Introduction to Spin Correlated States in Optical Lattices

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Outline

- Introduction
  i) Some general applications
  ii) S=1/2 Fermions and S=0 bosons in lattices
- **Spin-one bosons with anti-ferromagnetic interaction**
  i) Mott states (Nematic states)
  ii) Valence bond crystals and Spin singlet condensates
- **Magnetically Stabilized Nematic Order**
- **Correlated states as Hamming codes and stabilizer codes**

Information processor and Information storage

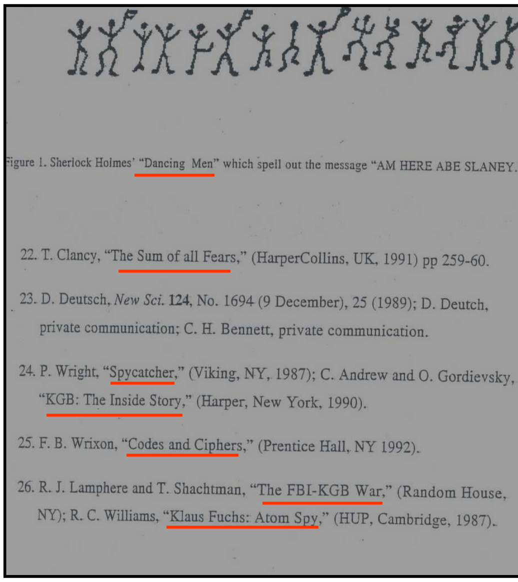

Figure 1. Sherlock Holmes' "Dancing Men" which spell out the message "AM HERE ABE SLANEY

22. T. Clancy, "The Sum of all Fears," (HarperCollins, UK, 1991) pp 259-60.
23. D. Deutsch, *New Sci.* **124**, No. 1694 (9 December), 25 (1989); D. Deutch, private communication; C. H. Bennett, private communication.
24. P. Wright, "Spycatcher," (Viking, NY, 1987); C. Andrew and O. Gordievsky, "KGB: The Inside Story," (Harper, New York, 1990).
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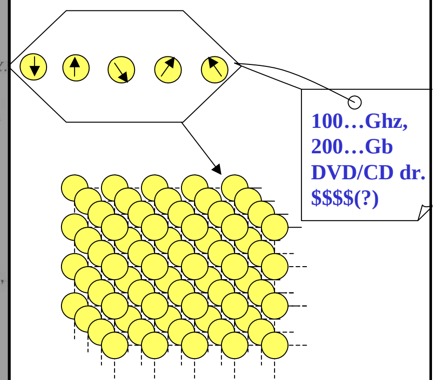
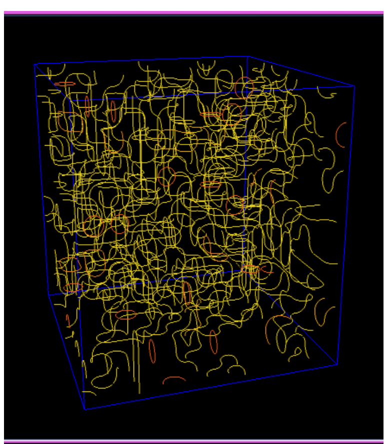
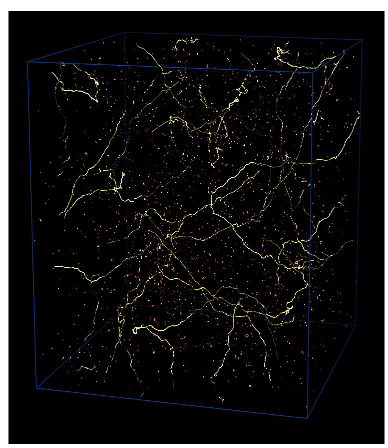


