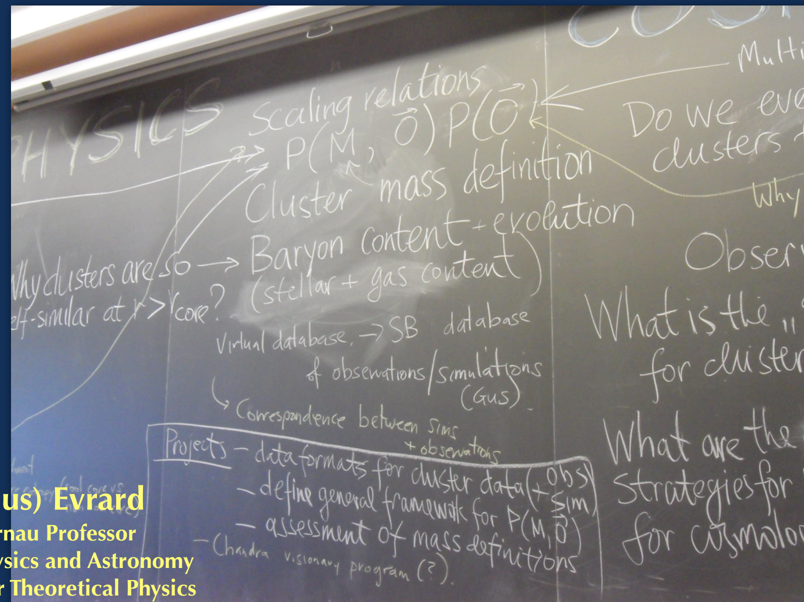
Size Matters: Similarity and Variety in Halos and Clusters



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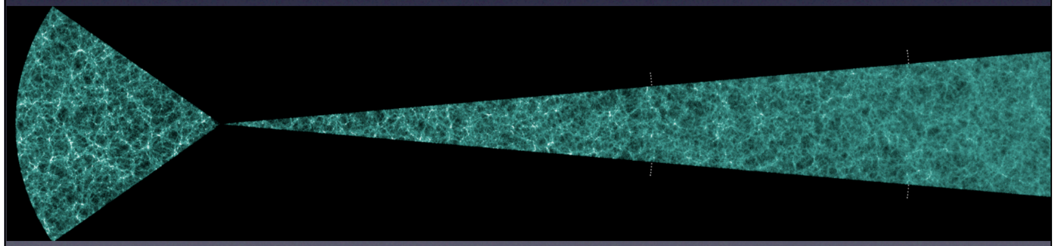
halo model paradigm of cosmological large-scale structure (LSS)

LSS = a hierarchical web of quasi-equilibrium bound structures - *halos* - that emerge via gravitational amplification from a noise field imposed during an early epoch of inflation.

Halo Model's key enabling ingredients:

– space density (*aka*, mass function), $n(M, z)$
– spatial N-point correlations (*e.g.*, 2-pt bias function), $b(M, z)$
– internal halo structure (kinematics, thermodynamics), $X(r/r_\Delta, M, z)$



