# The black hole story in 4 steps

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(Avery, Balasubramanian, Bena, Carson, Chowdhury, de Boer, Gimon, Giusto, Keski-Vakkuri, Levi, Lunin, Maldacena, Maoz, Niehoff, Park, Peet, Potvin,  Puhm,Ross, Ruef, Saxena, Simon, Skenderis, Srivastava,Taylor,Turton,Vasilakis,Warner ...)

Outline:

(A) Brief summary: The quantum theory of black holes

(B) How information comes out of the hole
(the fuzzball construction)

(C) How we can escape seeing a firewall
(fuzzball complementarity)

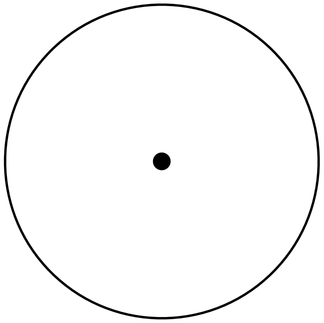
(D) Recasting local dynamics as nonlocal effects?

## (A) Brief summary: A story with 4 iterations

## First iteration

The Schwarzschild metric has a singularity at the horizon ...

$$ds^2 = -(1-\frac{2M}{r})dt^2 + \frac{dr^2}{(1-\frac{2M}{r})} + r^2(d\theta^2+\sin^2\theta d\phi^2)$$

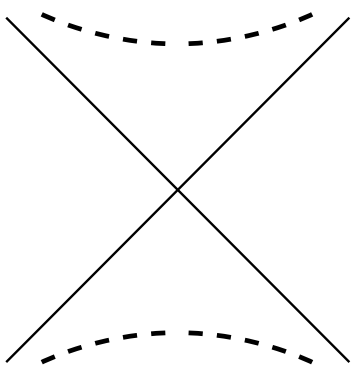


Thus if we do QFT in these coordinates, there will be violent fluctuations as we approach the horizon

# Second iteration

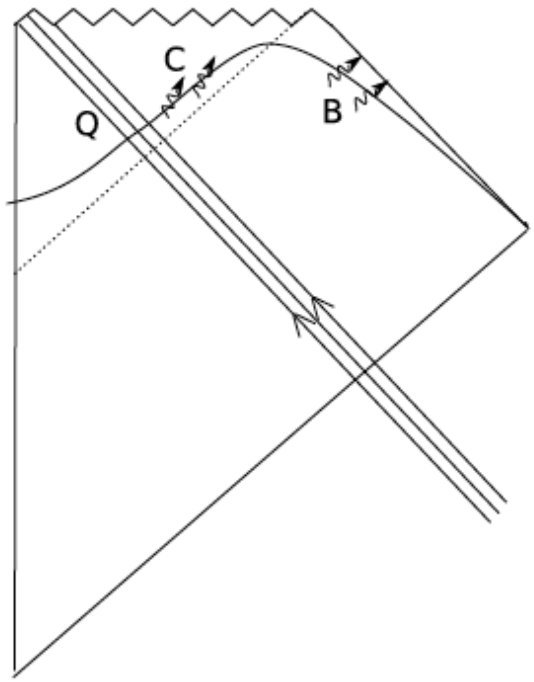
But it was soon realized that this is only a coordinate singularity, the metric continues through smoothly in Kruskal coordinates

In fact, it was found that 'black holes have no hair', so all solutions appeared to the standard one with vacuum at the horizon.



So now it seems that nothing happens at the horizon

But now we have Hawking's information paradox: the metric produces entangled pairs, so the entanglement keeps growing between the inside and the outside ...




Hawking: If the horizon is a normal place, then we have an entanglement problem

Hawking' (equivalent statement): If you don't want an
entanglement problem, you need something nontrivial to happen at the horizon

Using strong subadditivity, we can prove that the entanglement problem is stable to small corrections (SDM: arXiv: 0909.1038)

$$S_{N+1} \;>\; S_N + \ln 2 - 2\epsilon$$

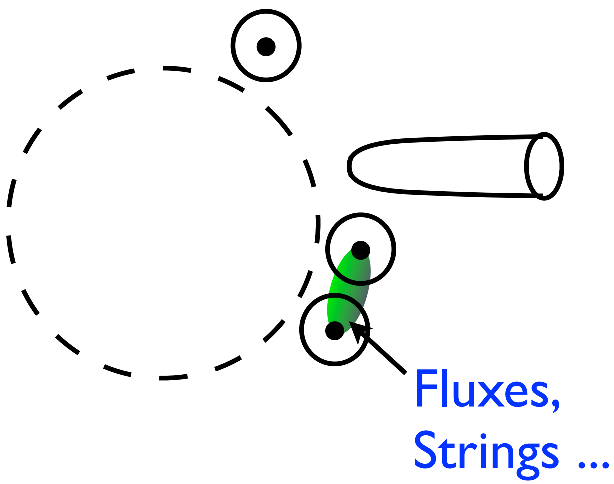
So now we are sandwiched between TWO problems

