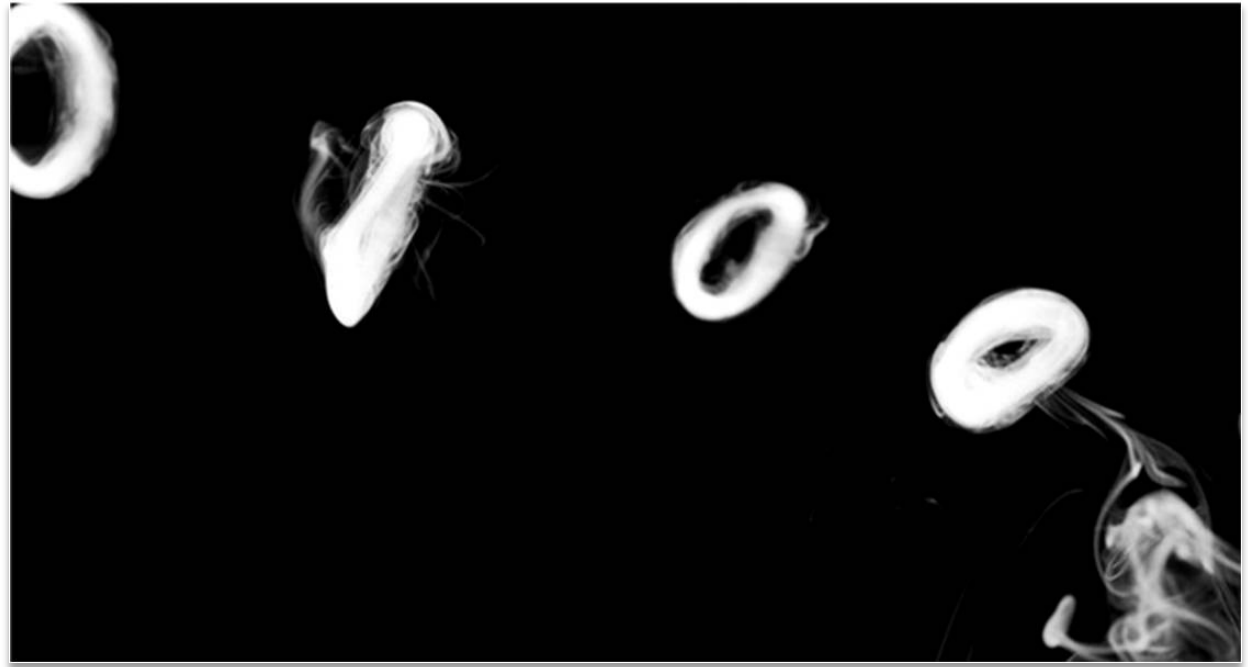
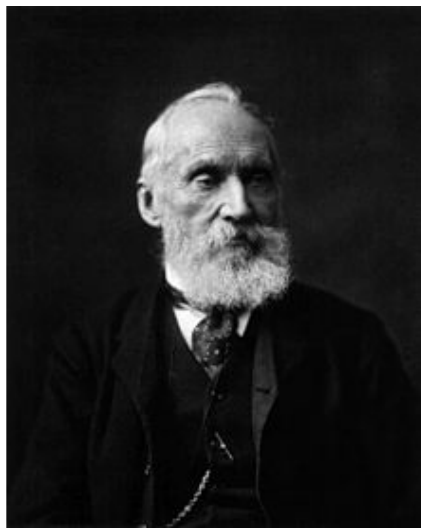
stability of smoke rings?

First encounters: topology and physics

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stability of smoke rings?

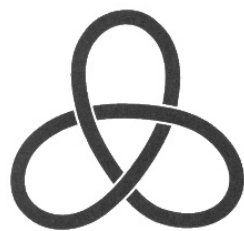
Thomson: Perhaps **atoms** are actually knots of swirling vortices in **aether**?

First encounters: topology and physics

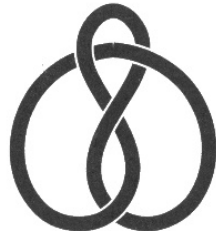
1867: William Thomson

atoms = knotted vortices in **aether**

inspired **Tait** to **classify knots**



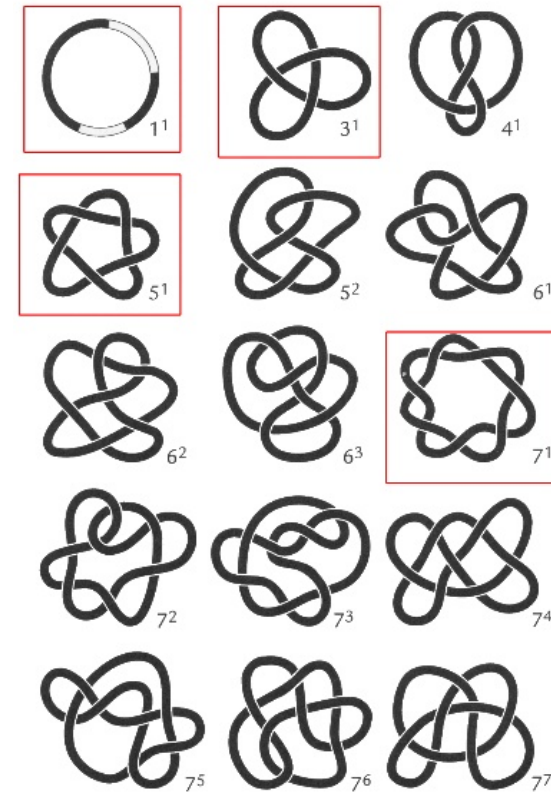
Carbon



Oxygen



Hydrogen

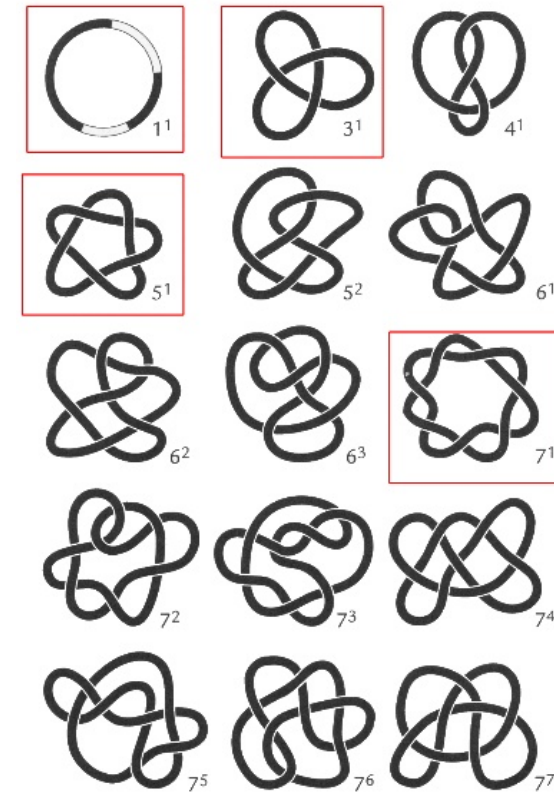
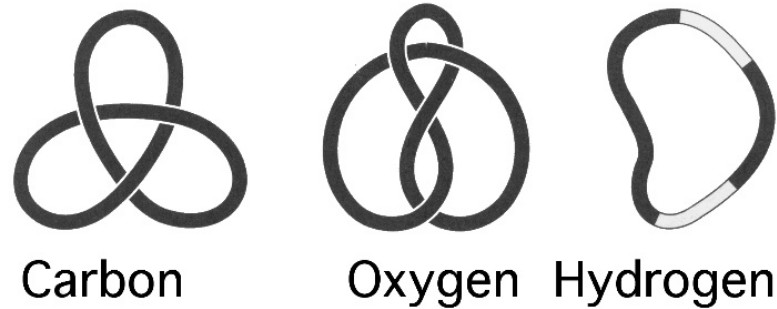


First encounters: topology and physics

1867: William Thomson

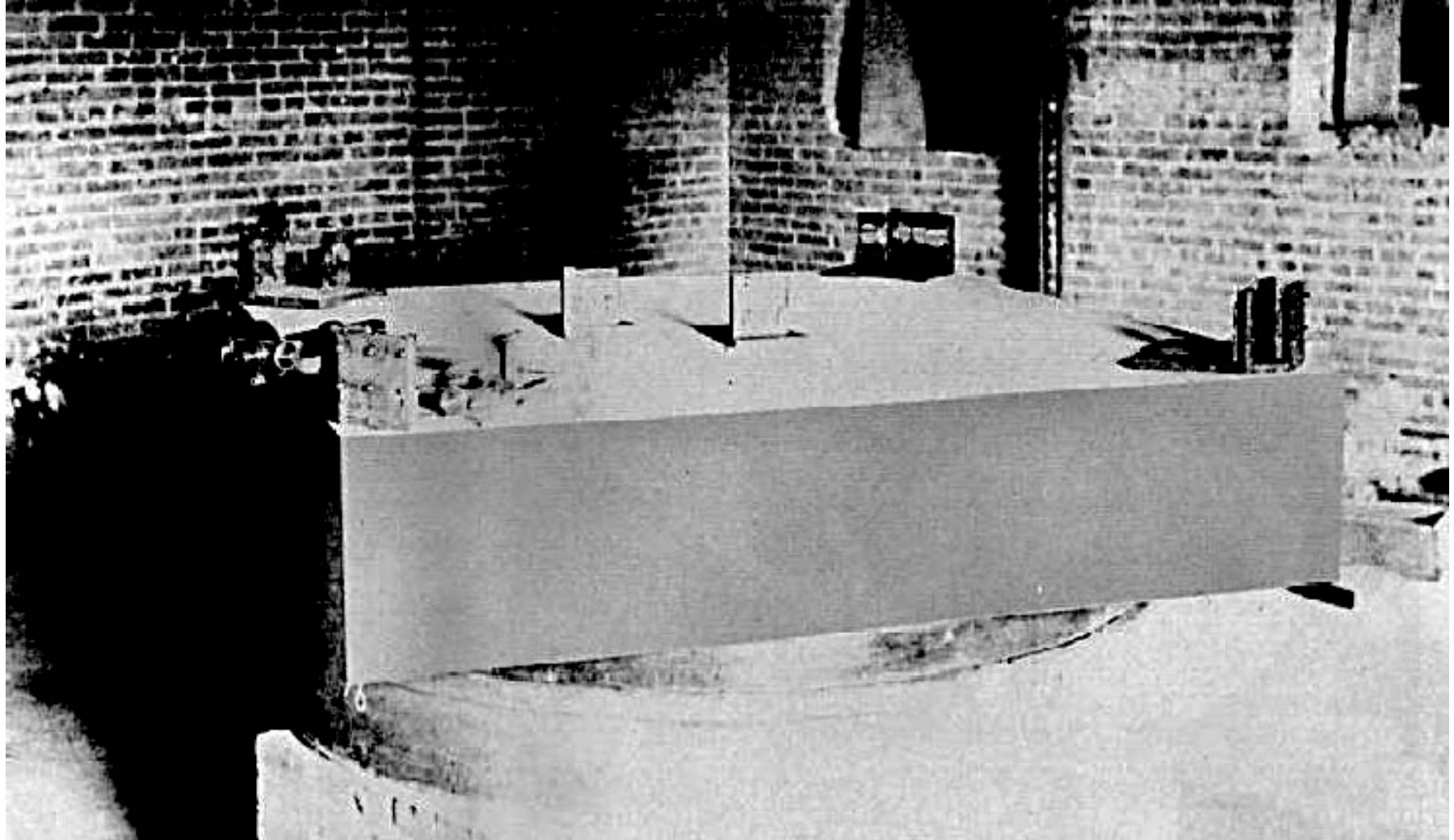
atoms = knotted vortices in **aether**

inspired **Tait** to **classify knots**



1887 Michelson-Morley experiment → inexistence of aether !!

Michelson-Morley experiment



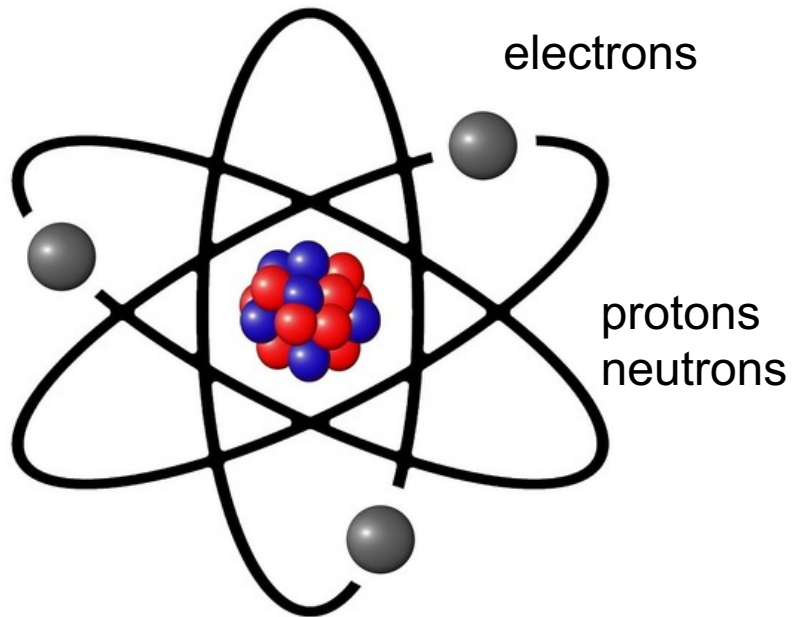
Cleveland, Ohio

1887 Michelson-Morley experiment → inexistence of aether !!

XXth century: Birth of quantum physics

Microscopic world (10^{-8} cm)