the lexicon

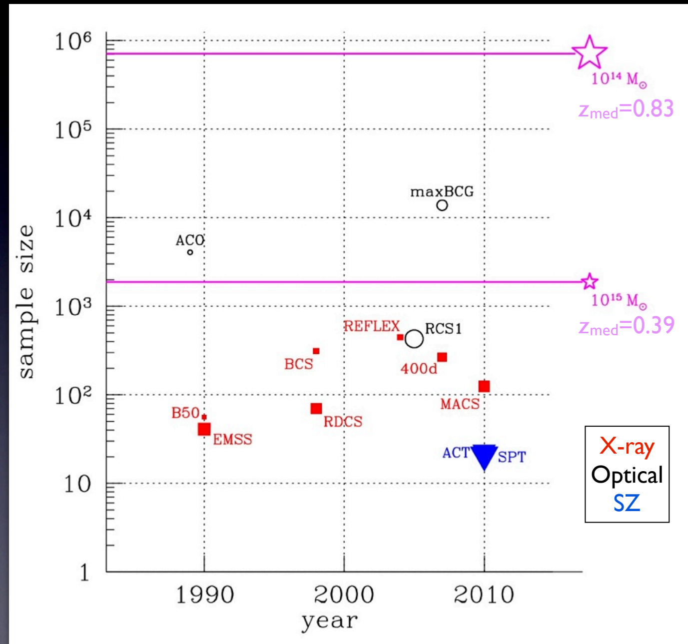
* **Halo :**

a self-bound, quasi-equilibrium structure comprised of multiple, interacting fluids (dark matter, multi-phase baryons, and radiation) formed via gravitational collapse within a cosmic web of random noise.

* **Cluster :**

a redshift-space projection of a massive halo, *and its line-of-sight neighbors*, with the resultant system containing multiple, bright galaxies and other visible components (multi-phase baryons, non-thermal matter, etc.).

cluster samples today are sparse relative to massive halos on the sky

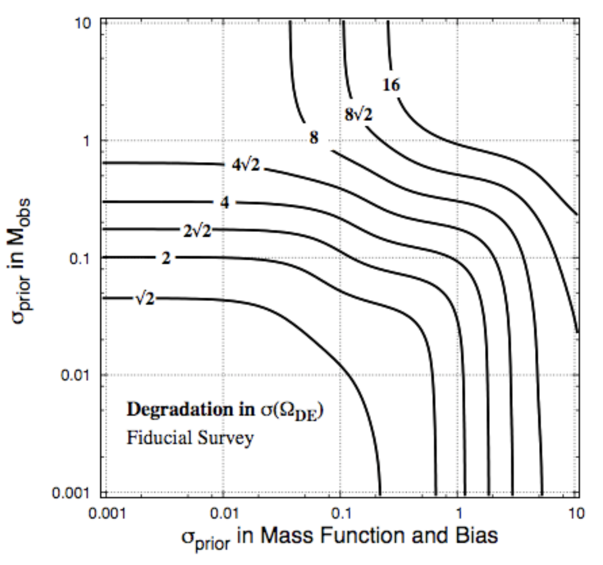


Allen, Evrard & Mantz 2011

Halo mass scale is M_{200m} (h = 0.7)

Fisher matrix motivation to understand scaling relations

Cunha + AE (2009)



scaling relation (Mobs) uncertainty is presently the dominant source of systematic error in DE studies

a PL+LN* multivariate signal model

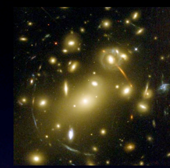
*Power Law + Log-Normal

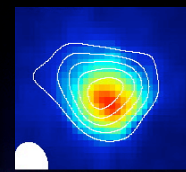
massive halo phenomenology: observable signal likelihoods

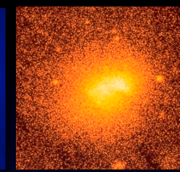
optical/lensing **sub-mm** **X-ray**

halo of
mass M
redshift z

=







"Astrophysics 101"

1. Dimensional analysis => mean relations are power-laws

2. Central Limit Theorem => deviations are log-normal

PL+LN covariance model for halo signals

1 A Local Model for Multivariate Counts

Consider a mass function described locally as a power-law in mass with slope $-\alpha$. Specifically, using $\mu \equiv \ln M$, define the mass function, $n(\mu, z)$, as the likelihood of finding a halo at redshift z in the mass range μ to $\mu + d\mu$ within a small comoving volume dV,

$$dp \;\equiv\; n(M,z)\, d\ln M\, dV = \; AM^{-\alpha} d\ln M\, dV = \; A\, e^{-\alpha\mu}\, d\mu\, dV. \qquad (1)$$

The local slope, α, and amplitude, A, implicitly depend on mass and redshift in a manner dependent on cosmology (*e.g.*, Tinker et al. 2008).

