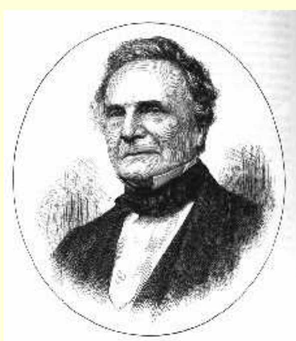
• Entanglement and precision probing with light and atomic ensembles

Entanglement and precision probing with light and atomic ensembles

KITP, May 2004

Klaus Mølmer

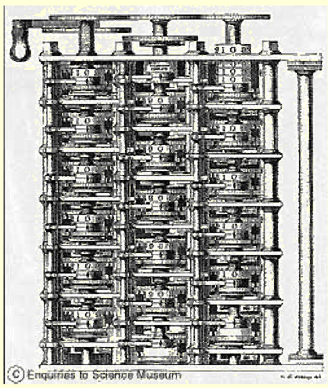


"… . Every atom, impressed with good and with ill, retains at once the motions which philosophers and sages have imparted to it, mixed and combined in ten thousand ways with all that is worthless and base. The air is one vast library, on whose pages are for ever written all that man has ever said or woman whispered."

Charles Babbage

Ninth Bridgewater Treatise, 1837

Information processing powered by steam.

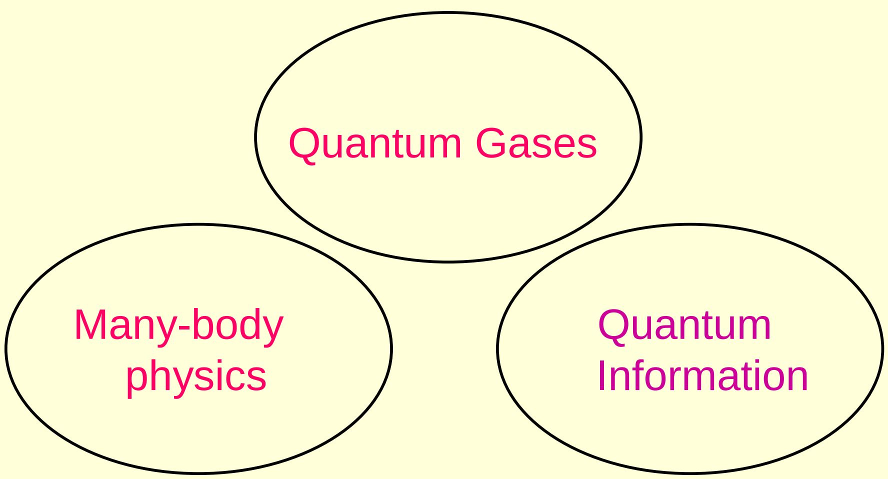


1832:

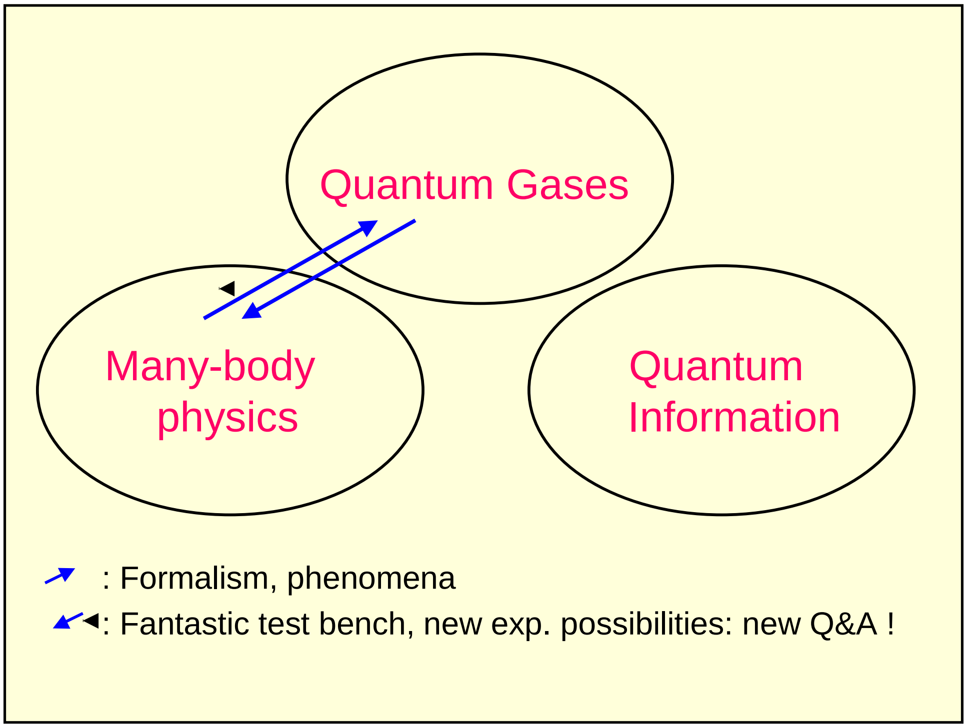
Babbage Difference Engine (2000 moving parts)