Bohr atom

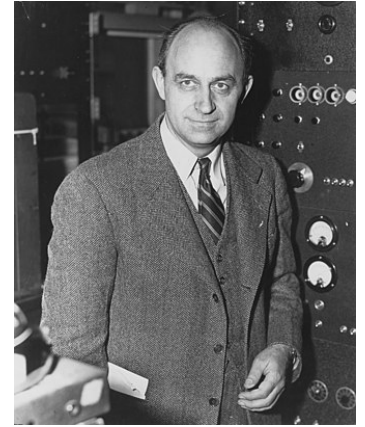
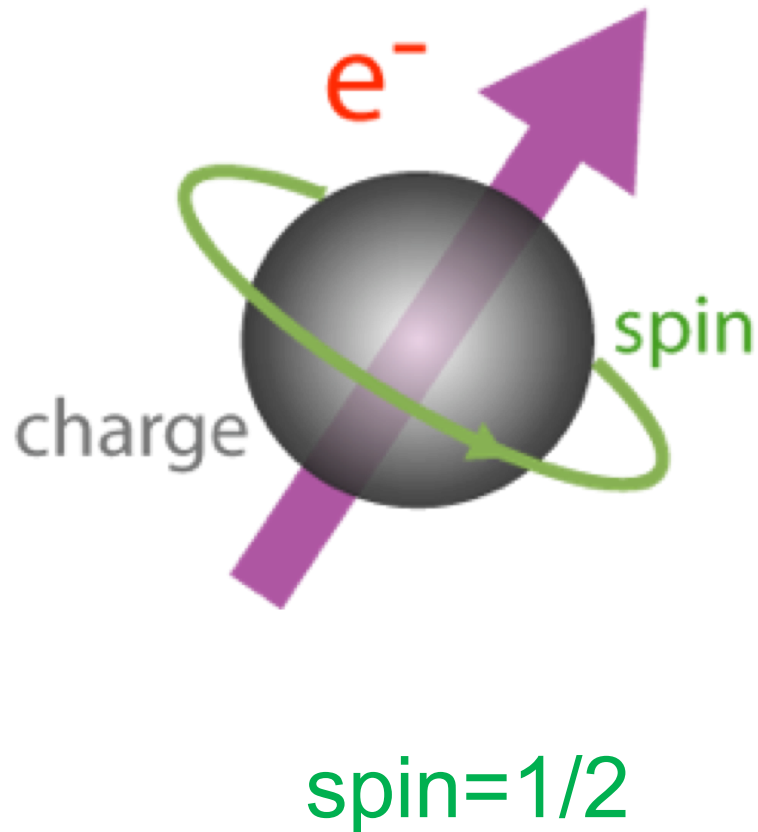


Nils Bohr
(1885-1962)

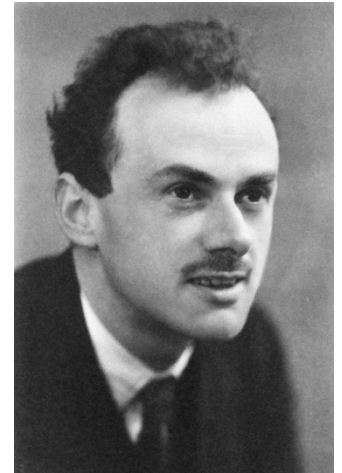
Nobel Prize 1922

XXth century: Birth of quantum physics

Microscopic world (10^{-8} cm)



Enrico Fermi
(1901-1954) Nobel Prize 1938



Paul Dirac
(1902-1984) Nobel Prize 1933

XXth century: Birth of quantum physics

Microscopic world (10^{-8} cm)

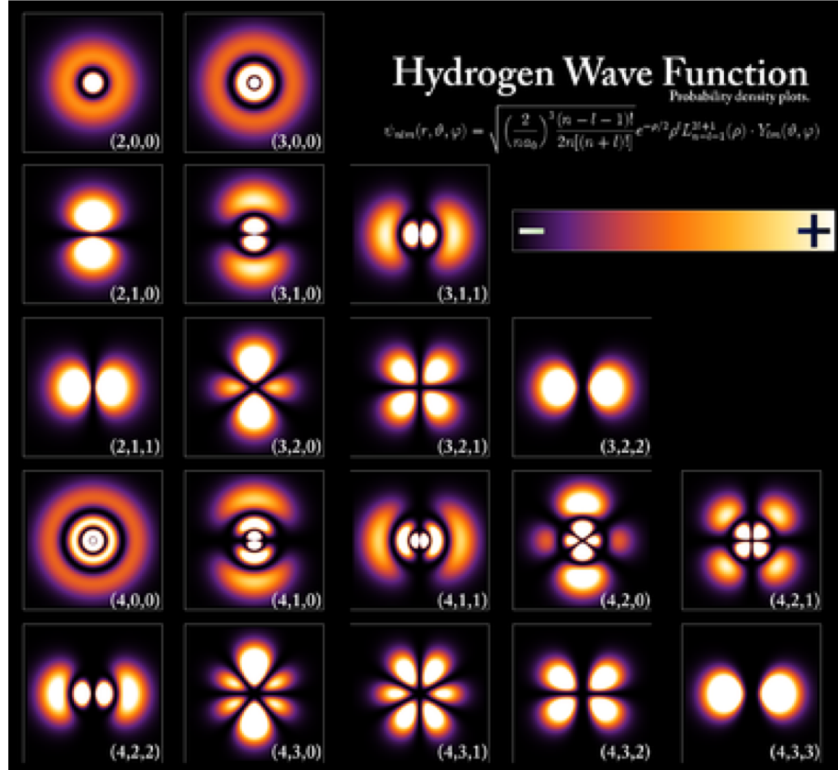
Electron wavefunction

$$\Psi(\mathbf{r}, t)$$

Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = H \Psi(\mathbf{r}, t)$$

$$\left| \Psi(\mathbf{r}, t) \right|^2$$



Werner Heisenberg

(1901-1976) Nobel Prize 1932

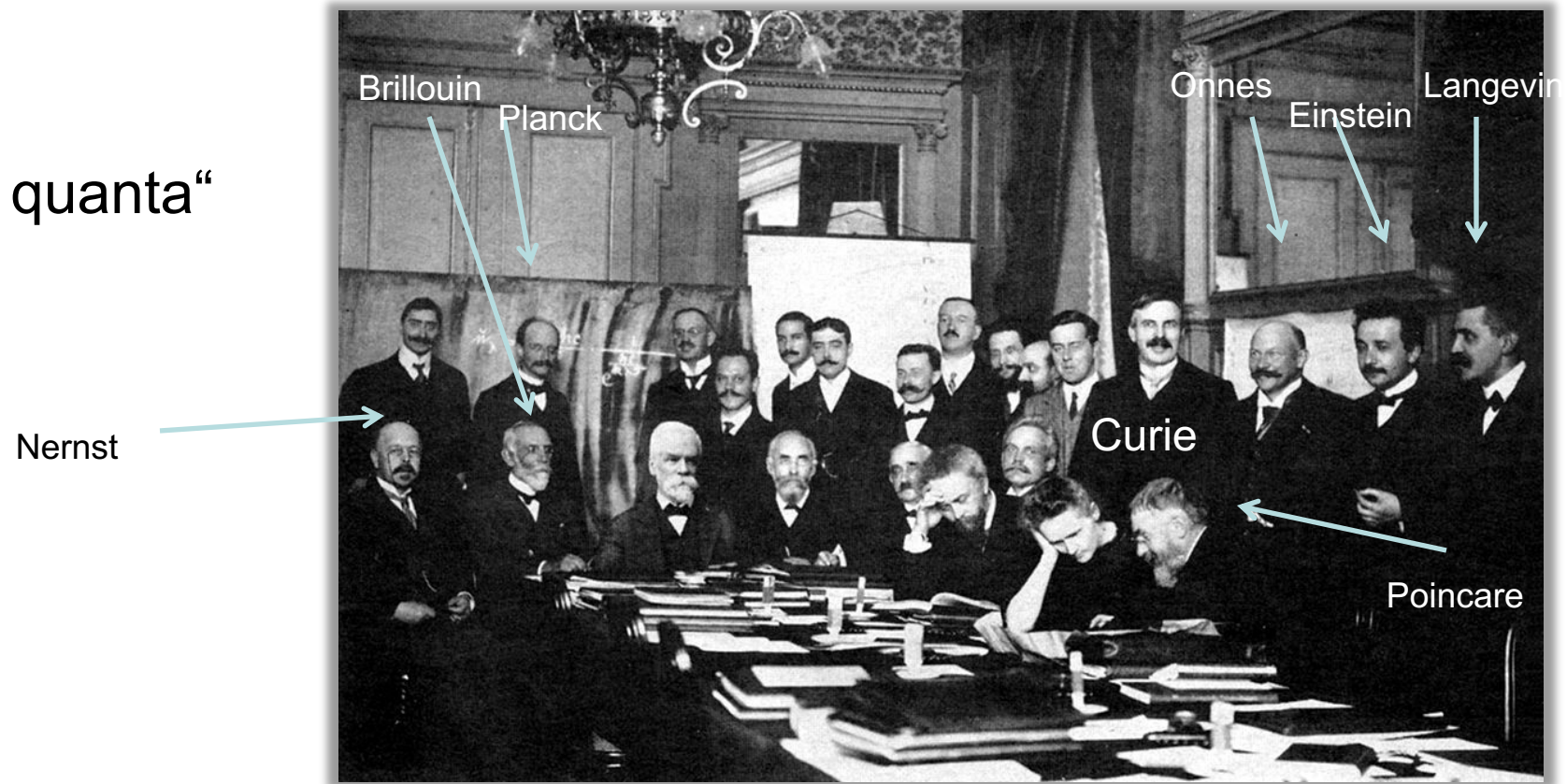


Erwin Schrödinger

(1887-1961) Nobel Prize 1933

XXth century: Birth of quantum physics

„Radiation and quanta“



1911: 1st Solvay Conference
(Brussels)

