

NUMERICAL MODELS OF BLACK HOLES ACCRETION FLOWS

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with

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Paradoxes of Massive Black Holes

KITP 16 April 2005

OUTLINE

Black Hole Accretion Models

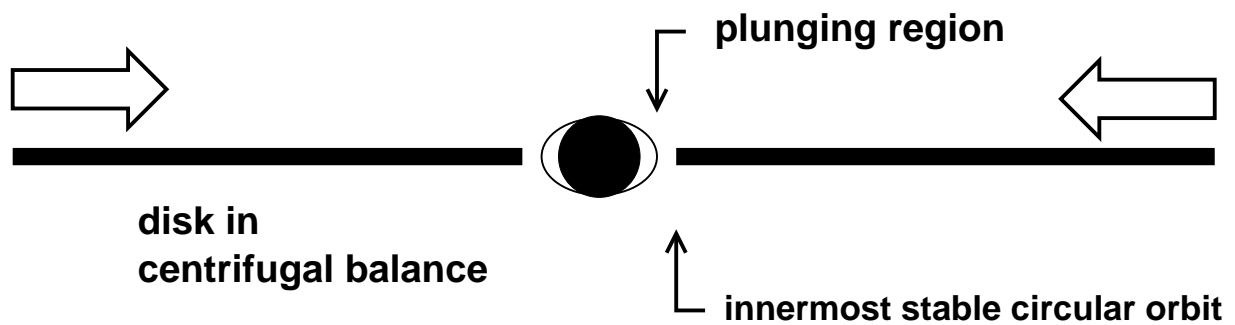
General Relativistic MHD

Black Hole Spinup/Spindown

Blandford-Znajek Effect

Summary

Thin Disk Model



Accretion luminosity:

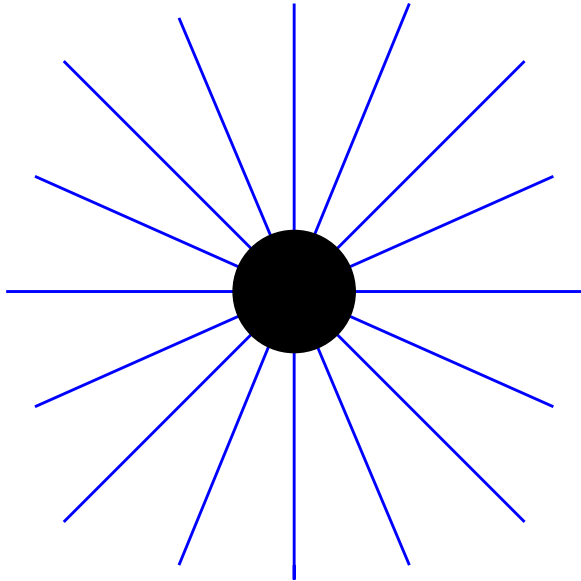
$$L = (1 - \mathcal{E})\dot{M}_0 c^2$$

Bardeen (1970): $a/M \rightarrow 1$

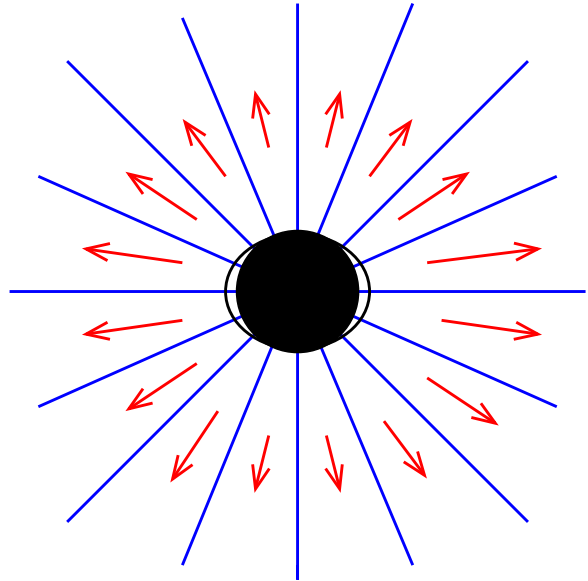
Thorne (1974): $a/M \rightarrow 0.998$

Many more sophisticated variants!

