Remarks about ER=ERP and Complexity

1. You can't tell from the density matrix of a black hole whether it has a firewall or a smooth Einstein Rosen bridge. You have to know how it is entangled with other systems. Here's an example.

   Consider the Thermofield Double state of a "two-sided' ADS setup. The right side black hole is entangled with the left side in a particular way that is characterized by two symmetries. The two CFT's don't interact with each other so that both Hamiltonians are conserved. The symmetry generated by $H_R - H_L$ is the Killing symmetry with orbits shown in fig 1.

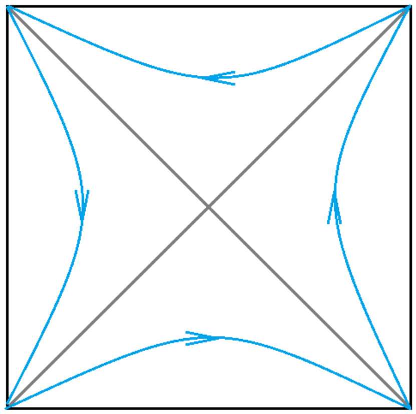


Fig 1

Not only is H_R – H_L conserved but the TFD state is invariant under its action:

(H_R – H_L) |TDF> =0.

Since the two CFT's are uncoupled there is also symmetry under

(H_R+H_L),

but the TFD state is not invariant under (H_R+H_L) . However, the density matrices of both outer regions are invariant.

The lack of full invariance is pretty obvious since the regions behind the horizon have no time-like Killing vector. The orbits of the partial symmetry on the outer regions are shown if fig 2

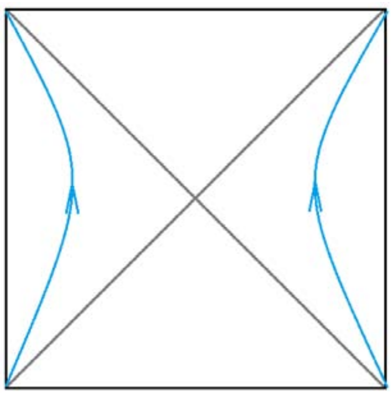


Fig 2

Now let’s follow Mark Van R and suppose a weak coupling is turned on between the two CFT’s. The two sides exchange energy until the bottled-up system comes to thermal equilibrium. One might expect the global state to be time-translation invariant. On the other hand the density matrices of either side are the same as in the TFD since they are thermal. That means that the two outer triangles are the same as in the TFD. But there doesn't seem to be any way to extend the action into the upper and lower triangular regions behind the horizons.

How can the whole system be stationary while the two outer triangles are the same is they were in the TDF? The easiest way to do this is to just get rid of the regions behind the horizon as in figure 3.

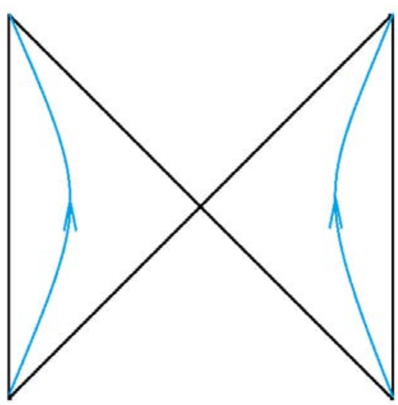


Fig 3.

In other words there are firewalls on both sides.

In this “butterfly” case correlations between the sides will be very small at all times. That’s exactly what Joe P. and Don M. found.

The precise statement is that at all times the correlations between the left and right CFT’s are exponentially small (in the entropy).

In both cases—the TFD—and the thermalized state, the density the two sides are maximally entangled and the density matrices of each side are thermal. And yet the ER bridges are as different as possible.

The implication is this: One **cannot** conclude from the fact that the state of one side is generic that there is a firewall. But the generic (thermal) ensemble of the two sided system does have firewalls. I’m not sure how much to trust this but it seems reasonable.