Letters Einstein → Max von Laue

BERLIN

Datum:
16. VIII. 1926

Datum:
12. VII. 1927

Datum:
29. IX. 1928

Datum:
3. X. 1928

Datum:
21. I. 1929

Datum:
30. I. 1929

OXFORD

Datum:
26. V. 1933

PRINCETON

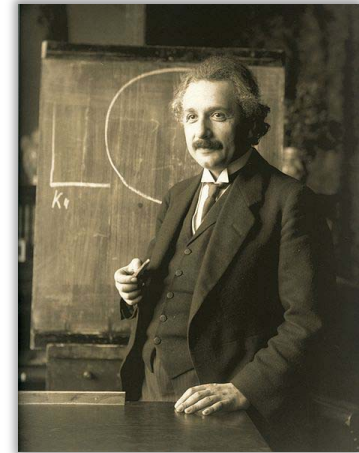
Datum:
23. III. 1934

Datum:
21. X. 1935
Elsa Einstein

Datum:
29. VII. 1936
Max von Laue

Datum:
29. VIII. 1936

Universitätsarchiv
Goethe University Frankfurt



Albert Einstein
(1878-1955) Nobel Prize 1921

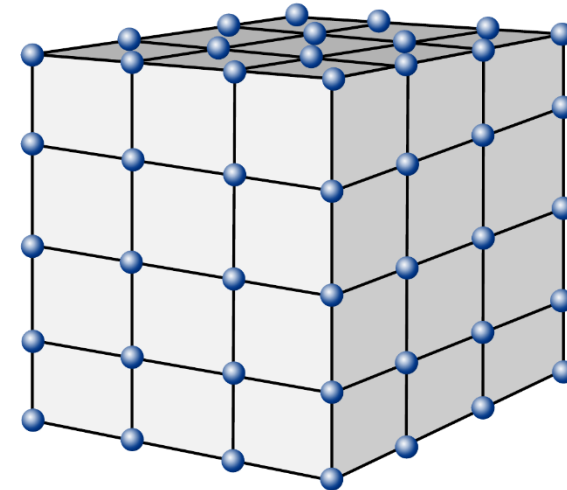
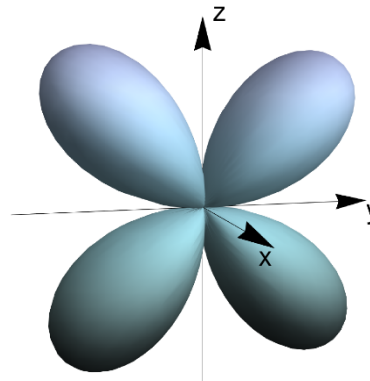
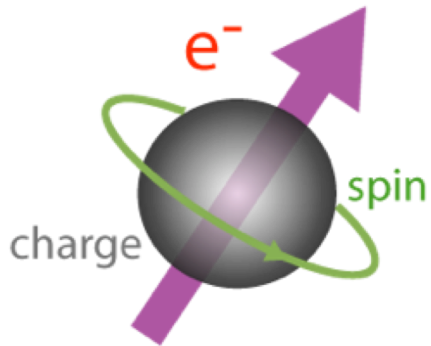


Max von Laue
(1879-1960) Nobel Prize 1914

XXIst century

Quantum Materials

- Interacting many-body systems:
 10^{23} electrons within a cm^3
- Simultaneous action of several degrees of freedom:



Emergence of distinct phases of matter

Emergence

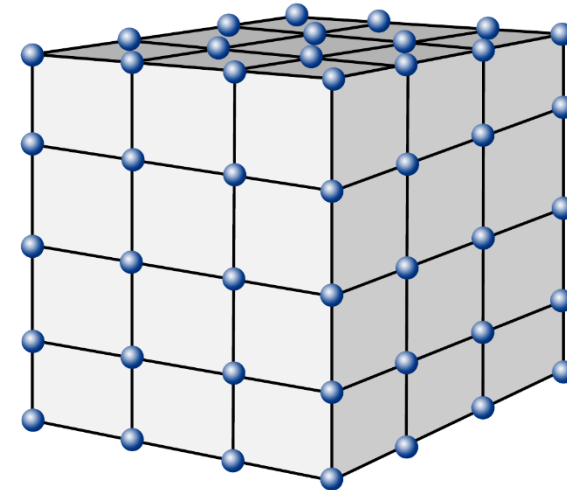
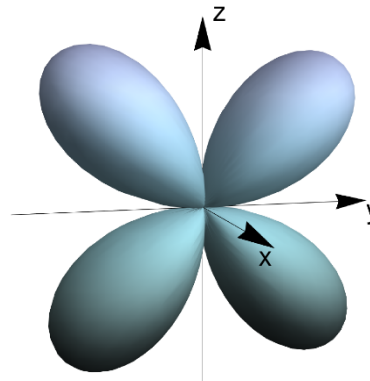
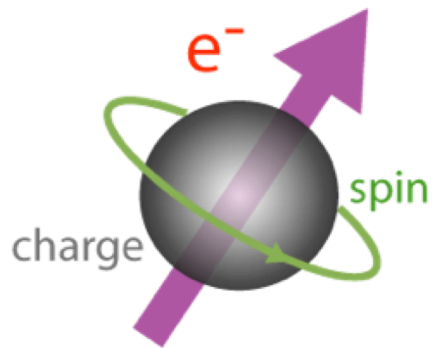
Flock of birds



Behavior not foreseeable from the knowledge of the constituents alone

Quantum Materials

- Interacting many-body systems:
 10^{23} electrons within a cm^3
- Simultaneous action of several degrees of freedom:



Emergence of distinct phases of matter

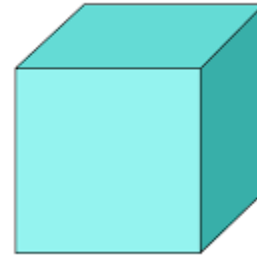
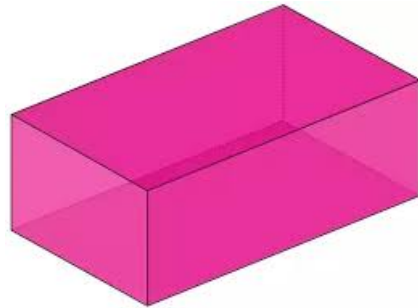
“Old way”

Quantum Materials

symmetry

invariance of an object under a transformation (rotation, translation,...)

useful to classify phases of matter through changes of symmetry (Landau)



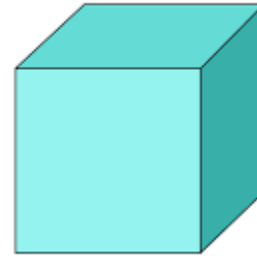
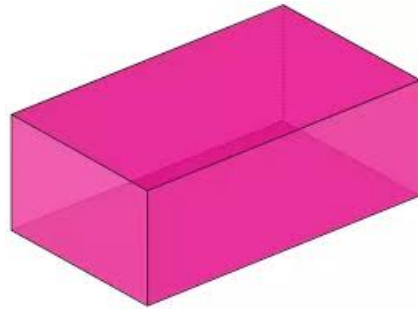
“Old way”

Quantum Materials

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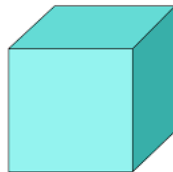


“New way”

topology

deals with properties that are preserved under continuous deformations

useful to classify topological phases of matter



pioneers in using the language
of topology to describe
quantum states

Physics Nobel Prize 2016



David Thouless
Washington University
Seattle USA



Michael Kosterlitz
Brown University
Providence USA

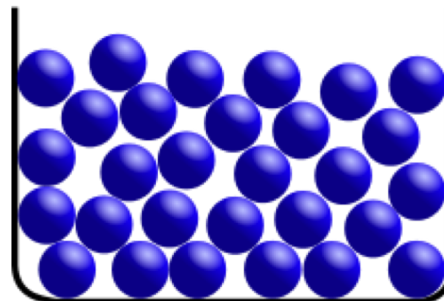
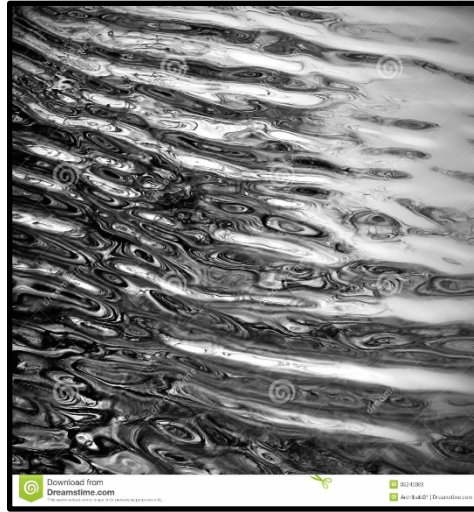


Duncan Haldane
Princeton University,
USA

„for theoretical discoveries of topological phase transitions and
topological phases of matter“

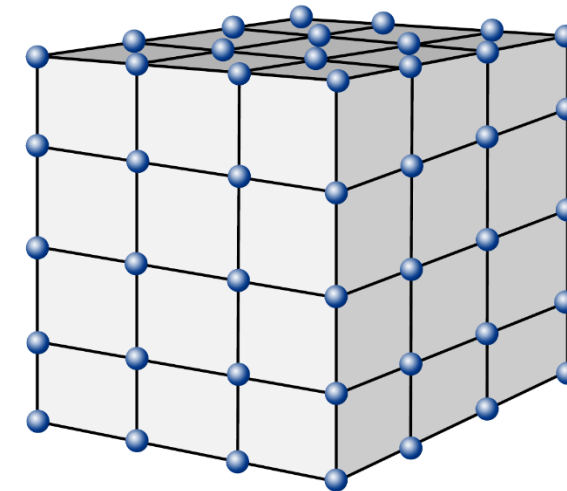
Phase transitions

liquid

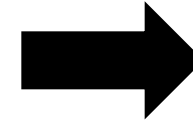


disordered atoms

solid



ordered atoms



lowering
Temperature

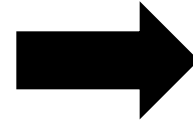


translational
symmetry

Phase transitions: paramagnet \rightarrow ferromagnet

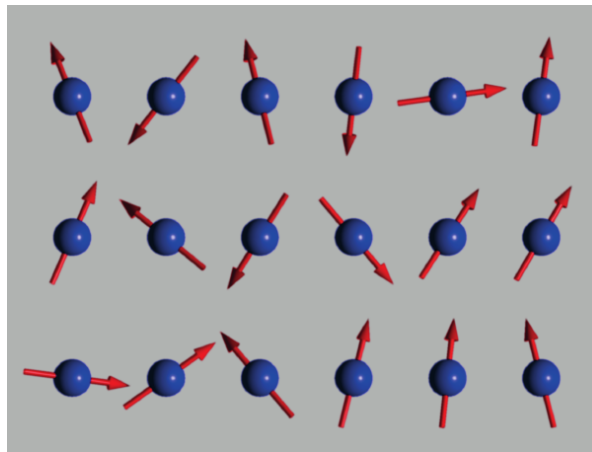
(Landau theory of phase transitions 1937)

paramagnet



lowering
Temperature

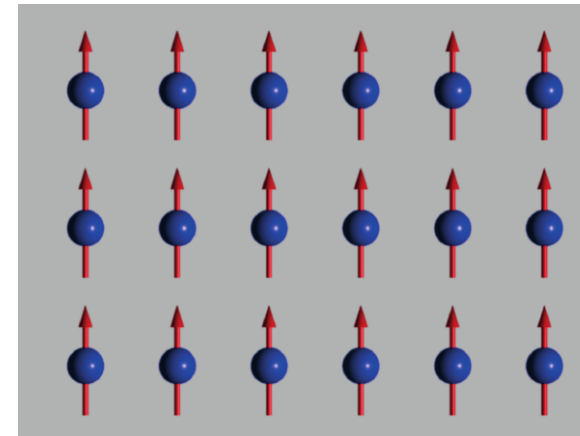
ferromagnet



disordered spins



Spin rotation
symmetry



ordered spins

Order Parameter = magnetization

1966: N.D. Mermin, H. Wagner \rightarrow in 2D a Heisenberg magnet cannot order

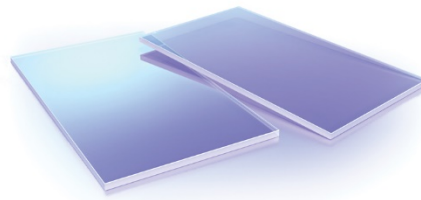
1967: P. Hohenberg \rightarrow in 2D superconductivity and superfluidity shouldn't exist

Problem:

Numerical evidence of phase transitions in 2D

Stanley, Kaplan (1966), Wegner (1967), Berezinskii (1970)

??





hair swirl

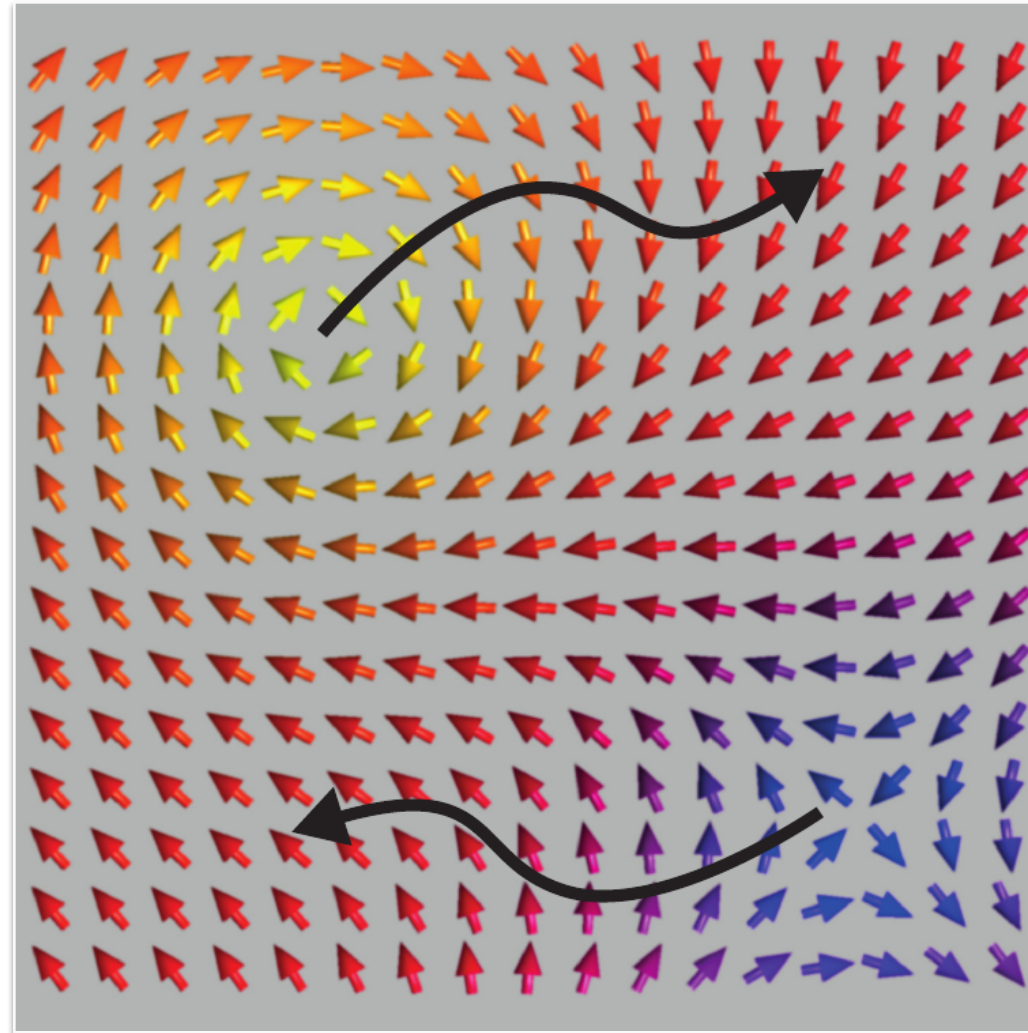
SYSTEM OF SPINS

Vortex-antivortex

Microscopic world (10^{-8} cm)

vortex:
 $G=1$

winding 2π



antivortex:
 $G=-1$

winding -2π

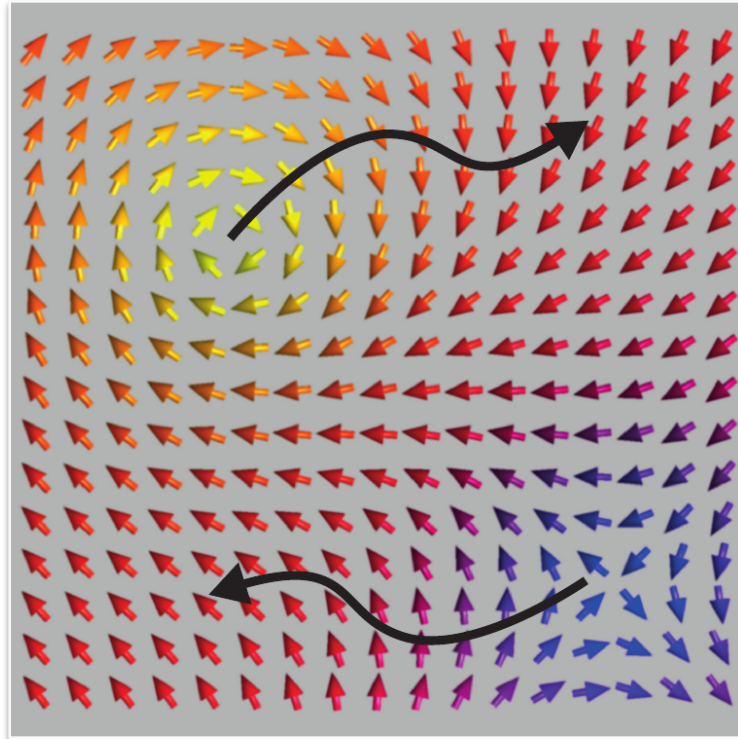
it is impossible to transform a **vortex** to an **antivortex** through a continuous transformation, but they can form pairs. **Pair $G=0$**

SYSTEM OF SPINS

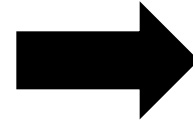
BKT phase transition

Kosterlitz and Thouless

J. Phys. C 5, L124 (1972); 6, 1181 (1973), Berezinskii (1972)



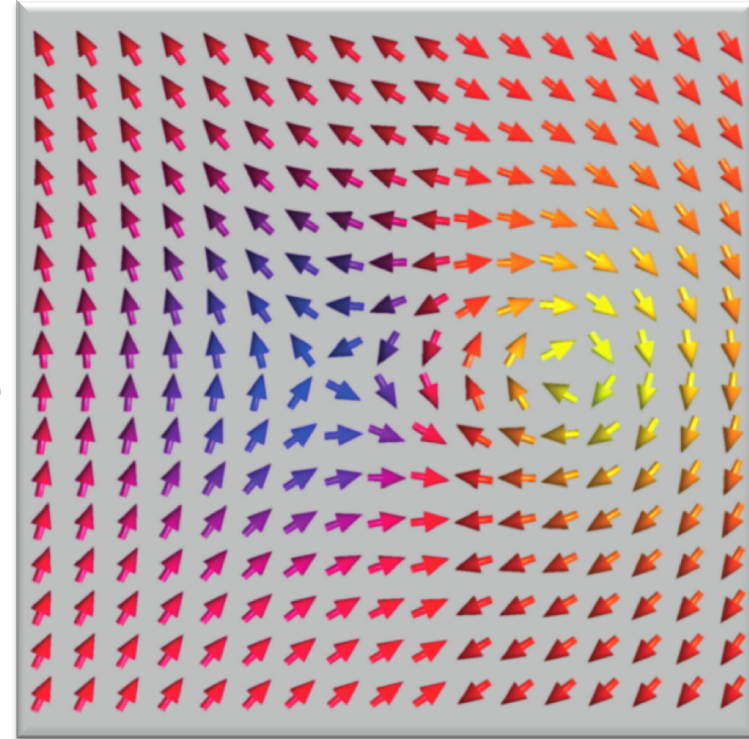
unpaired vortex – antivortex



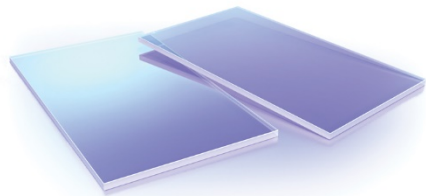
Lowering
Temperature

Change of
topology

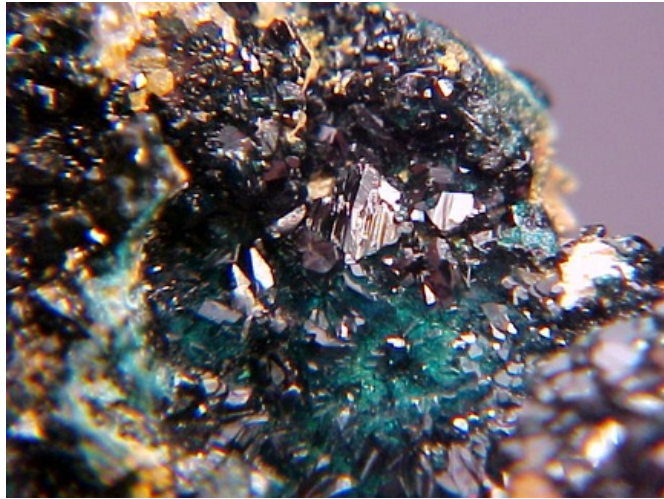
BKT



paired vortex – antivortex



In 2D, **topological defects (vortex)** can undergo a phase transition without symmetry change → new topological state



herbertsmithite

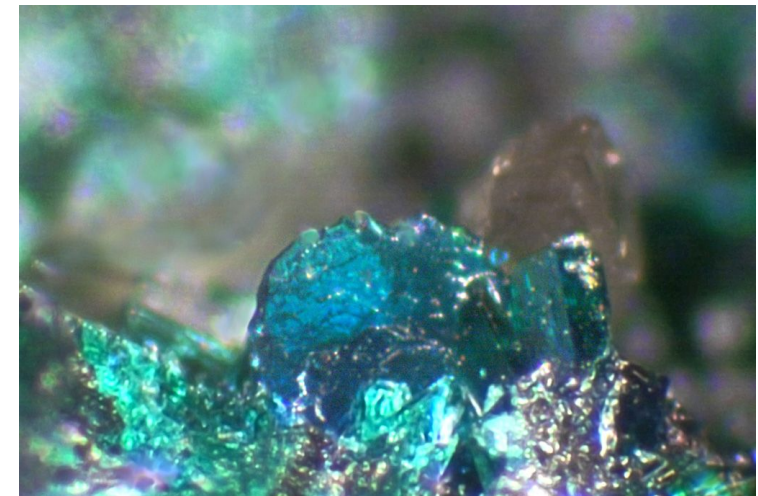


haydeeite

Quantum Materials



kapellasite



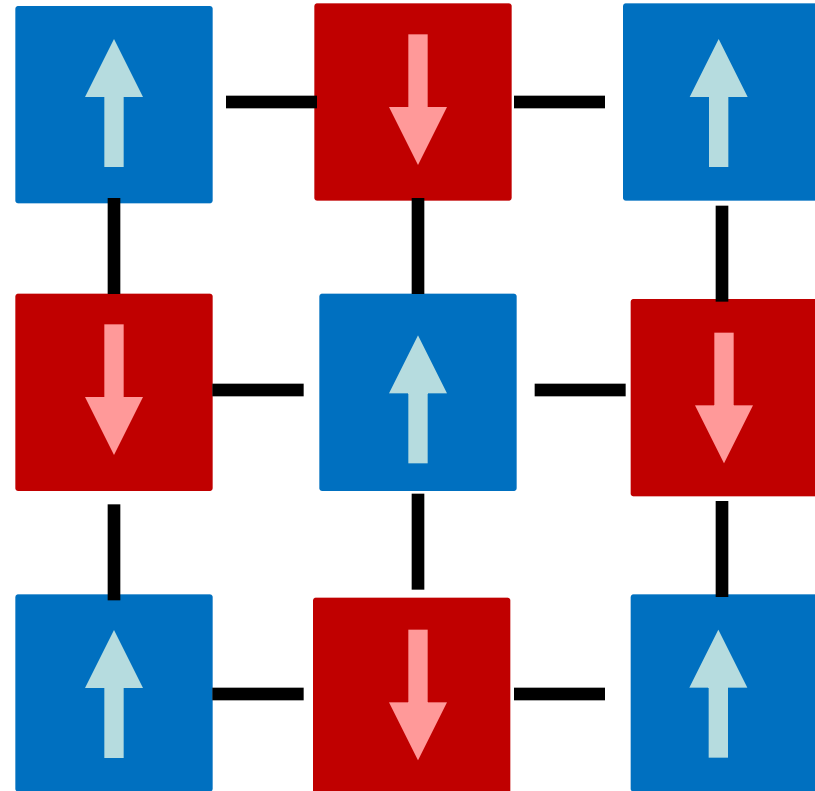
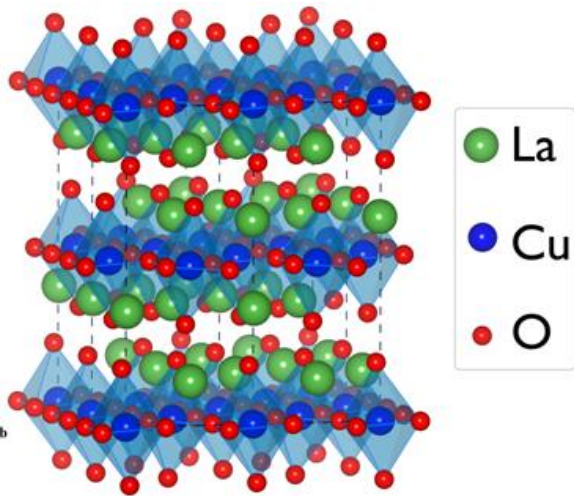
barlowite

