Warps, Waves, and Phase Space Spirals in the Milky Way

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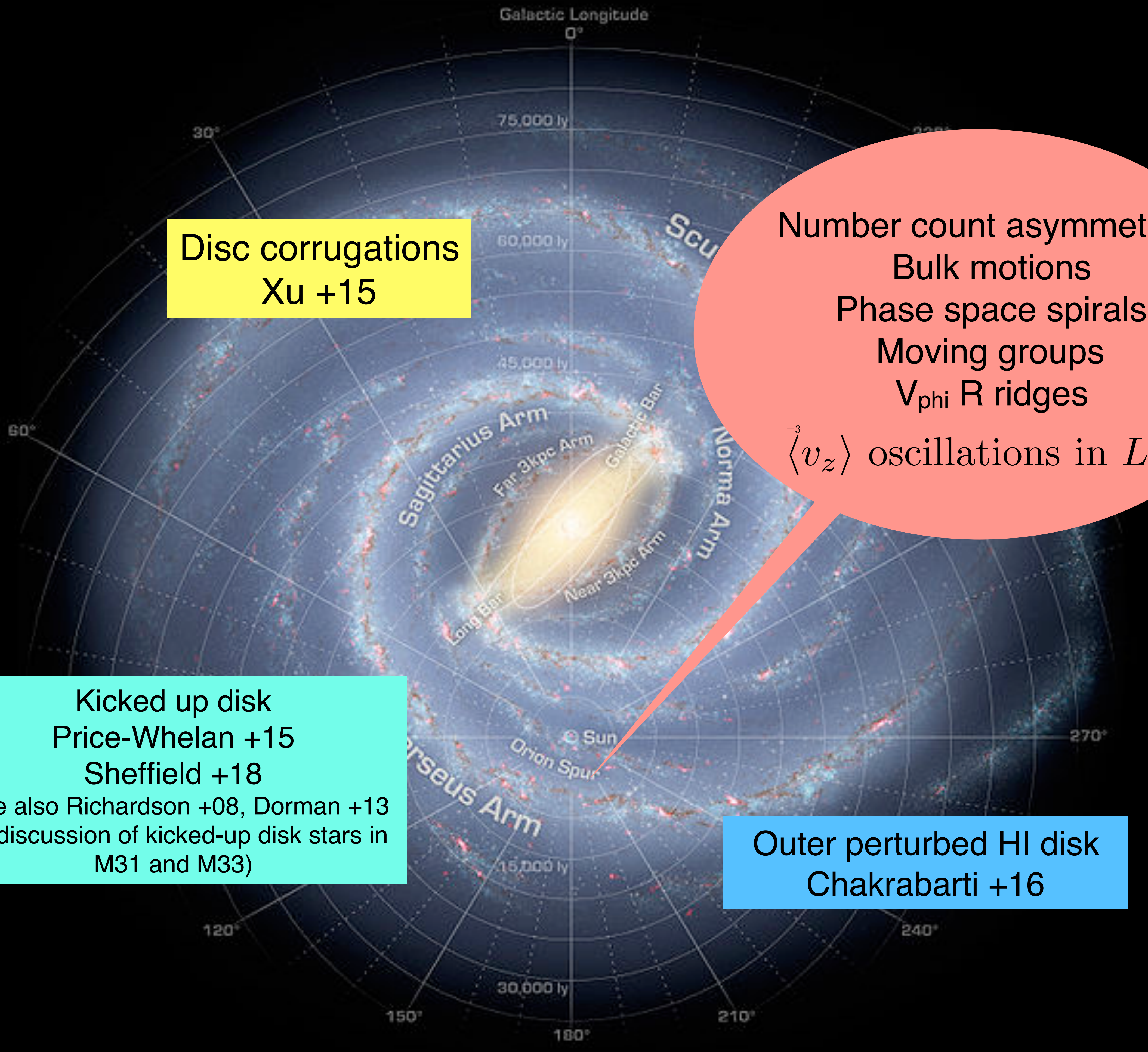
Queen's University

Disequilibrium @ KITP
April 2019



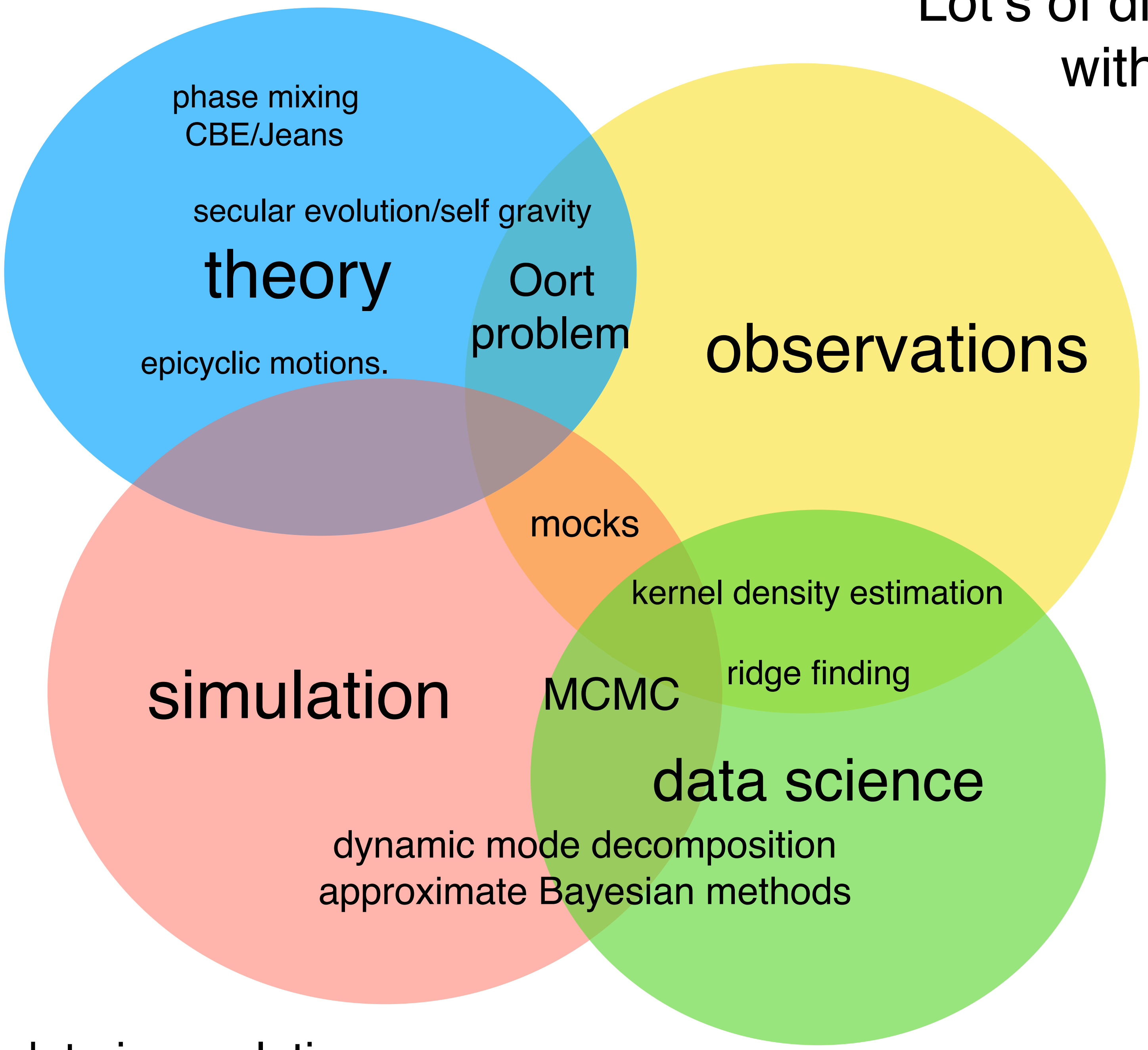
Disc corrugations
Xu +15

Number count asymmetries
Bulk motions
Phase space spirals
Moving groups
V_{phi} R ridges
$\langle v_z \rangle$ oscillations in L_z

Kicked up disk
Price-Whelan +15
Sheffield +18
(see also Richardson +08, Dorman +13 for discussion of kicked-up disk stars in M31 and M33)

Outer perturbed HI disk
Chakrabarti +16

Lot’s of discovery potential
with GDR2/3/4



phase mixing
CBE/Jeans

secular evolution/self gravity

theory

Oort
problem

epicyclic motions.

observations

mocks

kernel density estimation

ridge finding

simulation

MCMC

data science

dynamic mode decomposition
approximate Bayesian methods

Simulations lag data in resolution
but are just starting to reach the
high end of the stellar mass
function

Community is still finding its way
through data science methods to
see what is genuinely useful for
our field

disk disequilibria: historical context

One thread involves the study of bars, spiral structure and warps

Warps as bending modes in a disk
Linear theory applied to dynamical ring model
Hunter & Toomre 1969; Sparke & Casertano 1988

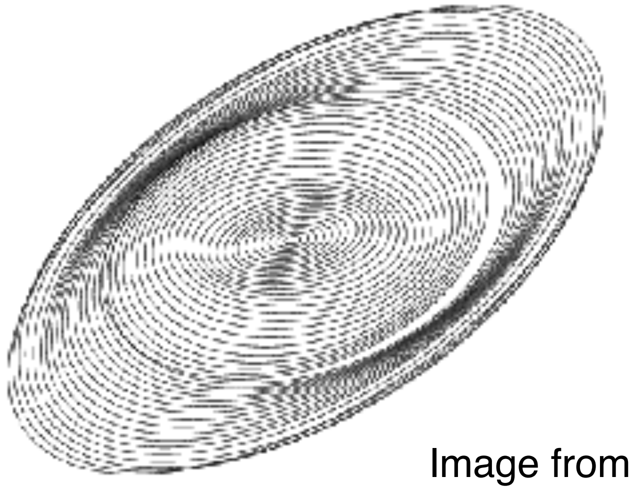
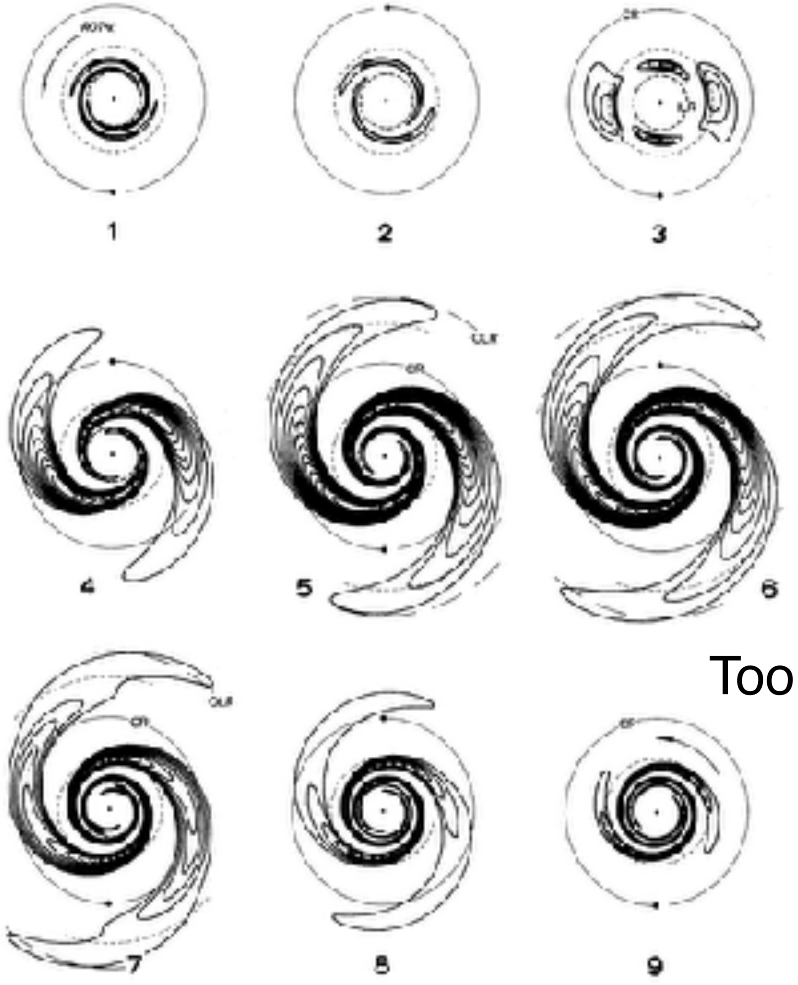
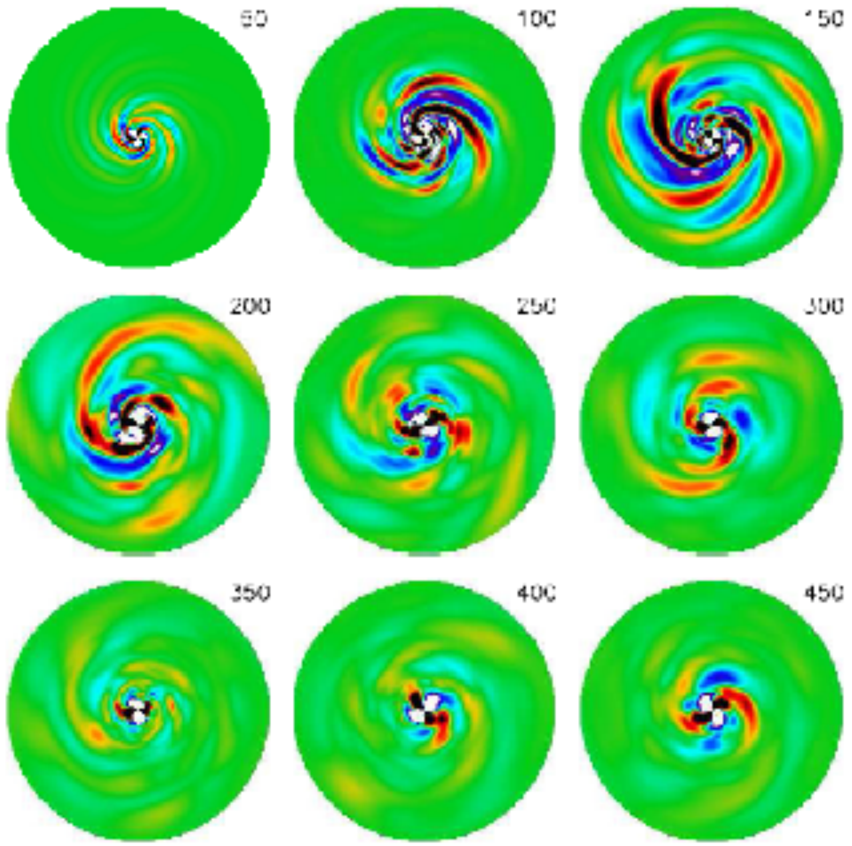


Image from Battaglia et al 2009



Swing amplification for spiral structure, a “conspiracy” between self-gravity and epicyclic motion

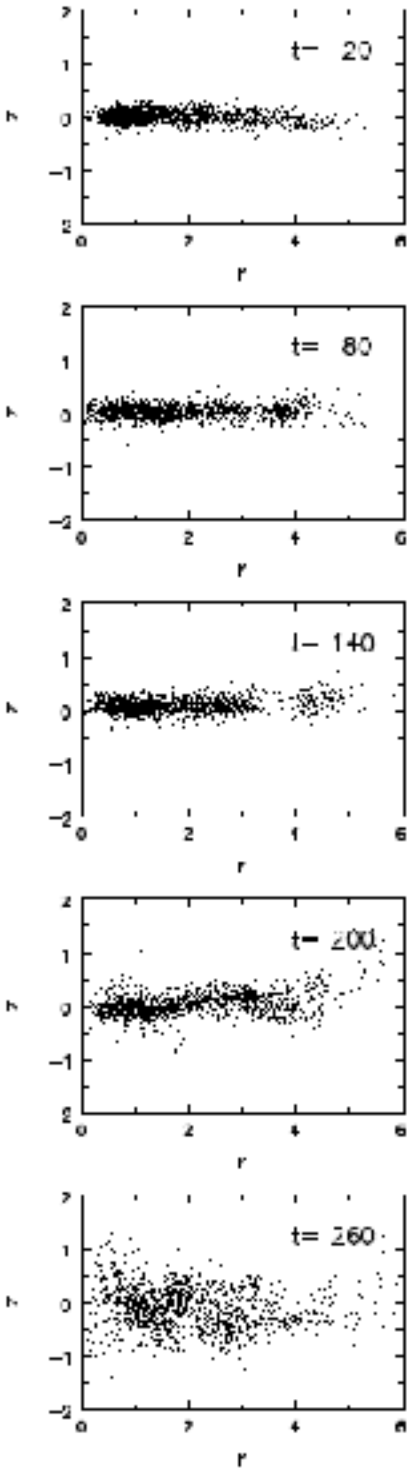
Toomre 1981



Sellwood & Carlberg 2014

A second thread focuses on disk heating

Dynamical friction (Chandrasekhar 1960)
Applied to heating by black holes in the halo (Lacey & Ostriker 1985)
Resonant heating of the disk via satellite encounter
(Sellwood, Nelson, & Tremaine 1998)



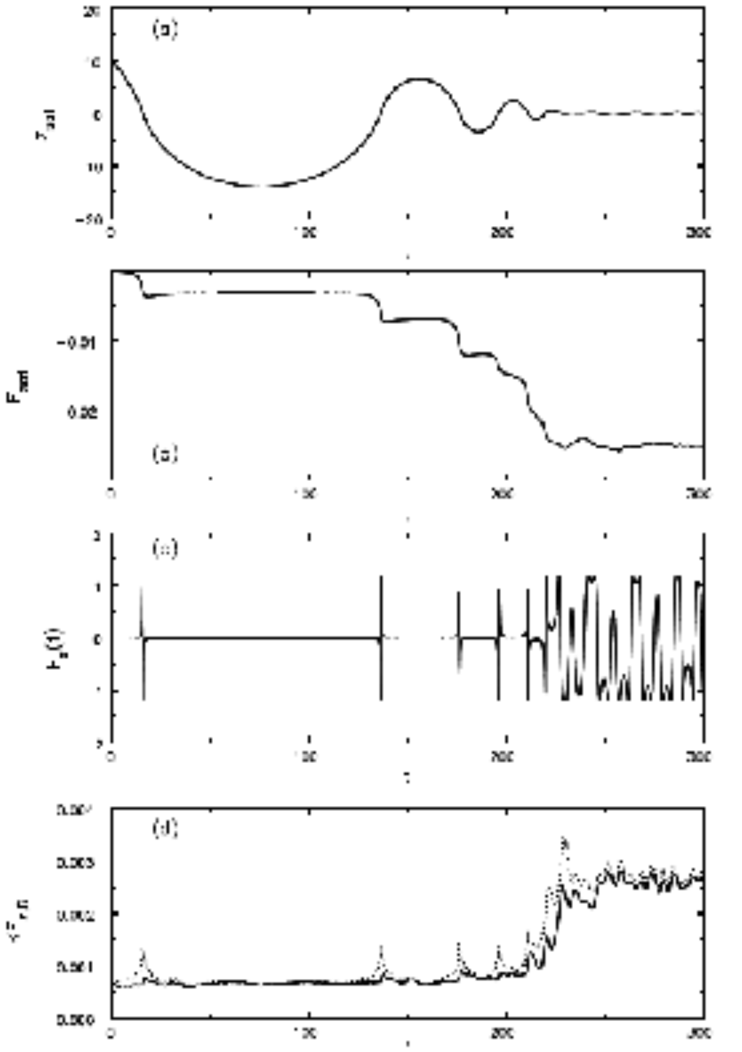
Edge on view of disk
during encounter with
satellite oscillating
through centre along
symmetry axis

Satellite position relative
mid plane

Satellite energy

Vertical force of satellite
star at centre of disk

Vertical energy of disk



Sellwood, Nelson, & Tremaine 1998

Takeaway: Phenomena we are observing may be the
early stages of a disc heating event

Outline

Disequilibrium in the disc: A brief and incomplete observational overview

Plausible scenarios for how these phenomena came to be.