Consider a set of N halo properties, $S_i \in \{N_{\rm gal}, L_{\rm X}, T_{\rm X}, M_{\rm gas}, Y_{\rm X}, Y_{\rm SZ}, \cdots\}$, let $\mathbf{s}$ be a vector containing their logarithms,

$$s_i \;=\; \ln(S_i) \qquad (2)$$

Assume that the mass scaling behavior of these properties are power-laws, so that the mean ln(signal) for a mass–complete sample scales as

$$\bar{\mathbf{s}}(\mu, z) \;=\; \mathbf{m}\mu + \mathbf{b}(z). \qquad (3)$$

The elements of vector $\mathbf{m}$ are the slopes of the individual mass-observable relations. (Note that, at some fixed epoch, we can always choose units such that the intercepts $b_i(z) = 0$.)

Assume that ln(signal) deviations about the mean are Gaussian, described by a likelihood

$$p(\mathbf{s}|\mu) \;=\; \frac{1}{(2\pi)^{N/2}|\Psi|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{s}-\bar{\mathbf{s}})^{\dagger}\Psi^{-1}(\mathbf{s}-\bar{\mathbf{s}})\right], \qquad (4)$$

where the covariance matrix has elements

$$\Psi_{ij} \equiv \langle (s_i - \bar{s}_i)(s_j - \bar{s}_j) \rangle, \qquad (5)$$

and the brackets denote an ensemble average over a (large) mass–complete sample.

PL+LN covariance model for halo signals

1.1 Multivariate Space Density

The space density as a function of the multivariate properties, $\mathbf{s}$, is found by the convolution, $n(\mathbf{s}) = \int d\mu\, n(\mu)\, p(\mathbf{s}|\mu)$. Using equations (1) and (4), the result is

$$n(\mathbf{s}) = \frac{A\Sigma}{(2\pi)^{(N-1)/2}|\Psi|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{s}^\dagger\Psi^{-1}\mathbf{s} - \frac{\bar{\mu}^2(\mathbf{s})}{\Sigma^2})\right], \quad (6)$$

where Σ^2 is the **multi-property mass variance** defined by

$$\Sigma^2 \;=\; (\mathbf{m}^\dagger\Psi^{-1}\mathbf{m})^{-1}, \quad (7)$$

and the mean mass is

$$\bar{\mu}(\mathbf{s}) \;=\; \frac{\mathbf{m}^\dagger\Psi^{-1}\mathbf{s}}{\mathbf{m}^\dagger\Psi^{-1}\mathbf{m}} - \alpha\Sigma^2, \quad (8)$$

$$\equiv\; \bar{\mu}_0(\mathbf{s}) - \alpha\Sigma^2. \quad (9)$$

The first term, $\bar{\mu}_0(\mathbf{s})$, is the mean mass for the case of a flat mass function, $\alpha = 0$, which corresponds to the mass expected from inverting the input log-mean relation.

The second term, $\alpha\Sigma^2$, represents the mass shift induced by asymmetry in the convolution when $\alpha > 0$. (Low mass halos scattering up outnumber high mass systems scattering down.) Note that **the magnitude of this effect scales with the variance**, not the rms deviation.

Applying Bayes' theorem in the form $p(\mu|\mathbf{s}) = p(\mathbf{s}|\mu)n(\mu)/n(\mathbf{s})$ leads to the result that the set of masses selected by a specific set of properties is Gaussian in the log with mean given by equation (9) and variance, equation (7).

PL+LN covariance model for halo signals

1.1.1 Explicit expressions for the one-variable case

For a single property, $s \equiv \ln(S)$, with slope, m, and logarithmic scatter at fixed mass, σ, the mass variance at fixed S is

$$\Sigma^2 = \left(\frac{\sigma}{m}\right)^2. \tag{10}$$

The mean mass for a sample complete in S is

$$\bar{\mu}(s) = \frac{s}{m} - \alpha\Sigma^2. \tag{11}$$

The property space density function is

$$n(s)\, ds = (A/m)\, \exp\{-\alpha \left(\frac{s}{m} - \alpha\Sigma^2/2\right)\}\, ds, \tag{12}$$

which is a power-law in the original property, $n(S) \propto S^{-(\alpha/m)}$.

Note that the effective shift in mass, $\alpha\Sigma^2/2$, is half that in the expression above. These expressions are consistent, in that they address different questions. Equation (11) gives the mean ln(mass) of a signal-selected sample while equation (12) gives the ln(mass) value that matches the local space density – in number per volume per ln(S) – of halos with property value, S.

