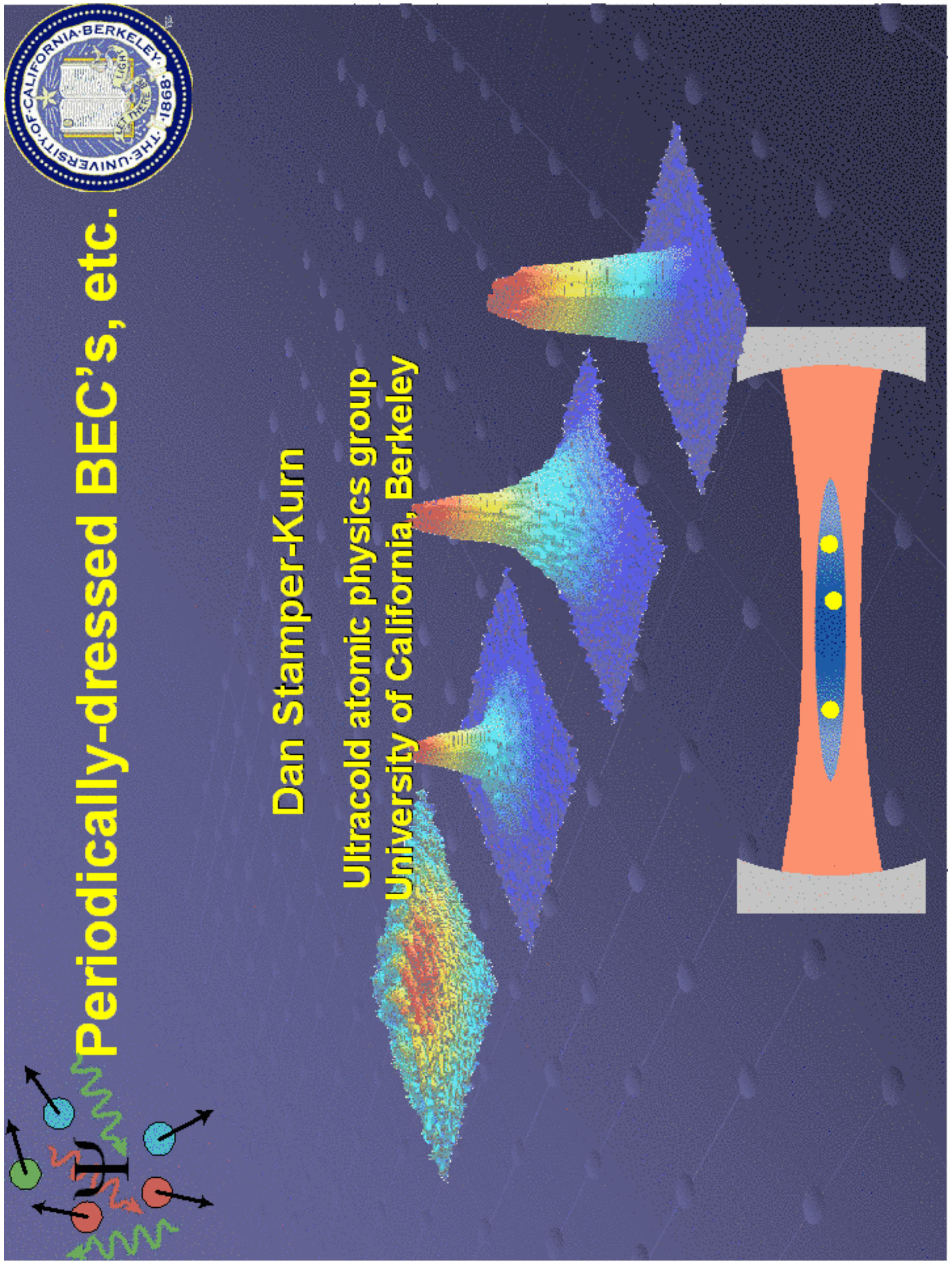
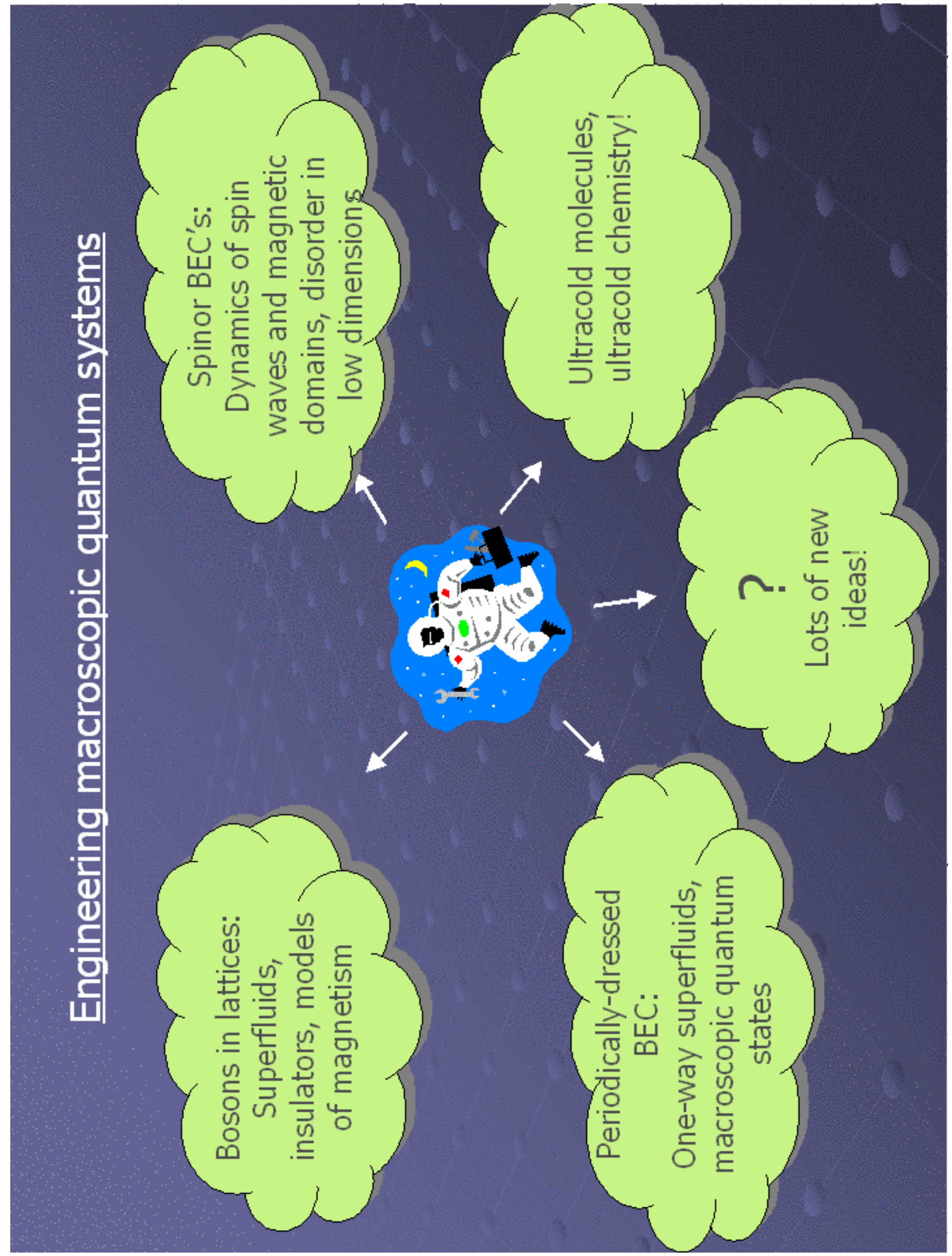
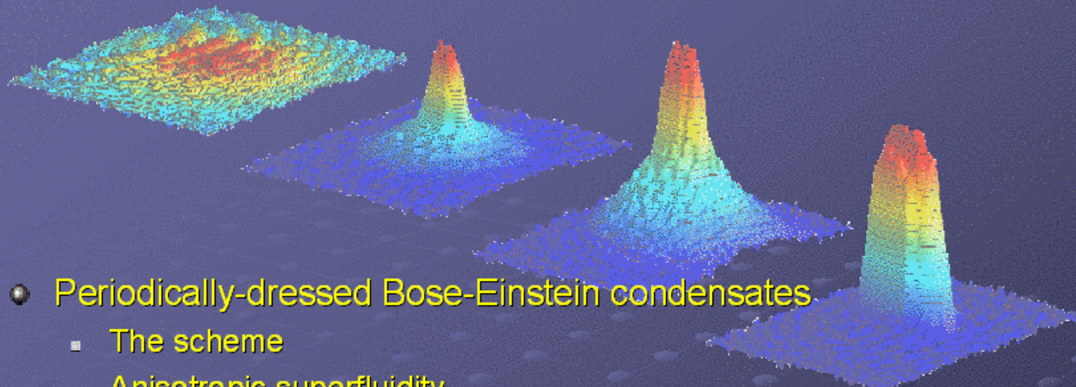
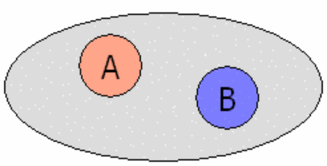
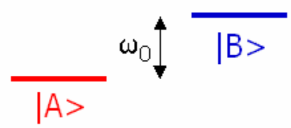

Periodically-dressed BEC's, etc.
Dan Stamper-Kurn
Ultracold atomic physics group
University of California, Berkeley


Engineering macroscopic quantum systems
Spinor BEC's: Dynamics of spin waves and magnetic domains, disorder in low dimensions
Ultracold molecules, ultracold chemistry!
?
Lots of new ideas!
Bosons in lattices: Superfluids, insulators, models of magnetism
Periodically-dressed BEC: One-way superfluids, macroscopic quantum states



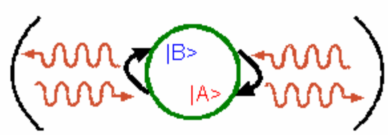
- Periodically-dressed Bose-Einstein condensates
  - The scheme
  - Anisotropic superfluidity
  - Impurity scattering as probe of anisotropy
  - Double-well in momentum space
  - Possible applications toward chronometry and interferometry
- Non-linear magneto-optics in a BEC – experiment
- Toward determinstic cavity quantum electrodynamics with magnetically trapped atoms

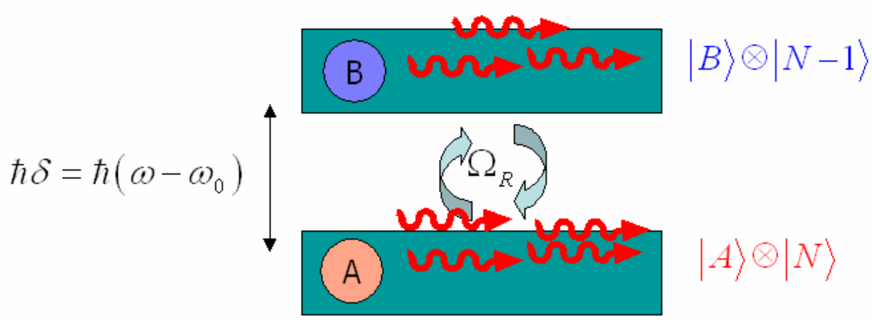




Add coupling A ⇔ B $V = \frac{\Omega_R}{2}(|A\rangle\langle B| + |B\rangle\langle A|)$

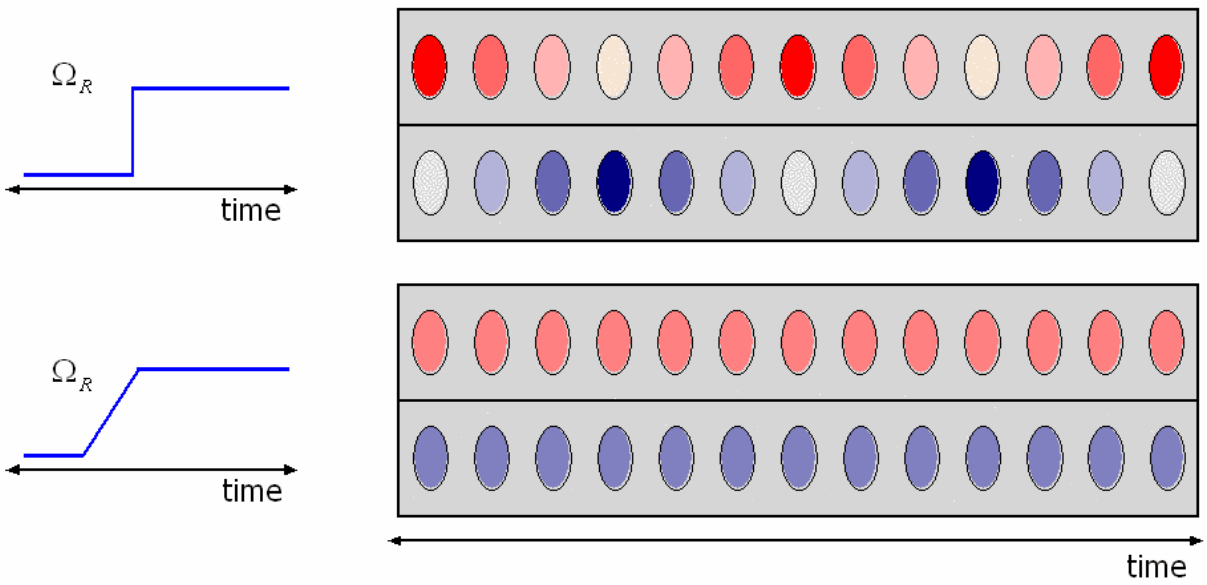
Dressed-state picture: view system in frame rotating with drive frequency





