Tutorial: AMO Hubbard "toolbox"

P. Zoller, Univ of Innsbruck, Austria

- Optical lattices
  - basic ideas, properties & special topics
- Hubbard models
  - naive derivation & microscopic picture, spin models, validity
- Lattice loading & Measurements
- Time-dependent aspects
- Impurities
- Phonons
  - cavities, (laser assisted) sympathetic cooling

This tutorial …

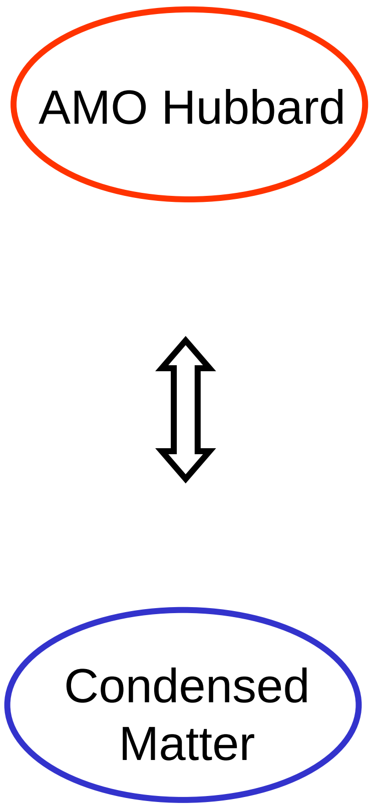
AMO Hubbard

- Hubbard Models with quantum degenerate atomic bose and fermi gases
  - which Hubbard models / interactions?
  - validity?
- AMO spectroscopic tools

⇕

Condensed Matter

- Condensed Matter / Hubbard models
  - solution
  - properties of phases, excitations etc.



Why?

- AMO experiment as simulator for (exotic) quantum phases
  - where no theoretical tools are available (e.g. sign problem)
  - where exact solutions are available (1D)
  - [application: quantum computing?]
- time dependence
- … ?

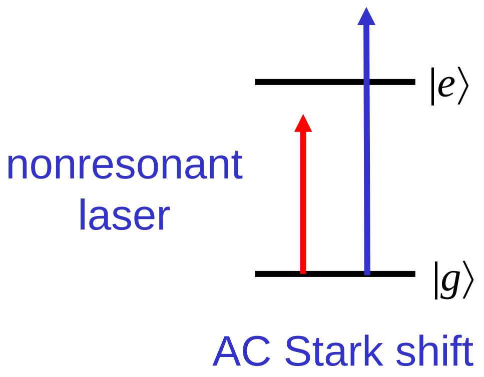
Which models are important?

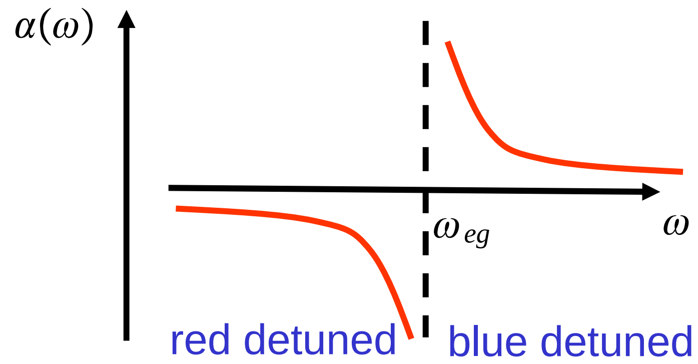
Expectations?

New tools?

1. Optical lattices: basics

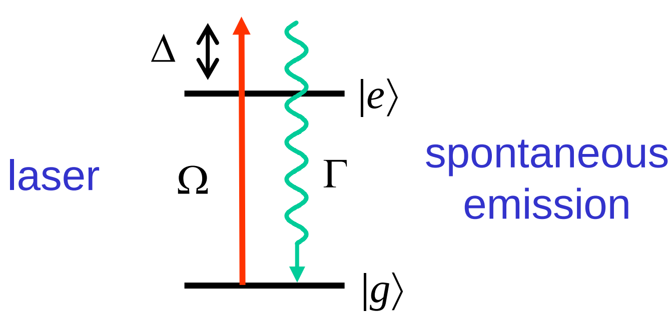
- AC Stark shift





$$\Delta E = \alpha(\omega) I \qquad (I \sim |\mathcal{E}|^2)$$

- decoherence: spontaneous emission



$$\Delta E_g = \frac{1}{4}\frac{\Omega^2}{\Delta - \frac{1}{2} i\Gamma} = \delta E_g - i\frac{1}{2}\gamma_g$$

$$\frac{\text{good}}{\text{bad}} = \frac{\delta E_g}{\gamma_g} \sim \frac{|\Delta|}{\Gamma} \gg 1$$

typical off-resonant lattice : $\gamma \sim \text{sec}^{-1}$

In a blue detuned lattice this can be strongly suppressed

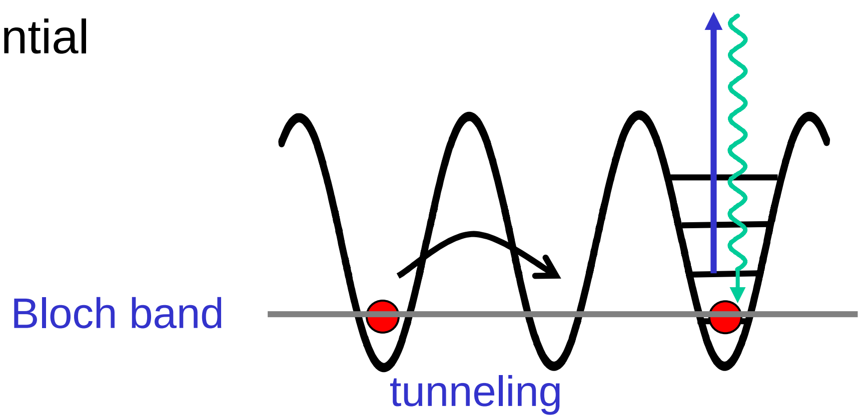
Zoller 2004

- standing wave laser configuration

laser → ← laser

$$\vec{E} = \vec{\epsilon}\mathcal{E}_0 \sin kz e^{-i\omega t} + \text{c. c.}$$

- optical potential



optical lattice as array of microtraps

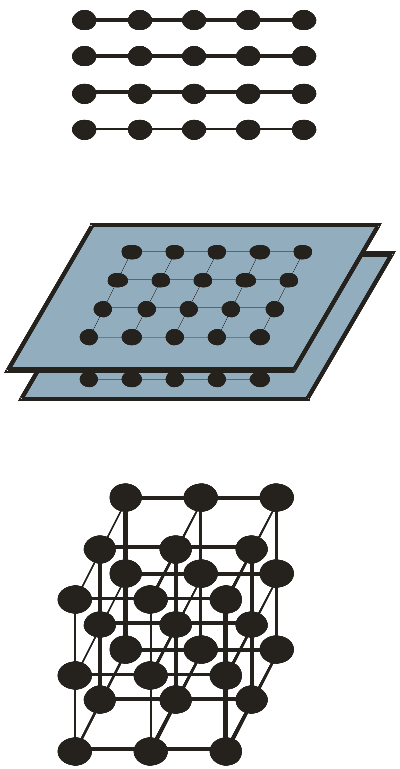
$$V(x) = V_0 \sin^2 kx \qquad (k = \tfrac{2\pi}{\lambda})$$

- Schrödinger equation for center of mass motion of atom

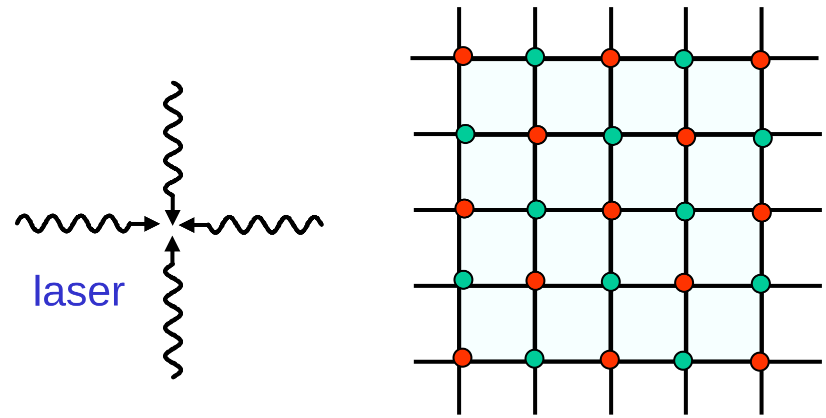
$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\psi(x,t)$$

Zoller 2004

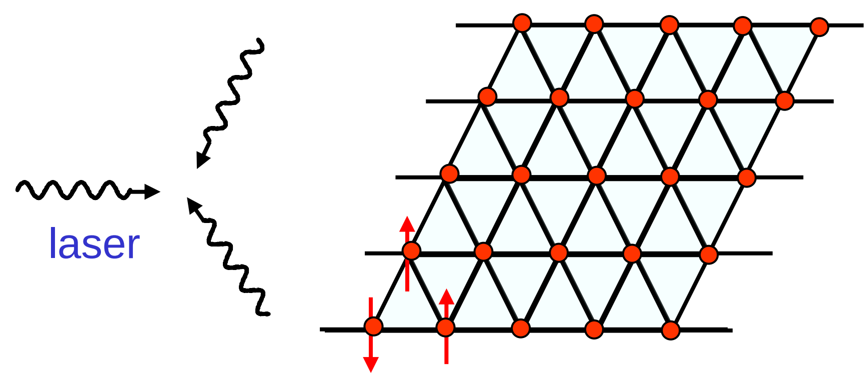
- 1D, 2D and 3D



- lattice configurations



square lattice



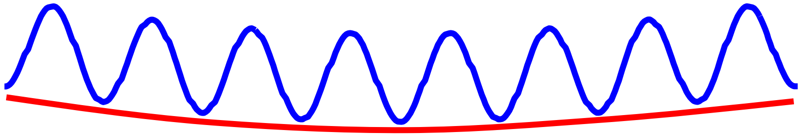
triangular lattice

Remarks:

✓ optical potentials generated by lattice beams with different frequencies add up incoherently

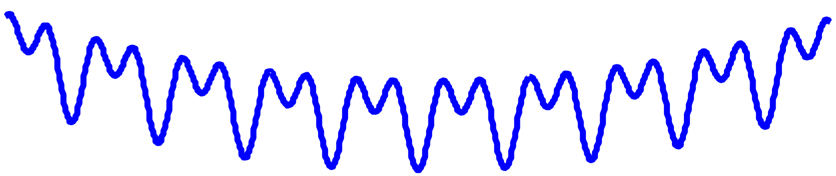
✓ interferometric stability (!?)

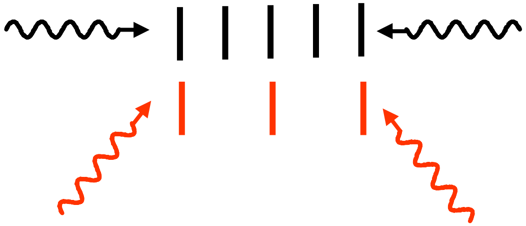
- harmonic background potential (e.g. laser focus, magnetic trap)



undo by inverse harmonic potential e.g. magnetic field

- superlattice





- random potentials
  - add more lasers from random direction or speckle pattern

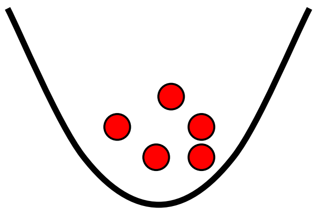
- Single atom coherent dynamcis studied …
  - Wannier-Bloch
  - quantum chaos: kicked systems

Zoller 2004

Jaksch et al 1998

2. Bose Hubbard in optical lattice: naïve derivation

- dilute bose gas



$$H = \int \psi^\dagger(\vec{x}) \left(-\frac{\hbar^2}{2m}\nabla^2 + V_T(\vec{x}) \right) \psi(\vec{x})\, d^3x$$