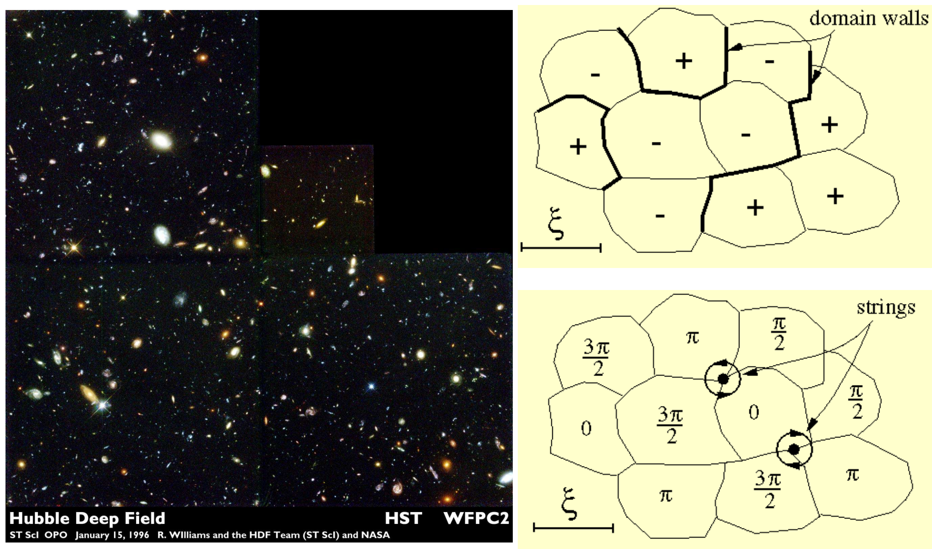
Simulations of a cosmic string network



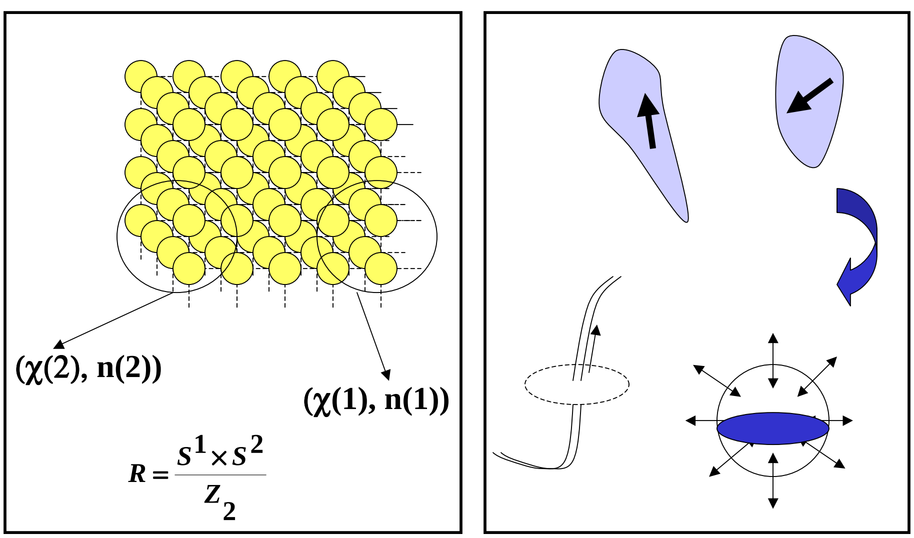


(Allen & Shellard, 00)

Large Scale Structure and Cosmic Strings

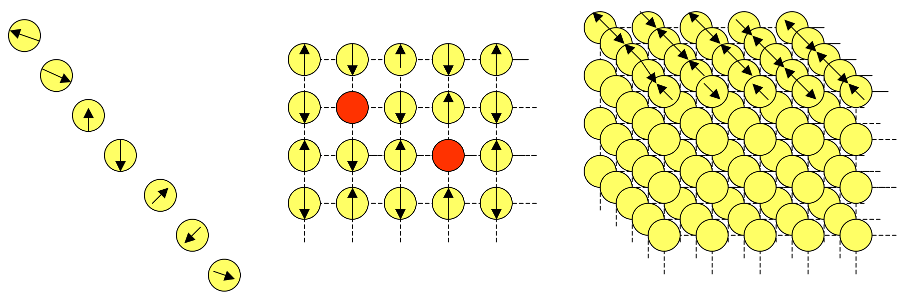


A lab for baby universe ?



Creation of vortices, monopoles and half vortices.