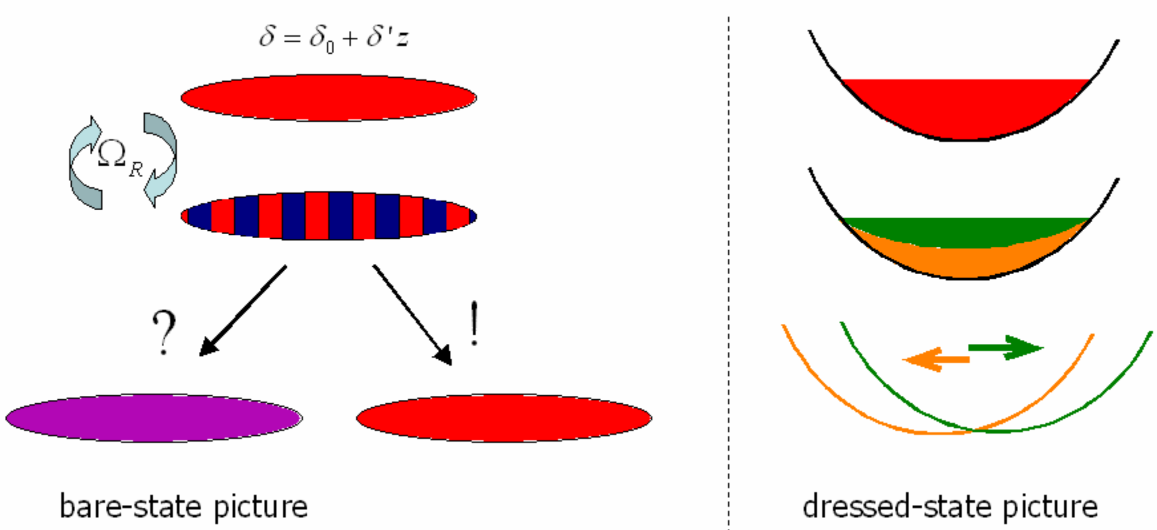
Dressed-states

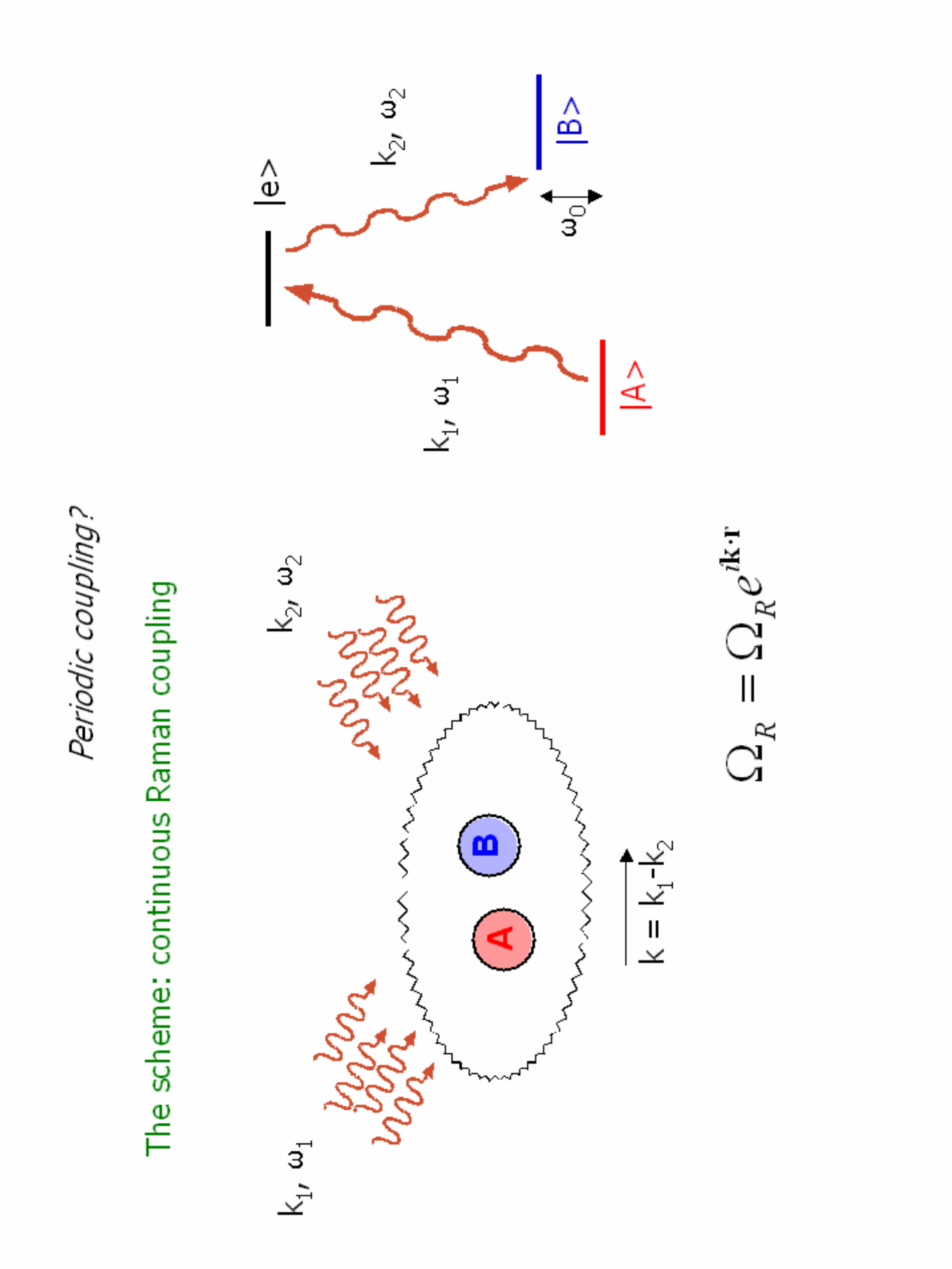
$$\begin{pmatrix} |a\rangle \\ |b\rangle \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} |\mu\rangle \\ |\pi\rangle \end{pmatrix} = R\left(\frac{\theta}{2}\right) \begin{pmatrix} |\mu\rangle \\ |\pi\rangle \end{pmatrix} \qquad \tan\theta = \frac{\Omega_R}{\delta}$$

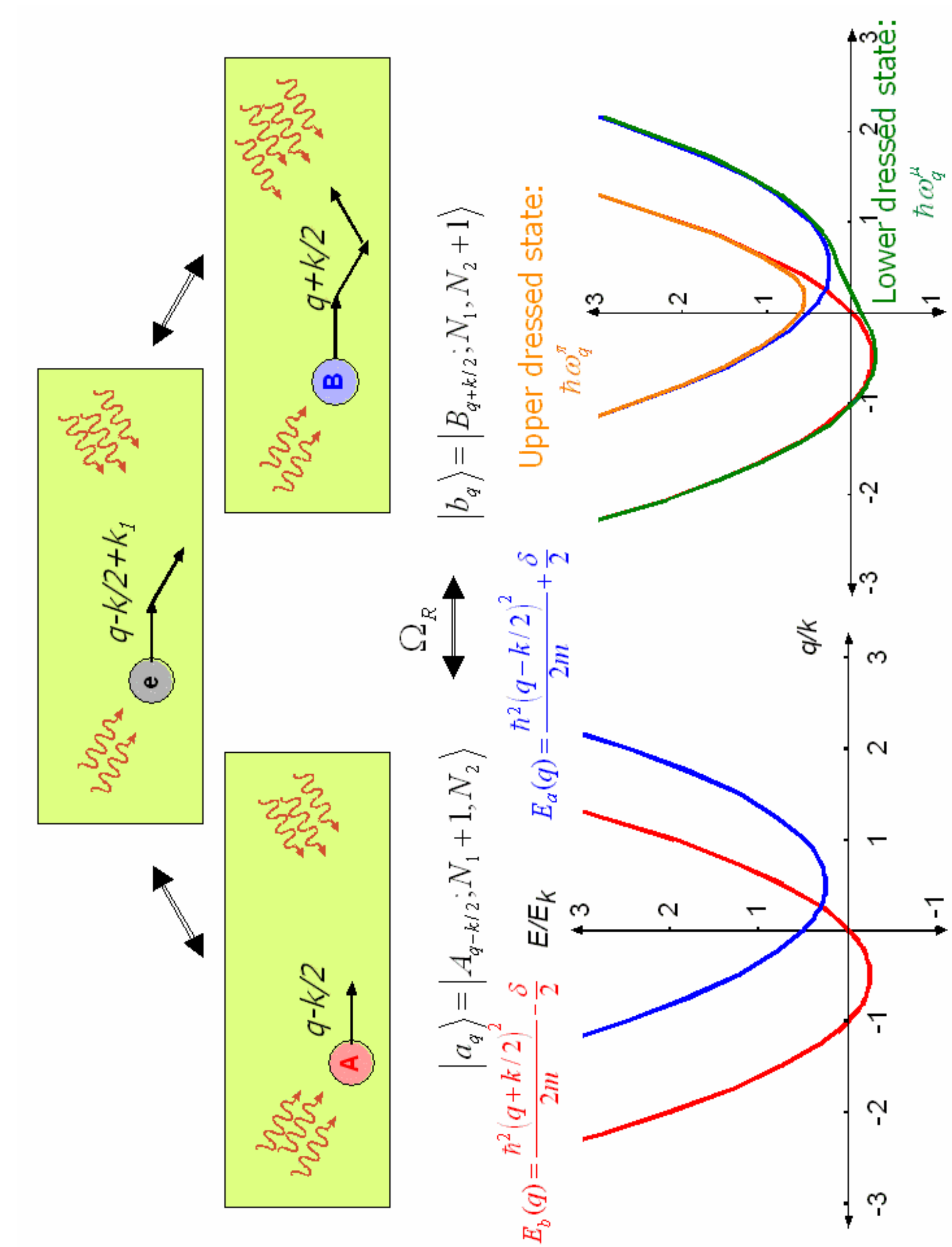


Spatially-dependent coupling

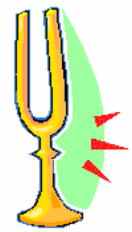
- Matthews et al., "Watching a superfluid untwist itself: Recurrence of Rabi oscillations in a BEC," PRL **83**, 3359 (1999)
- Williams et al., "Nonlinear Josephson-type oscillations of a driven, two-component Bose-Einstein condensate," PRA **59**, R31 (1999)
- Matthews et al., "Vortices in a BEC," PRL **83**, 3498 (1999)
- Williams and Holland, "Preparing topological states of a BEC," Nature **401**, 568 (1999)

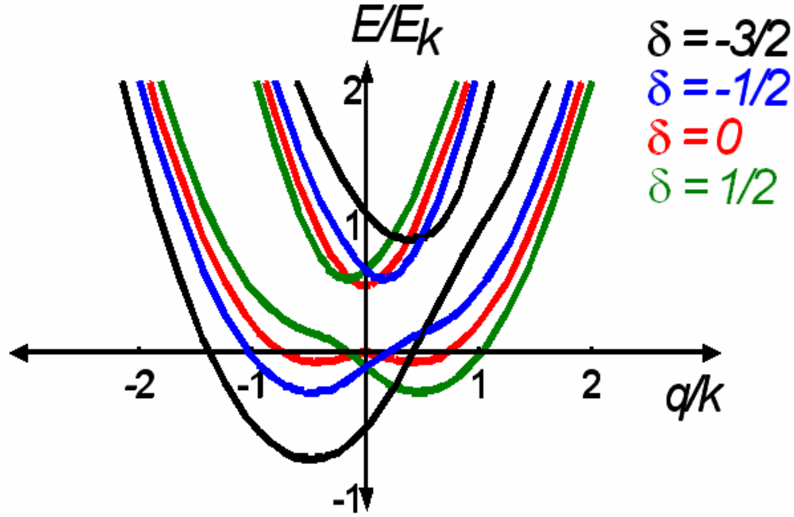



Periodic coupling?
The scheme: continuous Raman coupling
k_1, ω_1
k_2, ω_2
A
B
$k = k_1 - k_2$
|e>
|A>
|B>
ω_0
$\Omega_R = \Omega_R e^{i\mathbf{k}\cdot\mathbf{r}}$


q-k/2
$q-k/2+k_1$
q+k/2
e
A
B
Ω_R
$|a_q\rangle = |A_{q-k/2}; N_1+1, N_2\rangle$
$|b_q\rangle = |B_{q+k/2}; N_1, N_2+1\rangle$
$E_b(q) = \frac{\hbar^2(q+k/2)^2}{2m} - \frac{\delta}{2}$
$E_a(q) = \frac{\hbar^2(q-k/2)^2}{2m} + \frac{\delta}{2}$
E/E$_K$
q/k
Upper dressed state: $\hbar\omega_q^\alpha$
Lower dressed state: $\hbar\omega_q^\beta$

