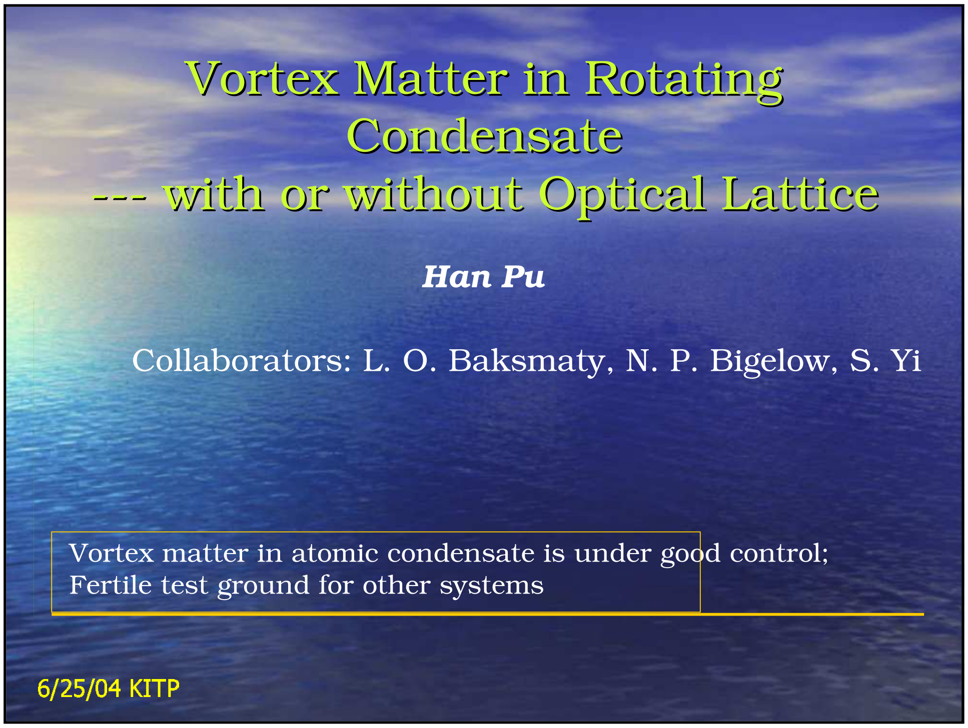
Vortex Matter in Rotating Condensate
--- with or without Optical Lattice
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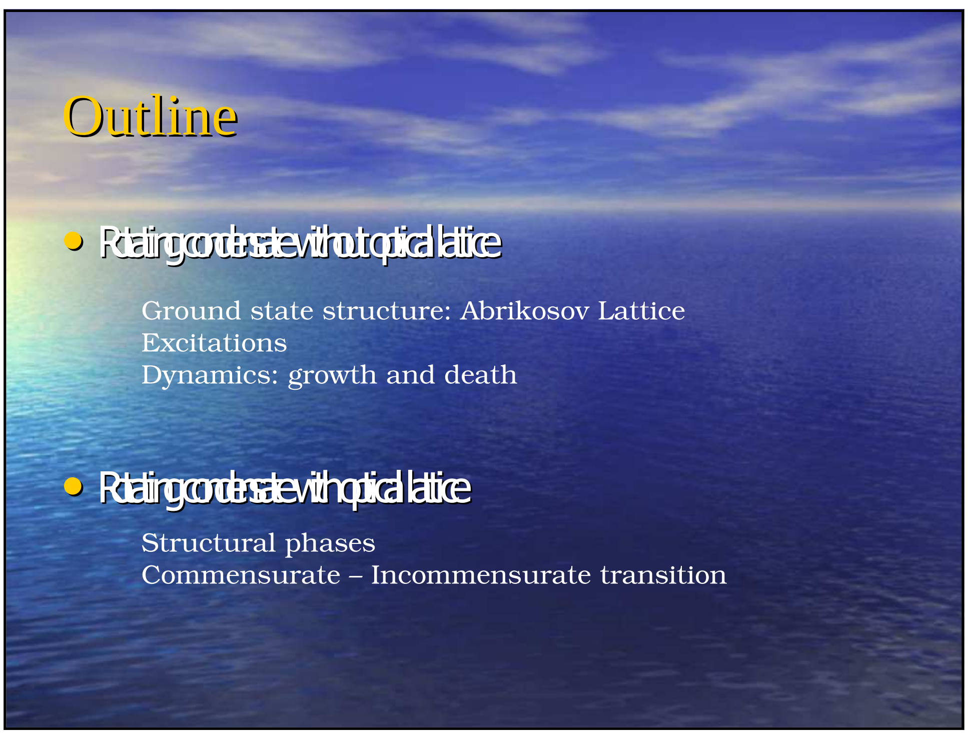
Collaborators: L. O. Baksmaty, N. P. Bigelow, S. Yi
Vortex matter in atomic condensate is under good control;
Fertile test ground for other systems
6/25/04 KITP

