



A Thermally Stable Magnetised ICM: From Plasma Microphysics to Global Dynamics

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Jeremy Sanders (*Cambridge*)

Schekochihin *et al.*, *ApJ* **629**, 139 (2005)

Schekochihin & Cowley, *Phys. Plasmas* **13**, 056501 (2006)

Schekochihin *et al.*, *PRL* **100**, 081301 (2008)

Schekochihin *et al.*, *MNRAS* **405**, 291 (2010)

Rosin *et al.*, *MNRAS*, in press; arXiv:1002.4017

Kunz *et al.*, *MNRAS* **410**, 2446 (2011)



Monsters, Inc.: Astro & Cosmology with G. Clusters, KITP, 14.03.11



“[some people] view all this ICM plasma physics as a nuisance :) ...so the hope is that you will educate us about all the possible dirt under the rug...”





Part I: Dirt Under the Rug

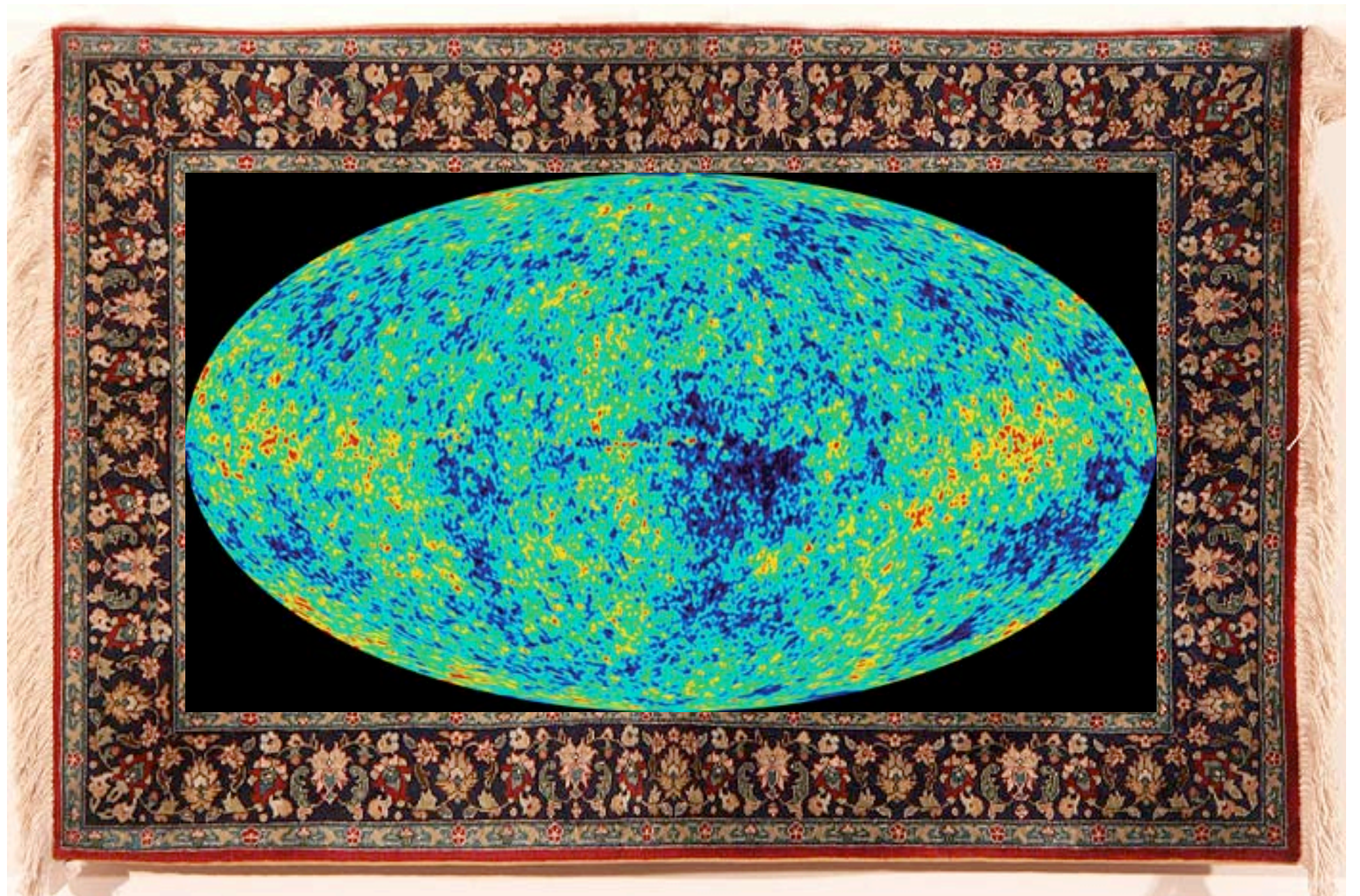
Schekochihin *et al.*, *ApJ* **629**, 139 (2005)