A tunable free-particle dispersion relation



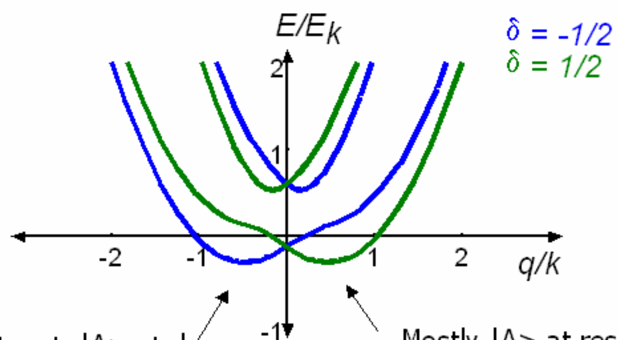


Other tunable parameters:

- interaction strength and sign
- coupling to molecular states
- trapping potential (harmonic, periodic)

$$\mathrm{H} = \sum_q \left(\hbar\omega_q^\mu \mu_q^\dagger \mu_q + \hbar\omega_q^\pi \pi_q^\dagger \pi_q \right) + \mathrm{H}_{\mathrm{int}}$$

Condensate forms in state of wavevector Q



Mostly |B> at rest, |A> at -k

Mostly |A> at rest, |B> at k

Interactions: $\mathrm{H}_{\mathrm{int}} = \frac{g}{2} \sum_q \left(n_q n_{-q} - N \right)$

$$n_q = \sum_l \begin{pmatrix} a_{l+q}^\dagger & b_{l+q}^\dagger \end{pmatrix} \cdot \begin{pmatrix} a_q \\ b_q \end{pmatrix} = \sum_l \begin{pmatrix} \mu_{l+q}^\dagger & \pi_{l+q}^\dagger \end{pmatrix} \overbrace{R\left(\frac{-\theta_{l+q}}{2}\right) R\left(\frac{\theta_q}{2}\right)}^{\Delta_q} \begin{pmatrix} \mu_q \\ \pi_q \end{pmatrix}$$

Collisions preserve A,B population but not μ, π population

Bogoliubov theory for a periodically-dressed condensate

The Bogoliubov recipe:
- condensate operator is treated as c-number
- retain terms of order N^2 and N
- canonical transformation to new quasi-particle excitations

$$H = N\hbar\omega_Q^{\mu} + \sum_{q\neq 0} \frac{w_i H_{ij} v_i}{2} + \text{other stuff}$$

Diagonalizing gives quasi-particles and their energies

$$a_{Q+q} \rightarrow \left(\mu_{Q+q}, \pi_{Q+q}\right) \rightarrow \left(\tilde{\mu}_q = u\mu_{Q+q} + v\mu^{\dagger}_{Q-q} + x\pi_{Q+q} + y\pi^{\dagger}_{Q-q}, \tilde{\pi}_q = \ldots\right)$$

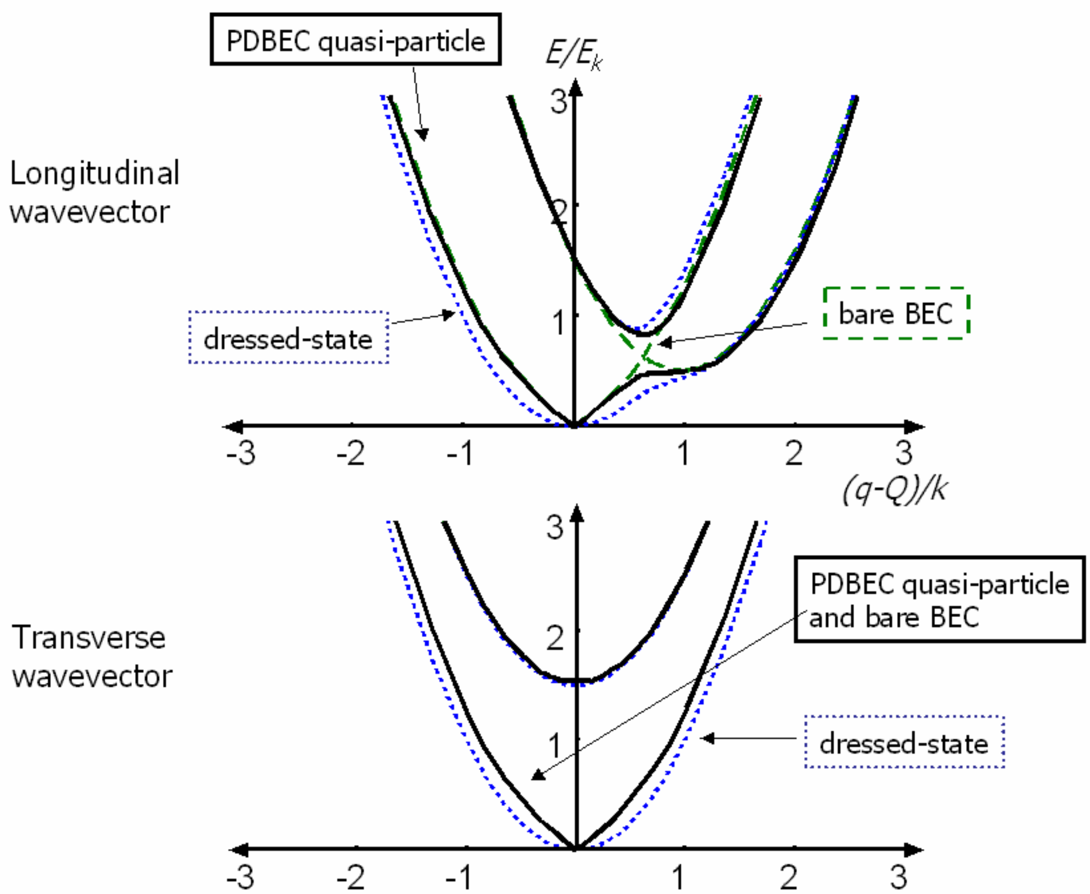
add Raman coupling

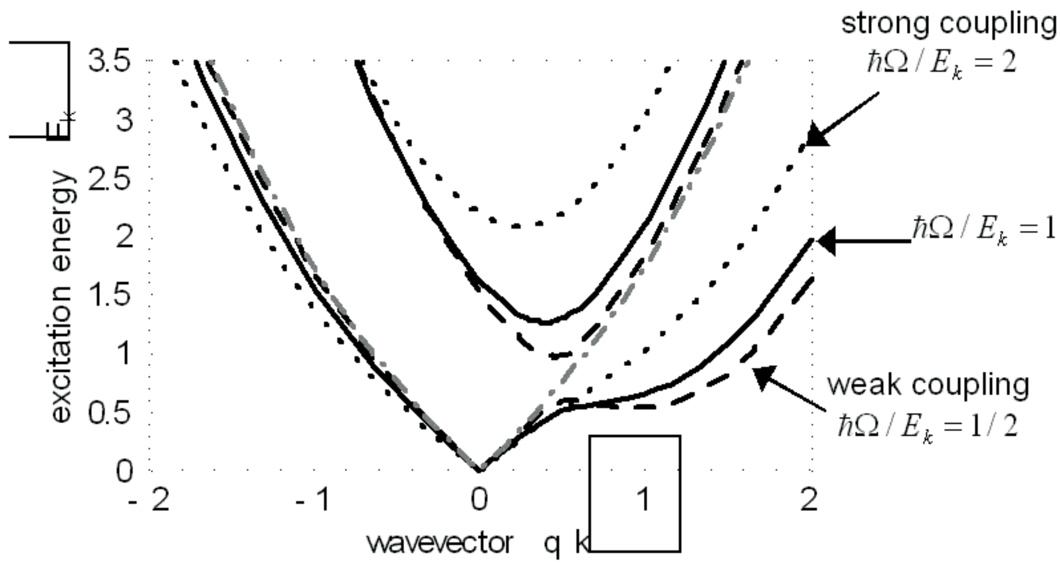
add a BEC and interactions

$$|\mathrm{BEC}\rangle : \begin{cases} \tilde{\mu}_q |\mathrm{BEC}\rangle = 0 \\ \tilde{\pi}_q |\mathrm{BEC}\rangle = 0 \end{cases}$$

Caution: read only in emergency

$$v = \left(\mu_{Q+q} \quad i\mu^{\dagger}_{Q-q} \quad \pi_{Q+q} \quad \pi^{\dagger}_{Q-q}\right)$$
$$w = \left(\mu^{\dagger}_{Q+q} \quad i\mu_{Q-q} \quad \pi^{\dagger}_{Q+q} \quad \pi_{Q-q}\right)$$
$$H = E_{ij} + \mu x_i x_j$$
$$E_{ij} = \hbar\,\mathrm{diag}\left(\omega^{\mu}_{Q+q} - \omega^{\mu}_{Q} \quad -\left(\omega^{\mu}_{Q-q} - \omega^{\mu}_{Q}\right) \quad \omega^{\pi}_{Q+q} - \omega^{\pi}_{Q} \quad -\left(\omega^{\pi}_{Q-q} - \omega^{\pi}_{Q}\right)\right)$$
$$x = \left(\cos\Delta_q \quad i\cos\Delta_{-q} \quad \sin\Delta_q \quad i\sin\Delta_{-q}\right)$$

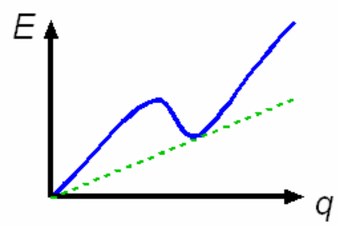




E.g. two-photon Raman transition on D2 line of ^{87}Rb

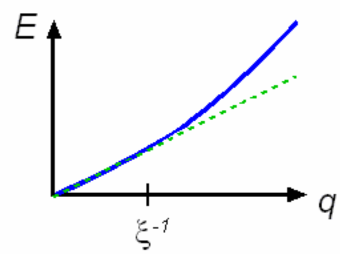
Parameter	Range	How tuned
Recoil energy	0 ⭘ E_k ⭘ h x15 kHz	relative angle of Raman beams
Rabi frequency	arbitrary*	Beam intensity
Detuning	arbitrary	laser frequencies
Chemical potential	μ ? h x kHz	density / Feshbach resonance

<u>Liquid ^{4}He</u>

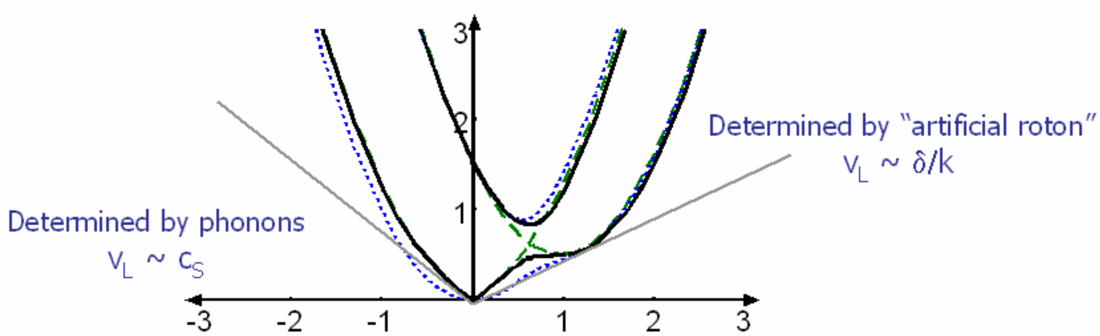


- V_L ~ 50 m/s
- Determined by rotons
- measured by dragging ions
  Meyer & Reif, McClintock et al.