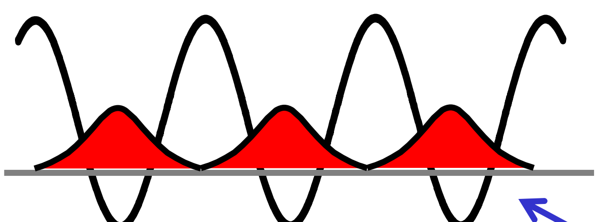
$$+\frac{1}{2} g \int \psi^\dagger(\vec{x})\psi^\dagger(\vec{x})\psi(\vec{x})\psi(\vec{x})\, d^3x$$

collisions

$$g = \frac{4\pi a_s \hbar^2}{m}$$ scattering length

- validity: dilute gas, $a_s \ll a_0 < \lambda/2$

- optical lattice



$$\psi(\vec{x}) = \sum_\alpha w(\vec{x} - \vec{x}_\alpha)\, b_\alpha$$

Wannier functions

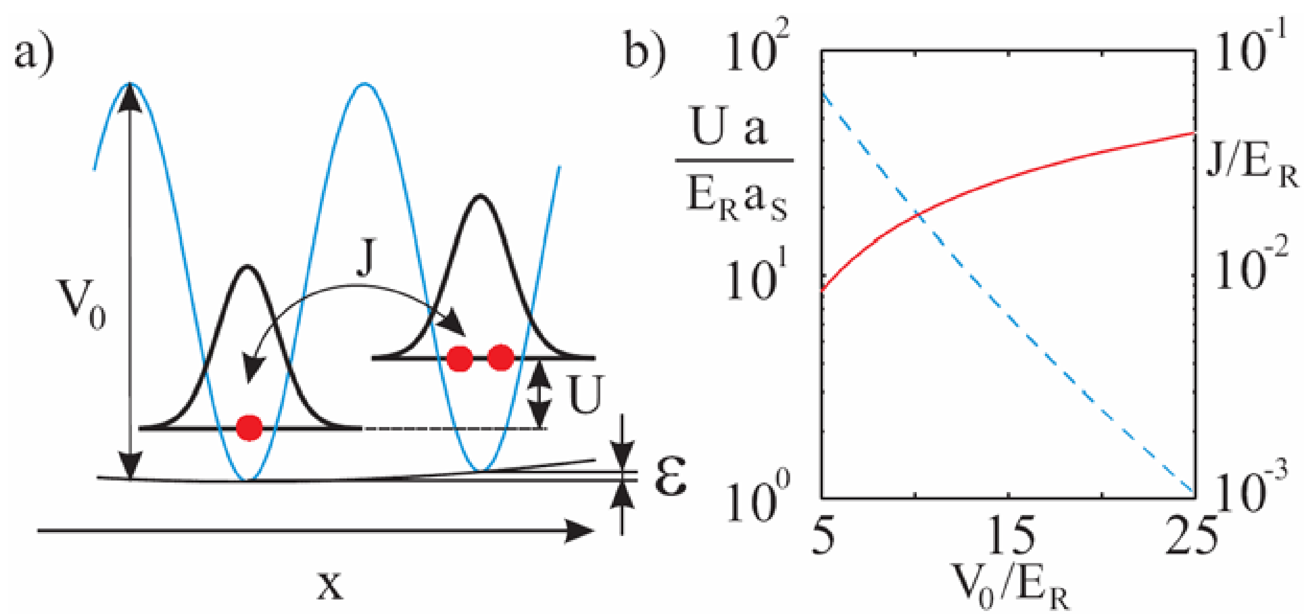
- Hubbard model

$$H = -J\sum_{<i,j>} b_i^\dagger b_j + \frac{1}{2}U\sum_i b_i^\dagger b_i^\dagger b_i b_i + \sum_i \epsilon_i b_i^\dagger b_i$$

kinetic engery: hopping

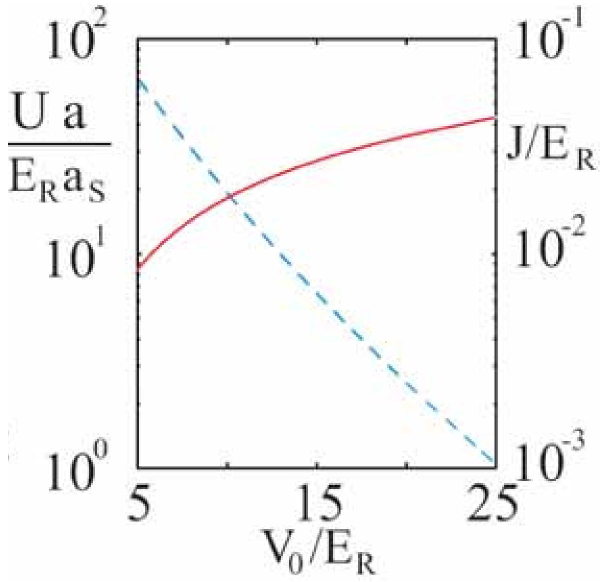
interaction: onsite repulsion

$$U = g\int |w(\vec{x})|^4 d^3x$$



- feature: (time dep) tunability from weakly to strongly interacting gas
- validity …

Zoller 2004



- parameters approx. SF-Mott transition:

  recoil energy $E_R = \hbar^2 k^2/2m,\ V_0 \sim 9E_R$

  Na: $E_R = 25$ KHz, $J \sim 1$ KHz, $U \sim 10$ KHz,

  Rb: $E_R = 3.8$ KHz ...

- validity:

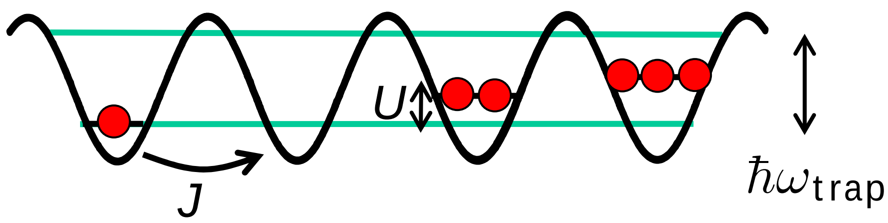
  $a_s \ll a_0 < \lambda/2,\ U \ll \hbar\omega_{\text{Bloch}}$ and $T \sim 0:\quad kT \ll J, U \ll \hbar\omega_{\text{Bloch}}$,

  density $n \sim 10^{14} - 10^{15}\text{cm}^{-3}$ (three particle loss)

Zoller 2004

Hubbard model: microscopic picture

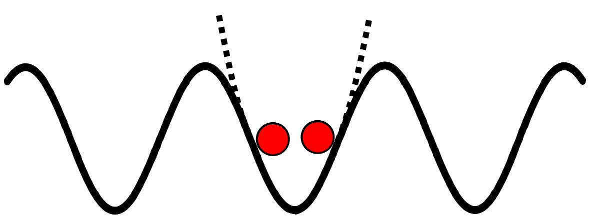
- Hubbard



✓ solve in n=1,2,3,… particle sector

✓ connect by tunneling (e.g in a tight binding approx)

- n=2 atoms on one lattice site: molecule



✓ molecular problem with added optical potential

- n=3 atoms on one lattice site: … e.g. Efimov-type problem
- [n>n_{max}~3 killed by three body etc. loss]

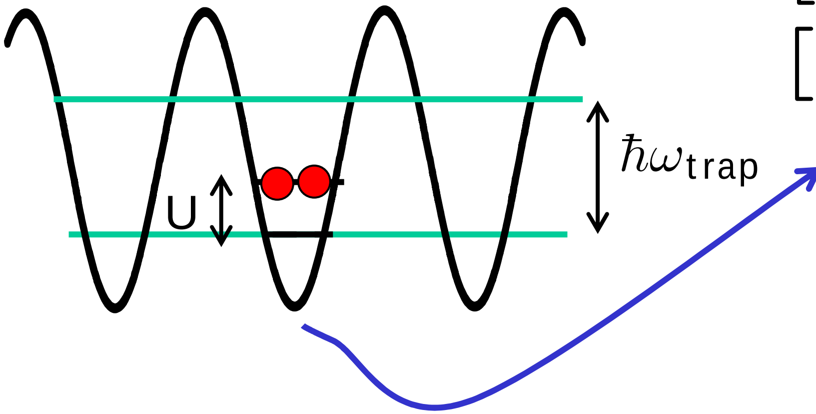
Zoller 2004

Julienne et al.

- two atoms on one site

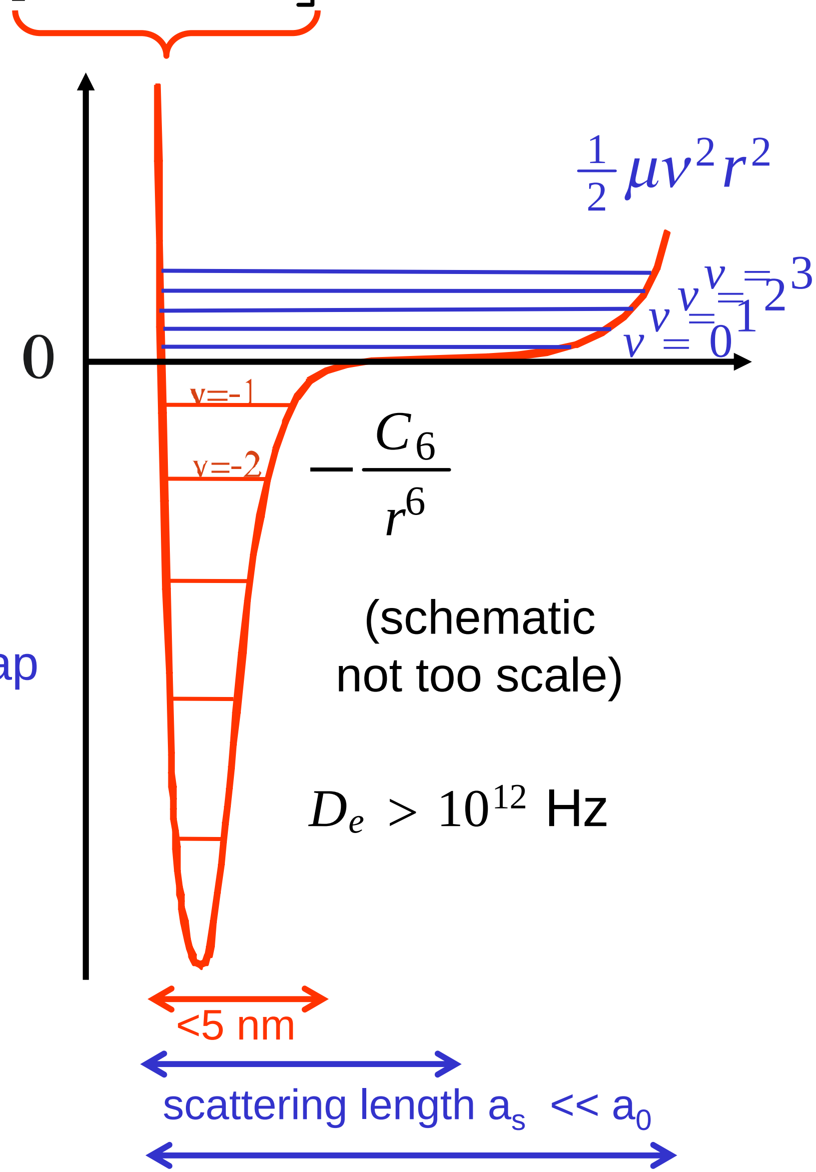
$$\left[-\frac{\hbar^2}{2(2m)}\nabla_R^2 + \frac{1}{2}(2m)\nu^2 R^2\right]\psi_{cm}(R) = E_{cm}\psi_{cm}(R)$$

$$\left[-\frac{\hbar^2}{2\mu}\nabla_r^2 + \frac{1}{2}\mu\nu^2 r^2 + V(r)\right]\psi(r) = E\psi(r)$$



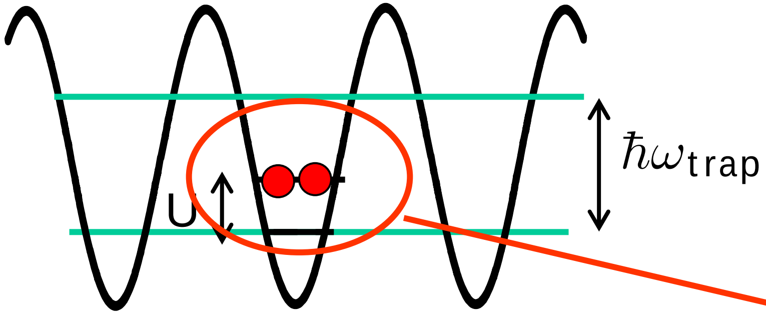
harmonic
approximation

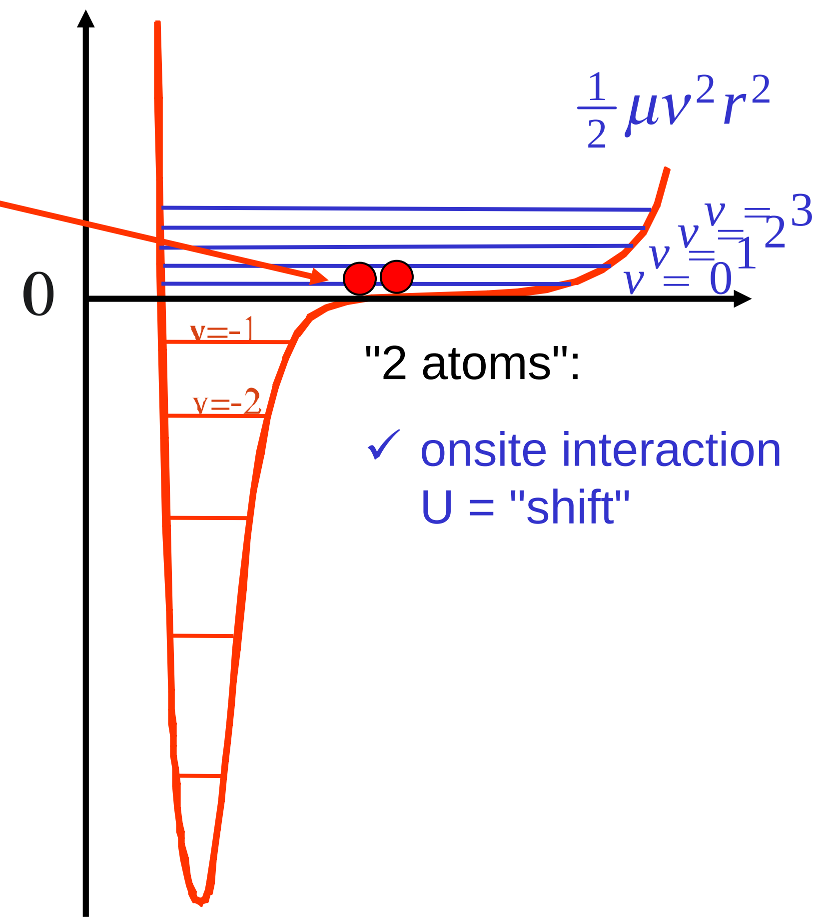
Born Oppenheimer
potentials including trap