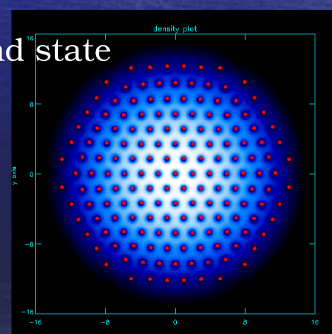

Outline
Rotating condensate without optical lattice
Ground state structure: Abrikosov Lattice
Excitations
Dynamics: growth and death
Rotating condensate with optical lattice
Structural phases
Commensurate – Incommensurate transition

Gross-Pitaevskii Equation

Rotating Frame, Quasi-2D

$$i\frac{\partial\phi}{\partial t}=\left[-\frac{\nabla^2}{2}+\frac{r^2}{2}+U\,|\phi|^2-\mu-\Omega L_z\right]\phi$$

Imaginary time evolution → Ground state



Free energy of a classical fluid with density n, in vessel rotating at frequency Ω, is given as

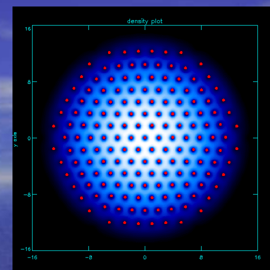
$$F=n\int\frac{1}{2}v^2-\Omega(\vec{r}\times\vec{v})d\vec{r}$$

$$\vec{v}(\vec{r})=\sum_{i=1}^{n}\frac{\hat{\Omega}\times(\vec{r}-\vec{r}_i)}{2\pi(\vec{r}-\vec{r}_i)^2}$$

$$F=\frac{n\kappa^2}{4\pi}\left(-\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\ln\left|\vec{r}_i-\vec{r}_j\right|^2-\Omega\sum_{i=1}^{m}\left(1-\left|\vec{r}_i\right|^2\right)\right)$$

vortex-vortex repulsion

Ground state structure



Length scales

Healing length (ξ) : $\frac{\hbar^2}{2m\xi^2} = Un$

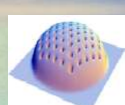
determines vortex core size

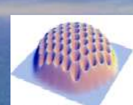
Magnetic length (ε) : $\frac{\hbar^2}{2m\varepsilon^2} = \hbar\Omega$

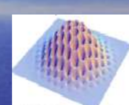
determines inter-vortex distance

Condensate radius (R): $R = \sqrt{\frac{2\mu}{m\left(\omega_\perp^2 - \Omega^2\right)}}$

From Hydrodynamic to Quantum Hall







Ω →

Stiff	Soft	Quantum Hall	Melting
$\Omega \ll c/R$	$c/R \ll \Omega \ll mc^2$	$\Omega \gg mc^2$	
Incompressible Rigid body Hydrodynamic		Lowest Landau Level	Strongly correlated system

← compressible → (Soft to Quantum Hall)

← Mean field → (Stiff to Quantum Hall)