Blandford-Znajek Model



$$a/M = 0$$



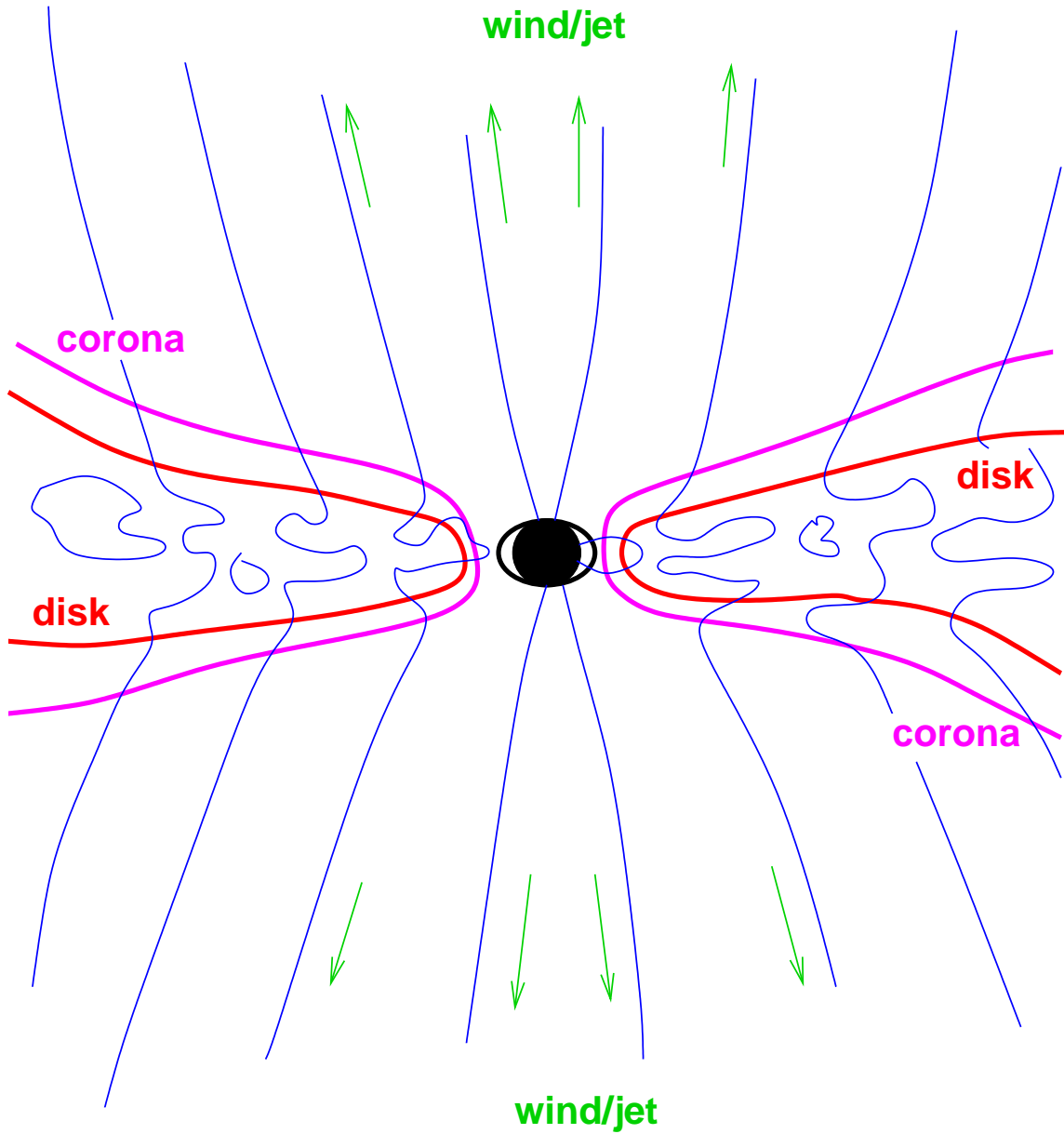
$$0 < a/M \ll 1$$

force-free, $b^2 \gg \rho_0 c^2 + u + p$

Spindown luminosity:

$$L = \frac{1}{12} \left(\frac{a}{M}\right)^2 (B^r)^2 c \left(\frac{GM}{c^2}\right)^2 + O(a^4)$$

The Real Central Engine?



**Magnetohydrodynamic
Models in
Full General Relativity**

General Relativistic MHD Equations

Particle number conservation:

$$\partial_t(\sqrt{-g} \rho_o u^t) = -\partial_i(\sqrt{-g} \rho_o u^i) \quad \partial_t \rho = -\nabla \cdot (\rho \mathbf{v})$$

Ideal MHD:

$$u_\mu F^{\mu\nu} = 0 \quad \mathbf{E} + \mathbf{v} \times \mathbf{B}/c = 0$$

Momentum and energy conservation:

$$\partial_t(\sqrt{-g} T^t{}_\nu) = -\partial_i(\sqrt{-g} T^i{}_\nu) + \sqrt{-g} T^\kappa{}_\lambda \Gamma^\lambda{}_{\nu\kappa}$$

$$\partial_t(\rho \mathbf{v}) = -\nabla \cdot \mathbf{T} - \rho \nabla \phi$$

$$T_{\mu\nu} = (\rho_o + u + p + \frac{b^2}{4\pi}) u_\mu u_\nu + (p + \frac{b^2}{8\pi}) g_{\mu\nu} - \frac{b_\mu b_\nu}{4\pi}$$

$$T_{ij} = \rho v_i v_j + (p + \frac{B^2}{8\pi}) \delta_{ij} - \frac{B_i B_j}{4\pi}$$

Induction equation:

$$\begin{aligned} \partial_t(\sqrt{-g} B^i) &= -\partial_j(\sqrt{-g}(u^j b^i - b^j u^i)) \quad \partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) \\ &= -\nabla(\mathbf{v} \mathbf{B} - \mathbf{B} \mathbf{v}) \end{aligned}$$

No monopoles constraint:

$$\partial_i(\sqrt{-g} B^i) = 0 \quad \nabla \cdot \mathbf{B} = 0$$

GRMHD Code

Basic Model

- ideal MHD
- stationary (Kerr) metric
- no cooling
- no radiative forces

HARM Algorithm

- Gammie, McKinney, Tóth (2003)
- conservative, LLF or HLL
- constrained transport: $\nabla \cdot \mathbf{B} = 0$
- axisymmetric (so far)
- covariant

Other GRMHD codes

- Koide, Meier et al. (Toyama)
- De Villiers, Hawley (Virginia)
- Komissarov (Leeds)
- Fragile, Anninos (UCSB)
- Duez, Shapiro (UIUC)