Zoller 2004

- two atoms on one site

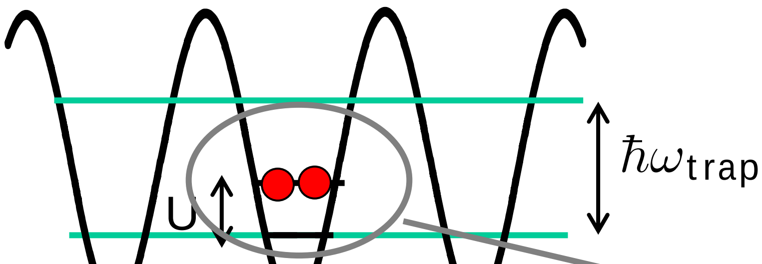


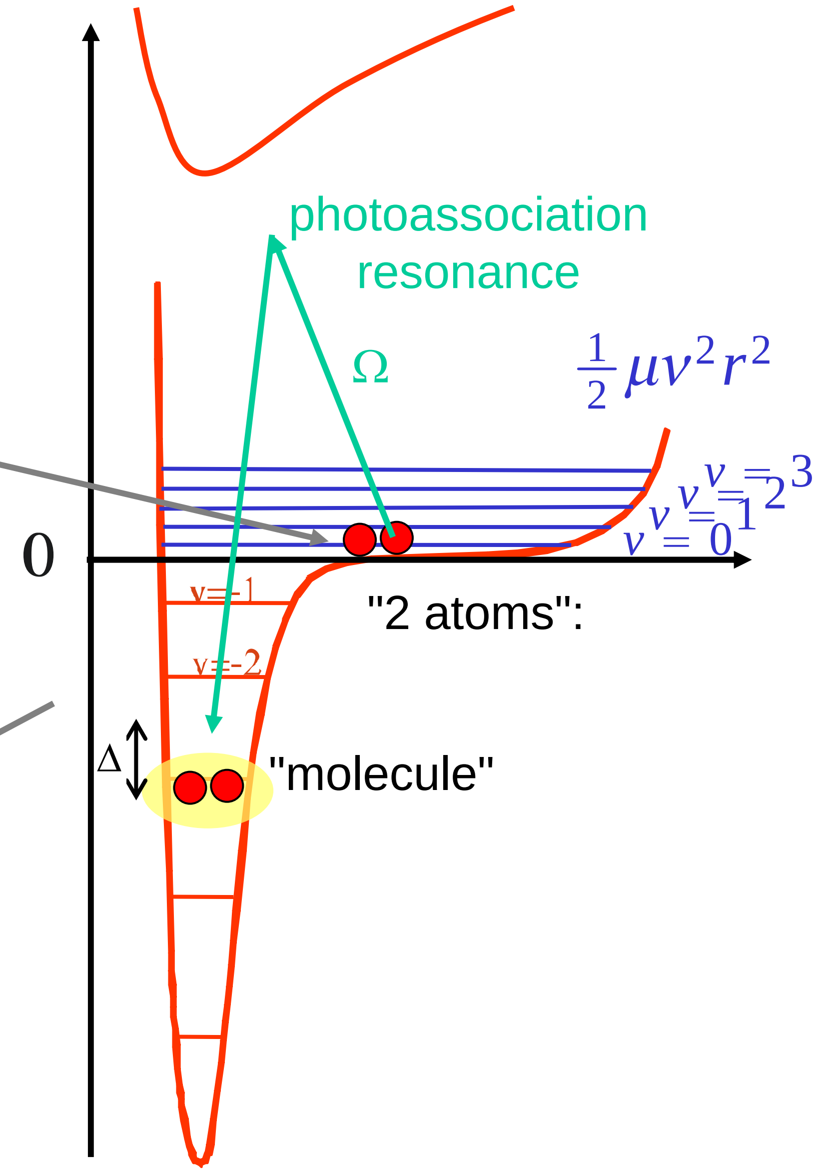


"2 atoms":

✓ onsite interaction
U = "shift"

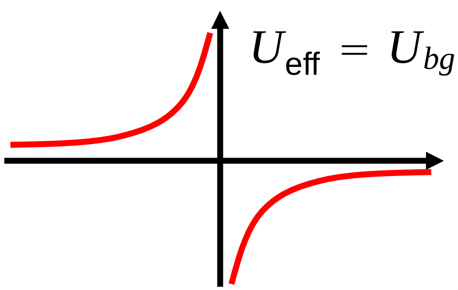
- two atoms on one site





AC Starkshift = optical Feshbach resonance

$$U_{\text{eff}} = U_{bg} + \frac{1}{4}\frac{\Omega^2}{\Delta}$$

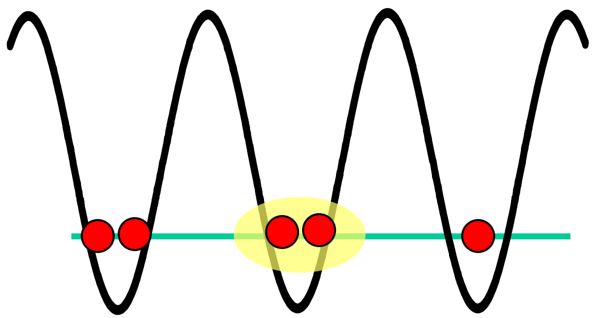


$$H = (U_{bg} + \frac{1}{4}\frac{\Omega^2}{\Delta}) b^{\dagger 2} b^2$$

Zoller 2004

Hubbard model including molecules

- Hamiltonian



$$H = -J_b \sum_{<i,j>} b_i^\dagger b_j + \frac{1}{2} U_b \sum_i b_i^\dagger b_i^\dagger b_i b_i$$
$$- J_m \sum_{<i,j>} m_i^\dagger m_j + \frac{1}{2} U_m \sum_i m_i^\dagger m_i^\dagger m_i m_i - \sum_i \Delta m_i^\dagger m_i$$
$$+ \frac{1}{2} \Omega \sum_i m_i^\dagger b_i b_i + \text{h.c.}$$

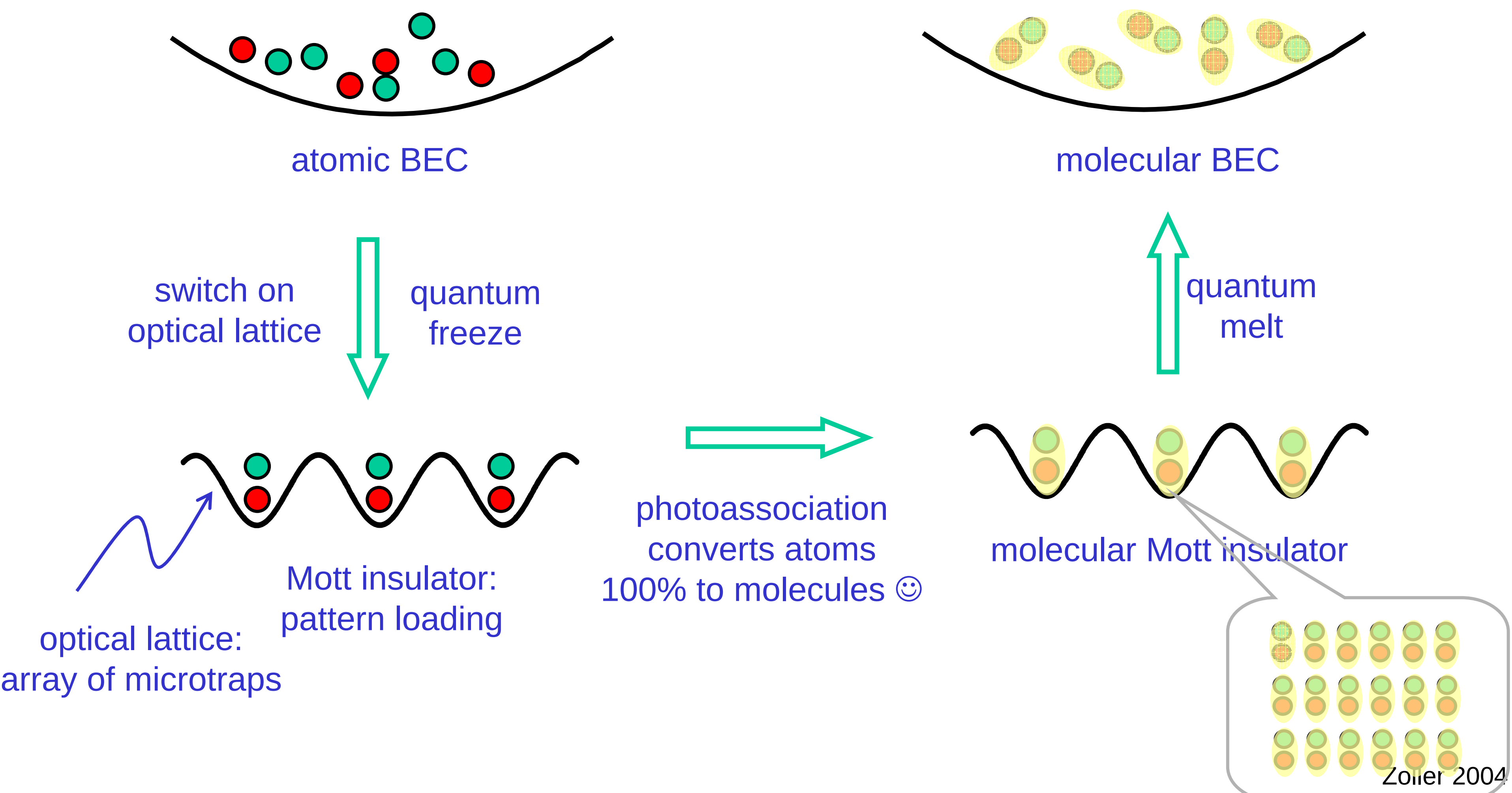
Remarks:

✓ we have derived this only for sector: 2 atoms or 1 molecule

✓ inelastic collisions / loss for >2 atoms and >1 molecules (?)

Remark: quantum phases of "composite objects"

- molecular BEC via a quantum phase transition



Remarks:

Straightforward generalization of these Hubbard *derivations* to …

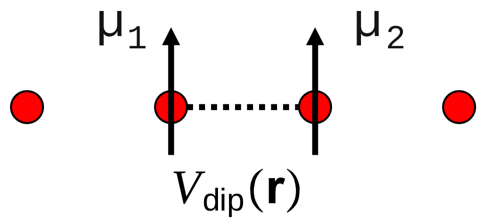
- bosons and / or fermions
- two-component mixtures of bosons / fermions
- dipolar gases / Hubbard models via heteronuclear molecules (long range dipolar forces)

Complaints:

- the time scales for tunneling are pretty long
- decoherence: spontaneous emission, laser / magnetic field fluctuations, …

Other ideas for interactions …

- optical dipole-dipole interactions … however loss ☹
- Rydberg-Rydberg interactions in a static electric field (huge)

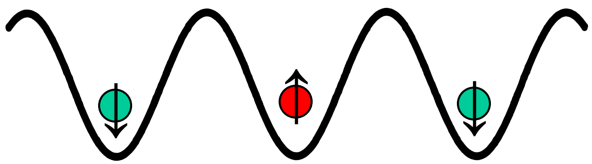


Zoller 2004

e.g. Duan et al.

