Chemical-potential standard for atomic Bose-Einstein condensates

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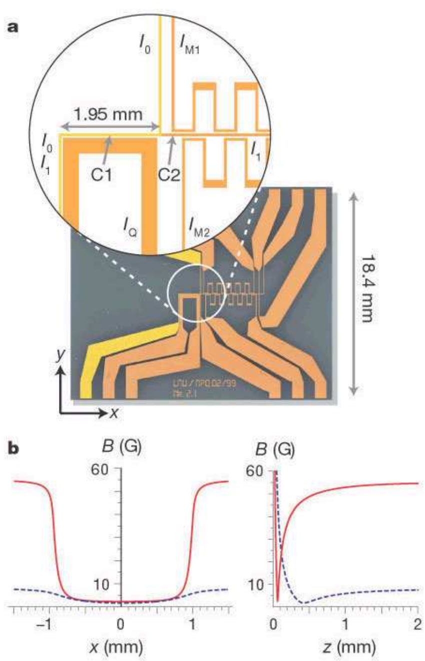
- Formulation of the problem
- Realization of phase-locked solutions
- Locking probability
- quantum fluctuations

[S. Kohler and F. Sols, New J. Phys. **5**, 94 (2003)]



BEC on a chip

W. Hänsel et al.
Nature 01

also e.g.
E. Andersson et al.
PRL 02

Introduction

[S. Shapiro, Phys. Rev. Lett. **10**, 80 (1963)]

steps in V-I characteristic

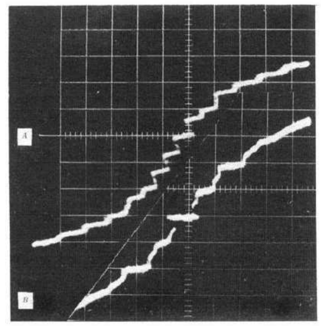


FIG. 3. Microwave power at 9300 Mc/sec (A) and 24850 Mc/sec (B) produces many zero-slope regions spaced at $h\nu/2e$ or $h\nu/e$. For A, $h\nu/e = 38.5\ \mu V$, and for B, 103 μV. For A, vertical scale is 58.8 $\mu V/cm$, horizontal scale is 67 nA/cm; for B, vertical scale is 50 $\mu V/cm$, horizontal scale is 50 $\mu A/cm$.

- plateaus at voltages $V = \frac{h}{2e} k f$
- voltage and frequency related by constant of nature

Formulation of the problem

- two-mode Hamiltonian

$$H = -\frac{\hbar\omega_R}{2}\left(a^\dagger b + b^\dagger a\right) + E_A(N_A, \xi) + E_B(N_B, \xi).$$

E_A, E_B: bulk energies of the fragments

ξ: time-dependent trap parameter

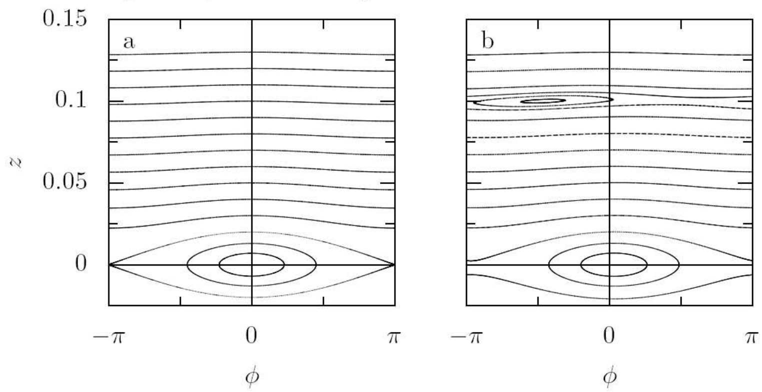
- corresponding equations of motion

$$\begin{aligned} \dot{\phi} &= \frac{z}{\sqrt{1-z^2}}\cos\phi + \Lambda z + \varepsilon\cos(\Omega t) = \Delta\mu/\hbar, \\ \dot{z} &= -\sqrt{1-z^2}\sin\phi - \gamma\dot{\phi}, \end{aligned}$$

- non-rigid driven pendulum

$$H(z,\phi,\lambda) = -\sqrt{1-z^2}\cos\phi + \frac{1}{2}\Lambda z^2 + \varepsilon z\cos(\Omega t)$$

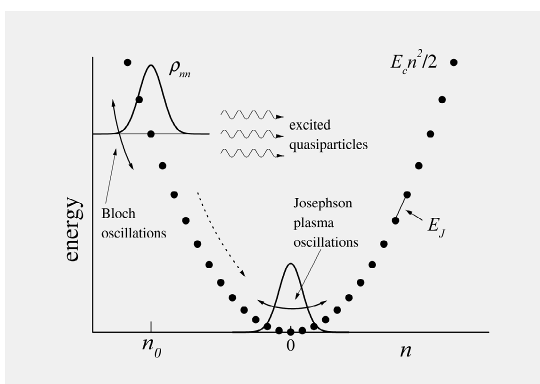
- non-dissipative, classical dynamics



- driving: resonance islands such that $\Lambda\bar{z} = k\Omega$, $\mathrm{d}\bar{z}/\mathrm{d}t = 0$
- correspondingly $\Delta\mu = k\Omega + \varepsilon\cos(\Omega t)$