(a) ‘Black holes have no hair’ so no structure at the horizon

(b) If no structure, we cannot solve the entanglement problem

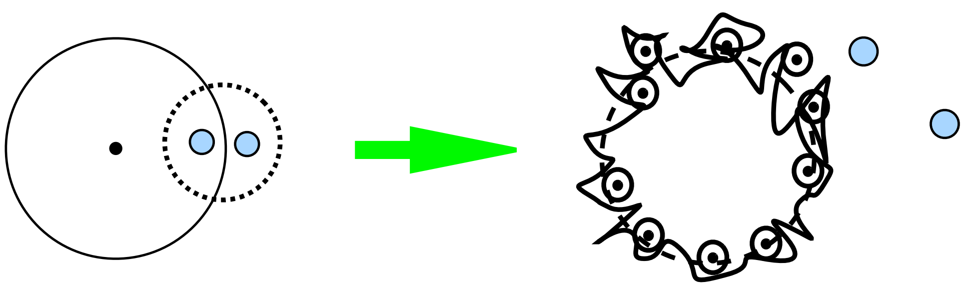
# Third iteration

Resolution of the information paradox: In string theory, we find hair (fuzzball construction)




Compact directions pinch off to make KK monopoles etc

Solution ends before horizon in a mess of string theory sources

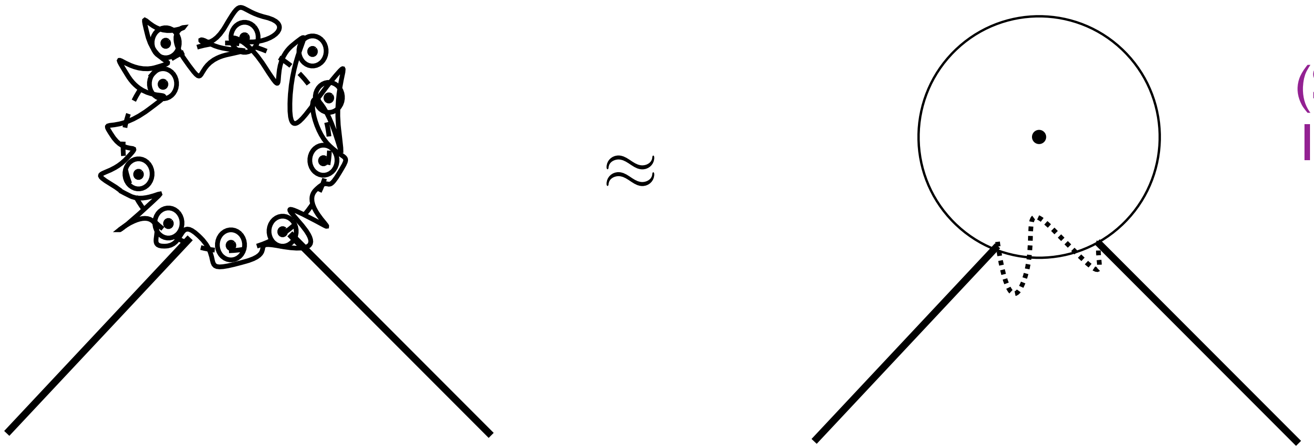


Radiation rate agrees with Hawking radiation

# Fourth iteration

Fuzzball complementarity

(SDM+Plumberg: 1101.4899)



For which operators is this approximation good ?

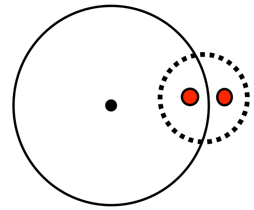
(a) High energy probes E>>T

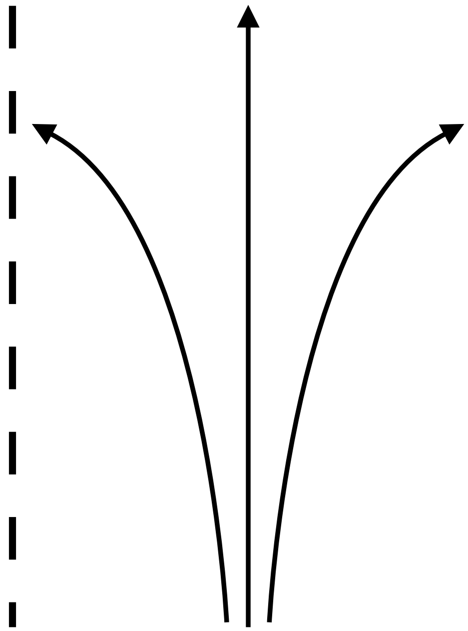
(b) Any energy, but low point functions (Page theorem on maximal local entanglement)

AMPS ask for equality rather than approximation, and this cannot work ...