Excitations

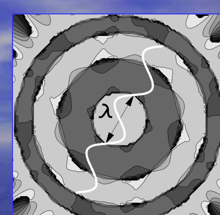
normal mode ansatz: $\Phi(t) = e^{-i\mu t}\left(\Phi_0 + u_n e^{-i\omega t} - v_n^* e^{i\omega t}\right)$

eigenvalue equation:

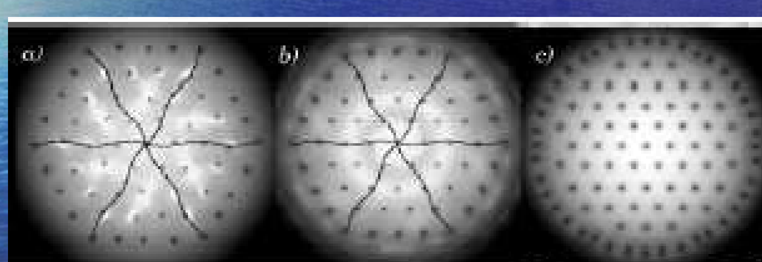
$$\omega u(\vec{r}) = \left[-\frac{\nabla^2}{2} + \frac{r^2}{2} - \Omega L_z - \mu + 2U\left|\Phi_0\right|^2\right] u(\vec{r}) - U\Phi_0^2 v(\vec{r})$$

$$-\omega v(\vec{r}) = \left[-\frac{\nabla^2}{2} + \frac{r^2}{2} - \Omega L_z - \mu + 2U\left|\Phi_0\right|^2\right] v(\vec{r}) - U\Phi_0^{*2} u(\vec{r})$$

Excitation Modes



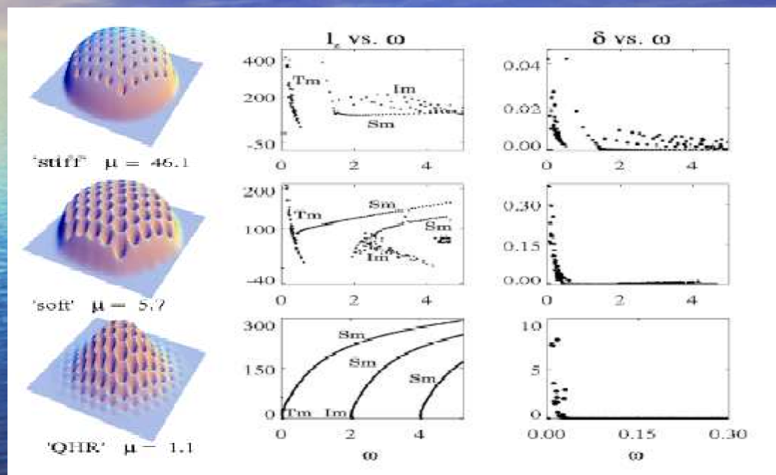
Density Fluctuation Amplitude: $\delta\rho_n = \Phi_0^* u_n - \Phi_0 v_n$



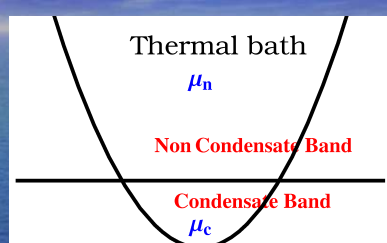
Tkachenko Mode — Inertial Mode — Surface Mode