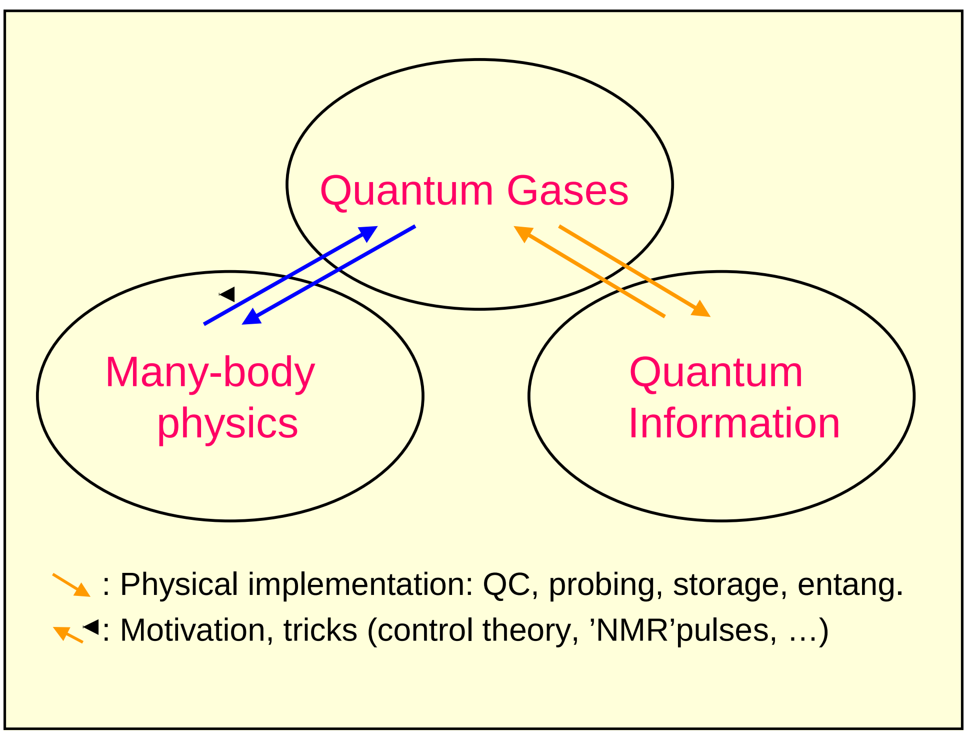
Mechanical → programmable design

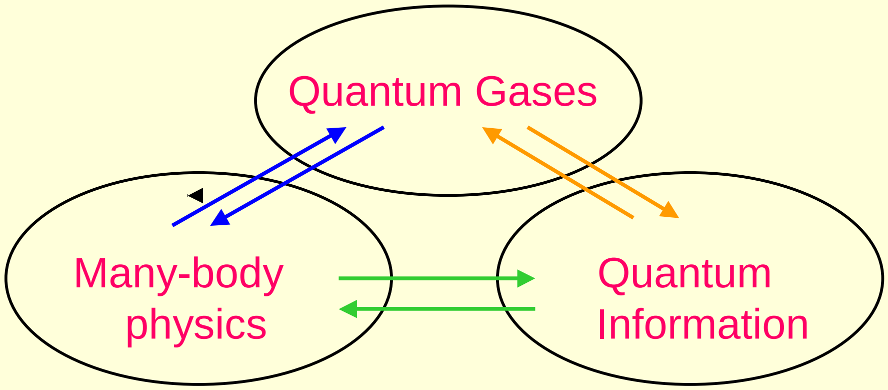
(Jacquard-loom)



- Entanglement and precision probing with light and atomic ensembles







→ : Physical implementation, QFT-contributions/ideas in QI

← : New order: entanglement length

New classical algorithms based on QI (DMRG++)

◄ ”RISQ”, the first useful quantum computer (?)

This talk

- Quantum Information with many-atom states
- Entanglement, spin squeezing, precision probing

Work done in collaboration with:

Isabelle Bouchoule, Antonio Di Lisi, Anders Sørensen
Lars Bojer Madsen, Vivi Petersen, Jacob Sherson,
Klemens Hammerer, Ignacio Cirac, Eugene Polzik

Hamiltonian evolution vs. measurement dynamics.

Solution of Schrödinger Equation:

$$\Psi \xrightarrow{t} U(t)\Psi$$

Measurement on quantum system:

$$\Psi \xrightarrow{\text{outcome } m} \Psi_m = P_m \Psi / \sqrt{p_m}$$

System must be intact in new state. BEC special case !

Plan for what to do for given, random *m:*

E.g.: accept only $m=m_0$, otherwise try again,