Topological phases in quantum materials

Spin liquids

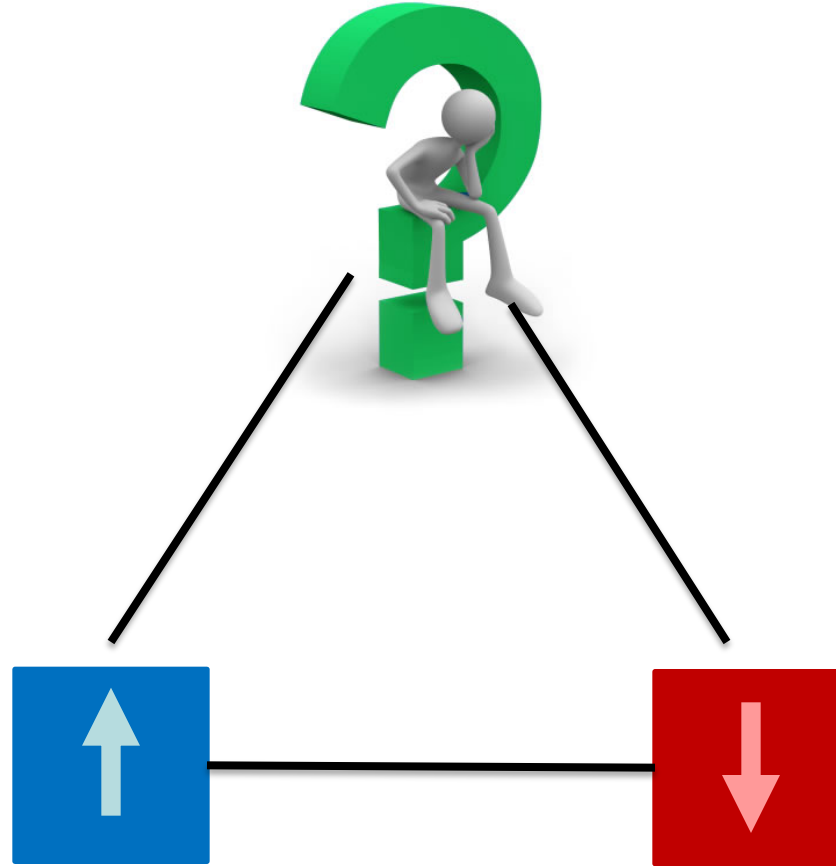
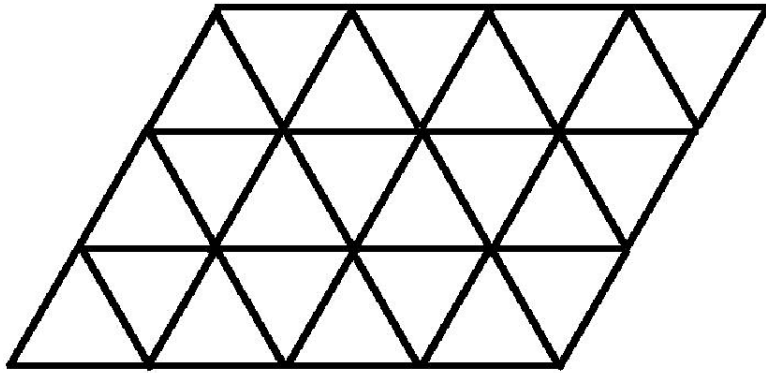
Antiferromagnet

square lattice

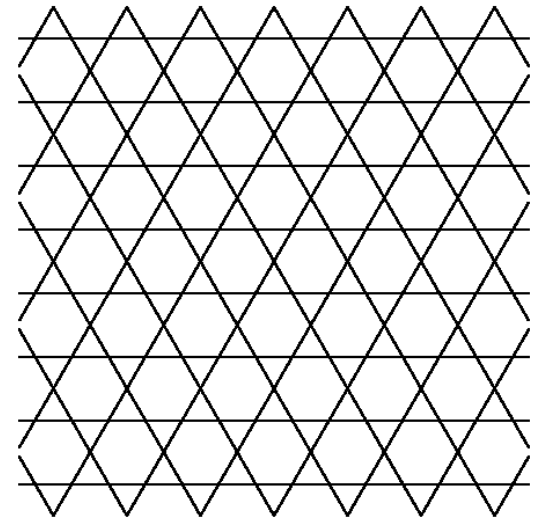


Geometric Frustration

triangular lattice



Kagome lattice



Spin liquid

- ◆ Importance of **quantum effects** to induce new types of states (*Spin liquid, resonating valence bond state, ...*)



*P.W. Anderson
Mat.Res.Bull 8, 153 (1973)*

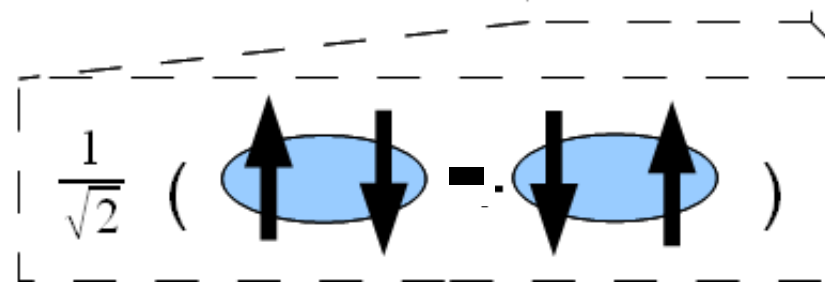
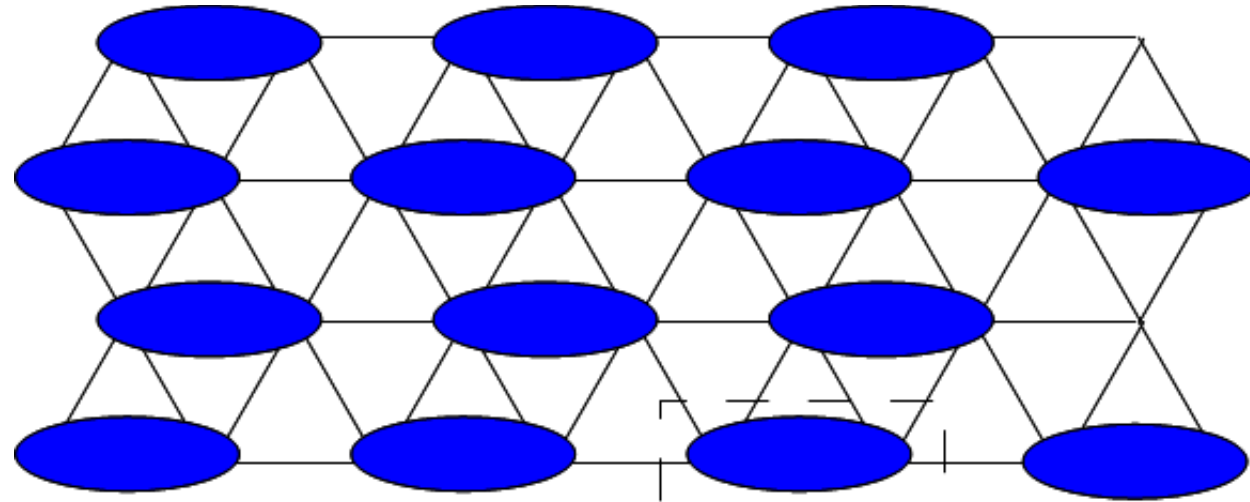
Nobel Prize 1977

T. Imai, Y. Lee Physics Today 69 (2016)

L. Balents, Nature 464, 199 (2010)

Spin liquid

triangular lattice



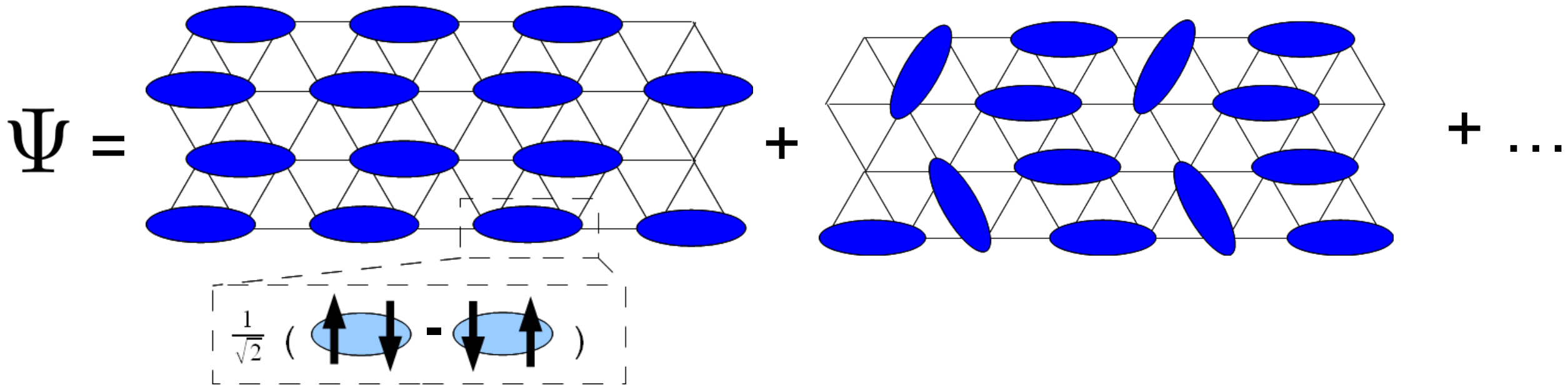
„valence bonds“



“I have good news and bad news”