Connection of two independent condensates

- E_J=0. Random phase. Well-defined number.
- connection $E_J \neq 0$ $I(\varphi) = E_J \sin\varphi$
- phase becomes defined in $t \sim \tau_D$
- Bloch oscillations (Josephson AC)
- Quasiparticle excitation
- Energy relaxation in $t \sim \tau_R >> \tau_D$



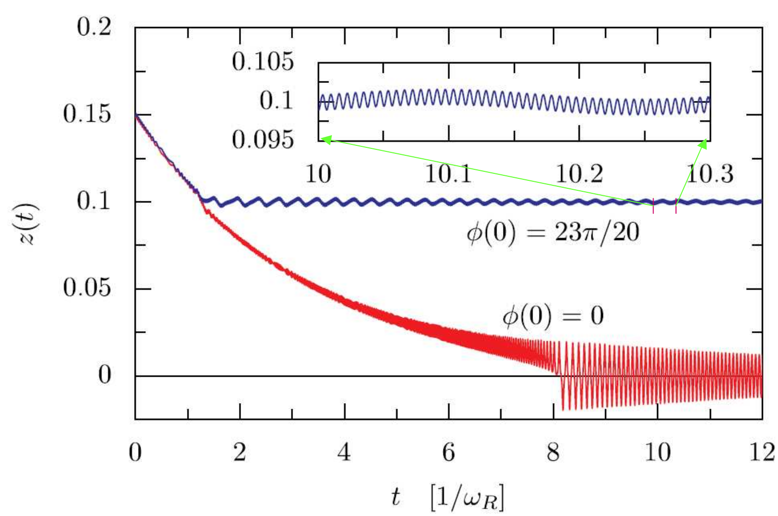
Zapata, Sols, Leggett PRA 03

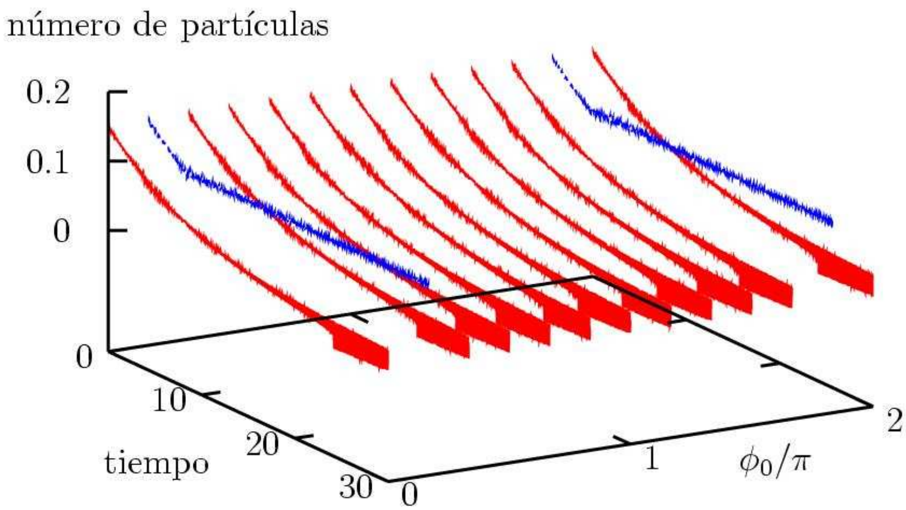
Phase measurement

- Well-defined number, random phase: $|\rho(\varphi,\varphi';t)| = \frac{1}{2\pi}$
- After connection, phase becomes defined in $t \sim \tau_D$

$$|\rho(\varphi,\varphi';t)| = \frac{1}{2\pi}\exp\left[-4\gamma_d t\ \sin^2\left(\frac{\varphi-\varphi'}{2}\right)\right]$$

- For $\gamma_d t \gg 1$ ⟹ $|\rho(\varphi,\varphi';t)| = (2\pi)^{-1}\exp[-(\varphi-\varphi')^2\gamma_d t]$
- Mixture of randomly centered pure gaussians of width $\sim(\gamma_d t)^{-1/2}$
- In a given run, one is chosen: $\sim\exp[-2(\varphi-\theta)^2\gamma_d t]$





Realization of phase-locked solutions

- ansatz

$$\begin{aligned} \phi &= k\Omega t + \frac{\varepsilon}{\Omega}\sin(\Omega t) + \delta\phi \\ z &= \frac{k\Omega}{\Lambda} + \alpha\cos(k\Omega t) + \delta z \end{aligned}$$

