Fermionic atom optics

Chris Search (→ Stevens Inst. Of Technology)
Han Pu (→ Rice U.)
Weiping Zhang (→ Tsinghua U.)
Takahiko Miyakawa

Dennis Douglas
Ben Kalafut
Dominic Meiser

Mike Moore (→ Ohio U.)

ARO NASA NSF ONR

Optical Sciences Center
The University of Arizona

KIT P 2004 1

Why atom interferometers ?

- Sagnac effect (atoms): $\Delta\Phi_{\text{Sagnac}} = \left(\frac{2M}{\eta}\right)\Omega A$
- Photons are massless particles: $\frac{\eta^2 k^2}{2M} \rightarrow \eta\omega = \eta k c$

$$M \rightarrow \frac{\eta k}{2c} = \frac{\eta\omega}{2c^2}$$

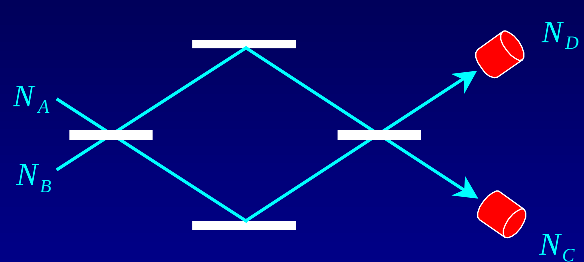
- Sagnac effect (photons): $\Delta\Phi_{\text{Sagnac}} \rightarrow \left(\frac{2\pi}{\lambda c}\right)\Omega A$

$$\text{sensitivity}\left(\frac{\text{atoms}}{\text{photons}}\right) \propto \frac{Mc^2}{\eta\omega} \approx 10^{11}$$

KITP 2004

3

A. Atom Interferometer—Bosons versus fermions

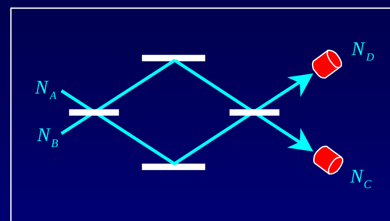


- Mach-Zehnder interferometer
- Quantum degenerate Bose or Fermi beam
- Collisions
- Pauli exclusion principle

KITP 2004

4

Phase fluctuations



Difference in number of counts:

$$\hat{N} = \hat{N}_D - \hat{N}_C$$

$$\langle \hat{N} \rangle = \langle \hat{N}_A - \hat{N}_B \rangle \cos\theta$$ (uncorrelated inputs, noninteracting atoms)

Fluctuations:

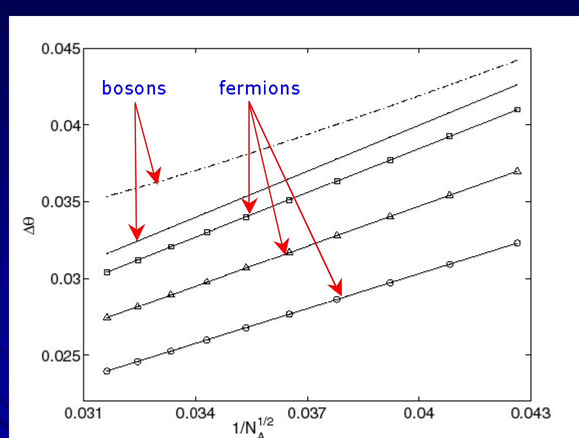
$$\Delta\theta = \frac{\Delta N}{\left| \partial \langle \hat{N} \rangle / \partial\theta \right|}$$

$$\Delta N = \sqrt{\langle N^2 \rangle - \langle N \rangle^2}$$

KITP 2004

See, e.g., M. O. Scully & J. P. Dowling, PRA **48**, 3186 (1993) 5

Bosons versus fermions



$\beta = 0.001$

$\beta = 0$

$\bar{n}_A = 1$

$\bar{n}_A = 2$

$\bar{n}_A = 3$

$$\bar{n}_A = \frac{2\pi n_A}{k_0} = \frac{2k_{F,A}}{k_0}$$

(Degree of monochromaticity)

KITP 2004

C. P. Search & PM, Phys. Rev. A (Rapid Commun.) **67**, 061601(R) (2003) 6

Experiment

VOLUME 92, NUMBER 23 PHYSICAL REVIEW LETTERS week ending 11 JUNE 2004

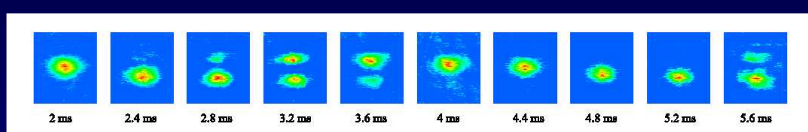
Atom Interferometry with Trapped Fermi Gases

G. Roati, E. de Mirandes, F. Ferlaino, H. Ott, G. Modugno, and M. Inguscio
LENS and Dipartimento di Fisica, Università di Firenze, and INFM, Via Nello Carrara 1, 50019 Sesto Fiorentino, Italy
(Received 11 February 2004; published 8 June 2004)

We realize an interferometer with an atomic Fermi gas trapped in an optical lattice under the influence of gravity. The single-particle interference between the eigenstates of the lattice results in macroscopic Bloch oscillations of the sample. The absence of interactions between fermions allows a time-resolved study of many periods of the oscillations, leading to a sensitive determination of the acceleration of gravity. The experiment proves the superiority of noninteracting fermions with respect to bosons for precision interferometry and offers a way for the measurement of forces with microscopic spatial resolution.

KITP 2004 7

Experiment (cntd)



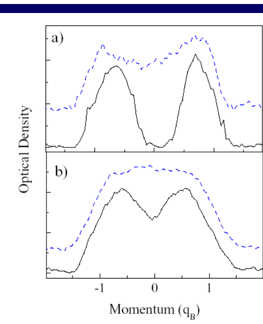


FIG. 4: a) Momentum distribution of fermions at two different holding times in the lattice: 1 ms (continuous line) and 252 ms (dashed line). b) Momentum distribution of bosons at 0.6 ms (continuous line) and 3.8 ms (dashed line). The much faster broadening for bosons is due to the presence of interactions.