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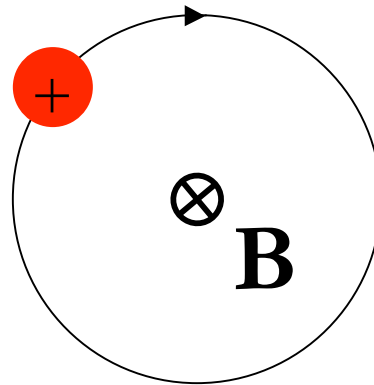
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$$B = 1 \mu\text{G}$$
$$T = 1 \text{ keV}$$

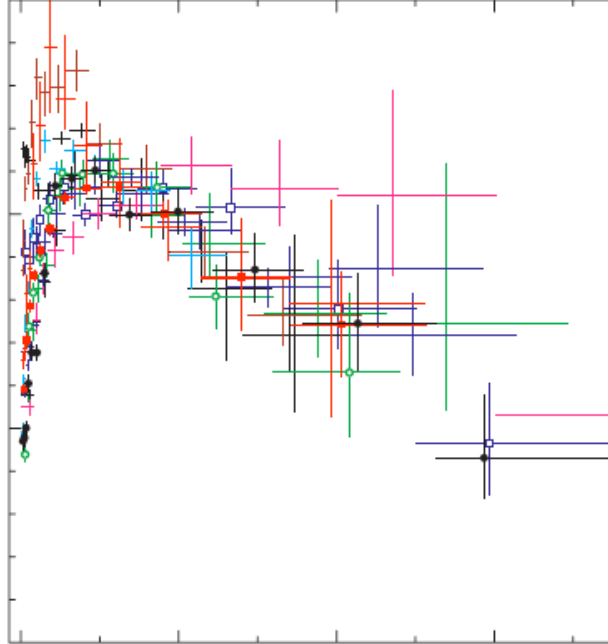
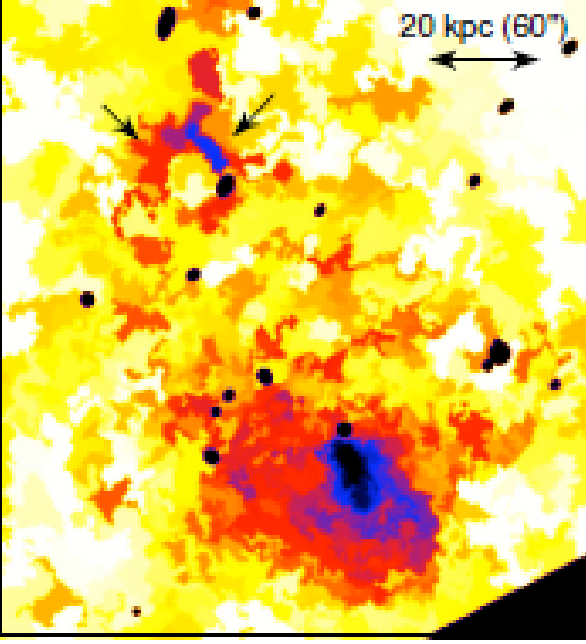


$$\rho_i = \frac{v_{\text{th}i}}{\Omega_i} = \frac{c\sqrt{2Tm_i}}{eB} = 4.6 \times 10^9 \text{ cm}$$
$$= \mathbf{1.5 \text{ nanoparsec}}$$

ICM Dynamics: A 3-Scale Problem

GLOBAL (profiles, transport)	TURBULENCE (+ dynamo, fluid instabilities, etc.)	PLASMA (micro- instabilities)
100 kpc	1–10 kpc	<i>a few times ρ_i</i> 10^4 – 10^6 km (1-100 npc)
1 Gyr	10 Myr	<i>a fraction of Ω_i</i> 10 hours