Code Verification

Nonrelativistic Tests

Ryu & Jones shock tubes

Orszag-Tang vortex

Special Relativistic Tests

Linear modes, up to $b^2/\rho = 10^6$

Komissarov's shocks

Transport

General Relativistic Tests

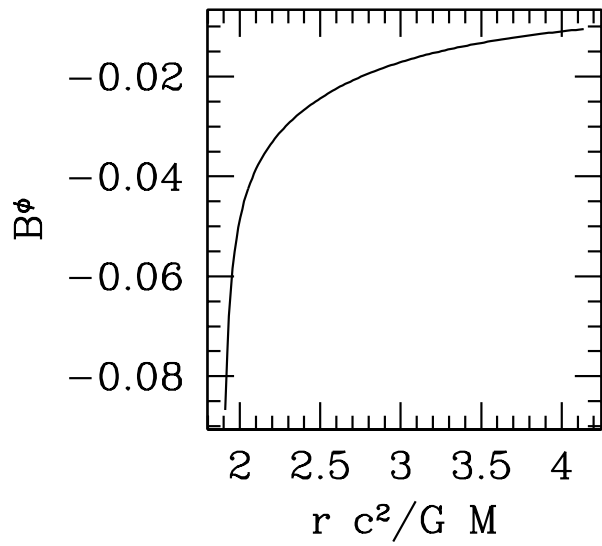
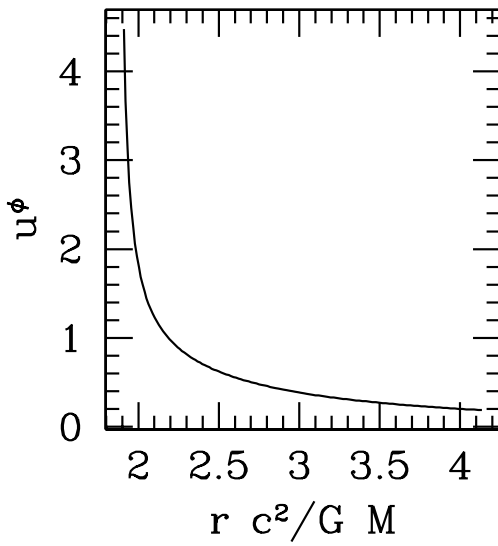
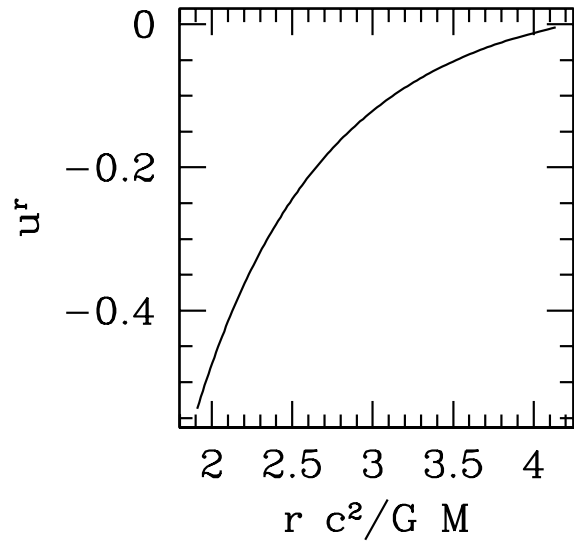
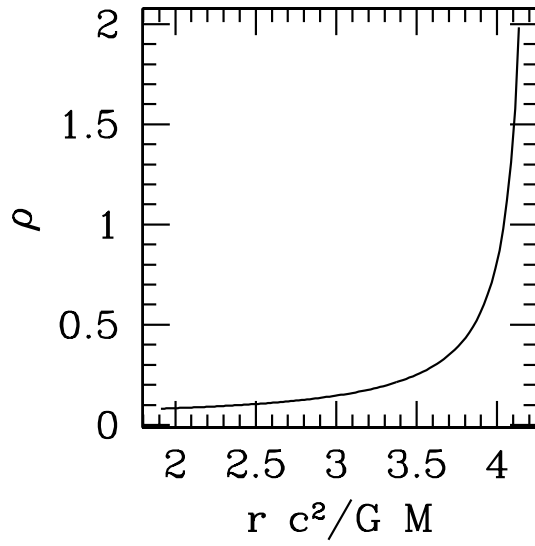
Bondi flow ($a/M = 0$)

Magnetized Bondi flow ($a/M = 0$)

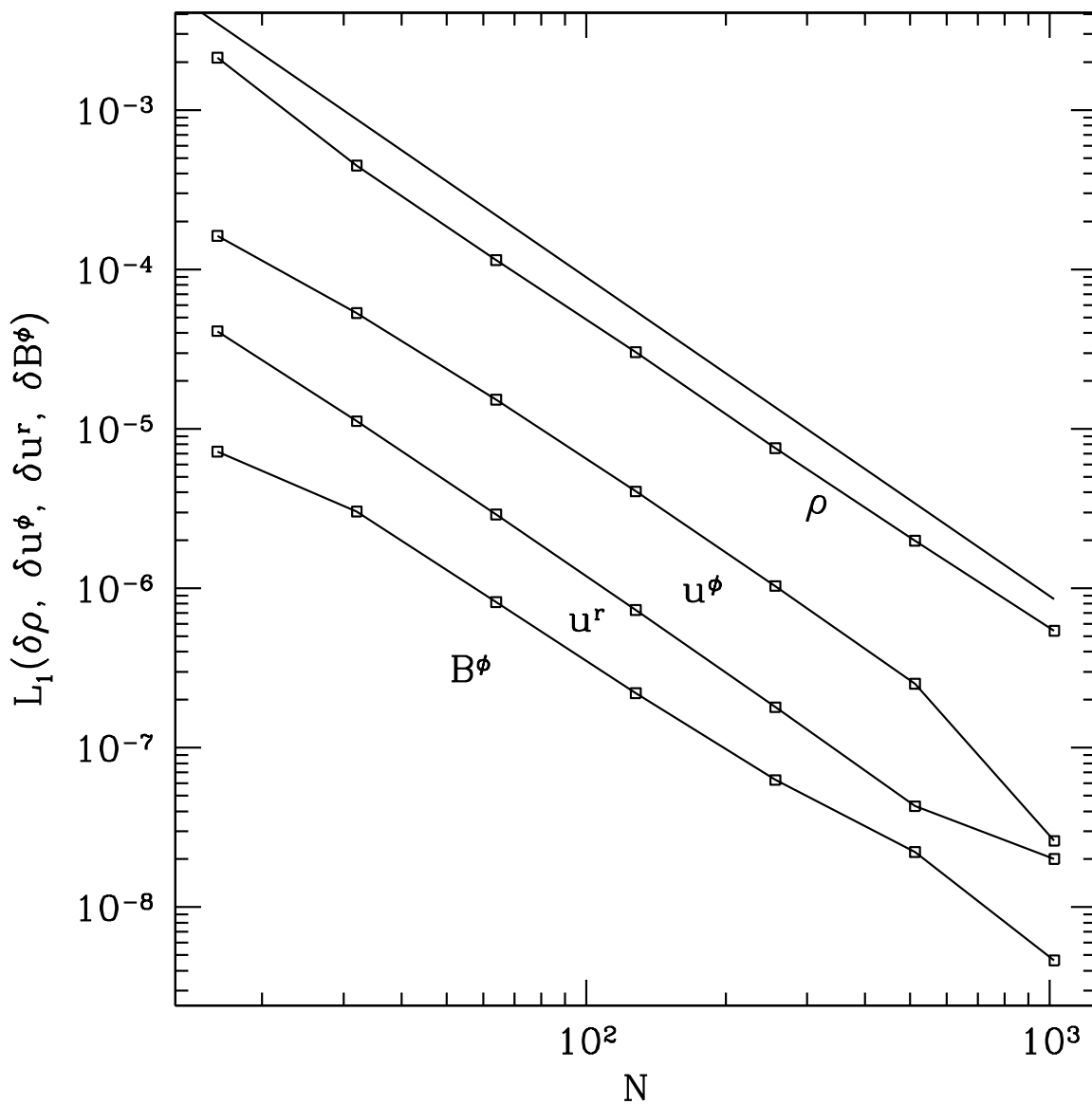
up to $b^2/\rho = 10^3$

Magnetized equatorial inflow ($a/M = 0.5$)