Excitation Spectra

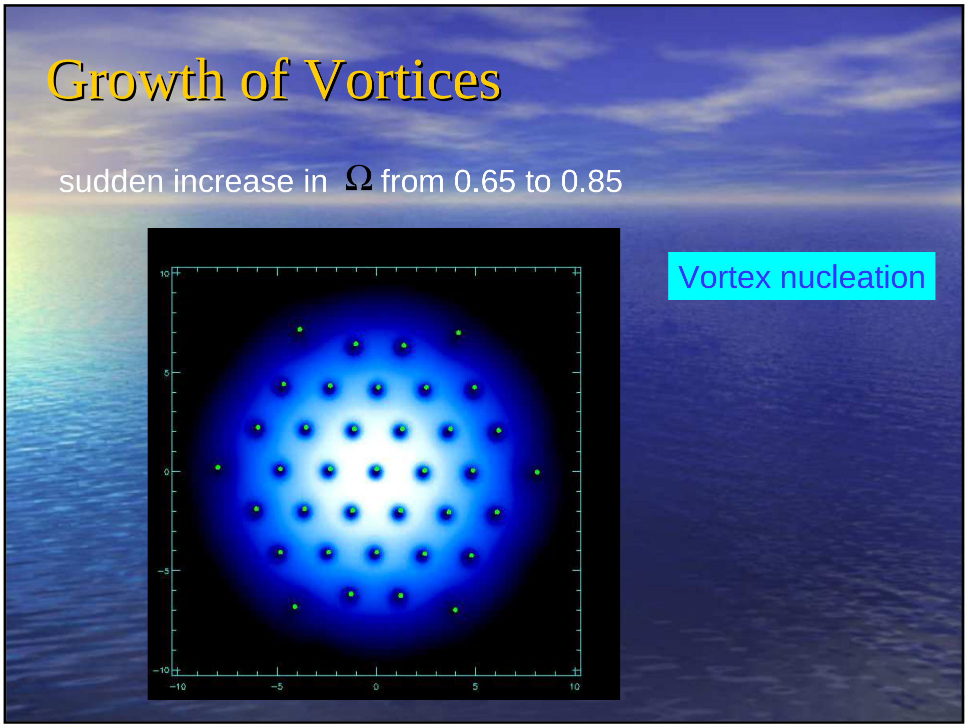
Angular Momentum Quantum Depletion

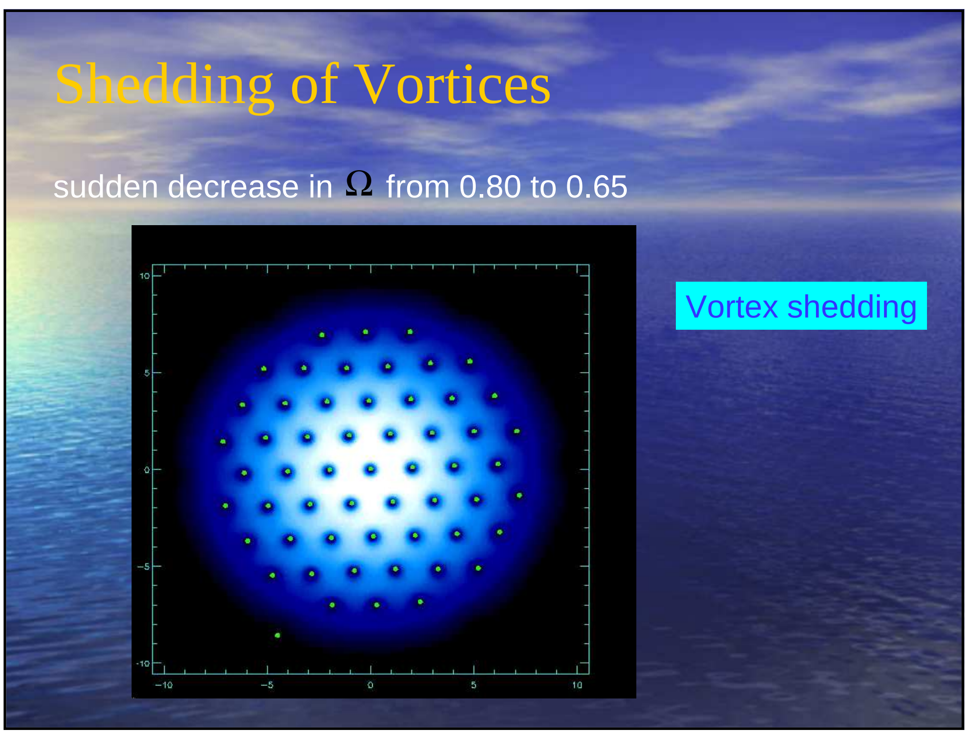


Vortex Dynamics: Birth and Death



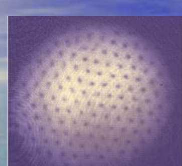
$$i\frac{\partial}{\partial t}\Phi_o = \left(-\frac{\nabla^2}{2}+\frac{r^2}{2}-\Omega L_z + U\left|\Phi_o\right|^2\right)\Phi_o + i\gamma\left(\mu_n - i\frac{\partial}{\partial t}\right)\Phi_o$$


