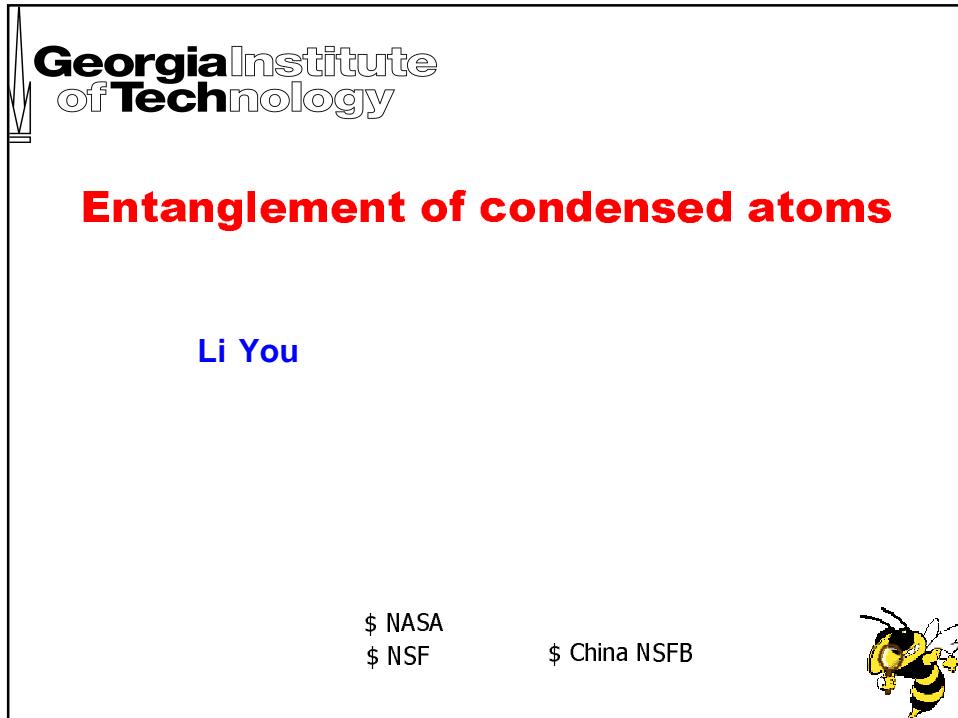


Quantum Entanglement of Condensed Atoms



Applications

Non-classical states of atoms ?

1. Precision measurement
2. Quantum information science

Quantum noise

$$\delta\theta : \frac{1}{\sqrt{N}} \Rightarrow \frac{1}{N}$$
$$\delta N = 1.$$


Quantum Entanglement of Condensed Atoms

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$$|N\text{-GHZ}\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow, \uparrow, \uparrow, \text{green}\rangle + |\downarrow, \downarrow, \downarrow, \text{green}\rangle e^{iN\varphi} \right)$$

$$|N\rangle = \frac{1}{\sqrt{2^N}} \left(|\uparrow\rangle + |\downarrow\rangle e^{i\varphi} \right)^N$$

M. J. Holland and K. Burnett, Phys. Rev. Lett. **71**, 1355 (1993)



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W. M. Itano et al., Phys. Rev. A **41**, 2295 (1990).
M. Kitagawa and M. Ueda, PRL **67**, 1852 (1991).



Manybody entanglement

correlated states from controlled dynamics

Atomic quantum gases

1. Many atoms (internal and external degrees of freedom),
2. Controlled interactions
3. Highly efficient quantum detections



Identical particle entanglement

Single particle (mode) entanglement ?

$$(a^\dagger + b^\dagger)|0,0\rangle_{+,-} \rightarrow \frac{1}{\sqrt{2}}(|1,0\rangle_{+,-} + |0,1\rangle_{+,-})$$

Separable for the modes,
but particle entangled?

$$\begin{aligned} a^{\dagger N} b^{\dagger N} |0,0\rangle &\rightarrow |N,N\rangle \\ &\rightarrow [\varphi_a(r_A) \varphi_b(r_B) + \varphi_b(r_A) \varphi_a(r_B)]^{\otimes N} \end{aligned}$$

L.-M. Duan, J. I. Cirac, and P. Zoller, Phys. Rev. A **65**, 033619 (2002)



Quantum Entanglement of Condensed Atoms

Two fermions

Schmidt expansion in terms of orthogonal
Slater determinant

$$\begin{aligned} |\Psi_F\rangle &= \sum_{i,j}^N \omega_{ij} f_i^\dagger f_j^\dagger |0\rangle \\ &= 2 \sum_k^{\leq N/2} Z_k f_{2k-1}^{*\dagger} f_{2k}^{*\dagger} |0\rangle \end{aligned}$$

John Schliemann, Daniel Loss, A. H. MacDonald, Phys. Rev. B **63**, 085311 (2001).

