KITP 1

ALKALI GASES vs. TRADITIONAL SUPERFLUIDS

Disadvantages:

- almost inevitably inhomogeneous
- v. dilute, so eg mass transport not usually directly measurable.

Advantages:

- geometry can be tailored
- variable parameters
- optical discrimination
- multiple species (though cf ^{3}He)
- * "near-instantaneous" change of parameters

$\Downarrow$

spin superpositions
fast rotation ("QHE")
restricted geometries
quantum information
* MI $\rightleftarrows$ sup.
* BEC $\rightleftarrows$ BCS

KITP 2

BEC IN A SYSTEM OF BOSONS:

(Penrose + Onsager 1955):

1p density matrix

$$\rho_1(r, r' : t) \equiv \sum_\nu p_\nu(t) \int dr_2 \ldots dr_N \, \Psi_\nu^*(r : r_2 \ldots r_N : t) \, \Psi_\nu(r' : r_2 \ldots r_N : t)$$

eigenvalues

$$= \sum_i n_i(t) \, \chi_i^*(rt) \, \chi_i(r't)$$

eigenfunctions

("SIMPLE") BEC if ONE AND ONLY ONE of $n_i(t) \equiv N_0(t) \sim N$, rest all $O(1)$. Then,

order parameter $\rightarrow$ $\Psi(r,t) \equiv \sqrt{N_0(t)} \, \chi_0(rt)$

WHY IS "SIMPLE" BEC THE NORM?

2-STATE SYSTEM: $\hat{\Delta N}, \hat{\Delta\varphi}$ conjugate variables

$$H \sim \tfrac{1}{2} U (\hat{\Delta N})^2 + t(1 - \cos \hat{\Delta\varphi}) \leftarrow \approx \tfrac{1}{2} t (\hat{\Delta\varphi})^2$$

$U/t \to \infty$: Fock state (eigenf'n of $\hat{\Delta N}$)
(fragmented)

$U/t \to 0$: coherent state (eigenf'n of $\hat{\Delta\varphi}$)
(simple BEC)

Dilute gas:

$$\text{K.E.} = f(\Delta\varphi) \sim \frac{\hbar^2}{2m} n^{2/3}$$

$$\text{P.E.} = f(\Delta N) \sim gn \sim \frac{4\pi\hbar^2}{m} a_s n$$

$$\text{P.E./K.E.} \sim (n a_s^3)^{1/3} \Rightarrow \text{approx. eigenf'n of } \Delta\varphi$$

$\ll 1$

KITP 3

SOME EXAMPLES OF GROUNDSTATES OF BOSE SYSTEMS WITHOUT "SIMPLE" BEC:

(a) no BEC at all:

solid ^{4}He

Mott-insulator phase in optical lattices

rapidly rotating bosons in QHE regime

(b) "fragmented" BEC: ← more than one $n_i(t) \sim N$

2-well system in Fock limit ($U \gg t$)

attractive interactions

LPB state of spin-1 bosons

KSA* " spin-1/2 "

$$\Psi \sim (a^+_{0\uparrow}a^+_{1\downarrow} - a^+_{0\downarrow}a^+_{1\uparrow})^{N/2}|vac\rangle$$

WHY ARE SPINLESS, EXTENDED BOSE GASES SO RARELY FRAGMENTED?

(a) EQUILIBRIUM: HF APPROX^n: >0

$$E = \sum_i n_i \epsilon_i + \frac{1}{2} U_0 \sum_{ij} (2 - \delta_{ij}) n_i n_j$$

(b) NONEQUILIBRIUM

e.g. p → s in annulus:

$$|M, N-M\rangle \sim \int \Psi_{coh}(\Delta\varphi)\, d(\Delta\varphi)$$

Fock, fragmented   simple BEC, coherent

"best" coherent state always at least as good as Fock!

KITP 2

THE "MOTT-INSULATOR ⇄ SUPERFLUID" TRANS^N

(cf: melting of solid ^{4}He into superfluid phase)

$$\Psi_{MI} \sim \mathcal{S}\,\varphi(r_1 - R_1)\varphi(r_2 - R_2) \dots \varphi(r_N - R_N)$$

particle   site

$$\Psi_{sup} \sim \prod_{i=1}^{N} \chi(r_i)\,. \qquad \chi(r) \sim \sum_{i=1}^{N} \varphi(r - R_i)$$

particles   sites

In "superfluid" (BEC) state, can take one particle right across sample, keeping the other N-1 fixed, and MBWF nowhere vanishes or becomes exp'ly small (and this is essential to explan^n of superfluidity)

In MI state, this is not true (hence not superfluid)

So, HOW LONG DOES THE SYSTEM TAKE to "find out" that it should be superfluid? (and how accurately does it do so?)

a) time for single atom to move across whole lattice

b) time to jump to nearest-neighbor site.

KITP 3

Mott Ins'r ⇄ Superfluid, cont.

Related problems:

A. Higgs-Kibble mechanism in cosmology

analog simulations of above (?) in sup. ^{3}He

Q: (a) interpret^n of (some) raw data open to question

(b) normal-superfluid counterflow apparently essential (and absent in early Universe!)

