An Infalling Observer and the Black Hole Information Paradox in AdS-CFT

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Complementarity, Fuzz or Fire
26 August 2013

Based on arXiv:1211.6767 (with Kyriakos Papadodimas)

Overview

- Construction of local operators outside the Black Hole.
- Construction of local operators behind the horizon.
- Complementarity and State Dependence.
- Addressing various counter-arguments:
  1. Suppressing commutators by system size. (AMPSS)
  2. Lack of a Left Inverse (AMPSS)
  3. $\langle N_a \rangle \neq 0$ (MP)
  4. Restoring unitarity with small corrections. (Mathur)
  5. Unfreezing the vacuum. (Bousso, van Raamsdonk)

Generalized Free Fields

- The boundary theory has generalized free fields, $\mathcal{O}(t, x)$ of low dimension.
- Correlators of these fields factorize:

$$\begin{aligned} &\langle \mathcal{O}(t_1, x_1) \ldots \mathcal{O}(t_{2n}, x_{2n}) \rangle \\ =&\langle \mathcal{O}(t_1, x_1) \mathcal{O}(t_2, x_2) \rangle \ldots \langle \mathcal{O}(t_{2n-1}, x_{2n-1}) \mathcal{O}(t_{2n}, x_{2n}) \rangle \\ &+ \text{permutations} + \frac{1}{N} \ldots, \end{aligned}$$

- $$\langle 0 | \mathcal{O}(t, x), \mathcal{O}(0, 0) | 0 \rangle = \left(\frac{-1}{t^2 - x^2 - i\epsilon t} \right)^{\Delta} = \left(\frac{-1}{(t - i\epsilon)^2 - x^2} \right)^{\Delta}.$$

- However, $\mathcal{O}$ does not obey an equation of motion.

Local Observables in empty AdS

- We can recast dynamics of $\mathcal{O}$ using a one-to-one map to another operator ϕ_{CFT}

  [Banks et. al., Bena, Kabat et al., 1998–2012]

  .

$$\mathcal{O} \Leftrightarrow \phi_{\text{CFT}}$$

- The precise definition is:

$$\phi_{\text{CFT}}^{\text{vac}}(t, x, z) = \int_{\omega>0} \frac{d\omega d^{d-1}k}{(2\pi)^d} \left[\mathcal{O}_{\omega,k}\xi_{\omega,k}(t, x, z) + \text{h.c.}\right]$$

  where ξ are appropriately chosen functions.

- $\phi_{\text{CFT}}^{\text{vac}}$ behaves like a free-field in AdS. For example:

$$[\phi_{\text{CFT}}^{\text{vac}}(t, x, z), \dot{\phi}_{\text{CFT}}^{\text{vac}}(t, x', z')] = \frac{i}{(2\pi)^d}\delta^{d-1}(x - x')\delta(z - z')z^{d-1}.$$

Emergent AdS

- This is the clearest way to understand the emergence of the bulk from the CFT.
- If we did not know AdS/CFT, this map would look miraculous.
- For example, no normalizable modes in the bulk with spacelike momenta. Correspondingly, the spacelike modes of generalized free-fields can be discarded at large N.
- So, the map is quite constrained even at infinite N.

Local Observables outside a Black Hole

- Consider the CFT in a pure state $|\Psi\rangle$ that is "close" to a thermal state.
- The same generalized free-fields $\mathcal{O}$ have different correlators in the state $|\Psi\rangle$.
- However, we can still construct perturbative local fields

$$\phi^{\beta}_{\text{CFT}}(t, x, z) = \int_{\omega>0} \frac{d\omega d^{d-1}k}{(2\pi)^d} \left[\mathcal{O}_{\omega,k} f_{\omega,k}(t, x, z) + \text{h.c.}\right]$$

More on Operators in front of the horizon

- This is even more miraculous and constrained. For example, at large-spacelike momenta, we require:

$$\mathcal{O}_{\omega,k} \to e^{\frac{-\beta|k|}{4}}$$

for spacelike momenta: $k >> \omega$. True because of general properties of thermal CFT correlators.

[Papadodimas, S.R., 2012]

- Moreover, the CFT-bulk map is state dependent. Expected, because we do not expect to formulate local observables in a background-independent way.