KITP 2004 8

Nonlinear atom optics (bosons)

Bosons: $\left[\hat{\psi}(\mathbf{r},t),\hat{\psi}^{\dagger}(\mathbf{r},t)\right]=\delta(\mathbf{r}-\mathbf{r}')$

$$\hat{\psi}(\mathbf{r},t) \Rightarrow \langle\hat{\psi}(\mathbf{r},t)\rangle \equiv \phi(\mathbf{r},t)$$

Then:

$$i\hbar\frac{d\hat{\psi}(r,t)}{dt}=H_0\hat{\psi}(r,t)+\left(\frac{4\pi\hbar^2 a}{M}\right)\hat{\psi}^{\dagger}(r,t)\hat{\psi}(r,t)\hat{\psi}(r,t)$$

$$i\hbar\frac{d\phi(r,t)}{dt}=H_0\phi(r,t)+\left(\frac{4\pi\hbar^2 a}{M}\right)|\phi(r,t)|^2\phi(r,t)$$

Nonlinear atom optics !

KITP 2004

G. Lenz, PM, and E. M. Wright, PRL **71**, 3271 (1993).

9

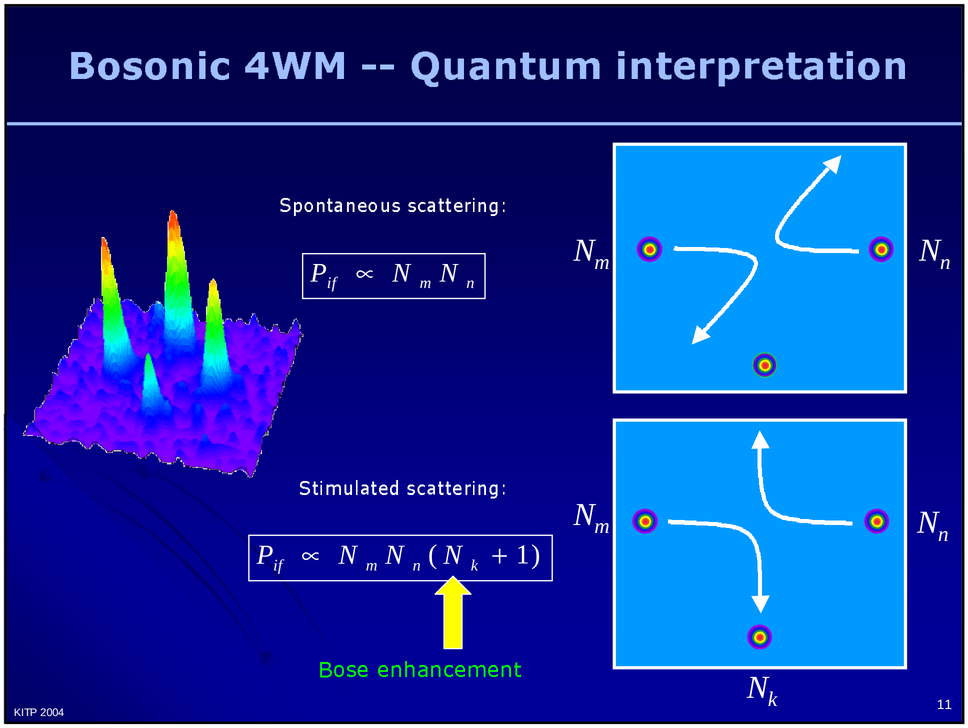
Examples

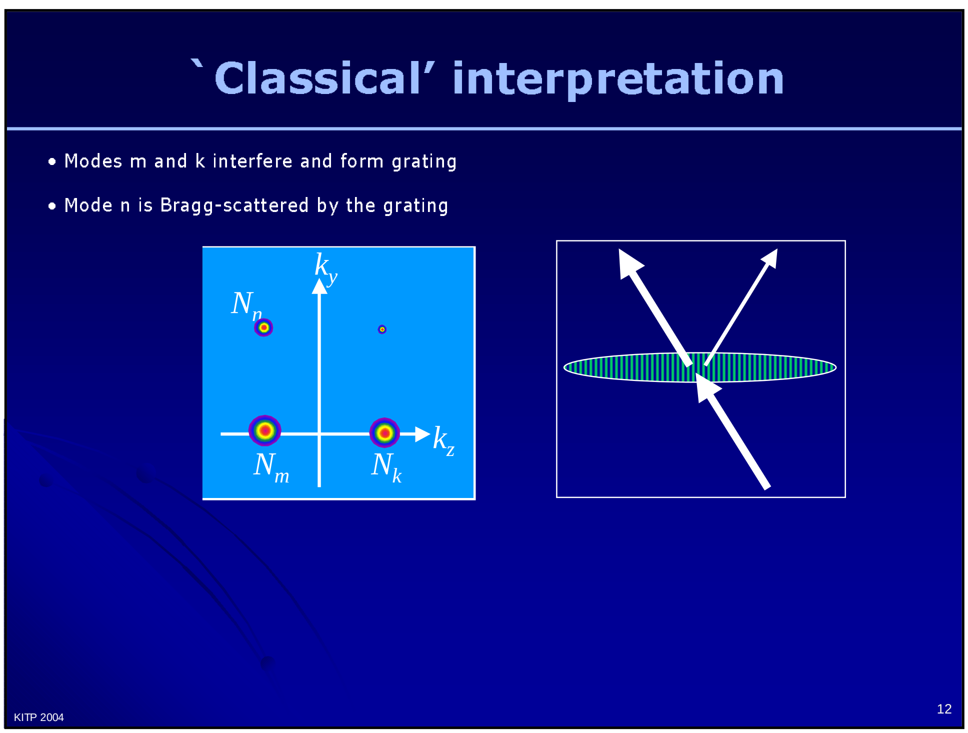
- Four-wave mixing of matter waves
- Matter-wave phase conjugation
- Atom holography
- Atomic solitons
- Second-harmonic generation
- Atom lasers
- Atom amplifiers
- Matter-wave superradiance
- Mixing of optical and matter waves
- Nonclassical and entangled states
- Coherence control
- Quantum information
- Sensors