PL+LN covariance model for halo signals

1.1.2 Explicit expressions for the two-variable case

For two properties, we introduce the correlation coefficient, $r \equiv \langle \delta_1 \delta_2 \rangle$, of the normalized deviations, $\delta_i \equiv (s_i - \bar{s}_i)/\sigma_i$, and write the covariance matrix,

$$\Psi = \begin{pmatrix} \sigma_1^2 & r\sigma_1\sigma_2 \\ r\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix},$$

and its inverse,

$$\Psi^{-1} = (1-r^2)^{-1} \begin{pmatrix} \frac{1}{\sigma_1^2} & -\frac{r}{\sigma_1\sigma_2} \\ -\frac{r}{\sigma_1\sigma_2} & \frac{1}{\sigma_2^2} \end{pmatrix}.$$

The mass variance is now a harmonic mixture

$$\Sigma^{-2} \;=\; (1-r^2)^{-1}\left(\sigma_{\mu 1}^{-2} + \sigma_{\mu 2}^{-2} - 2r\sigma_{\mu 1}^{-1}\sigma_{\mu 2}^{-1}\right), \qquad (13)$$

where $\sigma_{\mu i} = \sigma_i/m_i$ is the mass scatter at fixed signal S_i.

The zero-slope mean mass is

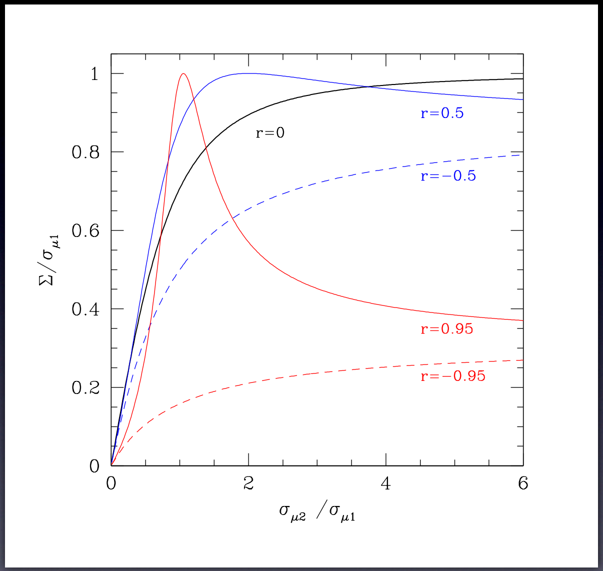
$$\bar{\mu}_0(s_1, s_2) \;=\; \frac{(s_1/m_1)\sigma_{\mu 1}^{-2} + (s_2/m_2)\sigma_{\mu 2}^{-2} - r(s_1/m_1 + s_2/m_2)\sigma_{\mu 1}^{-1}\sigma_{\mu 2}^{-1}}{\sigma_{\mu 1}^{-2} + \sigma_{\mu 2}^{-2} - 2r\sigma_{\mu 1}^{-1}\sigma_{\mu 2}^{-1}}, \qquad (14)$$

and the **joint space density** is

$$n(s_1, s_2) = \frac{A\Sigma}{\sqrt{2\pi(1-r^2)}\sigma_1\sigma_2} \exp\left[-\alpha\bar{\mu}_0 + \frac{\Sigma^2}{2}\left(\alpha^2 - \frac{(s_1/m_1 - s_2/m_2)^2}{\sigma_{\mu 1}^2\sigma_{\mu 2}^2}\right)\right]. \qquad (15)$$