or accept all states (but remember outcome !)

Motivation, what states to make ?

Entangled and squeezed states can be used for high precision purposes, quantum communication and computing.

"Schrödinger-strategies" exist, but they may be inefficient, sensitive to noise, or simply impractical – hence we look at measurement strategies.

Teleportation uses measurements,

Knill-Laflamme-Milburn proposal for quantum computing using linear optics + measurements (and feedback).

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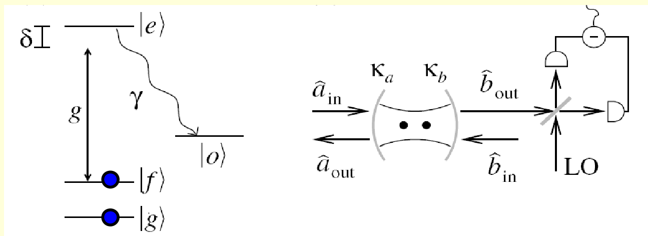
A simple example: a pair of atoms



Light does not change atomic state, but *measurement* of phase shift of light, gives *information* about the atoms !

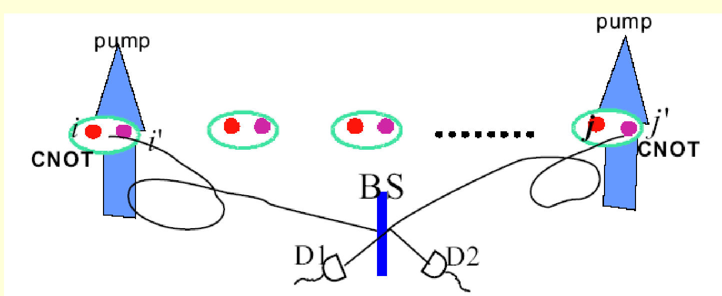
Product state, (|g> + |f>)(|g>+|f>) is changed:

n_f = 0 or 2 → product states |gg> or |ff> (try again)

n_f = 1 → entangled state (|gf> + |fg>)

A. Sørensen & KM (2003). (Cabrilo et al, Chris Monroe: experiments)

”Which-atom-decayed entanglement” (Cabrilo et al)



Recent experiments by Chris Monroe (Michigan)

