Postselection-free entanglement dynamics via spacetime duality

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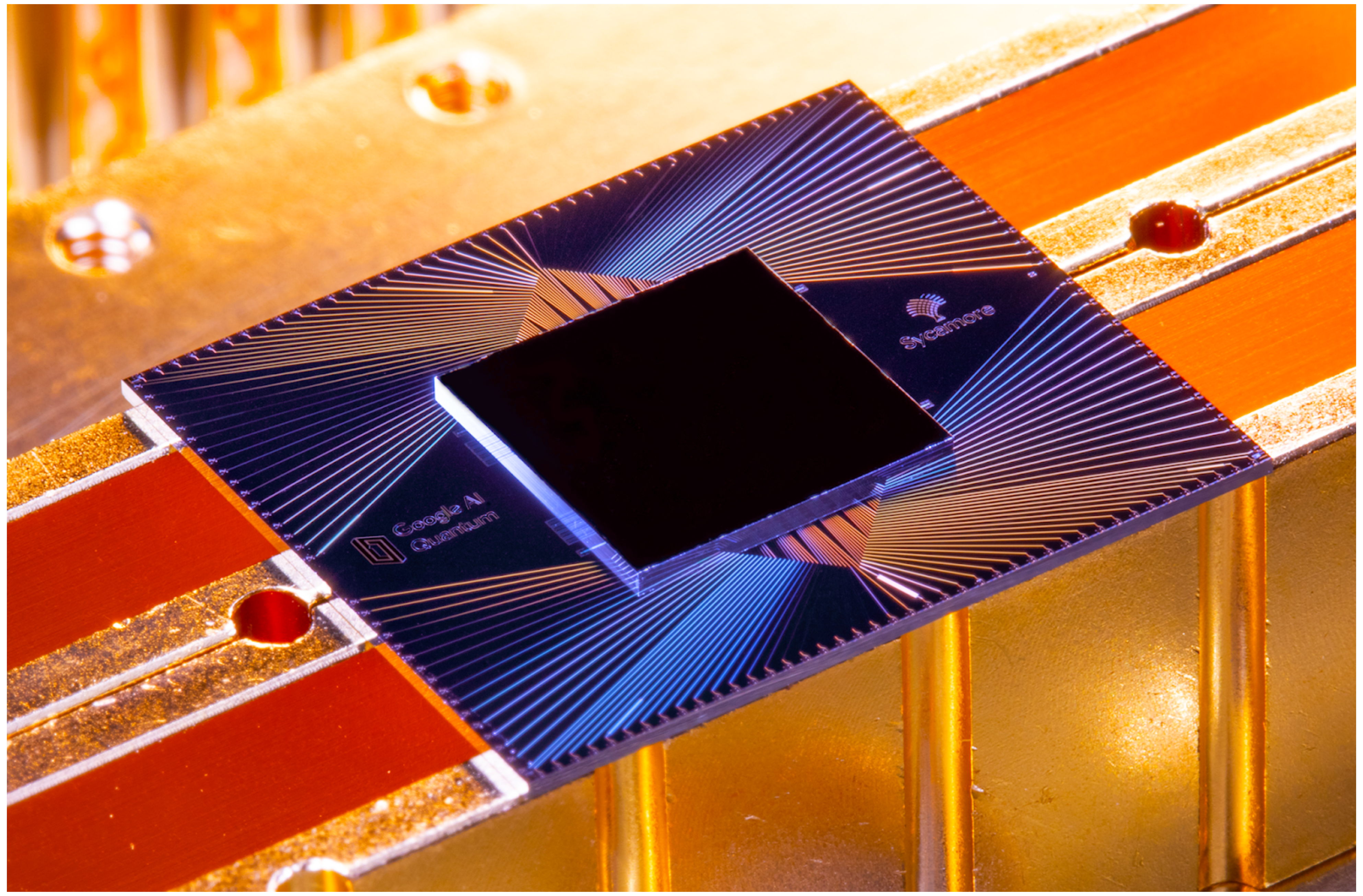
Based on arXiv:2010.xxxxx with V. Khemani

Frontiers of Quantum Computing and Quantum Dynamics, KITP, 10/20/2020

Introduction

Frontiers of quantum dynamics

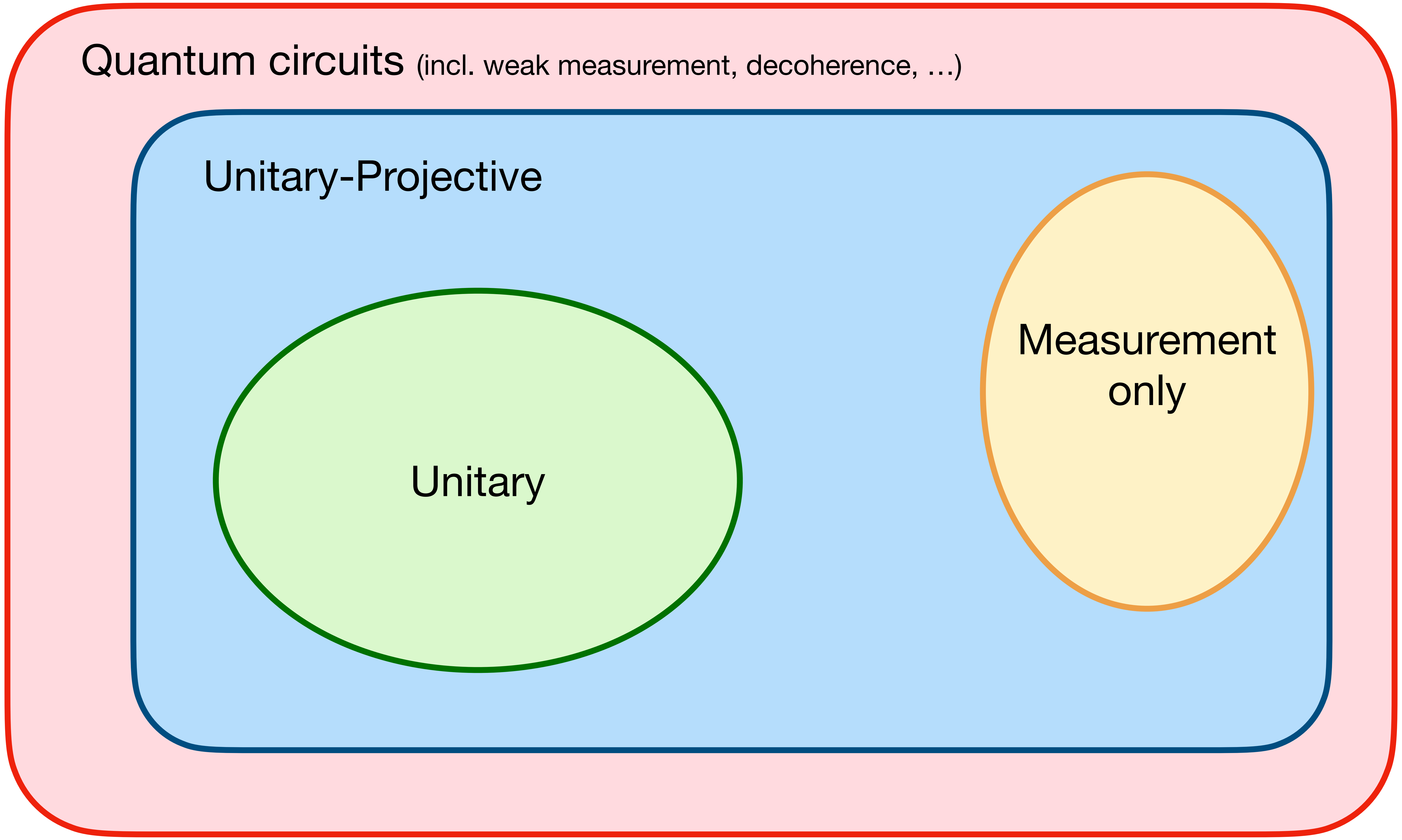
- NISQ devices: unprecedented control, measurement capabilities
- New focus on out-of-equilibrium dynamics, open systems
- Many exciting questions
  - Phases?
  - Universality?
  - Chaos?
  - Classical/quantum simulability?
  - **Entanglement**?



Google's Sycamore chip

Introduction

Entanglement dynamics in quantum circuits



Nahum, Ruhman, Vijay, Haah 16;
von Keyserlingk, Rakovszky, Pollmann, Sondhi 17;
Khemani, Vishwanath, Huse 17;
Zhou, Chen 18;
...

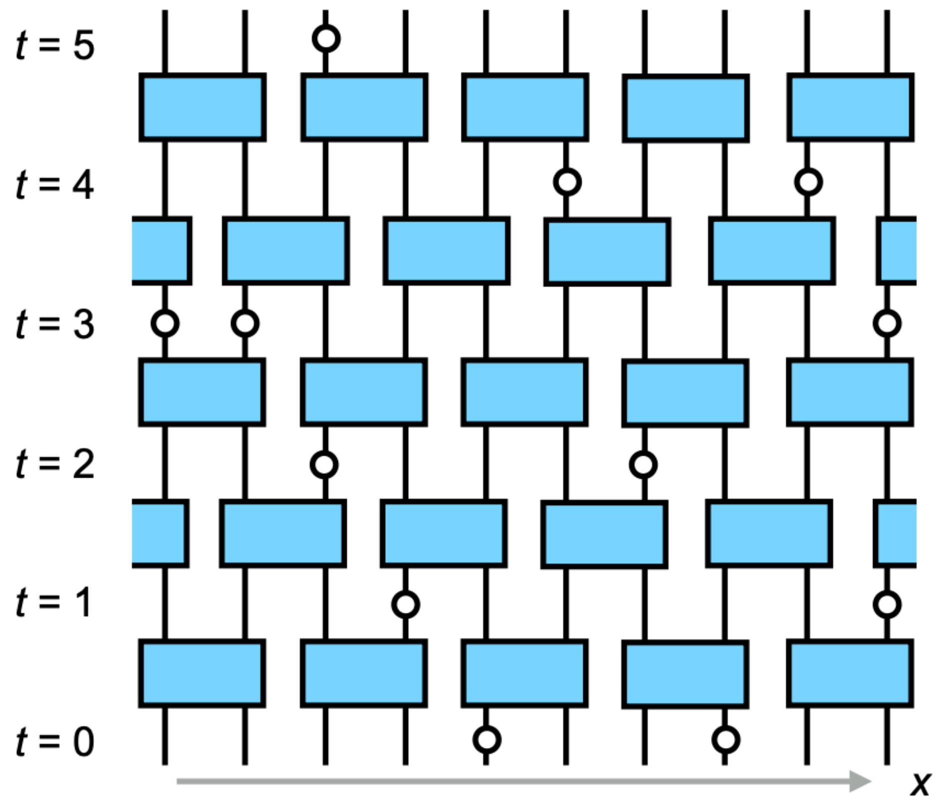
Li, Chen, Fisher 18;
Skinner, Ruhman, Nahum 18;
Choi, Bao, Qi, Altman 19;
Gullans, Huse 19;
...