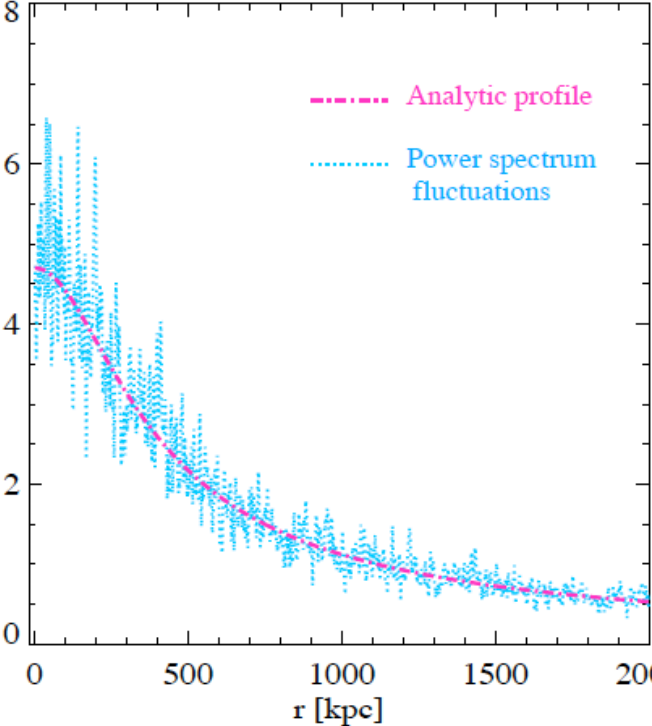
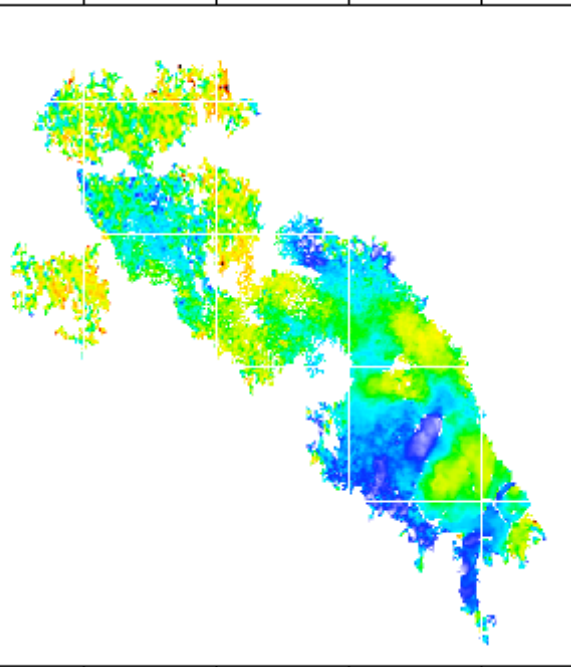
ICM Dynamics: A 3-Scale Problem

GLOBAL (profiles, transport)	TURBULENCE (+ dynamo, fluid instabilities, etc.)	PLASMA (micro- instabilities)
		<p><i>a few times ρ_i</i></p> <p>$10^4 - 10^6$ km</p> <p>(1-100 npc)</p>
<p>[Scaled profiles₃₈₀, Vikhlinin et al. 2005]</p>	<p>[A262, temperature map, Sanders et al. 2009]</p>	<p><i>a fraction of Ω_i</i></p> <p>10 hours</p>

OBSERVABLE

THEORY PARADISE
(but measurable in SW!)