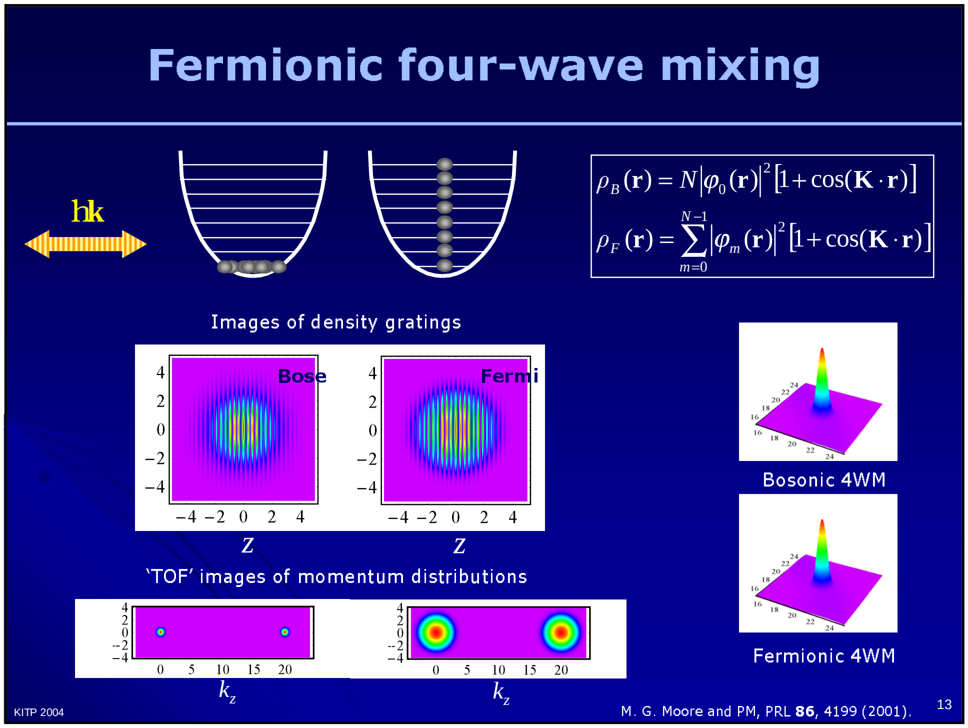
- Fermionic matter waves
- Fermi-Bose wave mixing
- Coherent molecular fields
- ...