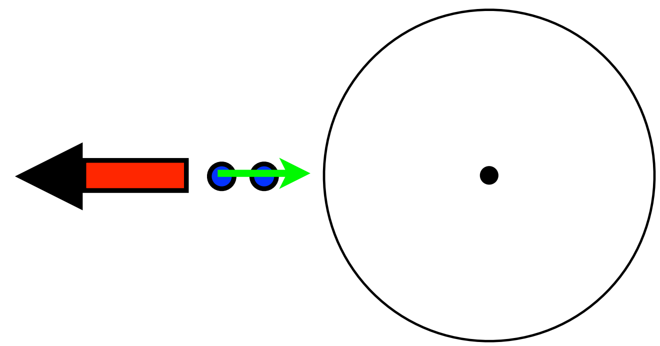
## (B) How information comes out of the hole

# Why is it hard to find hair?



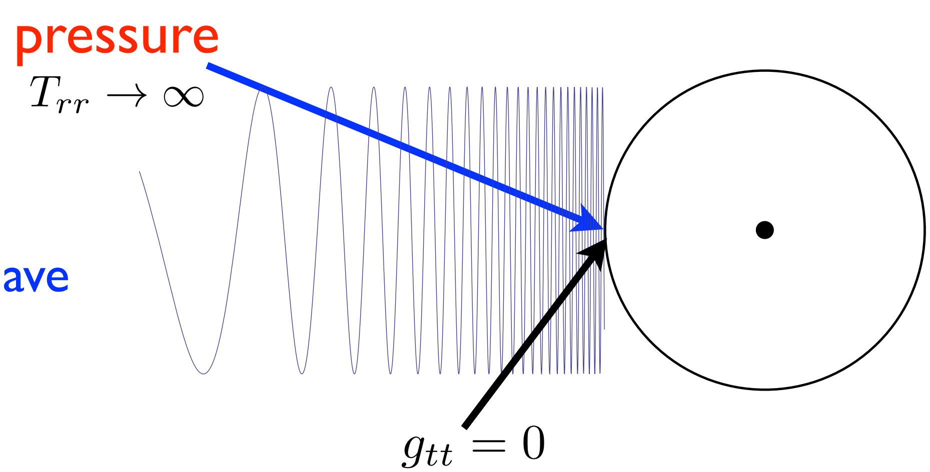


Large relative momentum needed to keep the rocket stationary

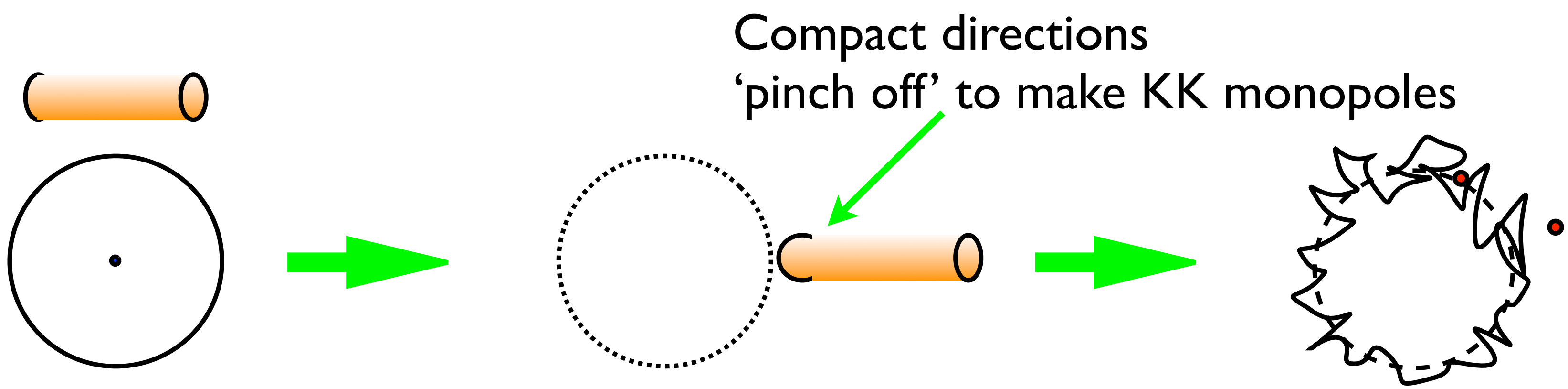


Horizon is an unstable place ...

Field modes have divergent stress-energy



In string theory, we find that states with the quantum numbers of the hole have a certain nonperturbative structure ...




All 2-charge extremal states are of this kind

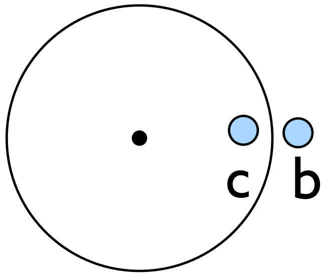
It now appears that all extremal and near extremal 3-charge states can also be understood this way (Bena-Shigemori-Warner, Lunin-SDM-Turton)

Some nonextremal states (JMaRT), neutral maximal Kerr (SDM+Turton, to appear)

Can it be that some states are fuzzballs, but others are traditional holes that have a vacuum at the horizon?

Question: Can we evade the Hawking argument if we have a traditional horizon?