Nahum, Skinner 19;
MI, Gullans, Gopalakrishnan, Huse, Khemani 20;
Lavasani, Alavirad, Barkeshli 20;
Sang, Hsieh 20;

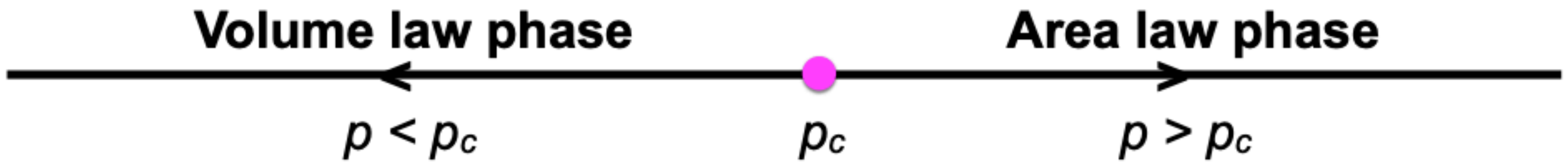
Introduction

Entanglement phases in unitary-projective circuits

- Random unitary circuits + projective measurements: **entanglement phases** [Li, Chen, Fisher 18; Skinner, Ruhman, Nahum 18]
- Many open questions on the nature of phases & transition



Li, Chen, Fisher arXiv:1901.08092



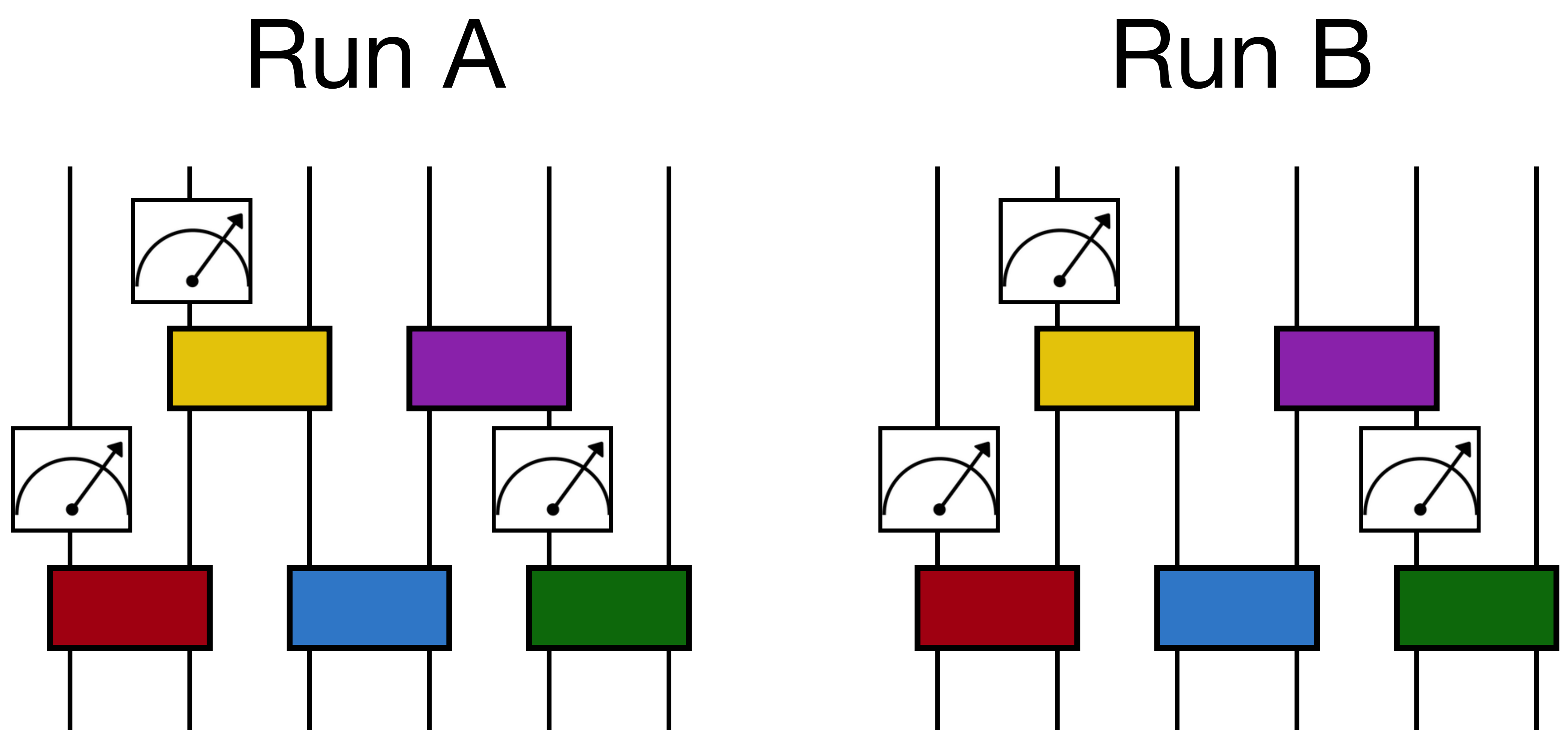
Nature of QEC code?
Subleading entropy terms
in Clifford models?

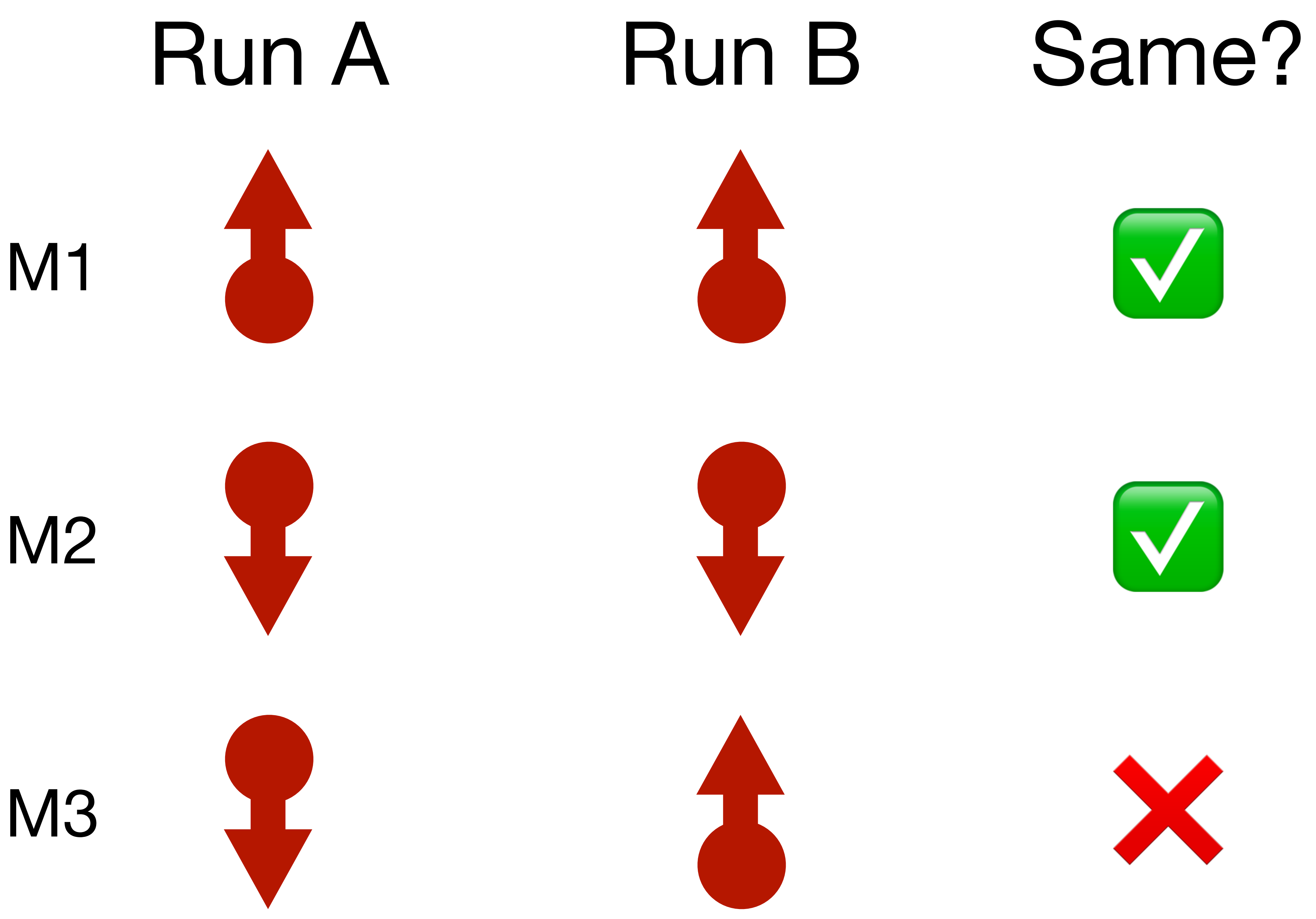
Order?
Phases?

Universality?
CFT?

The problem of postselection

- Measuring entropy: $\mathrm{Tr}(\rho \otimes \rho \cdot \mathrm{SWAP}) = \mathrm{Tr}(\rho^2)$ [Greiner et al 15]
- Producing two identical copies of the state is hard!
  - Quantum randomness of measurement outcomes requires **postselection**
  - Exponential overhead





The problem of postselection

- Measuring entropy: $\mathrm{Tr}(\rho \otimes \rho \cdot \mathrm{SWAP}) = \mathrm{Tr}(\rho^2)$ [Greiner et al 15]
- Producing two identical copies of the state is hard!
  - Quantum randomness of measurement outcomes requires **postselection**
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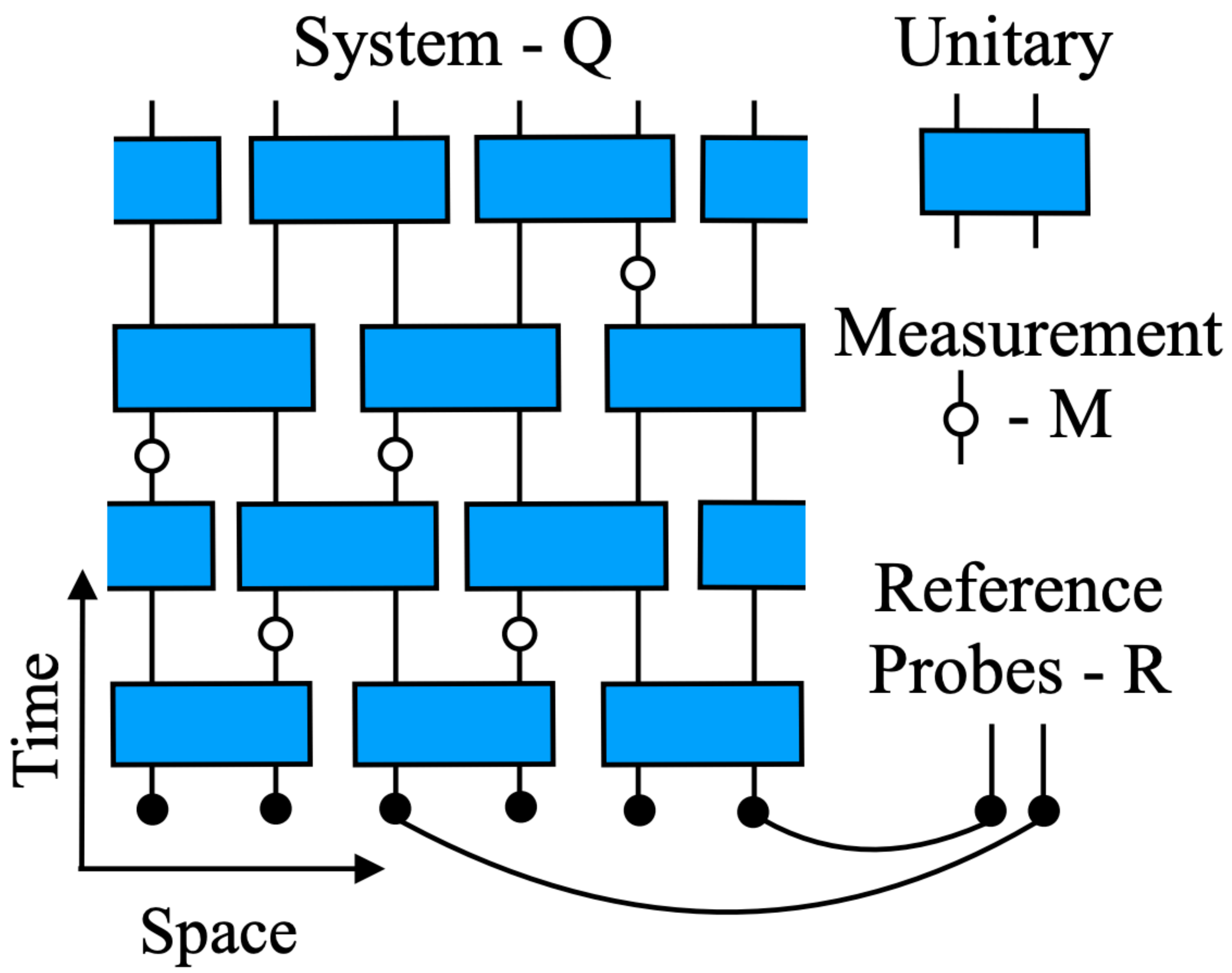
- Ways around this limitation [Gullans, Huse 19]
  - Local order parameter, "decoding lightcone"
  - Classical simulation + feedback (Clifford circuits)
- More direct access to dynamics of entanglement?