Spin models

- optical lattice



bosons in a Mott phase

$$\langle n_{i\uparrow}\rangle + \langle n_{i\downarrow}\rangle \approx 1$$

$$H = -J\sum_{<i,j>,\sigma} b^{\dagger}_{i\sigma} b_{j\sigma} + \frac{1}{2}U\sum_{i,\sigma} n_{i\sigma}(n_{i\sigma}-1) + U_{\uparrow\downarrow}\sum_i n_{i\uparrow} n_{i\downarrow}$$

⇓

$$H = \sum_{\langle i,j\rangle}\left[\lambda_z \sigma^z_i \sigma^z_j \pm \lambda_\perp (\sigma^x_i \sigma^x_j + \sigma^y_i \sigma^y_j)\right]$$

XXZ-model

$$\lambda \sim \frac{J^2}{U}$$

pretty small ☹

$$\sigma^z_i = n_{\uparrow i} - n_{i\downarrow}$$

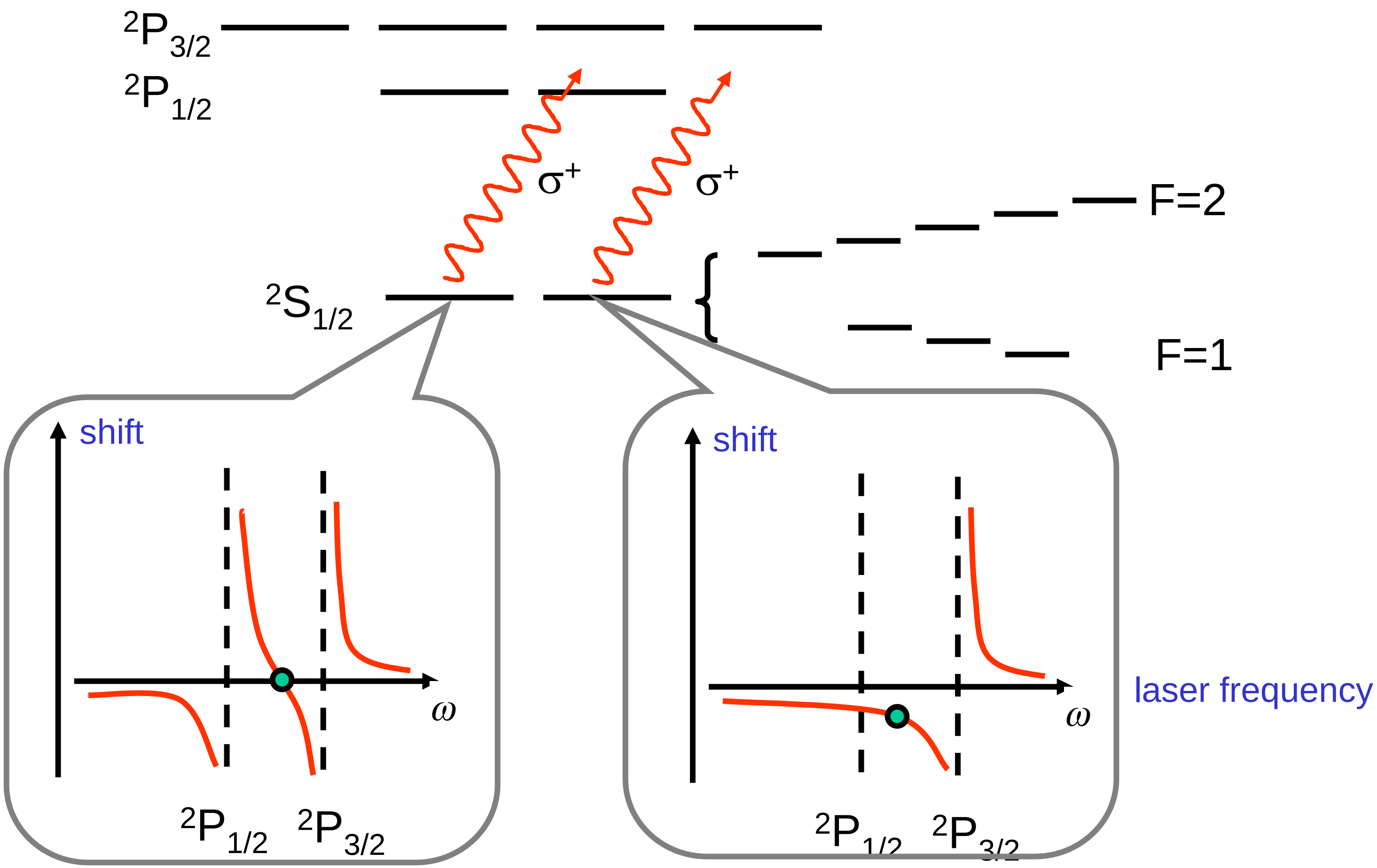
$$\sigma^x_i = b^{\dagger}_{\uparrow i} b_{i\downarrow} + b^{\dagger}_{i\downarrow} b_{\uparrow i}$$

$$\sigma^y_i = -i(b^{\dagger}_{\uparrow i} b_{i\downarrow} - b^{\dagger}_{i\downarrow} b_{\uparrow i})$$

- ideas for higher order H= σ σ σ interactions …

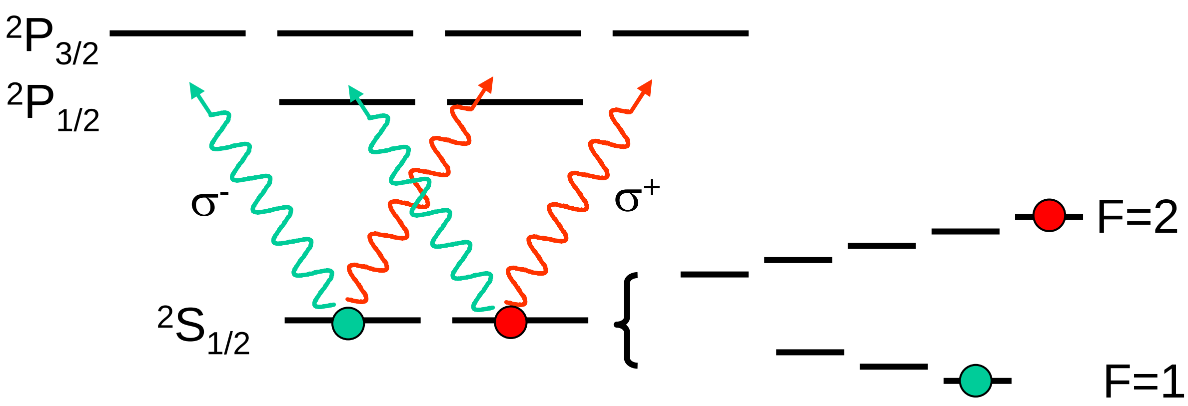
3. Optical Lattices … continued

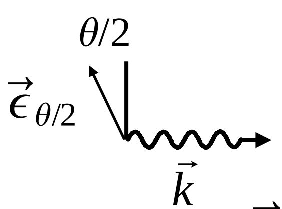
- multiple ground states & spin-dependent lattices

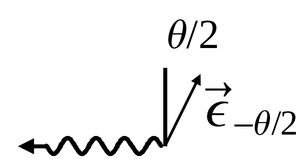


Zoller 2004

- multiple ground states & spin-dependent lattices





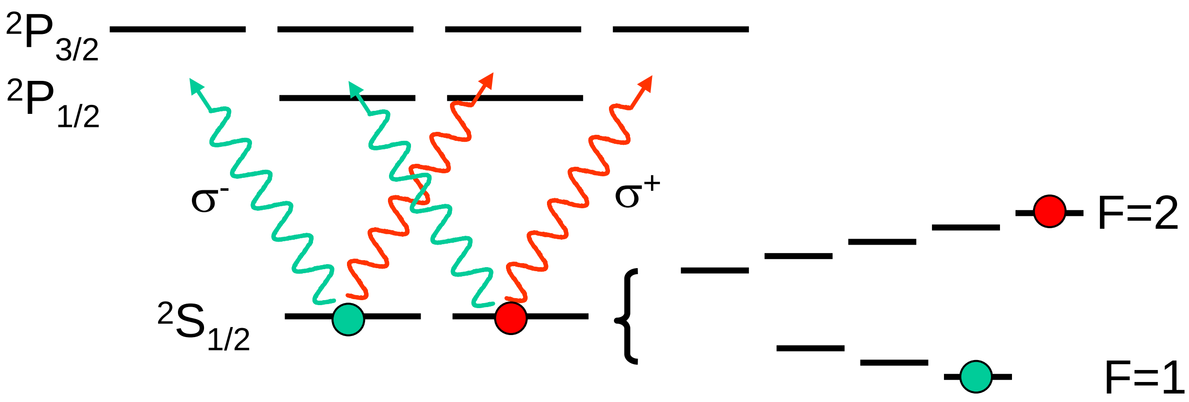


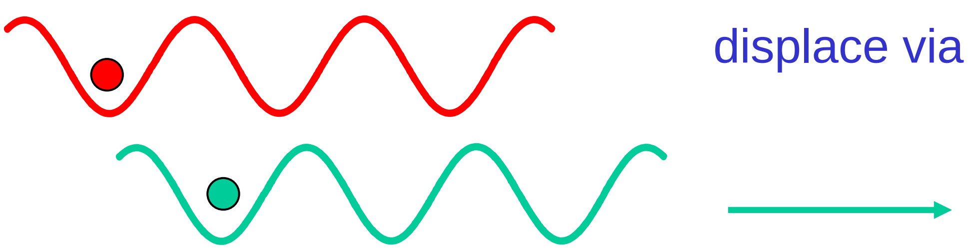
$$\vec{E} \sim \vec{\epsilon}_{\theta/2} e^{ikz-i\omega t} + \vec{\epsilon}_{-\theta/2} e^{-ikz-i\omega t}$$

$$\sim \vec{\epsilon}_{\sigma^+} \cos(kz - \theta/2) + \vec{\epsilon}_{\sigma^-} \sin(kz + \theta/2)$$

Zoller 2004

- multiple ground states & spin-dependent lattices

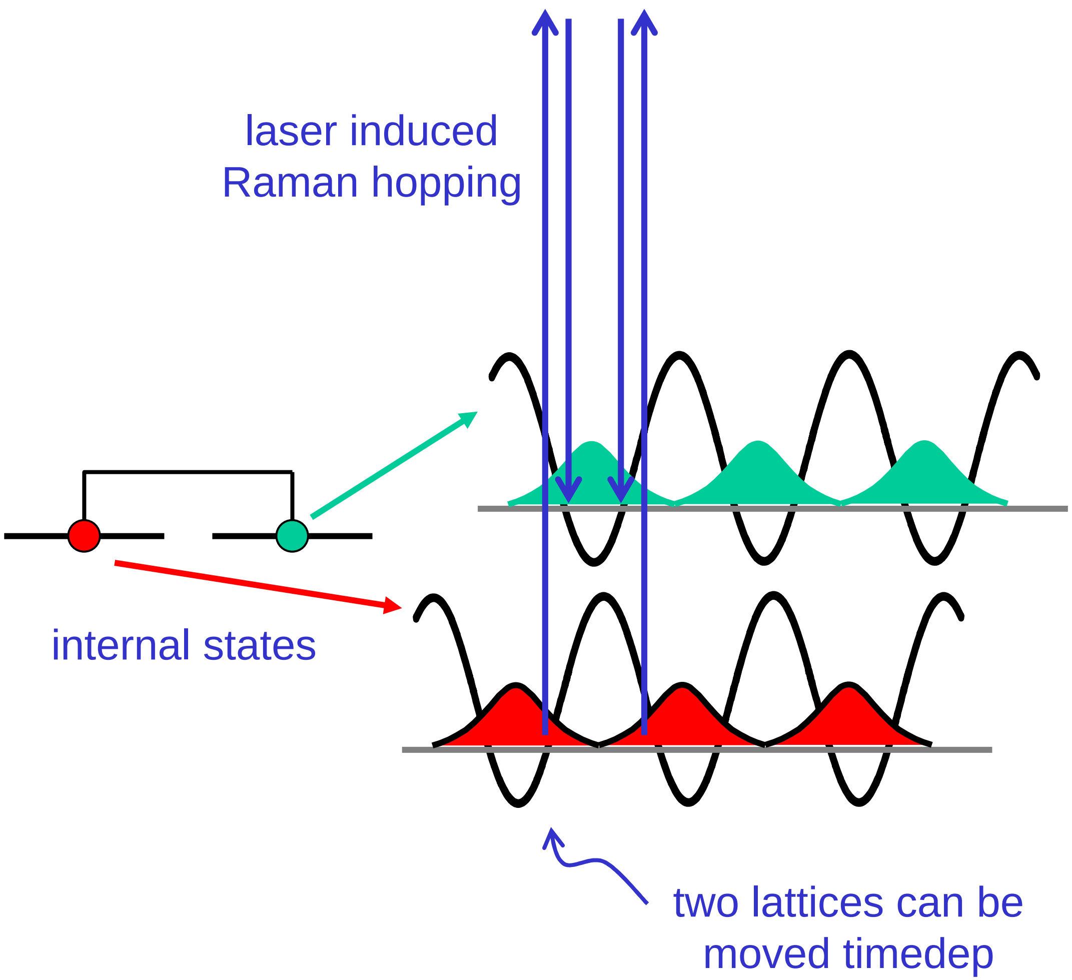




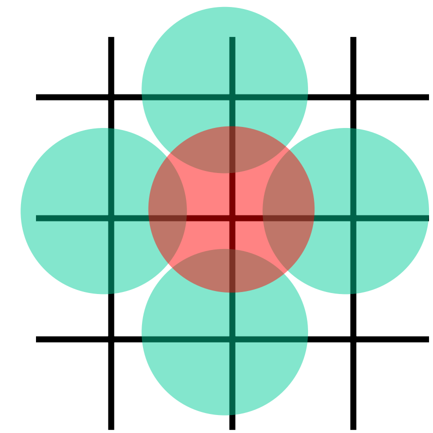
displace via polarization angle

Two component Hubbard models

- hopping via Raman transitions



- nearest neighbor interaction

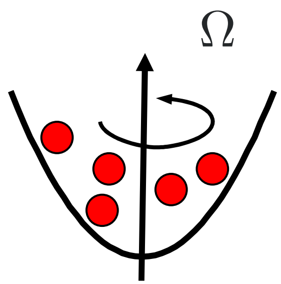


overlapping
wavefunctions
+
laser induced
Raman hopping

Zoller 2004

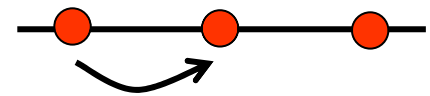
Adding “magnetic fields”