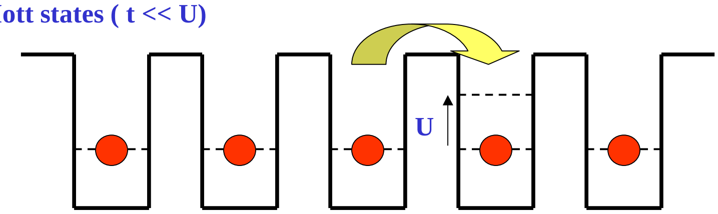
S=1/2 Fermions in optical lattices
(small hopping limit)



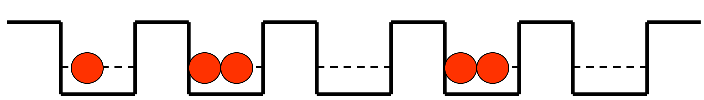
Gapless Spin liquid **HTcS made of cold atoms?** **Neel Ordered**

S=0 bosons in lattices

Mott states (t << U)



Condensates (t >>U)

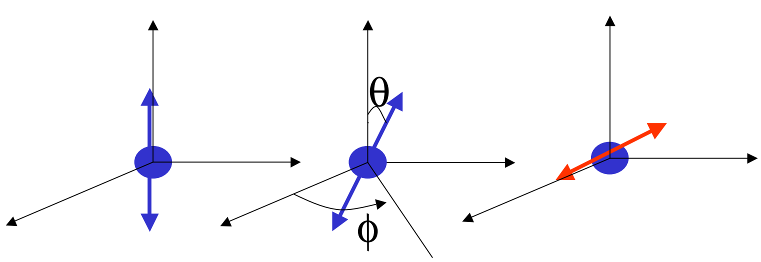


In (a) and (b), one boson per site. t is the hopping and can be varied by tuning laser intensities of optical lattices; U is an intra-site interaction energy. In a Mott state, all bosons are localized.

M. P. A. Fisher et al., PRB 40, 546 (1989);
On Mott states in a finite trap, see
Jaksch et al., PRL. 81, 3108-3111(1998).

Quantum Spin Nematic States and spin singlet states

S=1 bosons with Anti-ferromagnetic interactions



$$|\vec{n}(\theta,\varphi)>=\frac{\sin\theta e^{-i\varphi}}{\sqrt{2}}|1>+\cos\theta|0>-\frac{\sin\theta e^{i\varphi}}{\sqrt{2}}|-1>$$

$$:|-\vec{n}>=(-1)\ |\vec{n}>, \qquad <\vec{n}|S_\alpha|\vec{n}>=0.$$

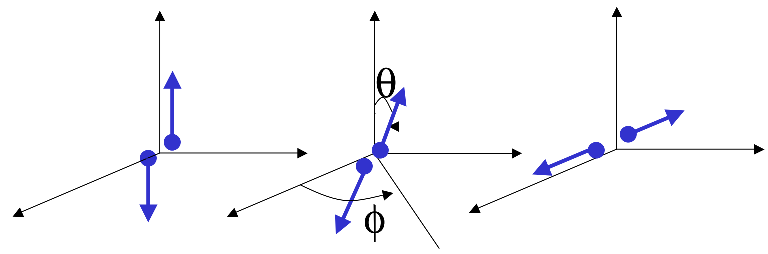
$$B:(0,\varphi)\Leftrightarrow\begin{pmatrix}0\\1\\0\end{pmatrix}, \quad R:(\frac{\pi}{2},0)\Leftrightarrow\begin{pmatrix}1/\sqrt{2}\\0\\-1/\sqrt{2}\end{pmatrix}.$$

$$U_F(r_1-r_2)=\delta(r_1-r_2)g_F,$$

$$g_F=\frac{4\pi\hbar a_F}{M}, g_2>g_0, F=0,2.$$

Ho, 98; Ohmi & Machida, 98; Law, 98.