Remote quantum computing:

Repeat until succesful (purple atoms)

Local operations → coupling of (red) qubit atoms

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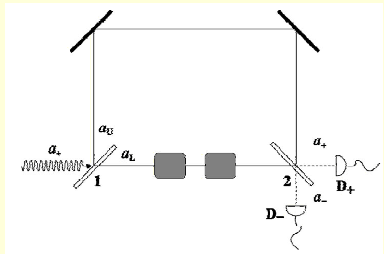
Entanglement of large atomic samples



Interaction → shift of optical phase ~ n_f

Product state, $(|g\rangle + |f\rangle)^N$ $(|g\rangle + |f\rangle)^N$

Measure phase shift → state with better defined $n_{f1}+n_{f2}$
(L.-M. Duan et al, Phys.Rev.Lett **85**, 5643 (2000))

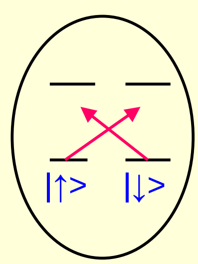
Demonstrated in Århus (and in Copenhagen) (Polzik)

Gaussian states of atoms and light

- All atoms in $(|\uparrow\rangle+|\downarrow\rangle)/\sqrt{2}$ → $\langle J_x\rangle=N_{at}/2$, $\langle J_y\rangle=\langle J_z\rangle=0$
  $Var(J_y)Var(J_z) = |\langle J_x\rangle|^2/4$ → binomial noise (M.U.S.).
  let $p_{at} = J_z/\sqrt{\langle J_x\rangle}$, $x_{at} = J_y/\sqrt{\langle J_x\rangle}$, $[x_{at},p_{at}]=i$
  harmonic oscillator ground state, Gaussian in x_{at},p_{at}

- x-polarized light has $\langle S_{x}\rangle = N_{ph}/2$, $\langle S_y\rangle = \langle S_z\rangle = 0$.
  let $p_{ph} = S_z/\sqrt{\langle S_x\rangle}$, $x_{ph} = S_y/\sqrt{\langle S_x\rangle}$, $[x_{ph},p_{ph}]=i$
  harmonic oscillator ground state, Gaussian in x_{ph},p_{ph}

- Dispersive atom-light interaction:
  σ^+ (σ^-) light is phase shifted more by $|\uparrow\rangle$ ($|\downarrow\rangle$) atoms
  → Faraday polarization rotation, proportional to $\langle J_z\rangle$
  $H_{int} = g S_z J_z = \kappa\, p_{at}\, p_{ph}$



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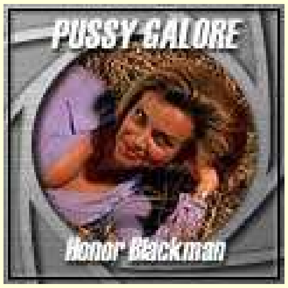
Entanglement theory is hard !



"Let us slip into something more comfortable"

Gaussian states !!!

G. Giedke & J. I. Cirac, Phys. Rev. A. **66**, 032316 (2002)
J. Eisert & M. Plenio, quant-ph/0312071 (2003), and others
J. Fiurasek, Phys. Rev. Lett. **89**, 137904 (2002)

Gaussian states

State characterized by mean values, **m**, and covariance matrix γ. (GP-wave function and Bogoliubov excitations)

Gaussian states (**m** and γ) transform under xx, xp and pp interactions (linear optics, squeezing), decay and losses.

(Kasevich, Chapman, Sengstock: $p_{at}p_{at}$)

Gaussian states (**m** and γ) transform under measurements of x's and p's (Stern-Gerlach and homodyne detection).

B. Kraus et al, Phys. Rev. A **67**, 042314 (2003)
K. Hammerer et al, quant-ph/0312156
K.M and L. Madsen, quant-ph/0402158

Gaussian distributions (classical)

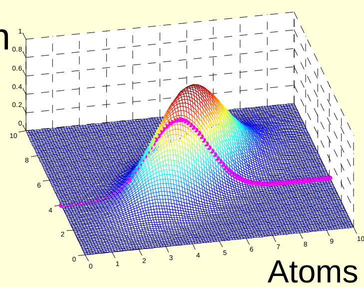
y = vector of (x's and p's) with mean values **m**.