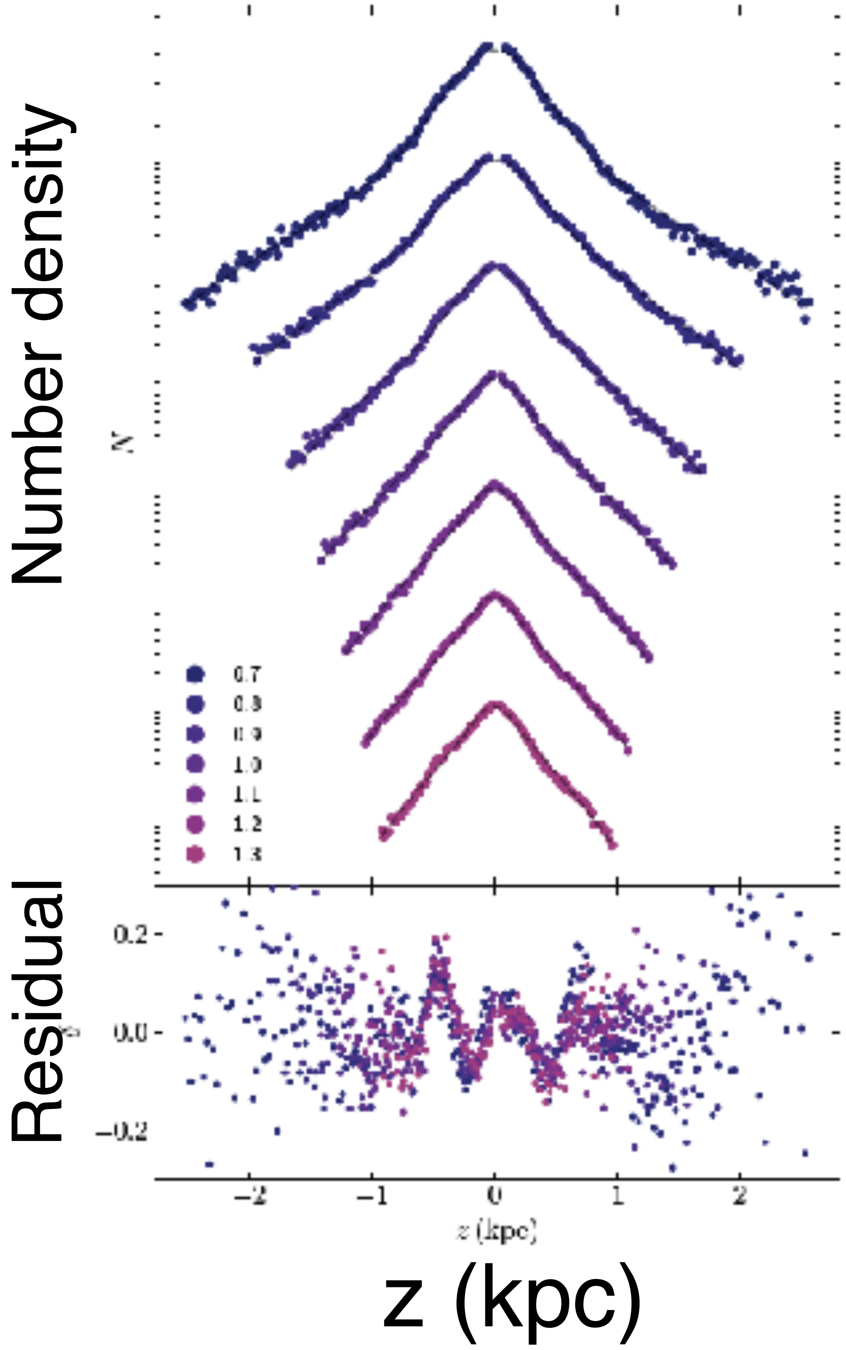
Relationship between the different observations

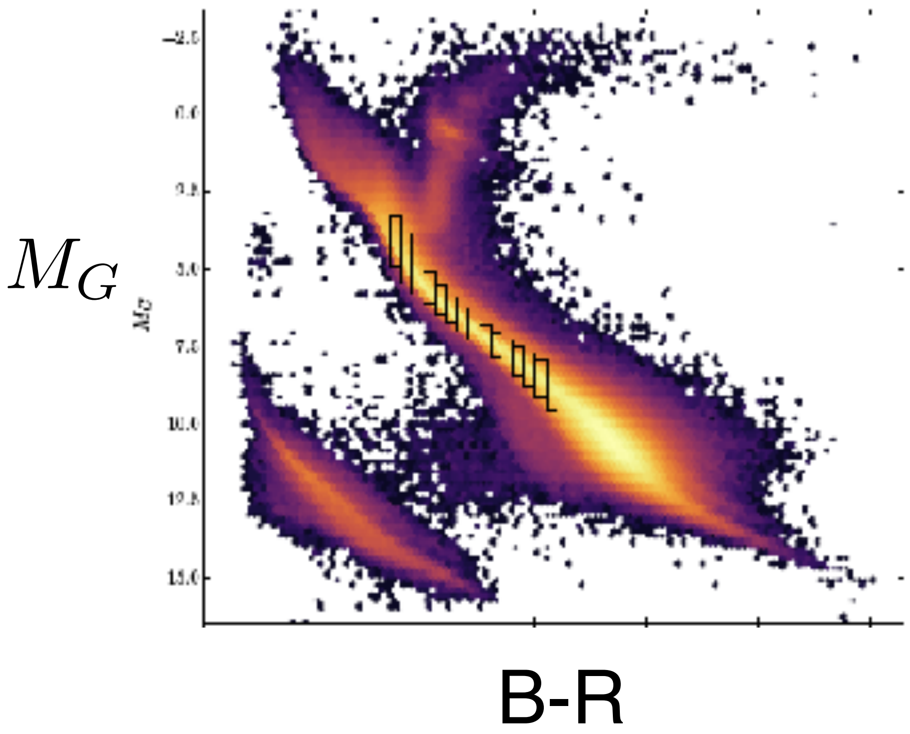
Physics of phase space spirals

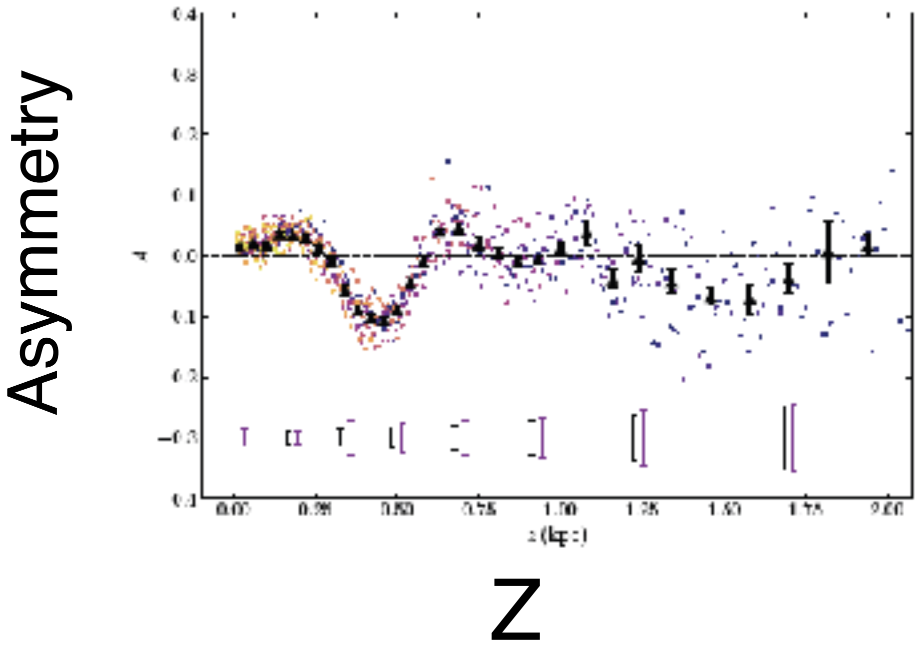
Two methods from data science that may prove useful

Number count asymmetry

LMW et al 2012, Yanny & Gardner 2013 with SEGUE
Bennett & Bovy 2018 with GDR2



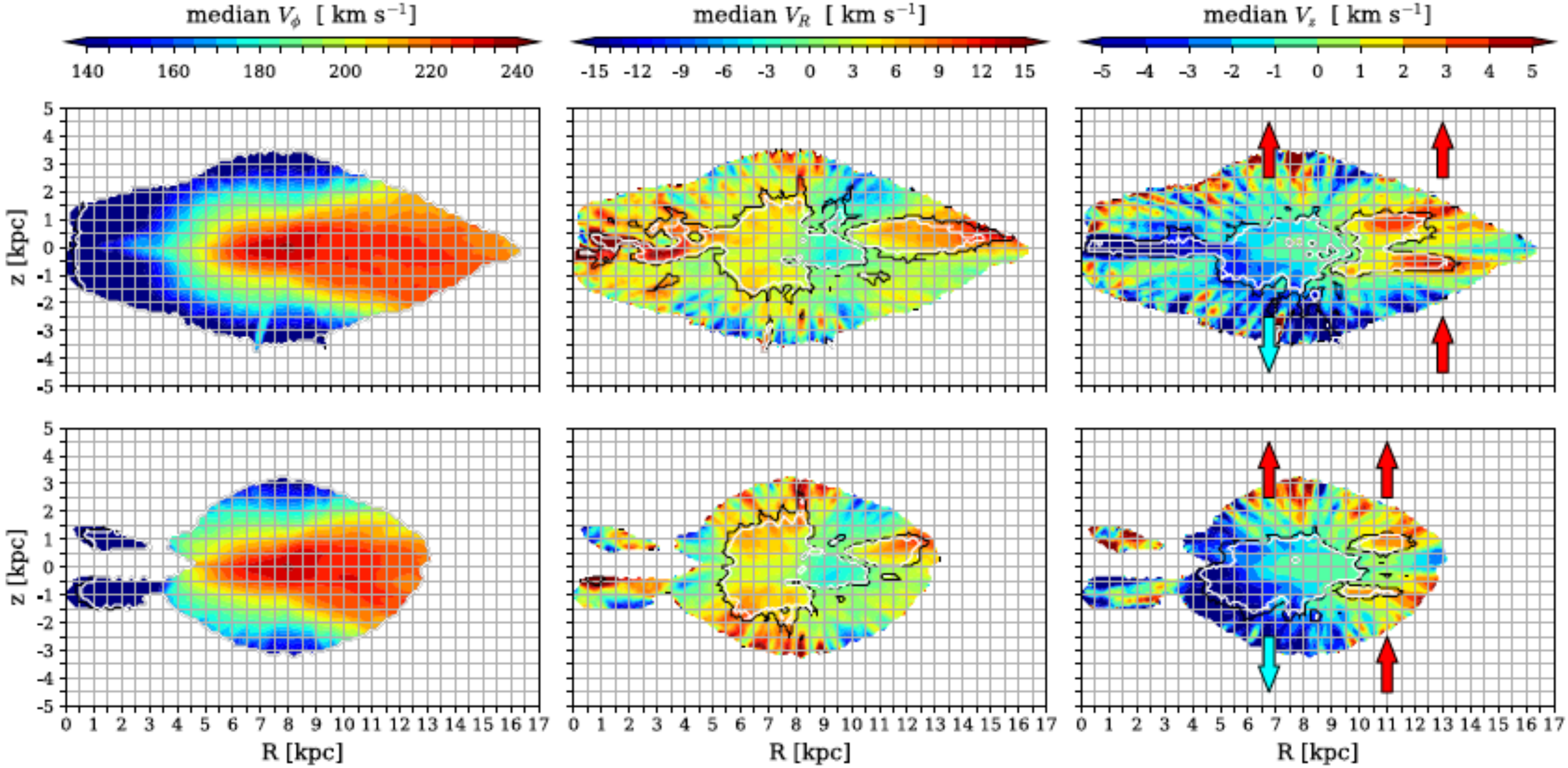




Figures from
B&B18

Bulk Motions of disk stars — R-z projection

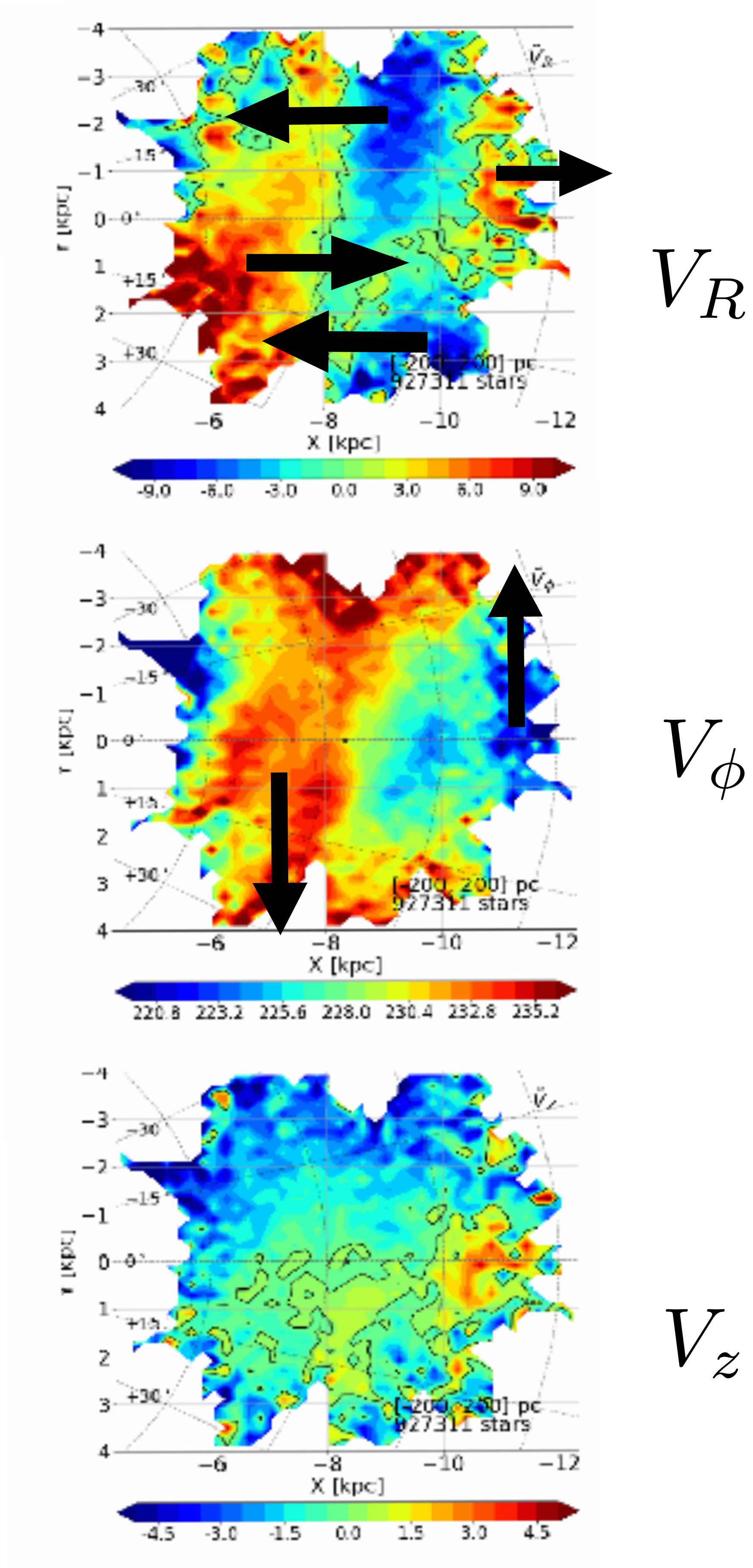
(LMW et al 2012 (SEGUE), Williams et al 2013 and Carrillo et al 2018(RAVE),
Carlin et al 2013 (LAMOST), Carrillo et al 2019, Laporte et al 2019

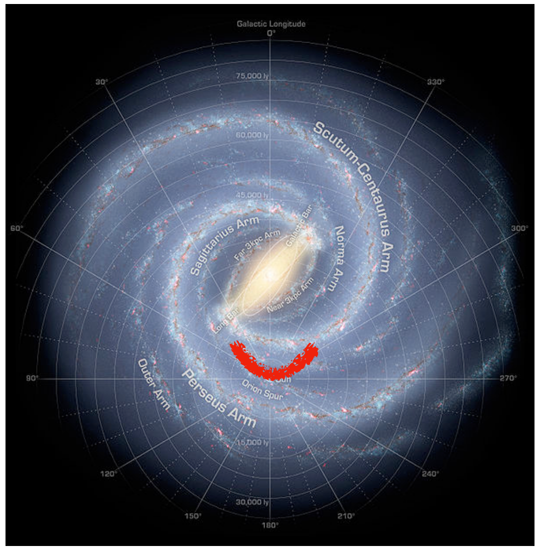


Carrillo et al 2019

Mean velocities in the GDR2 RV survey in the XY plane

Gaia Collaboration-Kinematics 2018
Corrillo et al 2019
Laporte et al 2019





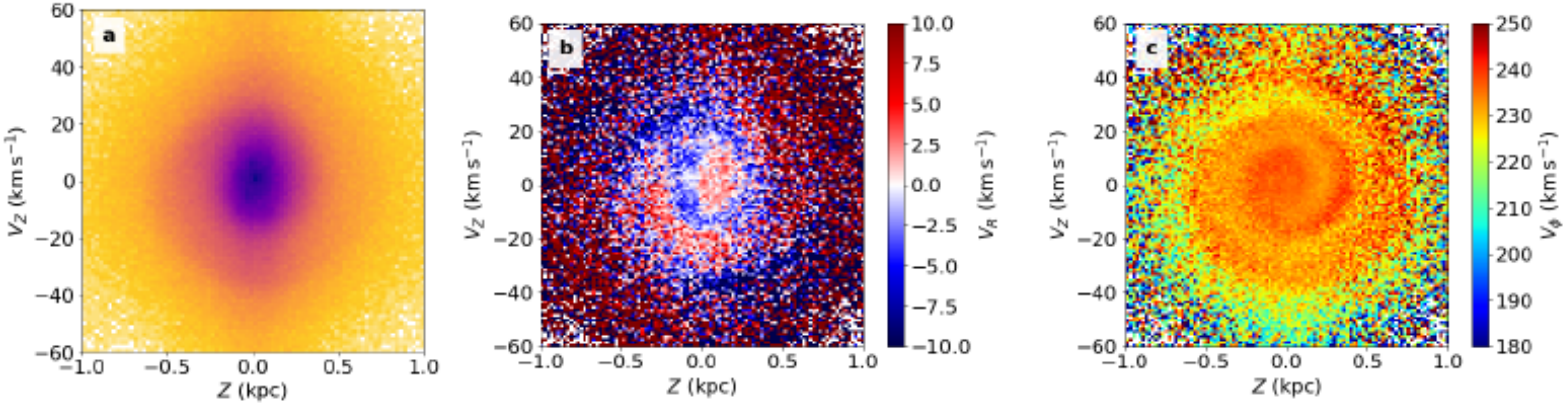
Phase Space Spirals in z-V_z plane

Antoja+18 (GDR2)