First step of emission $\frac{1}{\sqrt{2}}(|0\rangle_{b_1}|0\rangle_{c_1} + |1\rangle_{b_1}|1\rangle_{c_1})$




Second step of emission

$$\frac{1}{2}\Big(|0\rangle_{b_1}|0\rangle_{c_1}[(1+\epsilon_1)|0\rangle_{b_2}|0\rangle_{c_2} + (1-\epsilon_1)|1\rangle_{b_2}|1\rangle_{c_2}]$$
$$+|1\rangle_{b_1}|1\rangle_{c_1}[(1+\epsilon_1')|0\rangle_{b_2}|0\rangle_{c_2} + (1-\epsilon_1')|1\rangle_{b_2}|1\rangle_{c_2}]\Big)$$

2^N terms after N steps ... could we get an unentangled state at the end?

It turns out that the Hawking argument is stable to small corrections

Inequality (SDM 0909.1038)

If the horizon is a normal place where Hawking's semiclassical evolution is corrected only by order $\epsilon$ ,
then the entanglement keeps growing with each emission

$$S_{N+1} \ > \ S_N + \ln 2 - 2\epsilon$$

In other words, one needs order unity corrections at the horizon to resolve the information problem

(Proof has 3 preparatory lemmas, and then the use of the strong subadditivity)

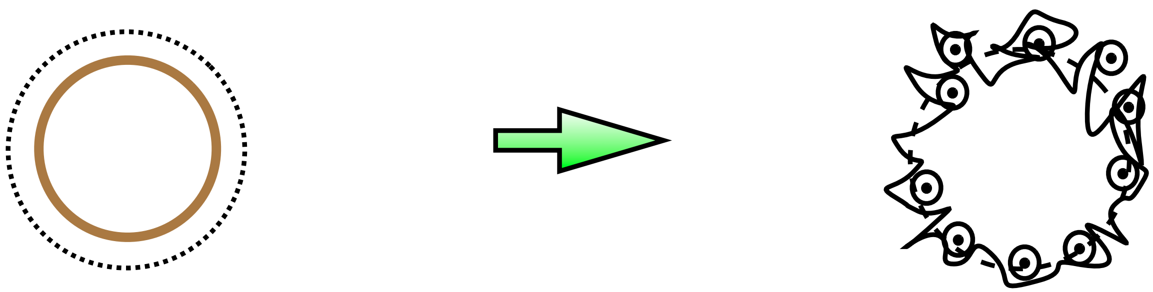
Thus no microstate can have a traditional horizon

AMPS use this growth of entanglement after the Page time, but I think there is more power in the full inequality for (e.g. for applications in papers of Chowdhury et al )

But why doesn't a collapsing shell make a traditional horizon?

A crude estimate gives an interesting observation ...

There is a small amplitude for the shell to tunnel into a fuzzball state ...



$$S_{tunnel} \sim \frac{1}{G}\int R d^4x \sim \frac{1}{G}\frac{1}{(GM)^2}(GM)^4 \sim GM^2$$

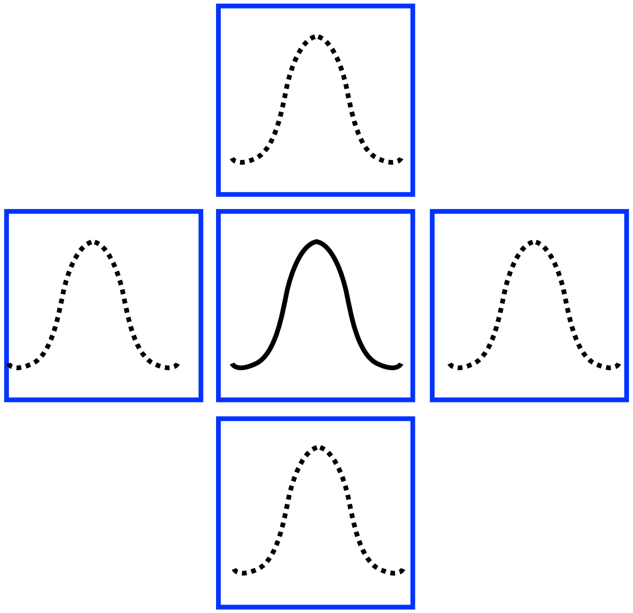
$\mathcal{A} \sim e^{-S_{tunnel}}$ Amplitude to tunnel is very small

$\mathcal{N} \sim e^{S_{bek}} \sim e^{GM^2}$ But the number of states that one can tunnel to is very large !

Smallness of amplitude can be cancelled by largeness of degeneracy ...

(SDM 0805.3716)

Toy model: Small amplitude to tunnel to a neighboring well, but there are a correspondingly large number of adjacent wells



In a time of order unity, the wavefunction in the central well becomes a linear combination of states in all wells

A crude estimate shows that this tunneling happens in a time much shorter than Hawking evaporation time (SDM 0905.4483)

## Summary of the fuzzball construction

(a) There needs to be nontrivial structure at the horizon for all microstates
(inequality proved by strong subadditivity, SDM 0909.1038)

(b) Microstates with this structure are found by the fuzzball construction

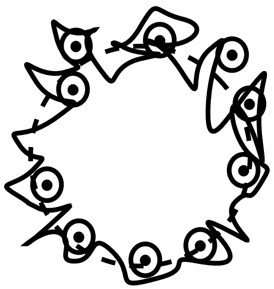
(c) Large measure of states destroys the semiclassical approximation

$$Z = \int D[g] e^{-\frac{1}{\hbar} S[g]}$$

The next question is:

Does the fuzzball behave like a firewall?