More on State Dependence

- This is not a modification of quantum mechanics: in different states, it is convenient to use different integral transforms of operators.
- What if we took the state $\frac{1}{\sqrt{2}}(|\Omega\rangle + |\Psi\rangle)$? Both $\phi_{\text{CFT}}^{\text{vac}}$ and $\phi_{\text{CFT}}^{\beta}$ are well-defined, but neither has a good interpretation as a bulk perturbative field.
- If we take $\sum_E c_E |E\rangle$, with the right c_E, $\phi_{\text{CFT}}^{\beta}$, but not $\phi_{\text{CFT}}^{\text{vac}}$, has a good bulk interpretation.
- The CFT-bulk kernel depends on more than the energy. Eg. for a unstable gas, before it collapses to a black hole, the kernel is different.

Operators behind the black hole

- To construct local operators behind the horizon, we need additional operators from an operator $\widetilde{\mathcal{O}}(t, x)$ in the CFT.
- This operator should almost commute with the original GFF:

$$[\widetilde{\mathcal{O}}(t_1, x_1), \mathcal{O}(t_2, x_2)] \sim \epsilon$$

- Results of measurements of $\mathcal{O}$ and $\widetilde{O}$ are correlated
- i.e. we want a generic equilibriated CFT state to look like

$$|\Psi\rangle = \frac{1}{\mathcal{Z}} \sum e^{\frac{-\beta\omega}{2}} |\omega, k\rangle \otimes |\tilde{\omega}, \tilde{k}\rangle$$

Local operators Behind the Horizon

- If we can find such operators in the CFT, local operators behind the horizon are

$$\phi^{\rm II}_{\rm CFT}(t, x, z) = \int_{\omega > 0} \frac{d\omega d^{d-1}k}{(2\pi)^d} \Big[\mathcal{O}_{\omega,k}\, g^{(1)}_{\omega,k}(t, x, z) + \widetilde{\mathcal{O}}_{\omega,k}\, g^{(2)}_{\omega,k}(t, x, z) + \text{h.c.} \Big],$$

where we can explicitly compute $g^{(1)}$ and $g^{(2)}$.

[Papadodimas, S.R., 2012]

So, the issue of whether or not we can describe the interior in AdS/CFT is an issue of finding such operators.

Resolving Various Puzzles

- If the $\mathcal{O}$ and $\widetilde{\mathcal{O}}$ operators are an overcomplete basis for the same space, then we can resolve the strong subadditivity puzzle
- Such a construction would also evade Mathur theorem on small corrections, which tacitly does not allow for this possibility of complementarity.
- AMPSS and MP have given several recent arguments in favour of firewalls.
- Later, we will discuss how many of these arguments are very naturally evaded if we allow the operators to be state dependent.
- The proposal is strong, precisely where these arguments meet it!

The Generic Commutator

- But first, let us address the following issue: can we ensure that $\widetilde{\mathcal{O}}$ operators live in the same space as the $\mathcal{O}$ operators and have a small commutator with them.

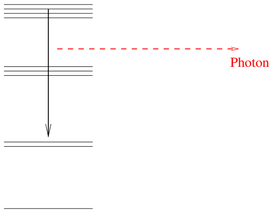
- We take a basis of operators $\mathcal{O}_n$ in a large Hilbert space. eg. imagine the spin operators of a spin chain.

- We now scramble it with a generic unitary matrix U to form $\widetilde{\mathcal{O}_n} = U\mathcal{O}_n U^{-1}$.

- Typically, the commutator $[\widetilde{\mathcal{O}}_m, \mathcal{O}_n]$ has $\text{O}\left(1\right)$ eigenvalues – Bad for locality.

A more realistic example

- We would like to see if these arguments hold true, in a more realistic case.
- Imagine an evaporating black-hole as a system coupled to a radiation field.



- The real puzzle is to find the operators $\widetilde{\mathcal{O}}$ when most of the energy is in the radiation field, so that $\mathcal{O}$ and $\widetilde{\mathcal{O}}$ are acting on the same space.

Dual operators in a radiation field

- Consider a one-dimensional field, living in a very long box of length $l = \frac{3}{2M_{pl}} \left(\frac{M}{M_{pl}} \right)^3$.