Density

V_R

V_{phi}

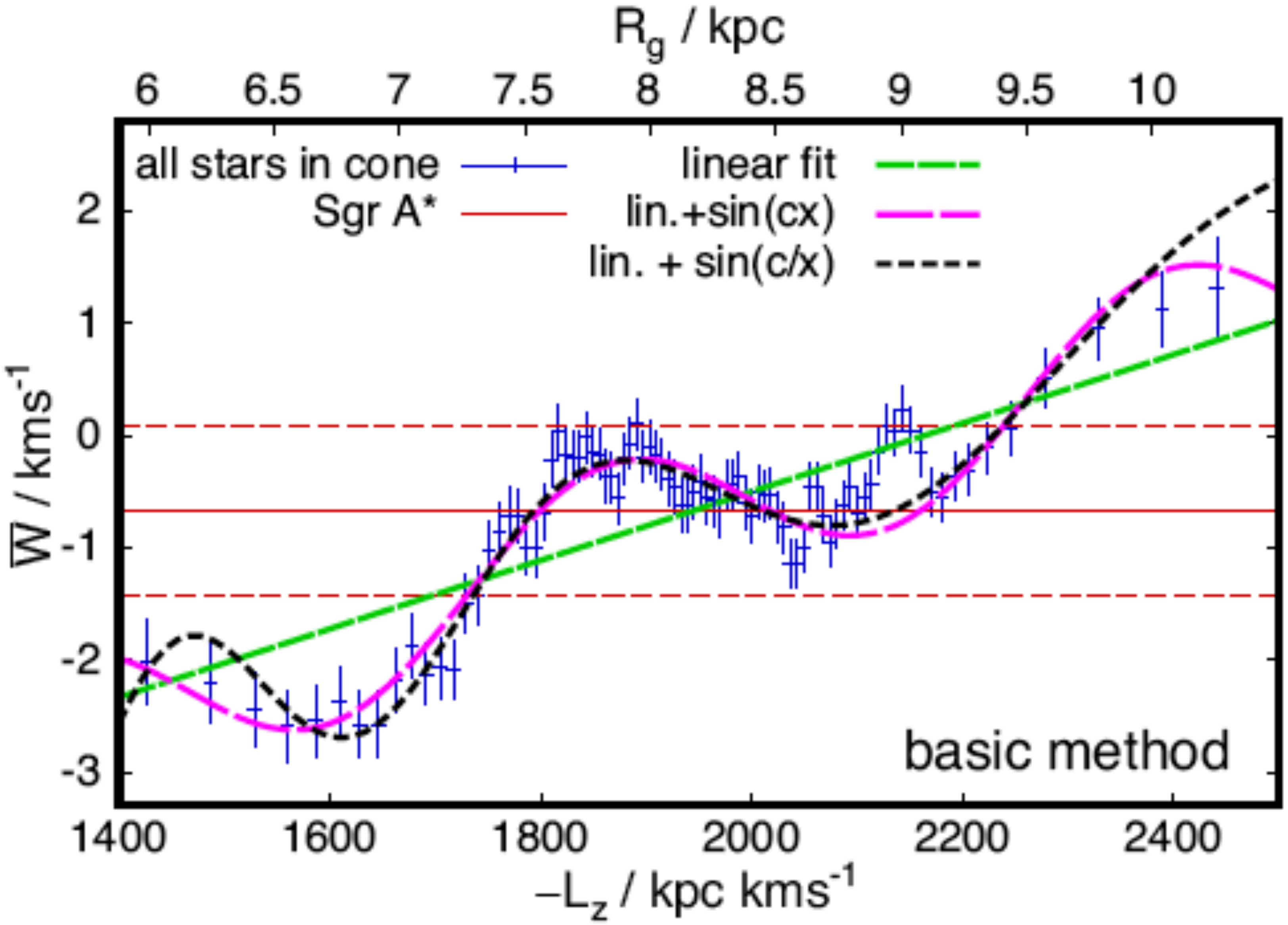


V_z

z

$\langle V_z \rangle$ as a function of L_z (or R) waves

Schoenrich & Dehnen 2018

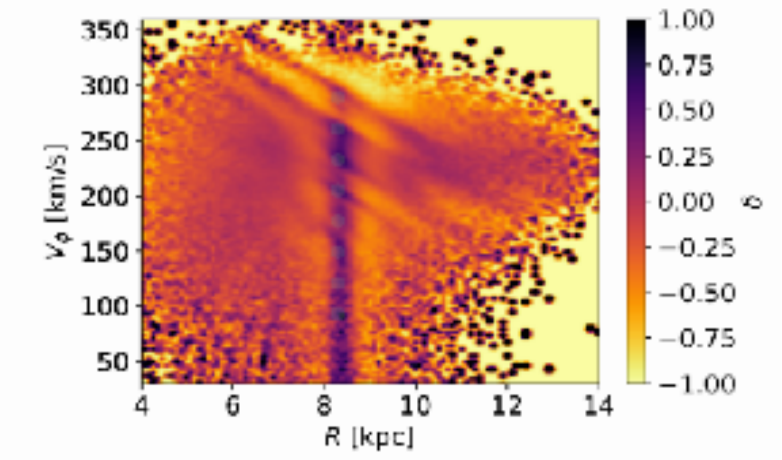


J_{phi}

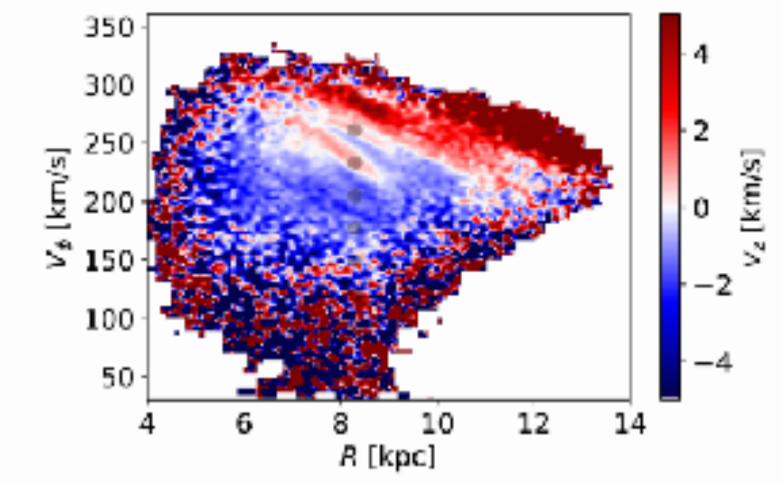
Phase space structures in R-V_{phi} plane

Laporte et al 2018

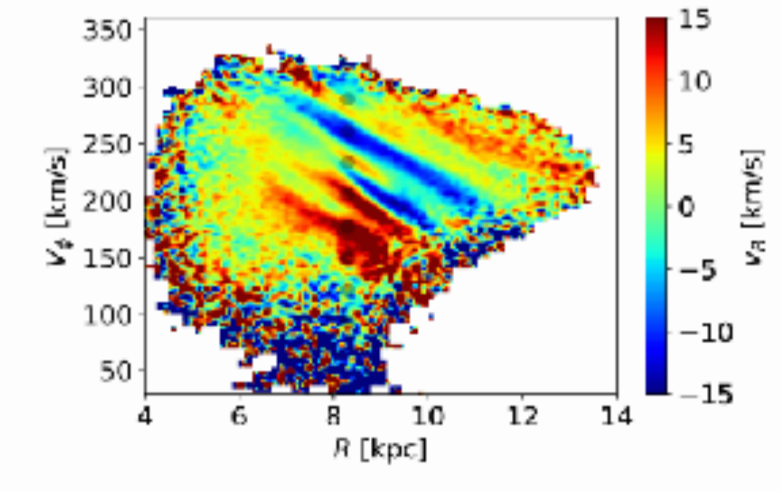
Density



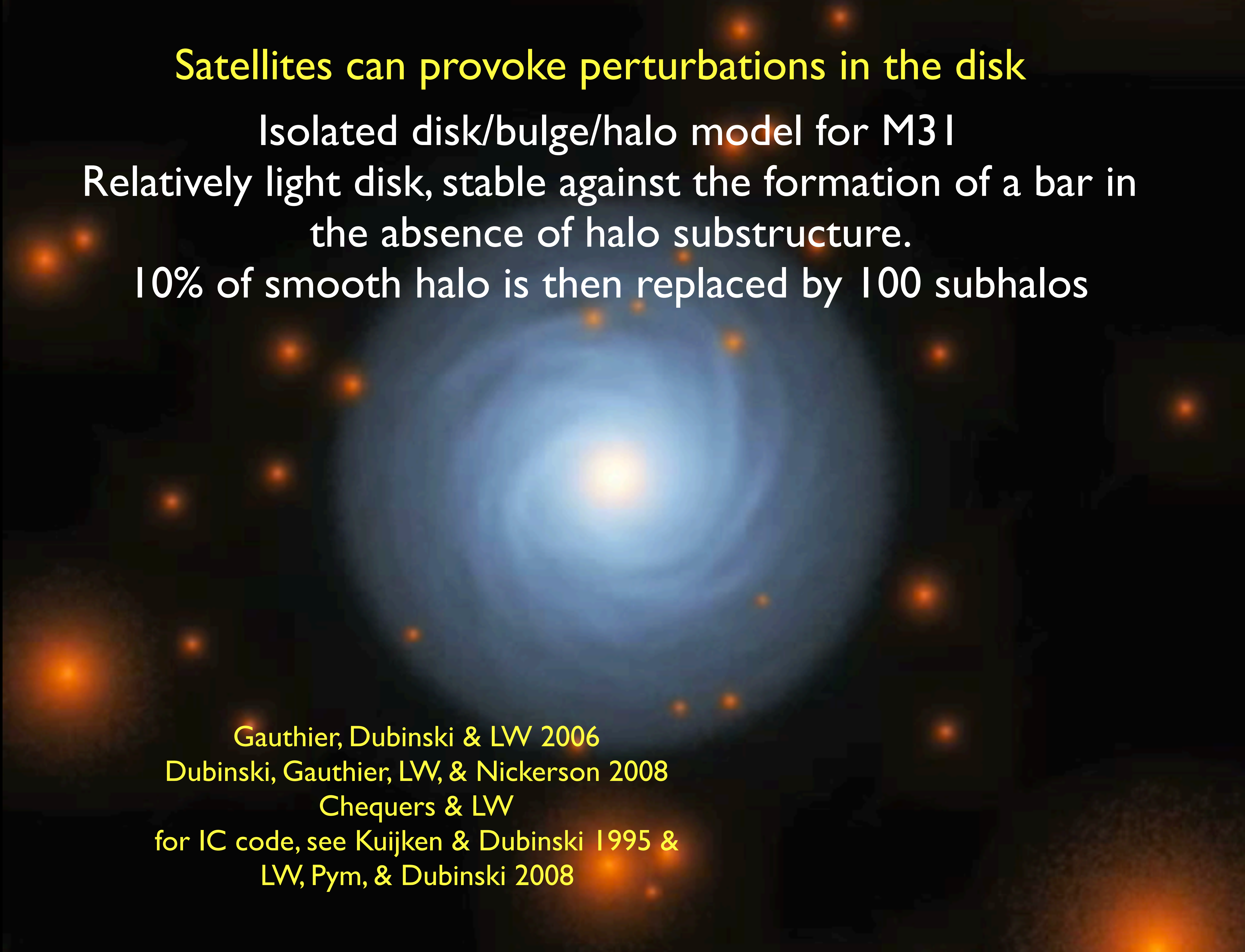
Mean V_z



Mean V_R



Satellites can provoke perturbations in the disk



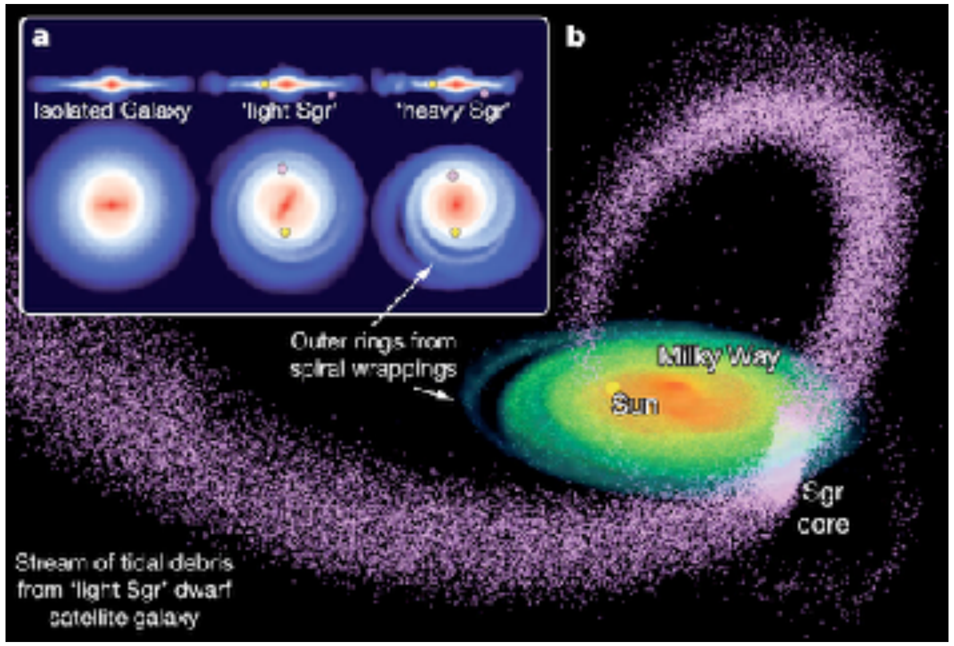
Isolated disk/bulge/halo model for M31

Relatively light disk, stable against the formation of a bar in the absence of halo substructure.

10% of smooth halo is then replaced by 100 subhalos

Gauthier, Dubinski & LW 2006
Dubinski, Gauthier, LW, & Nickerson 2008
Chequers & LW
for IC code, see Kuijken & Dubinski 1995 &
LW, Pym, & Dubinski 2008

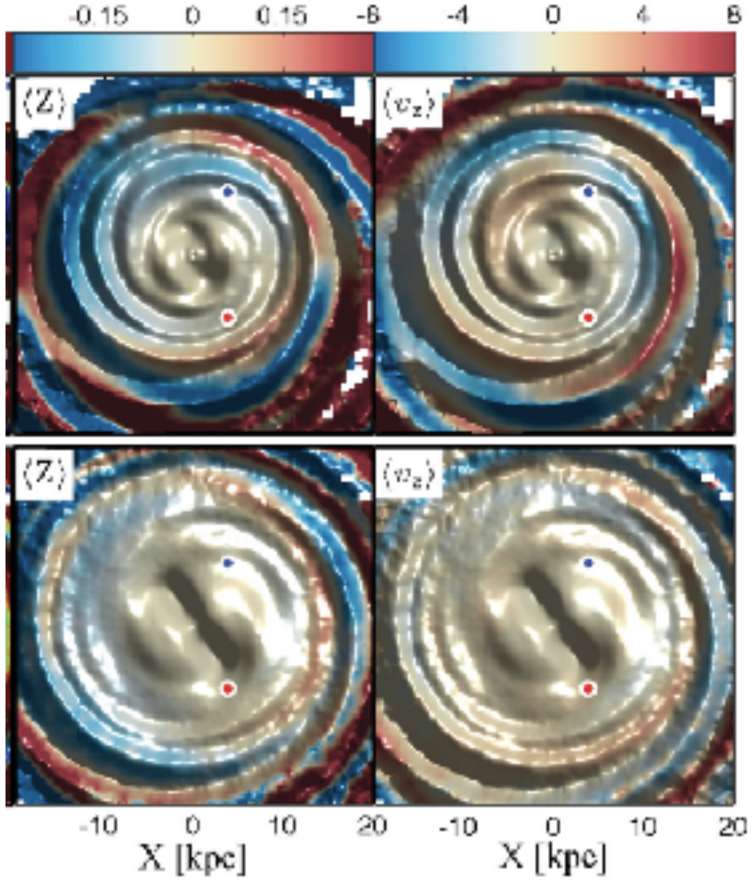
Sagittarius as architect of Milky Way Structure



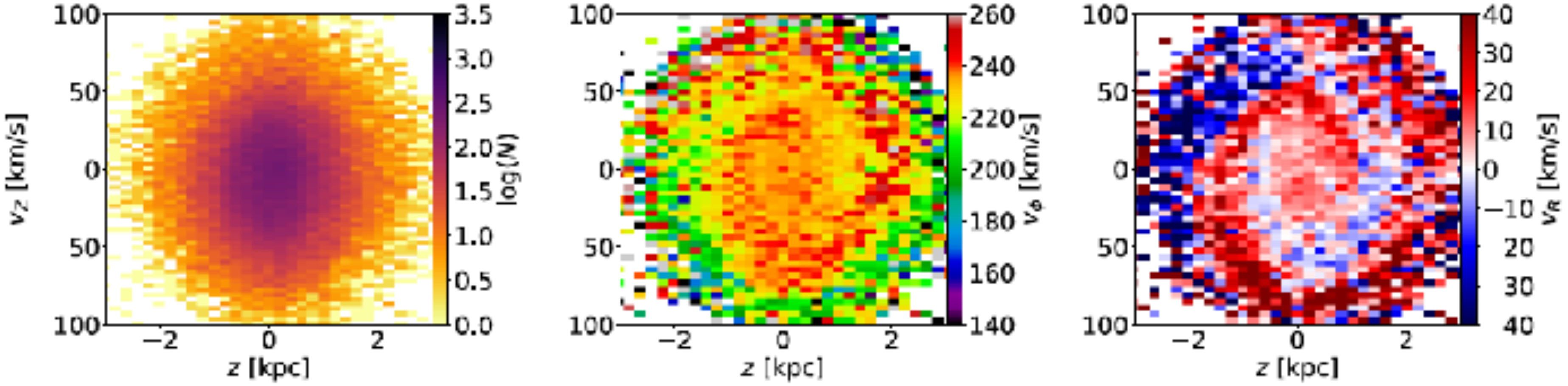
Purcell et al 2011

Heavy Sgr

Light Sgr



Gomez et al 2013…



Laporte et al 2019

Open questions

How are the different phase space structures related?
In what cases are we looking at the same distribution-function structures from different angles in 6D phase space?

Do we live in a special galaxy (i.e., one with a Sagittarius-like dwarf) at a special time and/or special place within the disc?
or
Are the structures that we see generic to stellar discs in clumpy, triaxial, time-dependent halos?

What do phase space structures tell us about the disk and what do they tell us about the perturbers?

The distribution function: What we want

$$f\left(\mathbf{x},\ \mathbf{v}\right)$$

Could also include population dependence
Modelled from simulations or theory

↓

Connection between simulations and
observations require a model for Gaia
selection function, etc..
See Sharma+11 (GALAXIA),
Kawata, Hunt +14, Sanderson +18

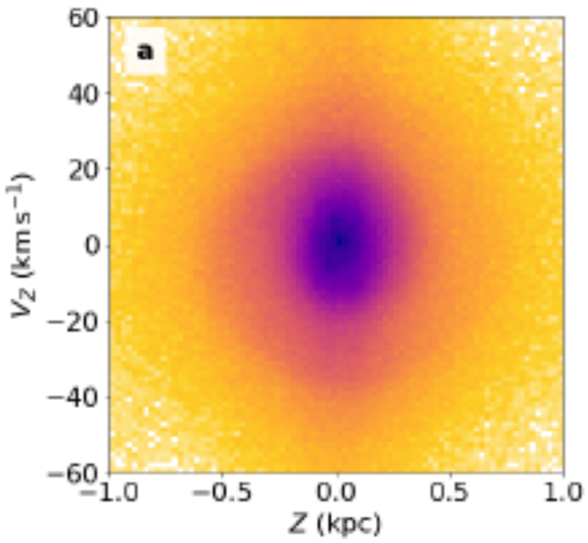
Observables

$$\left\langle \mathcal{O}\left(\mathbf{x}_p,\ \mathbf{v}_p\right)\right\rangle = \int_{\Omega} S(\mathbf{x})\, d^3x \int f\left(\mathbf{x},\ \mathbf{v}\right) \mathcal{O}\left(\mathbf{x},\ \mathbf{v}\right)\ d^3v$$

Selection function

Begin with $f(\mathbf{x},\ \mathbf{v})$

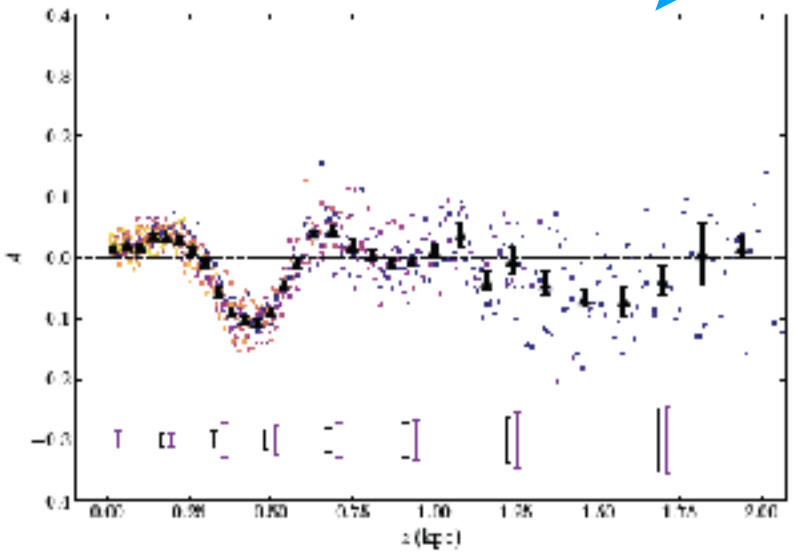
Arc in Galactocentric R/integrate over ϕ, V_R and V_ϕ



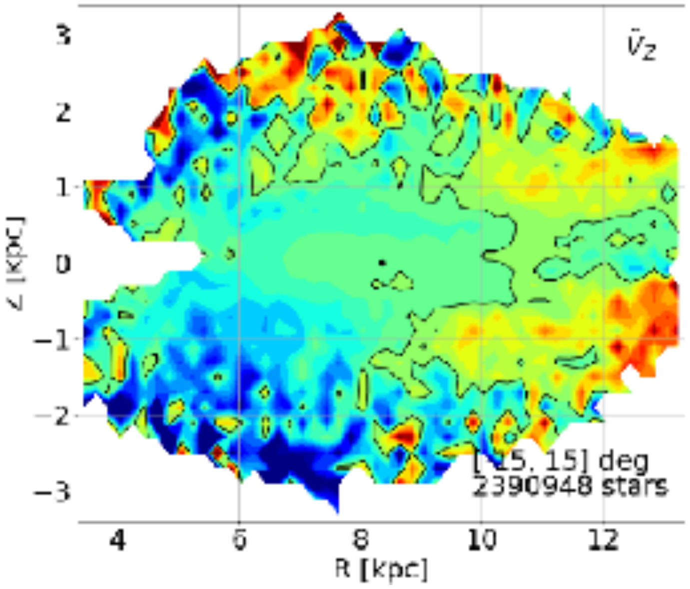
Integrate over v_z
difference across midplane