triangular lattice

Spin liquid

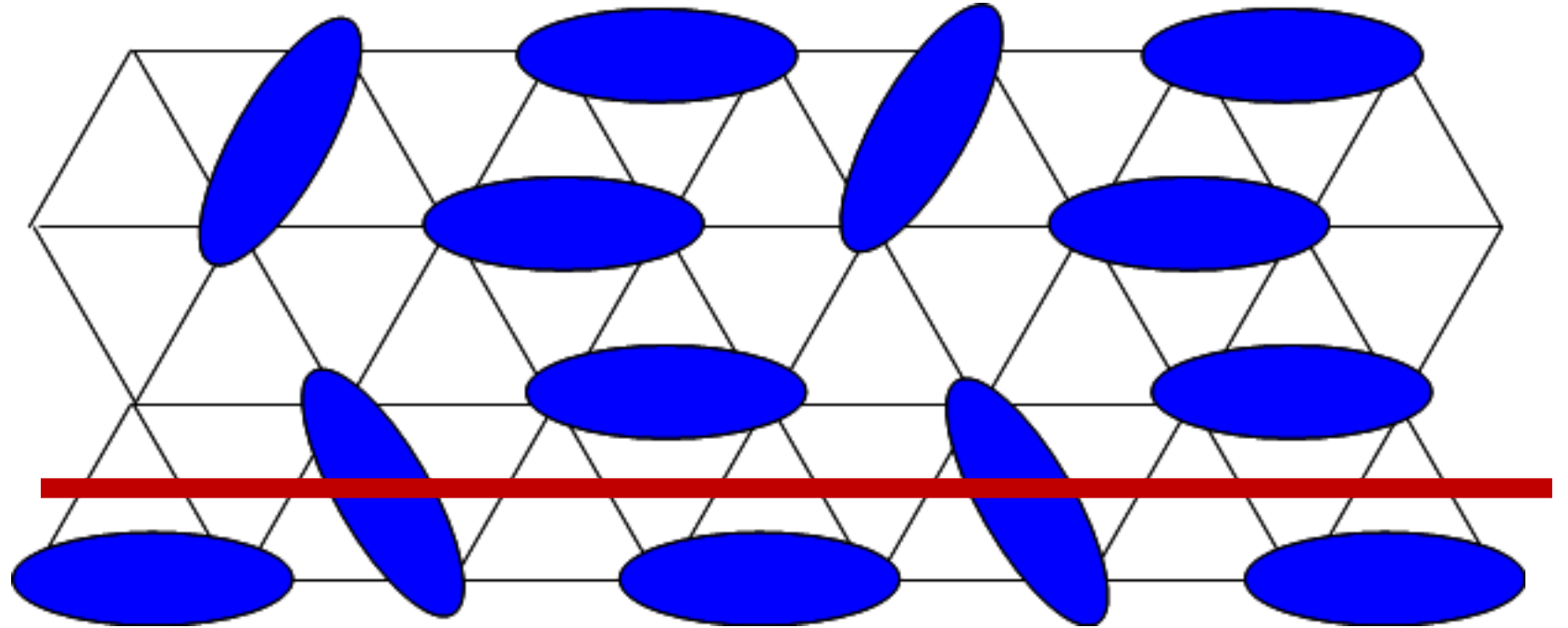


„resonating valence bonds“

„linear superposition
of quantum states“

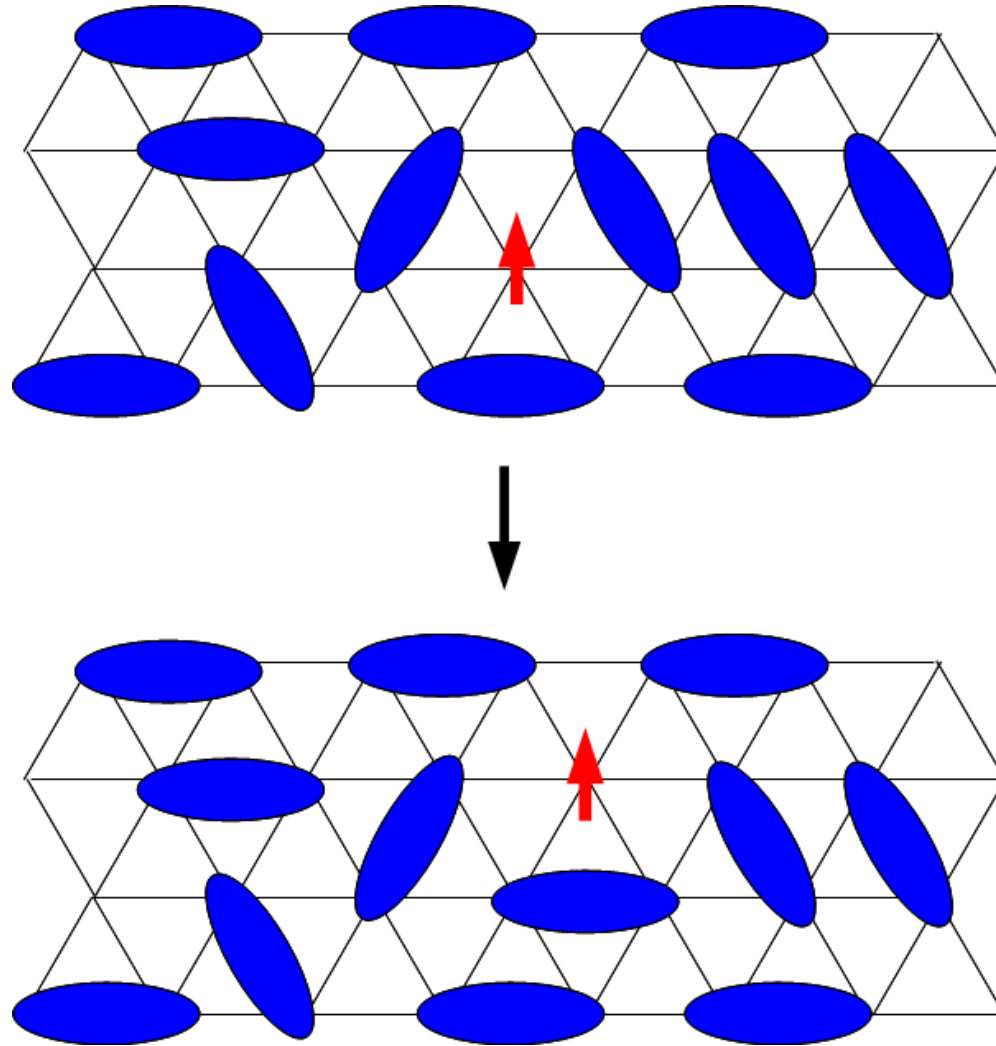
Spin liquid

Topological long range order



Topological invariant:
even or odd number of valence bonds crossing the red line

Emergence of exotic “quasi-particles”



spinon: particle with spin and no charge

Emergence of exotic “quasi-particles”

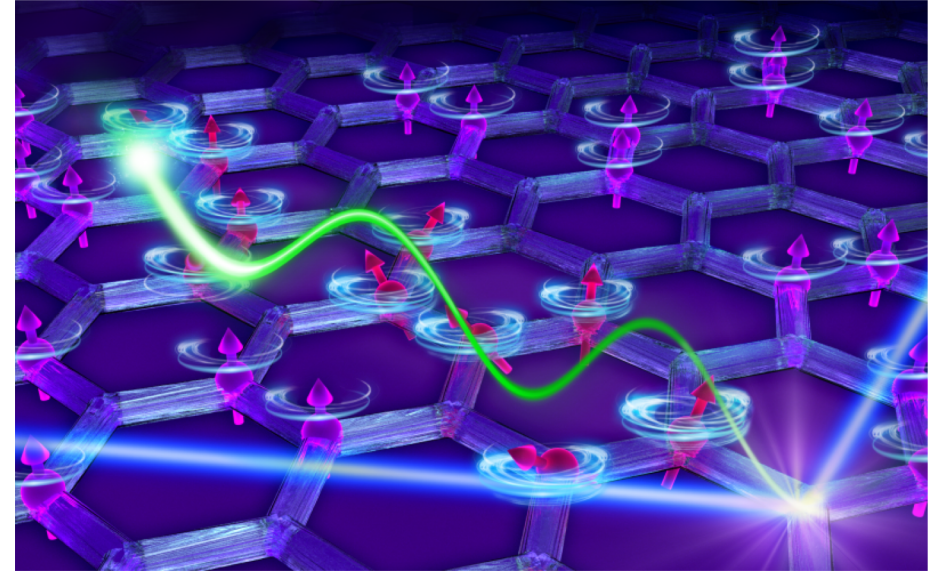
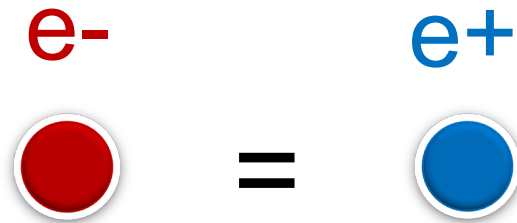


Image:ORNL/Jill Hemman

Majorana Fermions

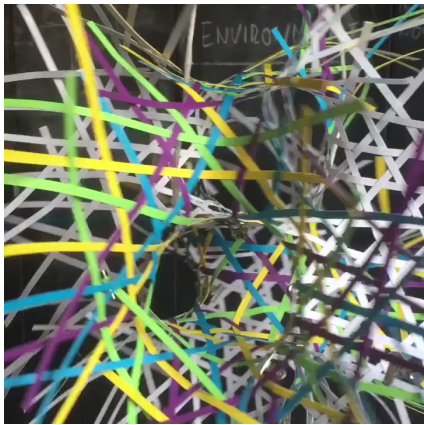


$$c_i^\dagger = c_i \quad , \quad c_i^2 = 0 \quad , \quad \{c_i, c_j\} = 0$$



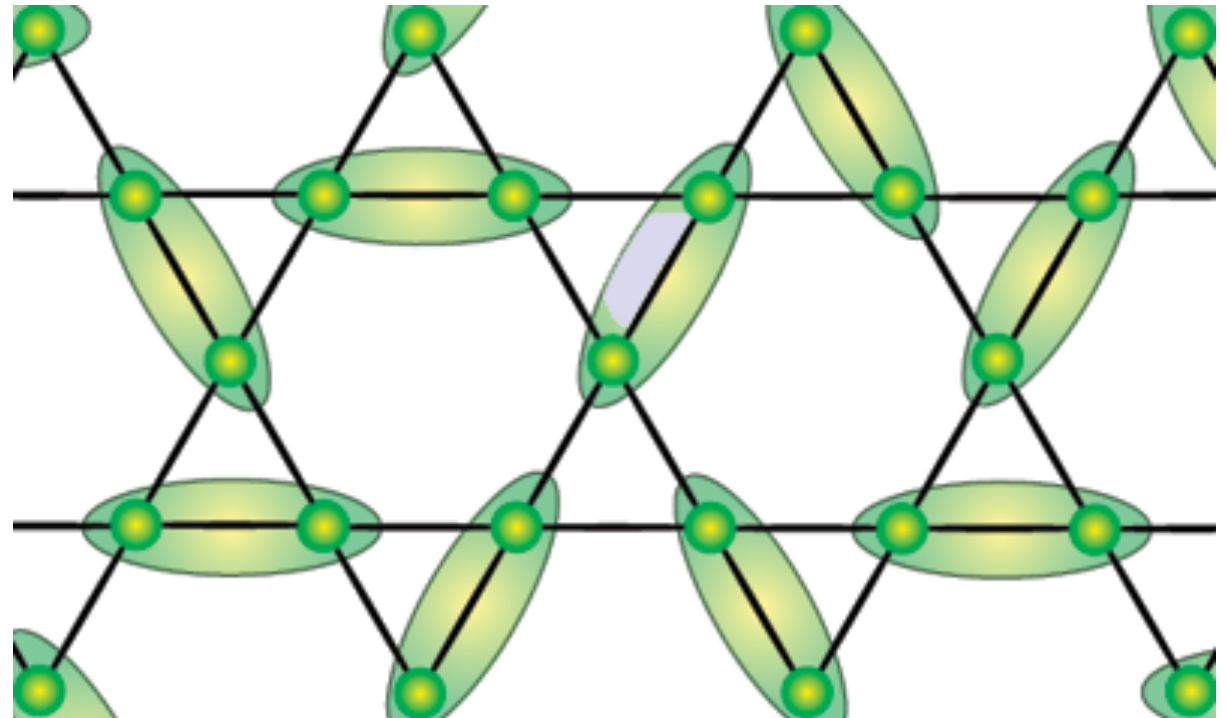
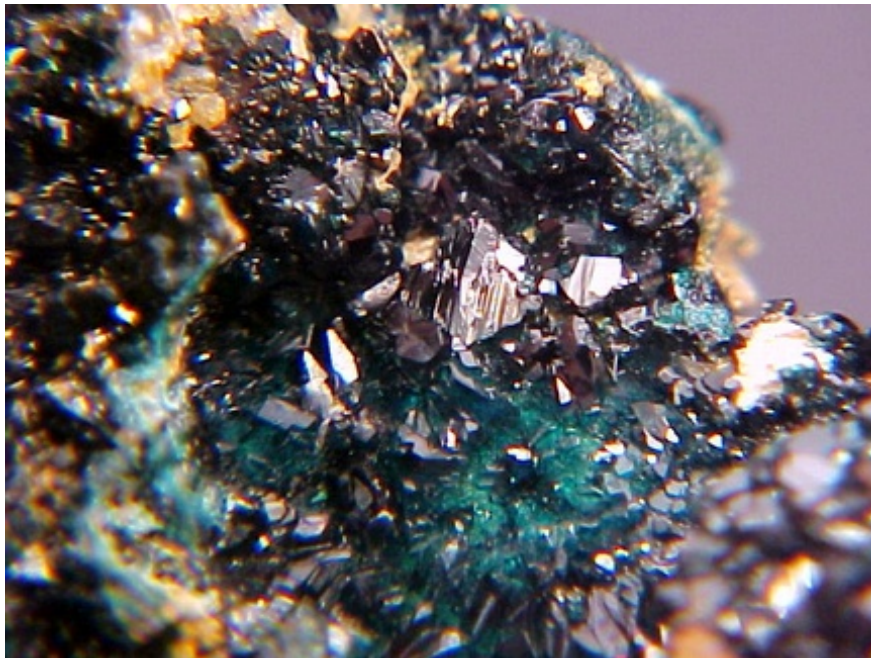
Ettore Majorana

where to find these quantum states in nature?