- effective magnetic field via rotation

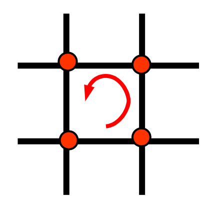


see: fractional quantum Hall effect

- effective magnetic field via lattice design



$$J_{\alpha\beta} \longrightarrow J_{\alpha\beta}\, e^{ie\int_{\alpha}^{\beta} \vec{A}\cdot d\vec{l}}$$

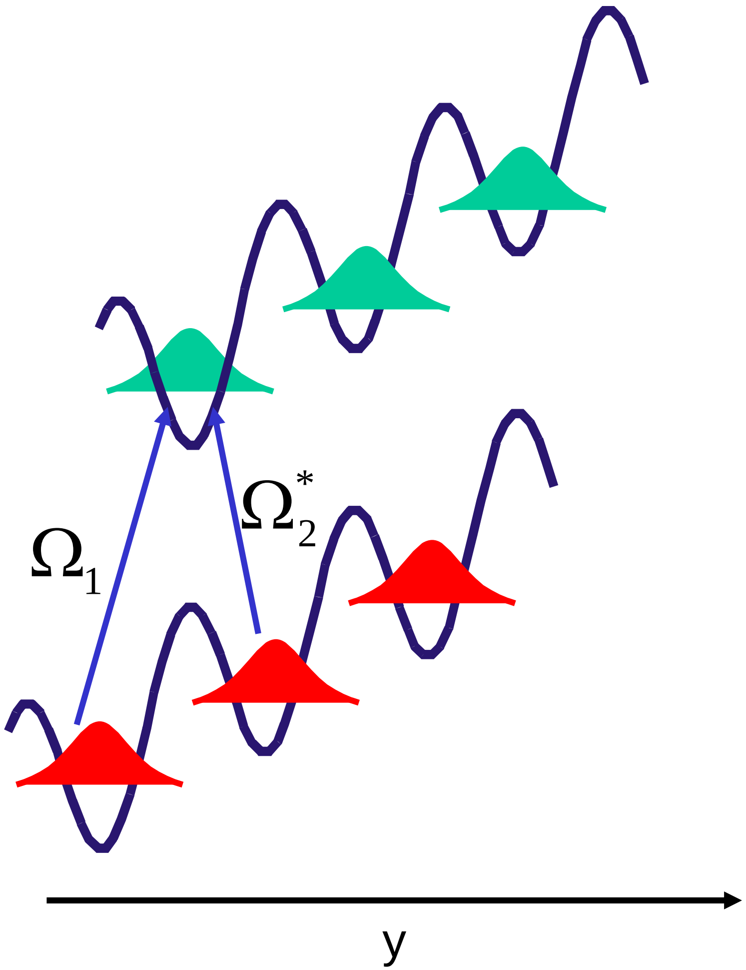


$$e^{ie\oint \vec{A}\cdot d\vec{l}} = e^{i\phi/\phi_0} \equiv e^{i\alpha\pi}$$

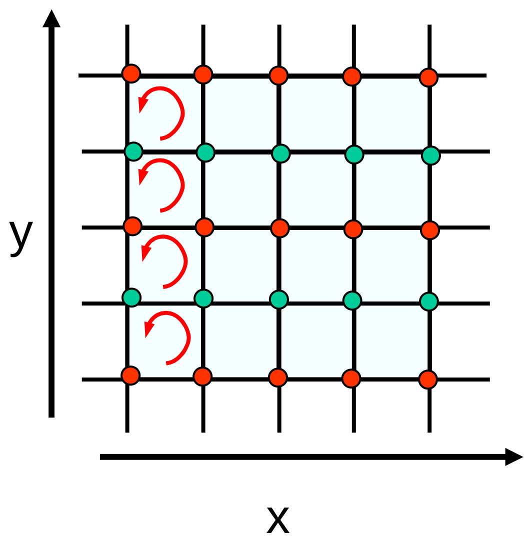
accumulate phase when walking around a plaquette

Zoller 2004

- effective magnetic fields: configuration

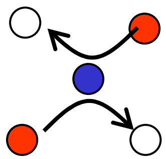


spatially varying phases
of the Raman lasers



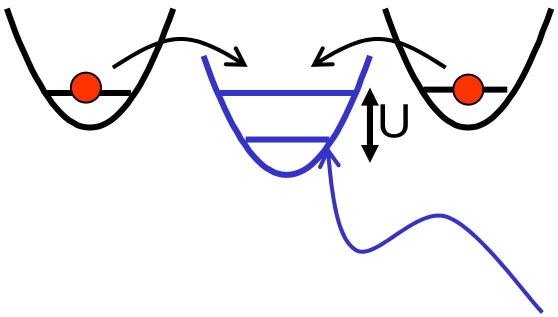
equivalent to a homogeneous
magnetic field

- two particle hopping



+ …

ring exchange

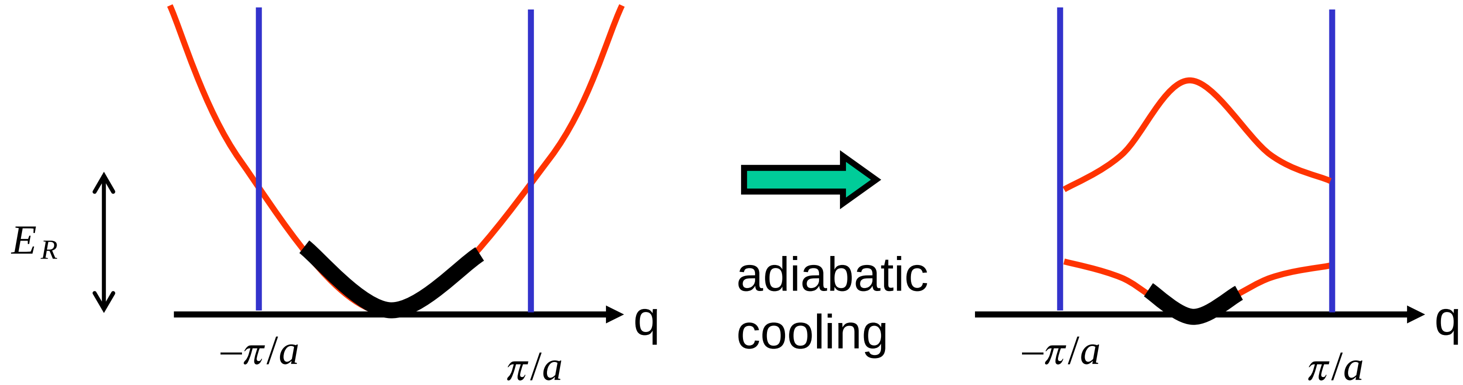


energy
conservation

Zoller 2004

4. Lattice Loading

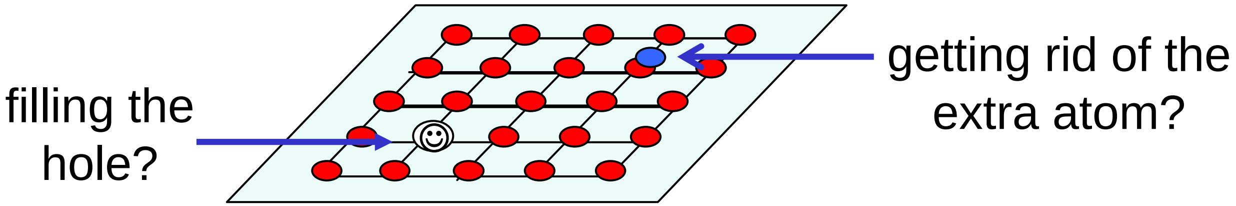
- [laser cooling: recent Sr experiments cool to fermi degeneracy]
- from BEC



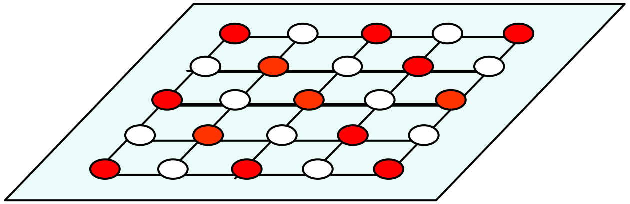
- thermal equilibrium? temperature? [no detailed exp results?]
- Integrate Schrödinger equation

Healing defects & Pattern loading

- Getting rid of the last defects …?



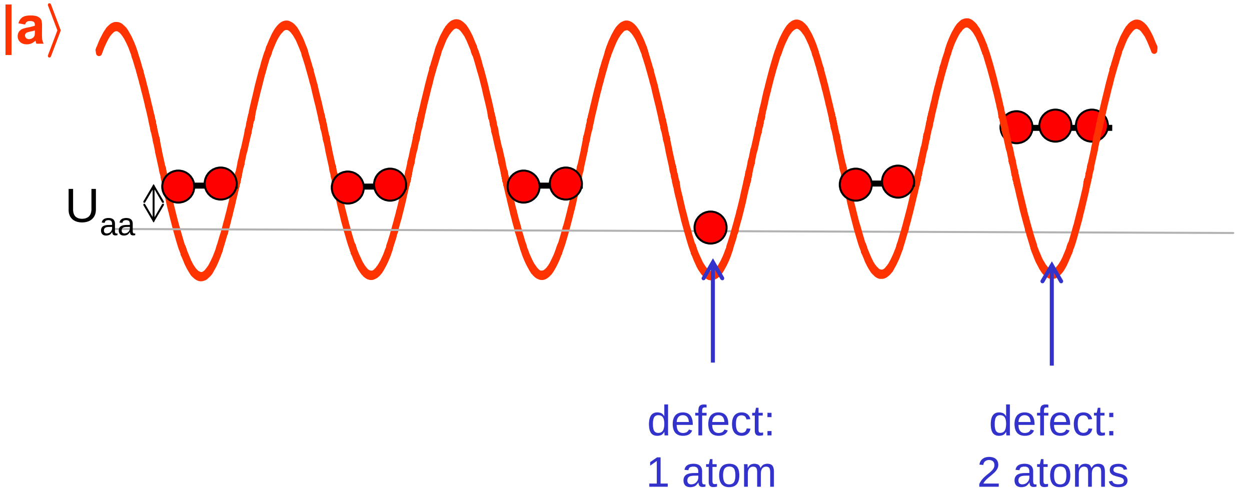
- loading spatial patterns



- fidelity of loading 1:10^4 or 10^5

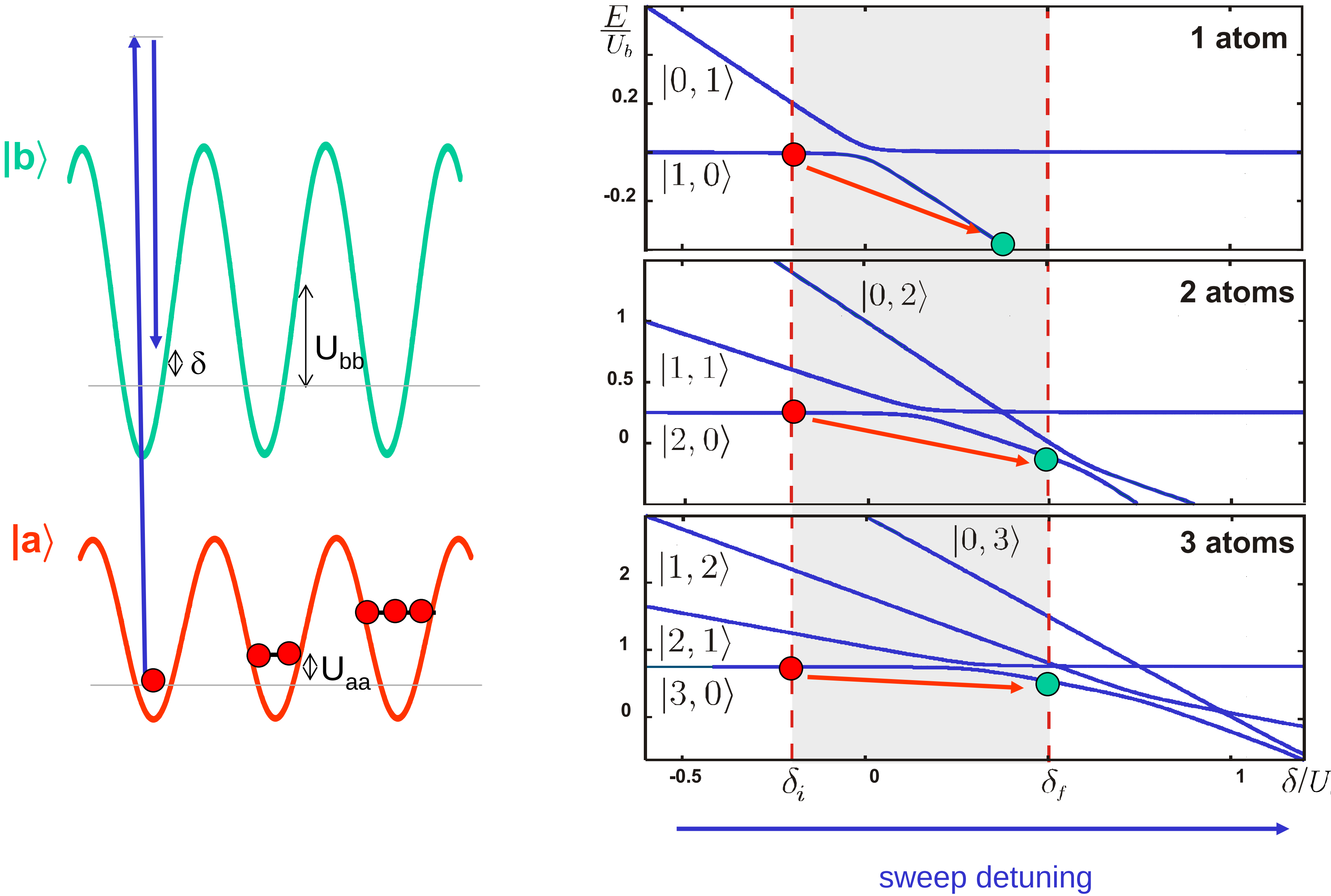
A filtering scheme

prepare a Mott insulator with n=2 atoms:

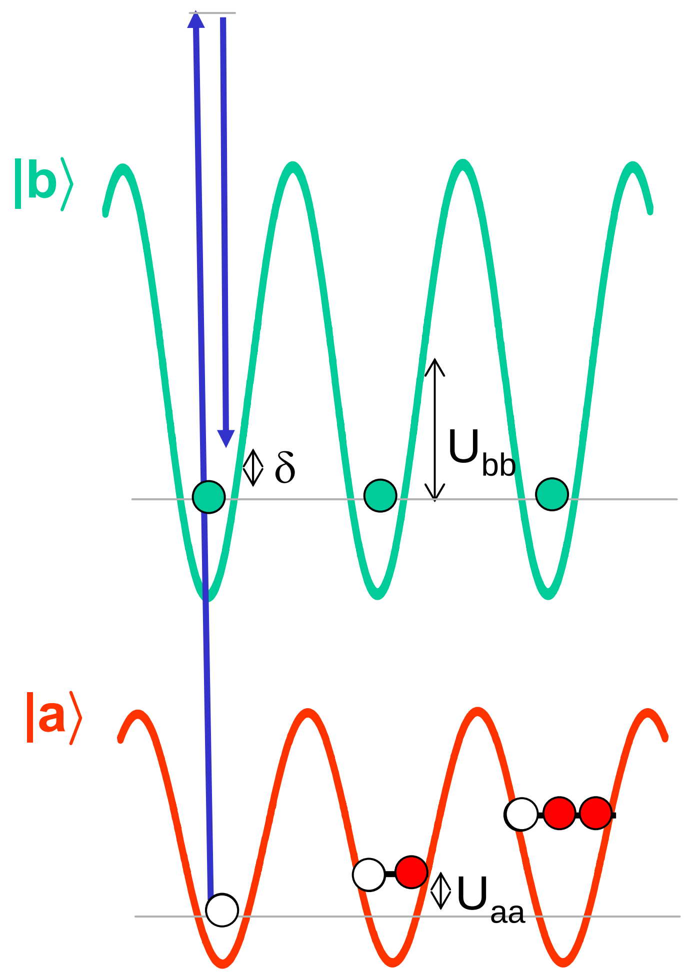


Zoller 2004

dressed energy levels



Zoller 2004



exactly one atom in b

$$|1,0\rangle \longrightarrow |0,1\rangle$$
$$|2,0\rangle \longrightarrow |1,1\rangle$$
$$|3,0\rangle \longrightarrow |2,1\rangle$$

$$\mathrm{Tr}_a\Big(\sum_n p_n |n\rangle_a\langle n|\Big) \otimes |0\rangle_b\langle 0|$$

$$\longrightarrow p_0|0\rangle_b\langle 0| + (1-p_0)|1\rangle_b\langle 1|$$

≈ 0

irregular → regular filling

mixed state → pure state:

“cooling from nanoK to picoK”

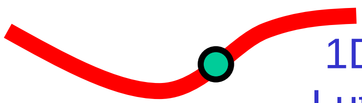
Zoller 2004

5. Measurements

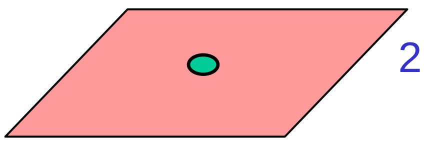
- releasing the atoms from the lattice $\langle a_i^\dagger a_j \rangle$
- Bragg scattering structure factor
- [borrowing ideas from lattice loading]

6. Impurities

- 1D, 2D and 3D BECs

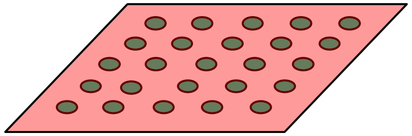


1D = bosonic
Luttinger liquid



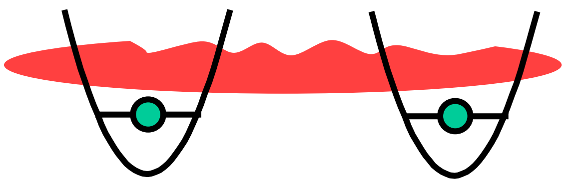
2D BEC

- arrays



qdots as array
of impurities

dot 1 dot 2



interaction via phonons

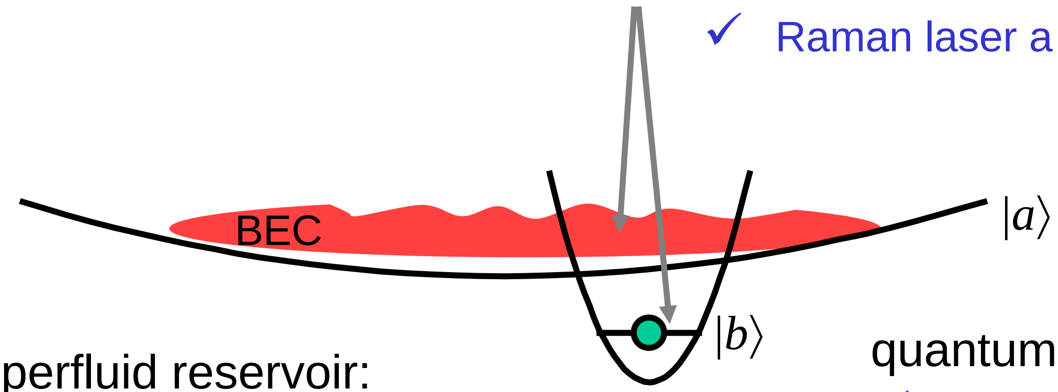
Atomic Quantum Dots coupled to BEC reservoirs

A. Recati
P. Fedichev
W. Zwerger
J. von Delft
P.Z.

- atomic quantum dot

in- and outcoupling:

✓ Raman laser a - b



BEC / superfluid reservoir:

✓ BEC represents a “phonon bath”

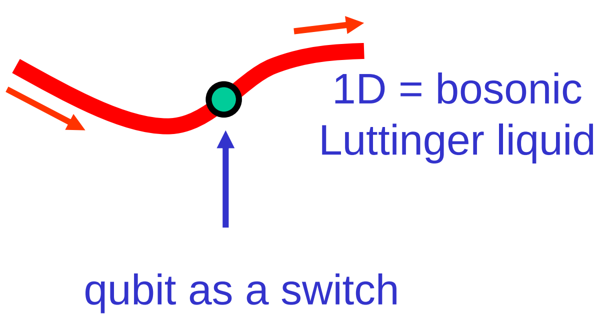
quantum dot:

✓ bosons in state b

✓ collisional blockade regime: n=0 or n=1 atoms (two-level system)

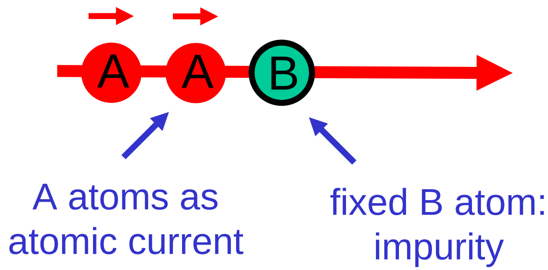
- Dynamics and spectroscopy: spin-boson model with controllable parameters
  - the spin can be decoupled from the bath by quantum interference
  - ohmic and superohmic phonon bath

Atomic Quantum Switch / Amplifier / Read Out

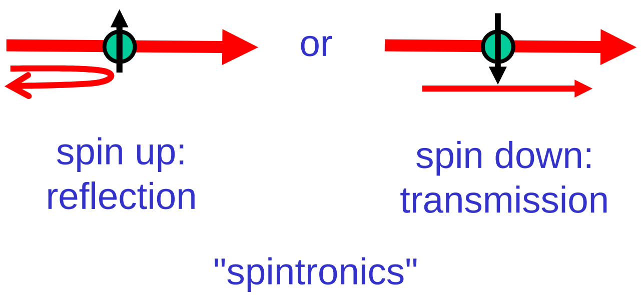


"Spintronics"

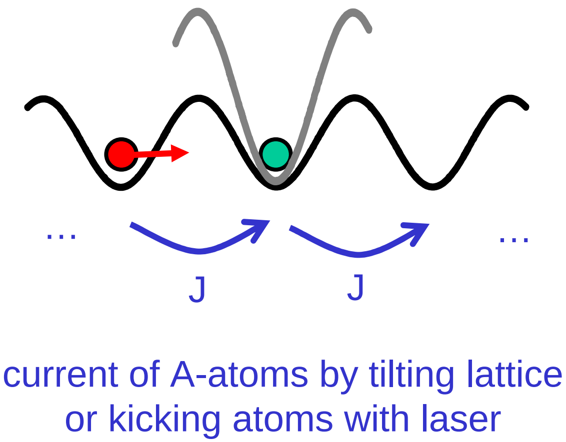
- impurity atom in a 1D atomic wire



- qubit B: spin or double dot

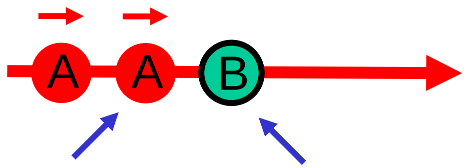


- realization: 1D optical lattice



Mesoscopic AMO

- impurity atom in a 1D atomic wire



A atoms as atomic current: bosons (or fermions)

fixed B atom: impurity

- qubit B: spin or double dot

spin up: reflection

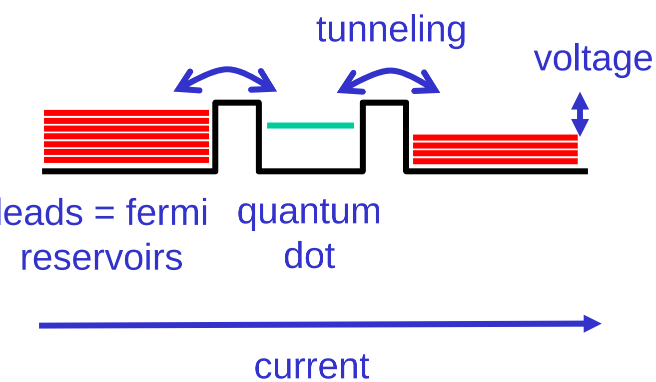
spin down: transmission

"spintronics"