V_z moment

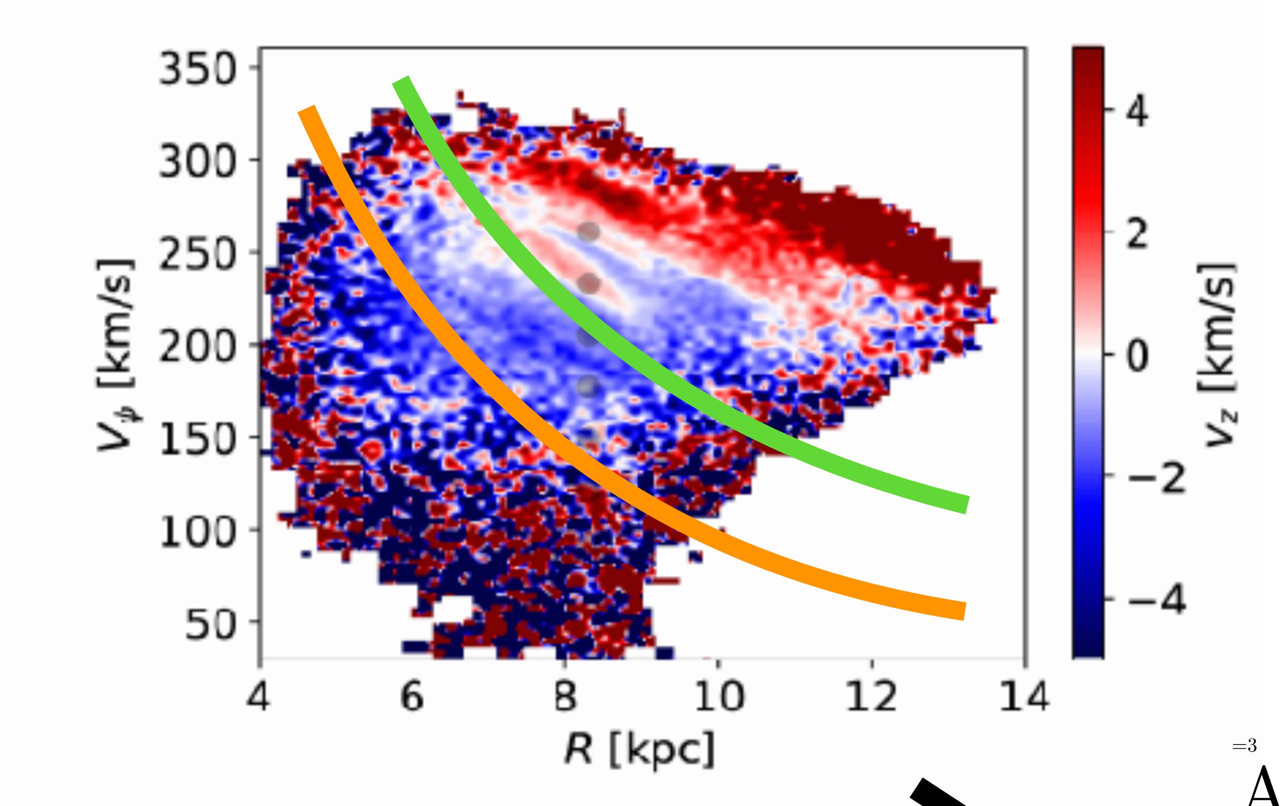
Asymmetry



Z



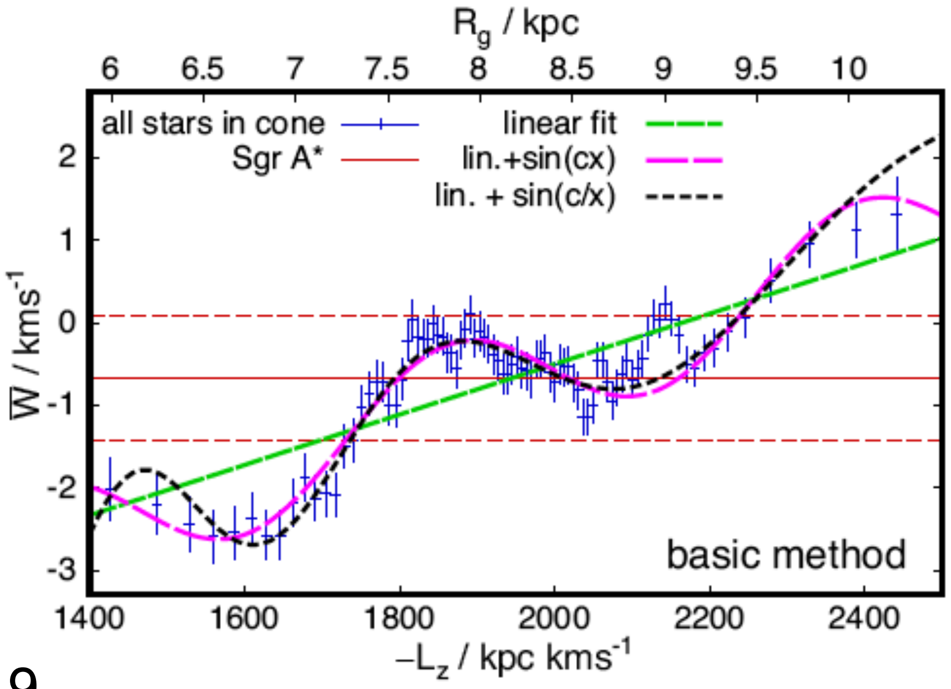
Bulk V_z motion in z-R plane
From GDR2-Kinematics
paper 2018



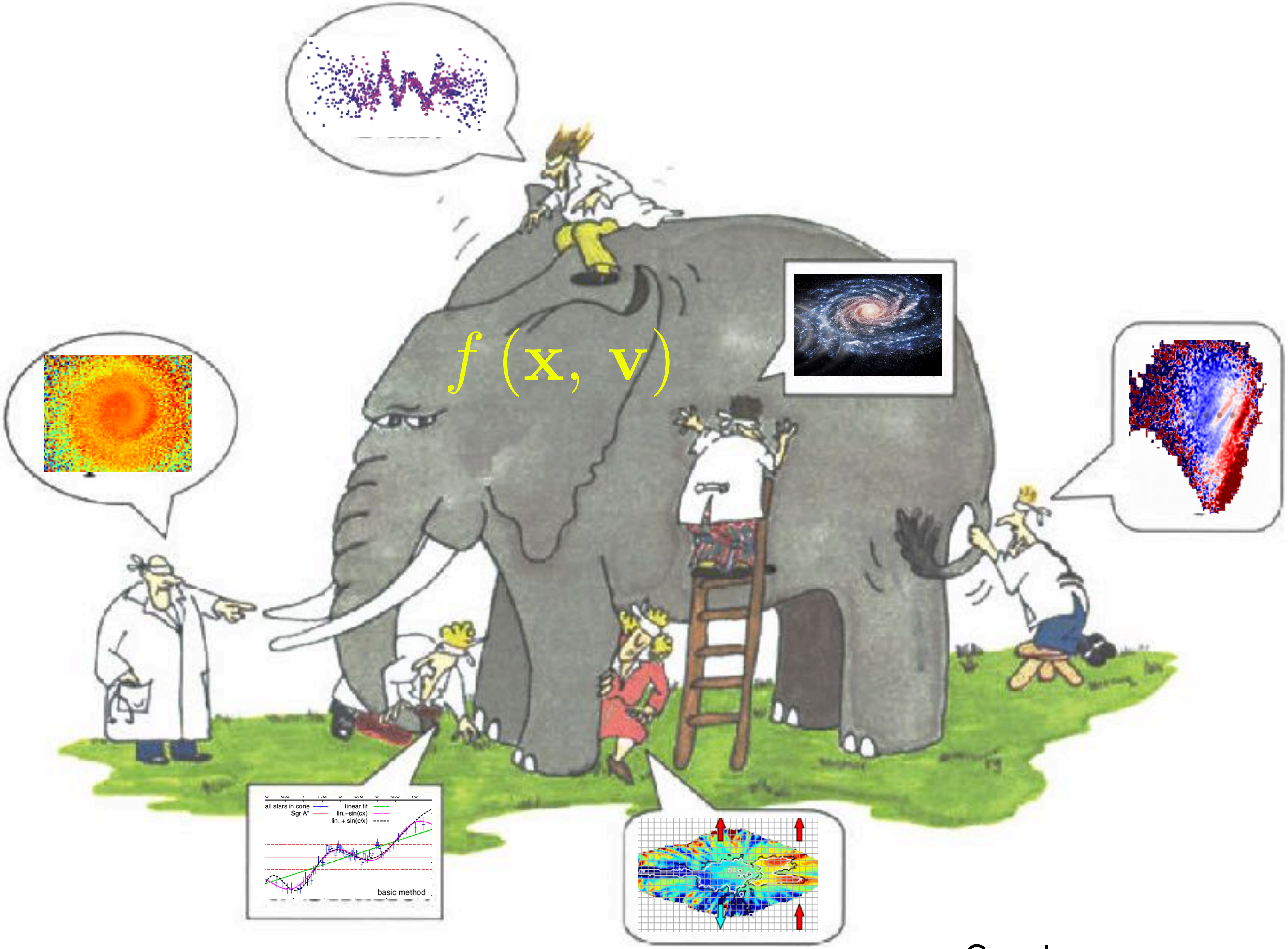
=3

Average over curves of constant J_ϕ

Laporte et al 2019



Schonrich & Dehnen 2019

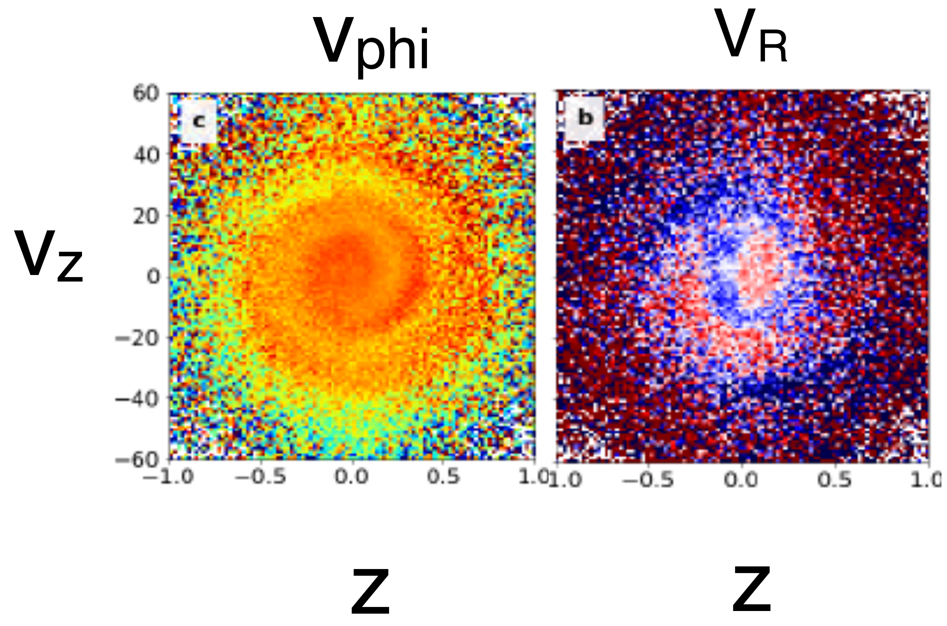

$f\,(\mathbf{x},\ \mathbf{v})$
all stars in cone
Sgr A*
linear fit
lin.+sin(cx)
lin. + sin(c/x)
basic method

Gary Larsen

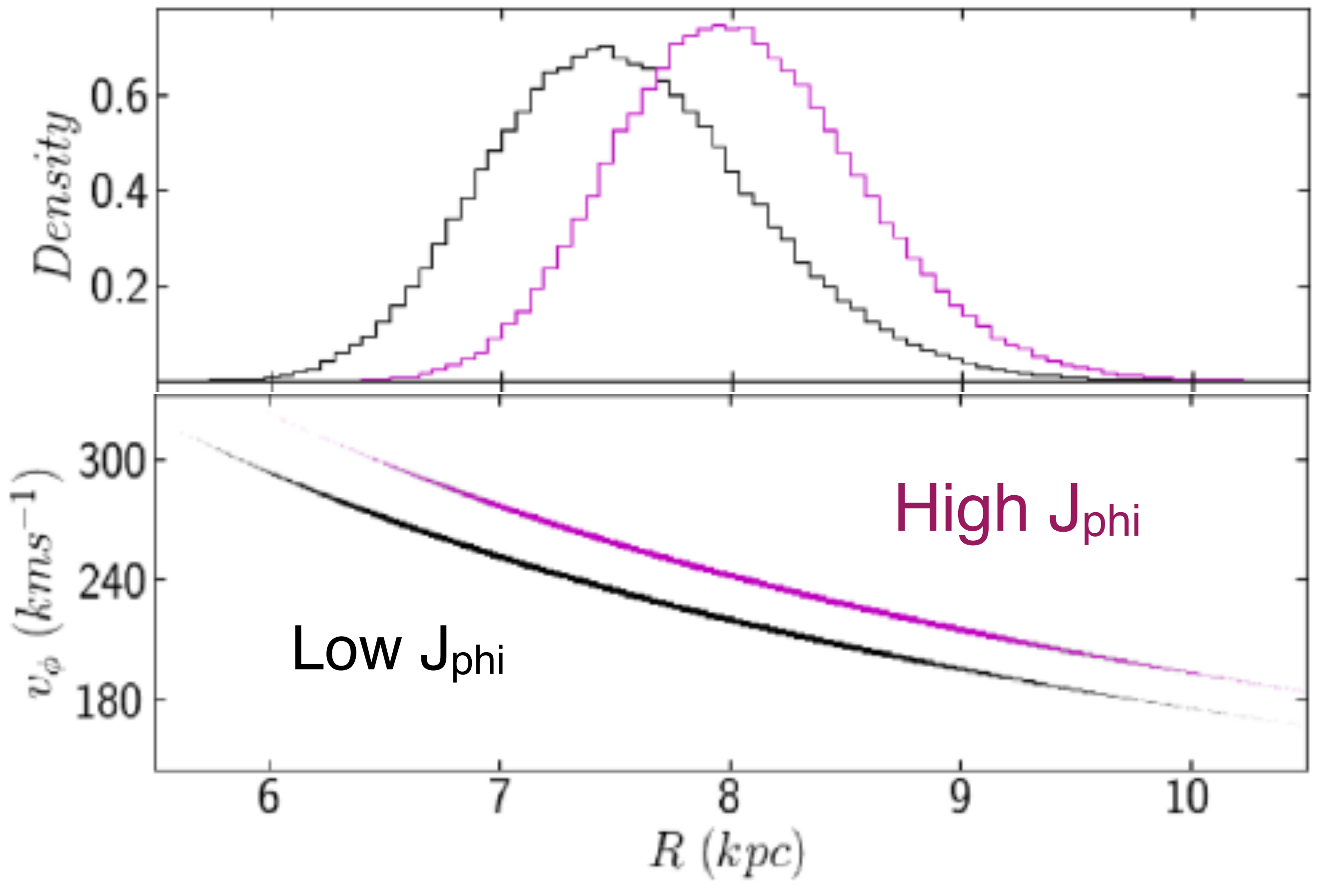
Ingredients for $z - v_z - v_\phi - v_R$ phase space spirals

Darling & LMW (2019), Binney & Schoenrich (2019) …

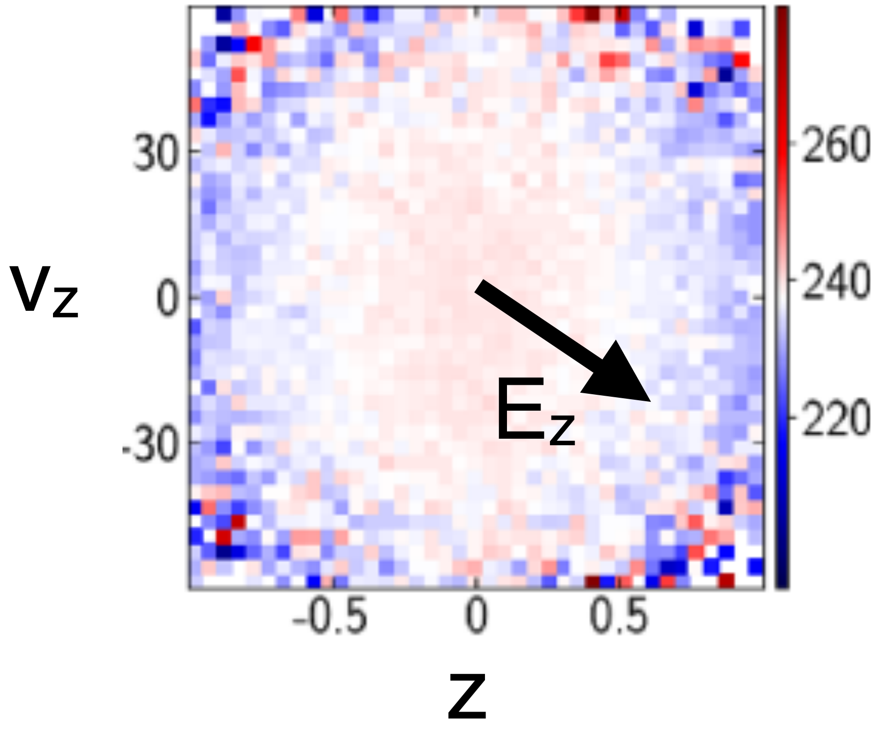
- Mixture of angular momenta at R_{sun}
- Stars with different angular momenta separate out in vertical energy E_z



Toy model with two pops at different J_{phi}



In equilibrium, J_{phi} pops separate in E_z

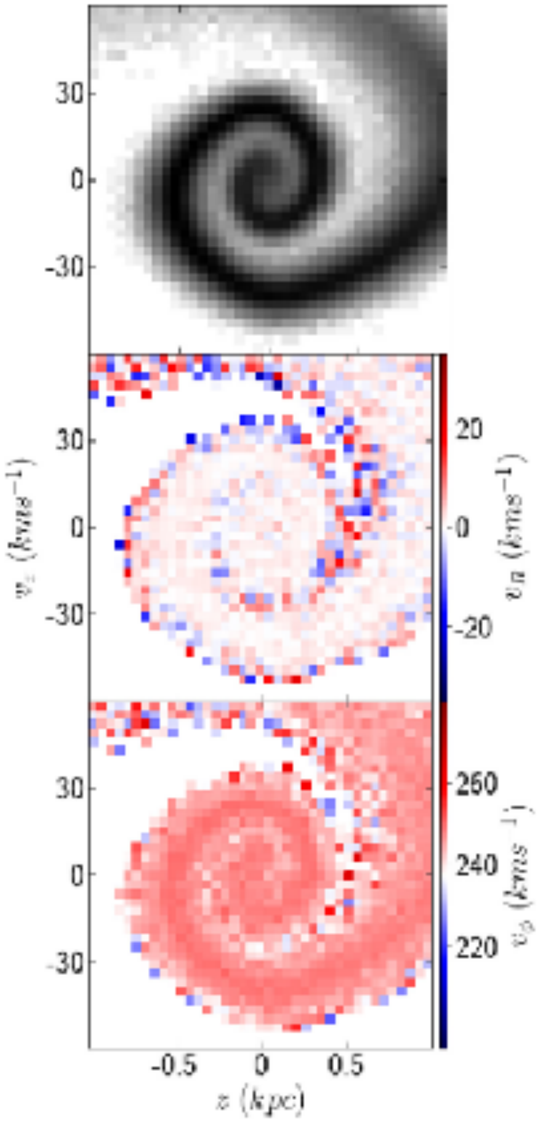


Coloured by V_{Phi}

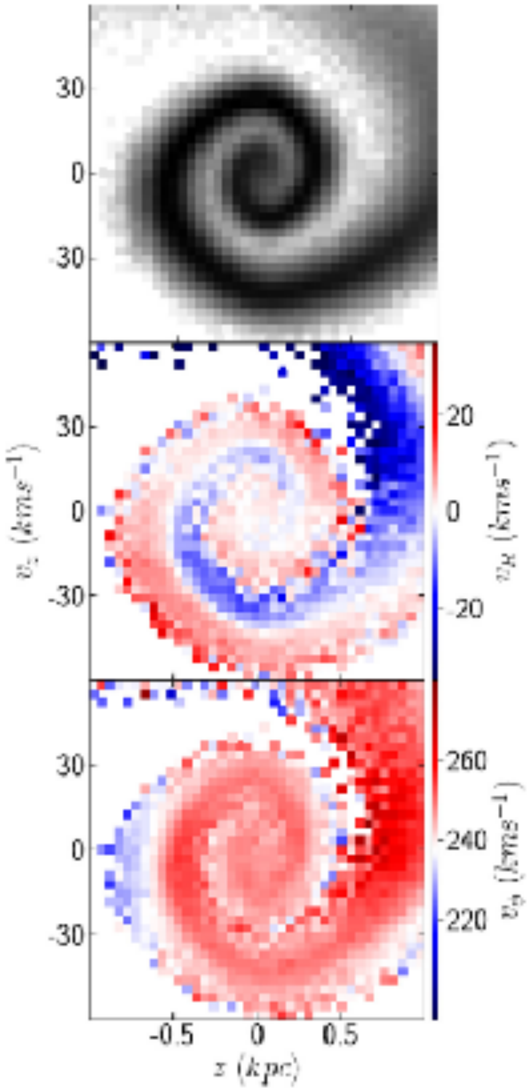
Figs from Darling & LMW

- Perturbation of system in z or v_z
- Anharmonic vertical potential; stars with high E_z rotate more slowly in the z-v_z plane
- R-z coupling in effective potential (terms beyond epicycle approximation) appears to be important to get the right morphology