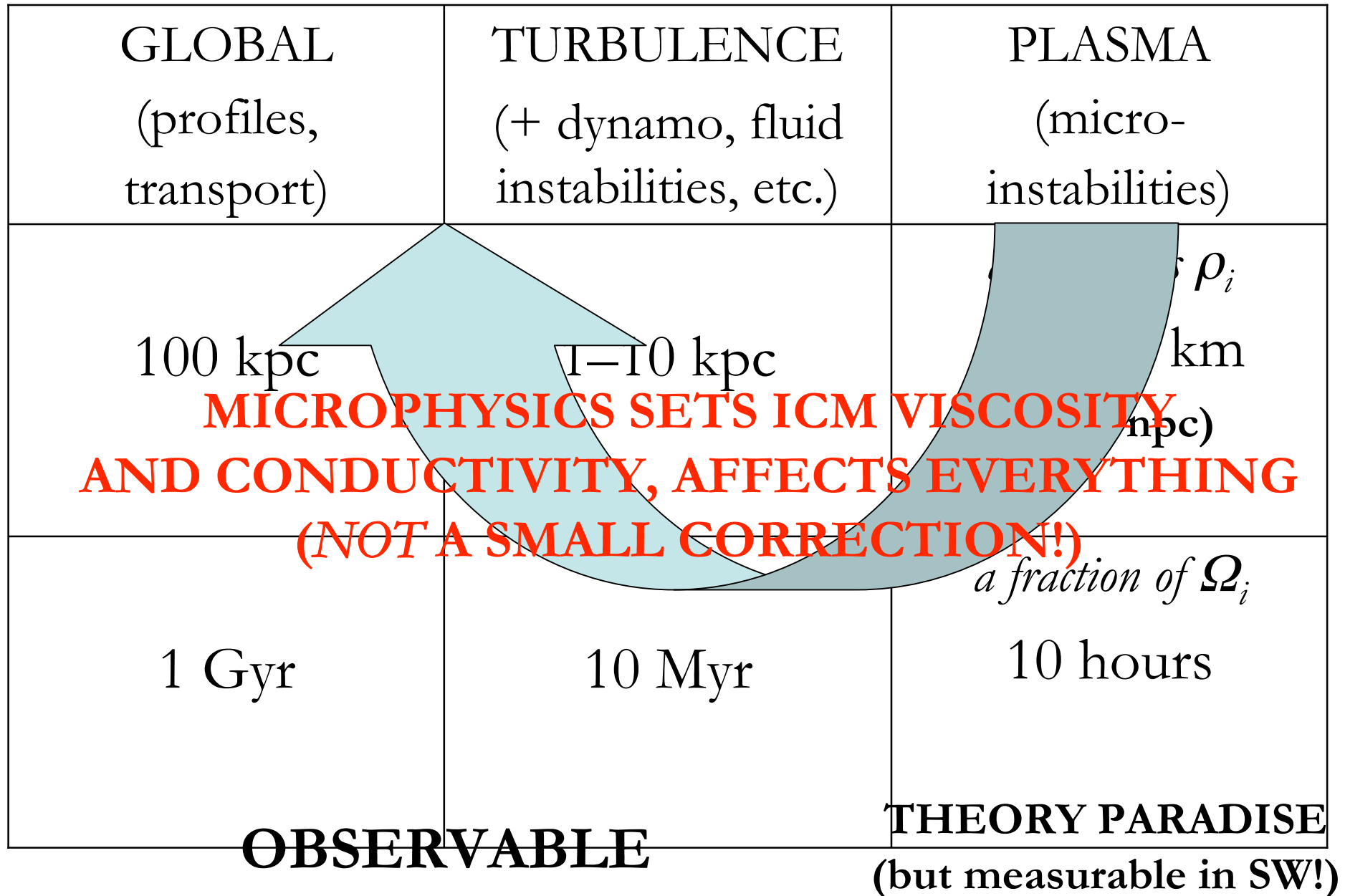
ICM Dynamics: A 3-Scale Problem

GLOBAL (profiles, transport)	TURBULENCE (+ dynamo, fluid instabilities, etc.)	PLASMA (micro- instabilities)
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<p>[Magnetic field in Coma, Bonafede et al. 2010]</p>	<p>[RM map, Hydra A, Vogt & Enßlin 2005]</p>	<p><i>a fraction of Ω_i</i> 10 hours</p>

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(but measurable in SW!)

ICM Dynamics: A 3-Scale Problem



Plasma Microinstabilities: Origin

First adiabatic invariant $\mu = \frac{mv_{\perp}^2}{2B}$ conserved provided $\Omega_i > \nu_{ii}$
holds already for $B > 10^{-18}$ G

Changes in field strength \Leftrightarrow pressure anisotropy

$$\sum_{\text{particles}} \mu = \frac{p_{\perp}}{B} = \text{const}$$

Plasma Microinstabilities: Origin

First adiabatic invariant $\mu = \frac{mv_{\perp}^2}{2B}$ conserved provided $\Omega_i > \nu_{ii}$
holds already for $B > 10^{-18}$ G

Changes in field strength \Leftrightarrow pressure anisotropy

$$\frac{d\Delta}{dt} \sim \frac{1}{B} \frac{dB}{dt} - \nu_{ii} \Delta \qquad \Delta \equiv \frac{p_{\perp} - p_{\parallel}}{p_{\perp}}$$

ignore evolution of p_{\parallel} anisotropy drives anisotropy relaxed by collisions