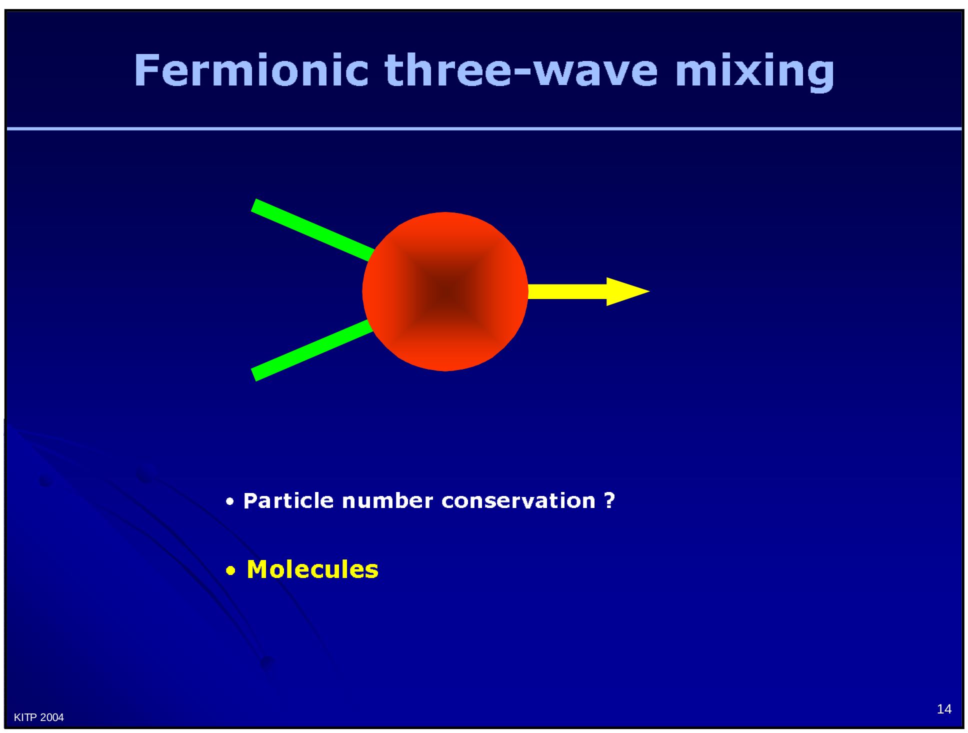
KITP 2004

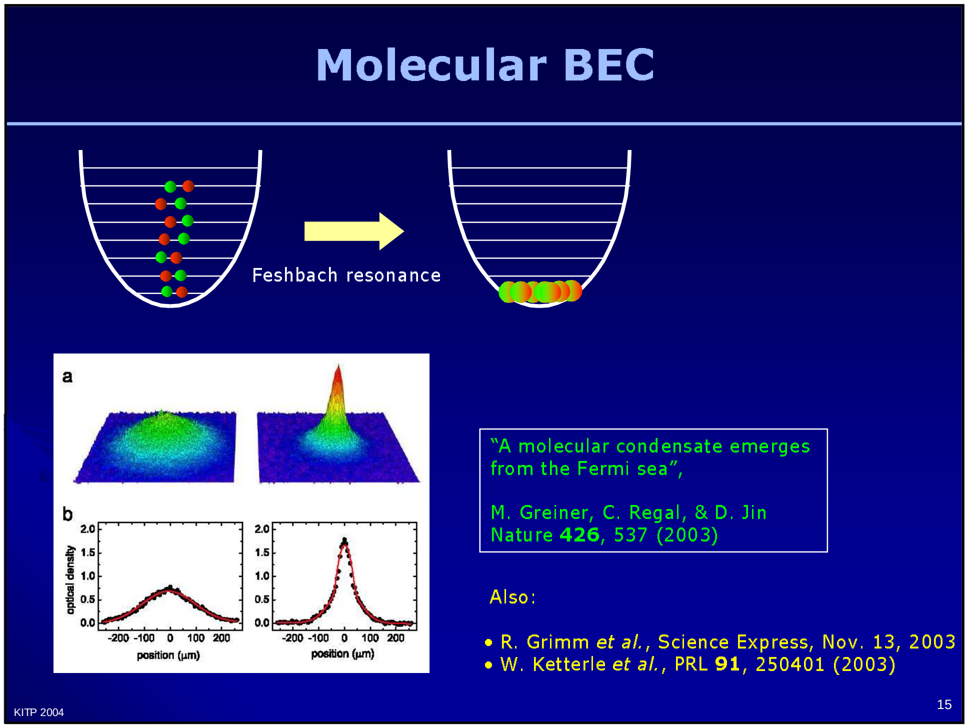
10

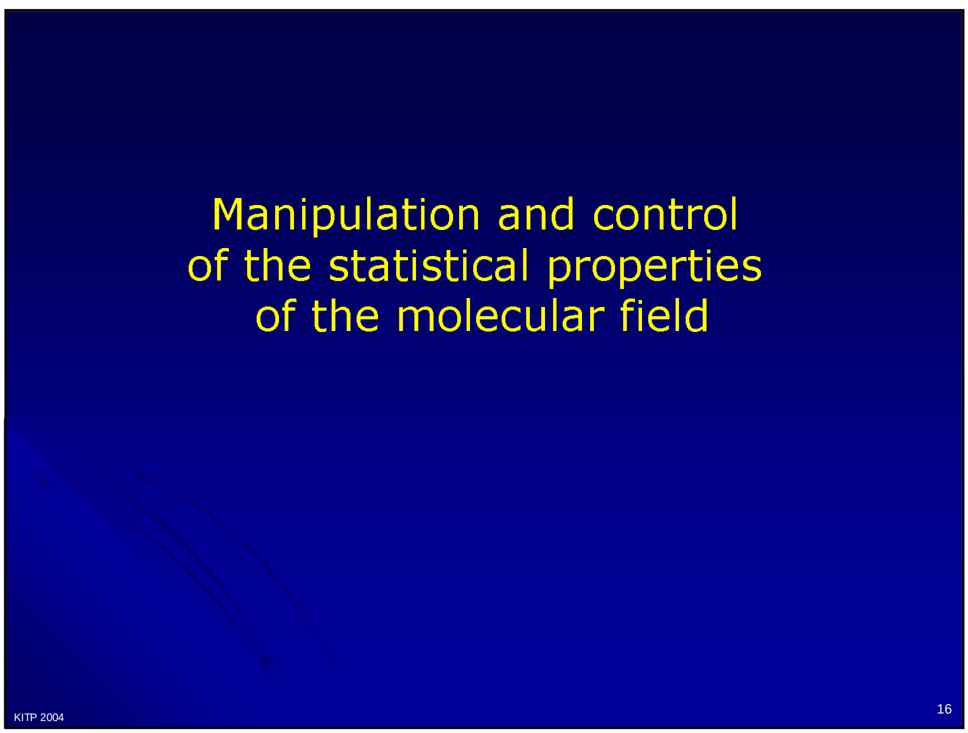

Bosonic 4WM -- Quantum interpretation
Spontaneous scattering:
$P_{if} \propto N_m N_n$
Stimulated scattering:
$P_{if} \propto N_m N_n (N_k + 1)$
Bose enhancement
N_m
N_n
N_k
KITP 2004
11


`Classical' interpretation
• Modes m and k interfere and form grating
• Mode n is Bragg-scattered by the grating
k_y
k_z
N_n
N_m
N_k
KITP 2004
12


Fermionic four-wave mixing
ħk
$\rho_B(\mathbf{r}) = N|\varphi_0(\mathbf{r})|^2[1+\cos(\mathbf{K}\cdot\mathbf{r})]$
$\rho_F(\mathbf{r}) = \sum_{m=0}^{N-1}|\varphi_m(\mathbf{r})|^2[1+\cos(\mathbf{K}\cdot\mathbf{r})]$
Images of density gratings
Bose
Fermi
z
z
'TOF' images of momentum distributions
k_z
k_z
Bosonic 4WM
Fermionic 4WM
KITP 2004
M. G. Moore and PM, PRL **86**, 4199 (2001).
13


Fermionic three-wave mixing
• Particle number conservation ?
• Molecules
KITP 2004
14

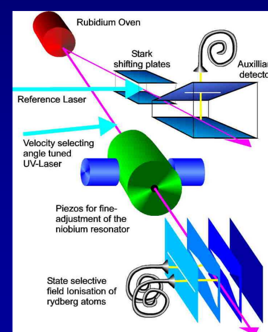

Molecular BEC
Feshbach resonance
a
b
optical density
position (μm)
"A molecular condensate emerges from the Fermi sea",
M. Greiner, C. Regal, & D. Jin
Nature **426**, 537 (2003)
Also:
• R. Grimm *et al.*, Science Express, Nov. 13, 2003
• W. Ketterle *et al.*, PRL **91**, 250401 (2003)
KITP 2004
15


Manipulation and control
of the statistical properties
of the molecular field
KITP 2004
16

Micromaser

- Single-mode lasers far above threshold produce coherent state.
- Micromasers can produce nonclassical states, Fock states, squeezed states, etc…
- Micromasers undergo a series of first-order-like phase transitions past the usual laser threshold.

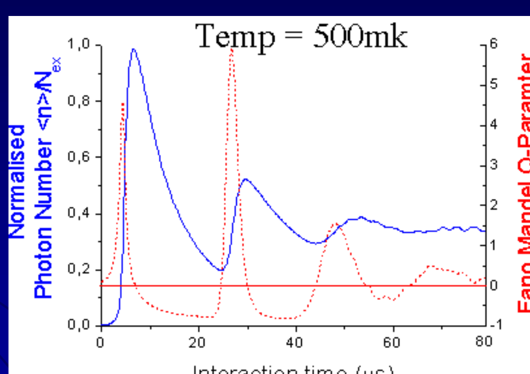
- **Matter-wave analog of a micromaser?**
- **Cavity matter-wave optics?**



H. Walther *et al.*, MPQ Garching

KITP 2004 17

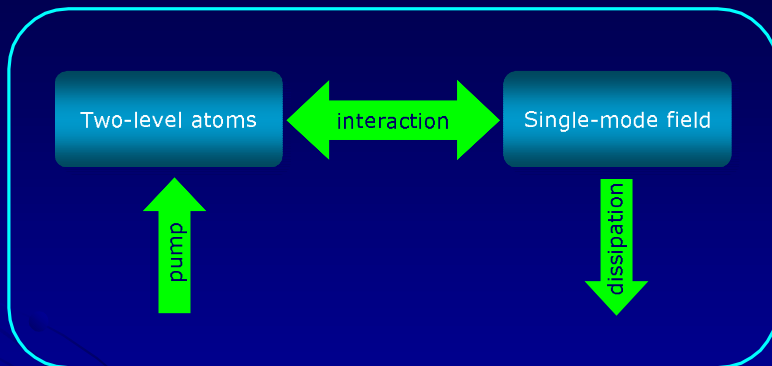
Micromaser photon statistics



H. Walther *et al.*, Max-Planck Institute for Quantum Optics

KITP 2004 18

Micromaser theory



Interaction: Jaynes-Cummings Hamiltonian

$$H = \hbar\omega_0 \sigma_z + \hbar\omega a^\dagger a + \hbar g\left(a\sigma_+ + a^\dagger \sigma_-\right)$$

KITP 2004

19

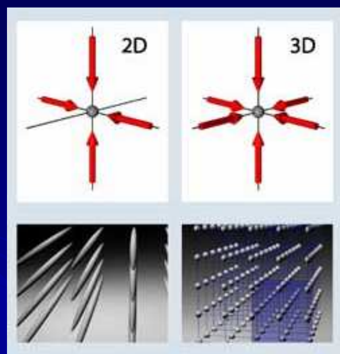
Is the analog of a micromaser for a molecular field possible?