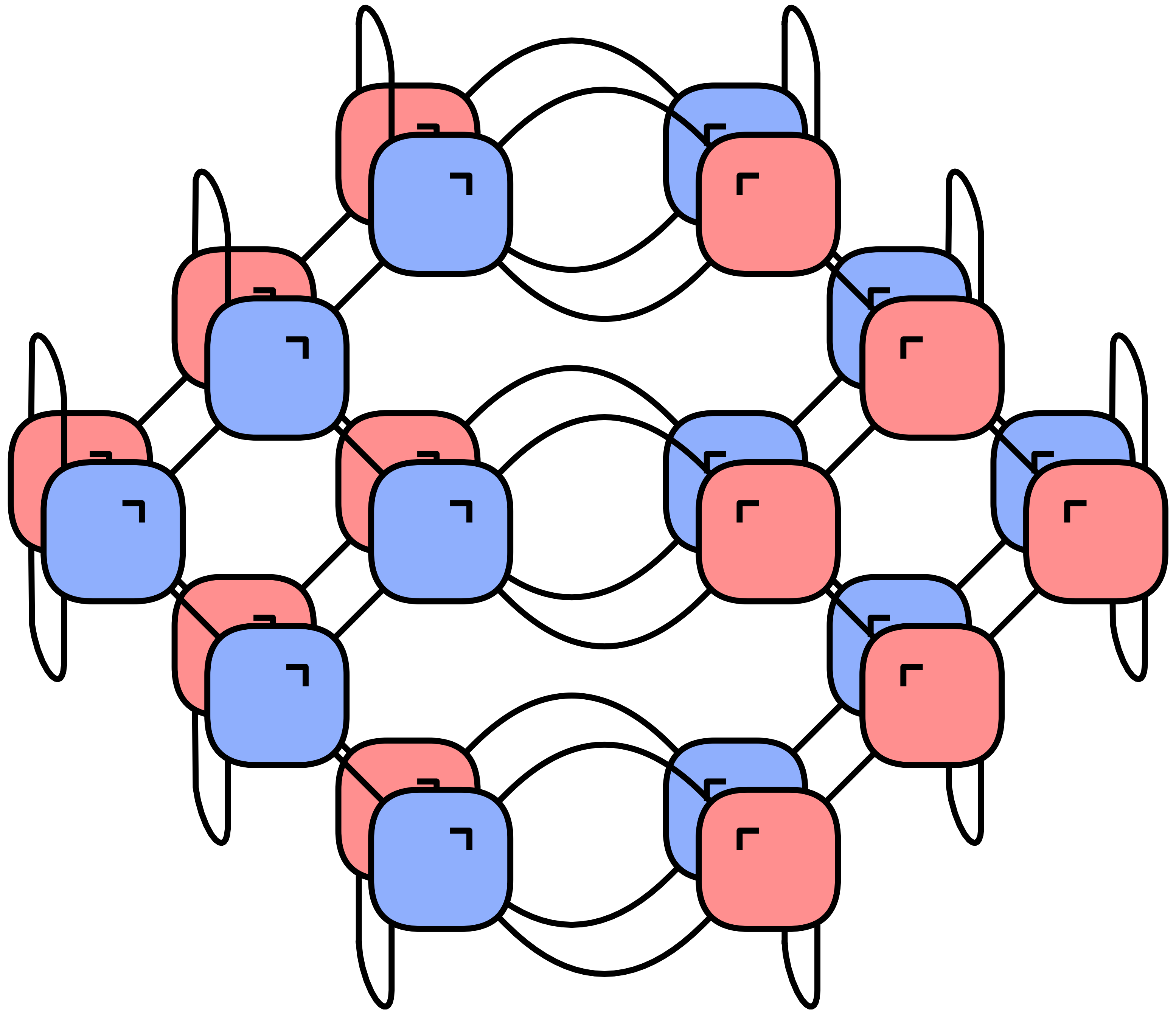


This talk

- A new way of realizing output states of a broad class of **non-unitary circuits:** *“spacetime duals”* of unitary circuits

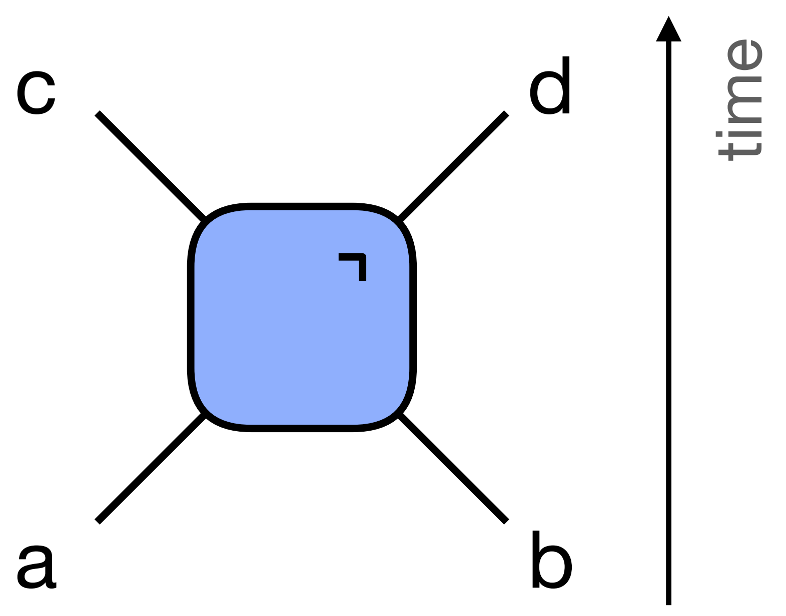
- Avoids **postselection problem** entirely

- Protocol for direct measurement of the **purity** / **entanglement entropy** of arbitrary subsystems as a correlation function in a unitary evolution

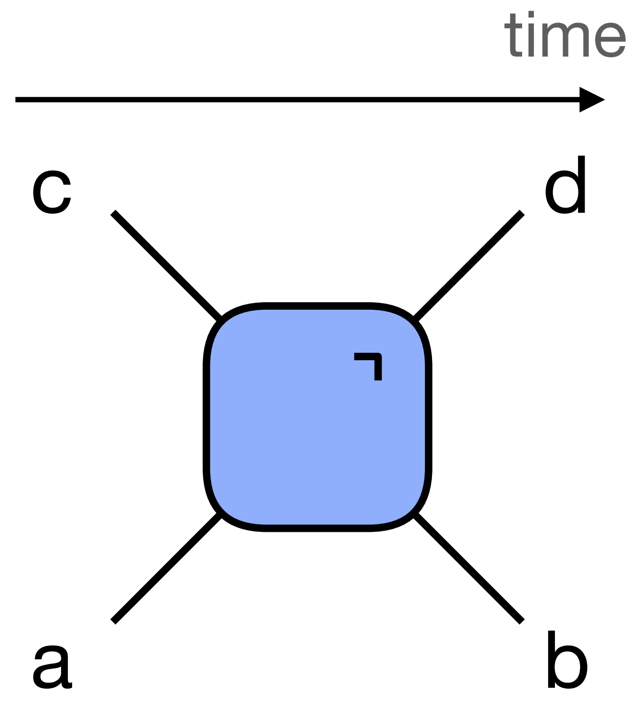


Spacetime duality

$U_{ab}^{cd} =$

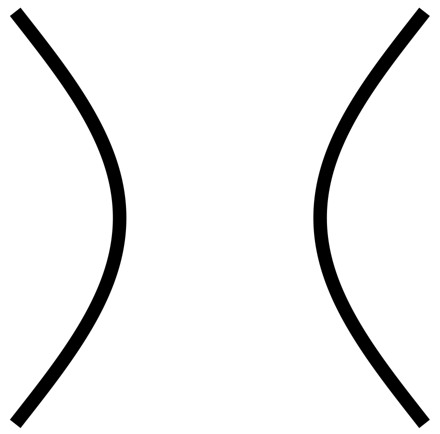


$\tilde{U}_{ca}^{db} =$



$$U = I$$

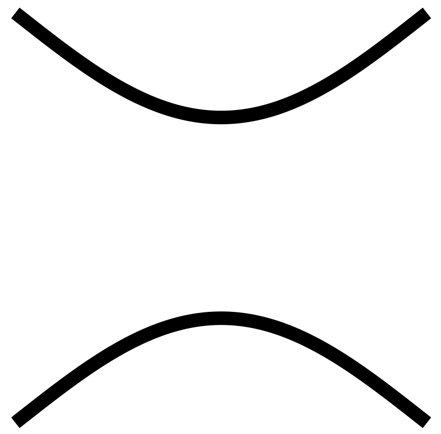
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



$$\tilde{U} = 2|\psi\rangle\langle\psi|$$

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

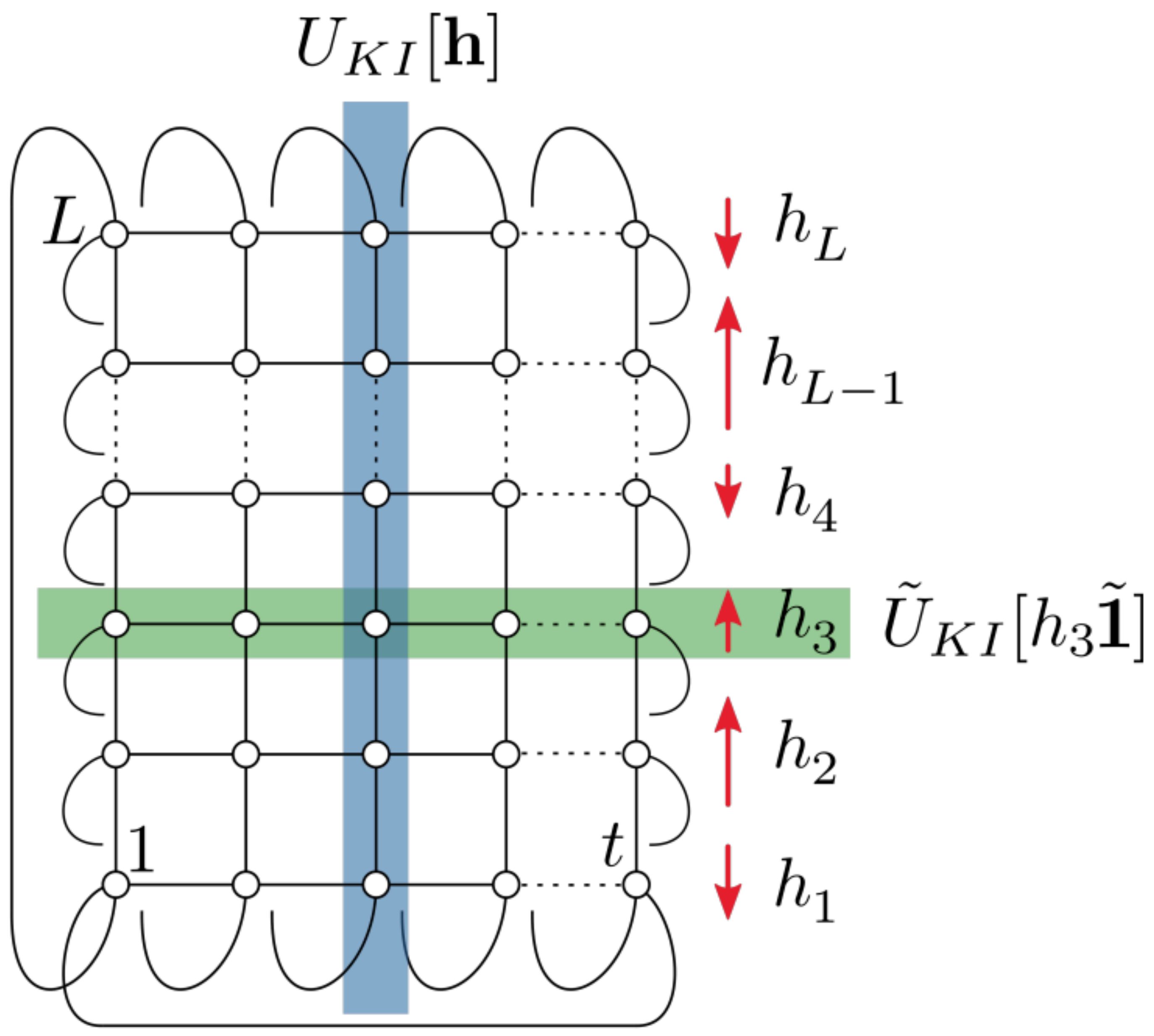


- Dualized gates need not be unitary! Generally, **weak measurements**
- **Forced** measurement - always same outcome!