Herbertsmithite $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$

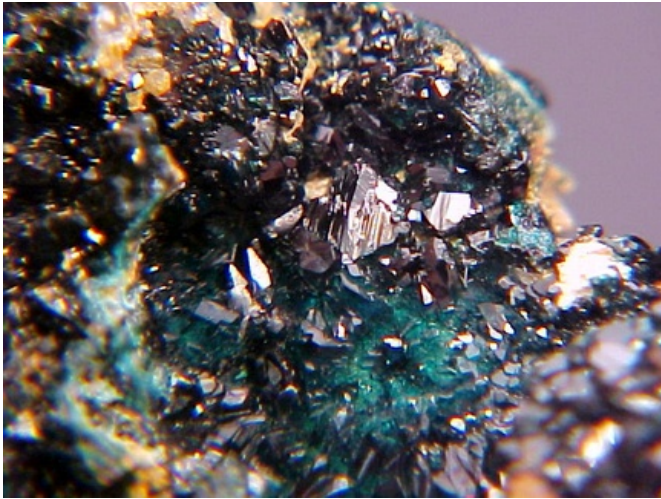
Kagome lattice



Picture: Lucile Clark

Spin liquid with spinon excitations

Quantum Materials



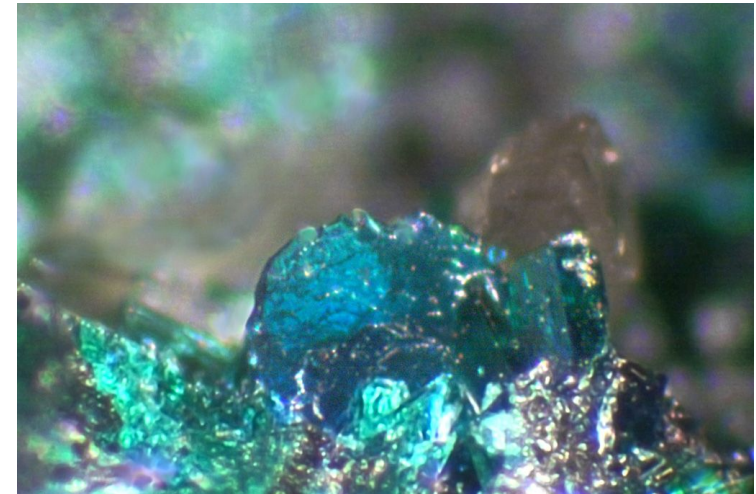
herbertsmithite



kapellasite

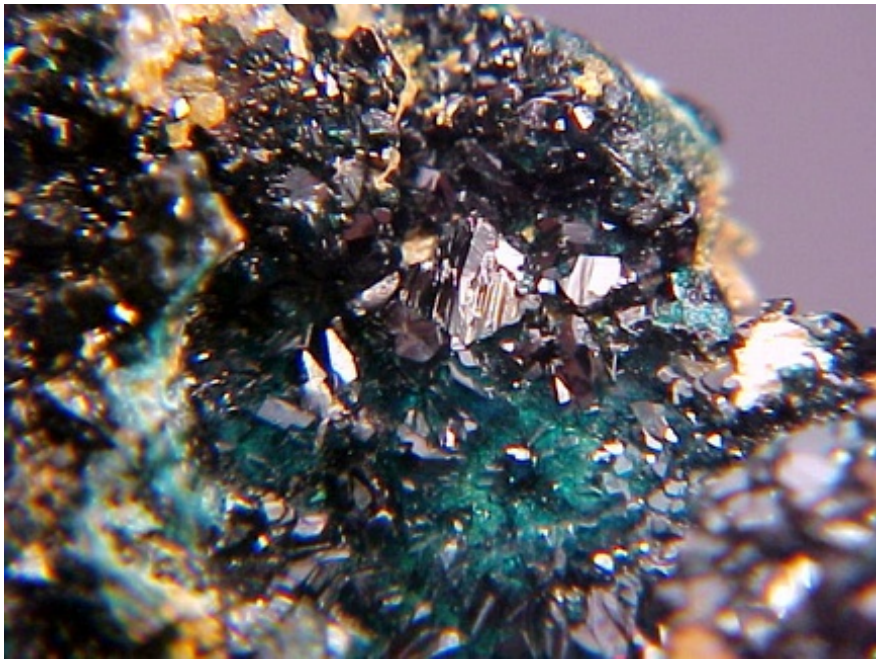


haydeeite



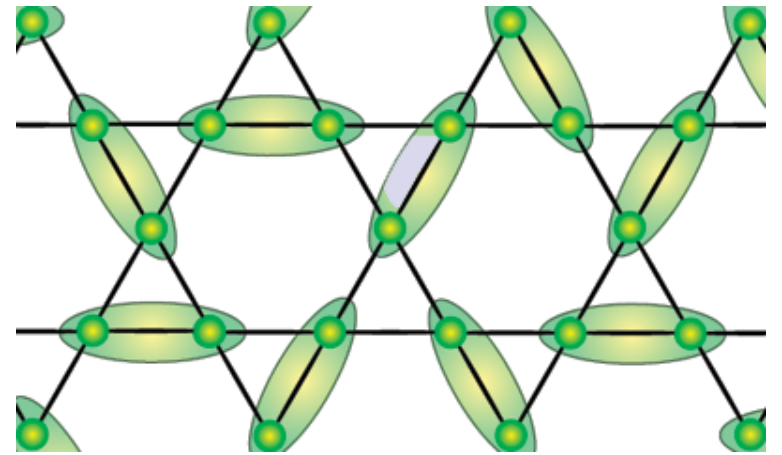
barlowite

Quantum Materials



challenge

?



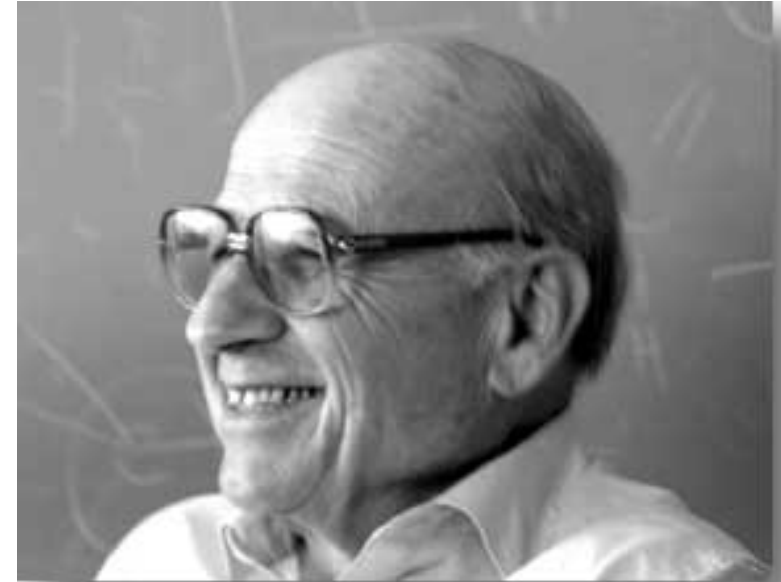
$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = H \Psi(\mathbf{r}, t)$$

- 10^{23} coupled equations!!!

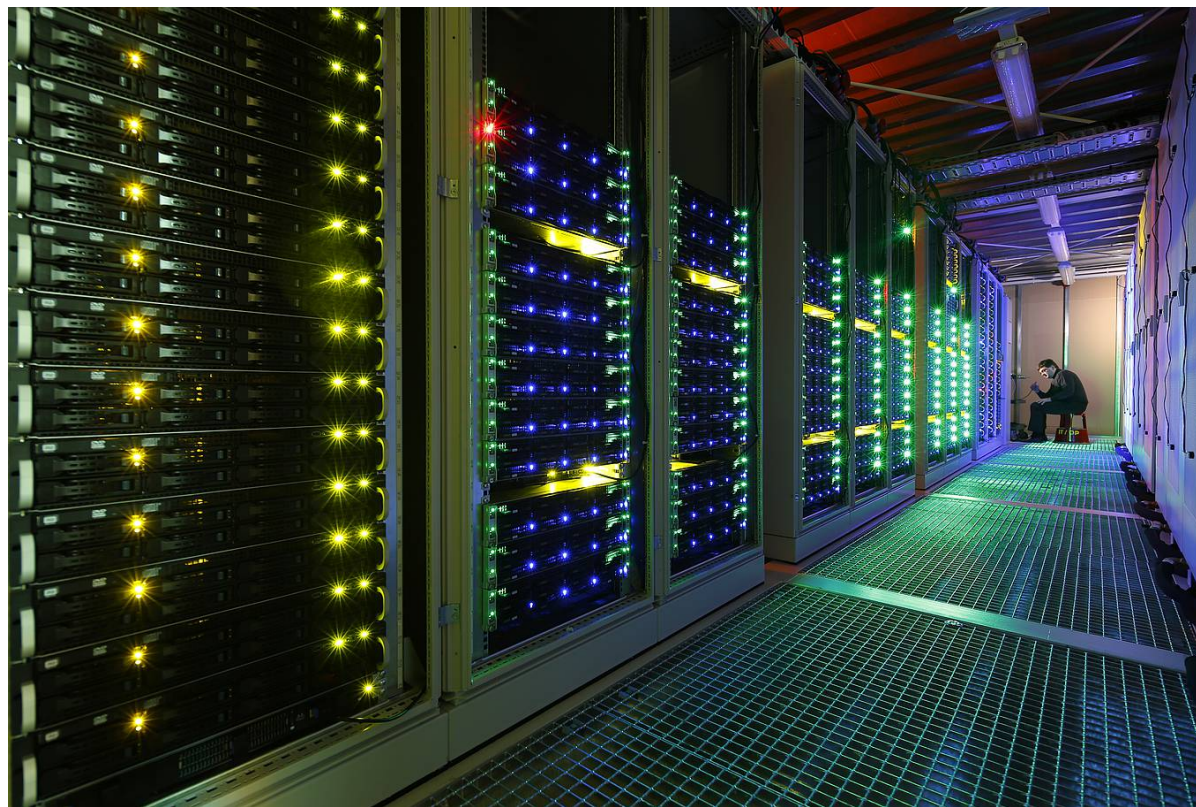
Density Functional Theory

Walter Kohn
(1923-2016)
Nobel Prize in 1998

Founding Director of ITP (now KITP)



Solve numerically the Schrödinger equations
for the constituent interacting electrons