- Generation of nonclassical states of molecular field (Fock states, squeezed states, etc…)
- Dissipative "quantum phase transitions"
- Quantum tunneling
- …

KITP 2004

20

1. "Resonator": Optical lattice



Number of atoms per site:

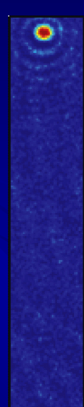
- 1D – several thousands
- 2D – several tens
- 3D – a few

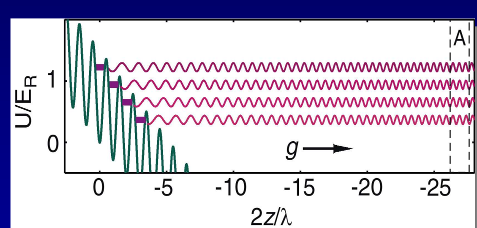
KITP 2004 21

First lattice experiment: tunnel array

Under appropriate conditions, atoms tunnel from lattice sites to the continuum.

Energy: $\eta\omega_0 + Mg\Delta z$



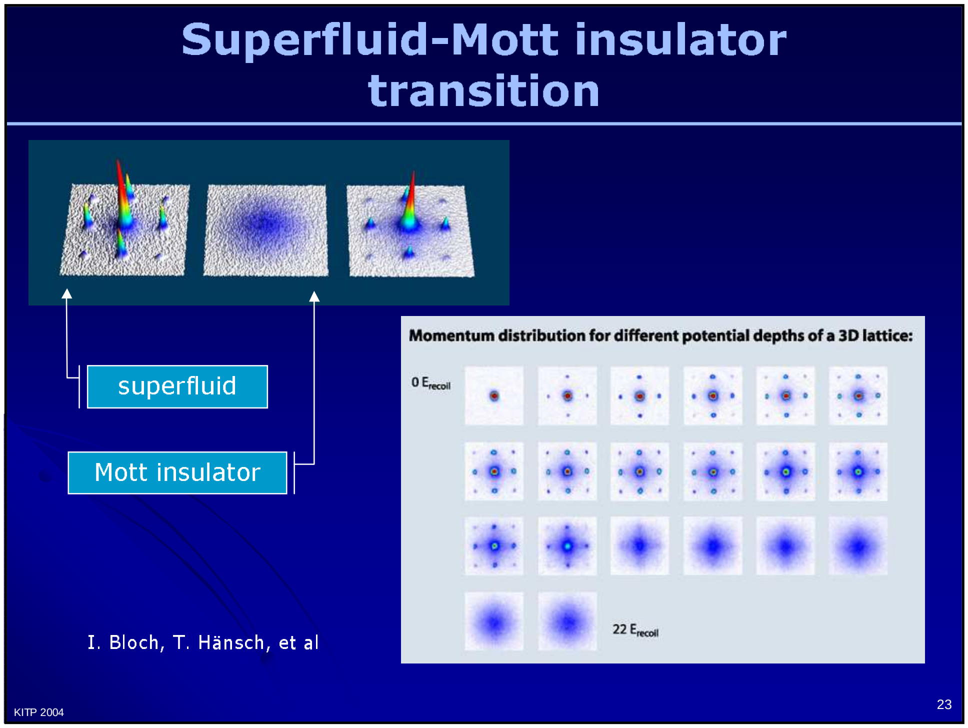


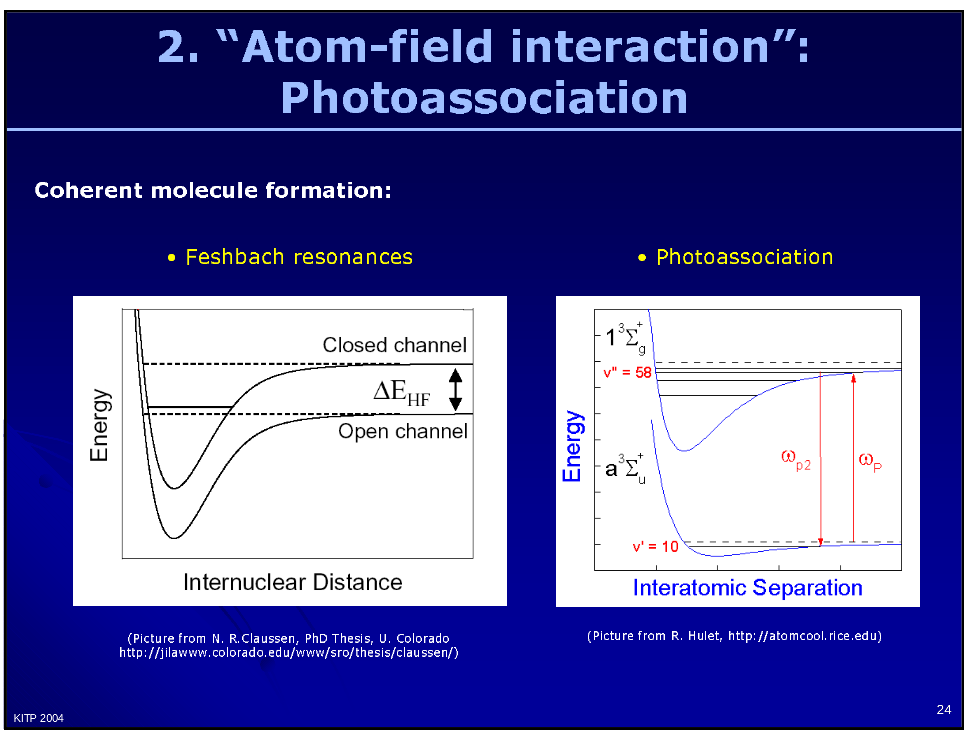
Result:

Periodic constructive interference of all de Broglie waves, mode-locked atom laser.