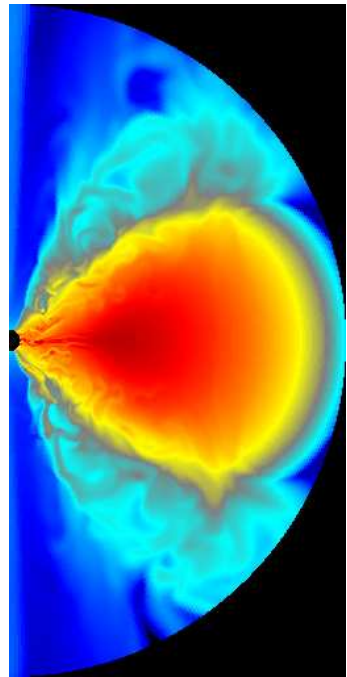
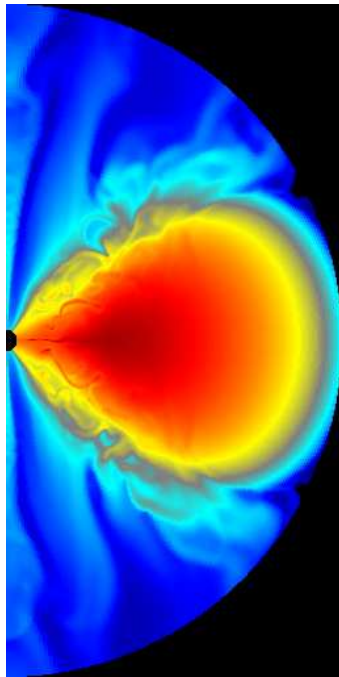
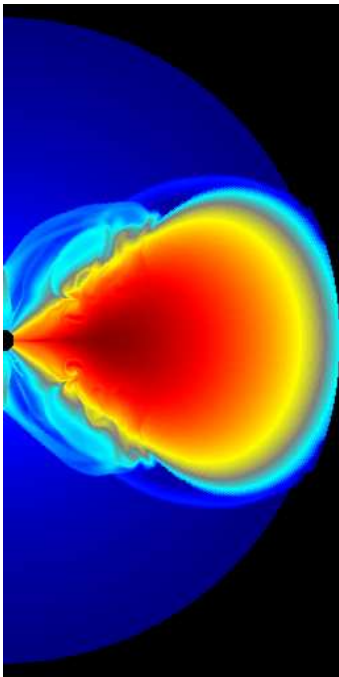
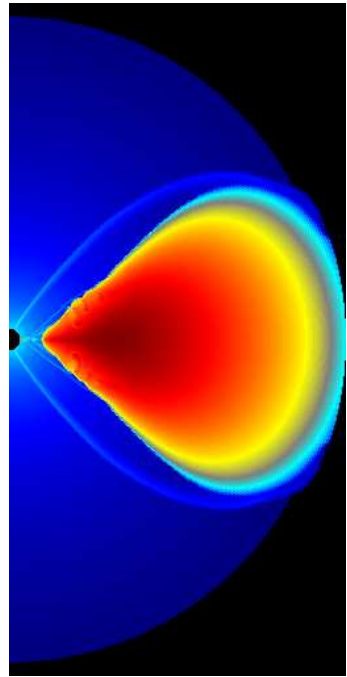
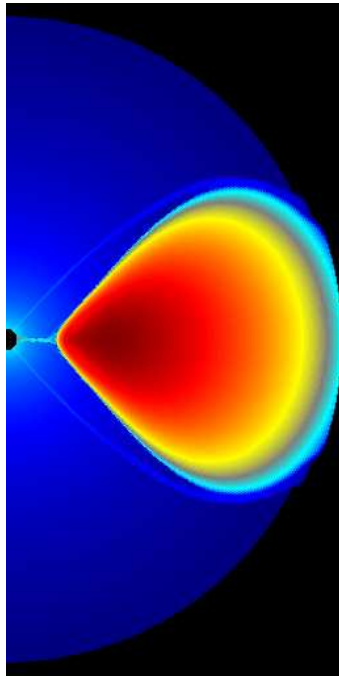
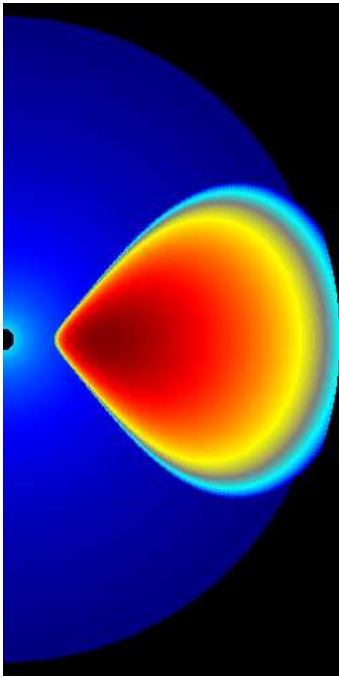
Fishbone-Moncrief torus ($a/M = 0.9$)