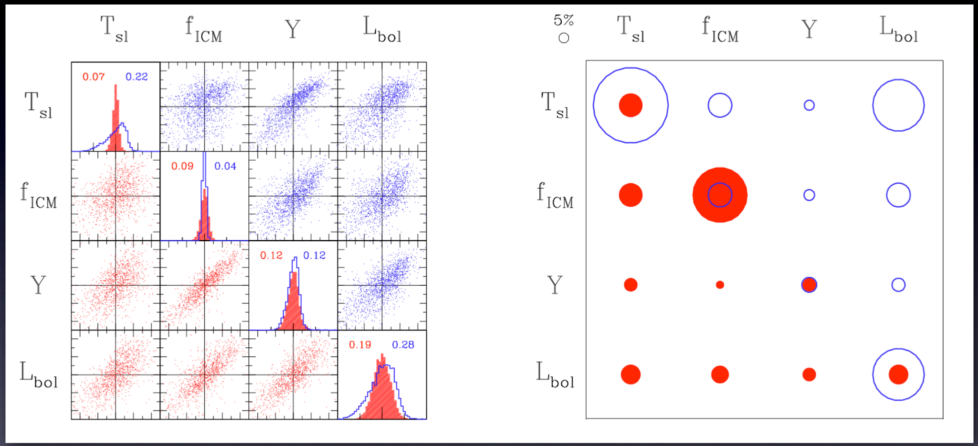
The first two terms in the exponent are analogous to those in the 1D expression, equation (12). For "reasonable" choices of (S_1, S_2) pairs — meaning values that pick out comparable mass scales, $s_1/m_1 \sim s_2/m_2$ — the space density remains effectively power-law. The third term in the exponent suppresses the number density for unreasonable pairings of s_1/m_1 and s_2/m_2, those lying out in the wings of the bivariate Gaussian.

mass scatter for two-property joint selection



covariance and implied mass scatter from Millennium Gas Simulations

Stanek et al 2010
Figure from Allen, Evrard, Mantz 2011



500 Mpc/h
1e9 gas+DM particles
m_p(DM) ~ 1.4e10 Msun
same cosmology as Mill Sim

preheating (200 kev-cm^2 @z=4)
gravity only

PL+LN covariance model for halo signals

1.2 Property-selected samples

For a halo sample selected with some property, s_1, we can now use Bayes' theorem to find the joint probability of those halos having a second property, s_2, and mass, μ. The result can be expressed as a bivariate Gaussian in terms of the two-element vector, $\mathbf{t} = [s_2\ \mu]$,

$$p(\mathbf{t}|s_1) = \frac{1}{(2\pi)|\tilde{\Psi}|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{t}-\bar{\mathbf{t}})^{\dagger}\tilde{\Psi}^{-1}(\mathbf{t}-\bar{\mathbf{t}})\right], \tag{16}$$

where the mean mass, $\bar{\mu}(s_1)$, is defined by equation (11) and the mean of the non-selection property is given by

$$\bar{s}_2(s_1) = m_2\left(\bar{\mu}(s_1) + \alpha r \sigma_{\mu 1}\sigma_{\mu 2}\right). \tag{17}$$

Note that, if $r < 0$, the non-selected property mean can be "doubly" biased low relative to a simple $m_2(s_1/m_1)$ expectation, with one shift coming from the extra $(-\alpha\Sigma^2)$ term in the mean mass and the second coming from the second term in the above expression.

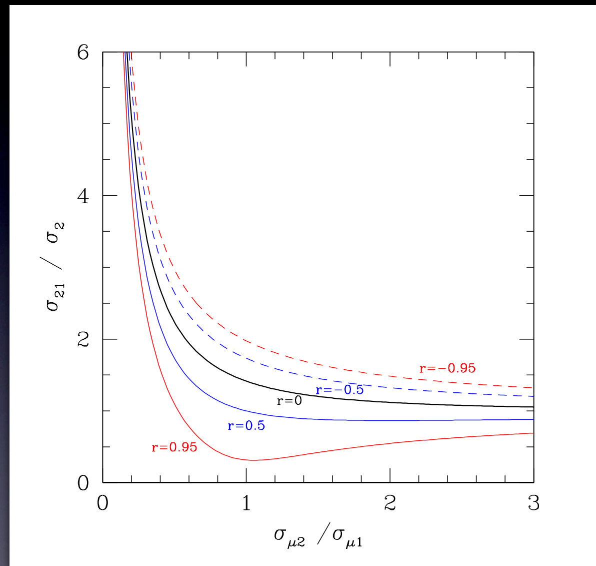
The covariance in s_2 and μ at fixed s_1 is given by