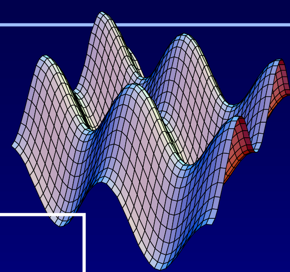
(Courtesy Brian Anderson)

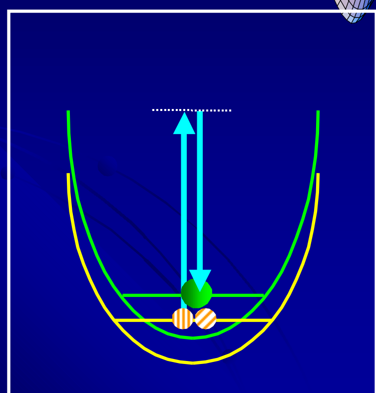
KITP 2004 22


Superfluid-Mott insulator transition
superfluid
Mott insulator
Momentum distribution for different potential depths of a 3D lattice:
0 E_recoil
22 E_recoil
I. Bloch, T. Hänsch, et al
KITP 2004
23


2. "Atom-field interaction": Photoassociation
Coherent molecule formation:
• Feshbach resonances
• Photoassociation
Energy
Closed channel
ΔE_HF
Open channel
Internuclear Distance
Energy
1³Σg+
v" = 58
a³Σu+
ωp2
ωp
v' = 10
Interatomic Separation
(Picture from N. R.Claussen, PhD Thesis, U. Colorado http://jilawww.colorado.edu/www/sro/thesis/claussen/)
(Picture from R. Hulet, http://atomcool.rice.edu)
KITP 2004
24

Photoassociation





Site *i* :

Two atomic fermions in Wannier ground state

$\longrightarrow$

One bosonic molecule in Wannier ground state

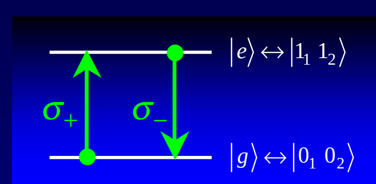
$$H = \eta\chi(t)\hat{b}_i^{\dagger}\hat{c}_{1,i}\hat{c}_{2,i} + H.c.$$

KITP 2004

25

Mapping

$$\sigma_- = \hat{c}_1\hat{c}_2$$
$$\sigma_+ = \hat{c}_2^{\dagger}\hat{c}_1^{\dagger}$$
$$\sigma_z = \hat{c}_1^{\dagger}\hat{c}_1 + \hat{c}_2^{\dagger}\hat{c}_2 - 1$$



- Molecules in Mott insulator state
- Neglect atomic intersite tunneling during photoassociation

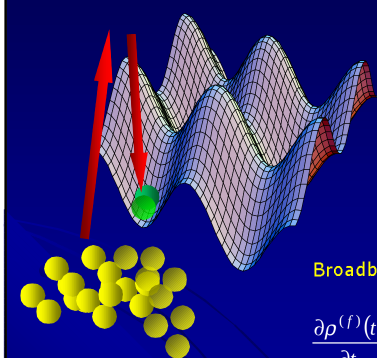
$$H_P = \hbar(\omega_b + U_x)\hat{n}_b + \hbar(\omega_f + U_x\hat{n}_b)\sigma_z + \frac{\hbar}{2}U_b\hat{n}_b(\hat{n}_b - 1)$$
$$+\hbar\left[\chi(t)\hat{b}^{\dagger}\sigma_- + \chi^*(t)\hat{b}\sigma_+\right]$$

Reduces to Jaynes-Cummings model for U_x and $U_b \to 0$

KITP 2004

26

3. "Pump": Raman transition



Two-photon Raman transition from untrapped states to trapped states of fermions

Broadband continuum of reservoir states (simplest case):

$$\left.\frac{\partial \rho^{(f)}(t)}{\partial t}\right|_{\text{pump}} = -\frac{\Gamma}{2}\sum_{\sigma=1,2}\left[\hat{c}_\sigma \hat{c}_\sigma^\dagger \rho^{(f)} - 2\hat{c}_\sigma^\dagger \rho^{(f)} \hat{c}_\sigma + \rho^{(f)} \hat{c}_\sigma \hat{c}_\sigma^\dagger\right]$$

$\Gamma t >> 1 \Rightarrow$ fermions are in state $|e\rangle = \hat{c}_2^\dagger \hat{c}_1^\dagger |0\rangle$ with unit probability

(Γ : pumping rate)

KITP 2004 27

4. Loss mechanisms

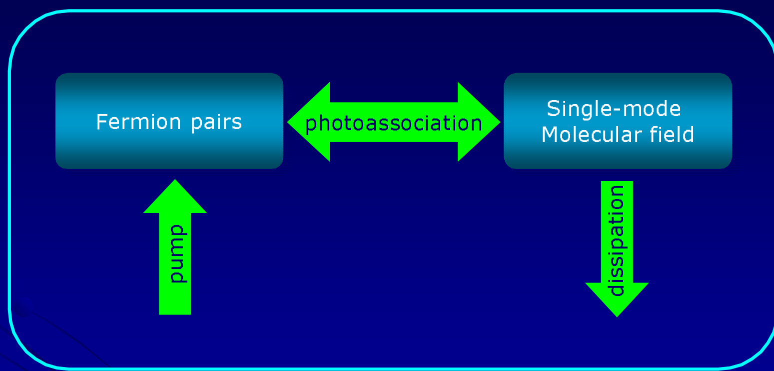
- Three-body inelastic collisions
- Spontaneous emission
- ...

Phenomenological master equation

$$\left.\frac{\partial \rho^{(b)}(t)}{\partial t}\right|_{\text{loss}} = -\frac{\gamma}{2}\sum_{\sigma=1,2}\left[\hat{b}^\dagger \hat{b} \rho^{(b)} - 2\hat{b}\rho^{(b)}\hat{b}^\dagger + \rho^{(b)}\hat{b}^\dagger \hat{b}\right] \equiv \mathcal{L}\left[\rho^{(b)}\right]$$

KITP 2004 28

Micromaser-like model



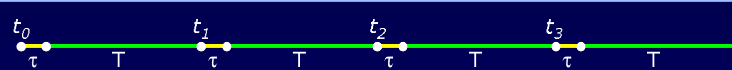
Photoassociation: Jaynes-Cummings Hamiltonian

$$H_P = \hbar\omega_0 \sigma_z + \hbar\omega a^\dagger a + \hbar g\left(a\sigma_+ + a^\dagger\sigma_-\right)$$ + nonlinear terms due to collisions