γ = matrix of covariances, $\gamma_{ij} = 2\langle (y_i - m_i)(y_j - m_j) \rangle$.

$P(\mathbf{y}) = \mathcal{N} \exp(-(\mathbf{y}-\mathbf{m})^T \ \gamma^{-1} (\mathbf{y}-\mathbf{m}))$

(cf. $P(y)=\mathcal{N} \exp(-(y-m)^2/2\sigma^2)$ for single variable).

Transformation of Gaussian



QUANTUM CASE:
When we measure an x, its conjugate becomes completely undetermined, Assign infinite variance to p and remove its correlations with all other elements, and update as before

Linear transformation $\mathbf{y} \rightarrow \mathbf{Sy}$:

$\mathbf{m} \rightarrow \mathbf{Sm}$, and $\gamma \rightarrow \mathbf{S} \gamma \mathbf{S}^T$, e.g., a rotation.

Measurement of one component $\mathbf{y}_2$ of $\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2)$:

$P(\mathbf{y}_1) \sim \exp(-(\mathbf{y}_1-\mathbf{m}_1)^T \ (\gamma^{-1})_{11} (\mathbf{y}_1-\mathbf{m}_1)$ + linear terms in $\mathbf{y}_1$)

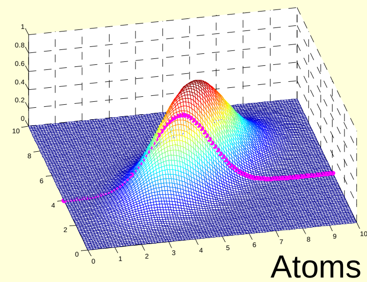
New covariance block: $\gamma_{11} \rightarrow \gamma_{11} - \gamma_{12} \ (\gamma_{22})^{-1} \ \gamma_{21}$,

New mean value: $m_1 \rightarrow m_1 + \gamma_{12} \ (\gamma_{22})^{-1} \ (y_2^* - m_2)$

- Entangelment and precision probing with light and atomic ensembles

Transformation of Gaussian states

Linear transformation
y → **Sy**:
m → **Sm**,
and $\gamma \rightarrow \mathbf{S}\, \gamma\, \mathbf{S}^T$, e.g.,
a rotation.



Measurement of one component $\mathbf{y}_2 = y_2^*$ of $\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2)$:
New covariance block: $\gamma_{11} \rightarrow \gamma_{11} - \gamma_{12}\, (\gamma_{22})^{-1}\, \gamma_{21}$,
New mean value: $m_1 \rightarrow m_1 + \gamma_{12}\, (\gamma_{22})^{-1}\, (y_2^* - m_2)$

Loss of light $\gamma_{ph} \rightarrow (1-\varepsilon)\gamma_{ph} + \varepsilon\, \mathbf{I}$
Atomic decay: $\gamma_{at} \rightarrow (1-\eta\Delta t)\gamma_{at} + 2(\eta\Delta t)\, \mathbf{I}$

Gaussian Quantum states II (loss and decay)

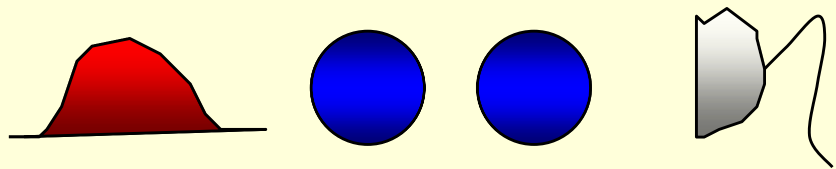
Loss of light:

$$\gamma_{ph} \rightarrow (1-\varepsilon)\gamma_{ph} + \varepsilon\, \mathbf{I}$$