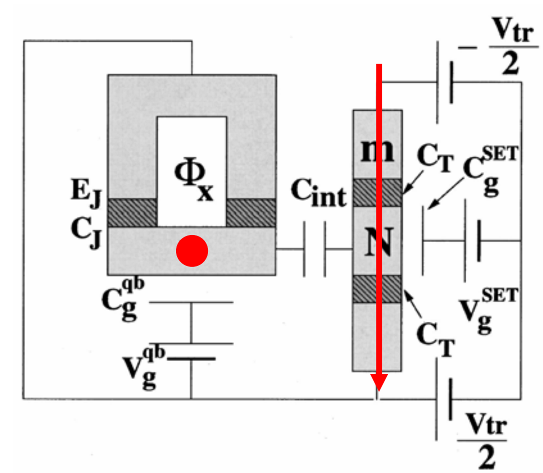
double dot

Mesoscopic CMP

- quantum dot



- single electron transistor: qubit read out

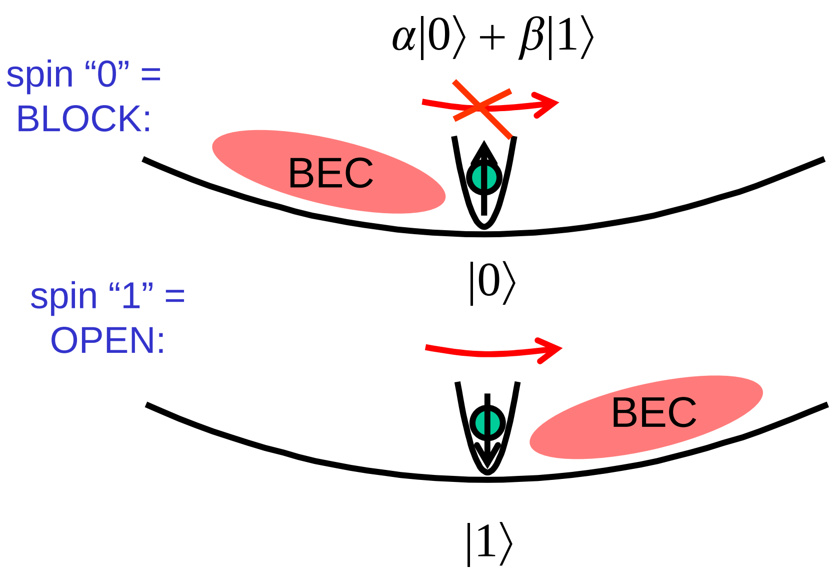


Zoller 2004

Mesoscopic AMO

- *macroscopic* superpositions:

spin-dependent "single atom mirror"

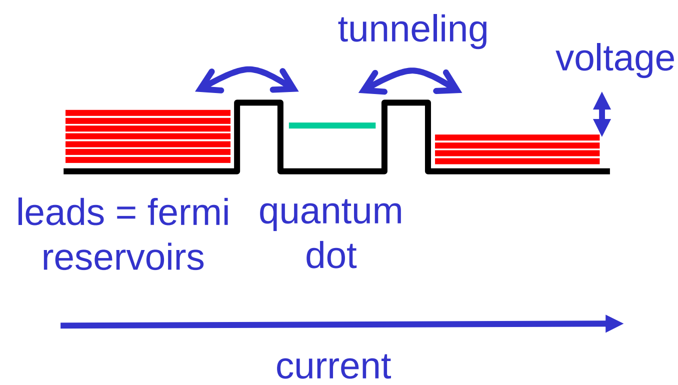


$\alpha|\text{BEC left}\rangle|0\rangle + \beta|\text{BEC right}\rangle|1\rangle$

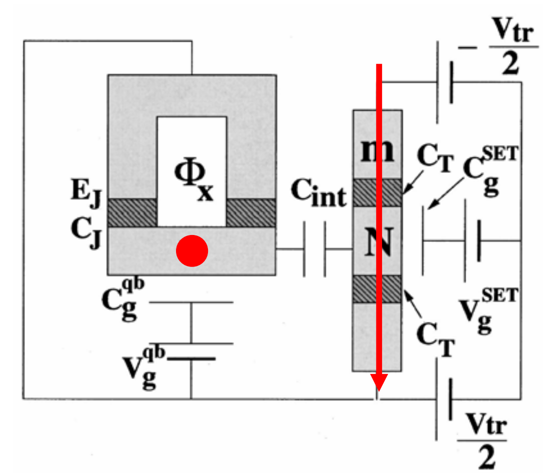
- qubit read out

Mesoscopic CMP

- quantum dot



- single electron transistor: qubit read out

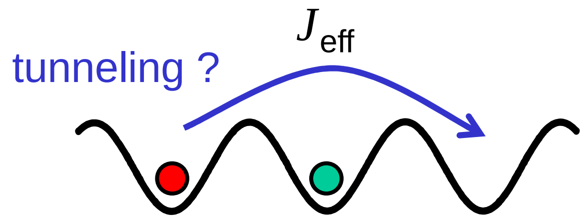


Zoller 2004

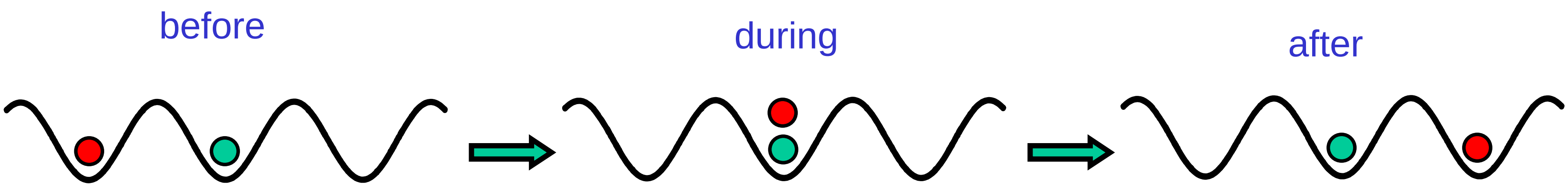
Atomic Switch (and Bloch band filter) by Quantum Interference

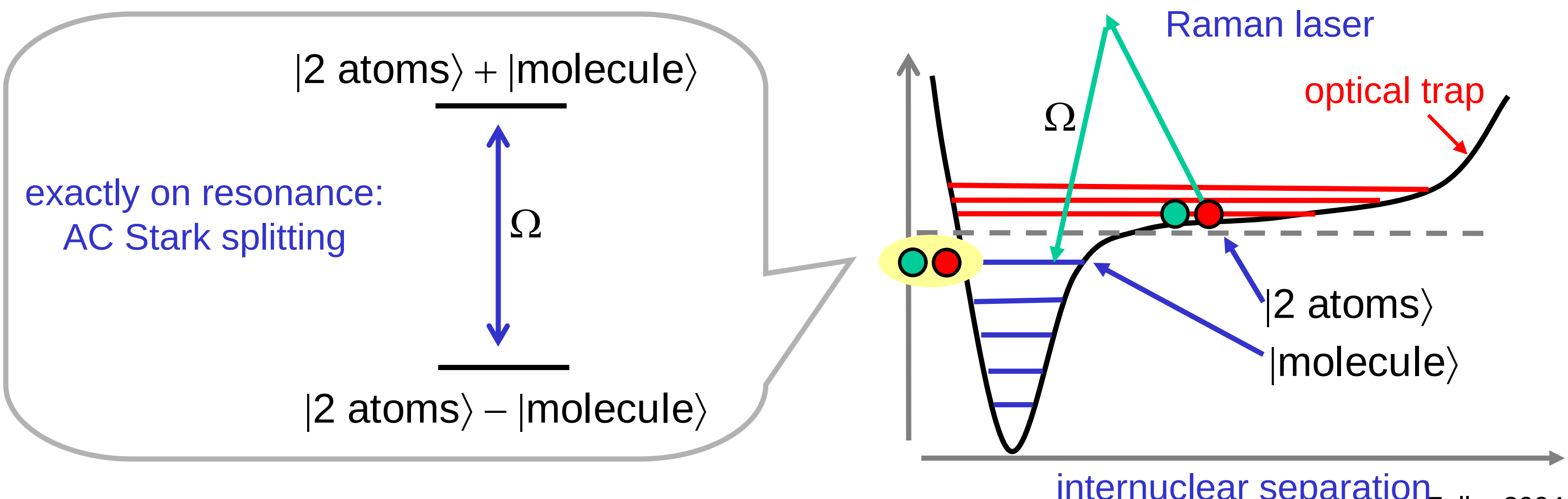
Micheli et al.

- tunneling through the quantum dot?



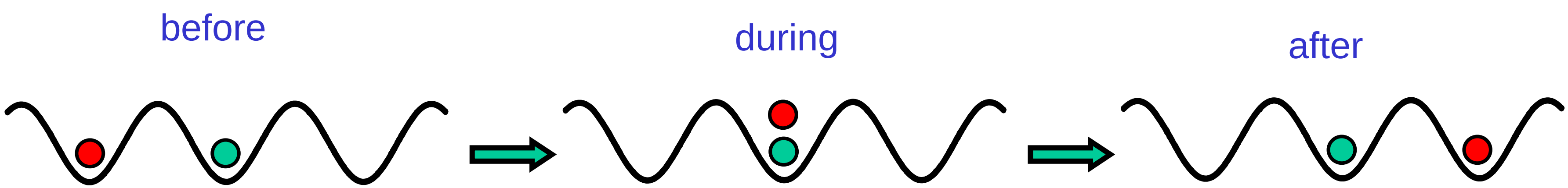
- process in an optical lattice

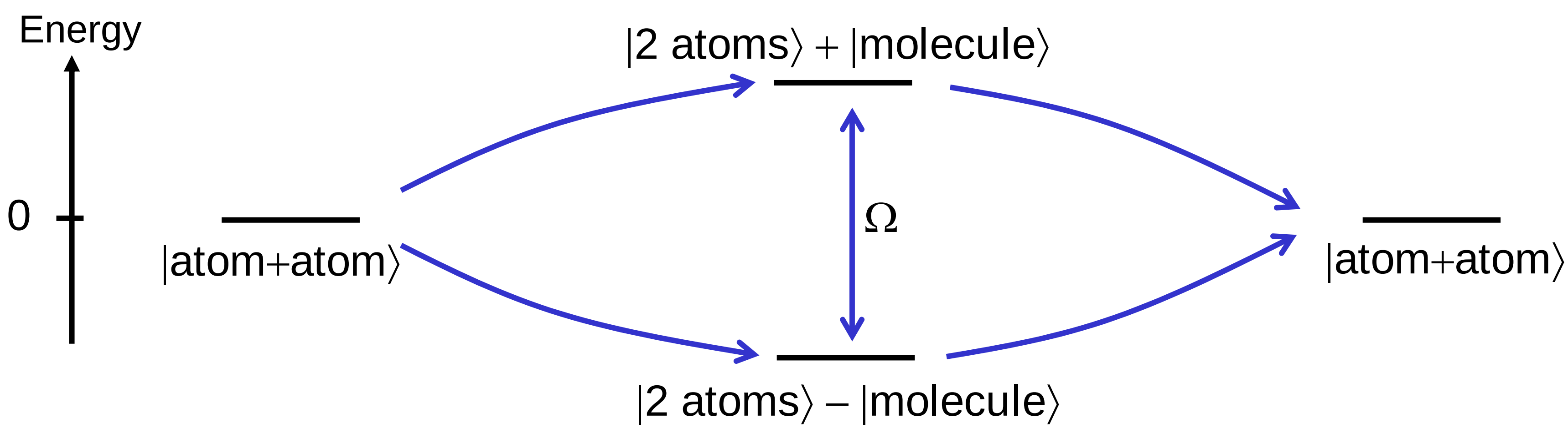




Zoller 2004

- blocking by quantum interference





$$J_{\text{eff}} \sim J\frac{1}{E-\frac{1}{2}\Omega}J + J\frac{1}{E+\frac{1}{2}\Omega}J \overset{!}{=} 0 \quad (\text{for } E = 0)$$

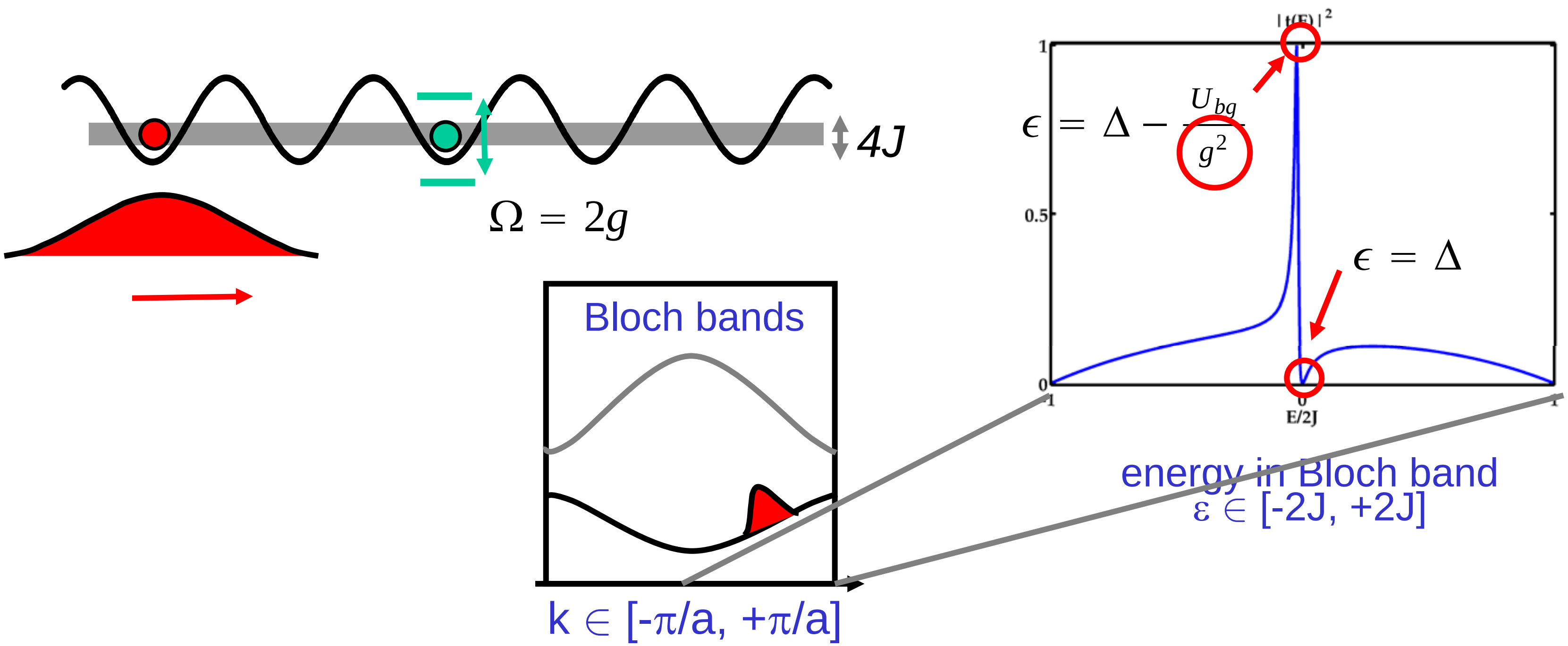
we kill the transport by quantum interference:
"infinite repulsion" (EIT)