Growth of Vortices
sudden increase in Ω from 0.65 to 0.85
Vortex nucleation


Shedding of Vortices
sudden decrease in Ω from 0.80 to 0.65
Vortex shedding

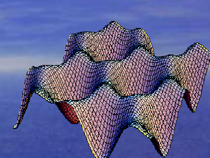
Now add an co-rotating optical lattice ..

Model



Condensate in rotating trap

+



Periodic optical potential (co-rotating)

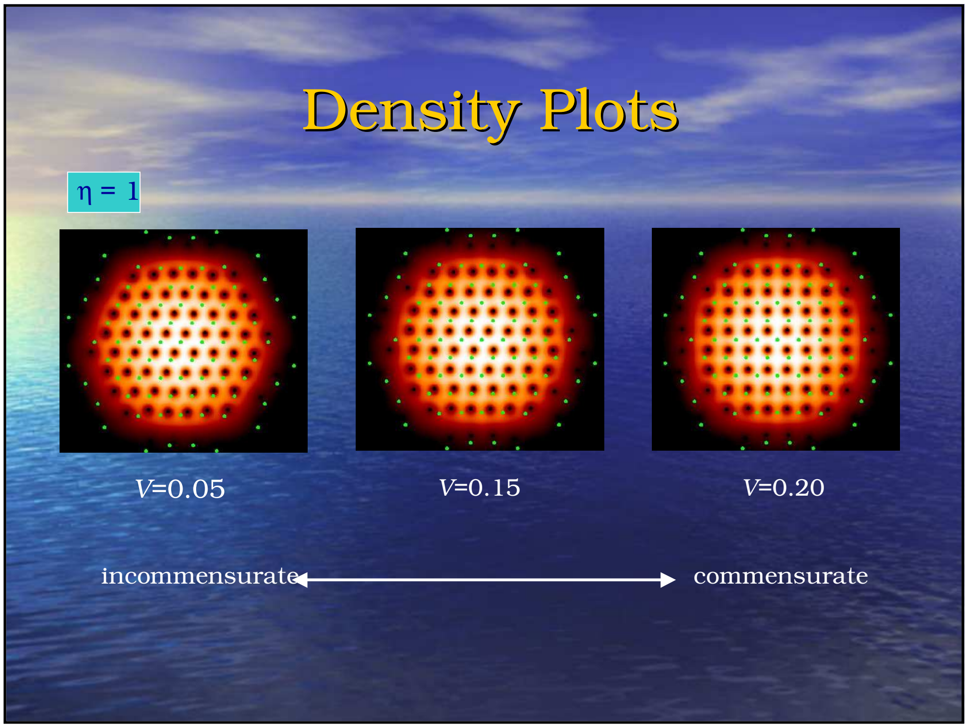
$$i\frac{\partial\phi}{\partial t}=\left[-\frac{\nabla^2}{2}+\frac{r^2}{2}+V_{lat}+U\,|\phi|^2-\mu-\Omega L_z\right]\phi$$