Now let’s go to the evaporating black hole in empty flat space. At any given time after the scrambling time the density matrix of the black hole is close to thermal. But it is definitely NOT bottled-up in equilibrium with its radiation. Presumably if the radiation were gathered in a container, and the black hole-container system isolated from everything else, the weak interactions between the container and the black hole would eventually bring the combined system to thermal equilibrium. If the above ideas are applicable the bottled-up black hole + radiation system would develop a firewall.

But if the radiation were just allowed to stream outward there is would not equilibrate with the black hole. In other words the system is a two-sided system that is not bottled up. So despite the fact that the black hole density matrix is generic, there is no reason for a firewall, at least from genericity arguments.

2. Do we have it upside down?

All of this suggests an interesting possibility. Instead of causing firewalls at the Page time by entangling the black hole with a distant system, Hawking radiation protects the smooth horizon by creating a ERB between two systems that are not bottled-up and forced to thermalize. If this is so, one would expect one-sided black holes in ADS to quickly develop firewalls unless the radiation were allowed to leak out of the ADS through the boundary.

3. What are precursors and what do they have to do with Black hole information?

Ted Jacobson gave a very good explanation of precursors and what they have to do with things. I think it bears repeating.

Perturbing the boundaries of the two CFT's creates local CFT perturbations. Call such a perturbation at time (t=0) **p**. In the Schrodinger picture at time 0 **p** is a local CFT operator .

But now let's run **p** back to time -t: **P** = e^{iHt} **p** e^{-iHt}. If **P** acts at time –t it has the same effect as **p** acting at time 0. **P** is generally very non-local while **p** is completely local. Typically **P** may be composed of very extended decorated Wilson loops.

In the present context we really want to run **p** forward in time,

P = e^{-iHt} **p** e^{iHt}.

In that case it can be called a "postcursor".

Precursors and postcursors are extremely complex operators but they evolve to simple operators at some particular time.

What about more general operators of similar complexity, but which don't ever look simple in the Schrodinger picture? In other words highly non-local operators involving ADS size Wilson loops of all kinds. Most will never evolve to local operators on the time-like boundaries. I am going to propose that at least some such operators can be identified with variations of the initial state on the past singularity. I don't have a good name for them so I'll temporarily call them singularcursors.

Now consider the Thermofield double and an operator A behind the horizon. Let A be the partner to B which is roughly near the symmetry time t=0. Both A and B are right-moving modes as in fig 4.

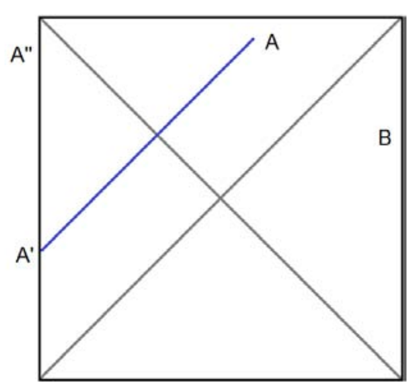


Fig 4

In some approximation we identify A with A’ on the left. It of course also contains contributions from the right but I am mostly interested in its connection with A’ We sometimes write A = A’.

Now A’ can be run forward to A’’ where it becomes a very complicated postcursor.

A’’ = e^{-iHt} **A’** e^{iHt}

where H refers to the left-side Hamiltonian.

One may write that A = A’’ although one has to be careful about the meaning. It is the analog of $A=R_B$. One may say that A’’ is the image of A in the boundary CFT system.

Note that we can move A, B, A’ and A’’ to the positions shown in the next figure, fig 5, by transforming with H_R-H_L.