John Schliemann, J. I. Cirac, M. Ku's, M. Lewenstein, and Daniel Loss, Phys. Rev. A **64**, 022303 (2001).

Is this useful entanglement or correlation ?



Two bosons

Schmidt expansion in terms of orthogonal
2-boson modes

$$\begin{aligned} |\Psi_B\rangle &= \sum_{i,j}^N \beta_{ij} b_i^\dagger b_j^\dagger |0\rangle \\ &= \sqrt{2} \sum_k^N B_k b_k^{*\dagger} b_k^{*\dagger} |0\rangle \\ &= B_1 |2,0,0,\text{ggg}\rangle + B_2 |0,2,0,\text{ggg}\rangle + B_3 |0,0,2,\text{ggg}\rangle + \text{ggg} \end{aligned}$$

R. Pausauskas and L. You, Phys. Rev. A **64**, 042310 (2001).

Y.S. Li, B. Zeng, X.S. Liu, and G. L. Long, Phys. Rev. A **64**, 054302 (2001).

**Inseparable correlations beyond what is required
from exchange symmetry among identical particles**



Quantum Entanglement of Condensed Atoms



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Entanglement and spin squeezing of three bosons in two modes

L. You

D. L. Zhou
B. Zeng, Z. Xu (Tsinghua Univ.)



Three distinguishable qubits

Generalized Schmidt decomposition:

$$l_0 |000\rangle + l_1 e^{ij} |100\rangle + l_2 |101\rangle + l_3 |110\rangle + l_4 |111\rangle$$

A. Acin, A. Andrianov, L. Costa, E. Jane, J.I. Latorre,
and R. Tarrach, Phys. Rev. Lett. **85**, 1560(2000).

3-qubit entanglement, SLOCC,
Two types of entanglement:

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$

W. Dur, G. Vidal, and J.I. Cirac, Phys. Rev. A **62**, 062314(2000).



Quantum Entanglement of Condensed Atoms

Three bosons in two modes

Generic form:

$$a |000\rangle + b(|100\rangle + |010\rangle + |001\rangle) \\ + c(|110\rangle + |101\rangle + |011\rangle) + d|111\rangle$$

Single particle basis rotation (unitary transformation),

$$|0\rangle \circledR a|0\rangle + b|1\rangle \\ |1\rangle \circledR -b^*|0\rangle + a^*|1\rangle$$



Three bosons in two modes

Standard form:

$$r|000\rangle + s\mathcal{C}^{ij}(|100\rangle + |010\rangle + |001\rangle) + t|111\rangle$$

Two types of entanglement:

$$|\text{GHZ}\rangle = z_a|aaa\rangle + z_b|bbb\rangle$$

$$t=0,$$

$$|\text{W}\rangle = \frac{1}{\sqrt{3(1+2|\langle a|b\rangle|^2)}}(|abb\rangle + |bab\rangle + |bba\rangle)$$

$$t=0;$$



Quantum Entanglement of Condensed Atoms

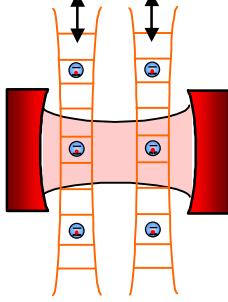
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Encoding a physical qubit into a logical qubit

L. You

D. L. Zhou,
B. Zeng, Z. Xu (Tsinghua Univ.)
C. P. Sun (ITP)



Errors (single qubit)

any 1-bit error:

$$E_i = e_{i0}I + e_{i1}X_1 + e_{i2}Z_1 + e_{i3}X_1Z_1$$

bit-flip: $X_1: \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|1\rangle + \beta|0\rangle$

phase-error: $Z_1: \alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle$



Error correcting codes

bit-flip code

$$|0\rangle \rightarrow |0_L\rangle \equiv |000\rangle$$

$$|1\rangle \rightarrow |1_L\rangle \equiv |111\rangle$$

phase-flip code

$$|0\rangle \rightarrow |0_L\rangle \equiv |+++ \rangle$$

$$|1\rangle \rightarrow |1_L\rangle \equiv |--- \rangle$$

Initial state

$$H = uJ_x^2$$

$$|\psi(0)\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |0, \underbrace{\dots}_{N}, 0\rangle$$