S=1 states as AM triplet Cooper pairs



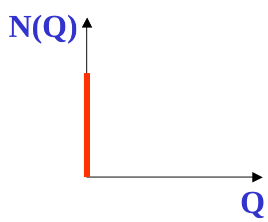
$$|S=1,\vec{n}> = \frac{1}{\sqrt{2}}(|\vec{n}>_1 |-\vec{n}>_2 + |-\vec{n}>_1 |\vec{n}>_2) \Leftrightarrow \Delta_{\alpha\beta} = (i\sigma_y \vec{\sigma}\cdot\vec{n})_{\alpha\beta}(k_x + ik_y).$$

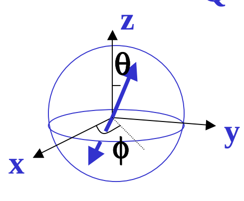
$$\vec{n} = \vec{e}_z \Rightarrow \frac{1}{\sqrt{2}}(|u>_1 |d>_2 + |d>_1 |u>_2);$$

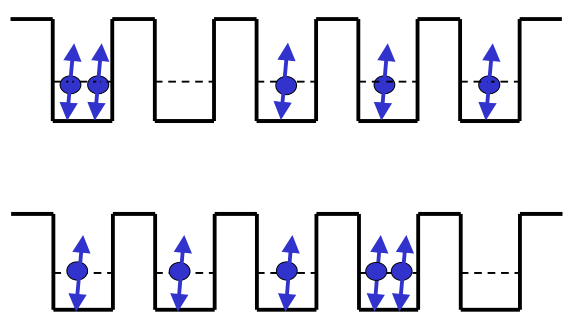
$$\vec{n} = \vec{e}_x \Rightarrow \frac{1}{\sqrt{2}}(|u>_1 |u>_2 + |d>_1 |d>_2).$$

Pairs given here are unpolarized, i.e. $<\vec{n}\,|\,S_\alpha\,|\,\vec{n}> = 0.$

Condensates of spin one bosons
(d>1)





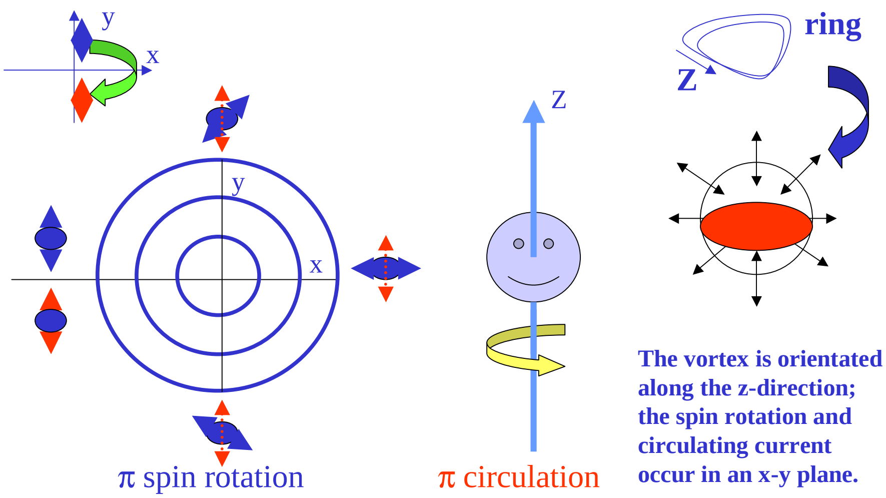


Snap shots

$$\psi_{pBEC} \sim \frac{(C^{+}_{Q=0,\alpha}\, n_\alpha)^{N\times V_T}}{\sqrt{(N\times V_T)!}}; C^{+}_{0,\alpha} = \frac{1}{\sqrt{V_T}}\sum_k C^{+}_{k\alpha}, \alpha = x, y, z.$$

Half vortices

In a half vortex, each atom makes a π spin rotation; a half vortex carries one half circulation of an integer vortex. A half vortex ring is also a hedgehog.



The vortex is orientated along the z-direction; the spin rotation and circulating current occur in an x-y plane.

Schematic of microscopic wave functions

a) NMI; b) SSMI (N=2k); c) SSMI (N=2k+1 in 1d).
Each pair of blue and red dots with a ring is a spin singlet.