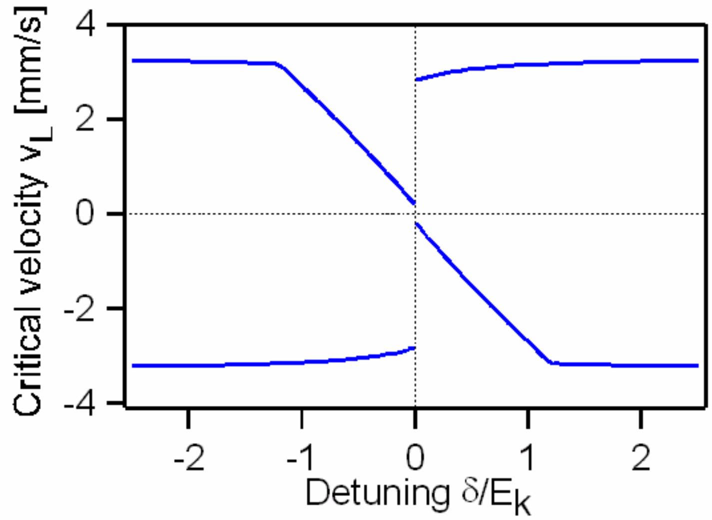
<u>Gaseous BEC</u>



- $V_L = c_S$ ~ 1 cm/s
- Determined by phonons
- measured by scattering of impurity gas
  Chikkatur et al., PRL **85**, 483 (2000)



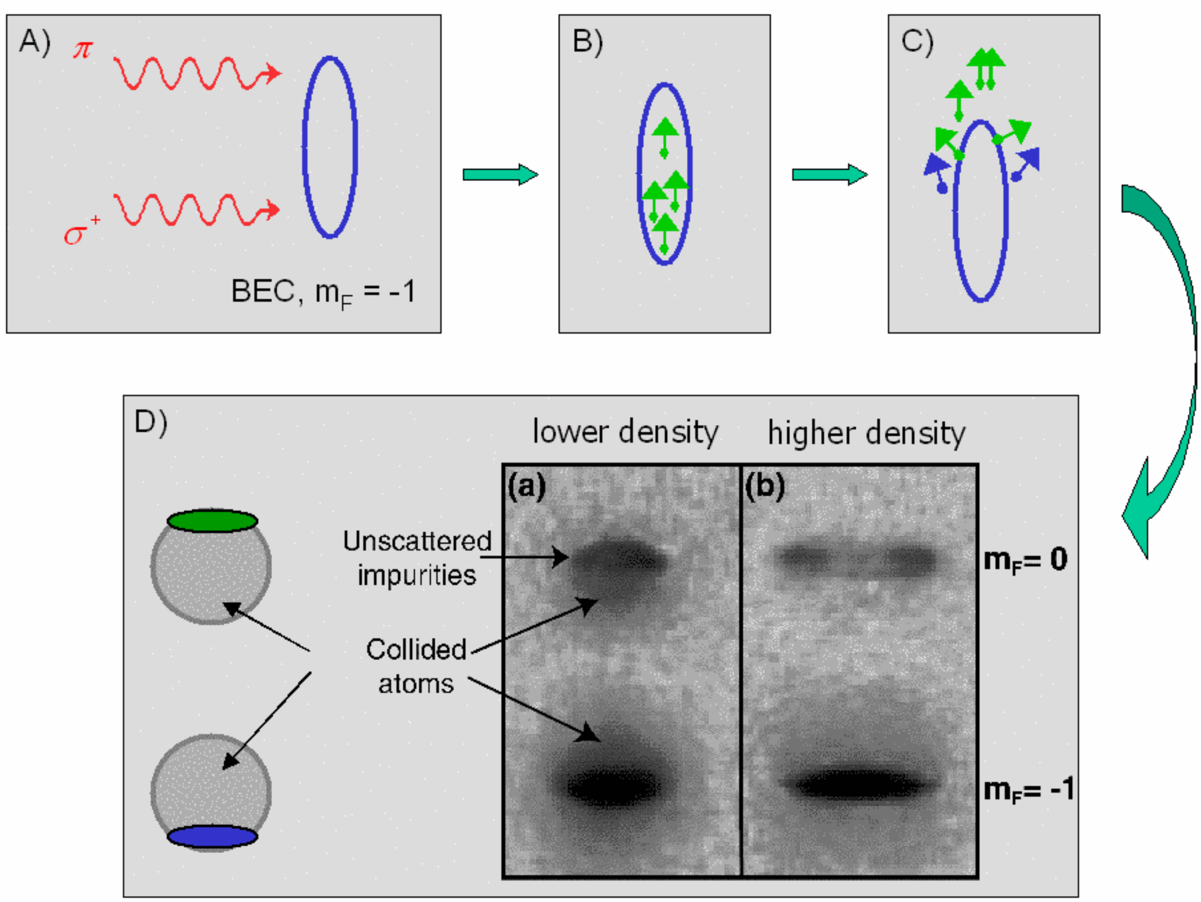
A "One-Way" Superfluid



"Experimental parameters" for ^{87}Rb

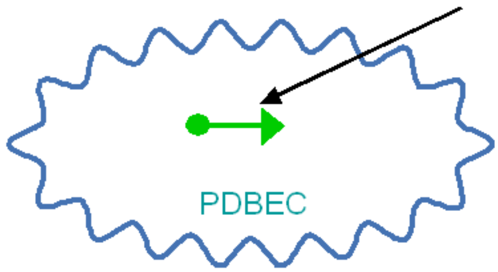
$|A\rangle = |F=1, m_F=-1\rangle$
$|B\rangle = |F=2, m_F=1\rangle$

Counter-propagating laser beams:	E_k = h x 15 kHz
Chemical potential at 3 x 10^{14} cm^{-3}:	m = h x 2.5 kHz
Bogoliubov speed of sound:	c_S = 3.3 mm/s



Chikkatur et al., PRL **85**, 483 (2000)

Impurity scattering



"non-magnetic" scattering

$$\mathrm{H}_{\mathrm{scat}} = \lambda \sum_{m,l,q} \left(a^\dagger_{m+q} a_m c^\dagger_{l+q} c_l + b^\dagger_{m+q} b_m c^\dagger_{l+q} c_l \right)$$

Applying Fermi's Golden rule:

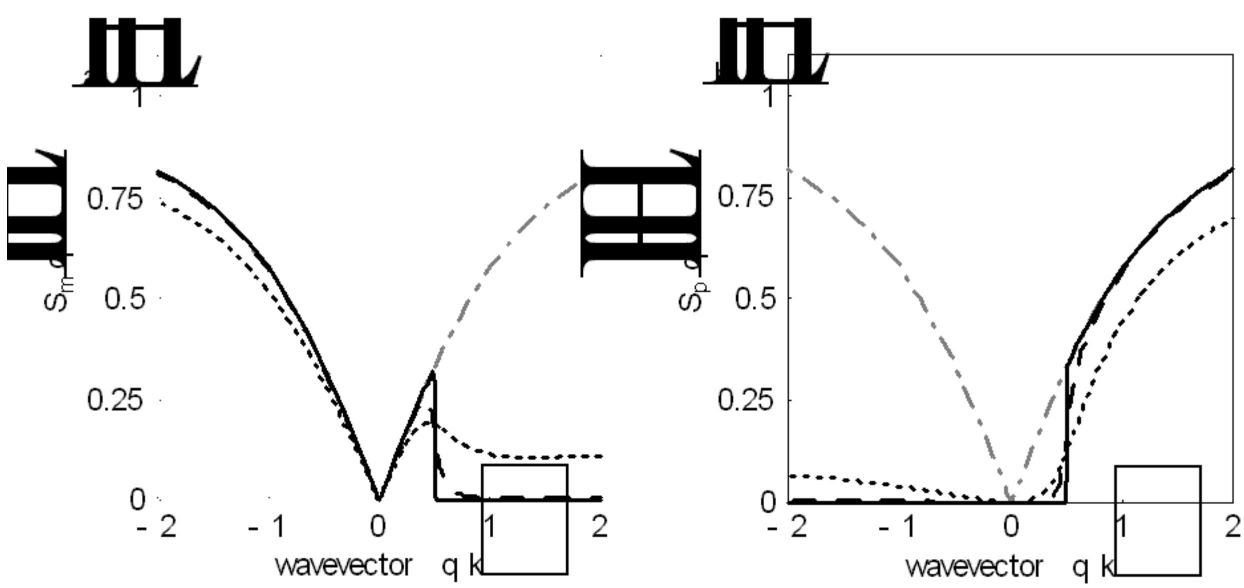
$$F_{\mu,\pi} = \frac{\hbar^2}{4\pi v M'^2} \int d^3\mathbf{q}\, S_{\mu,\pi}(\mathbf{q})\, \delta\left(\omega^{\mu,\pi}_{\mathbf{Q}+\mathbf{q}} + \frac{\hbar q^2}{2M_C} - \mathbf{v}\cdot\mathbf{q} \right)$$

static structure factors

$$S_{\mu,\pi}(\mathbf{q}) = \frac{1}{N_0} \langle 0 | n_{-\mathbf{q}} | \mathbf{q}^{\mu,\pi} \rangle \langle \mathbf{q}^{\mu,\pi} | n_{\mathbf{q}} | 0 \rangle$$

- Density vs. phase perturbation of quasiparticles
  - ⇒ structure factor suppressed for phonons
- Overlap with internal state of condensate
  - ⇒ Rabi frequency determines degree of mixing

Transition from scalar to PD BEC

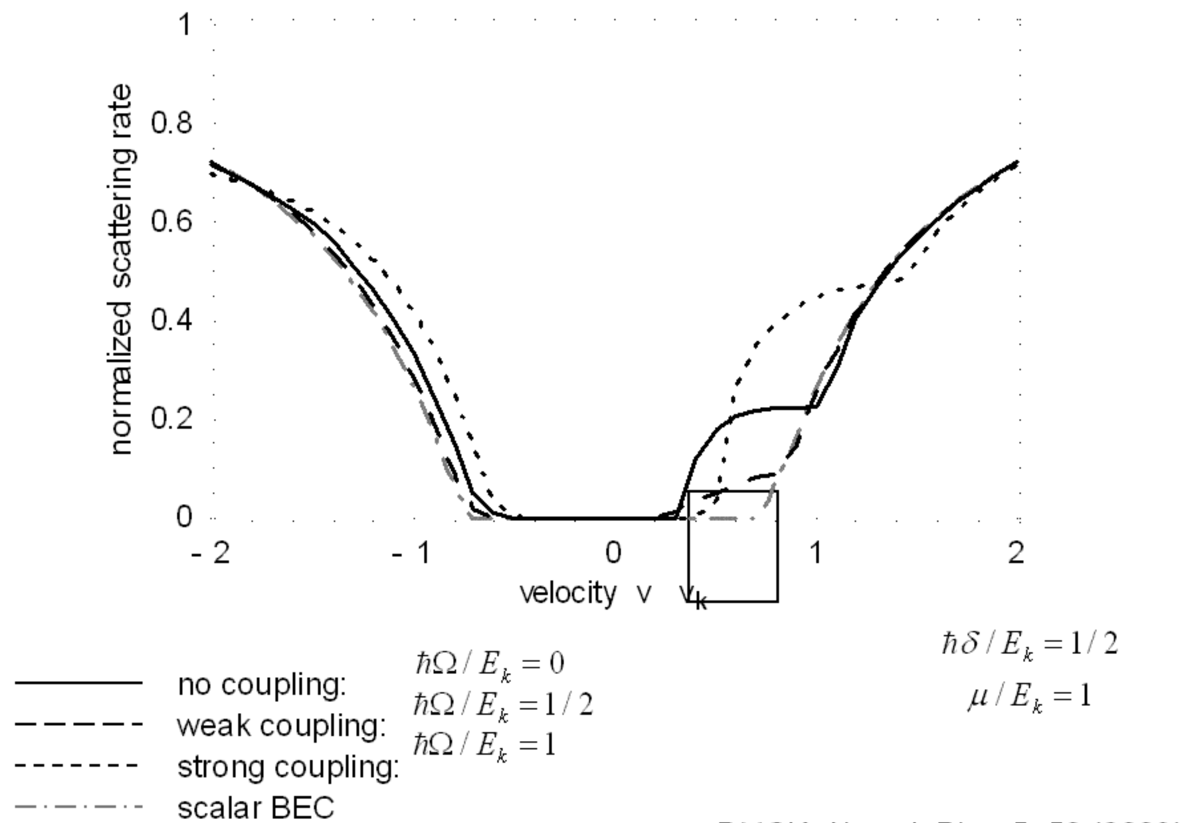


——	no coupling:	$\hbar\Omega / E_k = 0$
— — —	weak coupling:	$\hbar\Omega / E_k = 1/2$
- - - -	strong coupling:	$\hbar\Omega / E_k = 1$
-·-·-	scalar BEC	

$\hbar\delta / E_k = 1/2$

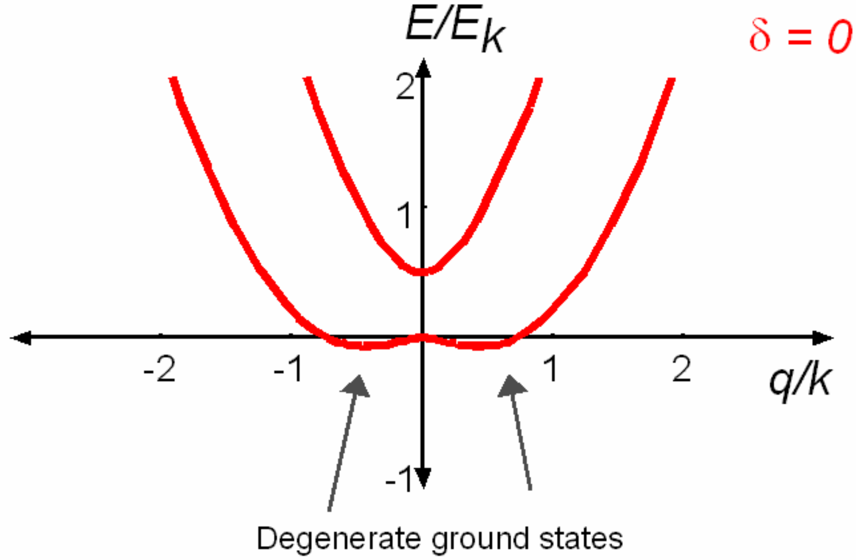
$\mu / E_k = 1$

Transition from scalar to PD BEC



DMSK, *New J. Phys* **5**, 50 (2003)

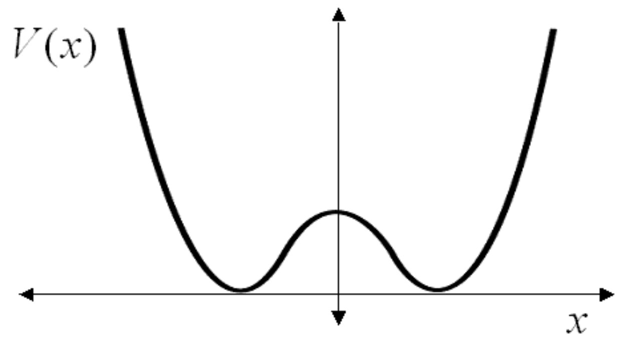
Raman Degeneracy and Schrödinger-Cat States