Zoller 2004

Exact solution of the scattering problem

- solve the Lippman-Schwinger equation

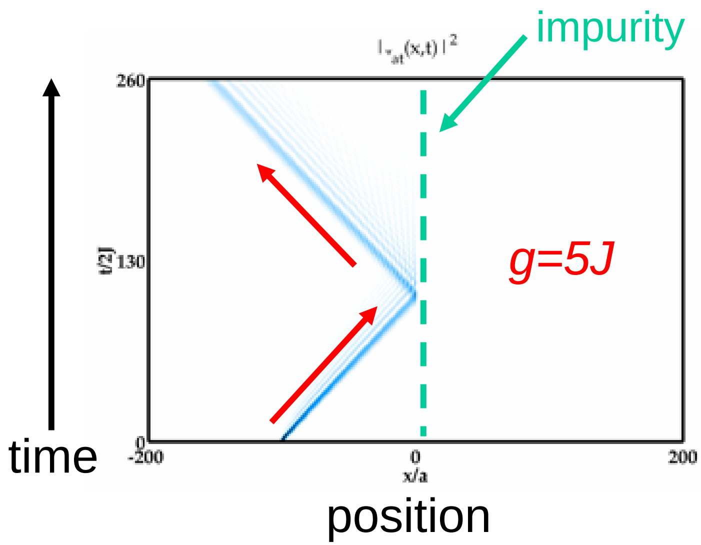
in general: Fano profile in transmission

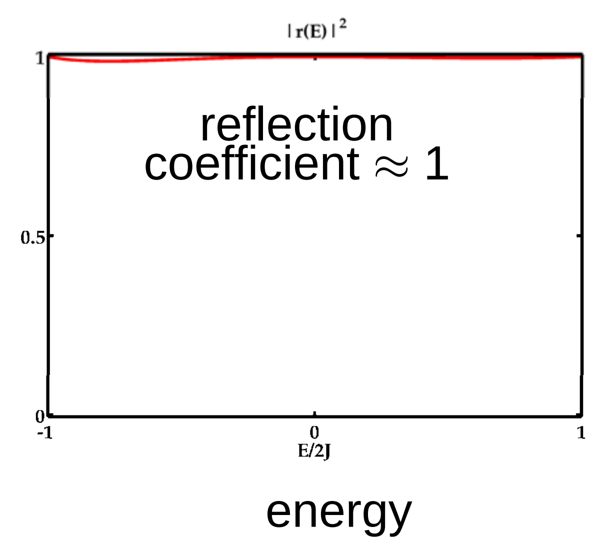


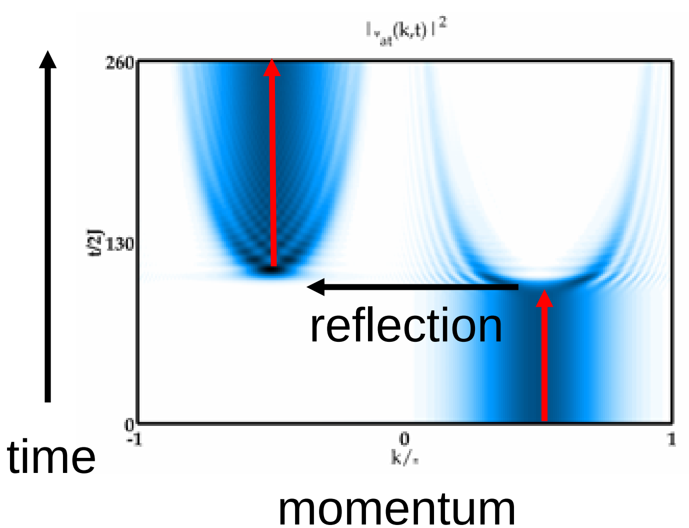
- Fano minimum and maximum depends on laser parameters:
  - for g >>J we have *only* an interference minimum
  - for g <J we have an *energy filter* in the Bloch band

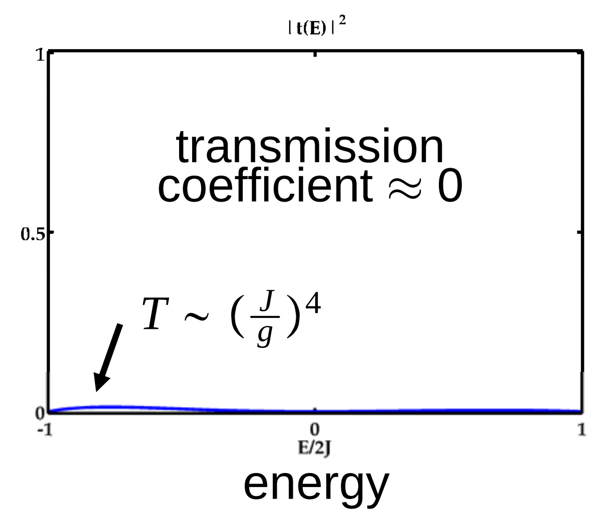
Zoller 2004

- numerical solution of the Schrödinger equation: wave packet dynamics: limit of strong PA laser



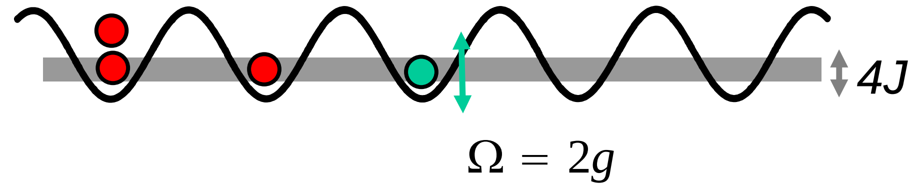






Zoller 2004

Many A atoms

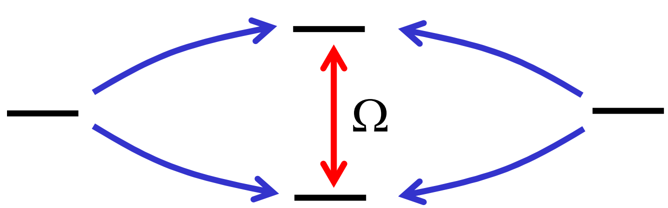


- Hubbard model with impurity on site "0"

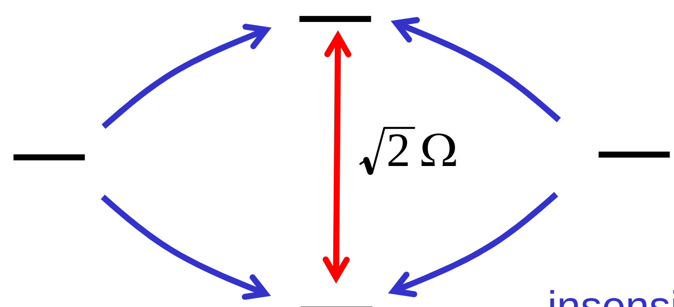
$$H = -\sum_i J(a_i^\dagger a_{i+1} + a_{i+1}^\dagger a_i) + \frac{1}{2} U \sum_i a_i^{\dagger 2} a_i^2$$
$$+ U_{bg} a_0^\dagger a_0 b^\dagger b - \Delta m^\dagger m + \frac{1}{2}\Omega(m^\dagger a_0 b_0 + a^\dagger b^\dagger m)$$

- assume valdity for many A atoms
- add loss term

- interference works also for many A atoms (if we believe the model):



1 A-atom



2 A-atoms

insensitive to loss

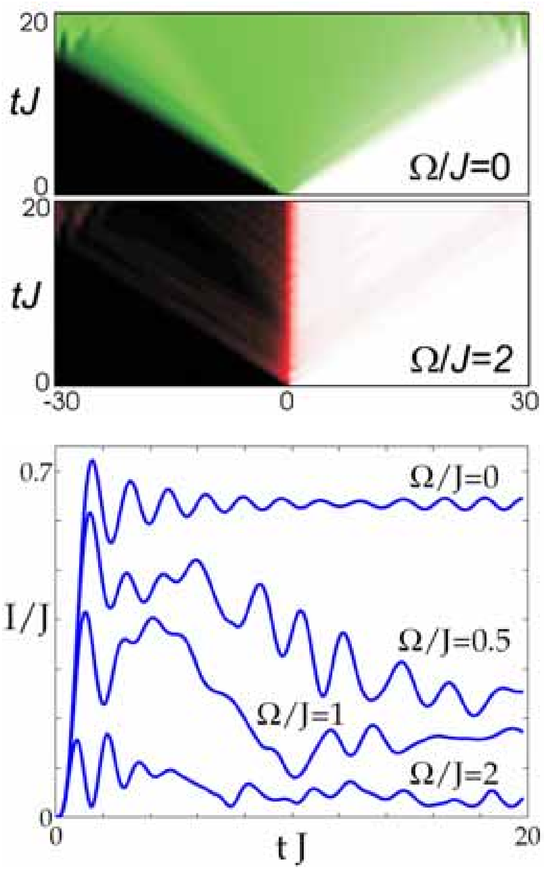
Zoller 2004

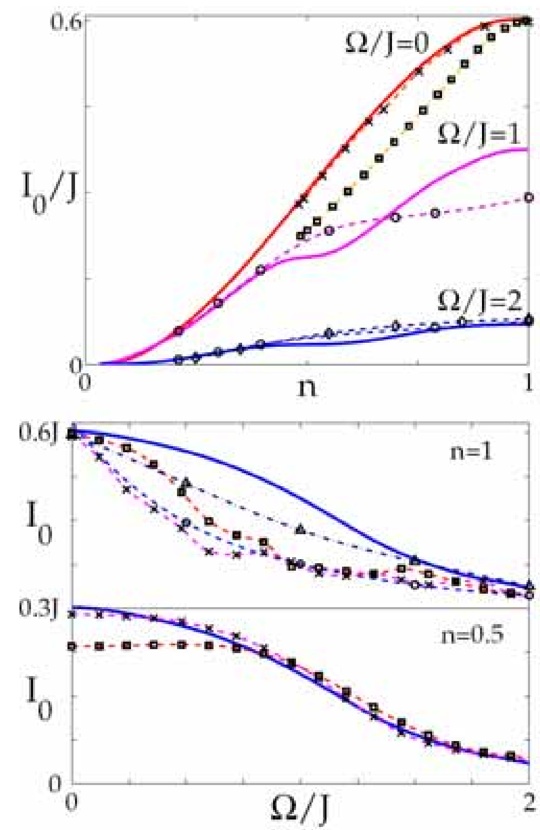
Micheli, Daley, Jaksch

Time dependent many body dynamics: results

- [Exact] Solution of *time dependent* many body Schrödinger equation
  - DMRG-type method G. Vidal, PRL 91, 147902 (2003)
  - hard core bose gas

N ~ 30 atoms on 61 lattice sites

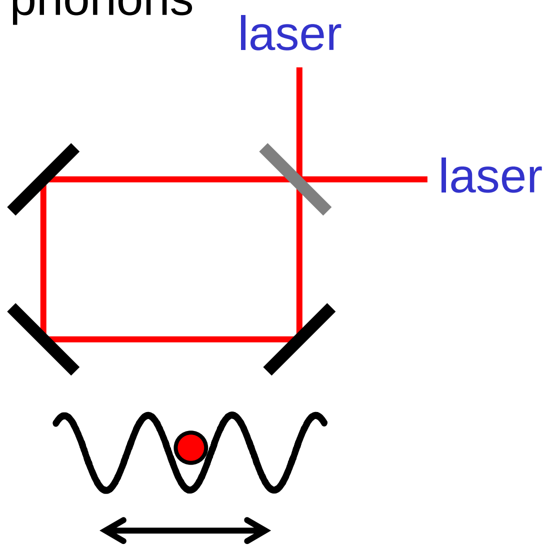




Jaksch, Griessner,
Daley, PZ

7. Phonons

- cavity mode dynamics as phonons



✓standing wave responds to the atomic motion

✓single mode (!)

- laser assisted phonon cooling
  - in analogy to laser cooling with photon → phonon