Towards a Weakly Interacting BCS Transition: Three Cooling Strategies

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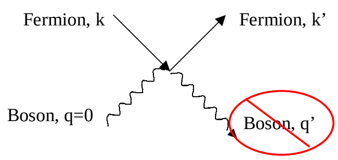
Overview: Towards a Weakly Interacting BCS Transition

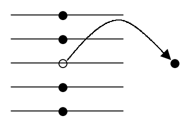
- Introduction: BCS & BEC in cold quantum gases
- I. Out of equilibrium cooling and loss in the Fermi sea
- II. Heat capacity in sympathetic cooling
- III. Entropic cooling through the BEC to BCS crossover
- Conclusions and Outlook

Cooling Scheme I: Loss in the Fermi Sea, Out of Equilibrium

- What is the ultimate lower limit on T/T_F that may be obtained by sympathetic cooling?
- "Hole heating" work by Timmermans
  - Phys. Rev. Lett. **87** 240403 (2001)



versus



→ How does the occupation number distribution of fermions evolve in time in the presence of loss?

→ Do the fermions reach thermal equilibrium?

A Model for Sympathetic Cooling

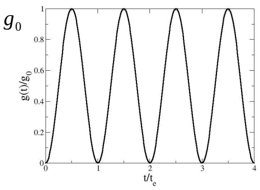
- Perfect Bose reservoir
  - Ideal gas, T=0, N_B ~ constant, excited bosons removed
- Non-interacting Fermi gas (no p-wave)
- Contact potential for Fermi-Bose interactions
- Uniform system
  - 3D box with periodic boundary conditions
- Discrete evaporation steps of duration t_e
  - $1/t_e$ ~ evaporation rate
- Constant fermion loss rate

The Hamiltonian

$$H = H_0 + V$$

$$H_0 = \sum_{\mathbf{k}} \frac{\hbar^2 k^2}{2m_F} c_{\mathbf{k}}^{+} c_{\mathbf{k}} + \sum_{\mathbf{q}} \frac{\hbar^2 q^2}{2m_F} b_{\mathbf{q}}^{+} b_{\mathbf{q}}$$

$$V = \frac{g(t)}{L^3} \sum_{\mathbf{k}',\mathbf{k},\mathbf{q}' \neq \mathbf{q}} b_{\mathbf{q}'}^{+} b_{\mathbf{q}} c_{\mathbf{k}'}^{+} c_{\mathbf{k}} \delta_{\mathbf{k}'+\mathbf{q}',\mathbf{k}+\mathbf{q}}$$



$g(t) \equiv g_0 \sin^2(t\pi / t_e)$; $b_{\mathbf{q}}^{+}$ ($c_{\mathbf{k}}^{+}$) creates boson (fermion)

- Apply perturbation theory (t_e small)
- Choice of g(t) avoids non-adiabatic effects

$$t_e \gamma_{coll} T / T_F << 1$$

$$\gamma_{coll} \equiv \frac{3}{8} \sigma n_B v_F$$

Discrete Quantum Boltzmann Equation

- Approximate iterative rate equation for the mean occupation numbers: evaporation steps of period t_e

$$N_{n+1}(\boldsymbol{k}) = (1 - \gamma_{loss} t_e) N_n(\boldsymbol{k}) \quad \leftarrow \text{loss}$$

$$+ \sum_{\boldsymbol{k}'} P(\boldsymbol{k}' \to \boldsymbol{k}) N_n(\boldsymbol{k}')[1 - N_n(\boldsymbol{k})] \quad \leftarrow \text{incoming fermions}$$

$$- \sum_{\boldsymbol{k}'} P(\boldsymbol{k} \to \boldsymbol{k}')[1 - N_n(\boldsymbol{k}')] N_n(\boldsymbol{k}) \quad \leftarrow \text{outgoing fermions}$$

$$P(\boldsymbol{k} \to \boldsymbol{k}') \equiv \frac{N_B}{\hbar^2 L^6} |g(\omega)|^2 \ , \ \hbar\omega \equiv \frac{\hbar^2 k'^2}{2m_F} + \frac{\hbar^2 (\boldsymbol{k} - \boldsymbol{k}')^2}{2m_B} - \frac{\hbar^2 k^2}{2m_F}$$