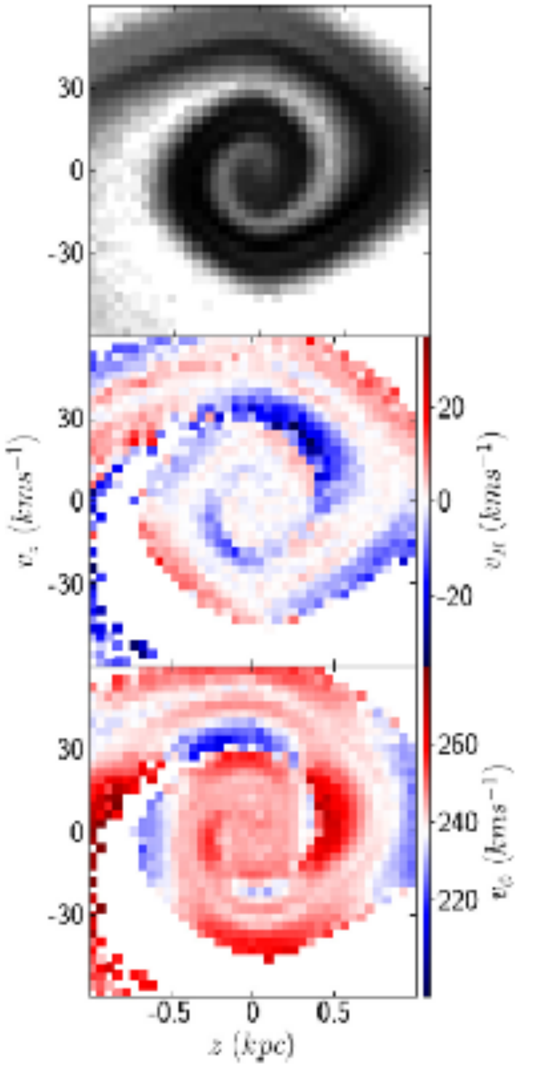
Separable potential



toy model with R-z coupling



Galpy potential



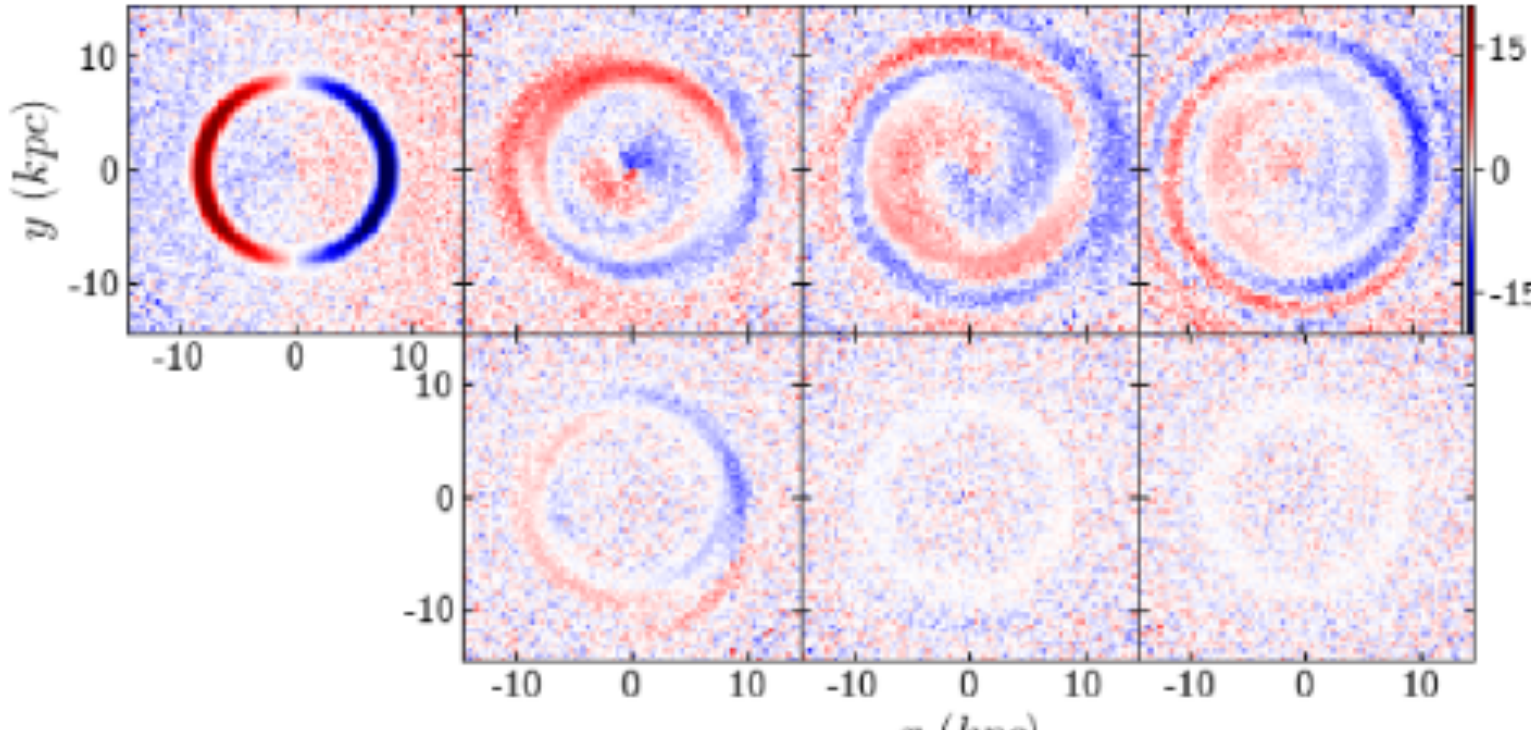
Suggests that the spirals provide a probe of the local potential

Importance of self-gravity

Perturb a disk by tilting a ring at these Solar circle

Use high-res, low mass particles in the ring to boost resolution where we want to probe phase space.

(This is somewhat dubious since particles mix in radius. However, the resolution in Gaia is 1 M_{sun}, which is well below what is possible in simulations.)



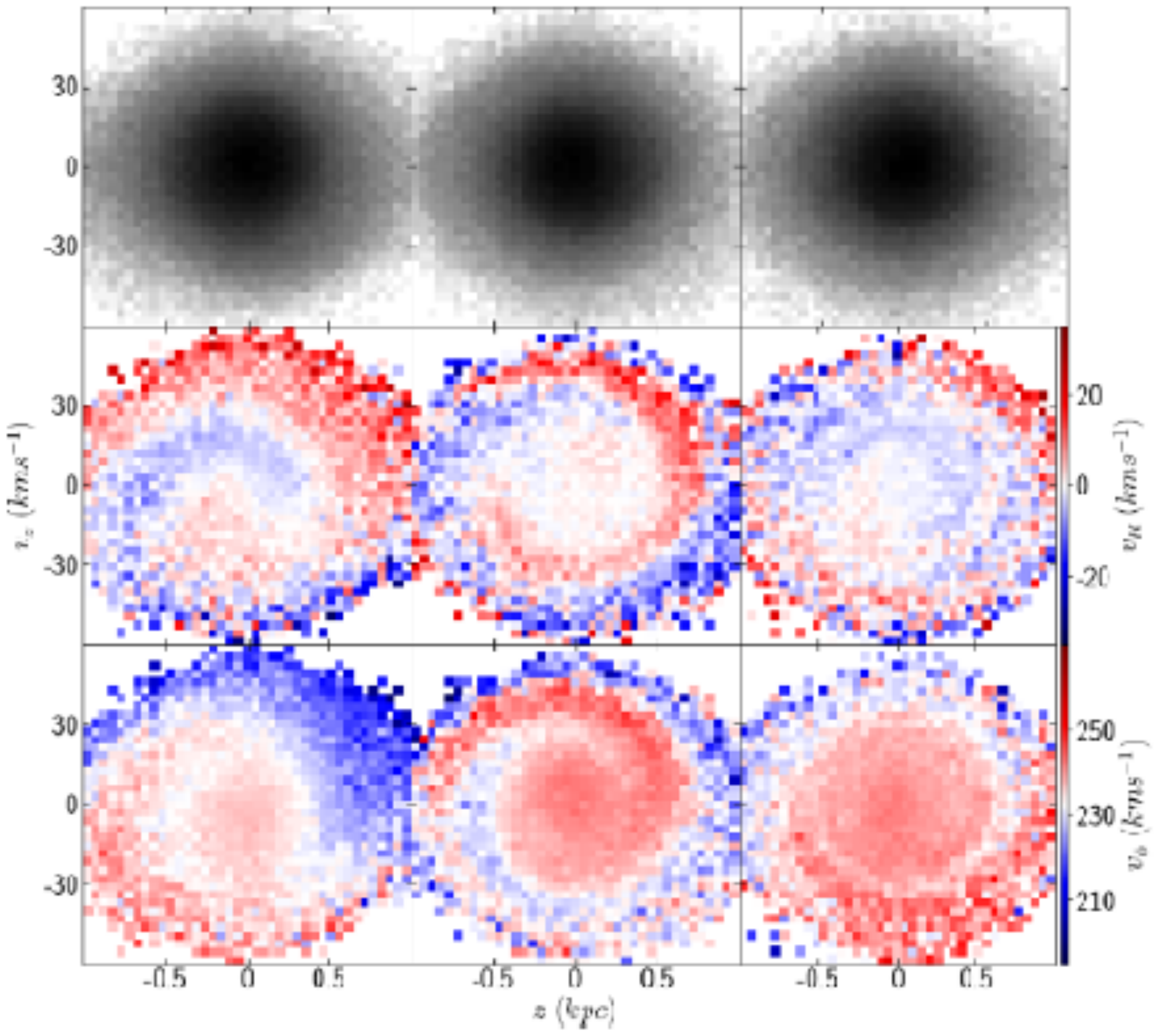
Fully live disk and halo
Includes self-gravity for perturbation

Test particles in static potential

Darling & LW 2019

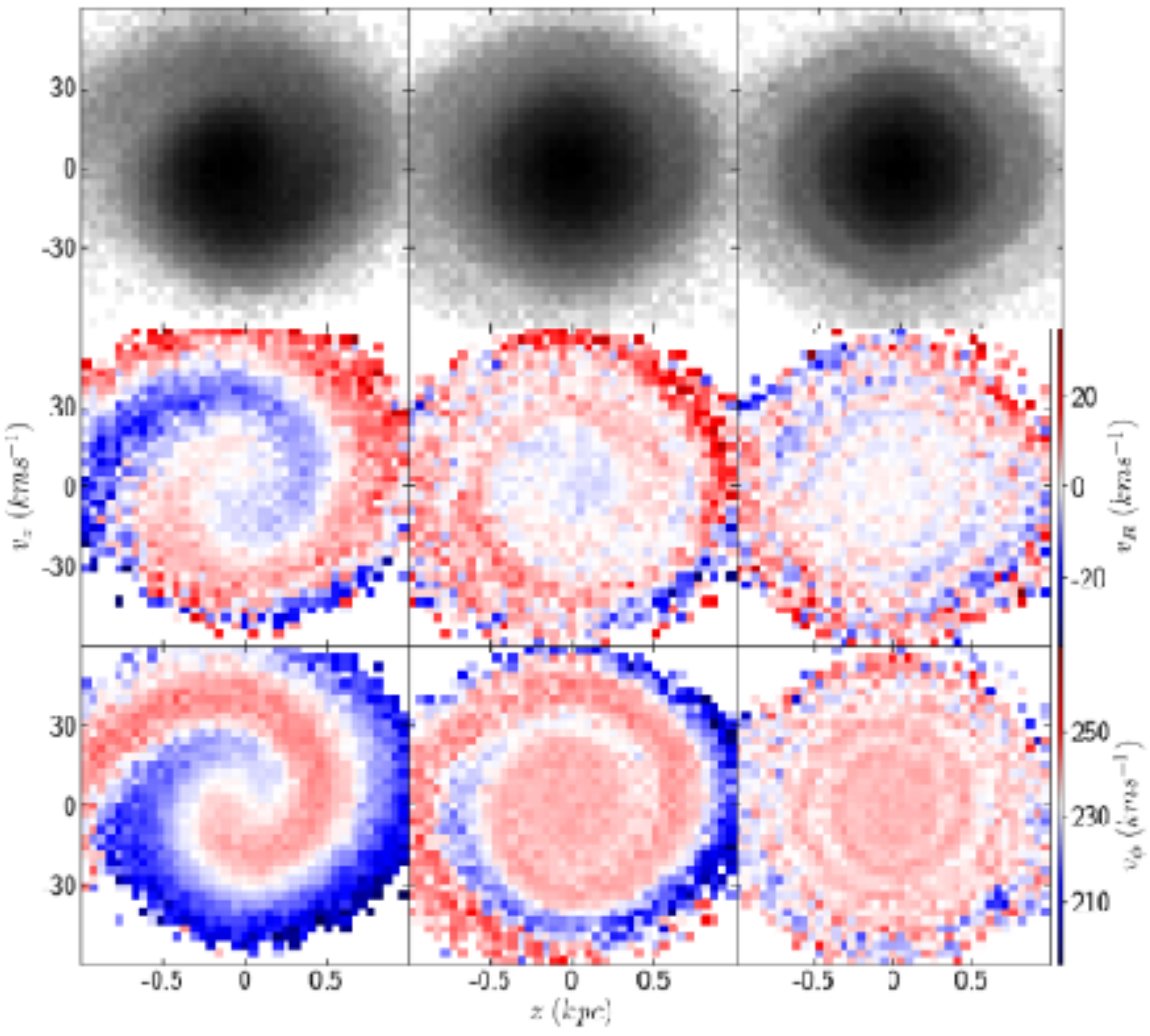
If you want to understand the Xu et al corrugation pattern and its connection to the warp, self-gravity is essential

Live disk and halo



250 Myr 500Myr 1Gyr

Test particle simulation



250 Myr 500Myr 1Gyr

Zeroth order problem: Find a suitable description for a perturbed DF

One possible framework is to assume phase mixing.

Begin with an unperturbed galaxy…Apply perturbation, which then undergoes phase mixing.

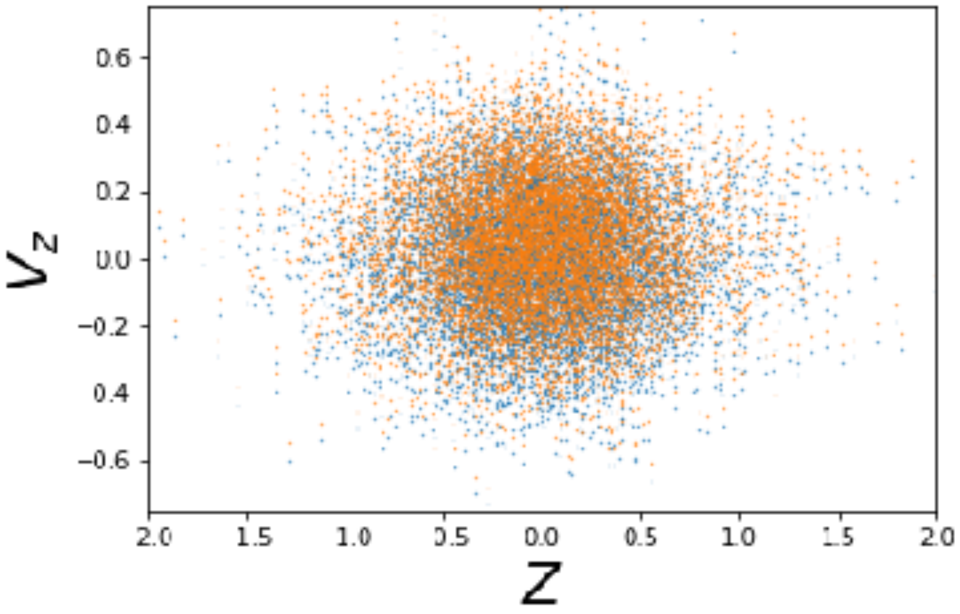
We should then be able to write:

$$f\left(\mathbf{x},\,\mathbf{v}\right)\;\;=\;\;f_{\mathrm{eq}}\left(\mathbf{x}_{\mathrm{eq}}\left(\mathbf{x},\,\mathbf{v}\right),\,\mathbf{v}_{\mathrm{eq}}\left(\mathbf{x},\,\mathbf{v}\right)\right)\mathcal{J}\left(\mathbf{x},\,\mathbf{v};\mathbf{x},\,\mathbf{v}\right)$$

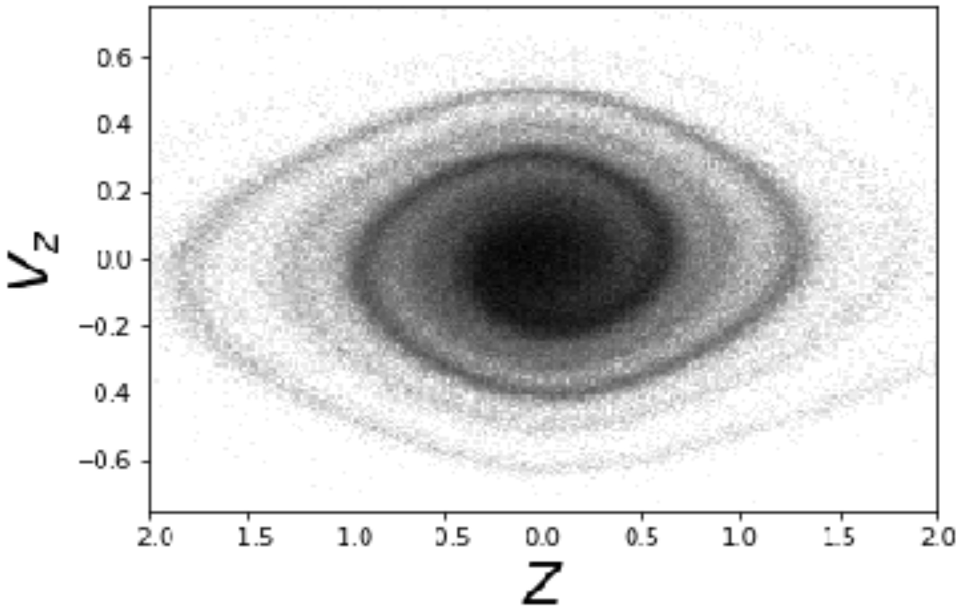
Lagrangian viewpoint; stars retain memory of the initial perturbation encoded in transformation between **x,v** and $\mathbf{x}_{\mathrm{eq}}$, $\mathbf{v}_{\mathrm{eq}}$

See de al Vega, Quillen et al 2015 and Antoja et al 2019; See, also Tremaine 1998

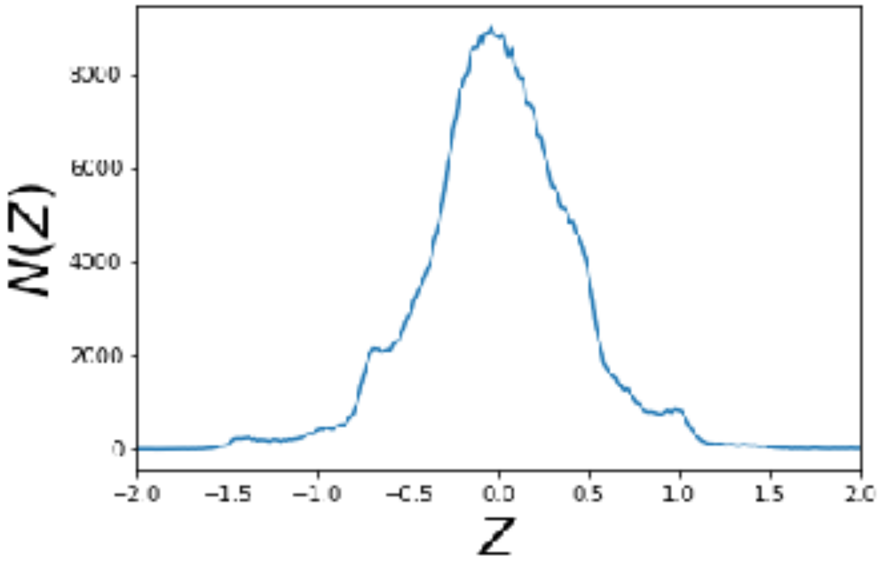
1D slab (Spitzer) model for vertical structure of the Galaxy
Here, particles evolve by phase mixing



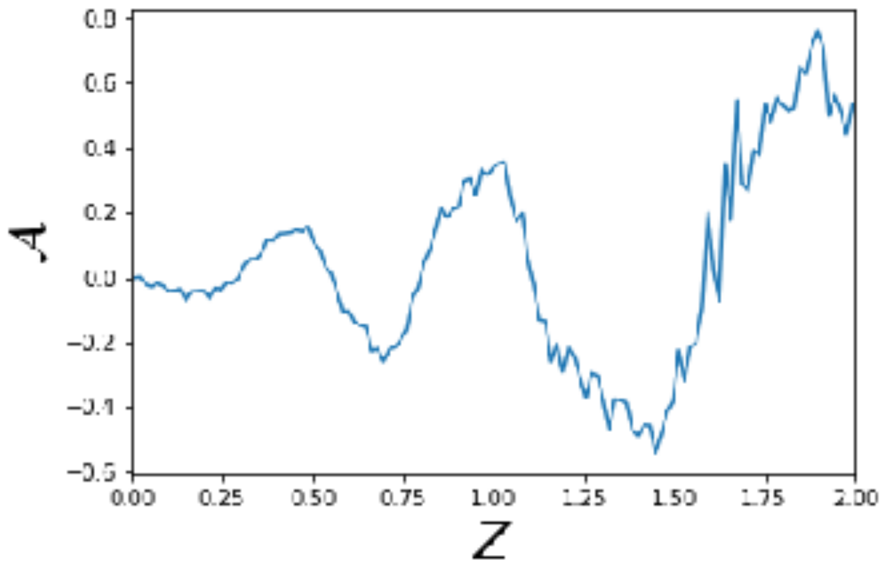
“bend” equilibrium DF by shifting mean velocity