Fractionation? (macroscopic occupation of multiple single-particle states)

Dual case: Bosons in position space double-well potential

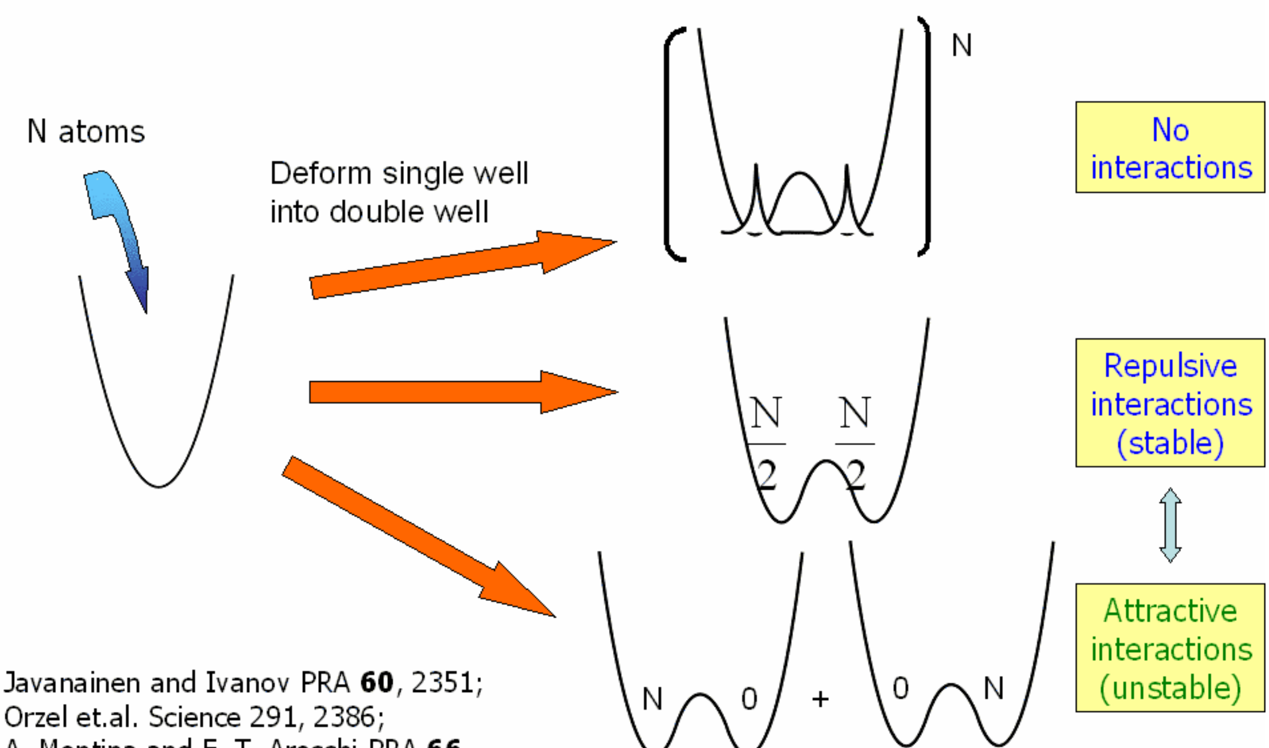
Paradigm for correlated many-body ground states (e.g. Mott insulator)



Two-mode approximation:

$$c_{R,L} = \frac{1}{\sqrt{2}}\left(\psi_{even} \pm \psi_{odd}\right)$$

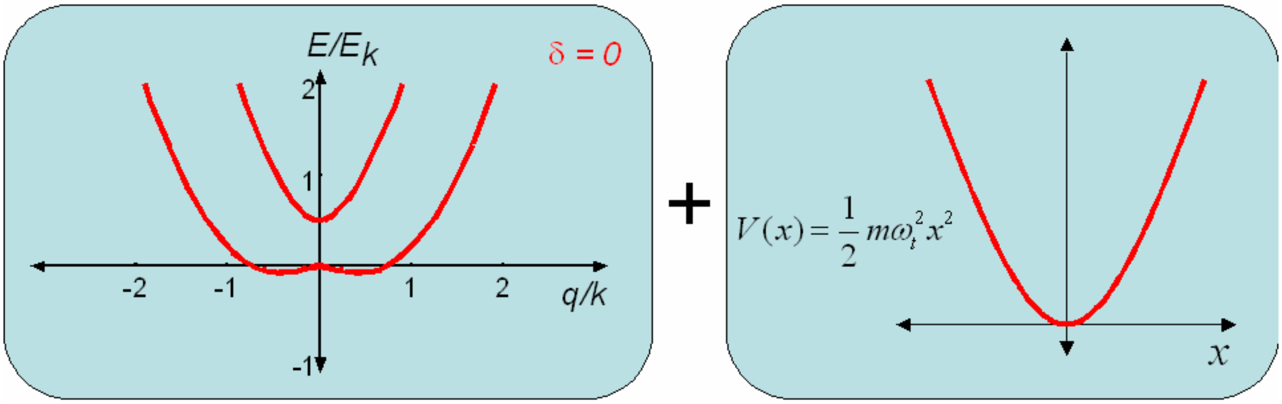
$$\mathrm{H} = -\frac{J}{2}\left(c_R^\dagger c_L + c_L^\dagger c_R\right) - U N_R N_L$$



Javanainen and Ivanov PRA **60**, 2351;
Orzel et.al. Science 291, 2386;
A. Montina and F. T. Arecchi PRA **66**, 013605

Needed:

- Momentum-space double-well potential (PDBEC)
- Tunneling between the wells (break conservation of momentum)



single-particle:

$$\mathrm{H}_{\mathrm{PD}}=\hbar\begin{pmatrix}\omega_\pi(\mathbf{q}) & 0\\ 0 & \omega_\mu(\mathbf{q})\end{pmatrix}-\frac{1}{M^2}R^\dagger(\mathbf{q})\vec{\nabla}_q^2R(\mathbf{q})$$

$$M=\frac{E_k}{\hbar\omega/2}=2\eta^2$$

many bosons:

$$J\propto e^{-\frac{M}{4}}$$

$$J_2\propto g e^{-\frac{M}{3}}$$

pair tunneling ??

$$\mathrm{H}=-UN_RN-\frac{(J-J_1)}{2}\left(c_R^\dagger c_L+c_L^\dagger c_R\right)+J_2\left(c_R^\dagger c_R^\dagger c_L c_L+c_L^\dagger c_L^\dagger c_R c_R\right)$$

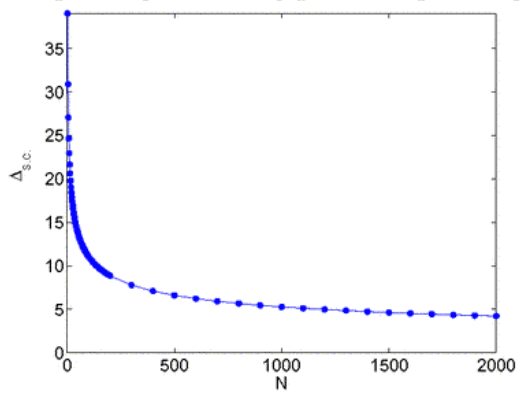
$$U\approx -g\langle n\rangle\sin^2\theta_{\mathrm{k}/2}$$

interactions tend to equalize energies of even and odd combinations

$$J_1\propto gNe^{-\frac{M}{8}}$$

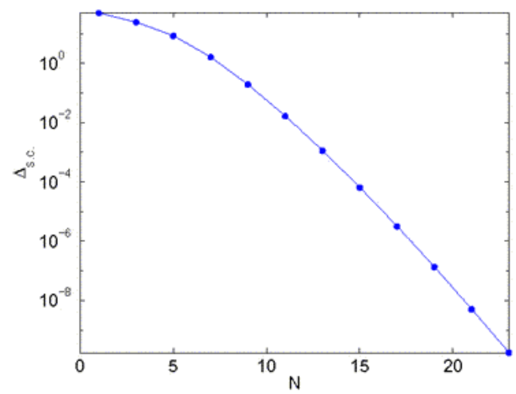
Cat State Energy Splittings

Splitting assuming partial squeezing



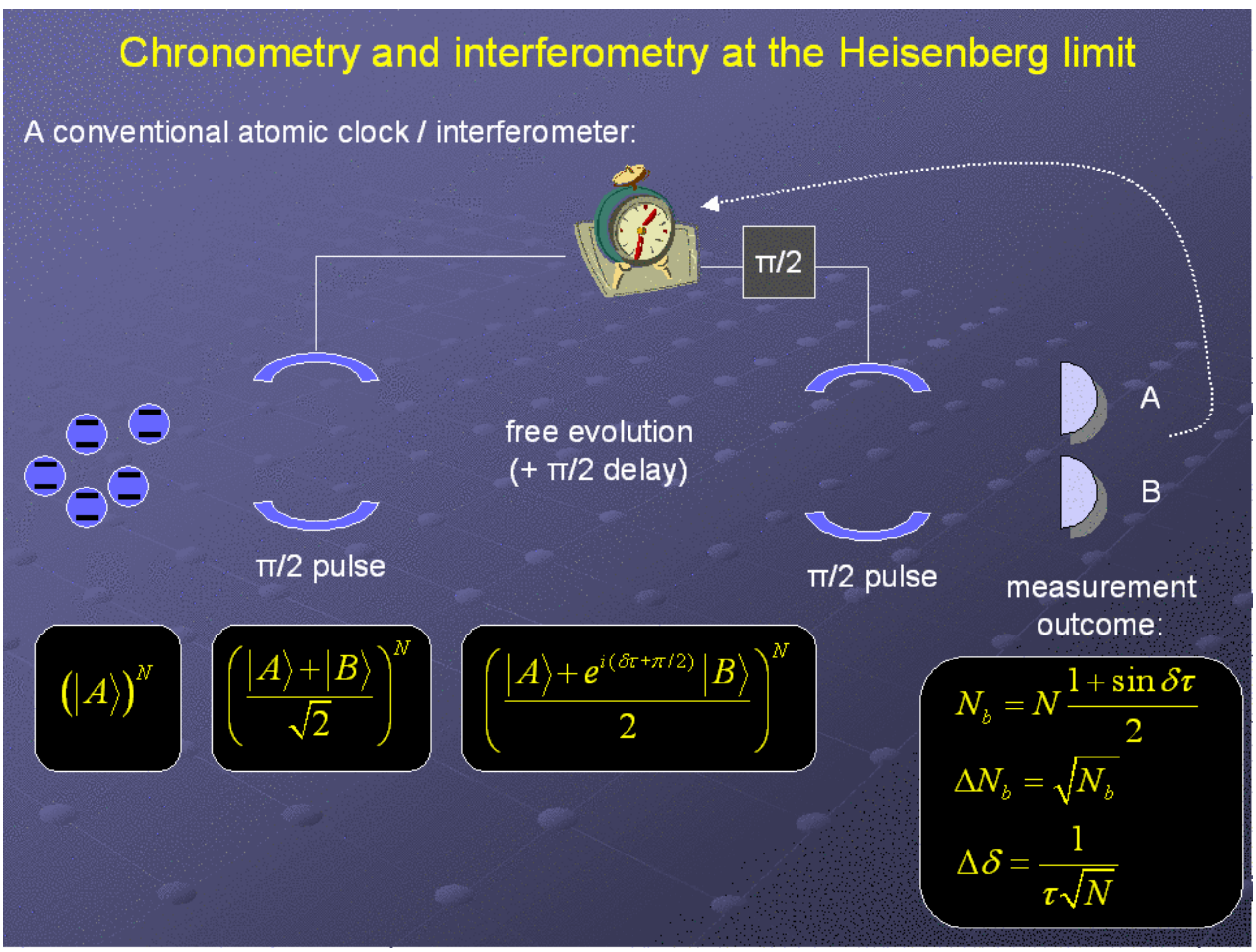
$\mathcal{J} = 50\text{Hz}$
$U = \mathcal{J}/N$