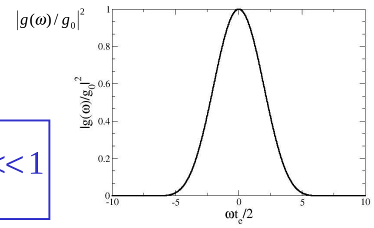
$$g(\omega) \equiv \int_0^{t_e} dt\, g(t) \exp(i\omega t) = \frac{\exp(i\omega t_e / 2) g_0 \sin(t_e \omega / 2)}{\omega [1 - (\omega t_e / 2\pi)^2]}$$

g(ω) is fourier transform of time dependent potential: a coherence window

Understanding the QBE

- Finite system:
  Continuous vs. Discrete
  - "Continuous regime":
    wide coherence window

$$\frac{\hbar}{m_F L^2}\pi t_e = \frac{t_e E_F}{\hbar}\frac{2\pi}{(6\pi N_F)^{2/3}} << 1$$



- Two requirements:

$$\frac{T_F}{T} << \frac{t_e E_F}{\hbar} << \frac{T_F}{T}\frac{E_F}{\gamma_{coll}\hbar}$$

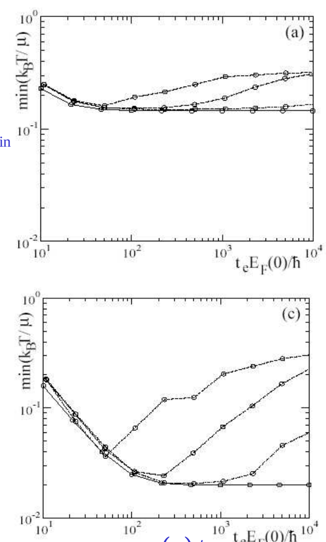
Due to width of coherence window function

Due to limits of perturbation theory

Minimum Temperature: Role of Evaporation Rate

- Minimum temperature as a function of the evaporation time step t_e
  - (a) $\gamma_{loss}/\gamma_{coll} = 8/3\times10^{-2}$
  - (c) $\gamma_{loss}/\gamma_{coll} = 8/3\times10^{-4}$
  - Dashed line: 100 atoms
  - Long dashed line: 1000 atoms
  - Dot-dashed line: 10000 atoms
  - Solid line: thermodynamic limit

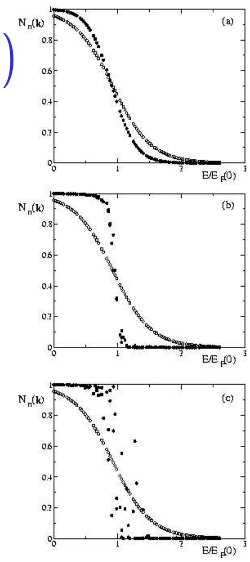
$\left(\frac{k_B T}{\mu}\right)_{min}$



$t_e E_F(0)/\hbar$

Meaning of the Thermodynamic Limit

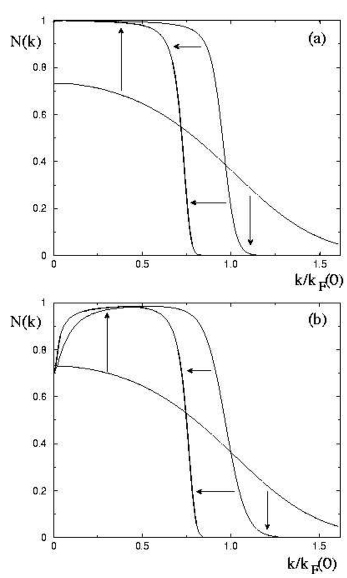
$N_n(\vec{k})$



$E/E_F(0)$

- Occupation number distribution
  - Open circles: initial state
  - Closed circles: final state
  - (a) $t_e E_F/\hbar = 10^2$
    - Continuous regime
  - (b) $t_e E_F/\hbar = 10^3$
  - (c) $t_e E_F/\hbar = 10^4$
    - Discrete regime
- In the continuous regime, the distribution depends only on |**k**|

Evolution of the Number Distribution

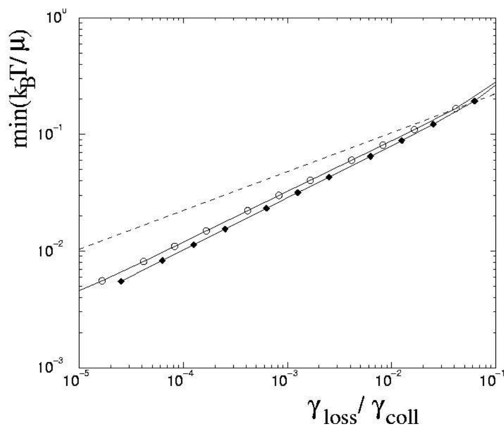


- Time evolution of the number distribution
  - (a) 7Li-Li6 mixture
  - (b) 23Na-6Li mixture
  - Parameters: $(k_B T/\mu)_{\min} \sim 0.08$

    $\gamma_{loss}/\gamma_{coll} = 10^{-2}$
- Three phases:
  - Cooling, quasi-static equilibrium, strong heating
- Hole at k=0 mass dependent

→ Non-thermal distribution!