Atomic decay:

$$\gamma_{at} \rightarrow (1-\eta\Delta t)\gamma_{at} + 2(\eta\Delta t)\, e^{\eta t}\, \mathbf{I}$$

Update of gaussian atomic state



$$\gamma = \begin{pmatrix}\gamma_A & 0 \\ 0 & I\end{pmatrix} \quad -> \begin{pmatrix}\gamma'_A & \gamma_{AF} \\ \gamma_{FA} & \gamma_F\end{pmatrix} \quad -> \begin{pmatrix}\gamma''_A & 0 \\ 0 & I\end{pmatrix}$$

Result: Spin squeezing

Entanglement of samples

Continuous limit

Frequent probing
(weak pulses/short segments of cw beam):
Update becomes continuous

Differential equation for covariance matrix

This, so-called, Ricatti equation is *non-linear*

… but it can sometimes be solved !

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This, so-called, Ricatti equation is *non-linear*

… but it can sometimes be solved !

Atomic spin squeezing due to optical probing.

For the simple atom-light example (binomial distribution):

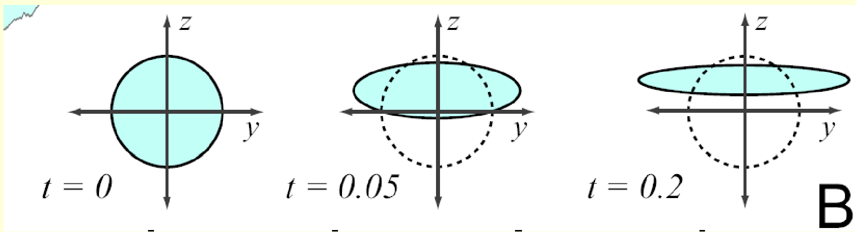
$$\frac{d}{dt}Var(p_{at}) = -2\kappa^2 Var(p_{at})^2 \Rightarrow Var(p_{at}) = 1/(2+2\kappa^2 t)$$

In presence of atomic decay η (ε<<1):

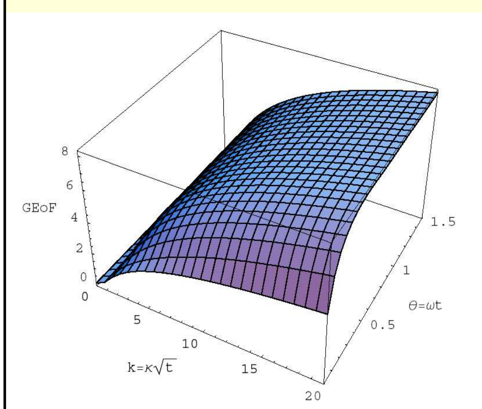
$$\frac{d}{dt}Var(p_{at}) = -2\kappa^2 e^{-\eta t} Var(p_{at})^2 - \eta Var(p_{at}) + \eta e^{\eta t}$$

Optimum of $Var(p_{at}) \sim \sqrt{(\eta/\kappa^2)} \sim 1/\sqrt{N_{at}}$
Occurring at $t=1/(\sqrt{\eta}\,\kappa) << 1/\eta$

Update of γ is deterministic
but displacement (mean value) varies from shot to shot.

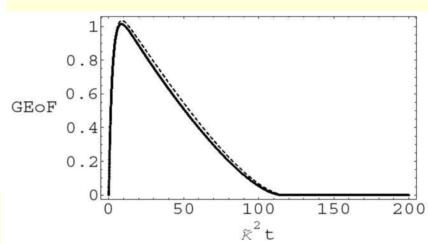


Entanglement of two gases
GEoF



Rotations → EPR state

Loss → optimum:



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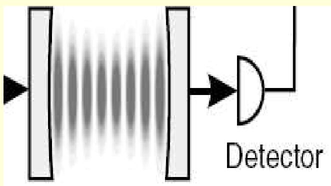
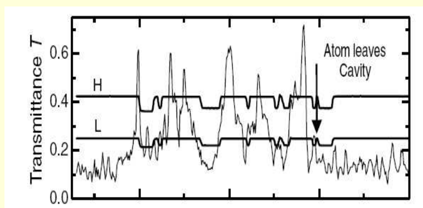
Probing of classical parameter

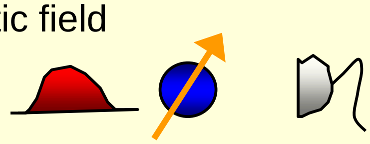




Optical transmission →
Existence and location of atom.
But experimental signal is noisy !
How well can we localize the atom ?