Splitting assuming pure S.C. state



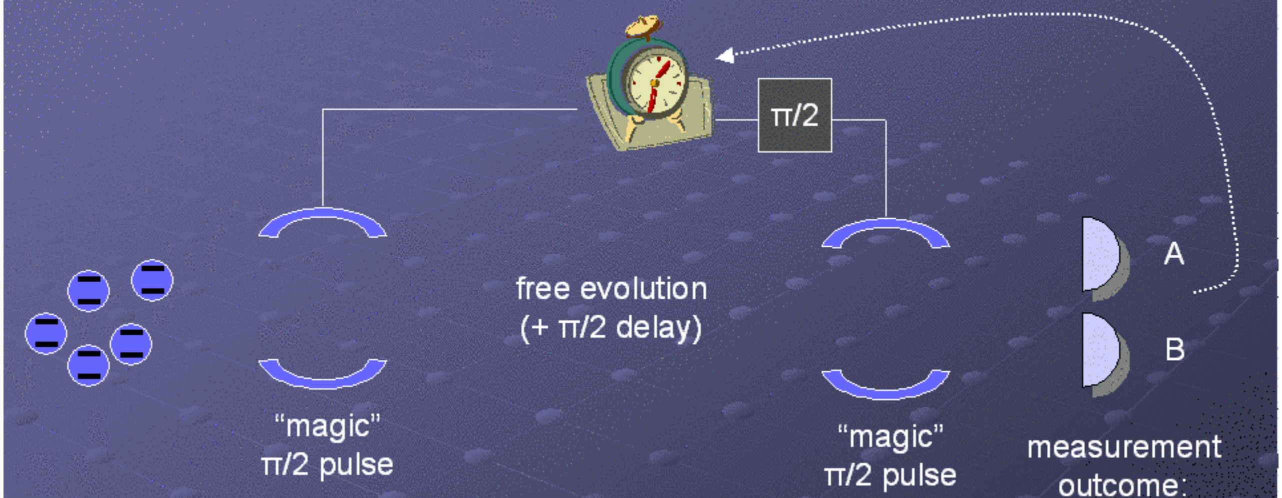
$\mathcal{J} = 50\text{Hz}$
$U = \mathcal{J}/\sqrt{N}$

- See also variational calculation by Montina et al. (**PRA** 67, 023616 (2003)) using Gross-Pitaevskii wavefunctions estimates even/odd S.C. energy splitting at $\sim 50\text{Hz}$ for $N \approx 1500$ using $\omega_{\text{trap}} \approx 100\text{Hz}$



Chronometry and interferometry at the Heisenberg limit

An unconventional atomic clock / interferometer:



$$\left(|A\rangle\right)^N \qquad \frac{\left(|A\rangle\right)^N + \left(|B\rangle\right)^N}{\sqrt{2}} \qquad \frac{\left(|A\rangle\right)^N + e^{i(N\delta\tau+\pi/2)}\left(|B\rangle\right)^N}{2}$$

$$N_b = N\frac{1+\sin N\delta\tau}{2}$$

$$\Delta N_b \approx \frac{N_b}{2}$$

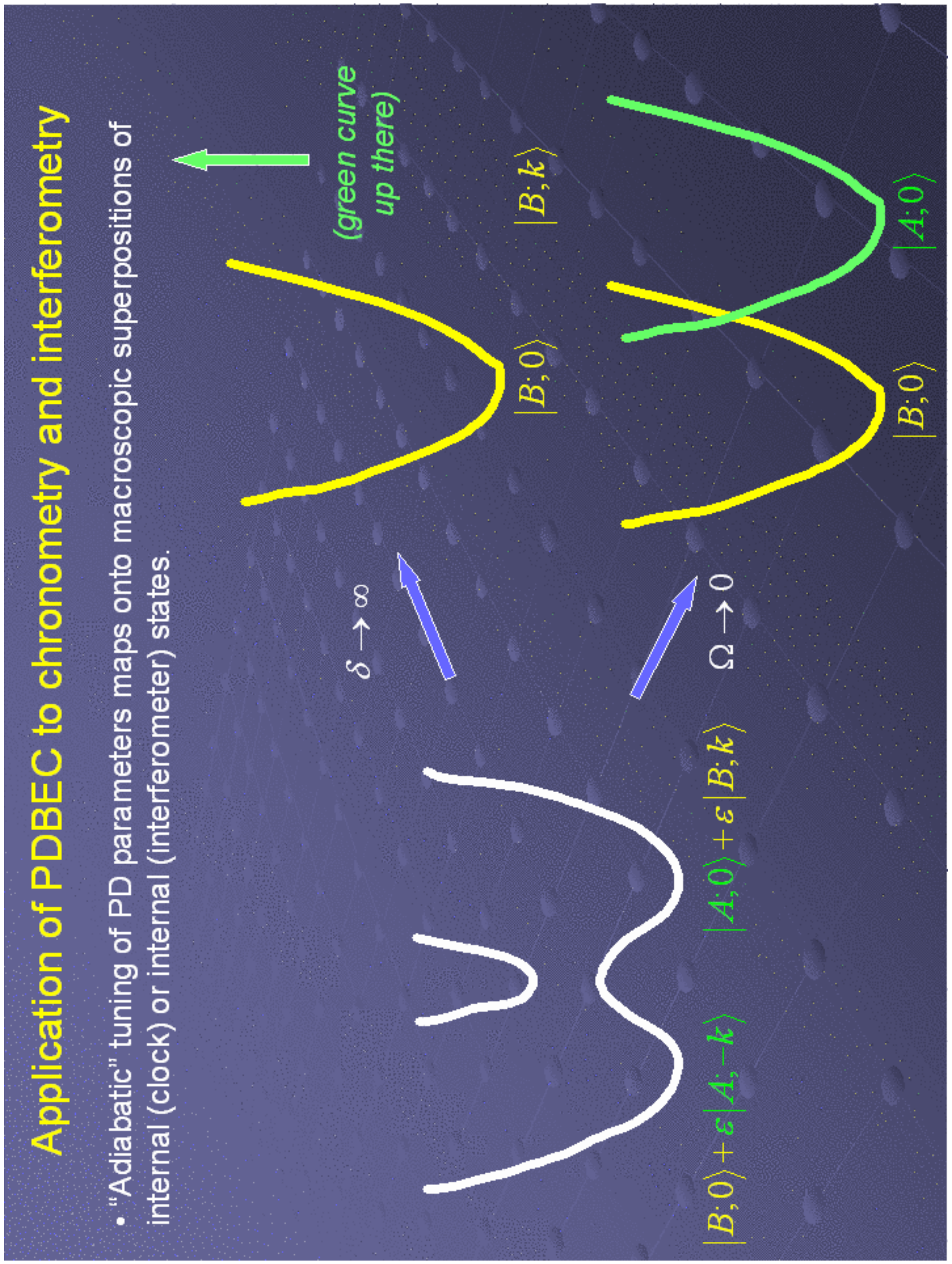
$$\Delta\delta = \frac{1}{\tau N}$$

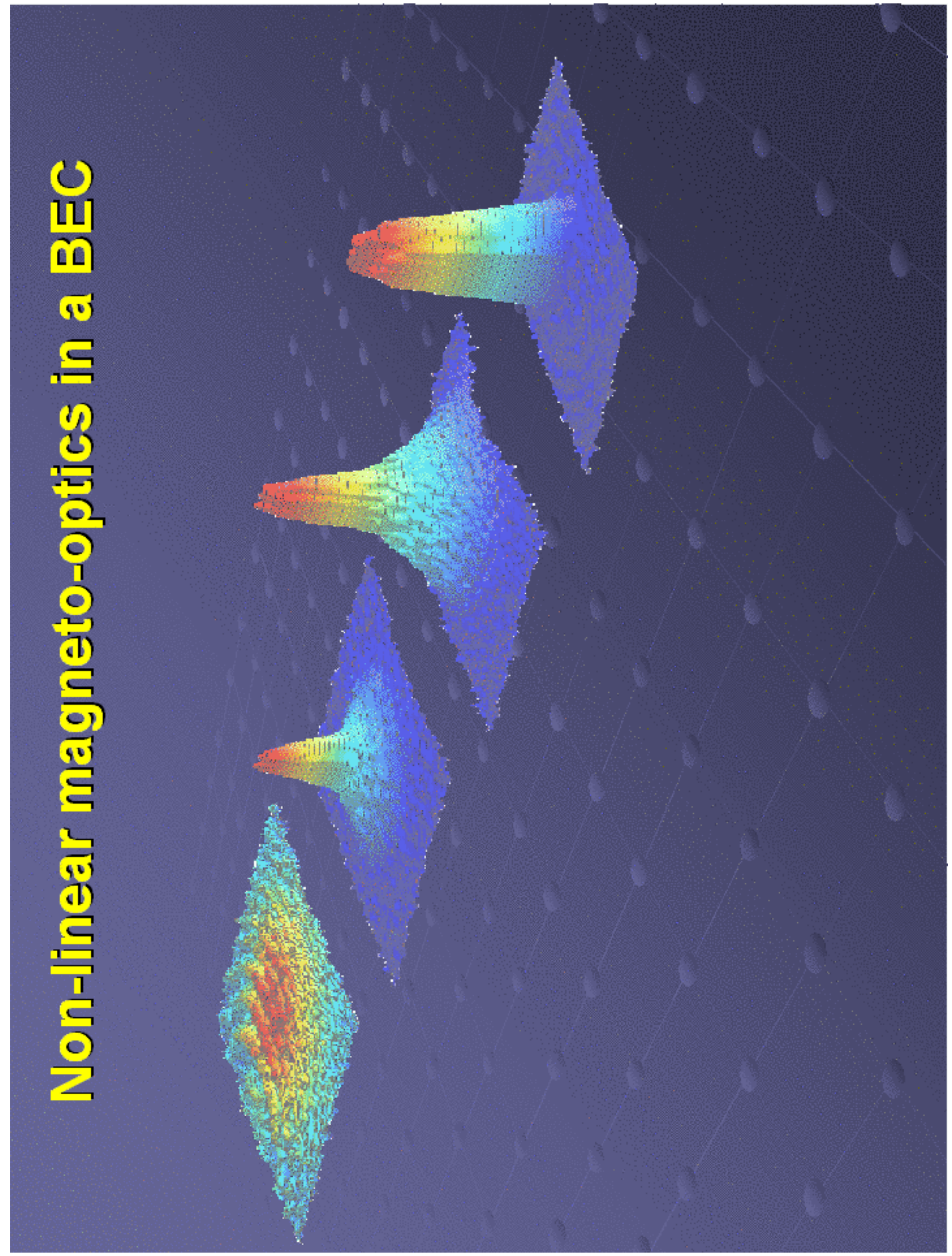
Application of PDBEC to chronometry and interferometry

- A "magic" π/2 pulse: Apply Raman lasers for ¼ period of macroscopic tunnel period

$$|i\rangle = \Psi_L \;\rightarrow\; \left[\Psi_0 = \frac{\Psi_R+\Psi_L}{\sqrt{2}} \quad \Psi_1 = \frac{\Psi_R-\Psi_L}{\sqrt{2}} \quad J_{Macro}(N)\right] \;\rightarrow\; |f\rangle = \frac{1-i}{2}\Psi_L + \frac{1+i}{2}\Psi_R$$

Raman beams off — Raman beams on — Raman beams off


Application of PDBEC to chronometry and interferometry
• "Adiabatic" tuning of PD parameters maps onto macroscopic superpositions of internal (clock) or internal (interferometer) states.
(green curve up there)
$\delta \to \infty$
$\Omega \to 0$
$|B;0\rangle$
$|B;k\rangle$
$|B;0\rangle$
$|A;0\rangle$
$|B;0\rangle + \varepsilon |A;-k\rangle$
$|A;0\rangle + \varepsilon |B;k\rangle$


Non-linear magneto-optics in a BEC

General view on non-linear optics and BEC

γ vs. $\omega_{Collective}$

Relaxation to equilibrium state

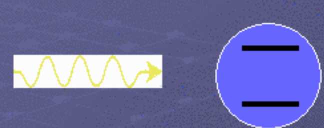
Collectively (Bose-) enhanced "ordering"
Pumping toward ordered state
Exponential/threshold behaviour
Link phase transition & NL optical response

saturation	spontaneous emission	vs.	Rabi oscillations
EIT	spin-relaxation, inhomogenous broadening (Doppler, Zeeman)	vs.	optical pumping, Rabi oscillations
non-linear Faraday rotation	spin-relaxation, magnetic inhomogeneity	vs.	optical pumping

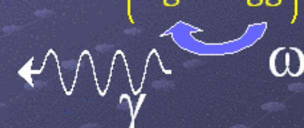
- How might this be generically different in a BEC?
- How might quantum statistics be relevant?
- Single-particle vs. many-body (collective) picture?