System evolves via phase mixing



Project DF on to z to get number count distribution



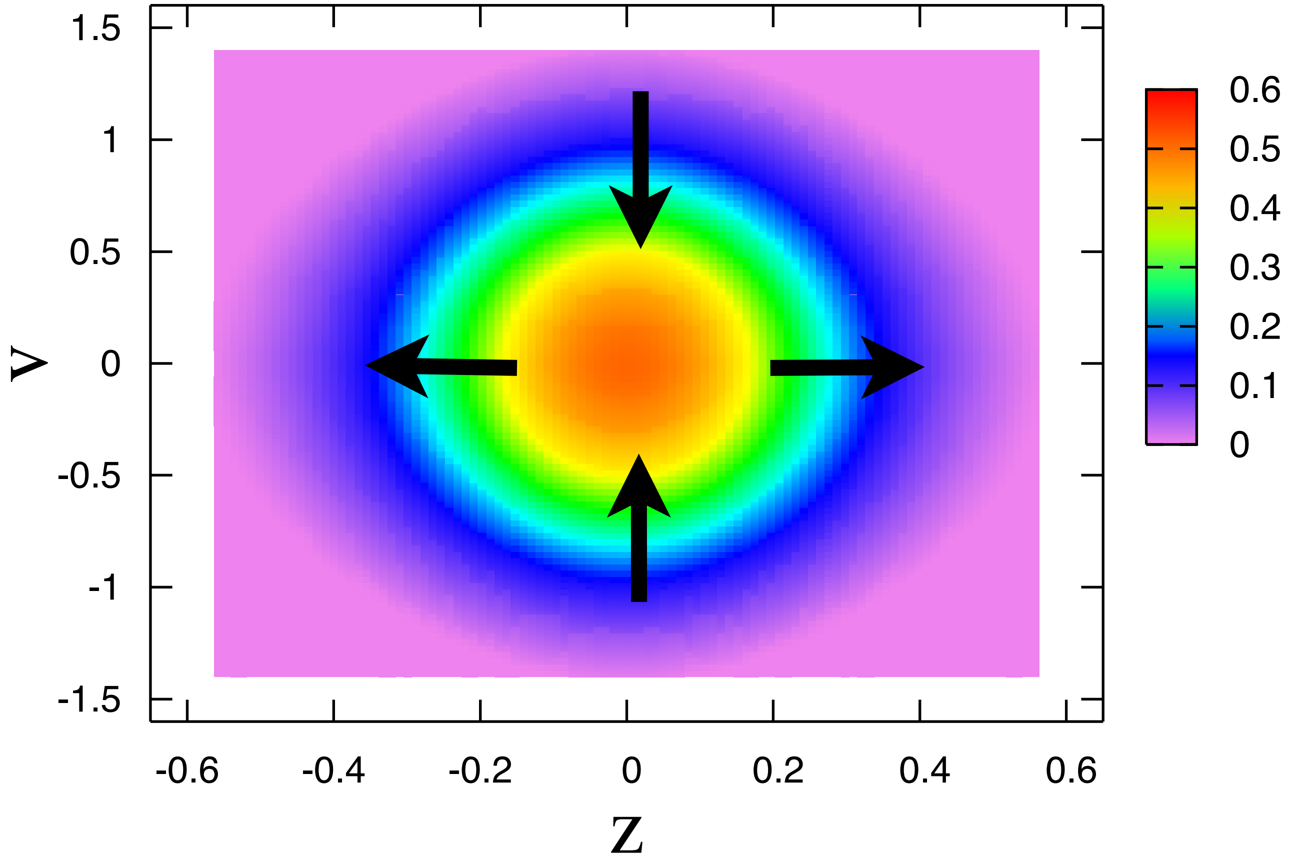
dimensionless N-S asymmetry parameter

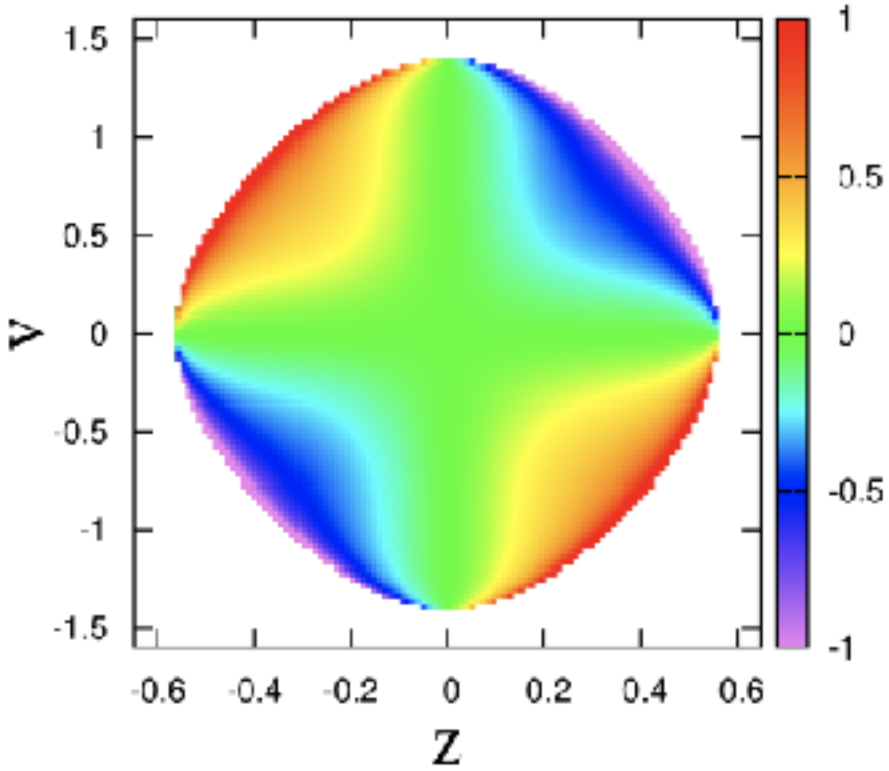
But perturbations of the disk imply a perturbation of the potential. Suggests we may have “modes”

(see Mathur 1990, Weinberg 1991, LMW et al 2014, LMW & Bonner 2015)

Breathing mode

phase space DF





Z-V_z pattern rotates clockwise without phase mixing

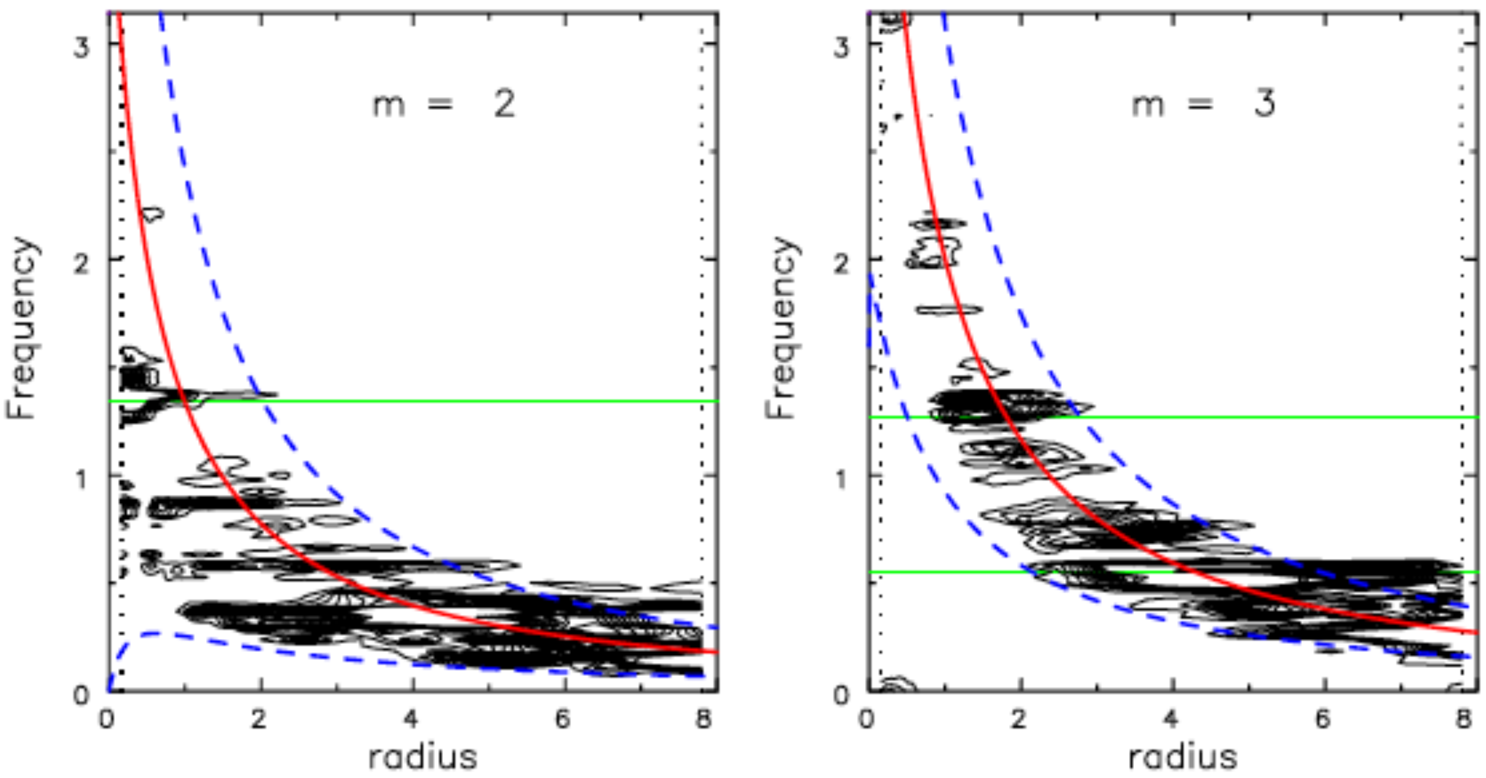
For modal decomposition, we might be tempted to try:

$$f(\mathbf{x},\,\mathbf{v}) \;=\; f_{\rm eq}(\mathbf{x},\,\mathbf{v}) \;+\; f_1(\mathbf{x},\,\mathbf{v},\,t)$$

$$\Sigma_1(R,\,\phi,\,t) \;=\; \int d^3v\,dz\,f(\mathbf{x},\,\mathbf{v})$$

$$\sim\; \sum_m \sum_k e^{im\phi} e^{i\omega_k t}\Sigma_{m,k}(R)$$

For spiral structure, see
Sellwood & Athanassoula 1986
Fig from Sellwood & Carlberg 2014

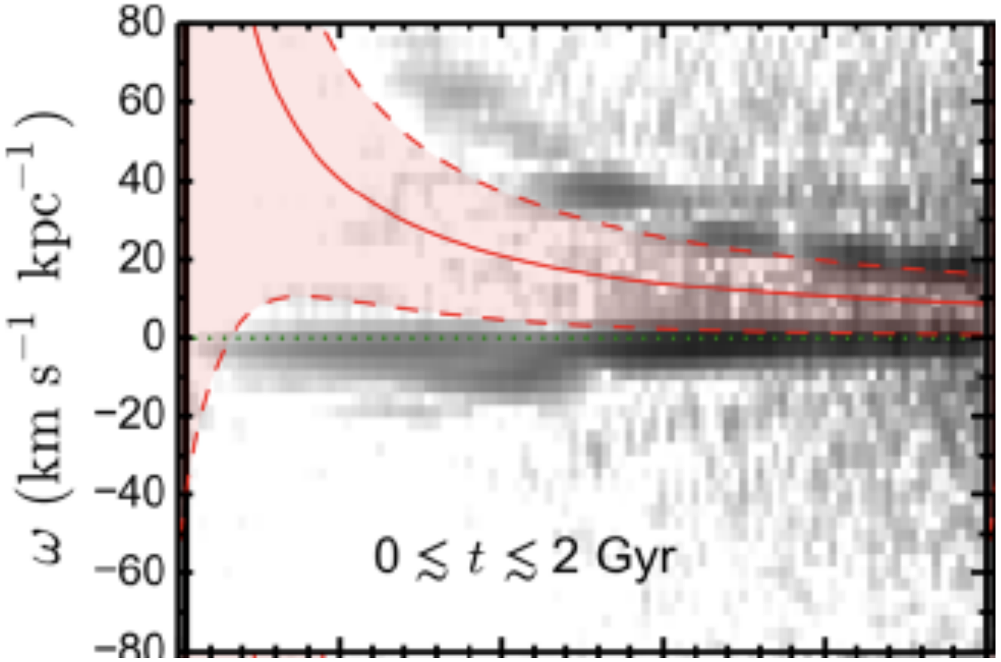


Can extend to moments in z and compare with linear theory of bending modes of a disk

Hunter & Toomre 1969, Sparke & Casertano 1988, Chequers, LMW 2017, 2018

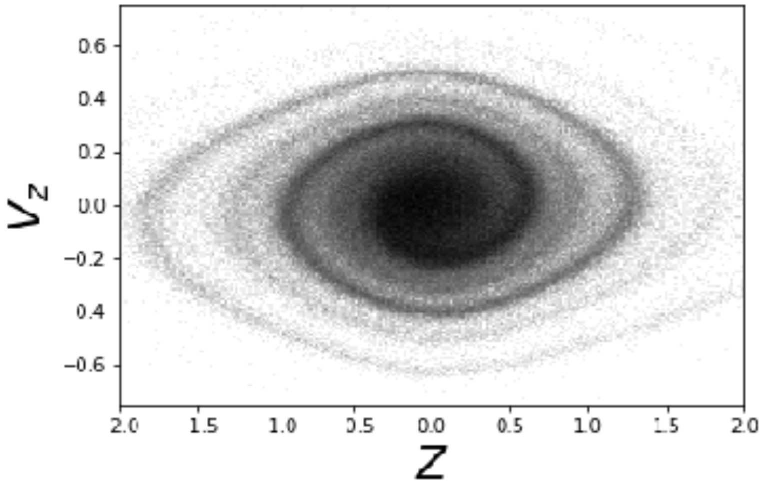
$$\bar{z}\left(R,\,\phi,\,t\right) \;=\; \int d^3v\,dz\,f\left(\mathbf{x},\,\mathbf{v}\right)\,z$$

$$\sim\;\sum_m \sum_k e^{im\phi} e^{i\omega_k t}\,\bar{z}_{m,k}(R)$$

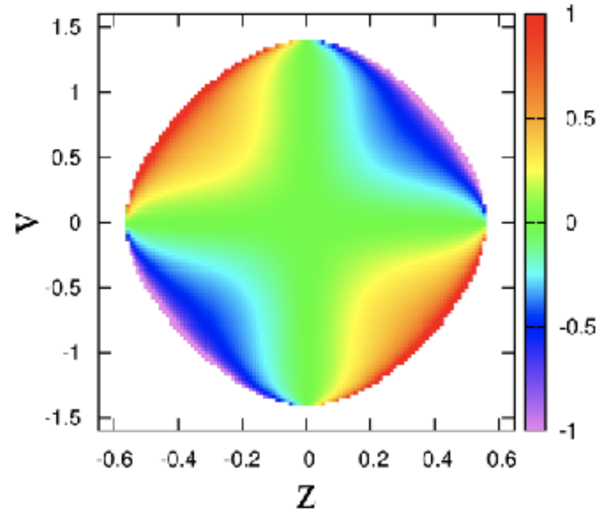


Spectral power for bending waves lies *outside* Linblad resonances, as predicted by linear theory. (Chequers & LW ’17, ’18)

In general, we expect both phase mixing and modal oscillations to be relevant for dynamics

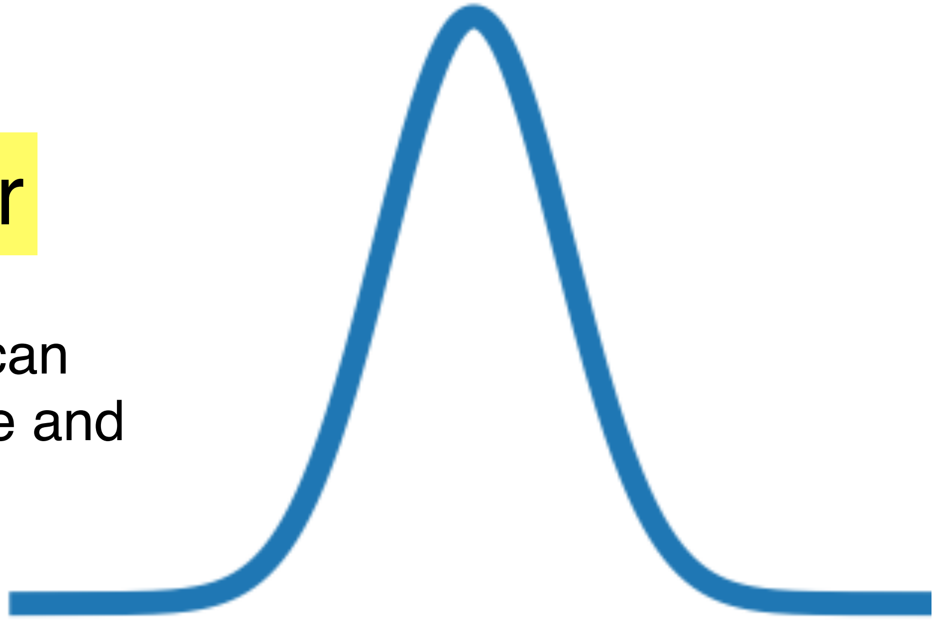


Weather

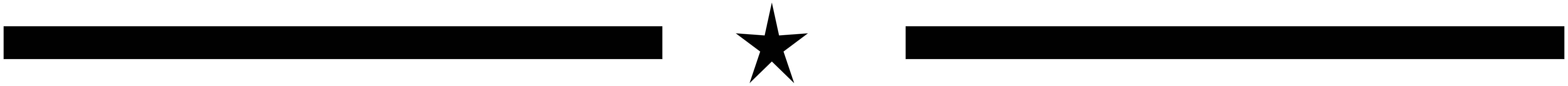


Climate

General perturbation can involve mix of pure mode and continuum



Frequency →



Continuum

discrete mode

Continuum

Explains why it is difficult to excite a pure mode in any realistic simulation (Weinberg 1991)