- The frequencies are quantized in units of $\omega_{IR} = \frac{2}{3} M_{\rm pl} \left(\frac{M_{\rm pl}}{M} \right)^3$.

- Consider putting an energy M into this box.

- Amusingly, the total entropy of this configuration can be calculated by Cardy's formula and is $S = \pi M^2$.

- We can think of this as the Hawking gas of s-waves emitted by an old-black hole, living in a box of the size of the Page time.

The natural observable

- The natural observable is the field

$$\phi(t, r) = \sum_n \frac{1}{\sqrt{n}} a_n e^{n \omega_{IR}(t-r)} + \text{h.c} + \text{left-movers}$$

- And we are interested in correlators $\langle \phi(r_1, t_1) \phi(r_2, t_2) \ldots \phi(r_p, t_p) \rangle$, where the number of points p does not scale with N

Coarse Graining the Field

- Consider an observer, with limited resolving power: cannot distinguish the frequency $p\omega_{IR}$ from $(p+1)\omega_{IR}$.
- Define the following linear combination of oscillators, where m can be any $O(1)$ number.

$$\alpha_p = \frac{1}{\sqrt{m}} \sum_{i=1}^{m} a_{mp+i}, \quad \beta_p^j = \frac{1}{\sqrt{m}} \sum_{i=1}^{m} \chi_i^j a_{mp+i},$$

- The only relevant property of the χ_i^j coefficients is that

$$\sum_i (\chi_i^{j_2})^* \chi_i^{j_1} = \delta^{j_1 j_2}; \quad \sum_i \chi_i^j = 0, \forall j,$$

so that the β_p^j oscillators are orthogonal to each other and to the α_p oscillators.

Coarse and Fine Spaces

- The physical idea is that the coarse observer sees effectively the space of excitations of the α_p, but there are fine-grained degrees of freedom in the β_p^j that he cannot access easily.

- Divide the full Hilbert space into

$$\mathcal{H} = \mathcal{H}_c \otimes \mathcal{H}_f$$

  where $\mathcal{H}_c$ consists of excitations of the α_p oscillators and $\mathcal{H}_f$ consists of excitations of the β_p^j oscillators.

- If we fix the total energy, then obviously we are restricted to certain entangled states in this product space.

Entanglement between coarse and fine d.o.f.

- Any state in the CFT can be written

$$|\Psi\rangle = \sum_{i,j} \alpha_{ij} |\Psi^c_i\rangle \otimes |\Psi^f_j\rangle,$$

where i runs over an orthonormal basis in $\mathcal{H}_c$ and j over an orthonormal basis in $\mathcal{H}_f$.

- We can perform a singular value decomposition of the matrix α

$$\alpha_{ij} = \sum_m U_{im} D_{mm} V_{mj}$$

where D is a rectangular diagonal matrix

$$D = \begin{pmatrix} D_{11} & 0 & 0 & 0 & \ldots \\ 0 & D_{22} & 0 & 0 & \ldots \\ 0 & 0 & D_{33} & 0 & \ldots \end{pmatrix}$$

Mirrored Operators

- In this basis, the state becomes

$$|\Psi\rangle = \sum_i D_{ii} |\hat{\Psi}_i^c\rangle \otimes |\hat{\Psi}_i^f\rangle$$

- For some matrix elements ω_{i_1, i_2}:

$$\alpha_p = \sum_{i_1, i_2} \omega_{i_1 i_2} |\hat{\Psi}_{i_1}^c\rangle \langle \hat{\Psi}_{i_2}^c|.$$

- Define a mirrored operator on the fine-grained space:

$$\widetilde{\alpha_p} = \sum_{i_1, i_2} \omega_{i_1 i_2}^* |\hat{\Psi}_{i_1}^f\rangle \langle \hat{\Psi}_{i_2}^f|,$$

Mirrored Field

- We can construct a mirrored field

$$\widetilde{\phi}(t,x) = \sum_n \frac{1}{\sqrt{nm}} \widetilde{\alpha}_n \sum_{j=1}^{m} e^{i\omega_{IR}(mn+j)(r-t)} + \text{h.c}$$