Or is some kind of complementarity ?



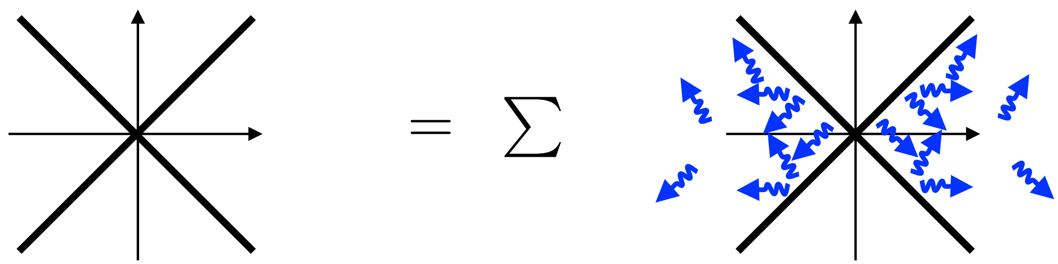
## (C) Getting complementarity

## Fuzzball complementarity (arxiv: 1101.4899, 1201.2079, 1205.0776, 1207.5431)

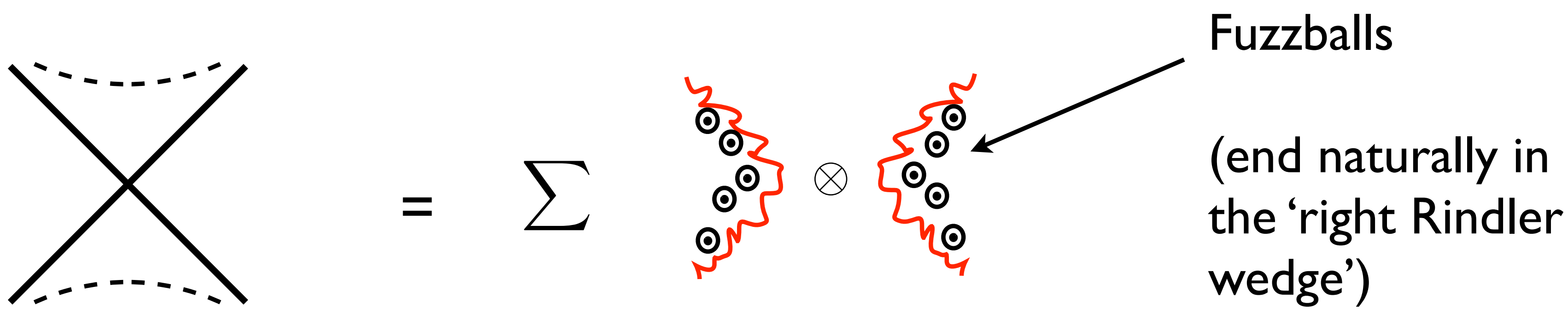
(We use earlier ideas of Israel, Maldacena, van Raamsdonk)

The Minkowski vacuum for a scalar field can be written as

$$|0\rangle_M = \sum_i e^{-\frac{E_i}{4\pi}} |E_i\rangle_L |E_i\rangle_R$$



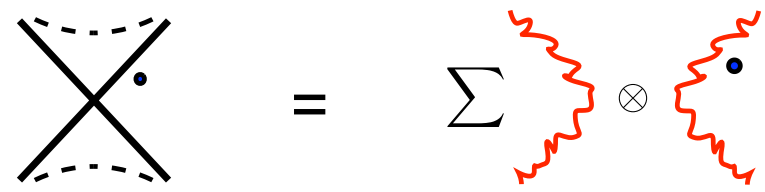
What are the Rindler states for gravity ?



(A) Expectation value of an operator in the right wedge is a thermal average over Rindler states ...

$$ {}_M\langle 0|\hat{O}_R|0\rangle_M = C^2 \sum_{i,j} e^{-\frac{E_i}{2}} e^{-\frac{E_j}{2}} {}_L\langle E_i|E_j\rangle_L \, {}_R\langle E_i|\hat{O}_R|E_j\rangle_R $$

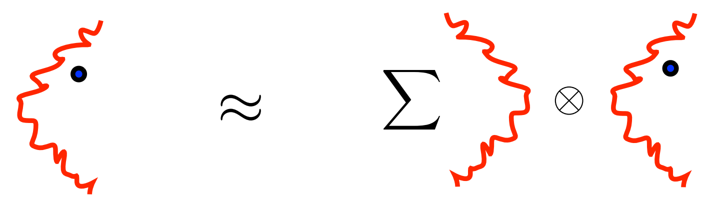
$$ = C^2 \sum_i e^{-E_i} {}_R\langle E_i|\hat{O}_R|E_i\rangle_R $$