B. "Supersolid" behavior of solid (?) ^{4}He

(Chan + Kim, Nature (2004))

destroyed by very small (10–100 ppm) of ^{3}He!

Can one introduce "chemical impurities" in optical-lattice problem?

KITP 4

"Pseudo-BEC" (Cooper pairing) in a Fermi System (Yang)

omit spin indices for simplicity

$$\rho_2(r_1r_2,r_1'r_2':t) \equiv \sum_\nu p_\nu \int dr_3 \dots dr_N\, \Psi_\nu^*(r_1r_2:r_3\dots r_N:t)\,\Psi_\nu(r_1'r_2':r_3\dots r_N:t)$$

$$\equiv \langle \psi^\dagger(r_1)\psi^\dagger(r_2)\psi(r_1')\psi(r_2')\rangle(t)$$

$$= \sum_i n_i(t)\,\chi_i^*(r_1r_2:t)\,\chi_i(r_1'r_2':t)$$

For unrestricted no. of single-particle states, max. value of any n_i is $N/2$.

(Fermi sea: all n_i 1 or 0)

"Simple" pseudo-BEC if one and only one of eigenvalues is $O(N)$, rest $O(1)$

BCS theory:

$\equiv \langle \psi_\uparrow^\dagger(r_1)\psi_\downarrow^\dagger(r_2)\rangle$

$$\chi_0(r_1r_2:t) \propto F(r_1,r_2) = F(\rho)$$

$\rho = r_1 - r_2$

$$F(\rho) \cong \frac{\sin 2k_F\rho}{k_F\rho}\exp -\rho/\xi$$

← pair radius, weakly $f(T)$

$$\frac{N_0(T)}{N} \sim \frac{\Delta}{\epsilon_F} \sim \frac{T_c}{T_F}$$ ← $\ll 1$ in BCS limit

KITP 7

WHY "SIMPLE" PSEUDO-BEC (i.e. NO FRAGMENT!) ?

NOT TRIVIAL! (cf. Gor'kov + Galitskii 196)

Generally, if 2 states possible and nonzero scattering between them, single coherent state formed in thermodynamic limit.

Ex.: ^{3}He-A:

poss. pair states have same orbital dependence, but can be $|\uparrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle$.

Which is correct:

(a) $\Psi_N \sim (|\uparrow\uparrow\rangle + e^{i\Delta\varphi}|\downarrow\downarrow\rangle)^{N/2}$ coherent

(b) $\Psi_N \sim |\uparrow\uparrow\rangle^{N/4} |\downarrow\downarrow\rangle^{N/4}$ Fock

Relevant terms in Hamiltonian:

polarization energy $S^2/2\chi$ — favors $\langle S^2\rangle = 0$, i.e. Fock state

dipole interaction $g_D(1-\cos\Delta\varphi)$ — favors $\langle\Delta\varphi^2\rangle = 0$, i.e. coherent state

in thermodynamic limit:

$\chi \sim N$, $g_D \sim N$

$\Rightarrow$ coherent state always formed

$\uparrow$: for small enough samples, Fock wins!
($\Rightarrow$ theory of NMR must be redone)

KITP 8

THE "BEC-BCS CROSSOVER" PROBLEM

1. What is optimal measure of "degree of condensation" f ?

(All reasonable measures give $f=1$ in BEC limit)

	N state	S state, T = 0
max. eigenvalue of $\hat{\rho}_2/N$	0	$\sim T_c/T_F \ll 1$
"superfluid" response (various)	0	1
"degree of pairing" $\sum_k n_{k\uparrow}n_{-k\downarrow}$	$1 - O(T_c/T_F)$	1

Which of the above (if any) is measured by various expts. in the crossover region?

2. Equilibrium state:

– is single-channel model adequate (for mid-life FR) in (a) strong-resonance limit
(b) weak-resonance limit ?

– is the "naive" Eagles ansatz
(a) quantitatively correct over whole crossover region? (no: GMB, PSS)
(b) qualitatively ("topologically") correct?

* Can we bound $T_c / N_0(T)$?

KITP 4

BEC ⇄ BCS CROSSOVER, cont.

3. Nonequilibrium (magnetic-field-sweep) expts.:

how many different timescales?

- $\hbar/\Delta(k_F a)$ ← inverse gap freq.
- τ_{pair} ← time to change N_0 (at const. $k_F a$)
- τ_{coll} ← time for internal equilibn of quasiparticle gas.
- τ_{ext} ← time for thermal equilibrium with "bath" (of doubtful relevance in real expts)

If sweep is fast compared to τ_{pair} and τ_{coll} but slow compared to $\hbar/\Delta$ (at relevant $k_F a$), would prima facie expect it to conserve "degree of pairing".

If slow compared to all of $\tau_{coll}, \tau_{pair}, \hbar/\Delta$, should conserve entropy (?)

4. NMR behavior:

to extent that $H(\underset{\sim}{S}) = f(S^2) - \underset{\sim}{S}\cdot\underset{\sim}{\mathcal{H}}_{eff}$, resonance must be δ-function at Larmor frequency $\gamma\mathcal{H}_{eff}$! (cf superfluid ^{3}He).

5. MISC: quench in BCS limit, p-wave.