Magnetized equatorial inflow solution for $a/M = 0.5$



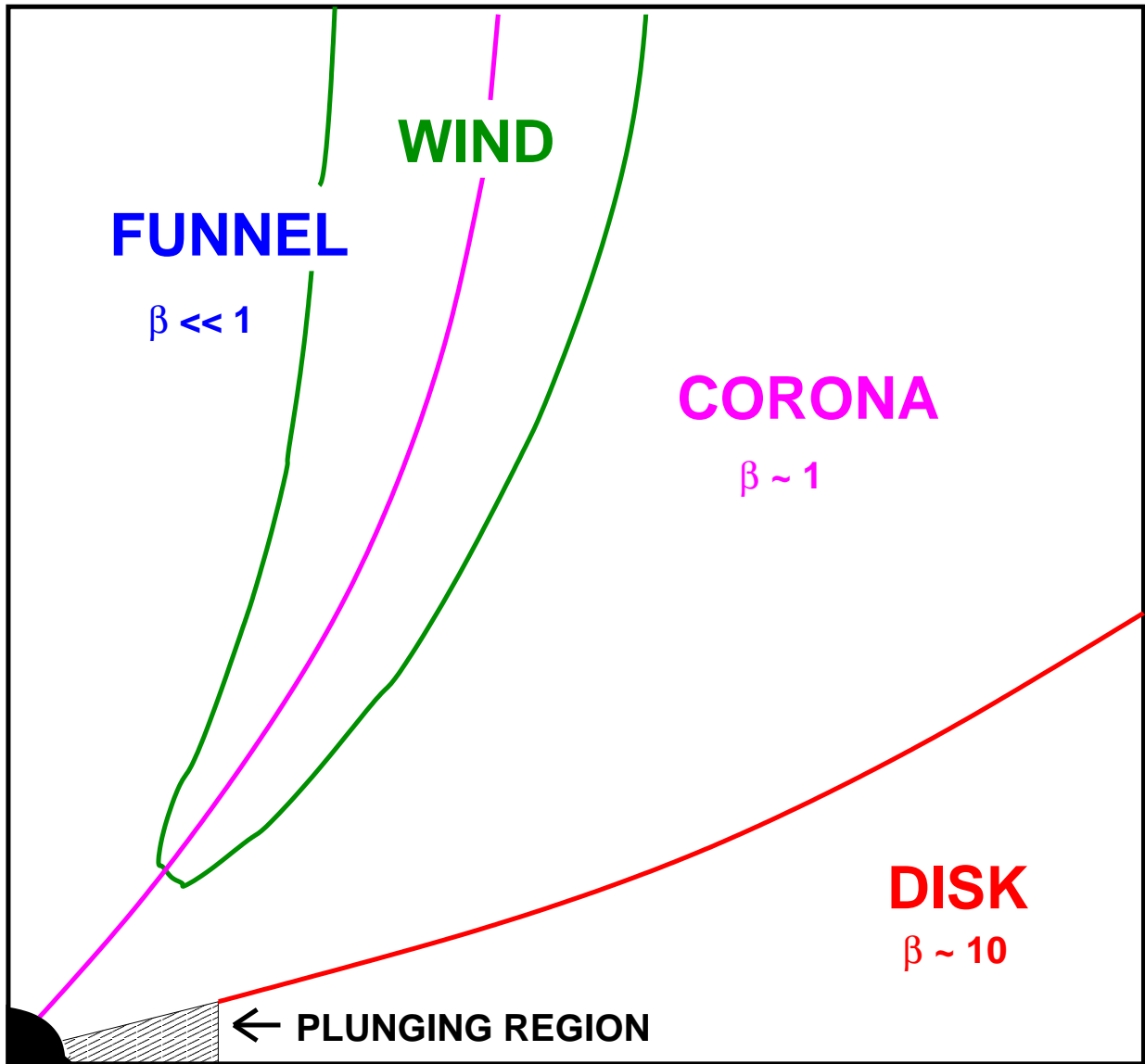
Convergence results for magnetized equatorial inflow in the Kerr metric with $a/M = 0.5$



color shows $\log(\text{density})$

movie (thick, thin)