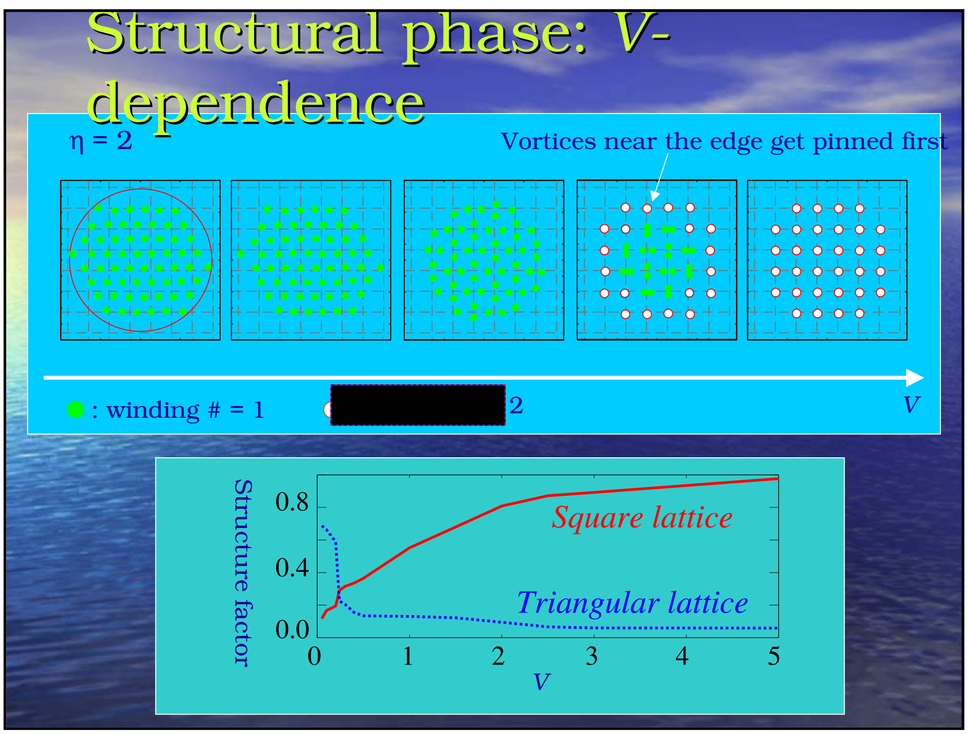
$$V_{lat}=V\left[\sin^2(kx)+\sin^2(ky)\right]$$

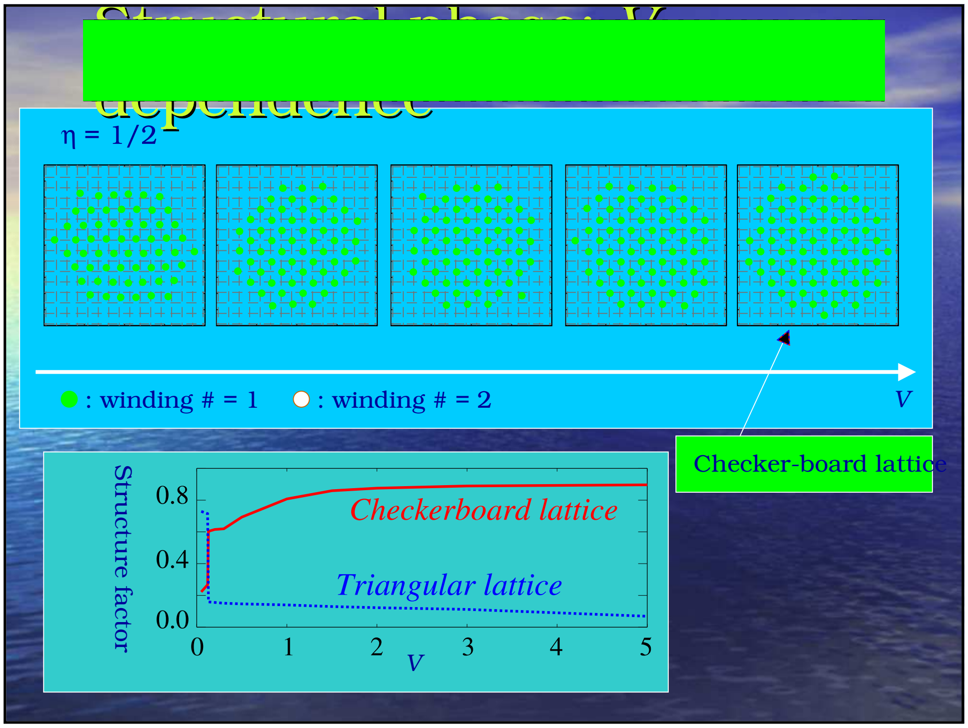
Control Knobs: *V*, *k*, (Ω, *trapping potential, etc.*)