Suggests that real discs have features described by some mixture of

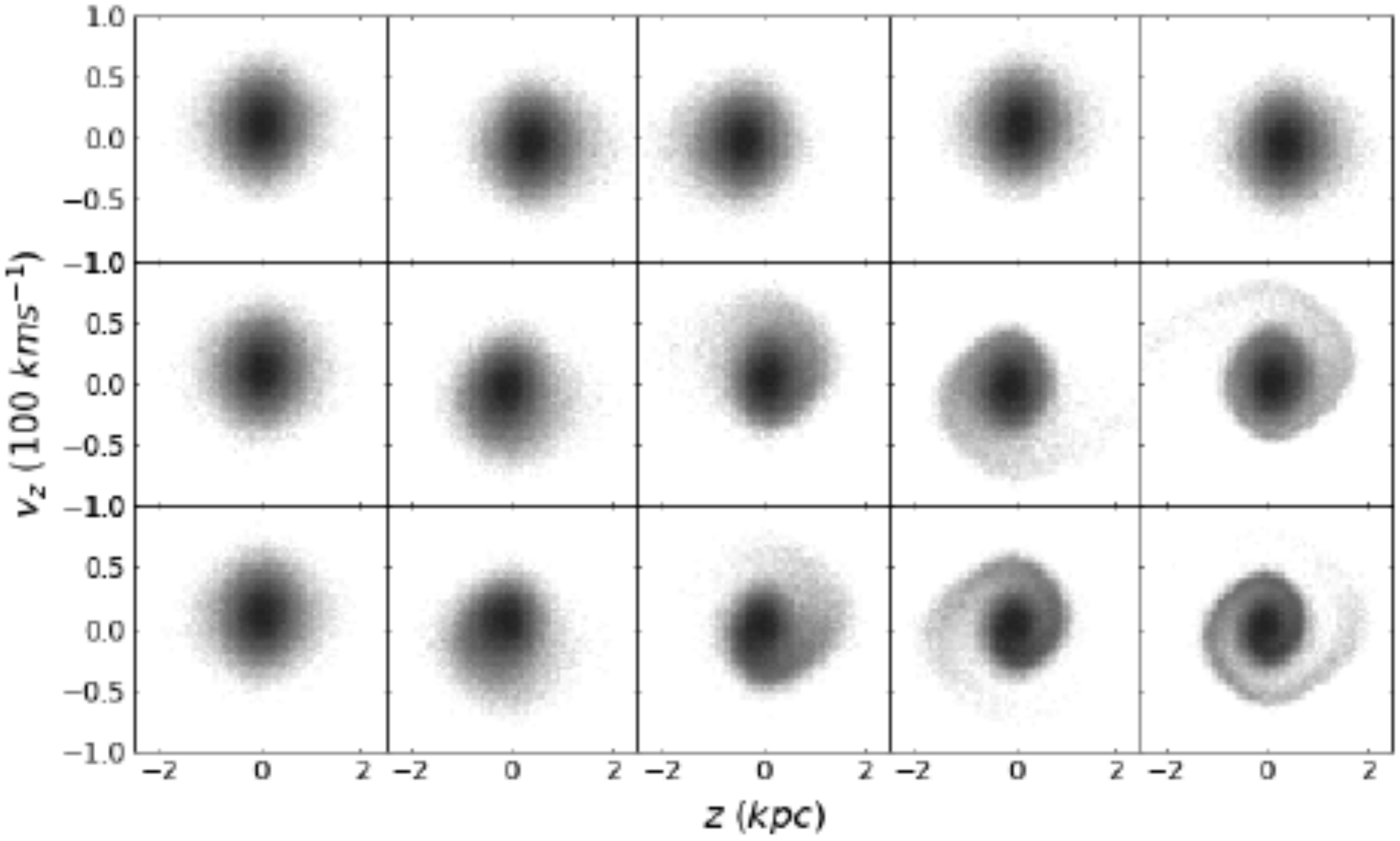
$$f(\mathbf{x},\ \mathbf{v}) \;=\; f_{\rm eq}(\mathbf{x},\ \mathbf{v}) \;+\; f_1(\mathbf{x},\ \mathbf{v},\ t)$$

and

$$f(\mathbf{x},\ \mathbf{v}) \;=\; f_{\rm eq}\left(\mathbf{x}_{\rm eq}(\mathbf{x},\ \mathbf{v}),\ \mathbf{v}_{\rm eq}(\mathbf{x},\ \mathbf{v})\right)\mathcal{J}(\mathbf{x},\ \mathbf{v};\mathbf{x},\ \mathbf{v})$$

$$\begin{aligned}\Sigma_1(R,\ \phi,\ t) \;&=\; \int d^3v\, dz\, f(\mathbf{x},\ \mathbf{v}) \\ &\sim\; \sum_m \sum_k e^{im\phi} e^{i\omega_k t}\Sigma_{m,k}(R)\end{aligned}$$

To explore the competition between phase mixing and self-gravity we consider a simple 1D (slab) model for stellar disk where we can tune the amount of self-gravity vs external potential



Self-gravitating

Test particles

Darling & LW, in prep

Dynamic Mode Decomposition (DMD)

For background on method, see books by Kutz et al
Application to stellar dynamics: Darling & LMW in prep

Method that uses simulation data to learn the “modes” of a complex system

In classical dynamics, we can solve for the modes of a Hamiltonian system by solving an eigenvalue problem.

$$\frac{d\mathbf{x}}{dt} = \mathcal{A}\mathbf{x} \qquad \longrightarrow \qquad \mathbf{x}_{j+1} = \mathbf{A}\mathbf{x}_j$$

Discretized dynamics in simulation

$$\mathbf{X} = \begin{pmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \ldots & \mathbf{x}_{m-1} \\ | & | & & | \end{pmatrix}$$

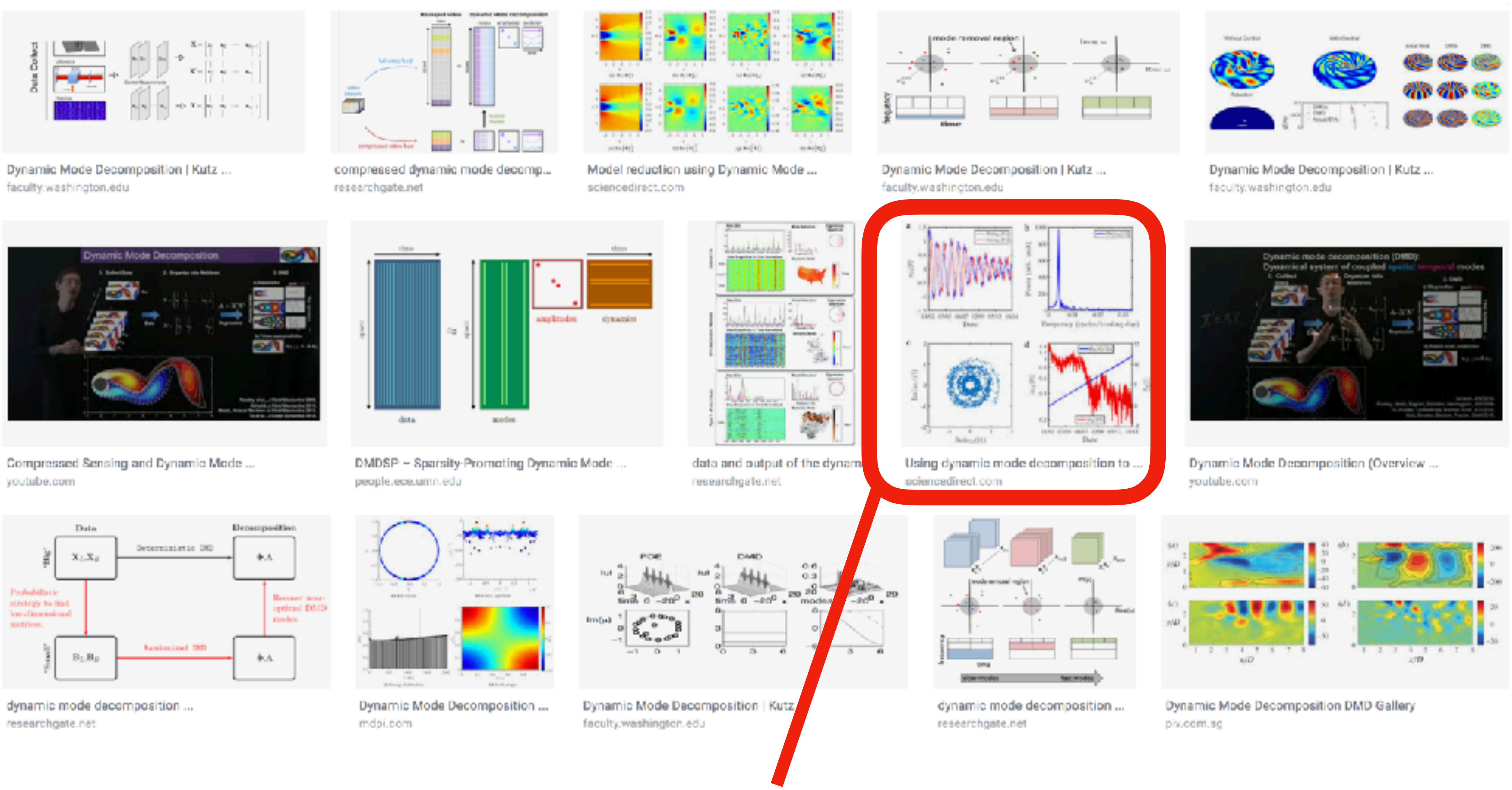
$$\mathbf{X}' = \begin{pmatrix} | & | & & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \ldots & \mathbf{x}_{m} \\ | & | & & | \end{pmatrix}$$

Columns are snapshot data

$$\mathbf{A} = \mathbf{X}'\mathbf{X}^{+}$$

Find dominant eigenvalues of A via linear algebra

DMD has its roots in the field of computational fluid dynamics



Dynamic Mode Decomposition | Kutz ...
faculty.washington.edu

compressed dynamic mode decomp...
researchgate.net

Model reduction using Dynamic Mode ...
sciencedirect.com

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Compressed Sensing and Dynamic Mode ...
youtube.com

DMDSP – Sparsity-Promoting Dynamic Mode ...
people.ece.umn.edu

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Using dynamic mode decomposition to ...
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mdpi.com

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Dynamic Mode Decomposition DMD Gallery
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Using dynamic mode decomposition to extract cyclic behavior in the stock market

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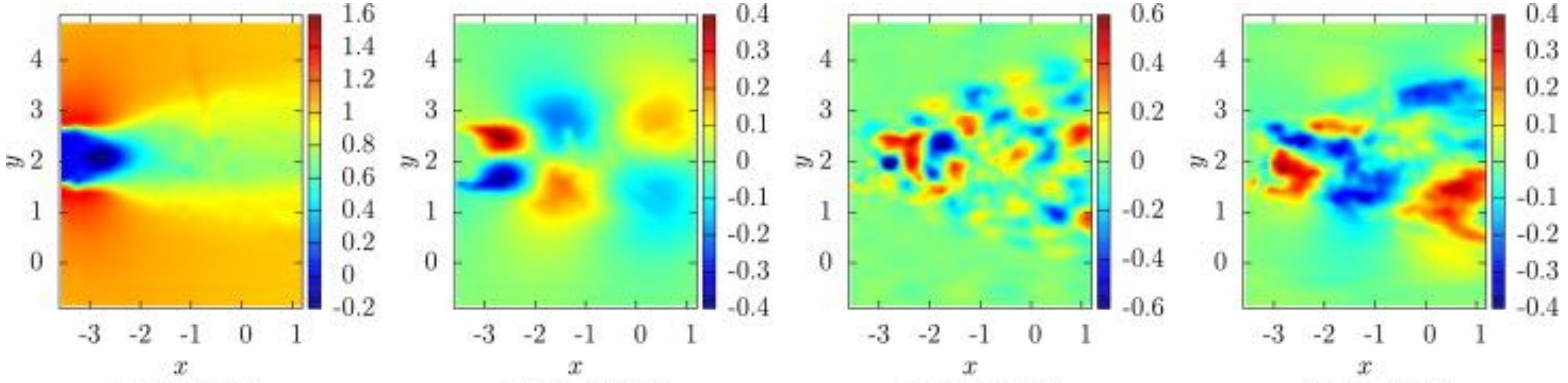
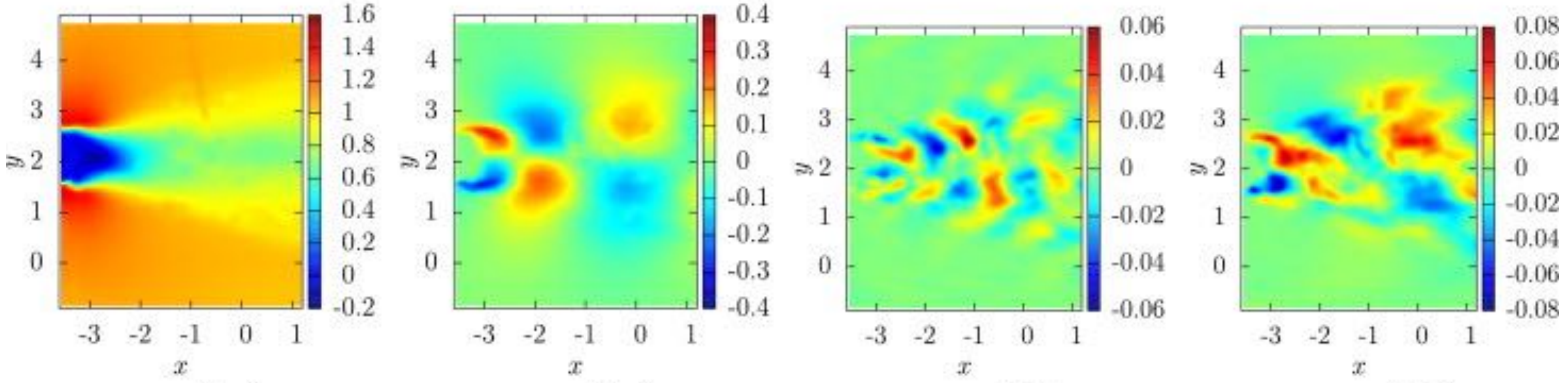
[b] Spectral Energies, LLC, Dayton, OH 45431, United States
[c] School of Electrical and Information Engineering, University of Sydney, NSW 2006, Australia

Flow around a cylindrical obstacle at high Reynolds number

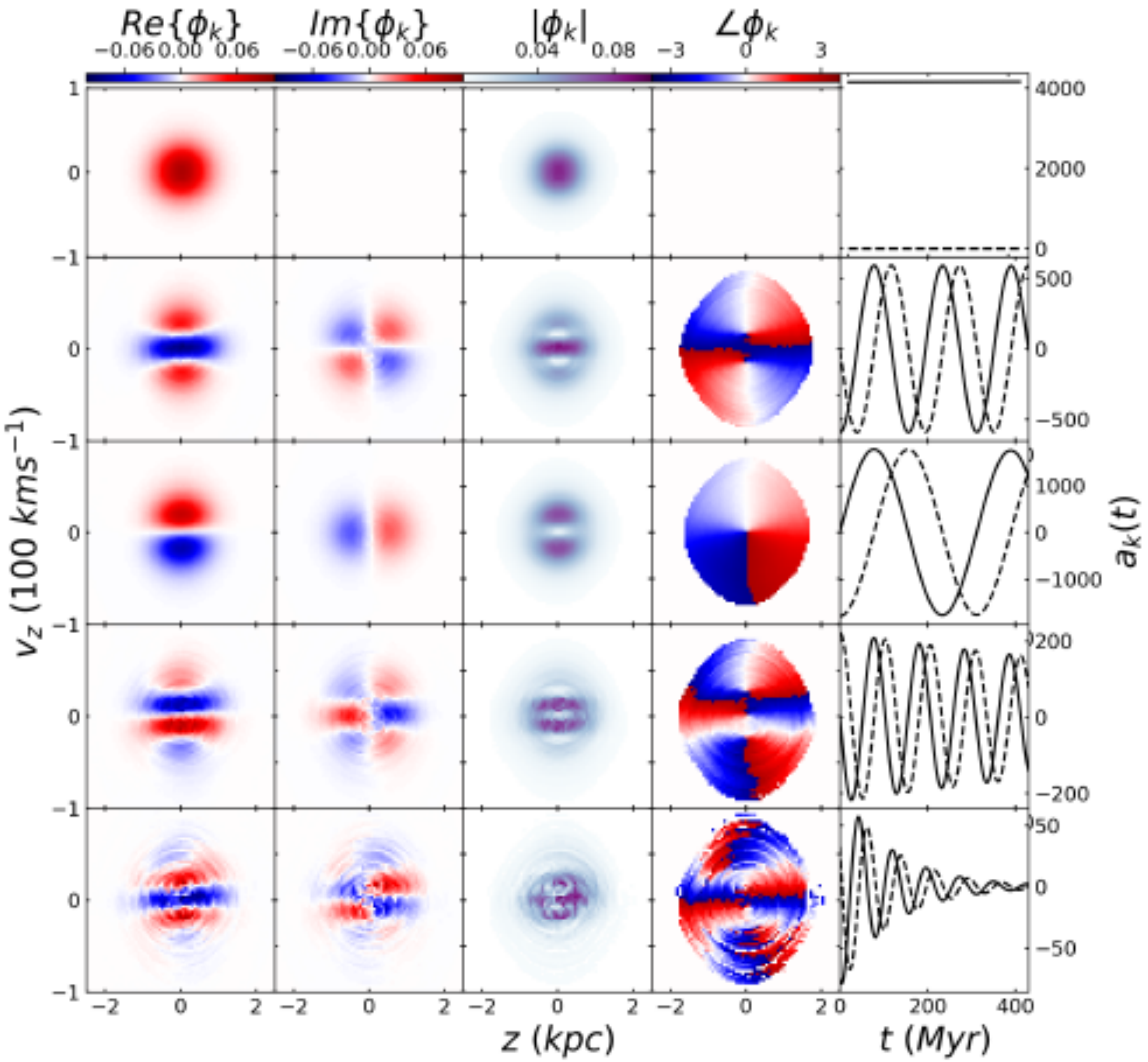


(a) Re $\left(\Phi_1^u\right)$ (b) Re $\left(\Phi_{27}^u\right)$ (c) Re $\left(\Phi_{79}^u\right)$ (d) Re $\left(\Phi_{29}^u\right)$

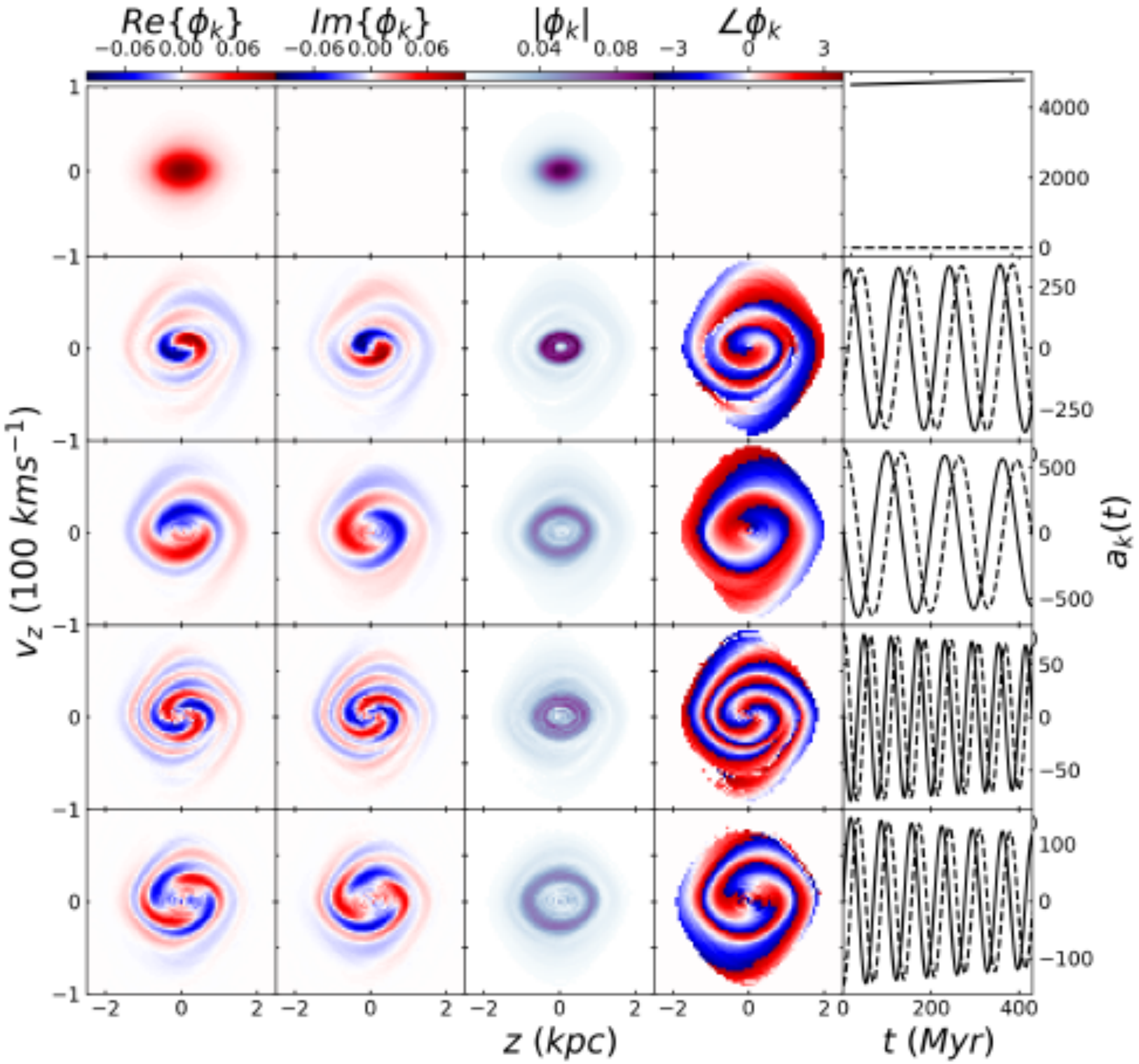


(e) Re $\left(\hat{\Phi}_1^u\right)$ (f) Re $\left(\hat{\Phi}_2^u\right)$ (g) Re $\left(\hat{\Phi}_4^u\right)$ (h) Re $\left(\hat{\Phi}_6^u\right)$