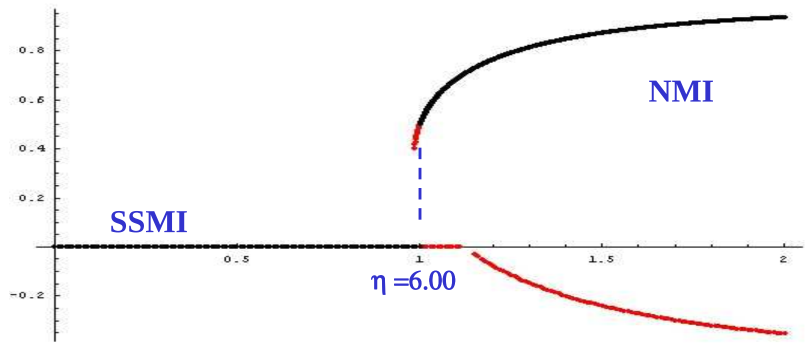
$$O^2_{\alpha\beta} = \langle c^+_\alpha c_\beta \rangle - \frac{1}{3}\delta_{\alpha\beta} \langle c^+_\gamma c_\gamma \rangle.$$

$$NMI: O^2_{\alpha\beta} = \varepsilon N(\vec{n}_\alpha \vec{n}_\beta - \frac{1}{3}\delta_{\alpha\beta});$$

$$SSMI: O^2_{\alpha\beta} = 0.$$

$$\eta = \frac{tz}{\sqrt{E_s E_c}}; \eta >> 1, \psi_{NMI} \sim \prod_k \frac{(c^+_{k\alpha}\vec{n}_\alpha)^N}{\sqrt{N!}} |vac>; \eta = 0, \psi_{SSMI} \sim \prod_k \frac{(c^+_{k\alpha}c^+_{k\alpha})^{N/2}}{\sqrt{N!}} |vac>.$$

Two particles per site



ε vs. η is plotted.

$$\Psi \propto \prod_k [\beta \,|\, 0,0 >_k + \sum_{m=\pm 2,\pm 1,0} \alpha_m \,|\, 2,m >_k];$$

$$|\beta|^2 + \sum |\alpha_m|^2 = 1$$

Constrained quantum rotor model

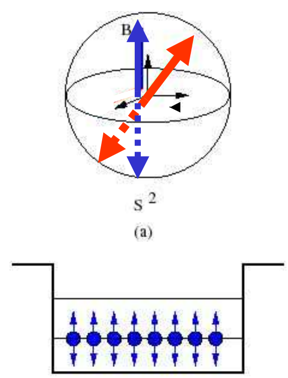
valid when the hopping is much less than "U".

$$H_{MI} = E_s \sum_k \vec{S}_k^2 - J_{ex} \sum_{<kl>} (\vec{n}_k \cdot \vec{n}_l)^2,$$

$$[\vec{S}_{k\alpha}, \vec{n}_{k'\beta}] = i\delta_{kk'} \varepsilon_{\alpha\beta\gamma} \vec{n}_\gamma.$$

$$1) J_{ex} = \frac{t^2 z}{E_c}, \eta = \frac{t\sqrt{z}}{\sqrt{E_c E_s}}.$$

$$2) (-1)^{S_k + N_k} = 1, \quad H_{MI}(\vec{n}_k) = H_{MI}(-\vec{n}_k).$$

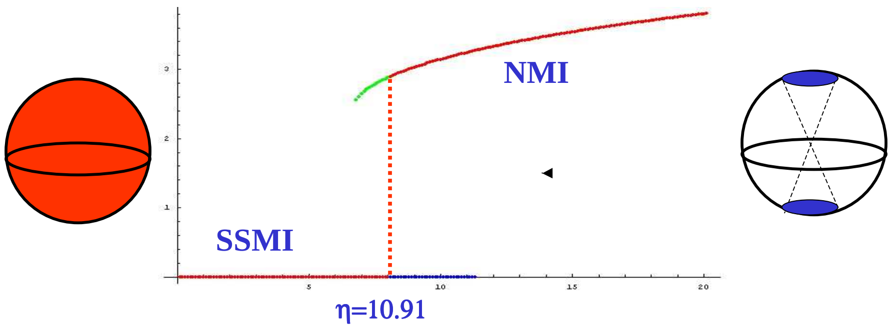


(a)