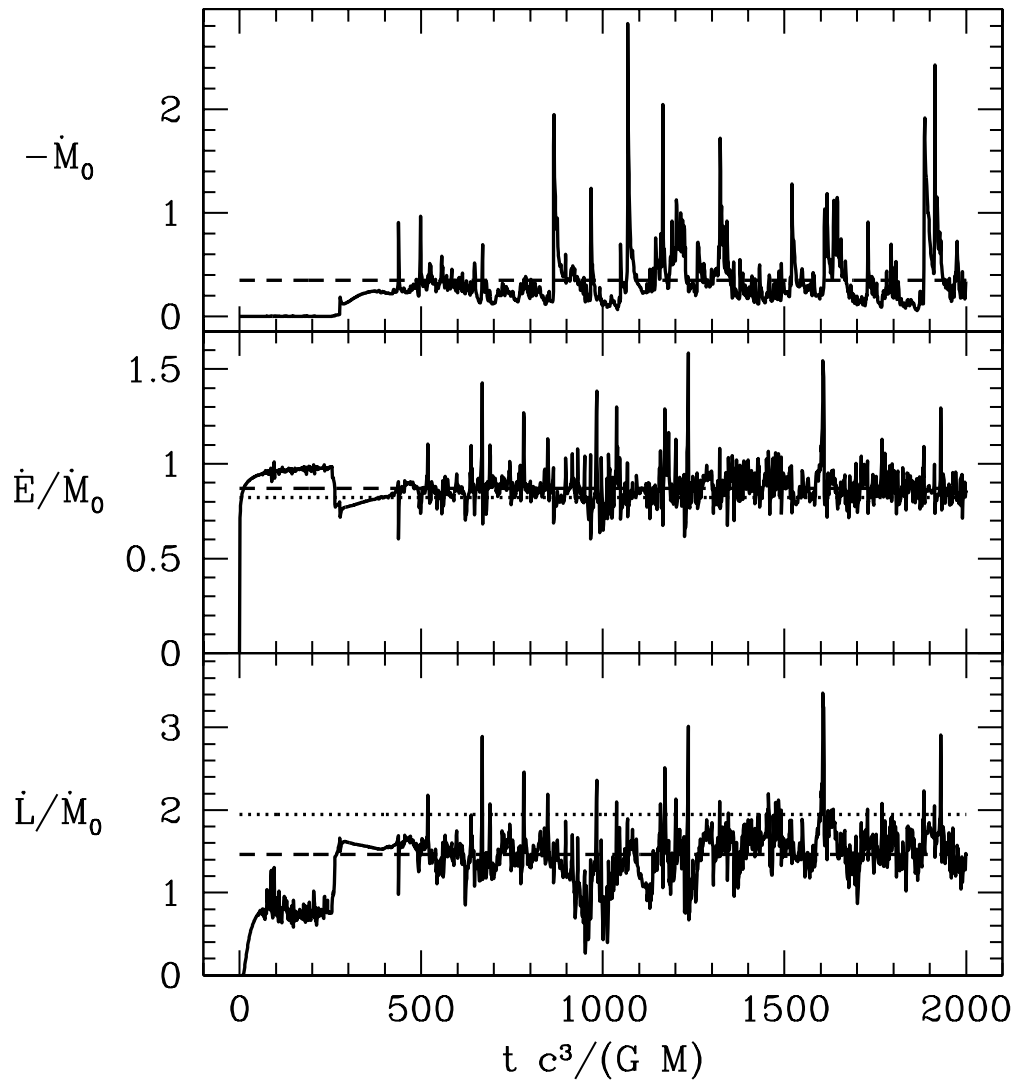
Accretion Flow Structure



McKinney & Gammie 2004

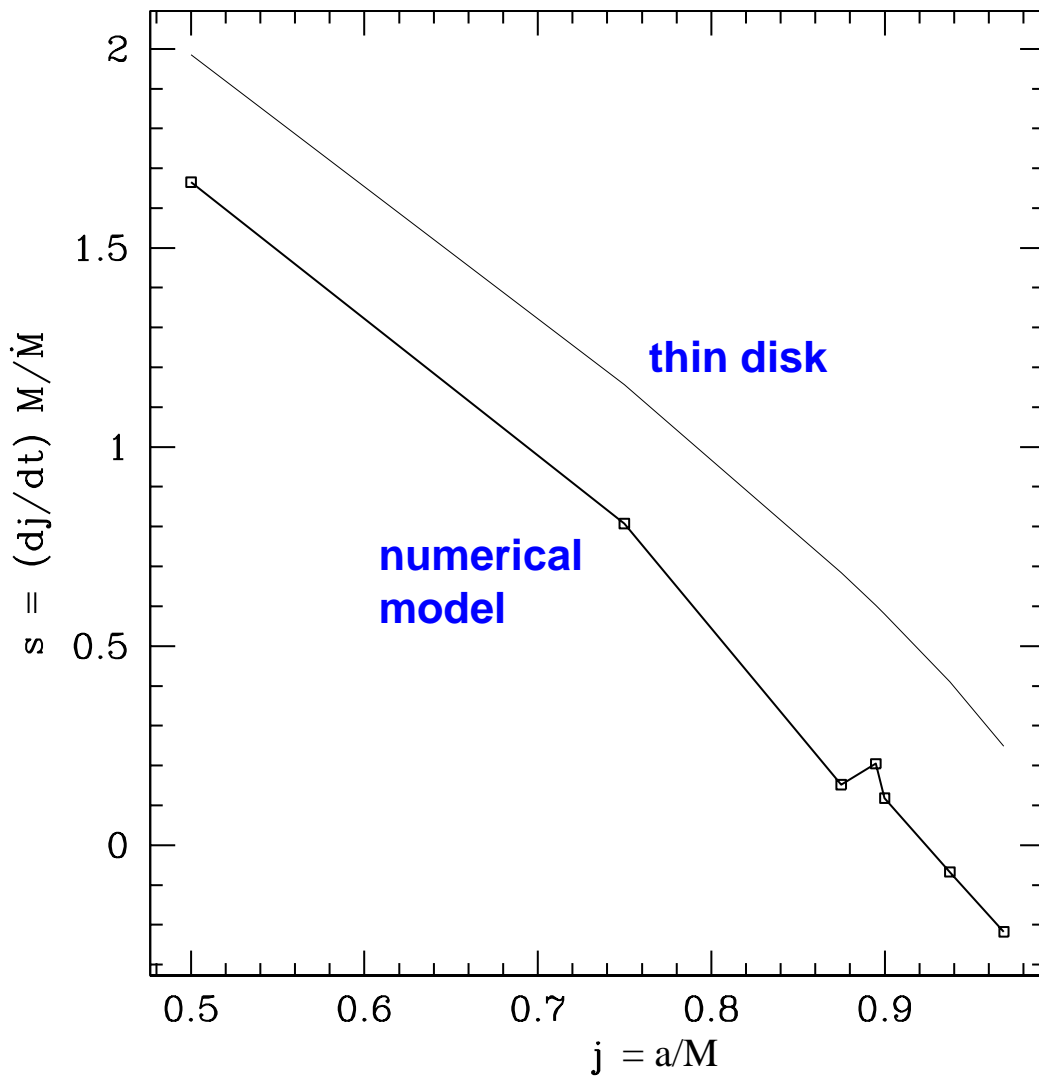
Black Hole Spinup/Spindown

Torque on Inner Edge of Disk



McKinney & Gammie 2004

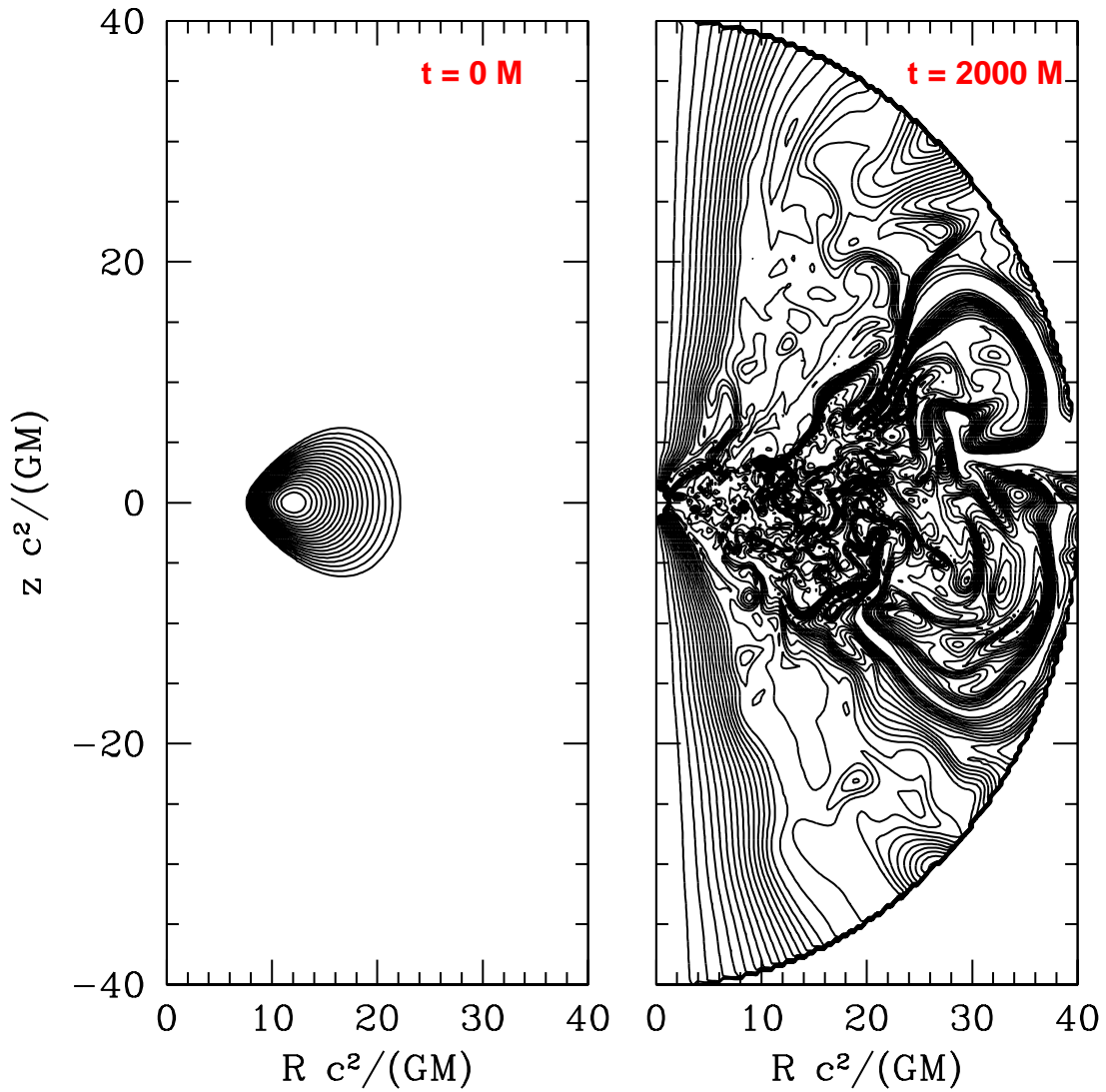
Black Hole Magnetic Spindown



Gammie, Shapiro, & McKinney 2004

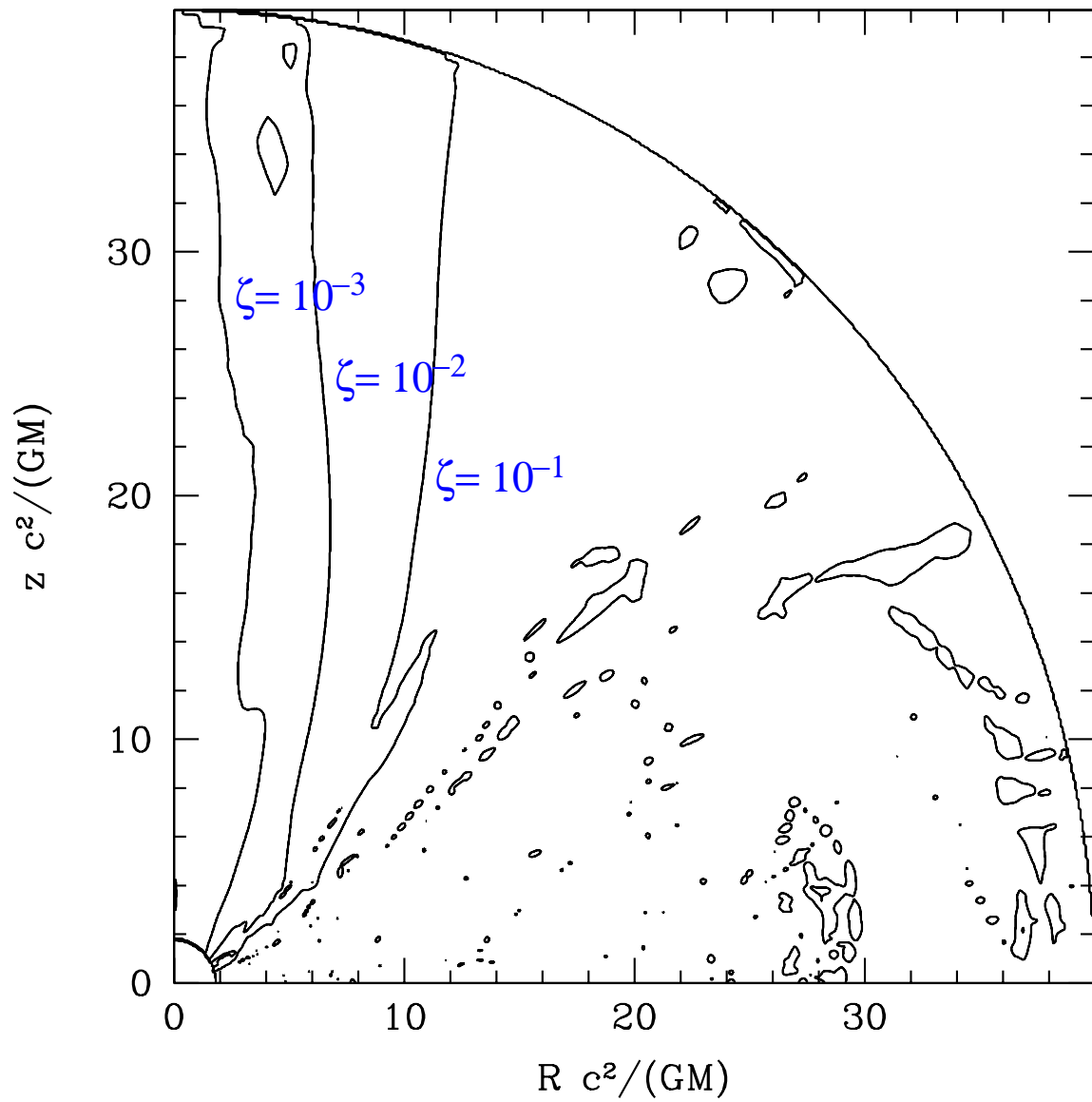