Minimum Temperature: Role of Loss



- Minimum temperature vs. loss rate
  - Circles: 7Li-6Li
  - Diamonds: 23Na-6Li
  - Dashed line:
    Analytical prediction

→ Power law

$$7\text{Li - 6Li}: \ (k_B T/\mu)_{\min} = 0.66 \times (\gamma_{loss}/\gamma_{coll})^{0.44} + 5.6 \times 10^{-4}$$

$$23\text{Na - 6Li}: \ (k_B T/\mu)_{\min} = 0.62 \times (\gamma_{loss}/\gamma_{coll})^{0.44} + 7.3 \times 10^{-8}$$

Results Not Shown

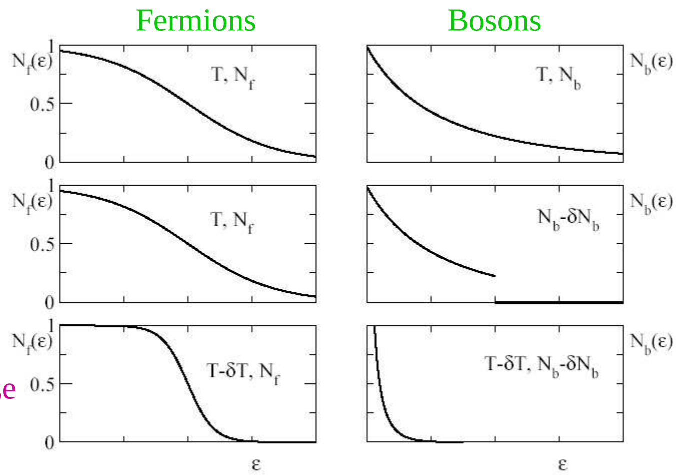
- Analytical Model
  - Fermi ansatz, continuous regime
  - Find exponent in power law of 1/3
- Discrete Quantum Boltzman *Master* equation
  - Monte Carlo simulations
  - No assumption of thermal equilibrium
  - Find exponent in power law of 0.51
    - NB: Contrast to 0.44 from Discrete Quantum Boltzmann Equation

Cooling Scheme II: Heat Capacity in Sympathetic Cooling

- Why (did) experiments seem limited to T/T_F~0.2?
- Explained by heat capacity:
  - Fermions (f) cooled by bosons (b)
  - Initial temperature: T_f and T_b
  - Conservation of energy: $T = (C_f T_f + C_b T_b)/(C_f + C_b)$
    $C_f \equiv \partial_T E_f \quad C_b = \partial_T E_b$
  - To cool need $T_b \ll T_f$ and $C_b \gg C_f$
- What is the real role of specific heat?
  - Evaporation
  - Bose statistics

Basic Model: Evaporation in Sympathetic Cooling

- Initial state
- Evaporate
- Rethermalize



Contrast to previous approach: mutual equilibrium, harmonic trap

Outline of General Approach

- Density of states: $\rho_{f,b}(\epsilon) = A_{f,b}\epsilon^{\delta+1/2}$
- Mean energy of evaporated particle:

$$\delta N_b = \int_{\eta k_B T}^{\infty} d\epsilon\, \rho_b(\epsilon) N_b(\epsilon)$$

$$\frac{\delta E_b}{\delta N_b} = f(\eta) k_B T \qquad \delta E_b = \int_{\eta k_B T}^{\infty} d\epsilon\, \rho_b(\epsilon) N_b(\epsilon)\epsilon$$

- Conservation of energy

$$E_f(N_f, T) + E_b(N_b, T) - \delta E_b = E_f(N_f, T - \delta T) + E_b(N_b - \delta N_b, T - \delta T)\,.$$

- Expand around small changes in T, N_b

$$\boxed{\frac{\delta T}{\delta N_b} = \frac{f(\eta)\, k_B T - \frac{\partial E_b}{\partial N_b}(N_b, T)}{C_f(N_f, T) + C_b(N_b, T)}} \qquad \begin{aligned} C_f &\equiv \partial_T E_f \\ C_b &= \partial_T E_b \end{aligned}$$