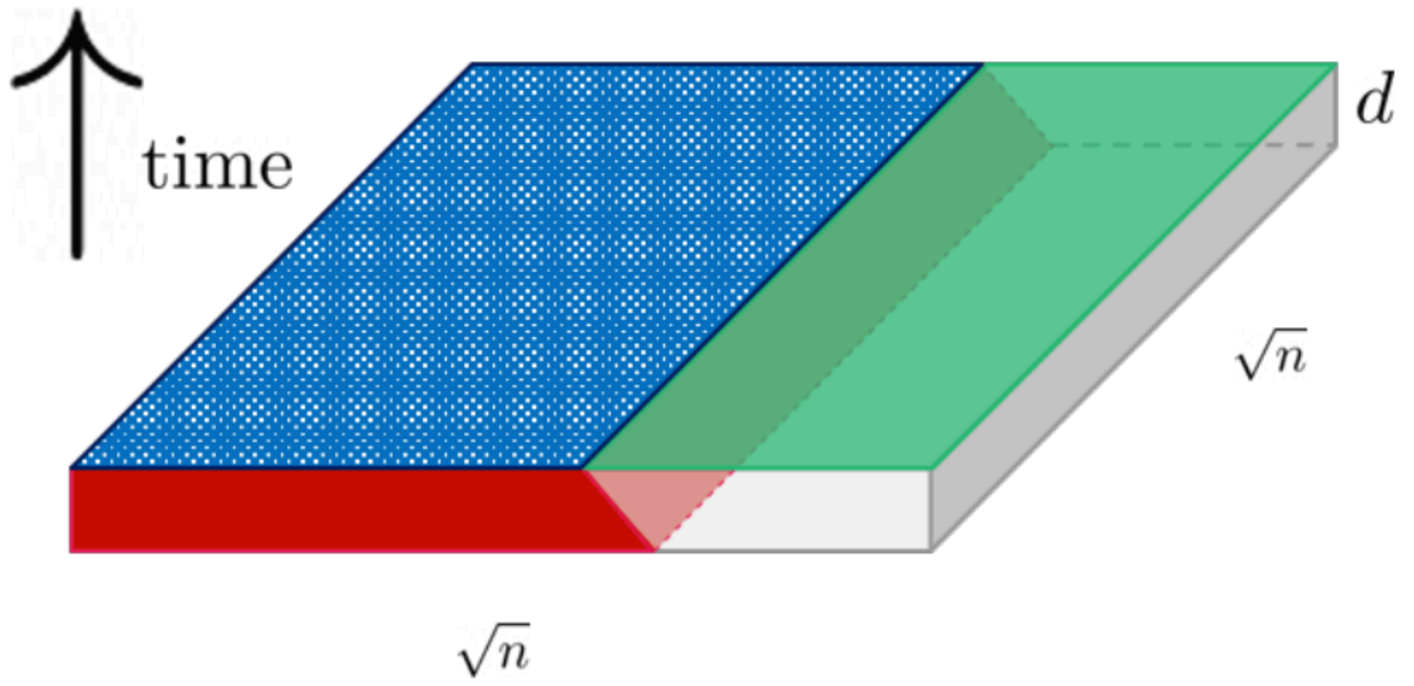
Spacetime duality

Some recent applications

- **Dynamics**
  - Dual-unitary circuits: exact connections to random-matrix theory, quantum chaos [Bertini, Kos, Prosen 18]
  - Analytical control over correlations, OTOCs, entanglement… [Prosen et al 18-20]
- **Complexity**
  - Optimized tensor network contractions [Harrow et al 20; Cirac et al 09]



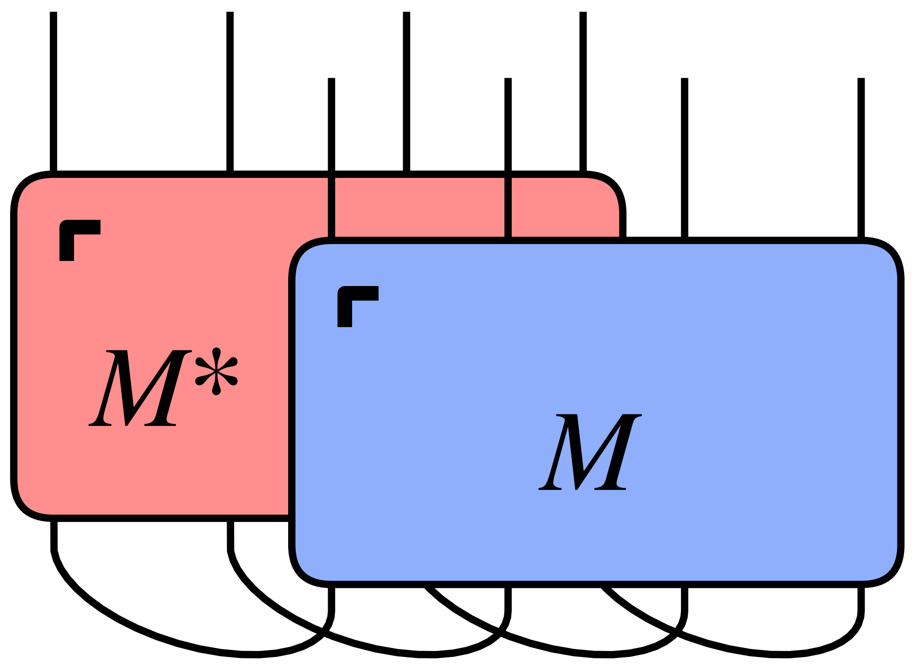
Bertini, Kos, Prosen
arXiv:1812.05090



Napp, La Placa, Dalzell, Brandao, Harrow,
arXiv:2001:00021

Purification dynamics

Definition

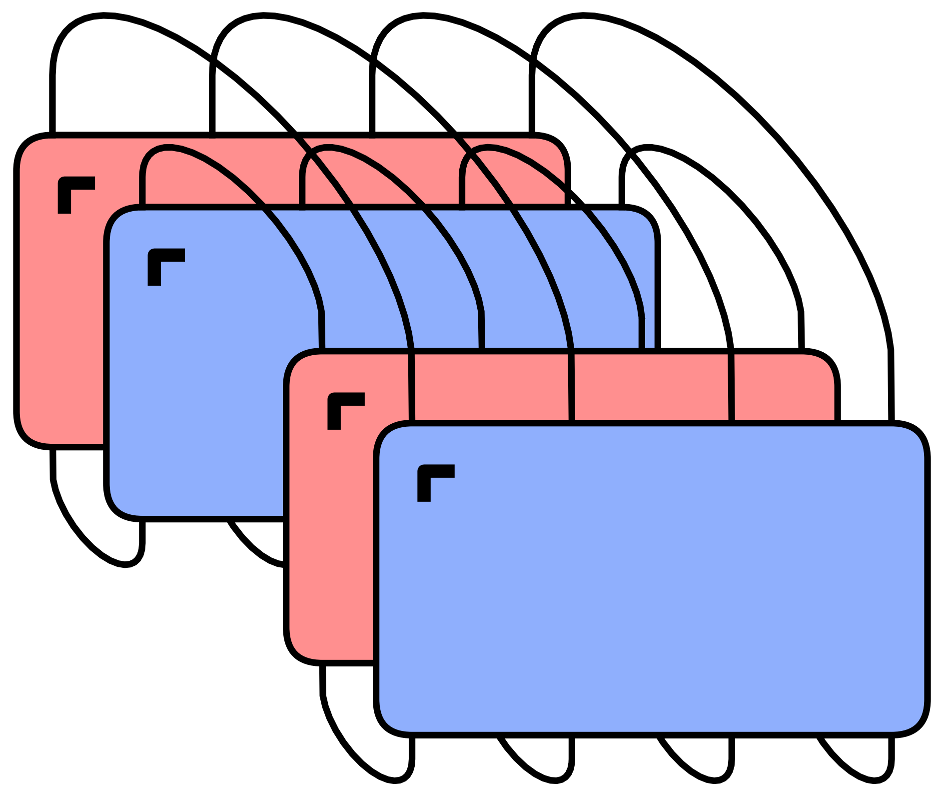


- Fully mixed state fed into a non-unitary circuit M
- The output state may or may not be purified:

$$\rho_{in} \propto I \mapsto \rho_{out} = M\rho_{in}M^\dagger \propto MM^\dagger$$

- **Purification phases** & transitions [Gullans, Huse 19]
  - Equivalent to the entanglement phases for pure-state inputs

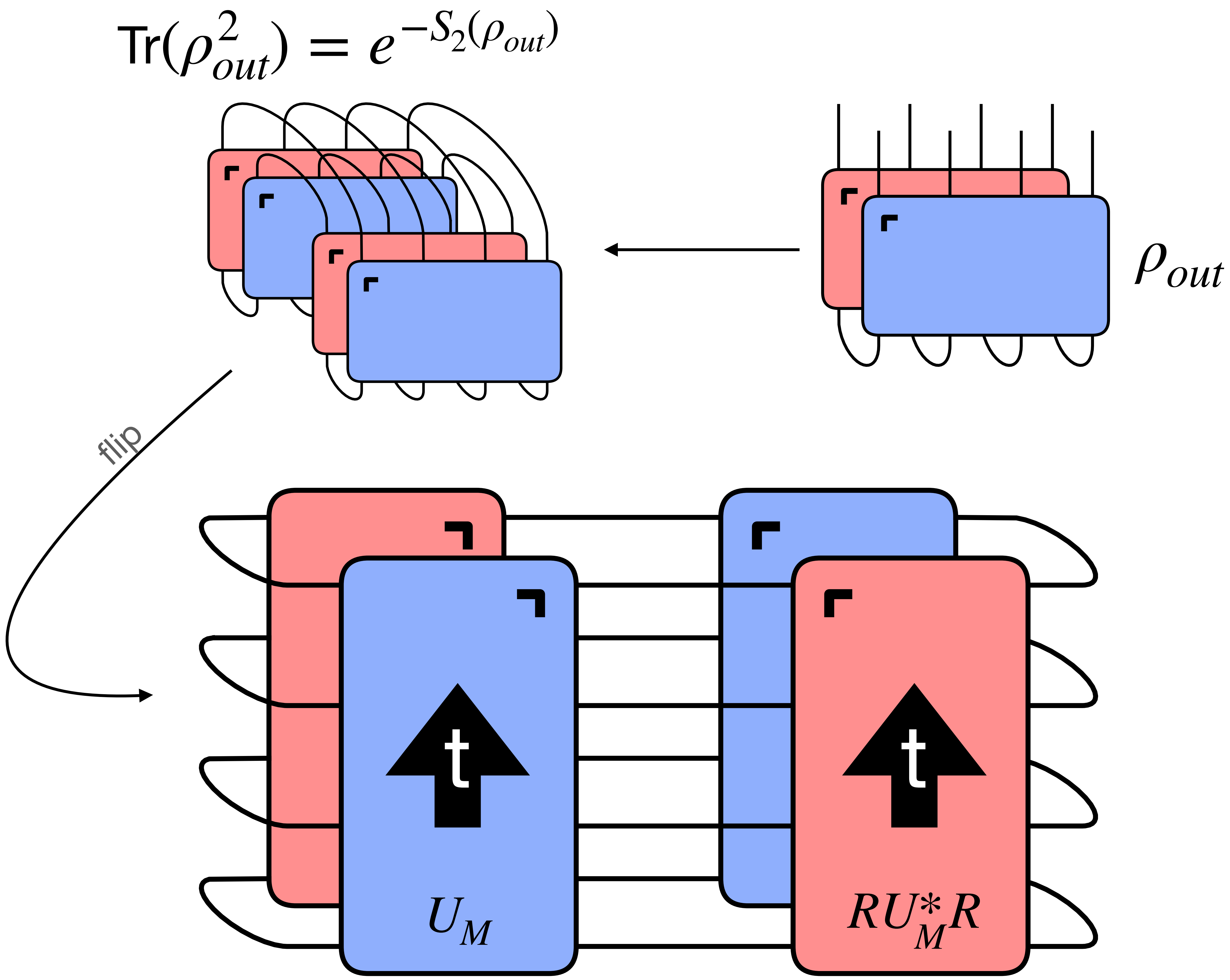
Purification phase	Related Entanglement phase	Purifies in time
Pure	Area-law	log(L)
Critical	Critical	O(L)
Mixed	Volume-law	exp(L)