Blandford-Znajek Effect

Magnetic Field Lines



McKinney & Gammie 2004

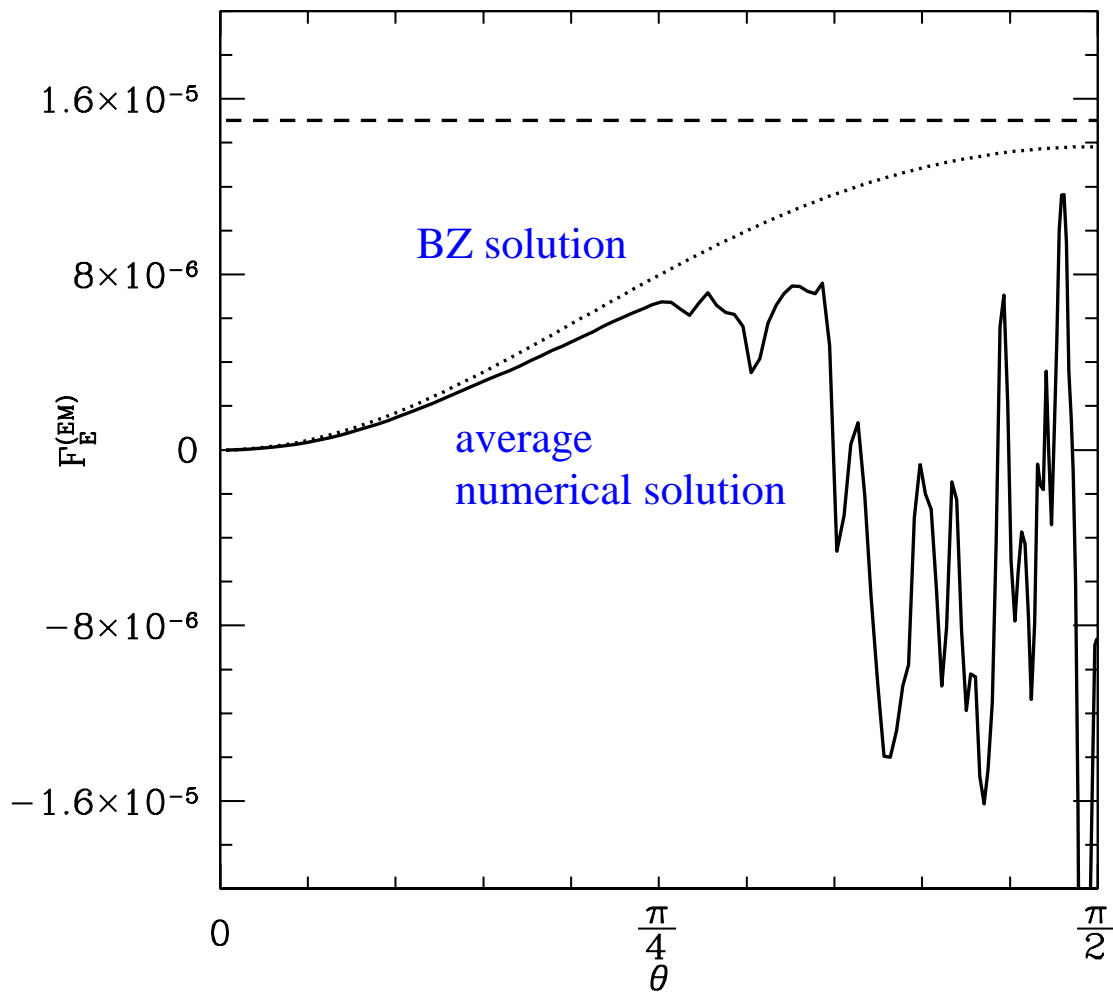
A Force-Free Funnel?



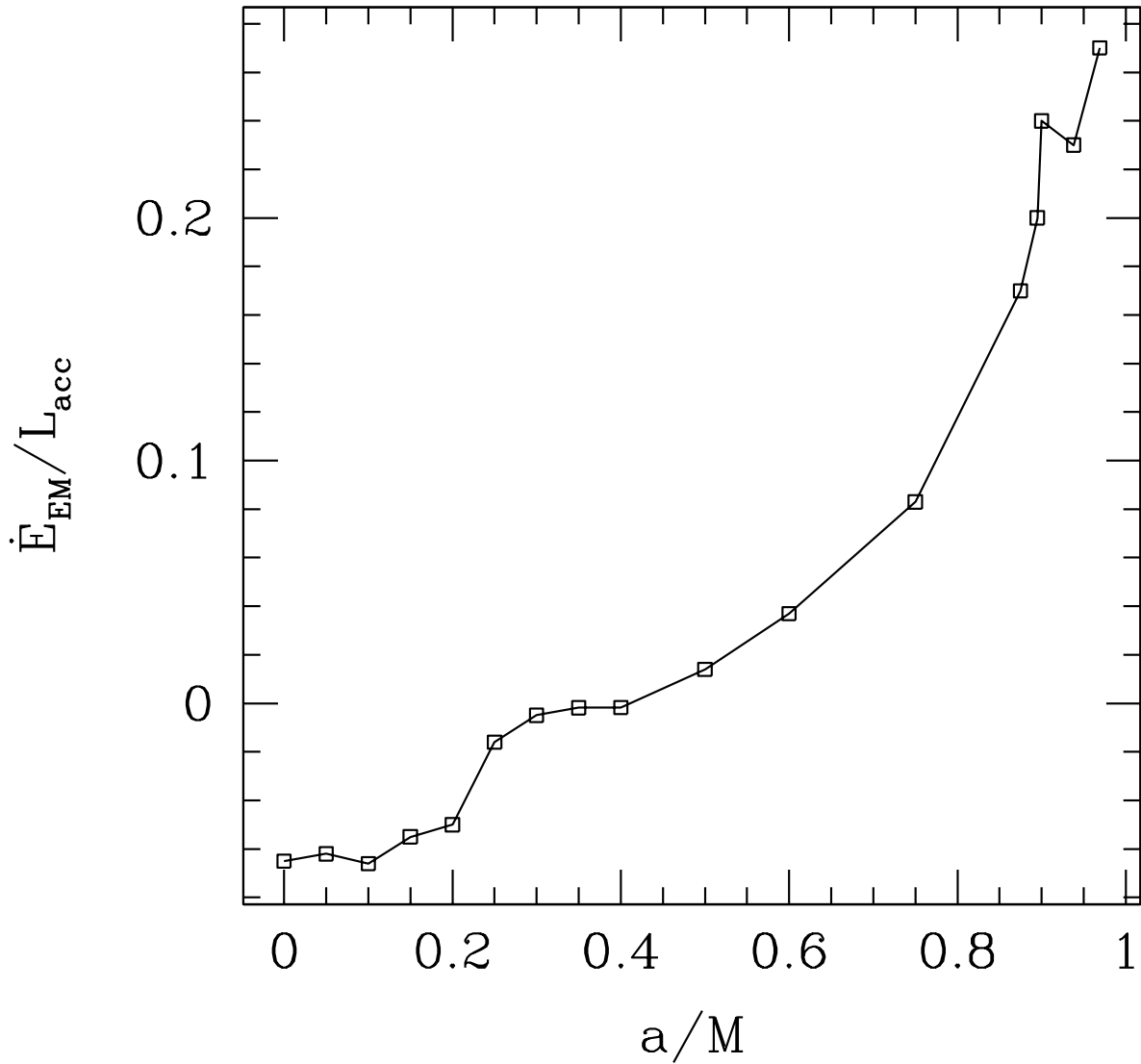
McKinney & Gammie 2004

Comparison with Blandford-Znajek

$$\langle \omega \rangle \approx 0.45 \Omega_H$$



Electromagnetic Flux on Horizon vs. Nominal Accretion Luminosity



McKinney & Gammie 2004

Conclusions

Robust, tested GRMHD codes now exist.

Black Hole Spin Up/Spin Down

Thick accretion flows produce $a/M \sim 0.9$.

Prediction: no holes with $a/M \sim 1$.

Blandford-Znajek Effect

No serious theoretical controversy.

Produces up to $\sim 30\%$ of nominal accretion
luminosity.