- This field commutes up to small corrections with the original field

$$\langle \phi(r_1,t_1)\phi(r_2,t_2)[\phi(r_3,t_3),\widetilde{\phi}(0,0)]\rangle$$
$$\approx \langle \phi(r_1,t_1)\phi(r_2,t_2)[\phi^c(r_3,t_3),\widetilde{\phi}(0,0)]\rangle + \mathrm{O}\left((t_3-r_3)\frac{M_{\text{pl}}}{S^{\frac{3}{2}}}\right)$$
$$= 0 + \mathrm{O}\left((t_3-r_3)\frac{M_{\text{pl}}}{S^{\frac{3}{2}}}\right)$$

- In fact, with a little more work, we can argue that only the square of the commutator is observable, so the smallest non-local effect is suppressed by $\frac{1}{S^3}$

No Left Inverse Argument

- So, this provides an explicit example of a model of the interior that can preserve unitarity, with small violations of locality, and small corrections to semi-classical correlators outside and inside the B.H.

- Consider, some other arguments for why the interior operators cannot exist.

- $\widetilde{a}^{\dagger}_{\omega,k}$ lowers the energy and has a left inverse.

- These is no such operator in the field theory.

[AMPSS, 13]

- But, we have an explicit construction of such operators in this toy model. What gives?

State dependence resolves left inverse argument

- Toy model:

$$\boxed{\text{coarse space}}$$

$$|0\rangle, \quad \text{with energy} = 0$$

$$|1\rangle, \quad \text{with energy} = 1$$

$$\boxed{\text{fine space}}$$

$$|0\rangle, \quad \text{with energy} = 0$$

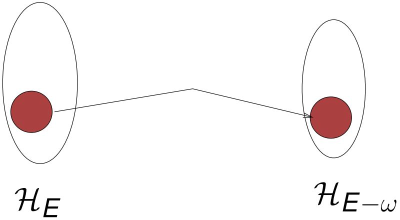
$$|1\rangle, |2\rangle, \ldots |n\rangle, \quad \text{with energy} = 1$$

- Consider some state with total energy = 1:

$$|\Psi\rangle = |0\rangle_{\text{coarse}} \otimes (\sum_{j=1}^{n}) c_j |j\rangle_{\text{fine}} + |1\rangle_{\text{coarse}} \otimes |0\rangle_{\text{fine}}$$

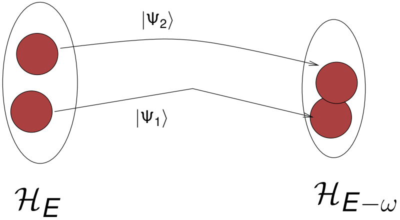
- The mirror of the coarse operator $a^\dagger = |1\rangle_{\text{coarse}}\langle 0|_{\text{coarse}}$ is $\widetilde{a}^\dagger = c_j |0\rangle_{\text{fine}} \langle j|_{\text{fine}}$

State Dependent Operators are Sparse



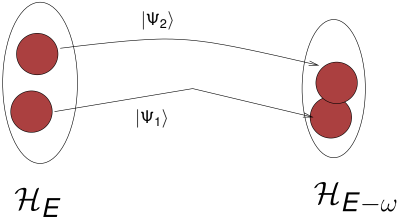
State Dependent Operators are Sparse!: No contradiction with linear algebra!

Union of all constructions?



- Can we use the "union" of all constructions to obtain a contradiction?

Union of all constructions?



- Can we use the "union" of all constructions to obtain a contradiction?
- NO!
- The explicit construction shows that if we try and cover $\mathcal{H}_E$ using different states $|\Psi\rangle_m$ with different structures of entanglement, we over-cover $\mathcal{H}_{E-\omega}$, as one would expect.

$\langle N_a \rangle \neq 0$ Argument

- Let N_b measure the occupation number of some b-mode outside the horizon.
- Marolf-Polchinski: If $\widetilde{N}_b$ is some fixed operator in the CFT, then we would expect that a generic state at fixed energy does not look like $|N_b, \widetilde{N}_b\rangle$ and so has $N_a \neq 0$.
- Compute the microcanonical expectation value $\langle N_a \rangle$ by going to the N_b basis: generically expect this to be non-zero.
- What is the contradiction with our explicit construction, where $\langle N_a \rangle = 0$?