Purity:
$\mathrm{Tr}(\rho_{out}^2)$

Purification dynamics

Spacetime duals of unitary circuits



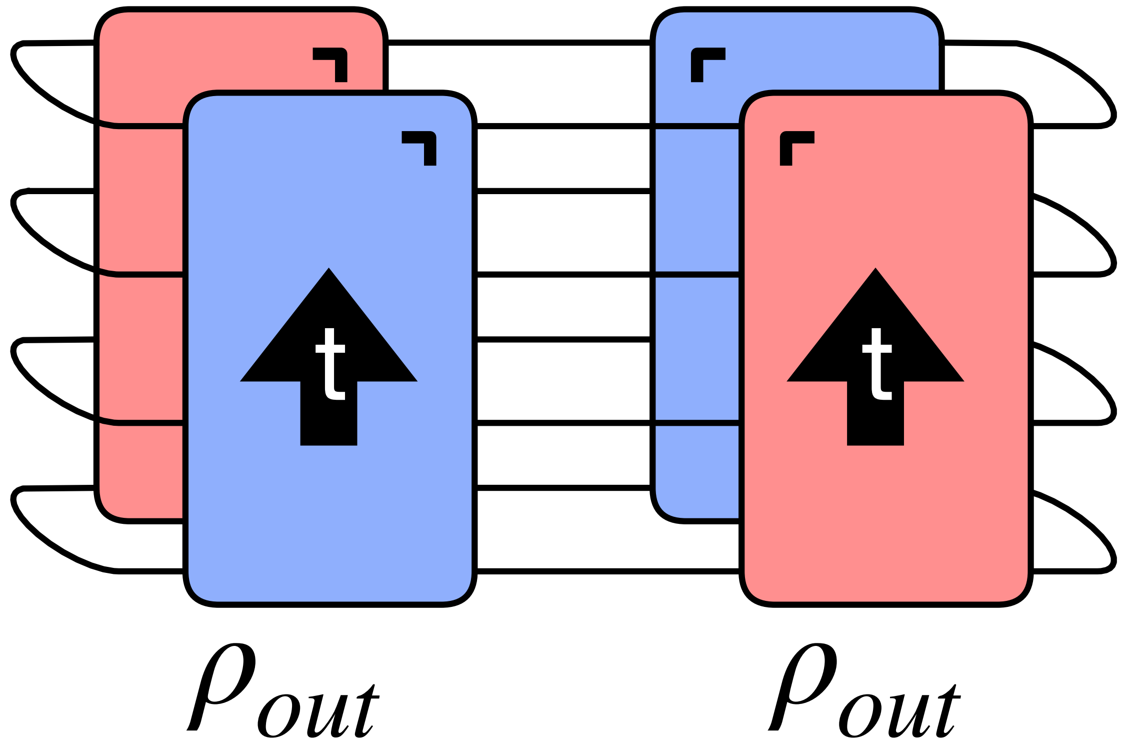
If $M = \tilde{U}_M$, with U_M **unitary:**

Purity of the output of **hybdrid circuit** M

Correlation function* in a **unitary circuit** $U_{left} \otimes U_{right}$

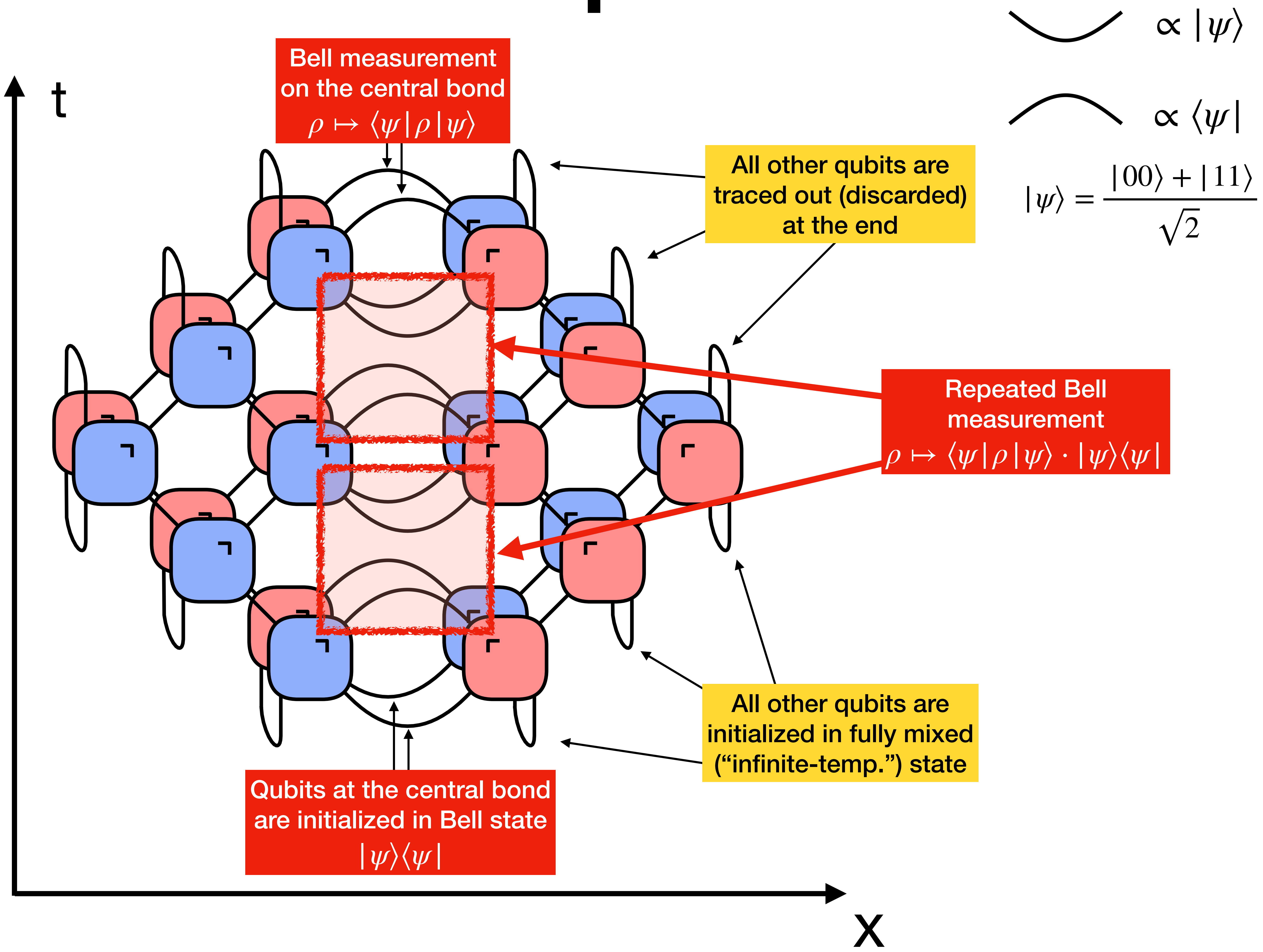
* [weird, multi-point]

Entanglement measurement protocol



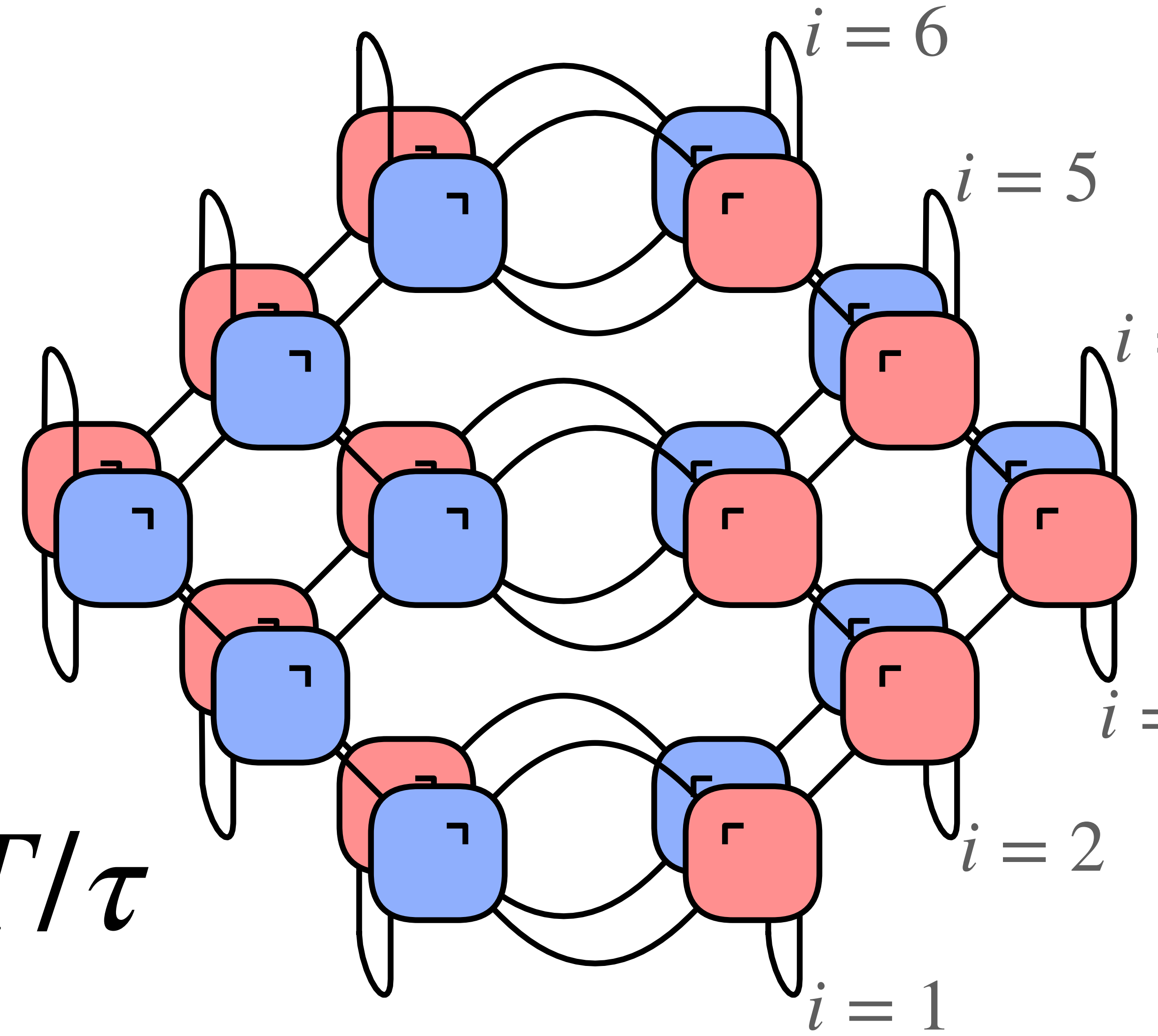
Operational protocol:

- Initialize $I \otimes |\psi\rangle\langle\psi| \otimes I$
- Every time a Bell measurement yields "wrong" outcome (not $|\psi\rangle$), **stop** and record a **failure**
- When T consecutive Bell measurements yield "right" outcome, record a **success**



Entanglement measurement protocol

- Run protocol N_{tot} times; $N_+(T)$ counts n. of runs successful up to time at least T
- Measures purification dynamics of $2T$ "dual" qubits evolved for time T



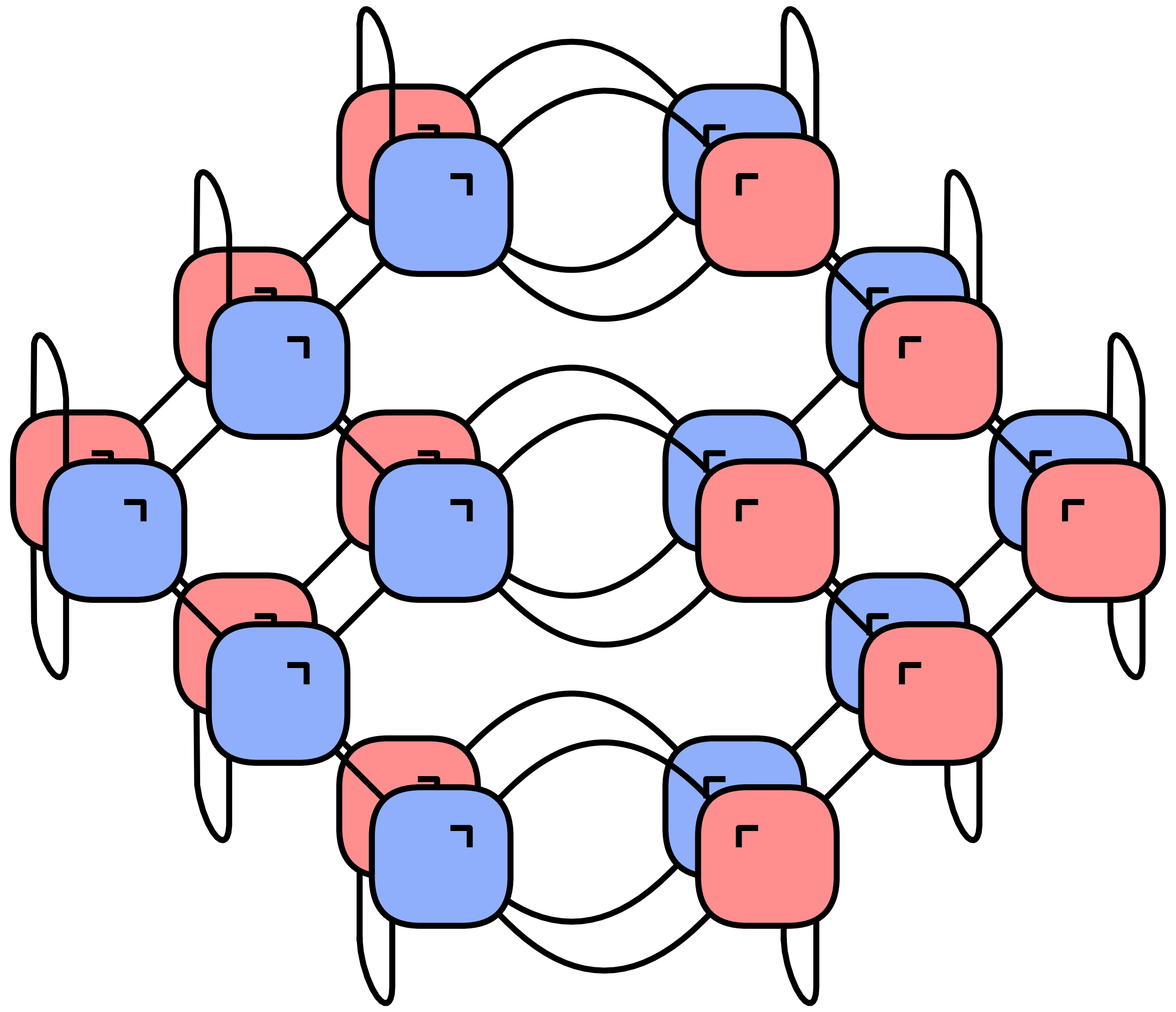
$$\mathrm{Tr}(\rho_{out}^2|_{2T}) = N_+(T)/N_{tot}$$

- Reasonable to expect $N_+(T) \sim Ne^{-T/\tau}$ in most cases…
  - …i.e. a **mixed phase**: $\mathrm{Tr}(\rho_{out}^2|_{2T}) \sim e^{-T/\tau} \implies S_2 \sim T/\tau$
  - **Entropy density** $s_2 \simeq (2\tau)^{-1}$ directly measurable!

Comments

- Doesn't use **postselection**
  - "Failed" runs contribute to N_{tot}, provide key information
  - Unitary circuits automatically prepare two copies of same state
- Constraint on which models can be implemented
  - **Wide class** - as big as space of unitary circuits
- Constraint on **maximum time** of hybrid dynamics
- The state ρ_{out} is never prepared all at once in space, exists on a **temporal slice** of the circuit
  - But it *can* be prepared in space, too!

$$\text{Tr}(\rho_{out}^2|_{2T}) = N_+(T)/N_{tot}$$

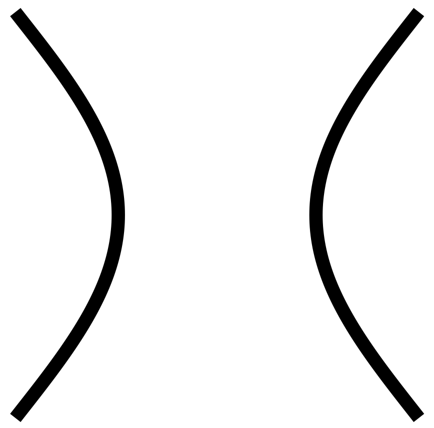


Purification phases and their signatures

Model

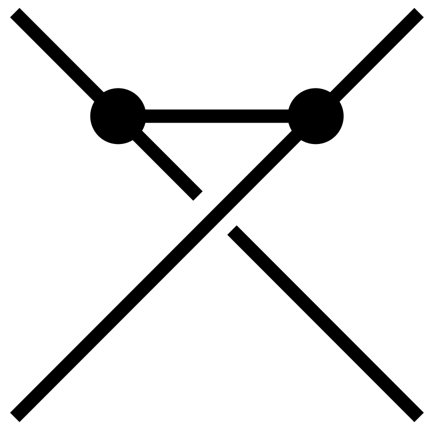
- Unitary Clifford circuit (efficient simulations); parameters $p, J \in [0,1]$
  - Two-qubit gates & their probabilities:

p:



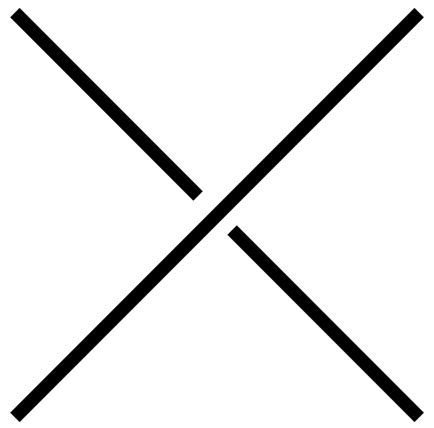
identity

$(1-p)J/2$:



iSWAP = SWAP · CZ

$(1-p)(1-J/2)$:



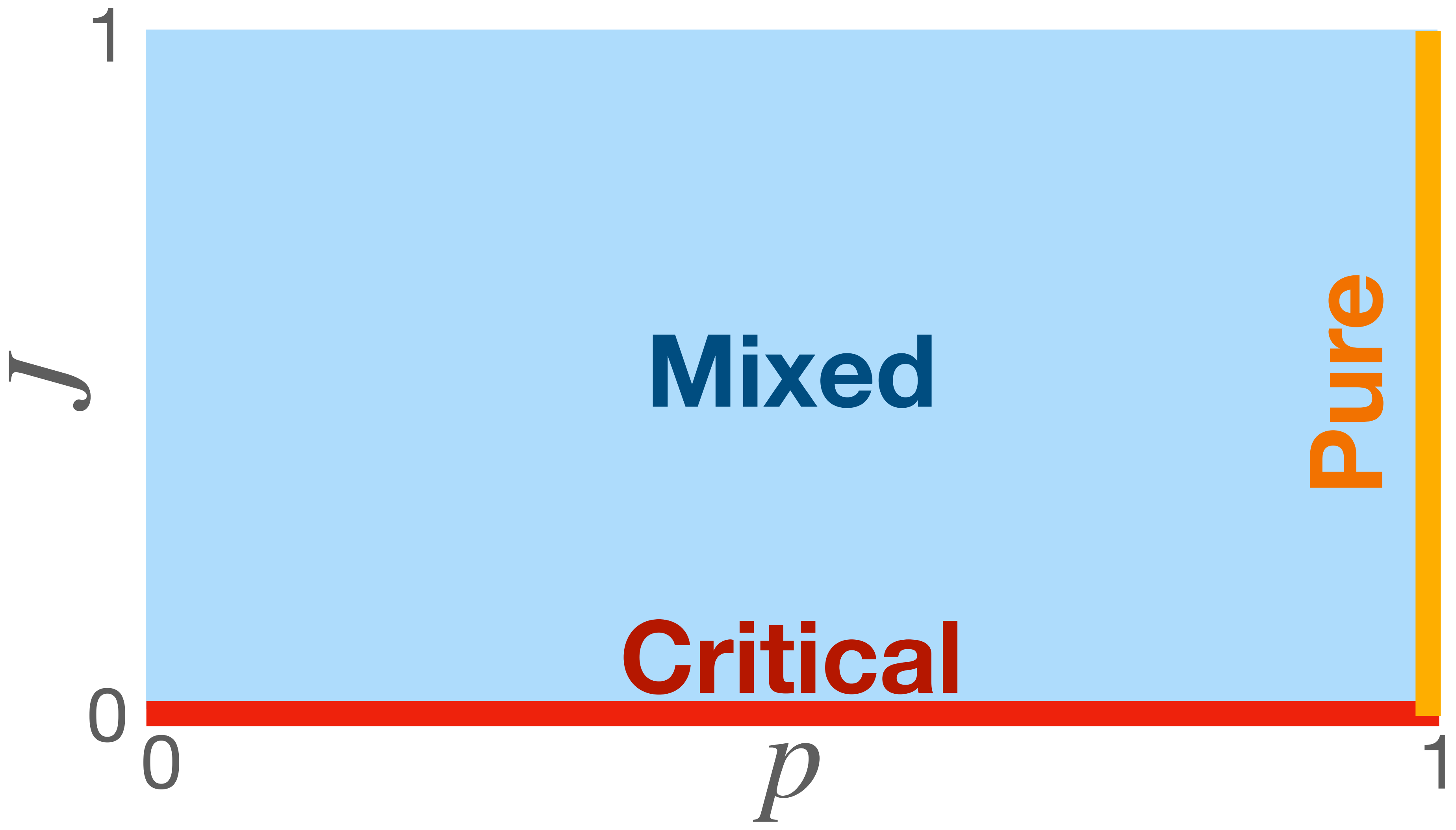
SWAP

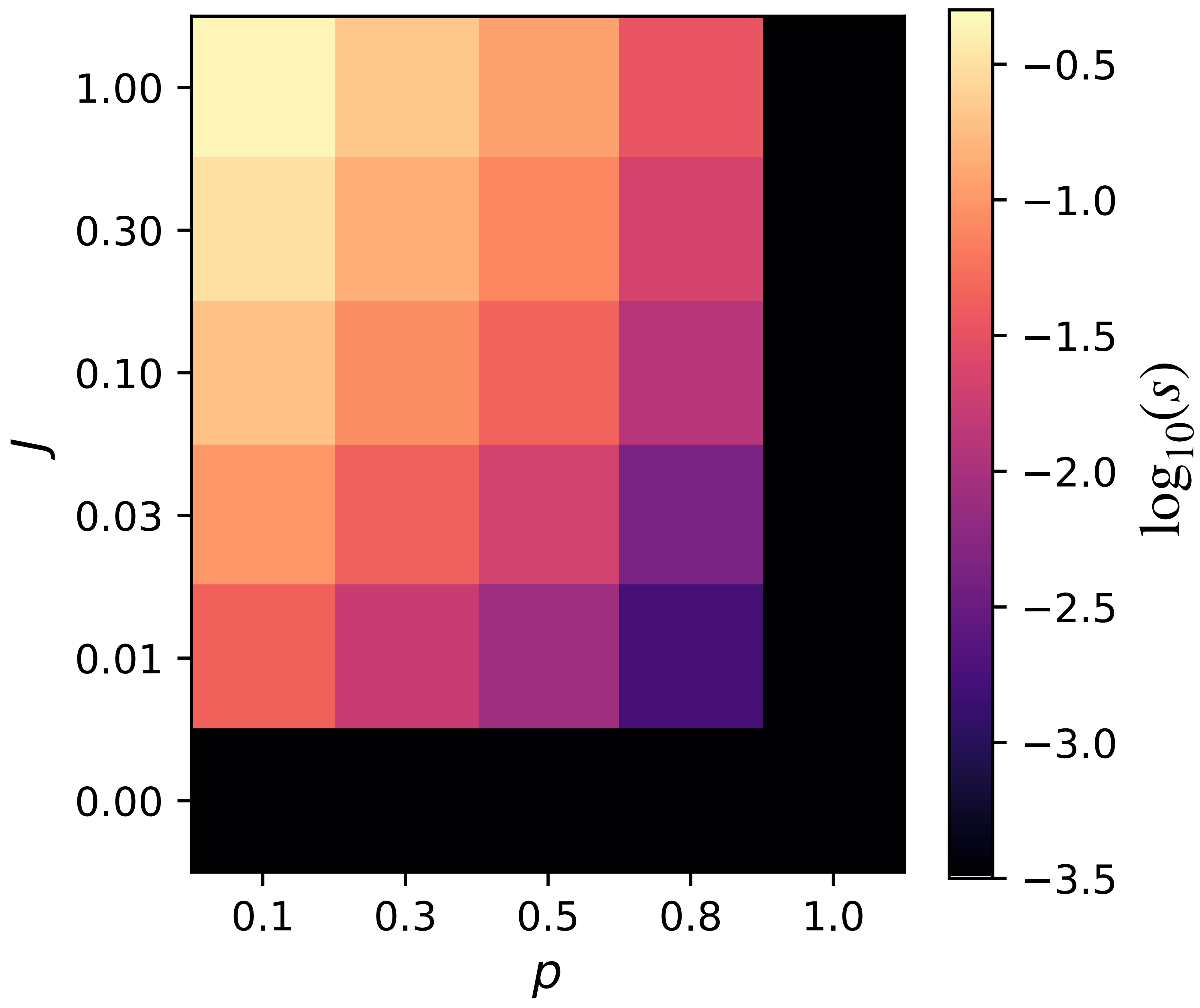
  - + random 1-qubit Clifford gates
- p tunes **measurement rate** in dual circuit
- J tunes **interaction rate** ($J = 0$ is noninteracting SWAP circuit)

Purification phases and their signatures

Phase diagram

- Numerical simulation (stabilizer method, $2T \leq 4096$ qubits)
- Entropy density: $s = \lim_{T\to\infty} S(\rho_{out}|_{2T})/2T$
- Purification phase diagram



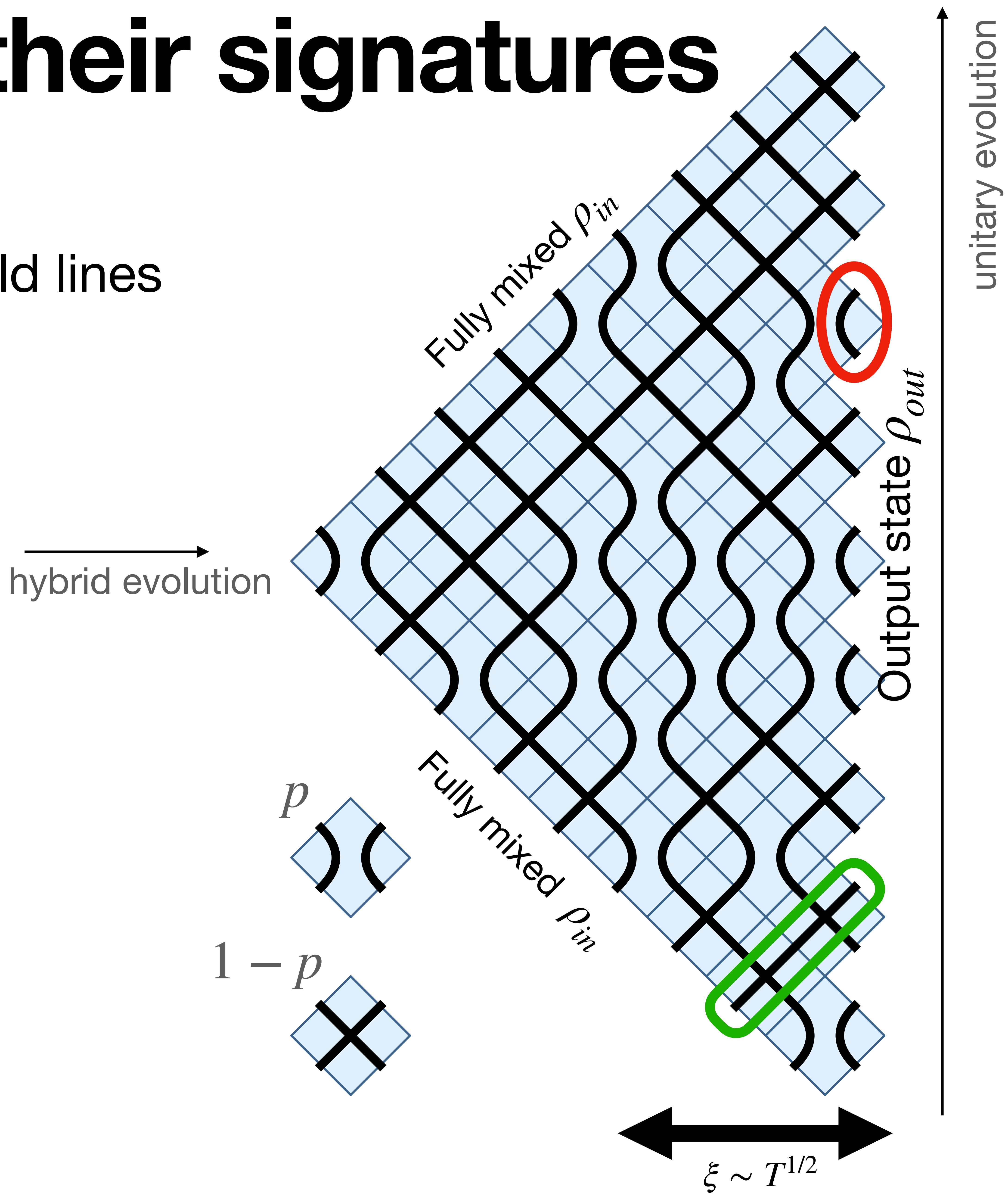


Purification phases and their signatures

Critical phase

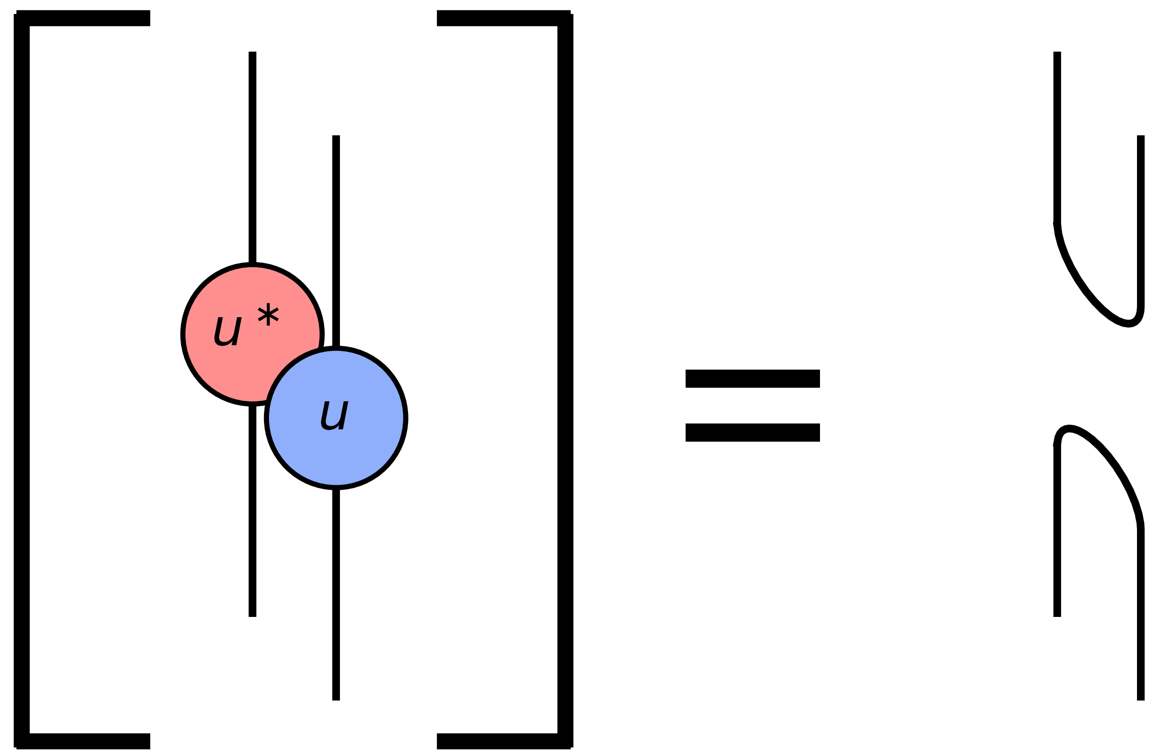
- $J = 0$: SWAP circuit, **random walk** of qubit world lines
- Qubit worldlines can:
  - start & end in ρ_{out}. Pure Bell pair, $S = 0$
  - start in ρ_{in}, end in ρ_{out} (or vice versa). Fully mixed qubit, $S = 1$
- **Diffusion:** $S \sim T^{1/2}$
- **Result:** Tr$(\rho_{out}^2|_{2T}) \sim e^{-\#\sqrt{T}}$