$$\tilde{\Psi} = \begin{pmatrix} \sigma_{21}^2 & \tilde{r}\sigma_{21}\sigma_{\mu 2} \\ \tilde{r}\sigma_{21}\sigma_{\mu 2} & \sigma_{\mu 2}^2 \end{pmatrix},$$

where the variance in s_2 at fixed s_1 is

$$\sigma_{21}^2 = m_2^2\left(\sigma_{\mu 1}^2 + \sigma_{\mu 2}^2 - 2r\sigma_{\mu 1}\sigma_{\mu 2}\right). \tag{18}$$

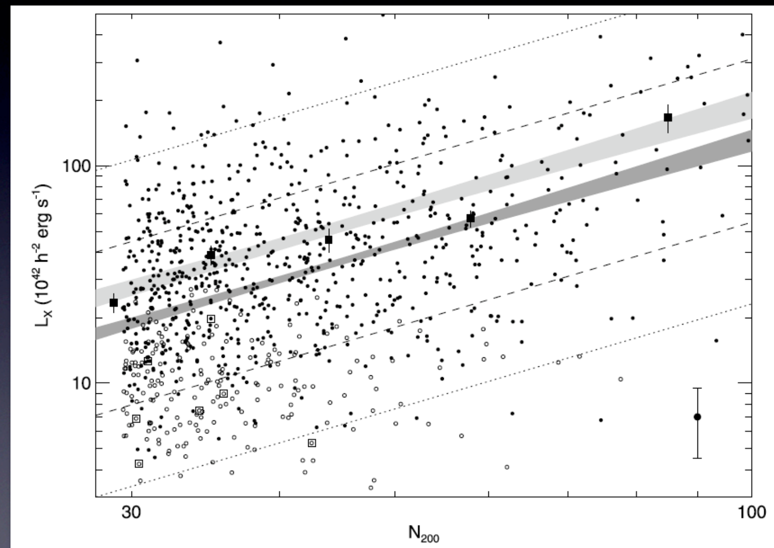
scatter in second property for one-property selection



RASS analysis of maxBCG sample

Rykoff et al 2008a

variance in Lx at fixed Ngal $\sigma_{\ln L_X \mid N_{gal}} = 0.83 \pm 0.03$



PL+LN covariance model for halo signals

The s_2–mass correlation coefficient, $\tilde{r}$, depends on both the intrinsic property correlation, r, as well as the ratio of scatter in mass for the two properties,

$$\tilde{r} \;=\; \frac{\sigma_{\mu 1}/\sigma_{\mu 2} - r}{\sqrt{1 - r^2 + (\sigma_{\mu 1}/\sigma_{\mu 2} - r)^2}}. \tag{19}$$

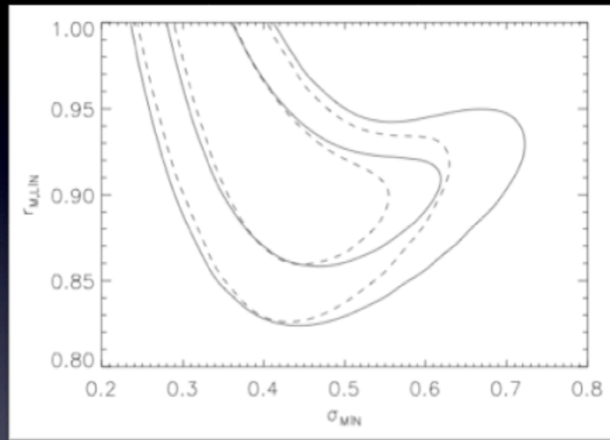
If the selection property is an excellent mass proxy ($\sigma_{\mu 1} \rightarrow 0$), then $\tilde{r} \rightarrow -r$.