(b)

$$\text{Ref}: H_{HAF} = J_{ex} \sum_{<kl>} \vec{S}_k \cdot \vec{S}_l; \; S = 1, \eta = \sqrt{Sz}.$$

Numerics I: Large **N=2k** *limit*

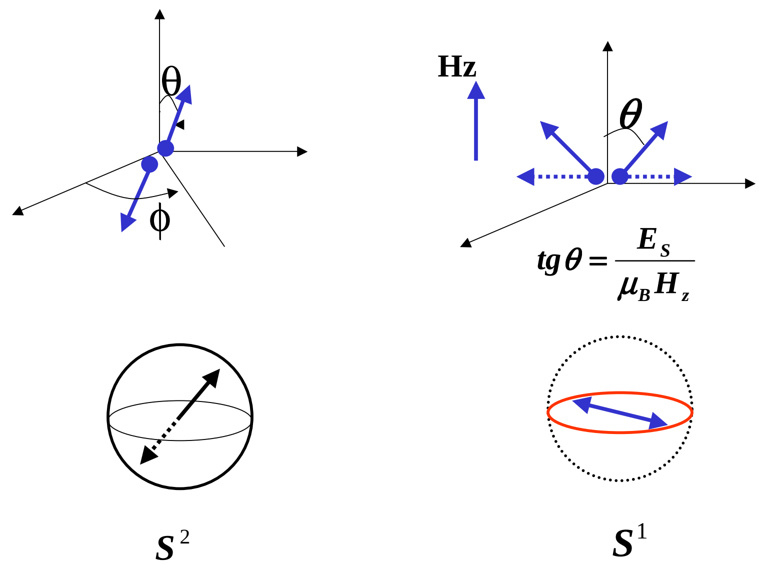


σ vs. η (proportional to hopping) is plotted here. Blue and Green lines represent metal stable states close to the critical point.

$$\Psi(\vec{n}) \sim \exp[-\sigma \vec{n}_\alpha \vec{n}_\beta Q^0_{\alpha\beta}]$$