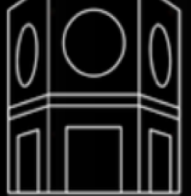


Computational simulations

Periodic Table of the Elements

1 1IA 11A	2 IIA 2A																	13 IIIA 3A	14 IVA 4A	15 VA 5A	16 VIA 6A	17 VIIA 7A	18 VIIIA 8A
1 H Hydrogen 1.0079	2 He Helium 4.00260																	5 B Boron 10.811	6 C Carbon 12.011	7 N Nitrogen 14.0074	8 O Oxygen 15.9994	9 F Fluorine 18.998403	10 Ne Neon 20.1797
3 Li Lithium 6.941	4 Be Beryllium 9.01218	11 Na Sodium 22.989768	12 Mg Magnesium 24.305	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8 VIII 8	9 VIII 9	10 VIII 10	11 IB 1B	12 IIB 2B	13 Al Aluminum 26.981539	14 Si Silicon 28.0855	15 P Phosphorus 30.973762	16 S Sulfur 32.066	17 Cl Chlorine 35.4527	18 Ar Argon 39.948				
19 K Potassium 39.0983	20 Ca Calcium 40.078	21 Sc Scandium 44.95591	22 Ti Titanium 47.88	23 V Vanadium 50.9415	24 Cr Chromium 51.9961	25 Mn Manganese 54.938	26 Fe Iron 55.847	27 Co Cobalt 58.9332	28 Ni Nickel 58.6934	29 Cu Copper 63.546	30 Zn Zinc 65.39	31 Ga Gallium 69.723	32 Ge Germanium 72.64	33 As Arsenic 74.92159	34 Se Selenium 78.96	35 Br Bromine 79.904	36 Kr Krypton 83.80						
37 Rb Rubidium 85.4678	38 Sr Strontium 87.62	39 Y Yttrium 88.90585	40 Zr Zirconium 91.224	41 Nb Niobium 92.90638	42 Mo Molybdenum 95.94	43 Tc Technetium 98.9062	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.9055	46 Pd Palladium 106.42	47 Ag Silver 107.8682	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.71	51 Sb Antimony 121.760	52 Te Tellurium 127.6	53 I Iodine 126.90447	54 Xe Xenon 131.29						
55 Cs Cesium 132.90543	56 Ba Barium 137.327	57-71	72 Hf Hafnium 178.49	73 Ta Tantalum 180.9479	74 W Tungsten 183.85	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.22	78 Pt Platinum 195.08	79 Au Gold 196.9665	80 Hg Mercury 200.59	81 Tl Thallium 204.3833	82 Pb Lead 207.2	83 Bi Bismuth 208.98037	84 Po Polonium (209 9424)	85 At Astatine 209 9871	86 Rn Radon 222.0176						
87 Fr Francium 223 9157	88 Ra Radium 226 0254	89-103	104 Rf Rutherfordium (261)	105 Db Dubnium (262)	106 Sg Seaborgium (266)	107 Bh Bohrium (264)	108 Hs Hassium (285)	109 Mt Meitnerium (288)	110 Ds Darmstadtium (289)	111 Rg Roentgenium (272)	112 Cn Copernicium (277)	113 Uut Ununtrium unknown	114 Fl Flerovium (289)	115 Uup Ununpentium unknown	116 Lv Livermorium (293)	117 Uus Ununseptium unknown	118 Uuo Ununoctium unknown						
Lanthanide Series		57 La Lanthanum 138.9055	58 Ce Cerium 140.115	59 Pr Praseodymium 140.90765	60 Nd Neodymium 144.24	61 Pm Promethium 144.9127	62 Sm Samarium 150.36	63 Eu Europium 151.9655	64 Gd Gadolinium 157.25	65 Tb Terbium 158.92534	66 Dy Dysprosium 162.50	67 Ho Holmium 164.93032	68 Er Erbium 167.26	69 Tm Thulium 168.93421	70 Yb Ytterbium 173.04	71 Lu Lutetium 174.967							
Actinide Series		89 Ac Actinium 227 0277	90 Th Thorium 232 0381	91 Pa Protactinium 231 03606	92 U Uranium 238 02891	93 Np Neptunium 237 04817	94 Pu Plutonium 244 0602	95 Am Americium 243 0604	96 Cm Curium 247 0607	97 Bk Berkelium 247 0609	98 Cf Californium 251 0762	99 Es Einsteinium (261)	100 Fm Fermium 267 0681	101 Md Mendelevium 268 1	102 No Nobelium 269 1098	103 Lr Lawrencium 262							
		Alkali Metals		Alkaline Earths		Transition Metals		Basic Metals		Semi-Metals		Nonmetals		Halogens		Noble Gases		Lanthanides		Actinides			

Design new interacting materials with exotic properties



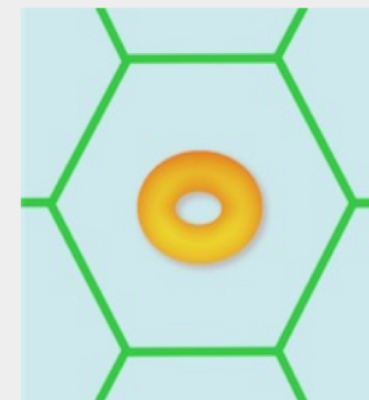
Topological Quantum Matter: Concepts and Realizations

Coordinators: Andriy Nevidomskyy, Nic Shannon, Ronny Thomale, and Roser Valenti

Scientific Advisors: Yukitoshi Motome, Natasha Perkins, Oleg Tchernyshyov

One of the recurring central themes in physics is the search for exotic phases of matter stemming from strong correlations between constituent particles. Among these, topological states have become a major research direction in the past decade, from quantum spin liquids, to topological insulators and superconductors, to examples in photonics and mechanical systems. Despite a plethora of promising visions towards application and implementation, a number of serious challenges remain. One of them is the lack of reliable models, with the exception of very few that lend themselves to an exact solution, to address the emergence and stability of topological phases in the presence of strong interactions. Another major challenge is the difficulty in elevating the description of topological phases from the zero-temperature ground state to finite temperatures, which is necessary to probe them experimentally, and ultimately render them accessible at technologically operable conditions.

To successfully address these problems, this program will bring together theorists, computational physicists, and a broad range of experimentalists. The goal is to stimulate the dialogue between practitioners of different approaches, to identify, as concretely as possible, the open problems in the field, and by bringing together experts from different communities, to help advance the research frontier.



DATES

Aug 26, 2019 - Nov 22, 2019

INFORMATION

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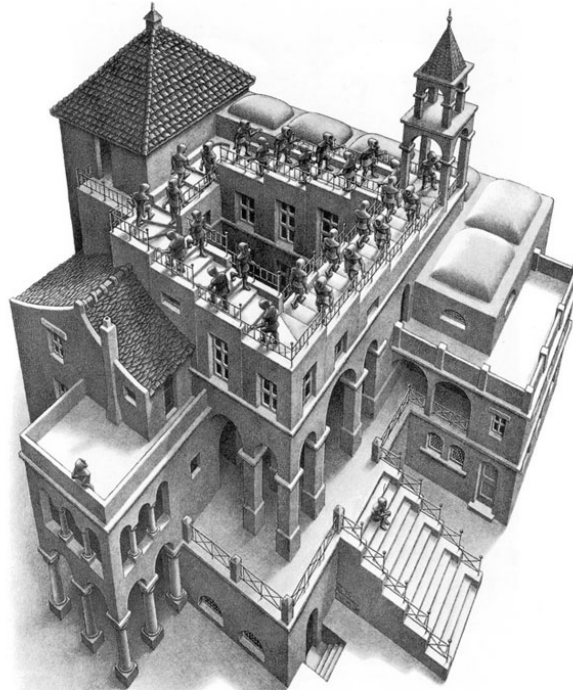
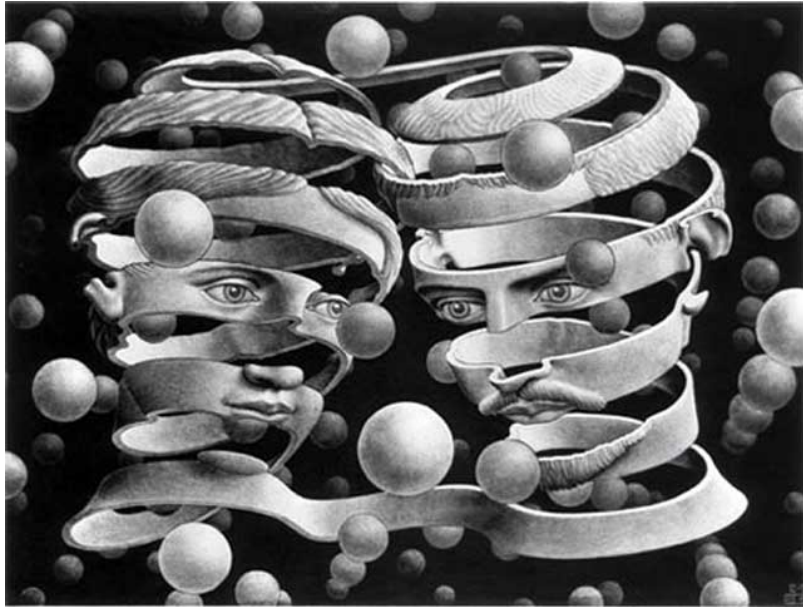
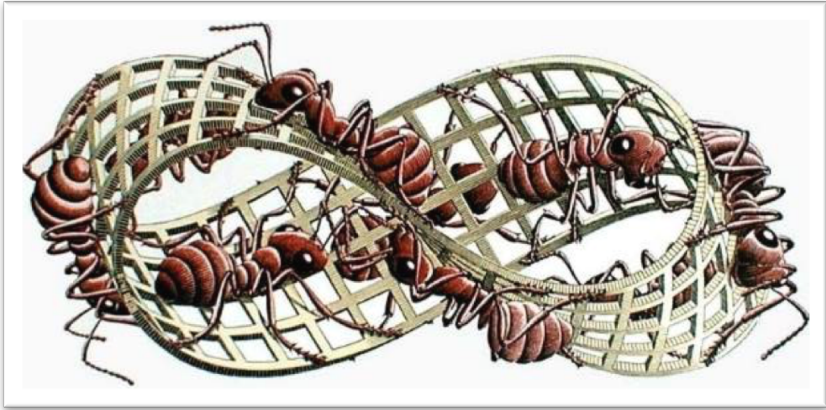
Application deadline has passed:

Sep 9, 2018

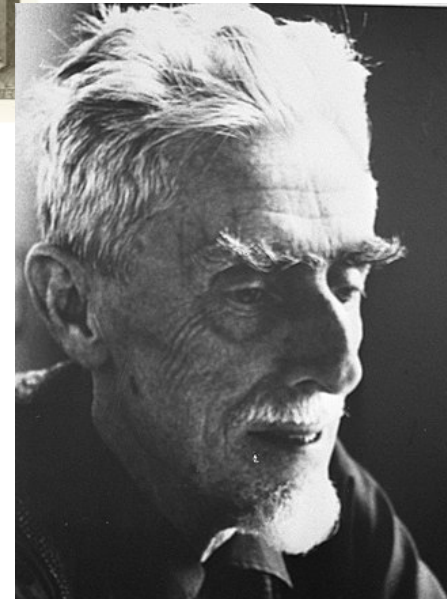
Primary deadline above date.

Rolling admissions after

Topology, Physics and Art



Maurits Cornelius
Escher (1989-1872)



THANK YOU!

Emergence of exotic “quasi-particles”

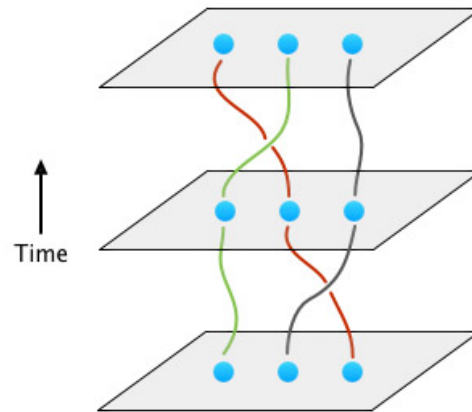
Anyons

(Alexei Kitaev)

Topological quantum computers

Braiding anyons

Qubit: 2 anyons



Operations protected by topology