Cooling by an Ideal Bose Gas Below T_c

- The coolant energy does not depend on N_b

$$E_b(N_b, T) = \frac{\nu A_b}{\delta + 5/2}(k_B T)^{\delta+5/2}$$

- Number of bosons required to cool fermions

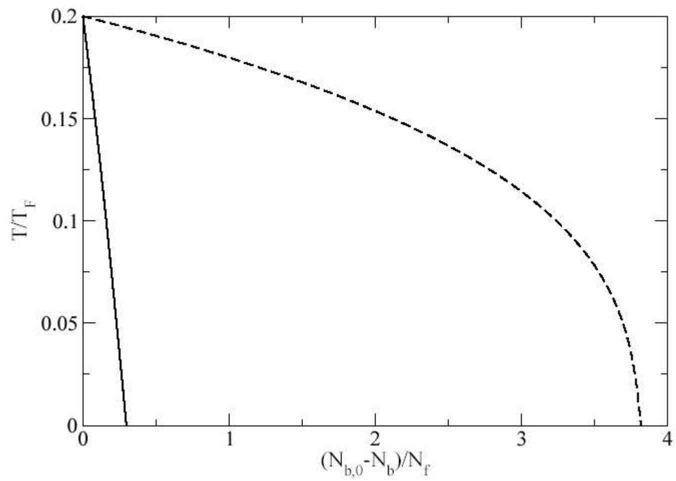
$$\frac{\delta T}{\delta N_b} = \frac{f(\eta)\, T_F}{2\alpha N_f \left[1 + \frac{3\nu}{\pi^2}\frac{A_b}{A_f}\left(\frac{T}{T_F}\right)^{\delta+1/2}\right]}$$

- Ratio of heat capacities

$$\frac{C_b}{C_f} = \frac{3\nu}{\pi^2}\frac{A_b}{A_f}\left(\frac{T}{T_F}\right)^{\delta+1/2}$$

- →Favorable to have $\boxed{C_b \ll C_f}$

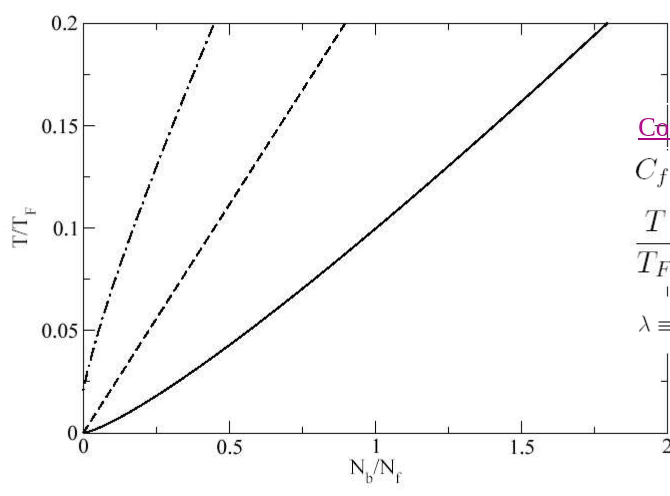
Bose Coolant II



$$N_{b,0} > \Delta N_b = \frac{C_{f,0}}{k_B f(\eta)} \left[1 + \frac{1}{\delta + 3/2} \frac{C_{b,0}}{C_{f,0}}\right]$$

Cooling by a Boltzmann Gas

- Key difference is energy: $E_b(N_b, T) = (\delta + 3/2) N_b k_B T$



Condition to cool to T=0

$$C_{f,0} < C_{b,0}(\lambda - 1)$$

$$\frac{T}{T_F} \propto \left(\frac{N_b}{N_f}\right)^{\lambda}$$

$$\lambda \equiv \frac{f(\eta) - (\delta + 3/2)}{\delta + 3/2} > 0$$

$$C_{b,0} >> C_{f,0}$$

Cooling Scheme III: Entropic cooling
The Basic Idea

- How does one get low enough temperatures to achieve a BCS transition for weak interactions?
  - $T_{BCS}/T_F = 0.277\exp(-\pi/2k_F|a|)$ ← Small parameter ($k_F|a|$)
- Our solution:
  - (1) Note that, for degenerate quantum gases
    - $S_{fermions} \sim T \;,\; S_{bosons} \sim T^3$
  - (2) Achieve molecular BEC (done!)
  - (3) Adiabatically dissociate bosonic molecules ← $a > 0$
    into their fermionic consituent atoms ← $a < 0$
  - (4) Adiabaticity implies entropy held constant
  - (5) Entropic advantage leads to cooling