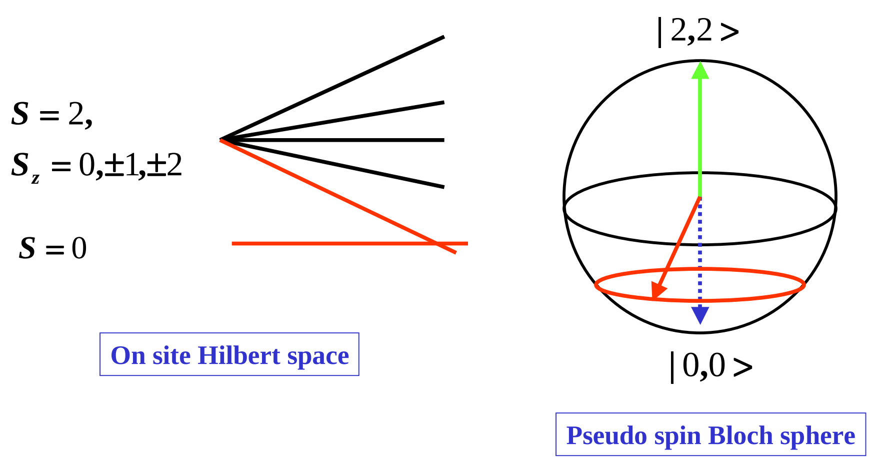
Magnetically stabilized nematic order

O(2) nematic order in external fields



Level Crossing and Nematic Order

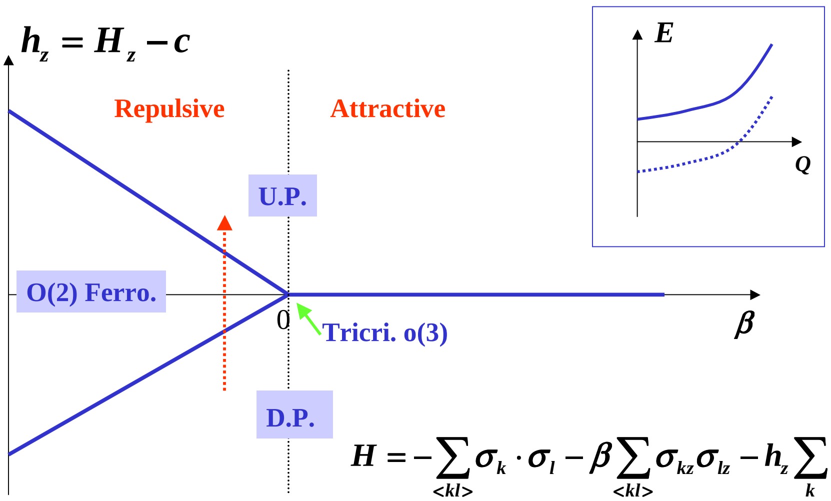
(from a spin singlet state to a nematically ordered state)



all spin coherent states except north and south pole states have nonvanishing nematic order.

Development of Nematic Order

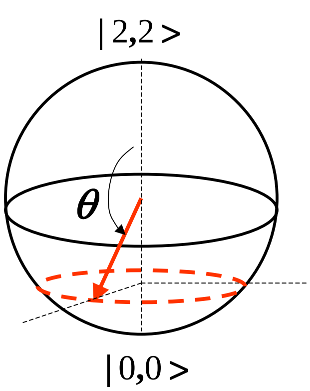
(mapping to a ferromagnetic XXZ model in a z-field)



$$H = -\sum_{<kl>} \sigma_k \cdot \sigma_l - \beta \sum_{<kl>} \sigma_{kz}\sigma_{lz} - h_z \sum_k \sigma_{kz}$$

U.P.= |2,2> Mott state; D.P.= |0,0> Mott state; O(2) Ferro=Nematic state

Magnetically stabilized nematic states



$$S_z = 2\hbar(1 - \cos\theta),$$

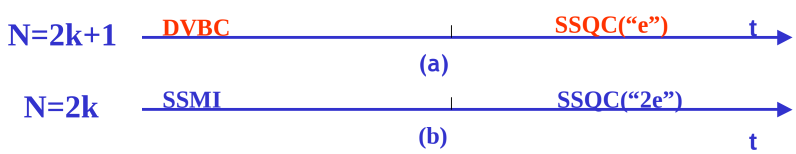
$$Q_{\alpha\beta} = \sin\theta(\Omega_\alpha \Omega_\beta - \frac{1}{2}\delta_{\alpha\beta}),$$

$$\Omega = (\cos\frac{\phi}{2}, \sin\frac{\phi}{2}), \sin\theta = \sqrt{1 - \frac{h_z^2}{h_c^2}}$$

Fractioanlization of atoms in 1D
and Hamming/Stabilizer codes

Spin singlet quantum "condensates" in 1D optical lattices
(SSQC)

S=1, "Q=e" bosons with AF interactions =>
S=0 , "Q=e" bosons interacting via Ising gauge fields



$$H_{fqc} = H_{m.} + H_{Z_2}$$

$$H_{m.} = -t_0 \sum_{<kl>} \sigma^z_{kl} (b^+_k b_l + h.c.) + E_C \sum_k (\hat{N}_{kb} - N)^2;$$

$$H_{Z_2} = \Gamma_a \sum_{<kl>} \sigma^x_{kl}$$

$$\hat{C}_k \Psi = \Psi, \hat{C}_k = \exp(i\pi[N_{kb} + \sum_{+} \sigma^x_{kl}]).$$

A projected spin singlet Hilbert space

$$|...N_k, d_{k,k+1}, N_{k+1}...>,$$
$$d_{k,k+1} = \frac{1-\sigma^x_{k,k+1}}{2}; d_{k,k+1} = 0,1.$$
$$N_k + \sum_{+} d_{k,l} = even.$$

In a projected spin singlet Hilbert space (states in a), b), d) and e)):
i) An atom forms a singlet pair with another atom either at the same site or at a nearst neighboring site;
ii) Each link is occupied by either one singlet pair of atoms or zero.