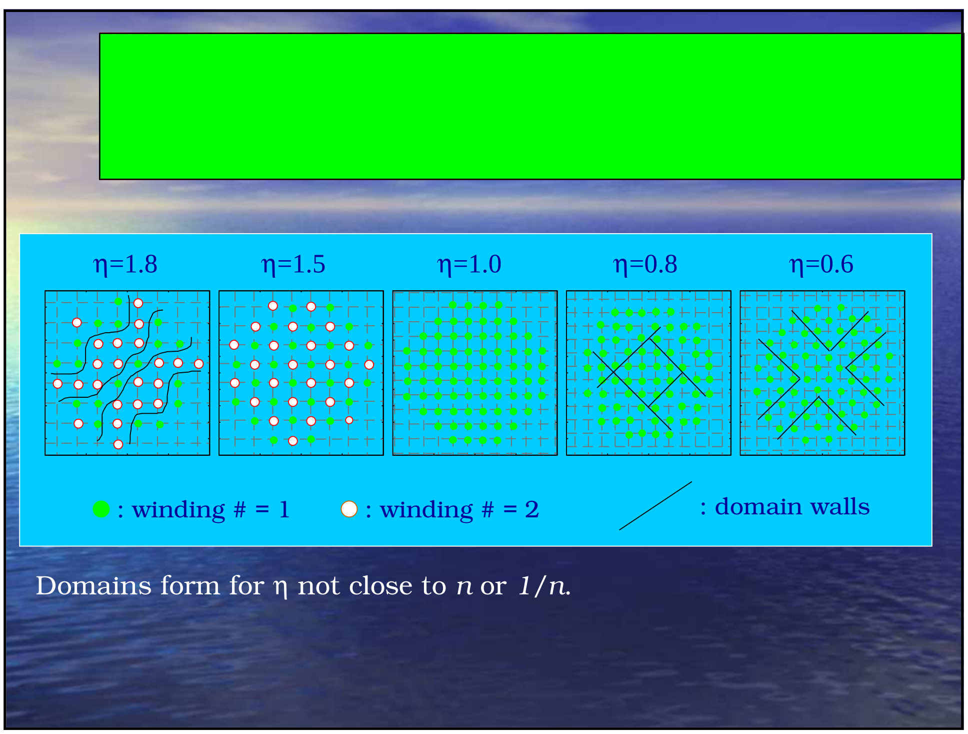
V determines the strength of pinning

k determines the filling factor: $\eta=\frac{N_{vortices}}{N_{sites}}\approx\frac{\pi\Omega}{k^2}$


Density Plots
η = 1
V=0.05
V=0.15
V=0.20
incommensurate
commensurate


Structural phase: V-dependence
η = 2
Vortices near the edge get pinned first
: winding # = 1
[redacted] 2
V
Structure factor
0.8
0.4
0.0
0
1
2
3
4
5
V
Square lattice
Triangular lattice


η = 1/2
: winding # = 1
: winding # = 2
V
Checker-board lattice
Structure factor
0.8
0.4
0.0
0
1
2
3
4
5
V
Checkerboard lattice
Triangular lattice


η=1.8
η=1.5
η=1.0
η=0.8
η=0.6
: winding # = 1
: winding # = 2
: domain walls
Domains form for η not close to n or 1/n.

Future directions

- Detailed dynamics of the structural phase transition
- Phase transition vs. Collective excitations
- Effect of optical potential on quantum melting
- Different types of optical potentials (disordered, non-rotating, etc…)
- ……

These quantitative studies are enabled by the well-known microsco
physics of the atomic condensate systems.