Cavity QED

Josephson junctions

Atoms in an optical lattice

N ancilla bits



Three qubit bit-flip code

$$|0\rangle \rightarrow |0_L\rangle \equiv |000\rangle$$

1 bit error rate $p < 1$,

$$|1\rangle \rightarrow |1_L\rangle \equiv |111\rangle$$

(3 bit) error rate $p^3 + 3p^2 (< p)$,

error-detection (quantum measurement):

$$P_0 \equiv |000\rangle\langle 000| + |111\rangle\langle 111|$$

no error

$$P_1 \equiv |100\rangle\langle 100| + |011\rangle\langle 011|$$

bit flip on qubit one

$$P_2 \equiv |010\rangle\langle 010| + |101\rangle\langle 101|$$

bit flip on qubit two

$$P_3 \equiv |001\rangle\langle 001| + |110\rangle\langle 110|$$

bit flip on qubit three

Recovery: flip the flipped qubit again



Quantum Entanglement of Condensed Atoms

Three qubit phase-flip code

$$|0\rangle \rightarrow |0_L\rangle \equiv |+++ \rangle$$

$$|1\rangle \rightarrow |1_L\rangle \equiv |---\rangle$$

error-detection (quantum measurement):

$$P_0 \equiv |+++ \rangle \langle +++| + |---\rangle \langle ---| \quad \text{no error}$$

$$P_1 \equiv |---\rangle \langle -++| + |++-\rangle \langle +--| \quad \text{phase flip on qubit one}$$

$$P_2 \equiv |+-+\rangle \langle +--| + |-+-\rangle \langle -+-| \quad \text{qubit two}$$

$$P_3 \equiv |++-\rangle \langle ++-| + |--+ \rangle \langle +-+| \quad \text{qubit three}$$

Recovery: flip the flipped qubit again



The Shor code

$$|0\rangle \rightarrow |0_L\rangle \equiv \left(\frac{|000\rangle + |111\rangle}{\sqrt{2}} \right)^{\otimes 3}$$

$$|1\rangle \rightarrow |1_L\rangle \equiv \left(\frac{|000\rangle - |111\rangle}{\sqrt{2}} \right)^{\otimes 3}$$

P. Shor, PRA **52**, 2493 (1995).



Our encoding scheme

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|_{\substack{0 \\ 1 \\ 4 \\ 2 \\ N+1}}\cdots|_{\substack{0 \\ 1 \\ 2 \\ 3 \\ N+1}} + \beta|_{\substack{1 \\ 0 \\ 1 \\ 2 \\ N+1}}\cdots|_{\substack{0 \\ 1 \\ 2 \\ 3 \\ N+1}}$$

Initial state

$$|\psi(0)\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |_{\substack{0 \\ 1 \\ 4 \\ 2 \\ N}}\cdots|_{\substack{0 \\ 1 \\ 2 \\ 3 \\ N}}$$

N ancilla bits



First protocol

$$(\alpha|0\rangle + \beta|1\rangle) \otimes |_{\substack{0 \\ 1 \\ 4 \\ 2 \\ N}}\cdots|_{\substack{0 \\ 1 \\ 2 \\ 3 \\ N}}$$

$$H = uJ_x^2$$

1 4 2 4 4

$$\sigma_z^{(1)} \sigma_z^{(2)}$$

1 4 2 4 4

$$H = uJ_x^2$$



Quantum Entanglement of Condensed Atoms

Alternative protocol

$$(\alpha|0\rangle + \beta|1\rangle)_{\frac{1}{4} \frac{1}{2} \frac{1}{4} \frac{3}{4}} \otimes |0, \dots, 0\rangle$$

$$H = uJ_x^2$$

6 4 7 48

Measure 1st qubit



Experimental systems

Ion traps

$$(\alpha|0\rangle + \beta|1\rangle)_{\frac{1}{4} \frac{1}{2} \frac{1}{4} \frac{3}{4}} \otimes |0, \dots, 0\rangle_{N=8}$$

$$\rightarrow \alpha|0,0,0\rangle|0,0,0\rangle|0,0,0\rangle + \beta|1,1,1\rangle|1,1,1\rangle|1,1,1\rangle$$

$$\rightarrow \alpha \left[\frac{1}{\sqrt{2}}(|0,0,0\rangle - |1,1,1\rangle) \right]^{\otimes 3} + \beta \left[\frac{1}{\sqrt{2}}(|0,0,0\rangle + |1,1,1\rangle) \right]^{\otimes 3}$$

Cavity QED
Josephson Junctions
Atoms in an optical lattice