State dependence resolves the $N_a \neq 0$ argument

- The operator N_a depends on N_b and $\widetilde{N}_b$.
- So, in taking the expectation value $\sum_\Psi \langle \Psi | N_a | \Psi \rangle$, the precise operator N_a varies as we change $|\Psi\rangle$
- In fact, by construction, for generic states:

$$|\Psi\rangle = \sum_{N_b} e^{-\frac{1}{2}\beta N_b \omega_b + \ldots} |N_b, \widetilde{N}_b, \ldots\rangle$$

  So, $N_a |\Psi\rangle = 0$.

- So, the Marolf-Polchinski argument should be interpreted as a strong argument in favour of state-dependence of the interior operator.

Frozen Vacuum?

- Another counter-argument (specific to this construction): If the interior operator is always defined to be the one chosen by entanglement, then the vacuum is always feature-less (or frozen).

  [Bousso, van Raamsdonk, 13]

- The rule is not that the interior operator is chosen by entanglement.

- Rather it is that the interior operator is chosen to be the one that gives perturbative local correlators.

- For a generic equilibrium state, the two rules coincide.

Interior Operators for a Non-Equilibrium State

- For a slightly out of equilibrium state $\mathcal{O}(x,t)|\Psi\rangle$, where $|\Psi\rangle$ is an equilibrium state, the rule is that we define the operators $\widetilde{\mathcal{O}}$ with respect to $|\Psi\rangle$, and then do not change them.
- It would be nice to prove that this rule is uniquely fixed by the criterion that we should get local operators.
- This rule does give local operators with the correct behaviour and no frozen vacuum. So, we conjecture that it is correct.

Small Corrections

- How do have a unitary model of evaporation, with only small corrections to correlators?

[Mathur, 2009–13]

- If we look at the full wave-function, it appears to evolve as

$$|\Psi\rangle \to \frac{1}{\sqrt{2}}|\Psi'\rangle \otimes \left(|0\rangle_c|0\rangle_b + |1\rangle_c|1\rangle_b\right)$$
$$\to \frac{1}{2}|\Psi''\rangle \otimes \left(|0\rangle_{c_1}|0\rangle_{b_1} + |1\rangle_{c_1}|1\rangle_{b_1}\right) \otimes \left(|0\rangle_{c_2}|0\rangle_{b_2} + |1\rangle_{c_2}|1\rangle_{b_2}\right)$$

- The excitations b_1, c_1, b_2, c_2 are all excitations of the same collective modes. They are not independent excitations

- This is what allows us to evade the theorem, and obtain unitary evolution with small corrections.

Summary

- We want to reorganize boundary correlators in terms of perturbative fields propagating on a spacetime with an extra dimension.

- Our construction suggests that, to leading order in $\frac{1}{N}$, this successfully describes both the interior and the exterior of the black hole.

- The operators, both outside and inside are state-dependent. (At leading order, the exterior operators depend only on the energy, while interior operators seem to have stronger state dependence.)

- This seems to resolve several arguments raised against the existence of such operators.

Open question 1: size of non-local effects

- The current model has power-law (in entropy) suppression for commutators. It is difficult to extend this to exponential suppression, without making the black-hole too long lived.

- On the other hand, even for points r_{out} outside the black-hole:

$$[h_{00}(t, \infty), \phi(t, r_{\text{out}})] = \mathrm{O}\left(\frac{1}{N}\right)$$

  because the asymptotic metric can measure the energy inside.

- So, we need to understand which operators (if any) should have exponentially suppressed commutators.

Open questions 2: $\frac{1}{N}$ corrections

- We have made a guess for interior operators. How we know that this is correct?
- Again, the same question exists for exterior operators.
- We conjecture that a careful study of $\frac{1}{N}$ corrections should allow us to better fix both interior and exterior operators.

Open questions 3: Bulk Interpretation

- It seems clear that the bulk theory will require some (preferably exponentially small) degree of non-locality to resolve the information paradox.
- If we formulate the theory using a path integral, we should expect such corrections since the semi-classical spacetime ceases to make sense at this level of accuracy.
- But, can we find some explicit non-perturbative effect that produces the non-local corrections required to restore unitarity?