← exp. MPQ

Probing of a classical magnetic field



- B-field causes spin precession $\Psi(t)$
- Farady rotation of polarization $\sim \langle \Psi(t) | J_z | \Psi(t) \rangle \sim B_y$
- Detection of light
  what is B_y, what is the error-bar ?

- Light detection is random
- Stochastic evolution of atomic quantum state
- Bayesian update for classical probability P(B):
- P(random signal | B) → P(B | signal)   (H. Mabuchi)

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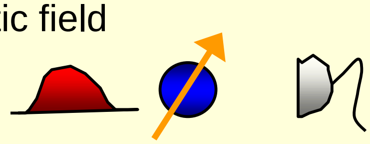
Probing of a classical magnetic field

(K.M & L. Madsen, quant-ph/0402158)



Our approach:
Treat atoms AND light AND B field as a quantum system
Covariance matrix for $(B, x_{at}, p_{at}, x_{ph}, p_{ph})$.

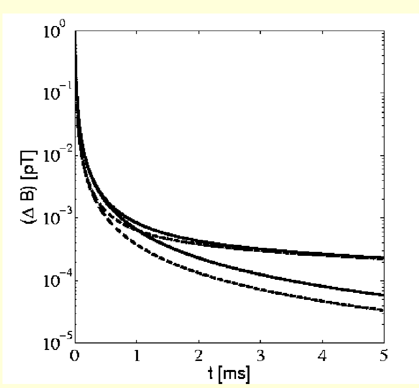
$$\Delta B(t)^2 = \frac{(1+\kappa^2 t)\Delta B_0^2}{1+\kappa^2 t + \frac{2}{3}\kappa^2\mu^2(\Delta B_0)^2 t^3 + \frac{1}{6}\kappa^4\mu^2(\Delta B_0)^2 t^4}$$

Long times: $\Delta B \sim 1/(N_{at}\, t^{3/2})$

- Independent of ΔB_0
- not as $1/\sqrt{N_{at}}$,$1/\sqrt{t}$

fT-magnetometry with atomic probe

(Recent experiments in Nature 422, 596 (2003))



ΔB as function of time.
(2 10^{12} Cs atoms, μW laser power
2 mm^2 cross section, κ^2=1.8 10^6 s^{-1})

Lower solid curve: analytical result
Upper solid curve: include spon. em.
(η = 1.8 s^{-1} ,,GHz detuning)
Dashed curves: polarization squeezed optical probe

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More field components: divide and conquer

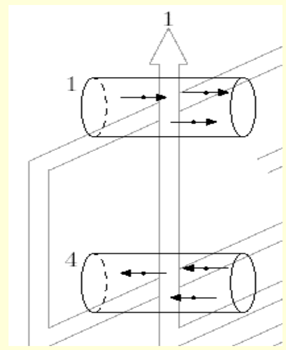
Mean spin along x:

J_z, J_y sensitive to B_y, B_z,

but we cannot measure both !

Divide gas in two, orient along +/- x, and measure sums and differences of z,y components (EPR-pair).