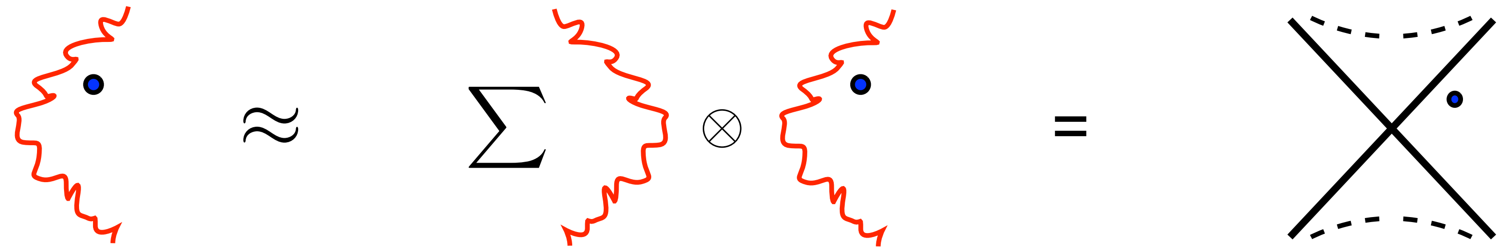


(B) For appropriate operators, in generic states, we have

$$ {}_R\langle E_k|\hat{O}_R|E_k\rangle_R \approx \frac{1}{\sum_l e^{-E_l}} \sum_i e^{-E_i} {}_R\langle E_i|\hat{O}_R|E_i\rangle_R = {}_M\langle 0|\hat{O}_R|0\rangle_M $$

(Just the usual statement that measurements in one sample can be replaced by the ensemble)





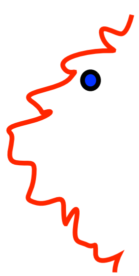
picture 1

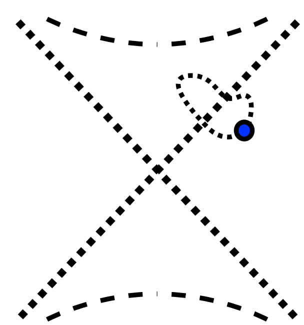
picture 2

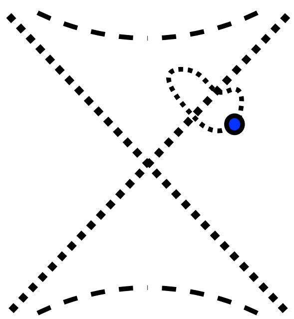
This is fuzzball complementarity

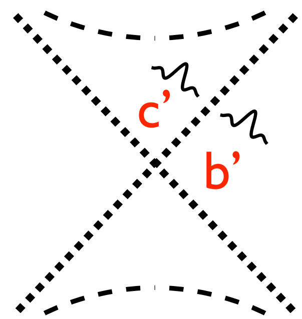
In picture 1 the operator excites the degrees of freedom of the fuzzball. There is no ‘interior’

In picture 2, there is vacuum at the horizon, and the dynamics is one of free passage across the horizon





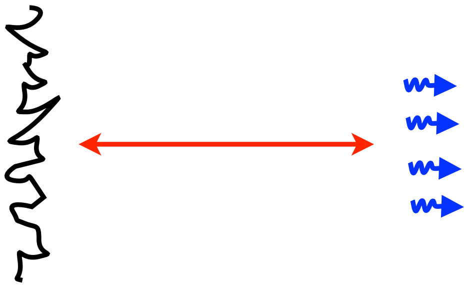





b', c' are correctly entangled in picture 2 to make the vacuum

But this is what AMPS had claimed could NOT happen in picture 2

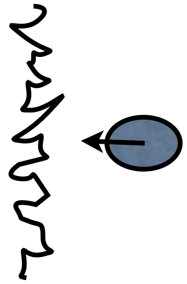
What is different now ? .... They did not take E>>kT



Start with a black hole of mass M

This has $e^{S(M)}$ states

Suppose these states are maximally entangled with the radiation at infinity



Suppose an object of energy E>>kT falls in

Now there are $e^{S(M+E)}$ possible states of the hole

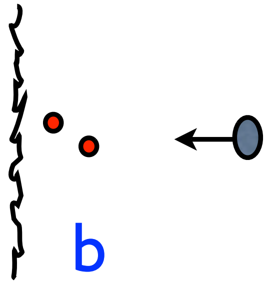
$$\frac{N_f}{N_i} = \frac{e^{S(M+E)}}{e^{S(M)}} = \frac{e^{S(M)+\Delta S}}{e^{S(M)}} = e^{\Delta S} \approx e^{\frac{E}{kT}} \gg 1$$

So most of the new states created after impact are not entangled with the radiation at infinity

(This is just like the entanglement before the halfway evaporation point)

Complementarity is the dynamics of these newly created degrees of freedom, and says that this dynamics is captured by the physics of the black hole interior

AMPS worry only about experiments with Hawking modes b, c, but these have E~kT

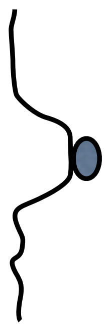



Can the infalling object get burnt by the Hawking quanta b before the new degrees of freedom are accessed ?