If the selection property is a much poorer mass proxy compared to the second property, then $\tilde{r} \rightarrow 1$, irrespective of the intrinsic correlation, r.

first measurement of property covariance for clusters

Rozo et al 2009

correlation, $r_{M,L|N}$



scatter in ln(mass) at fixed Ngal

From SDSS-RASS:

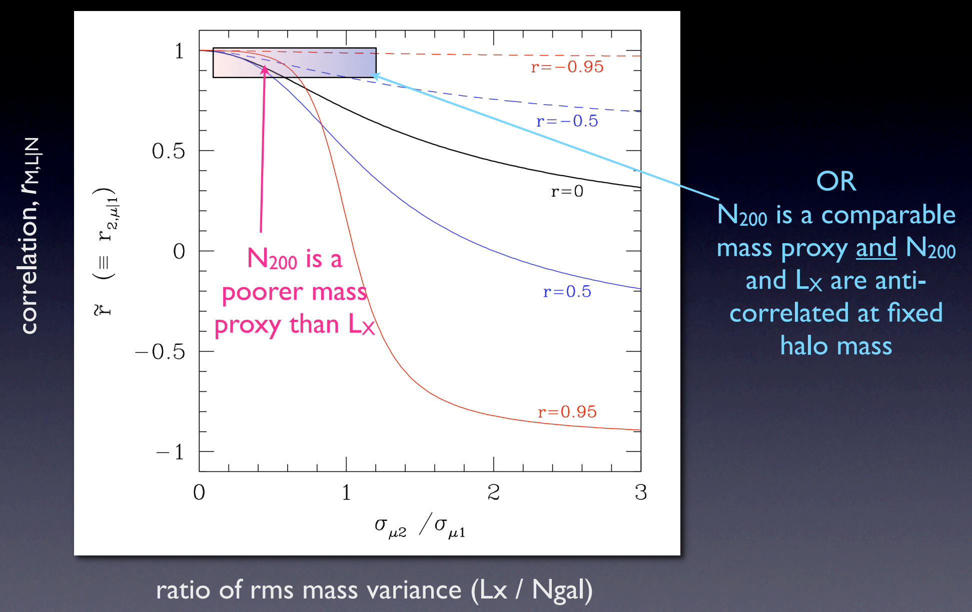
- $dn(N_{200})/dN_{200}$
- L_X–N_{200} scaling
  slope, norm, scatter
- M_{200}–N_{200} scaling
  slope, norm

missing:
M_{200}–N_{200} scatter
M_{200}, L_X | N_{200} correlation

Extra information:
400d survey
L_X –M_{500} scaling
slope, norm, scatter

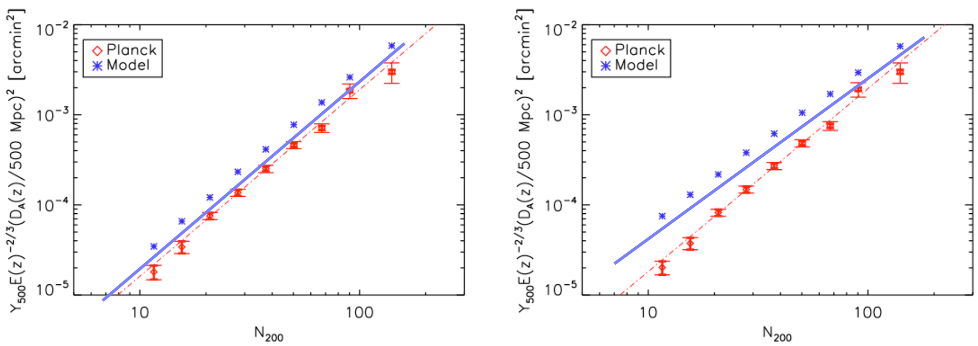
Vikhlinin et al 2008

what does a large covariance in mass and Lx mean?



Planck measurements of Y for maxBCG sample

Planck Collaboration 2011



One likely error source: model assumes $M_{X\text{-ray}} = M_{lens}$

Blue lines : $M_{X\text{-ray}} = 0.8\ M_{lens}$

summary

era of large, overlapping multi-wavelength surveys
- is getting nearer
- will enable stringent tests of the basic PL+LN model

Are we doing all we can now?

note that
- selection effects (that include projection) must be carefully modeled
- effect of projection on covariance measurements needs study

How to "world average" slopes, intercepts, covariance?
What are the limits of the basic PL + LN model?