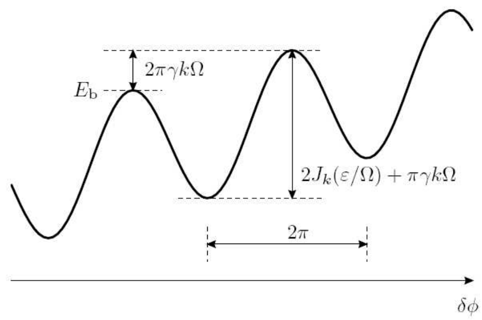
- results in (time-averaged over many cycles)

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\delta\phi &= \Lambda\,\delta z \\ \frac{\mathrm{d}}{\mathrm{d}t}\delta z &= -J_k(\varepsilon/\Omega)\sin(\delta\phi) - \gamma k\Omega - \gamma\Lambda\,\delta z \end{aligned}$$

- δz, $\delta\phi$ shown to vary slowly

- tilted washboard potential with dissipative force $F_{\rm diss} = -\gamma\Lambda\,\delta z$ and renormalized plasma frequency

$$H(\delta\phi, \delta z) = \frac{1}{2}\Lambda\,\delta z^2 - J_k(\varepsilon/\Omega)\cos(\delta\phi) + \gamma k\Omega\,\delta\phi$$



- $\delta\phi$ at rest: phase locking $\bar{\mu} = k\Omega$

- restrictions for parameters (first resonance: $k = 1$)

$\varepsilon \ll \Omega$	chemical potential difference much larger than its modulation
$\varepsilon > 2\gamma\Omega^2$	existence of stable wells in the washboard potential
$\Lambda \ll \Omega^2$	operation within MQST regime, Bloch oscillations small
$\Lambda \gg \Omega$	number imbalance $\bar{z} = \Omega/\Lambda$ small

Locking probability

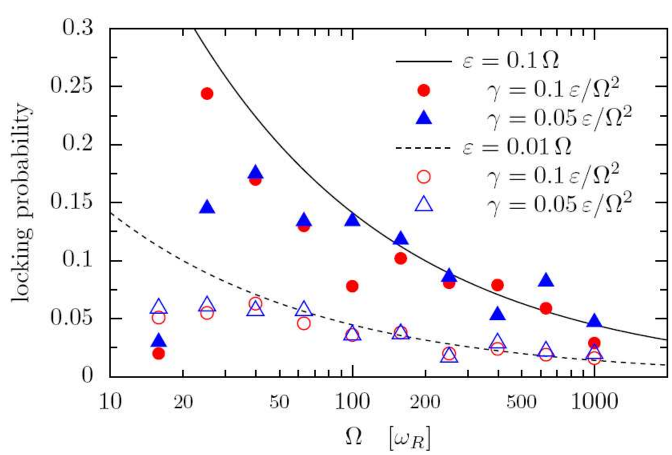
- probability of reaching a stable well by starting with a random initial phase?
- assumption: random initial phase equivalent to entering well with random energy in effective washboard potential
- energy dissipated within well

$$E_{\mathrm{diss}} \approx -\int F_{\mathrm{diss}}\,\mathrm{d}\phi \approx 2\pi\gamma\Lambda\sqrt{2\varepsilon/\Lambda\Omega}$$

- locking if $E_{\mathrm{diss}} > E_0 - E_{\mathrm{barrier}}$
- trapping probability for random $E_0 - E_{\mathrm{barrier}}$:

$$w = \frac{E_{\mathrm{diss}}}{E_0 - E_{\mathrm{barrier}}} \approx \sqrt{\frac{2\varepsilon\Lambda}{\Omega^3}}$$

- comparison to numerical simulation (1000 runs)



- typical locking probability: 10%

Relevance of quantum fluctuations?

- number-phase uncertainty: $[z, \phi] = -2i/N$
- tunneling rate out of metastable well: $\kappa = \omega_0 e^{-2\pi E_0/\hbar\omega_0}$
- κ small and classical limit valid for

$$\text{number of bound states} = \frac{E_0}{\hbar\omega_0} = N\sqrt{\frac{\varepsilon}{2\Omega\Lambda}} \gg 1$$

- $N = 10^5$ atoms, $\epsilon = 0.01\Omega$, $\Lambda = 10\Omega \longrightarrow \frac{E_0}{\hbar\omega_0} \approx 100$

  i.e., approximately 100 states supported by stable well

Summary

- visibility condition $\Lambda\,\Delta z < \Omega \ll \Lambda$ sufficient for precision measurement
- e.g. 10^6 ^{23}Na atoms, $\omega_{\text{ho}} = 2\pi \times 100\,\text{Hz}$, $\mu \approx 70\,\hbar\omega_{\text{ho}}$,
  mean field energy $E_A(N) \approx \hbar N^{7/5} \times 0.01\,\text{s}^{-1}$,
  Gaussian shape of width $6\,\mu\text{m}$,
  barrier height $1.05\,\mu$
- Thus,
  Rabi frequency $\omega_R \approx 2\pi \times 0.05\,\text{Hz}$
  plasma frequency $\omega_{\text{JP}} = 2\pi \times 5\,\text{Hz}$
  $\longrightarrow$ phase locking sets in after $5\,\text{s}$