(SDM+Turton 1306.5488)

AMPS assumed that the stretched horizon does not react before it is actually hit

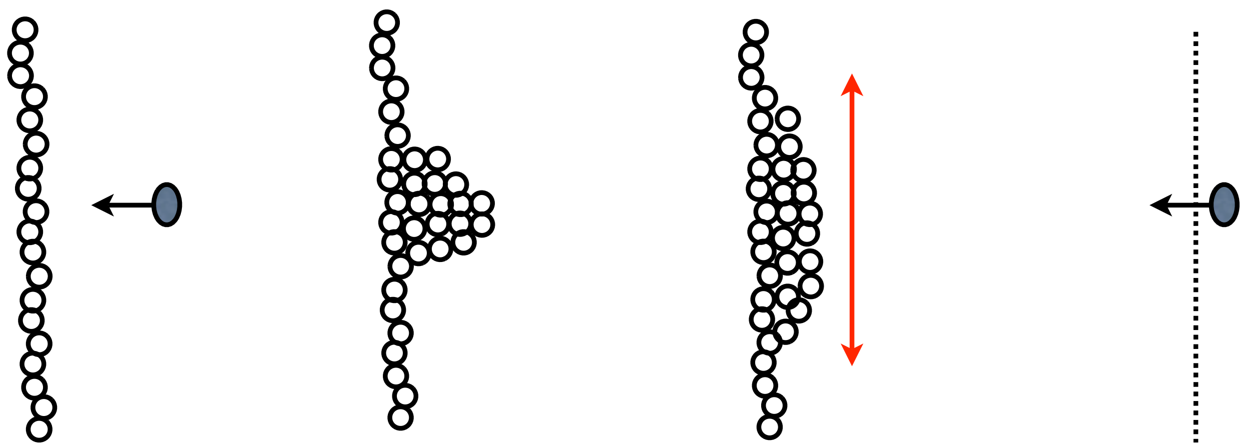
But the tunneling estimate indicates that the stretched horizon (fuzzball surface) moves out a certain distance before it is actually hit



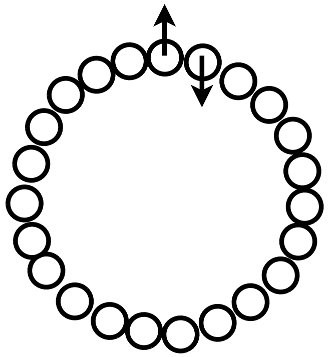
The interaction cross section with Hawking modes b is too small (for E>>T) at this distance from the horizon

Thus the Hawking quanta b do not burn the infalling object before the new (unentangled) degrees of freedom on the stretched horizon are accessed

Complementarity is the dynamics of these new unentangled degrees of freedom

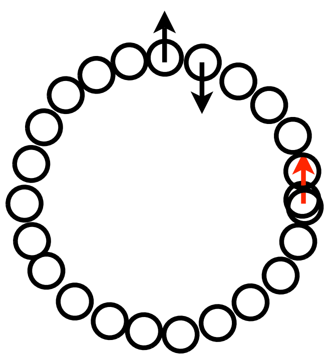


Even on abstract grounds, perhaps we should have anticipated that the stretched horizon will move out ...

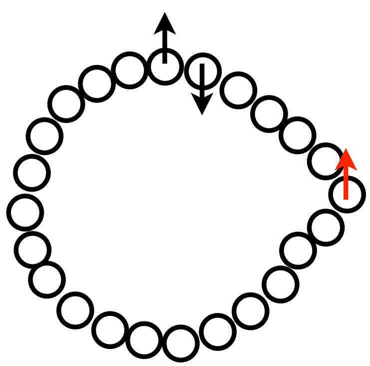


$e^S = 2^N$ states

A new bit falls to the horizon, which has area A

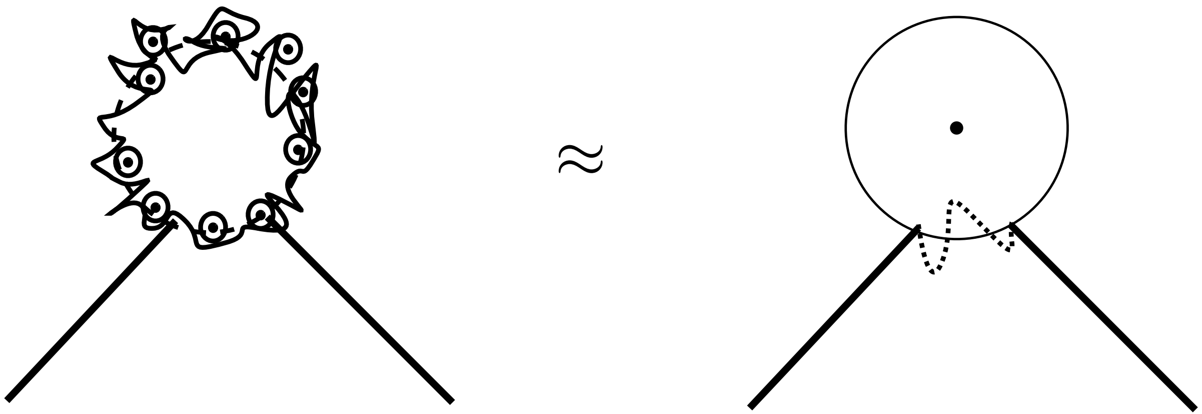


If the horizon does not expand, we would have more than $e^S$ states on a horizon of area A