KITP 2004

29

Solution



- Assume that photoassociation is only for times interval τ short compared to decay time of molecular field

- During photoassociation intervals:

$$\rho^{(b)}(t_\lambda + \tau) = \mathrm{Tr}_{\mathrm{atoms}}\left[e^{-iH_P\tau}\rho(t_\lambda)e^{iH_P\tau}\right] \equiv F(\tau)[\rho(t_\lambda)]$$

- Between photoassociation intervals:

$$\mathcal{L}\left[\rho^{(b)}\right]$$

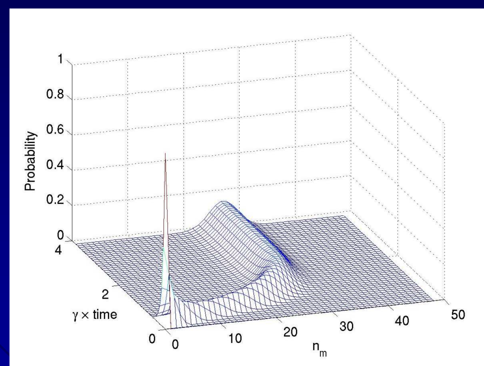
- **Return map description:**

$$\boxed{\rho^{(b)}(t_{\lambda+1}) = \exp(\mathcal{L}T)\mathcal{F}(\tau)\rho^{(b)}(t_\lambda)}$$

KITP 2004

30

Build-up of molecular field



KITP 2004

31

Mean molecule number

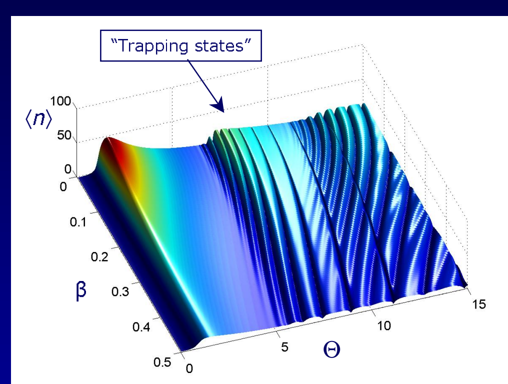
Steady state:

$$p_n = p_0 \prod_{\lambda=1}^{n} \frac{C_\lambda(\tau)}{\gamma\tau\lambda}$$

$$C_\lambda(\tau) = \frac{4|\chi|^2\lambda}{R_{\lambda-1}^2}\sin^2(R_{\lambda-1}\tau/2)$$

$$R_\lambda = \sqrt{[2\omega_f - \omega_b + (2U_x - U_b)\lambda]^2 + 4|\chi|^2(\lambda+1)}$$

$$N_{ex} = 1/\gamma T$$
$$\beta = (2U_x - U_b)/2|\chi|$$
$$\eta = (2\omega_f - \omega_b)/2|\chi|$$



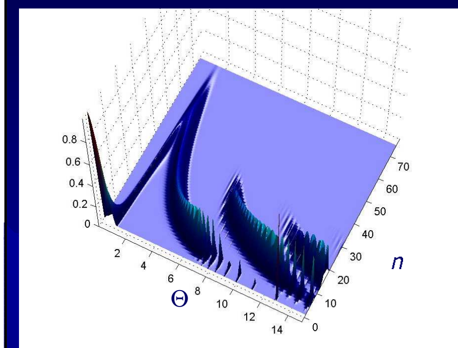
Average number of molecules versus $\theta = N_{ex}^{1/2}\chi\tau$ and β for N_{ex}=50 and η=0

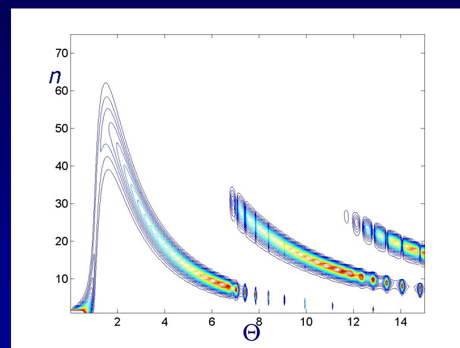
KITP 2004

C. P. Search, W. Zhang, and PM, PRL **91**, 190401 (2003).

32

Molecule statistics





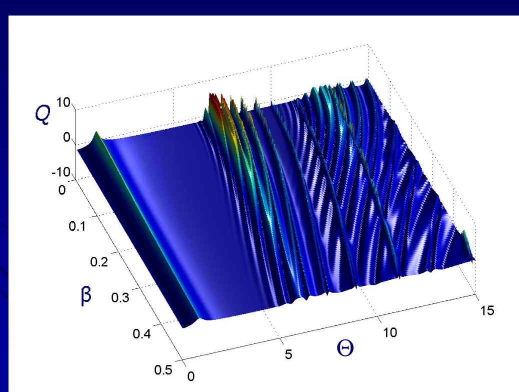
Molecule statistics as a function of Θ for $\beta = \eta = 0$ and $N_{ex} = 50$

KITP 2004 33

Mandel *Q*-parameter

$$Q = \frac{\langle \hat{n}_b^2 \rangle - \langle \hat{n}_b \rangle^2}{\langle \hat{n}_b \rangle} - 1$$

- $Q > 0$: Classical field
- $Q = 0$: Coherent field
- $Q < 0$: Nonclassical field



$N_{ex} = 1/\gamma T$

$\beta = (2U_x - U_b)/2|\chi|$

$\eta = (2\omega_f - \omega_b)/2|\chi|$

Q-parameter of the molecular field versus $\theta = N_{ex}^{1/2}\chi\tau$ and β for N_{ex}=50, η=0.