Entropy of Interacting Fermions and Bosons

- Grand potential + Fermi statistics
  + Density of states + Semi-classical approx.
  - Fermion Entropy

$$S_F = k_B N \pi^2 \frac{T}{T_F}\left(1 + \frac{64}{35\pi^2} k_F a\right)$$

- Grand potential + Bose statistics + Boguliubov energy + Thomas-Fermi profile + Density of states
  - Boson Entropy

$$S_B \cong k_B N_{mol}\left(\frac{T}{T_{BEC}}\right)^3\left(\frac{2\pi^4}{45\zeta(3)} + 3\frac{\mu_{mol}}{k_B T}\right)$$

Final Temperature

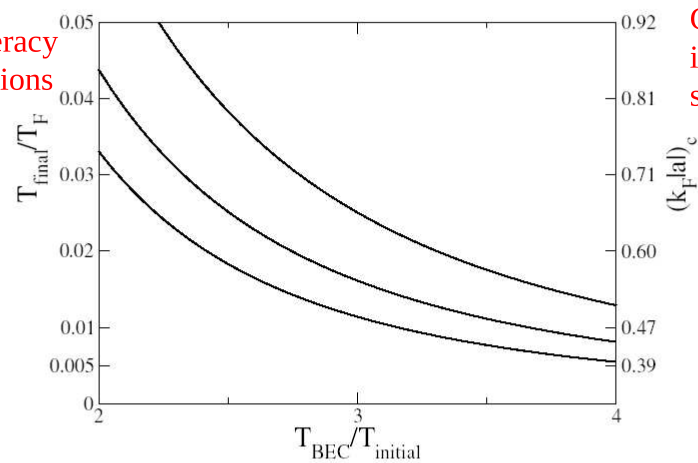
- Set entropies equal and solve for temperature
  - $S_B = S_F$

$$\Rightarrow \left(\frac{T_{final}}{T_F}\right) \cong \left(\frac{\pi^2}{45\zeta(3)} + \frac{3}{2\pi^2}\frac{\mu_{mol}}{k_B T}\right)\left(\frac{T_{initial}}{T_{BEC}}\right)^3$$

  - For example, $\mu \cong k_B T \Rightarrow$
    - $T_{initial} = T_{BEC}/2 \Rightarrow T_{final} = 5\times 10^{-2}\, T_F$
    - $T_{initial} = T_{BEC}/4 \Rightarrow T_{final} = 5\times 10^{-3}\, T_F$
  - Recall the semiclassical BCS transition temperature
    - $T_{BCS}/T_F = 0.277 \exp(-\pi/2k_F|a|)$

Results of adiabatic switching

Final Degeneracy of fermions



Critical interaction strength

Initial Degeneracy of molecular BEC

Answers to Practical Questions

- Can it be adiabatic? $\gamma_{eff} t >> 1$
  - Molecules (a>0) $\gamma_{eff}^{mol} = \gamma_{coll}^{mol} N_T / N_{mol}$
  - Atoms (a<0) $\gamma_{eff}^{atom} = \gamma_{coll}^{atom} (T/T_F)^2$
  - Experimental parameters ➔ t ~ 300 ms
- What about an excess of one spin state and Fulde-Ferrell-Larkin-Ovchinnikov phases?
  - Remove leftover atoms on molecular side
- Can adiabaticity beat heating due to loss?
  - $\left[\gamma_{loss} / \left(2\pi^2 \gamma_{coll}\right)\right]^{1/4} \leq T_{final} / T_F \sim 10^{-2}$

Conclusion: How to Get to a Weakly Interacting BCS Transition

- I. Minimize the loss rate w.r.t. the collision rate and try out of equilibrium scheme
  - Power law: $(k_B T / \mu)_{\min} \sim 0.66 \times (\gamma_{loss} / \gamma_{coll})^{\alpha}, \ \alpha = 0.33, 0.44, 0.5$
  - Typical parameters: $\gamma_{loss} / \gamma_{coll} = 10^{-3} \Rightarrow (k_B T / \mu)_{\min} \sim 0.03$
- II. *Minimize* the specific heat of the coolant
  - In an ideal model, can cool to T=0
- III. Take advantage of statistics in BEC-BCS
  - Temperatures T ~ 10^{-2} to 5 x 10^{-3} T_F
  - Interaction strength k_F |a| ~ 1/3 or ½

Where Did This Work Go?

- Idea #3 implemented in 3 experiments
  - Innsbruck: Bartenstein *et al.*, PRL 92 120401 (2004).
  - JILA: Regal *et al.*, PRL 92 040403 (2004).
  - MIT: Zwierlein *et al.*, PRL 92 120403 (2004).

- Published as
  - I. Carr, Bourdel, and Castin, PRA **69** 033603 (2004).
  - II. Carr and Castin, PRA **92** 043611 (2004).
  - III. Carr, Shlyapnikov, and Castin, PRL **92** 150404 (2004).