Non-linear Rayleigh scattering/superradiance

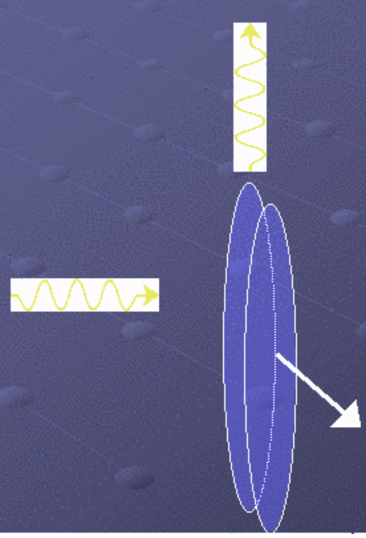
linear scattering:

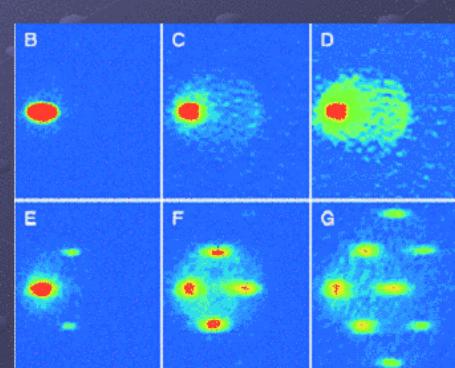


$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix}$$

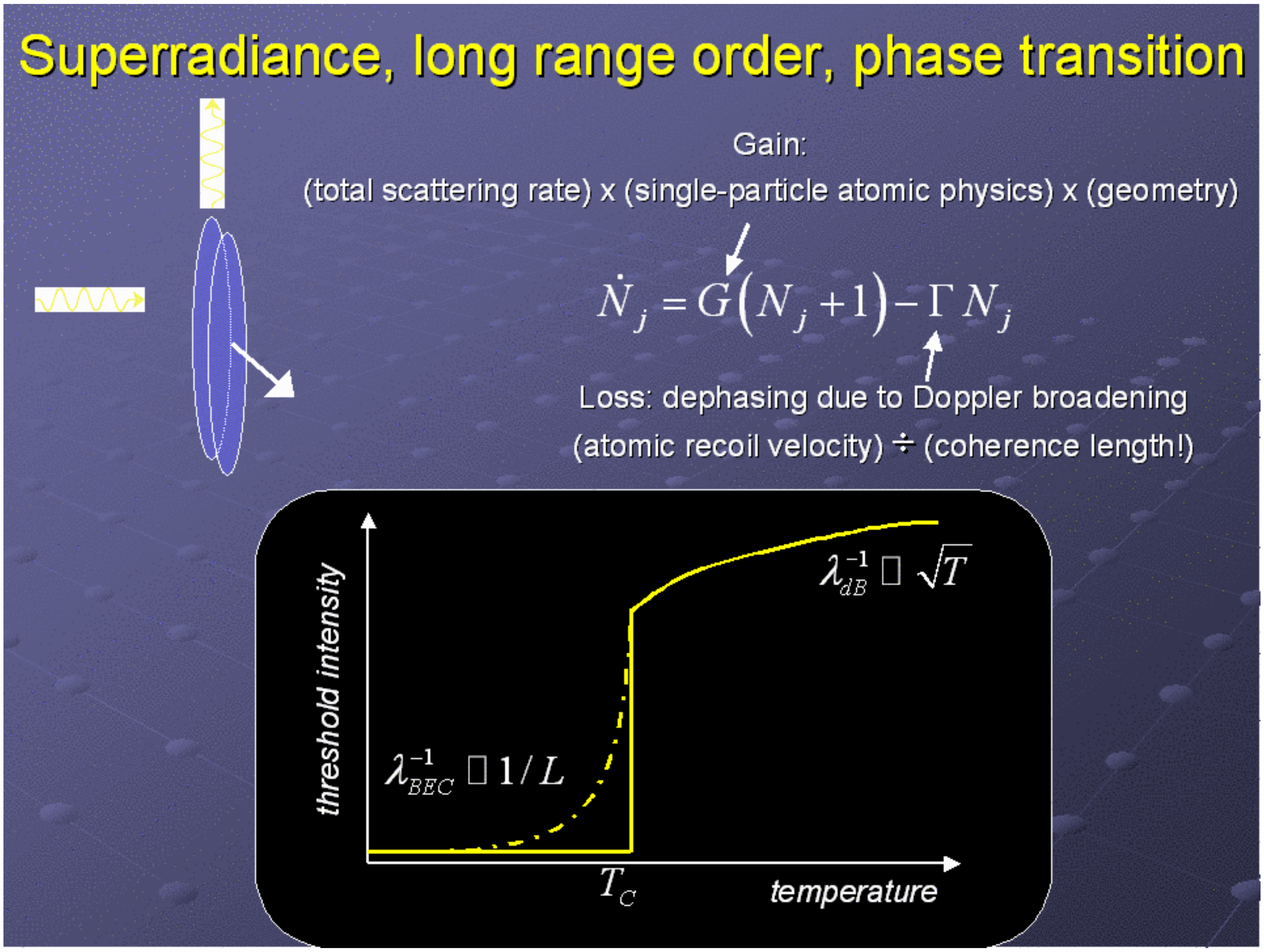


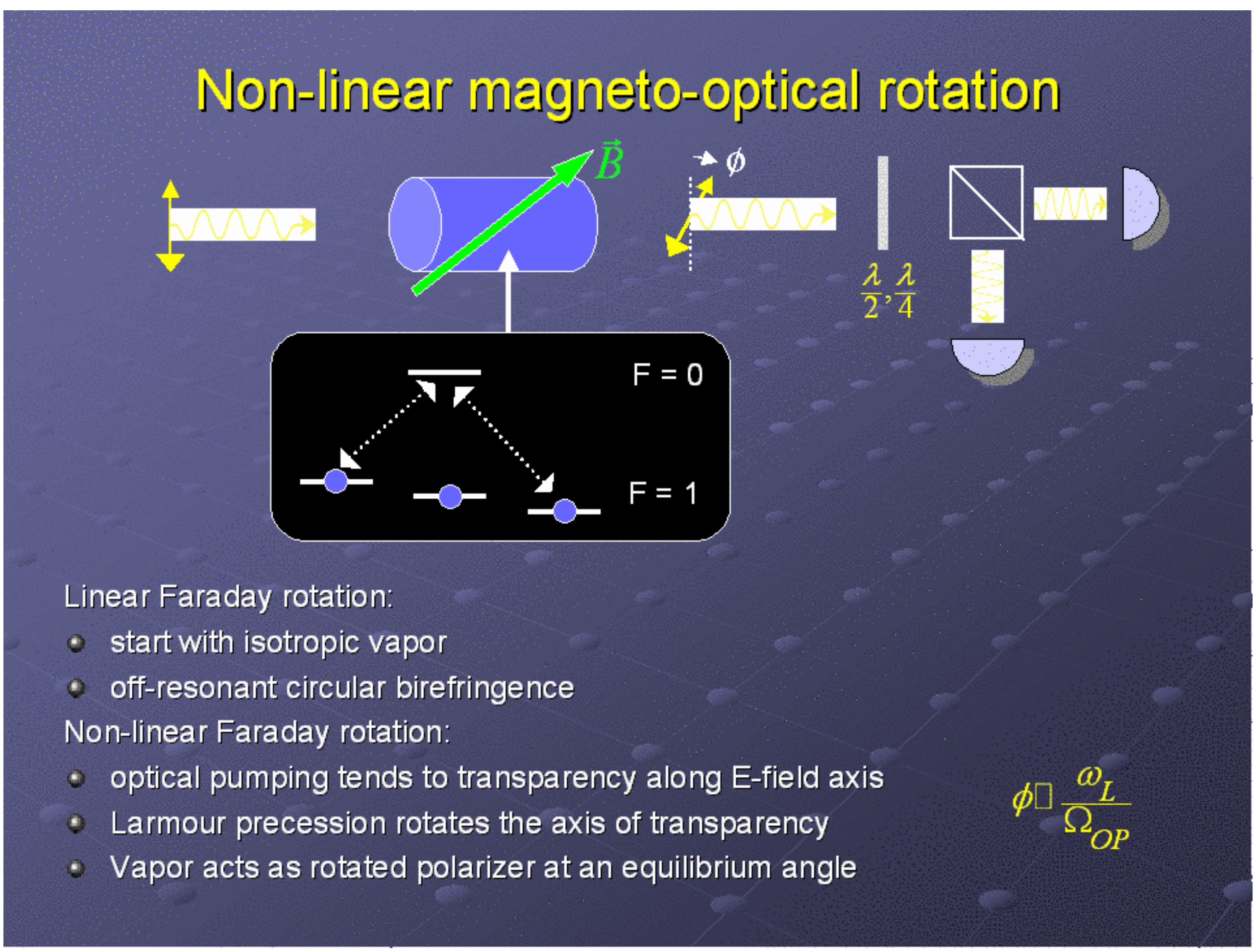
Collective scattering: Dicke superradiance





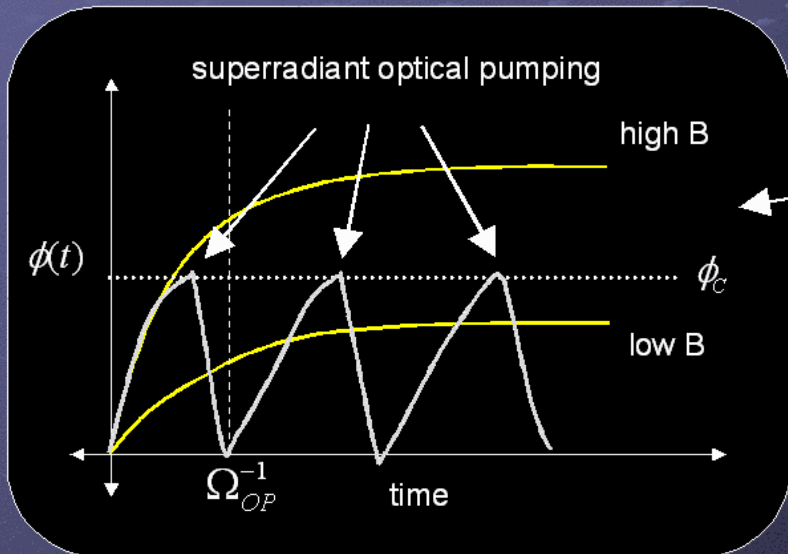
Rayleigh: Inouye et al., Science (1998)
Raman: Schneble et al., Yoshikawa et al. (2003)


Superradiance, long range order, phase transition
Gain:
(total scattering rate) x (single-particle atomic physics) x (geometry)
$\dot{N}_j = G(N_j + 1) - \Gamma N_j$
Loss: dephasing due to Doppler broadening
(atomic recoil velocity) ÷ (coherence length!)
threshold intensity
$\lambda_{BEC}^{-1} \sim 1/L$
$\lambda_{dB}^{-1} \sim \sqrt{T}$
T_C
temperature


Non-linear magneto-optical rotation
$\vec{B}$
ϕ
$\frac{\lambda}{2}, \frac{\lambda}{4}$
F = 0
F = 1
Linear Faraday rotation:
start with isotropic vapor
off-resonant circular birefringence
Non-linear Faraday rotation:
optical pumping tends to transparency along E-field axis
Larmour precession rotates the axis of transparency
Vapor acts as rotated polarizer at an equilibrium angle
$\phi \sim \frac{\omega_L}{\Omega_{OP}}$

A magneto-optical effect in a BEC

- Atoms prepared in anisotropic (spin-polarized) state
- observe rotation angle vs. time

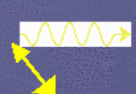


- critical angle ϕ_c or critical B vs. intensity

BEC selects optical/atomic recoil modes

The problem (opportunity):

- Arrange for superradiance to enhance optical pumping to dark state
- Arrange for dark state to rotate into the bright



Ψ

input polarization | input atomic mode | output propagation direction

$\Rightarrow$ two optical polarizations and two corresponding output atomic recoil modes
$\Rightarrow$ polarization with maximum intensity: preferred optical scattering/atomic recoil mode

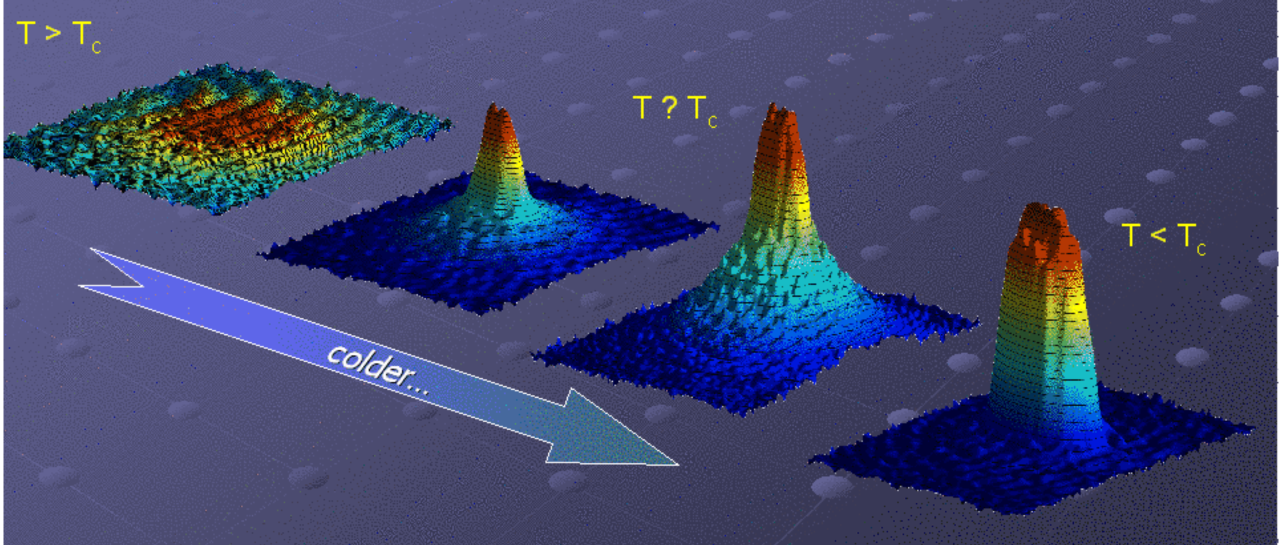
Orientation of anisotropic BEC:
$\Rightarrow$ Orientation specifies direction;
$\Rightarrow$ superradiance amplifies preferred mode?