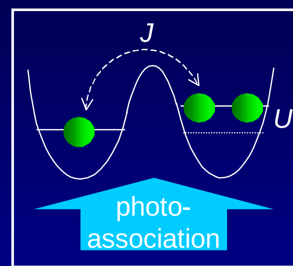
KITP 2004 34

Double-well system

Hamiltonian for molecular field

$$H_m = -J\left(b_l^\dagger b_r + b_r^\dagger b_l\right) + \frac{U}{4}\left(b_l^\dagger b_l - b_r^\dagger b_r\right)^2$$

J : Tunneling coupling
enhances coherence between wells
U : Two-body collisions
suppress coherence between wells



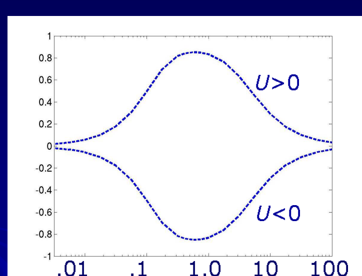
Focus:

- Coherence/Correlation properties of steady state
- Buildup of relative phase coherence

KITP 2004 35

Steady-state coherence and correlations

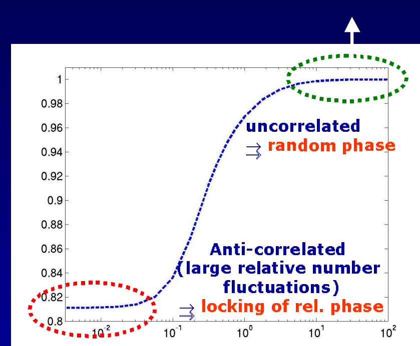
- First-order coherence



$$G_1 \propto b_l^\dagger b_r + b_r^\dagger b_l = b_s^\dagger b_s - b_a^\dagger b_a$$

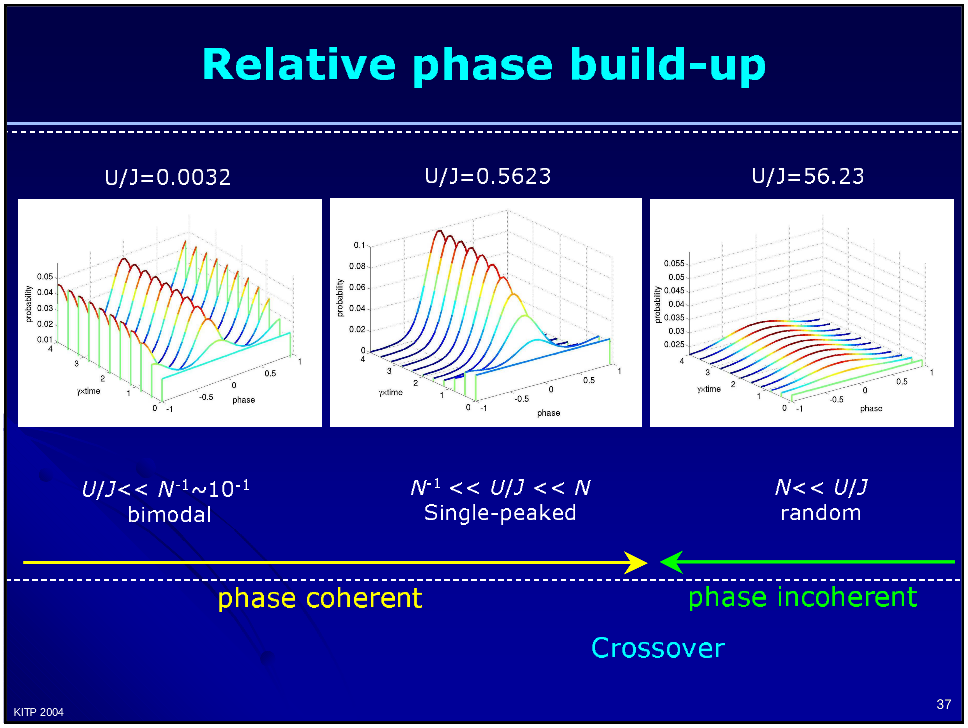
symmetric (0)/anti-symmetric (π)

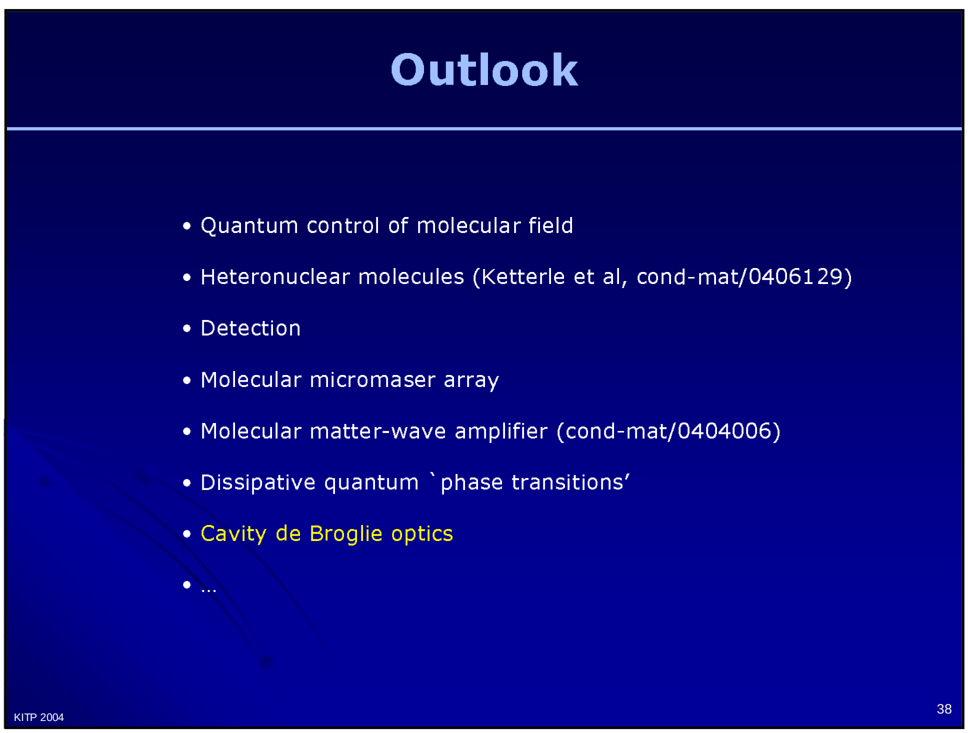
- Second-order correlation



$N_{ex} = 1/\gamma T = 10,\ \Theta = \sqrt{N_{ex}}\,|\chi|\,\tau = \pi,\ J/\gamma = 2.5$

KITP 2004 36


Relative phase build-up
U/J=0.0032
U/J=0.5623
U/J=56.23
U/J<< N-1~10-1
bimodal
N-1 << U/J << N
Single-peaked
N<< U/J
random
phase coherent
phase incoherent
Crossover
KITP 2004
37


Outlook
• Quantum control of molecular field
• Heteronuclear molecules (Ketterle et al, cond-mat/0406129)
• Detection
• Molecular micromaser array
• Molecular matter-wave amplifier (cond-mat/0404006)
• Dissipative quantum `phase transitions'
• Cavity de Broglie optics
• …
KITP 2004
38