An effective Hamiltonian

$$|...N_k, d_{k,k+1}, N_{k+1}...>\Rightarrow$$
$$|...N_k + 1, \tilde{d}_{k,k+1}, N_{k+1} - 1...>;$$
$$d_{k,k+1} \neq \tilde{d}_{k,k+1}.$$

$$H_{fqc} = H_{m.} + H_{Z_2};$$
$$H_{m.} = -t_0 \sum_{<kl>} \sigma^z_{k,l}(b^+_k b_l + h.c.) + E_C \sum_k (\hat{N}_{kb} - N)^2;$$
$$H_{Z_2} = \Gamma_a \sum_{<kl>} \hat{d}_{k,l}.$$

Hopping of S=1 Bosons

a) and d) SSMI states for even and odd numbers of atoms.

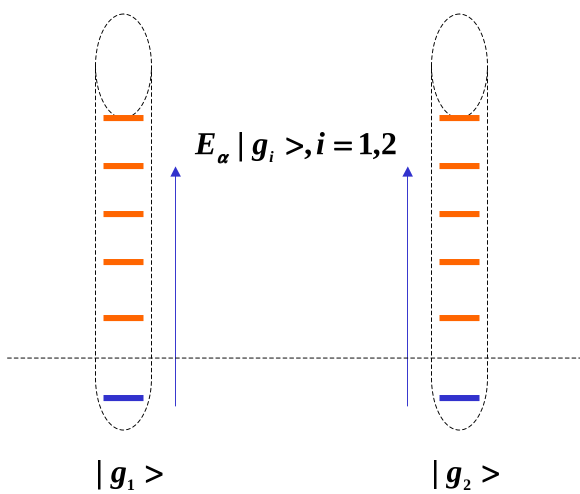
b) A kink-like S=0, Q=1 excitation in an "odd" lattice.

e) A string-like Q=1 excitation in an "even" lattice.

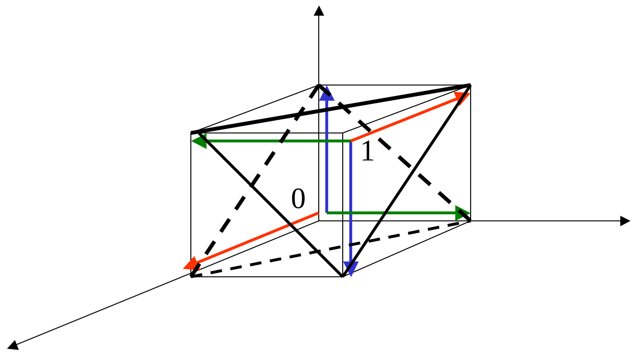
c) Hopping in an "odd" lattice leads to kink-anti kink excitations.

f) In an "even" lattice hopping is suppressed because of a string of valence bonds between particle and hole excitations. Red dots are "charged".

Tower-like Hilbert space for "odd" Mott states



A 3-bit Hamming code [3,1,3]



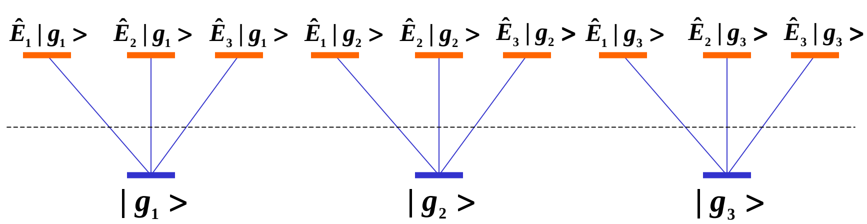
(0,0,0) $\rightarrow$ 0 (1,1,1) $\rightarrow$ 1

Quantum Error Correction Code

Requirement for a QECC
1) Errors in different code words are distinguishable;
2) Subspace of errors are indistinguishable so that there will be no information leakage.

$$< g_i | \hat{E}_\alpha^+ E_\beta | g_j >= \delta_{ij} C_{\alpha\beta}$$

Error Space



Code Space

Spin one bosons in optical lattices

We have discussed

1) Half vortices in condensates
2) Nematic Mott insulators and spin singlet Mott insulators
3) Valence bond crystals (N=2k+1,1D) and Spin singlet condensates

Work in progress

- Magnetically stabilized nematic order
- Towards fault tolerant quantum information storage

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