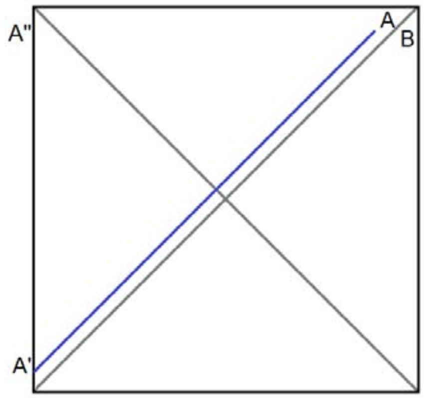


Fig 5

Next consider a slight perturbation on the right boundary represented by the dotted orange line in fig 6. It represents a change in the initial state of the right black hole. It occurs at time t=0 and consists of a single particle of energy equal to the Hawking temperature. One would expect the effect to be negligible and indeed it is on the right-sided black hole.

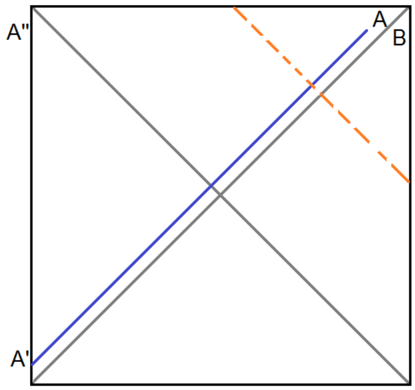


Fig 6

The single low energy particle adds one bit to the right side black hole and has a negligible effect.

But it changes the two sided system dramatically. In fact the diagram above is quite wrong.

Suppose with transform the diagram by H_L – H_R so that B winds up near t=0. Shenker and Stanford show that the diagram changes to the one below (fig 7).

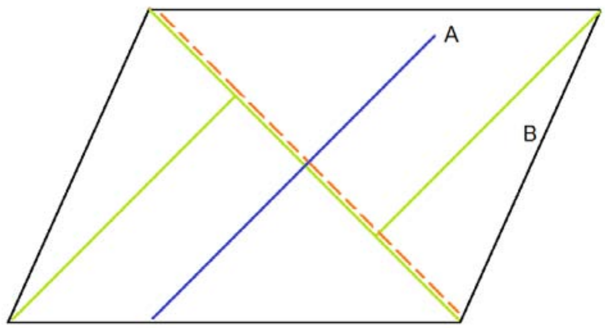


Fig 7

The interesting point is that A does not evolve from a boundary point, but rather from the past singularity. The representation of A in the CFT system is no longer a postcursor but a singularcursor.

The important point is that a slight change of one bit in the initial state has caused a huge change in the way A is encoded in the CFT. This is an example how a small change in the initial state can change the **code subspace** by a lot. It is also an example of the state dependence in the encoding of A. But in the infalling frame A has hardly changed at all.

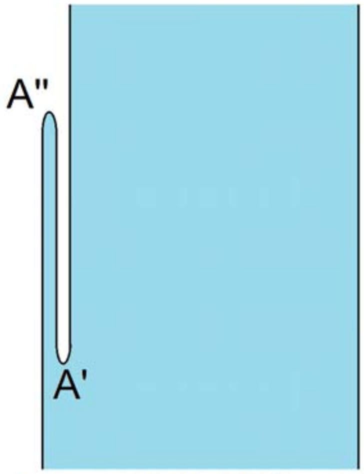
4. Postcursors and folds.

Here is something I learned by talking to Stefan Leichenauer and Douglas Stanford. Pre/postcursors are closely connected with time-folds (Bulk and Transhorizon Measurements in AdS/CFT Idse Heemskerk, Donald Marolf, Joseph Polchinski, James Sully). They are also connected with the discussion in Maldecena-Susskind—sections 3.3 and 3.4—about Alice restoring the Thermofield state.

Consider the operators A’ and A’’:

A’’ = e^{-iHt} **A’** e^{iHt}.

This is exactly the situation discussed by Heemskerk, Marolf, Polchinski, and Sully. I won’t go into detail but you can visualize it in terms of a time-fold. Here is the picture.



Two sided ADS with a fold on the left side.

The fold represents the action of the operators e^{-iHt} and e^{iHt} where H is the left side Hamiltonian. This leads some strange paradoxes that I will explain if anyone asks.