Not entirely supported by data

- Some modes have higher loss, e.g. high optical density
- Predicted superposition recoil states not seen

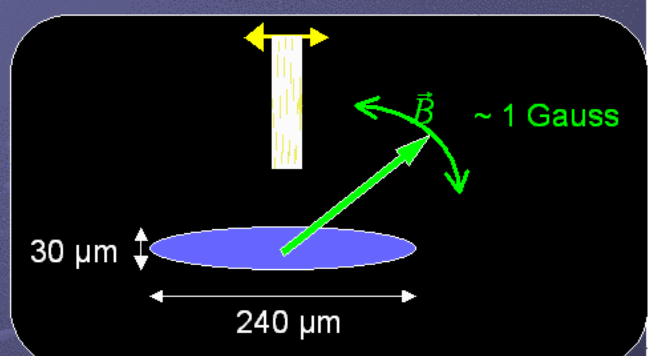
Vital statistics on Berkeley's Rb BEC

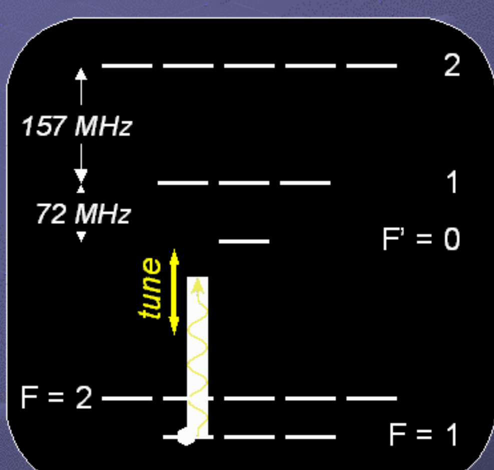
- Atom source: recirculating Rb oven and increasing-field Zeeman slower
- Ioffe-Pritchard trap, compressed to 6 x 160 x 160 Hz in F=1
- Evaporative cooling to T_c ? 500 nK, N_c ? 10^7
- "Pure" BEC of ? 2 x 10^6 produced, limited by 3-body losses

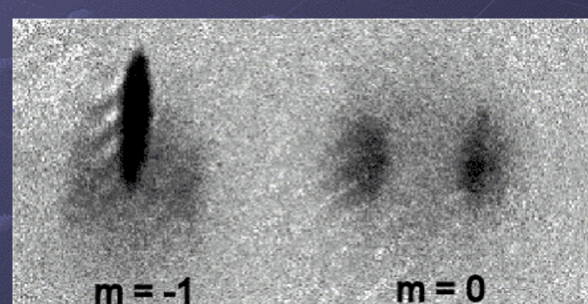


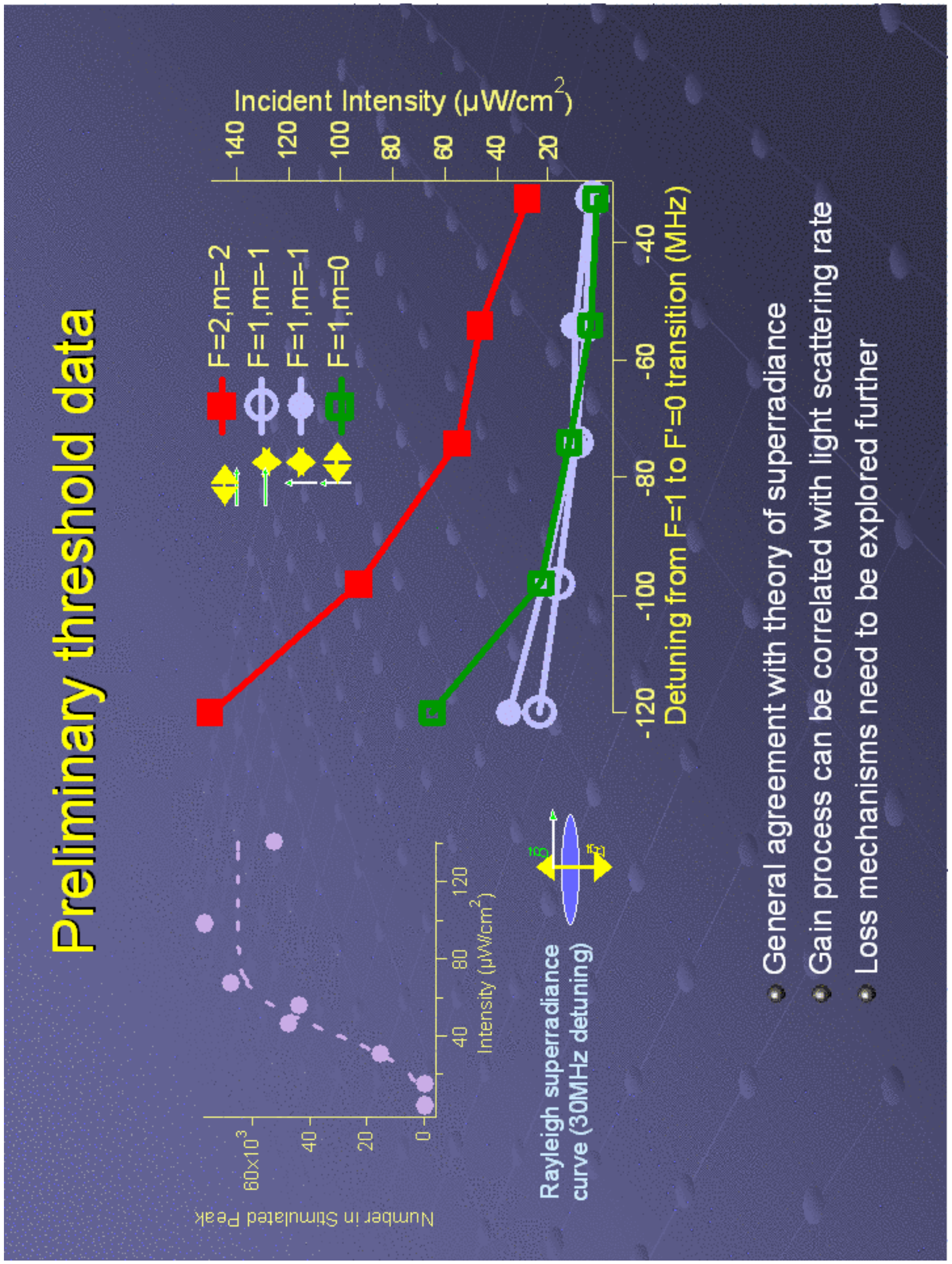
Implementation with ^{87}Rb BEC

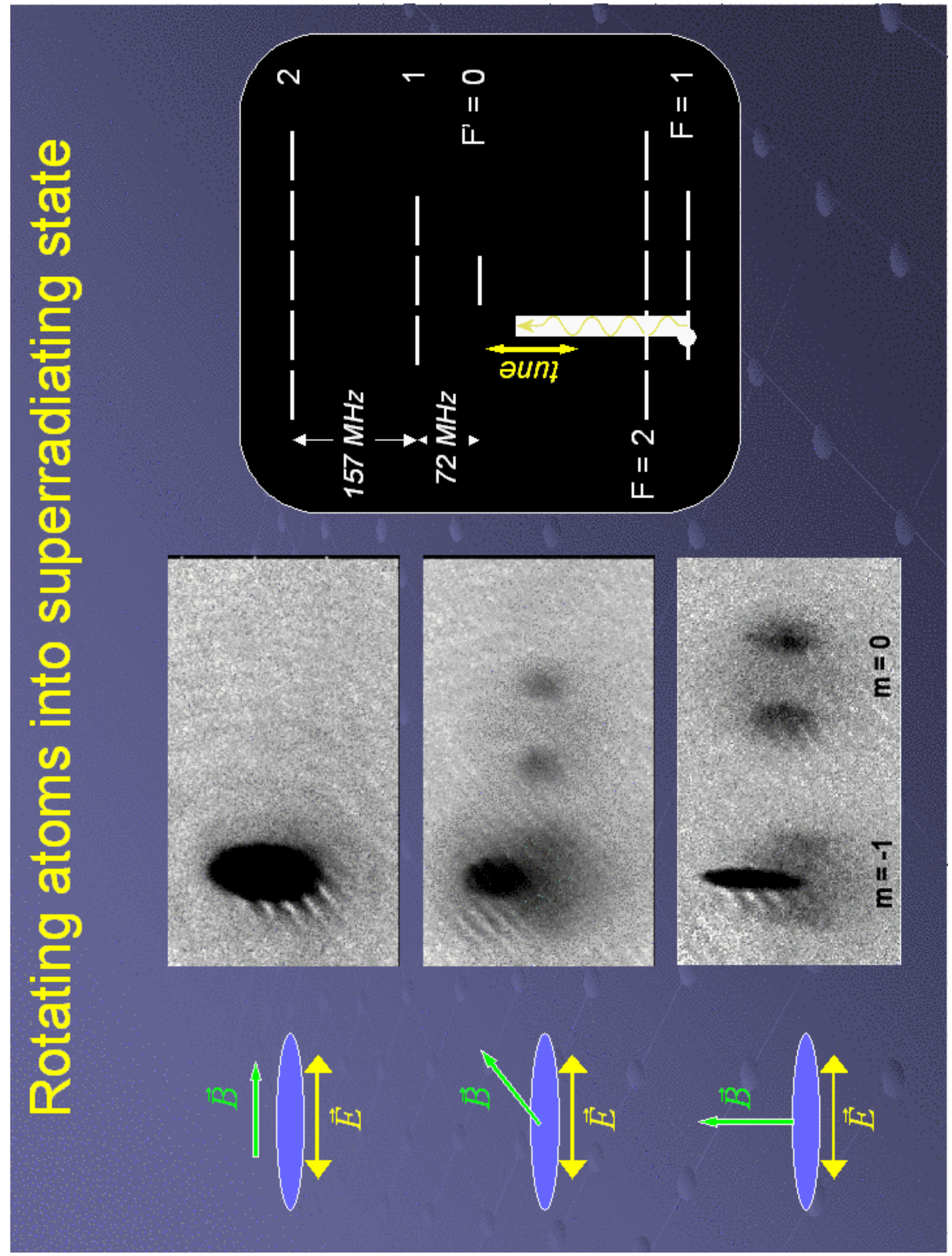
- BEC after 3.5 ms free expansion
  - optical density (1 ? 0) ~ 560 x 80
- Guide field controls internal state
- 0 to -120 MHz from F = 1 ? F' = 0
- time-of-flight + Stern-Gerlach separation



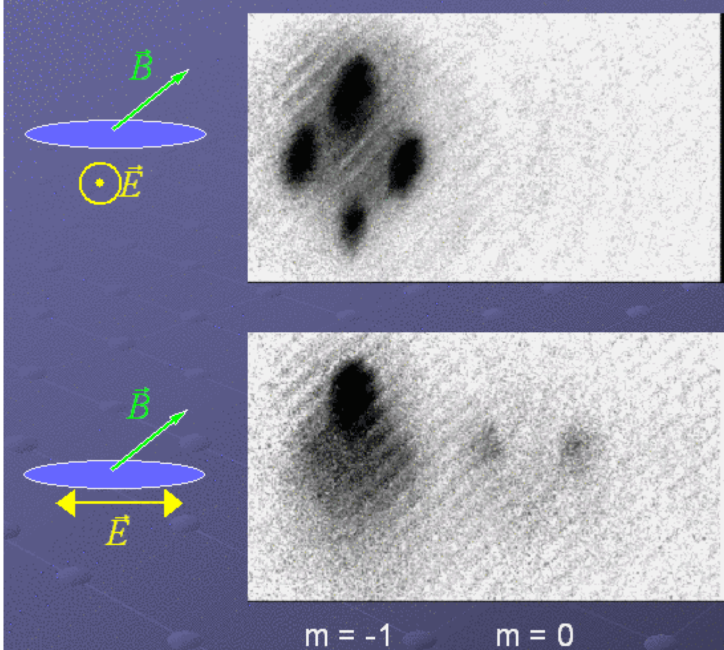





Preliminary threshold data
Incident Intensity (μW/cm2)
140 120 100 80 60 40 20
-40 -60 -80 -100 -120
Detuning from F=1 to F'=0 transition (MHz)
F=2,m=-2
F=1,m=-1
F=1,m=-1
F=1,m=0
Number in Stimulated Peak
60x10³ 40 20 0
0 40 80 120
Intensity (μW/cm2)
Rayleigh superradiance curve (30MHz detuning)
General agreement with theory of superradiance
Gain process can be correlated with light scattering rate
Loss mechanisms need to be explored further


Rotating atoms into superradiating state
2
1
F' = 0
157 MHz
72 MHz
tune
F = 2
F = 1
m = -1
m = 0

Searching for superposition recoil modes



Expected recoil mode populations
(m = -1) : (m = 0) : (m = +1):

preferred:	41 : 21 : 25
un-preferred:	27 : 55 : 4

- How do we understand superradiance from non-static superpositions?

Conclusions

- Begun examining general connection between Bose-Einstein condensation and non-linear optics
  - superradiant optical pumping, non Faraday rotation
- Further examination of Raman superradiance
  - optical density effects / role of dark states
  - preferred atomic recoil modes?
  - fate of superpositions?

BEC team:
Current: James Higbie, Lorraine Sadler, Ananth Chikkatur;
Past also: Craig Hetherington, Fabien Lienhart, Kevin Moore, Matt Pasienski, Veronique Savalli

http://physics.berkeley.edu/research/ultracold

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