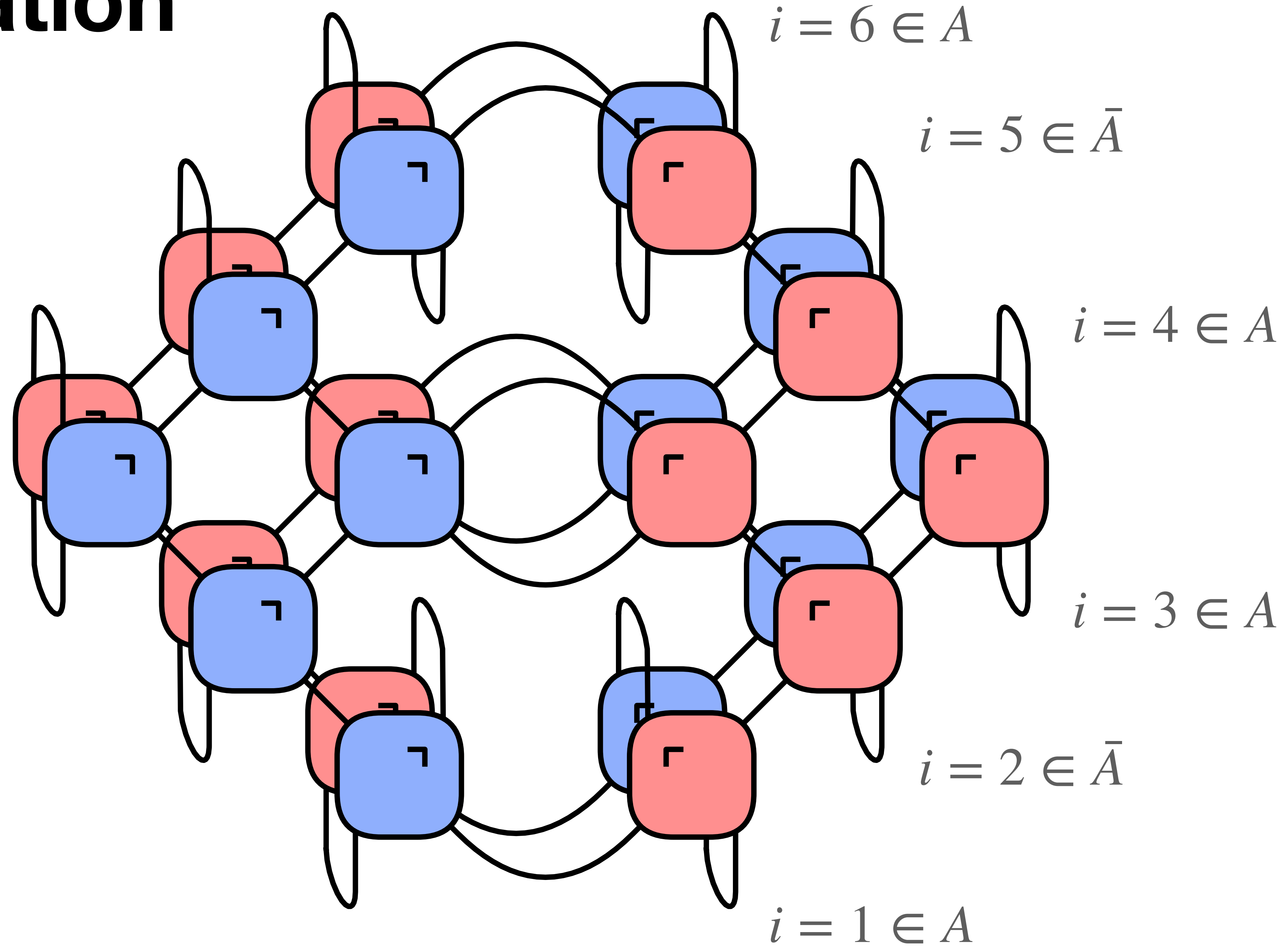
(cf. $e^{-T/\tau}$ for mixed phase)



Subsystem entropies

- Method can be generalized to measure $\mathrm{Tr}(\rho^2_{out,A})$ for **arbitrary subsystem** A
- How to "trace out" a physical qubit? **Depolarization**
  - Random Clifford gate + averaging many runs

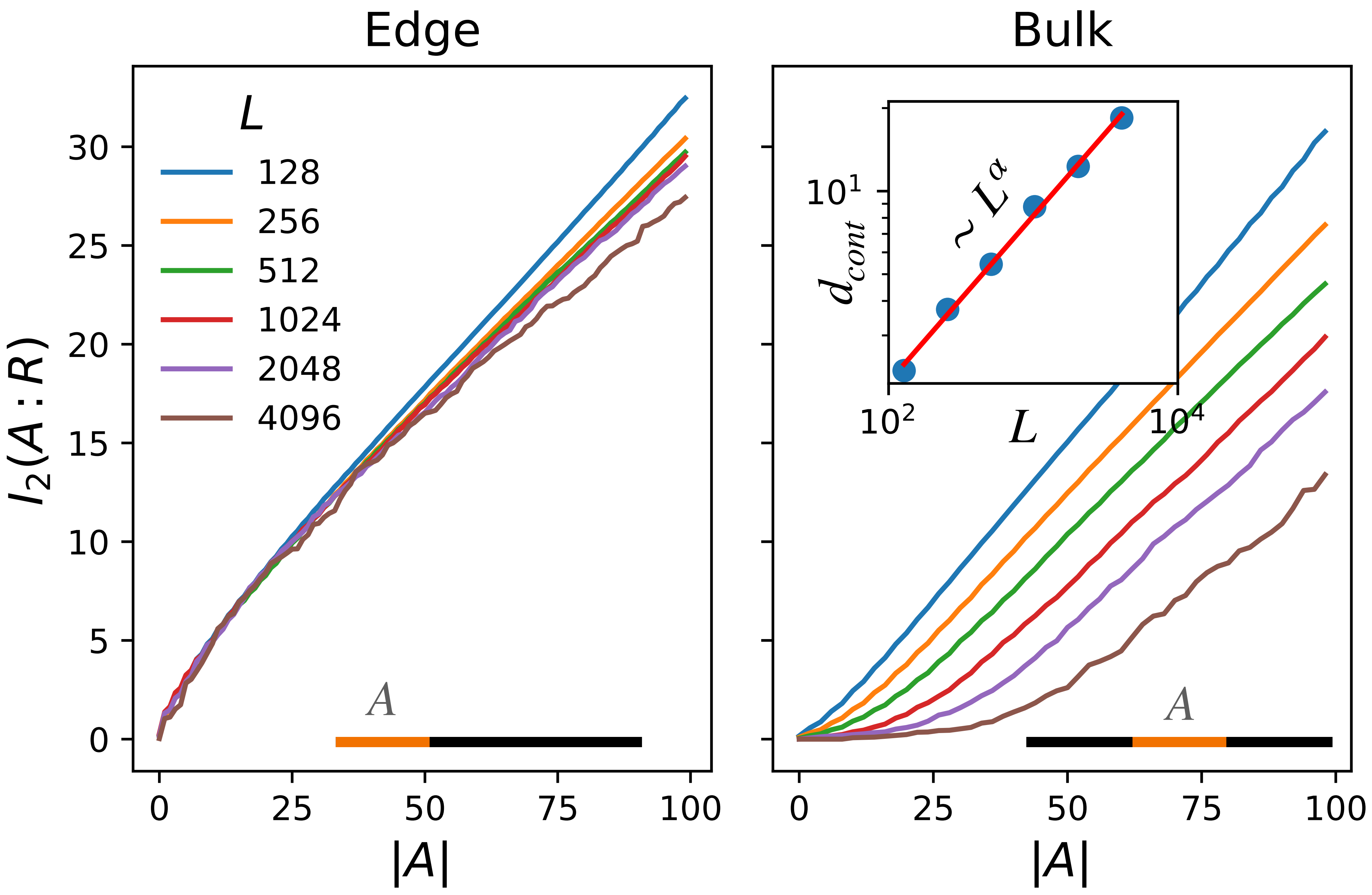
$\mathbb{E}_{u\in U(2)}$ 

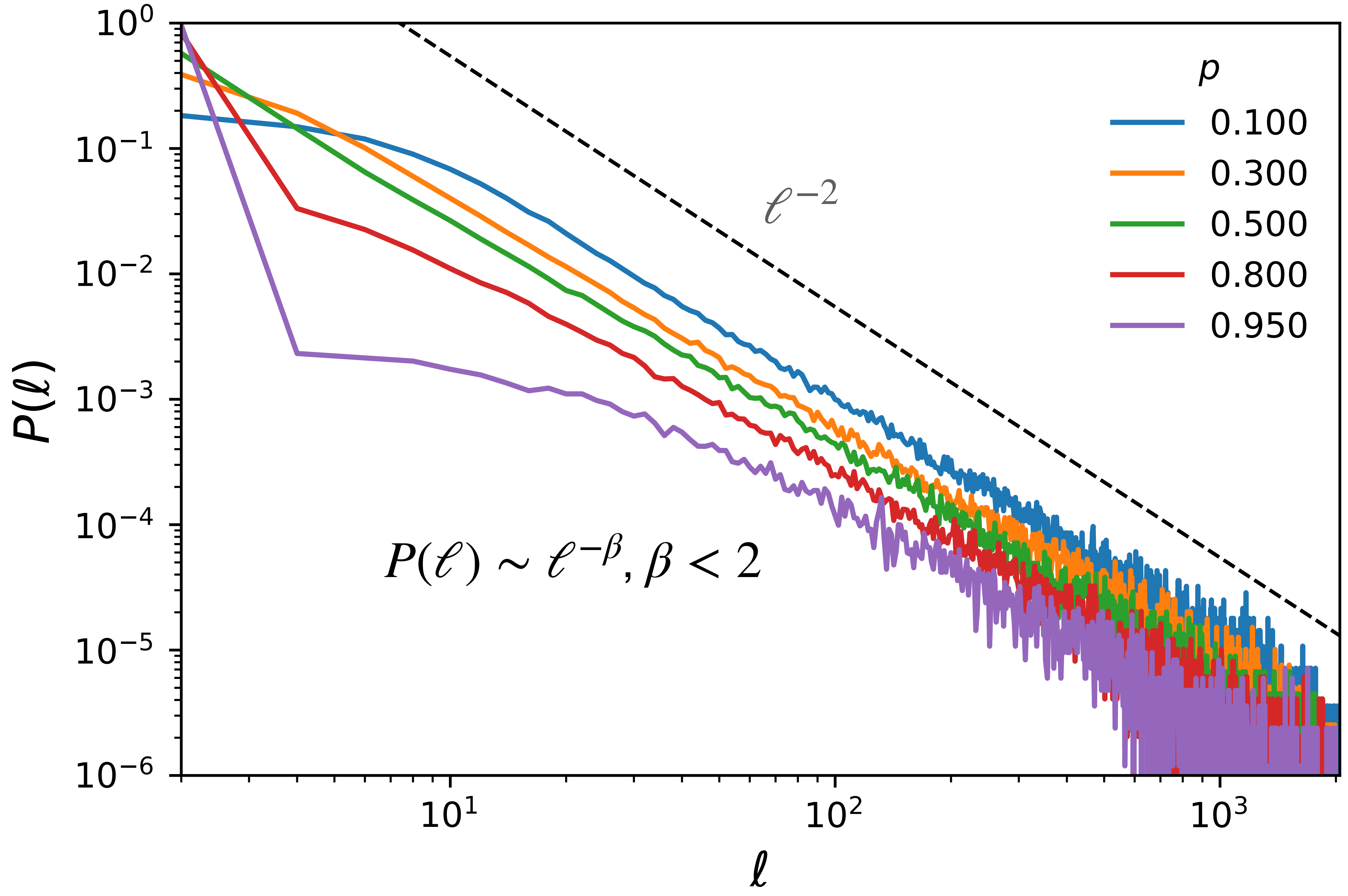


- Application: direct access to (Renyi) mutual information, **code properties** in mixed phase

Quantum code properties of the mixed phase

- How special is the mixed phase in this model?
  - Power-law divergence in **mutual information** $I_2(A:\bar{A})$
  - Contiguous **code distance**: $d_{cont} \sim 1$ near edge, $d_{cont} \sim L^{\alpha}$ in bulk





Conclusion

- Spacetime duality enables forced measurements, sidesteps issue of **postselection**: novel route to investigate broad class of hybrid circuits
- Global & subsystem purity directly measurable: enables access to **quantum code properties** in the mixed phase
- Interesting **constraints** on entanglement/purification dynamics from unitarity in “transverse” direction
- Broader applications for **entanglement dynamics**?

Thank you!

MI, V. Khemani, arXiv:2010.xxxxx

Thanks to M. Gullans, D. Huse and
S. Gopalakrishnan for discussion

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State preparation protocol