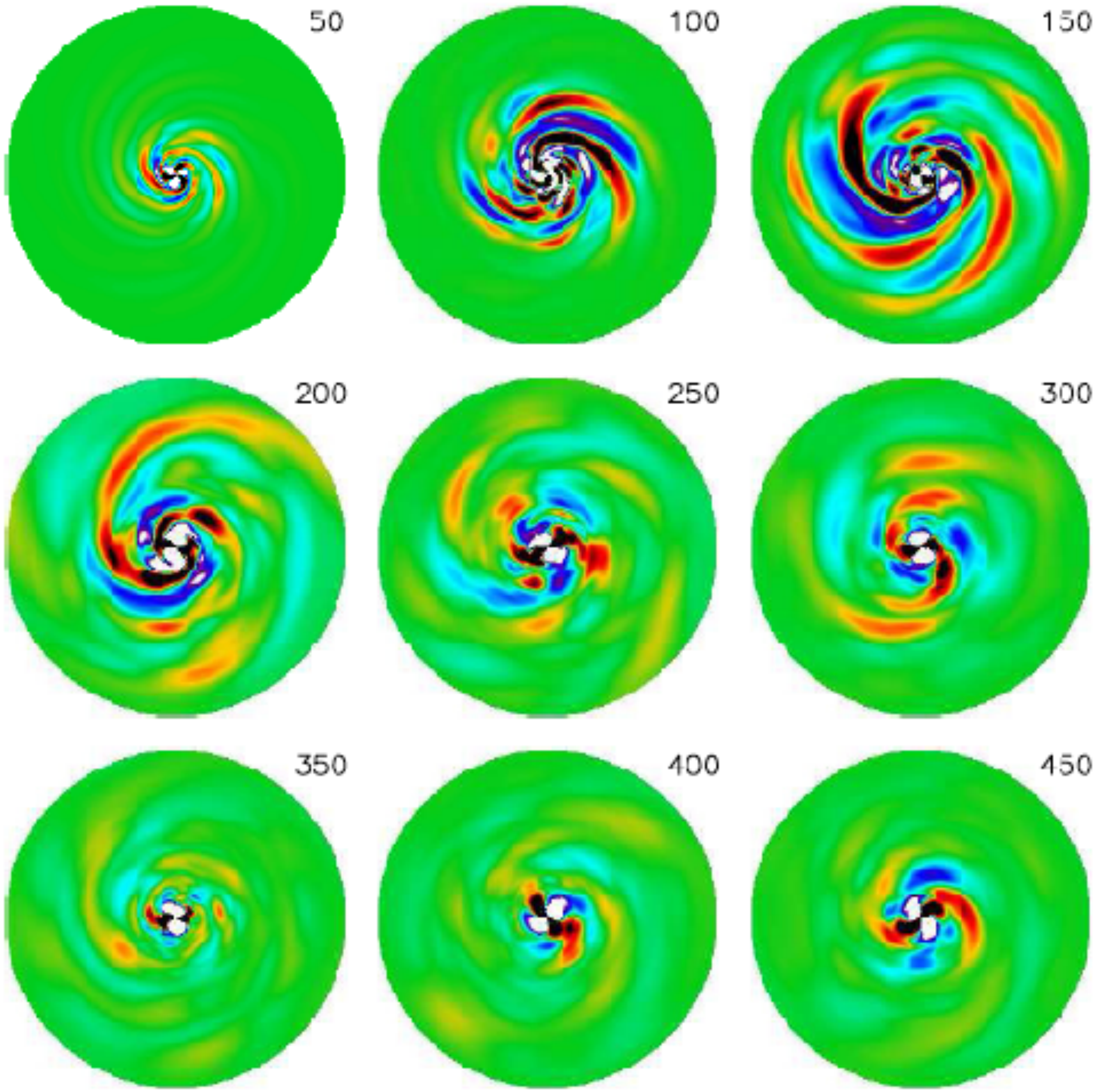
Dominant modes for a nearly self-gravitating disk



Dominant modes for nearly test-particle case



Seems well-suited to problems like formation of spiral structure

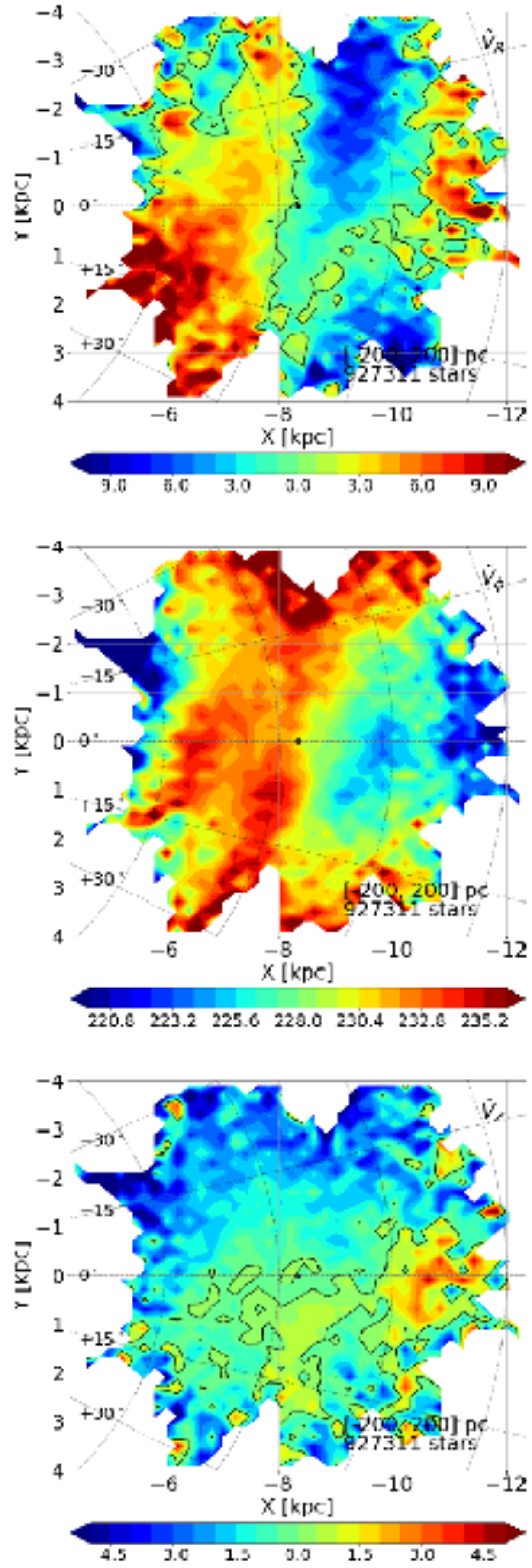


Sellwood & Carlberg

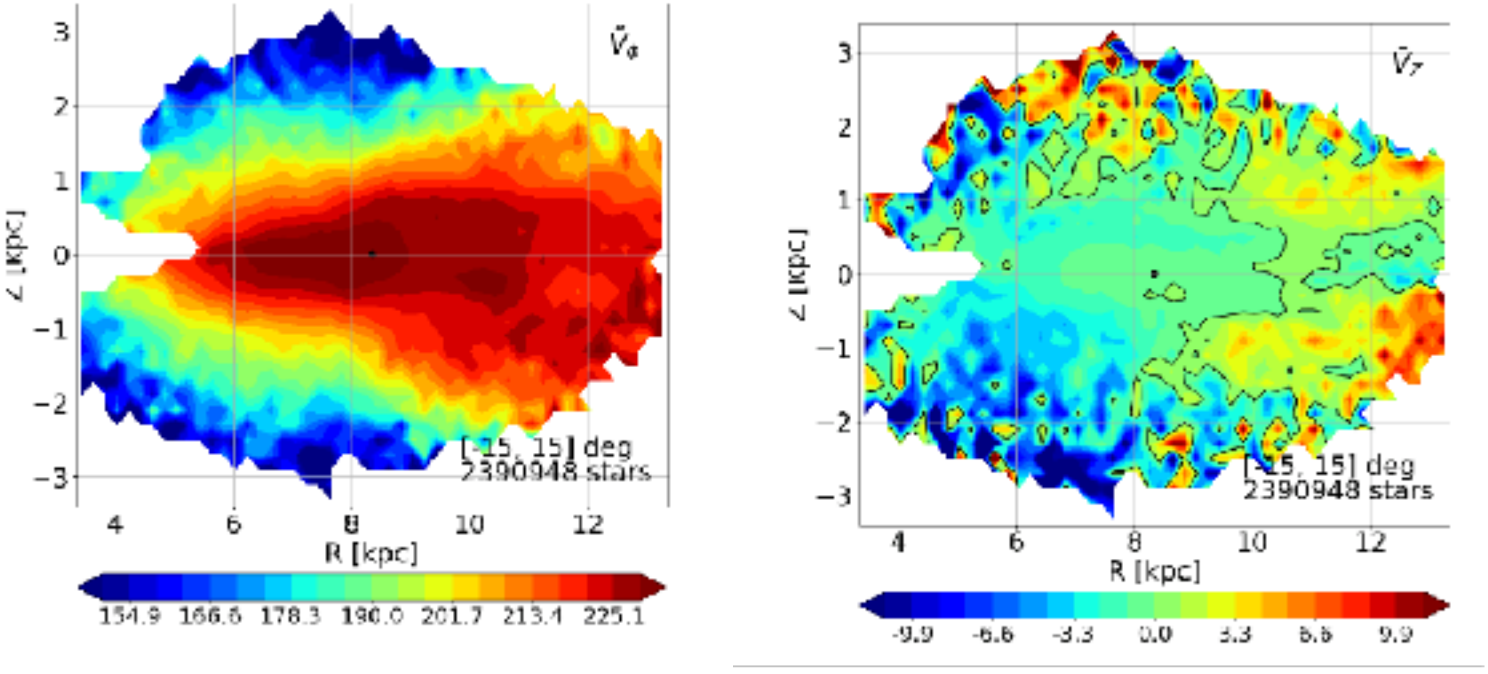
See also Martin Weinberg’s talk at the beginning of the workshop on Multi Singular Spectral Analysis

Stellar Velocity Field

Standard method involves slicing and binning in phase space. E.g. Take stars within 200 pc of mid plane, bin in X and Y and computer mean V_R , V_{Phi} ,V_z

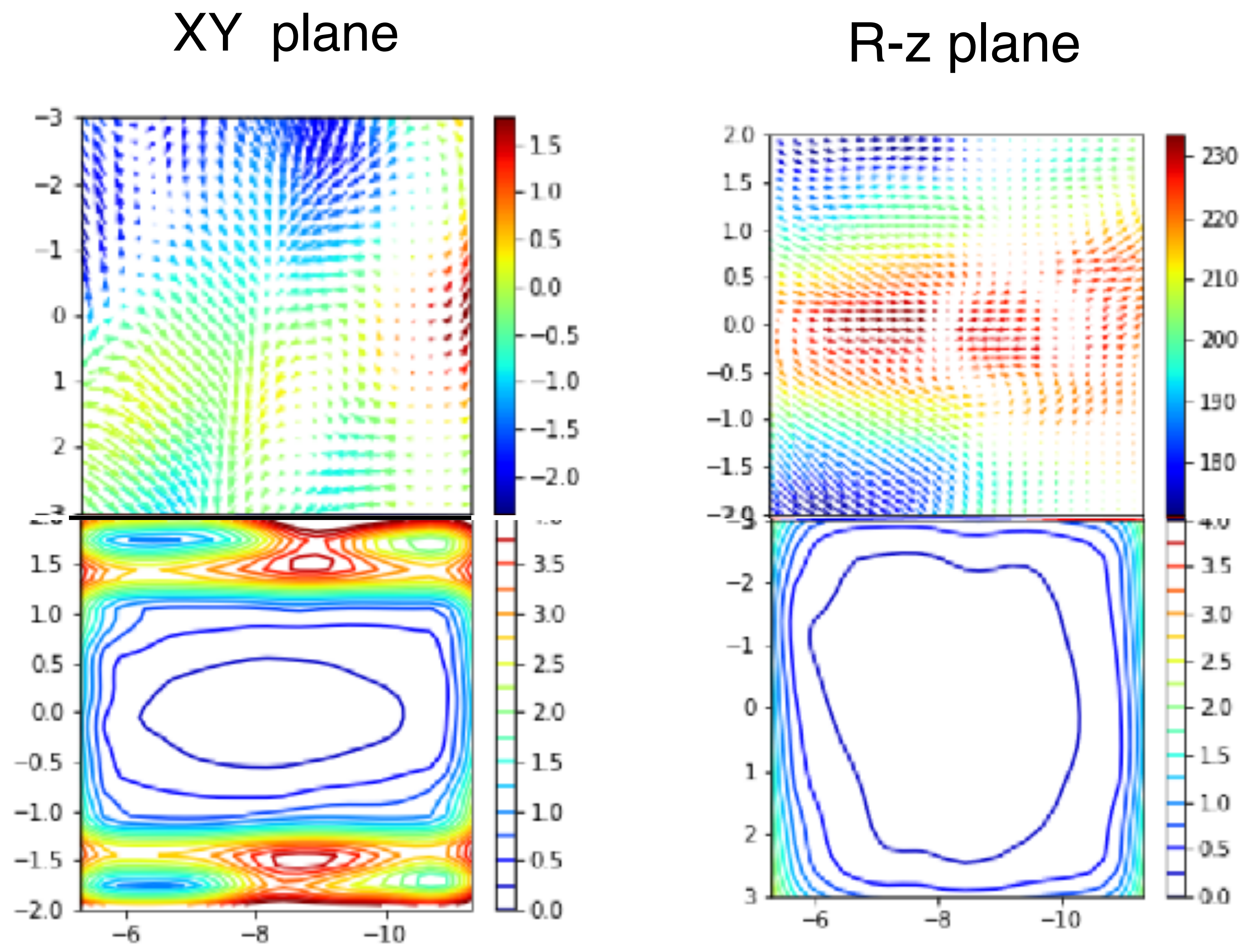


Or slice and bin in R-z plane



>50 separate maps in GDR2-Kinematics paper

Gaussian Process Regression provides a path to an alternative approach where we build a smooth and differentiable model for the 3D velocity field using all the data. We also get uncertainties.

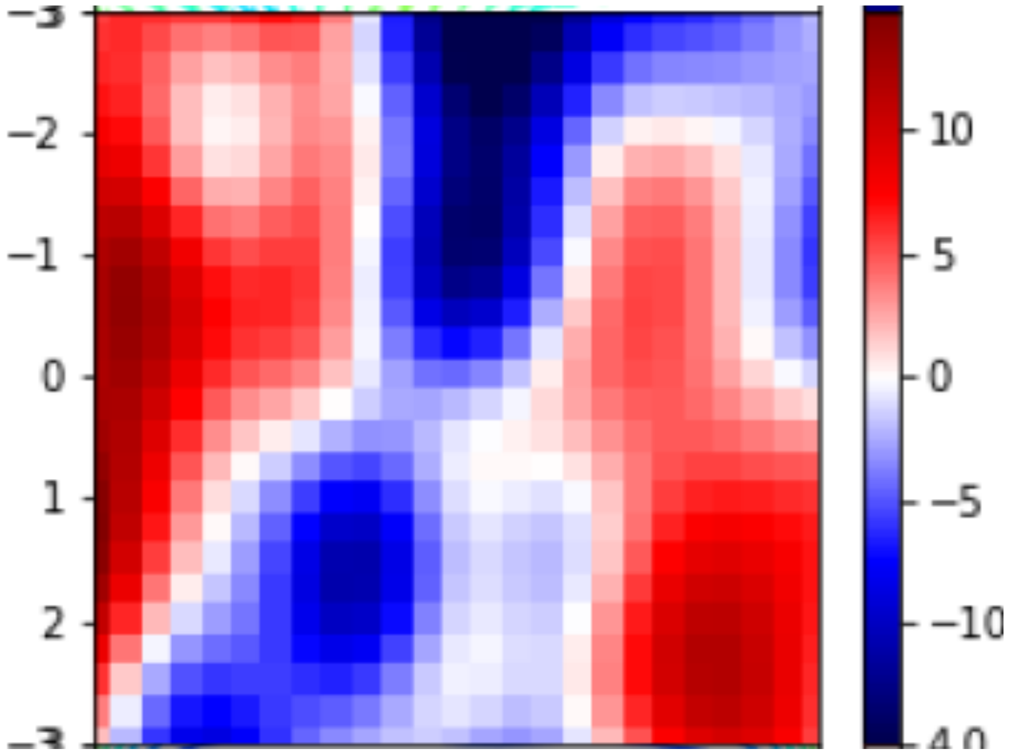


Velocity field coloured by component normal to plane

Uncertainty

One can also infer derivatives of the velocity field

Divergence in the XY plane



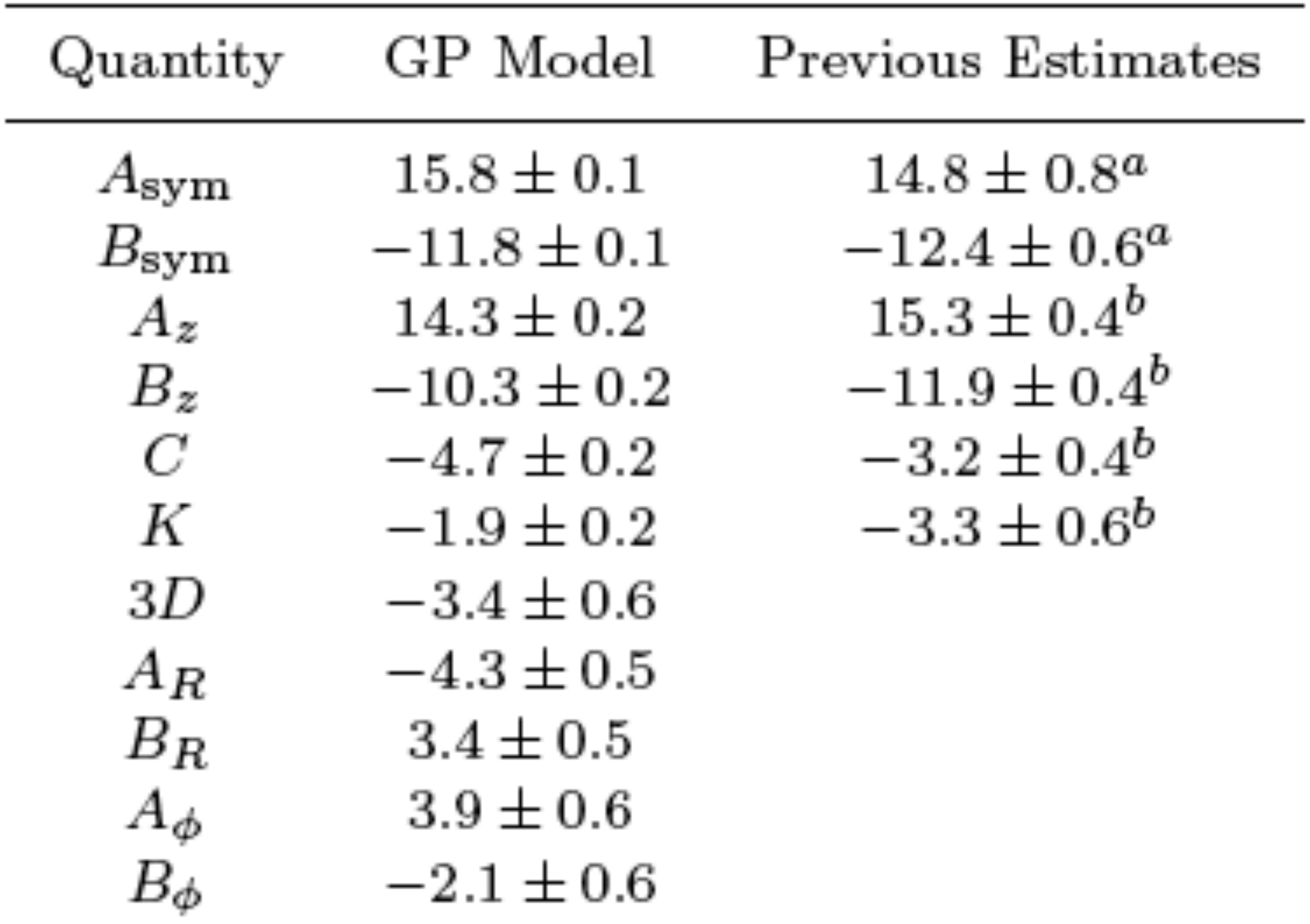
Continuity equation

$$\nabla \cdot \mathbf{v} \;=\; -\frac{1}{\rho}\frac{d\rho}{dt}$$

div(V) therefore tells us something about compression and expansion in the disc — another handle on disc disequilibrium

In principle, this method could be extended to higher moments of the velocity field (Jeans equation) or to the phase space DF itself (requires selection function) and the CBE

We can also estimate the generalized Oort constants



	Quantity	GP Model	Previous Estimates
	A_{sym}	15.8 ± 0.1	14.8 ± 0.8^a
	B_{sym}	-11.8 ± 0.1	-12.4 ± 0.6^a
	A_z	14.3 ± 0.2	15.3 ± 0.4^b
	B_z	-10.3 ± 0.2	-11.9 ± 0.4^b
	C	-4.7 ± 0.2	-3.2 ± 0.4^b
	K	-1.9 ± 0.2	-3.3 ± 0.6^b
divergence	$3D$	-3.4 ± 0.6	
shear in phi-z plane	A_R	-4.3 ± 0.5	
curl in phi-z plane	B_R	3.4 ± 0.5	
shear in R-z plane	A_ϕ	3.9 ± 0.6	
curl in R-z plane	B_ϕ	-2.1 ± 0.6	

Summary and Open Problems

A wide range of observations in the vicinity of the Sun and beyond point to a stellar disc in a state of disequilibrium.

The hope is that by studying these phenomena, we can learn something about the structure of the disc and its environment.

While we have attractive phenomenological models that reproduce these observations, we still lack a coherent picture of the perturbed DF and the physics of how it evolves.

The hope is that a combination of more observations, theory, simulations, and perhaps data science will guide us to a more complete understanding of disc dynamics.