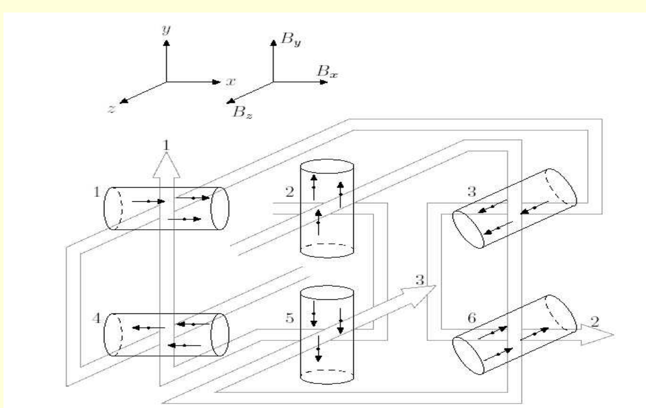
Local use of entanglement !!!



Same scaling for long times:

$\Delta B_{x,y} \sim 1/(N_{at}\, t^{3/2})$ (t → t/2)

Measure three B-field components divide gas in 6, and probe 4 commuting obs.



Same scaling for long times:

$\Delta B_{x,y,z} \sim 1/(N_{at}\, t^{3/2})$ (t → t/3, N_{at} → 2/3 N_{at})

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Outlook I, Gaussian states.

- Many atoms and many photons are ”easy experiments” (classical fields, homodyne detection)
- Many atoms and many photons are ”easy theory” (readily generalized at low cost to more samples/fields)
- Gaussian states: squeezing, entanglement, … and also: finite bandwidth sources, finite bandwidth detection
- Gaussian states unify quantum and classical variables: classical B-field + atoms + light probe

  → other observables: interferometry, … .

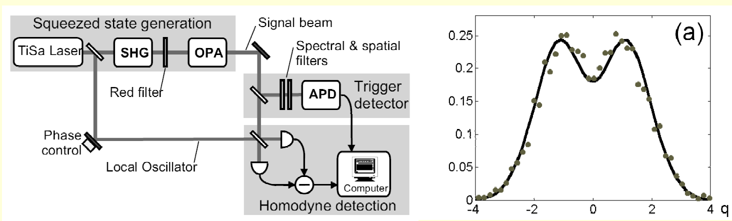
Outlook II, Gaussian states.

- Condensates are good atomic systems for precision probing: well localized, good optical depth, … .
- Condensate and fermion physics with Gaussian states
- Gross-Pitaevskii equation + Bogoliubov excitations
- Probing and dynamics/interaction
- Current project: quantum state tomography from spatial densities recorded at different times

• Entangelement and precision probing with light and atomic ensembles

Outlook III, non-gaussian states

- Gaussian states → qubits
  - conditioned: cf. example with ions, squeezed light on atoms
  - unconditioned: (B. Kraus, I. Cirac, quant-ph/0307158)
- Distillation of Gaussian states requires non-gaussian intermediate state
  - non-homodyne detection: photon counting
  - J. Wenger et al, quant-ph/0402192:



Is parameter estimation by 'quantization' correct ?

1. Any classical variable is a quantum variable, that happens to be tractable by classical theory (QM is still correct).

   Here QND measurement: conjugate variable π_B not relevant.
2. The "quantum state" is a representation of our knowledge about a system.
   - it does not underestimate variances (it is correct)
   - more knowledge would be a "hidden variable"

The conditioned mean and the quantum variance is the correct estimator of the parameter !

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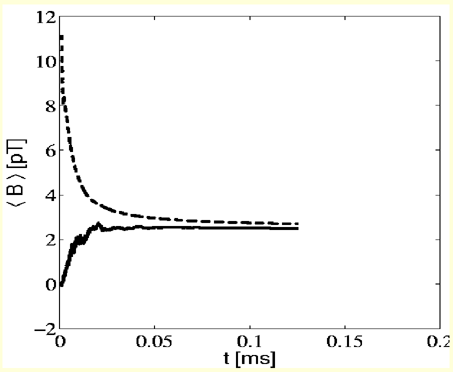
Conditioned estimator (mean)



Random photocurrent I(t)

- varies from shot to shot
- different B_{est}=<B>
- converges (Var(B) → 0)

- Upper curve:
  Simple estimator
  Integrated current
  $Q \sim B\ \mu\kappa^2\ t^3/6$