Since the bits are packed to maximal density, the horizon has to expand to admit the new bit

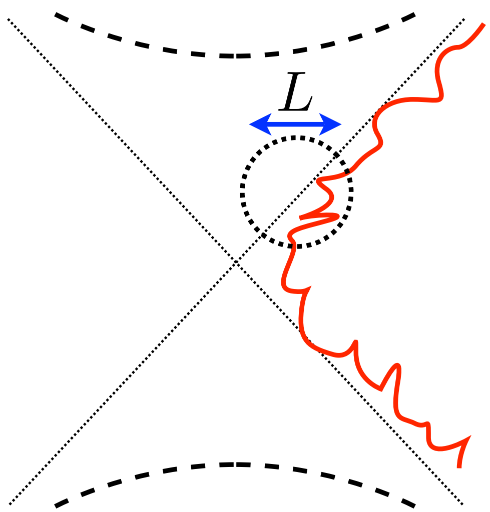
(D) Writing local effects in terms of nonlocal ones?



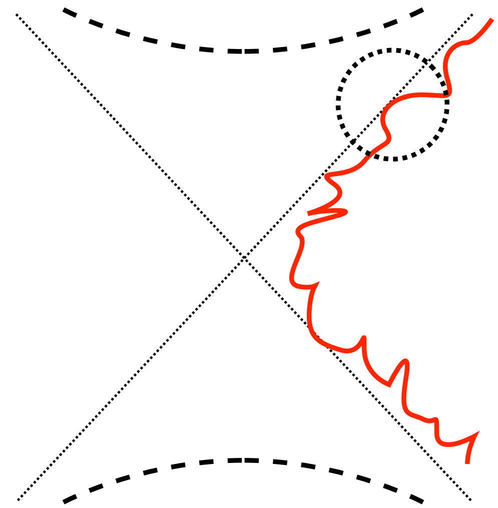
Exact answer

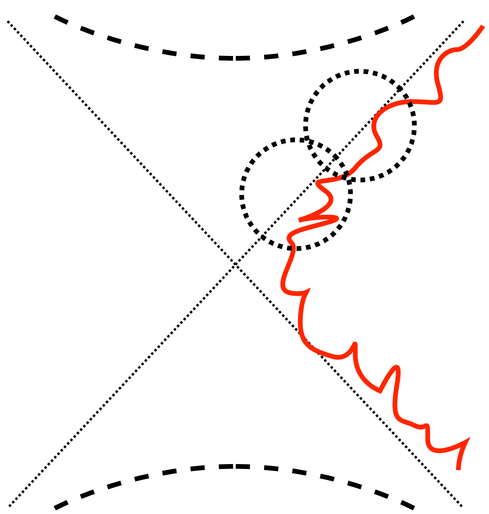
Approximate answer obtained from auxiliary system

Question: Can we add corrections to the auxiliary picture, to recover the exact picture?




$$E \gg T \;\rightarrow\; L \ll M$$





Cannot join two patches of size L to make a larger patch

Mismatch can be recast as a gentle low energy effect, very nonlocal, depending on a large part of the state

# Summary

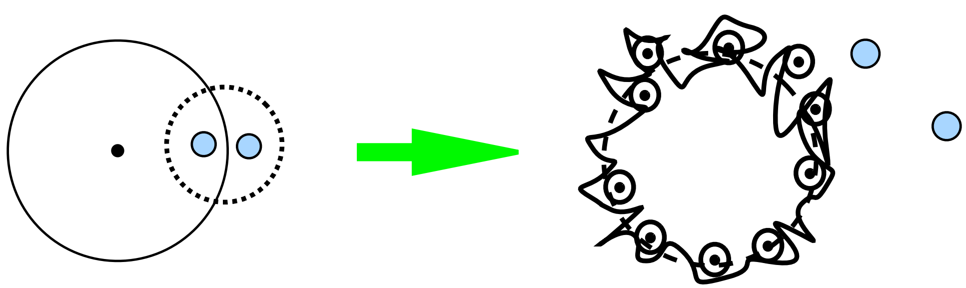
(a) Hawking: If there is no structure at the horizon, and physics is assumed local on good slices, then we have an entanglement problem ...

(b) Is the Hawking argument rigorous? It is indeed stable to small corrections (SDM 0909.1038)

Thus we need nontrivial structure at the horizon to solve the entanglement problem

(c) How can we get this structure? We appear to have a no-hair theorem

In string theory, the fuzzball construction appears to give a complete set of hair



(d) Will an infalling observer feel burnt when he hits the fuzzball surface (firewall) or will there be some kind of complementarity ?

We can only have an approximate complementarity since different fuzzballs have different wavefunctionals.

This 'fuzzball complementarity' evades the AMPS argument because the stretched horizon moves out by a sufficient distance for E>>T