Plasma Microinstabilities: Origin

First adiabatic invariant $\mu = \frac{mv_{\perp}^2}{2B}$ conserved provided $\Omega_i > \nu_{ii}$
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Changes in field strength \Leftrightarrow pressure anisotropy

$$\frac{d\Delta}{dt} \sim \frac{1}{B} \frac{dB}{dt} - \nu_{ii} \Delta \quad \longrightarrow \quad \Delta \sim \frac{1}{\nu_{ii}} \frac{d \ln B}{dt} = \frac{\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla \mathbf{u}}{\nu_{ii}}$$

ignore evolution of p_{\parallel} change in B drives anisotropy anisotropy relaxed by collisions

because $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$

and so $\frac{1}{B} \frac{dB}{dt} = \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla \mathbf{u}$

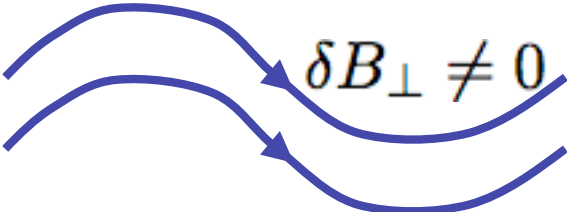
Plasma Microinstabilities: Taxonomy

First adiabatic invariant $\mu = \frac{mv_{\perp}^2}{2B}$ conserved provided $\Omega_i > \nu_{ii}$
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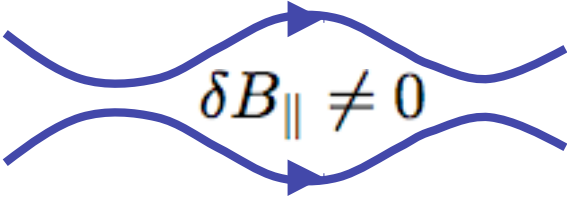
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Magnetic field **decreases**: $\Delta < 0$

FIREHOSE: $\omega^2 = \frac{k_{\parallel}^2 v_{thi}^2}{2} \left(\Delta + \frac{2}{\beta_i} \right)$

 destabilised Alfvén wave

Magnetic field **increases**: $\Delta > 0$

MIRROR: $\gamma = \frac{|k_{\parallel}| v_{thi}}{\sqrt{\pi}} \left(\Delta - \frac{1}{\beta_i} \right)$

 resonant instability

Plasma Microinstabilities: Where and When?

Typical structure of magnetic fields
generated by turbulence
(MHD simulations with $Pm \gg 1$
by A. B. Iskakov & AAS)
for details see
Schekochihin *et al.* 2004,
ApJ **612**, 276



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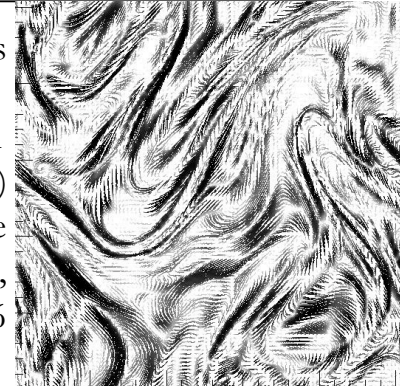
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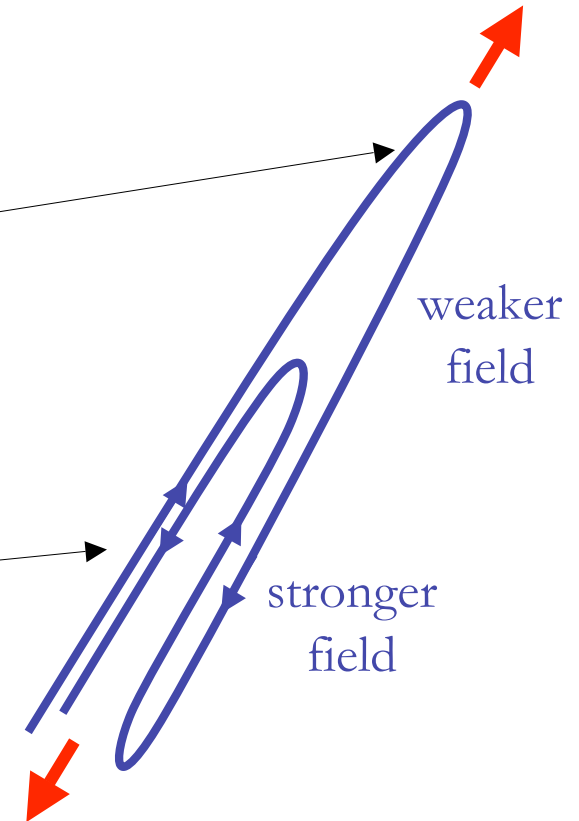


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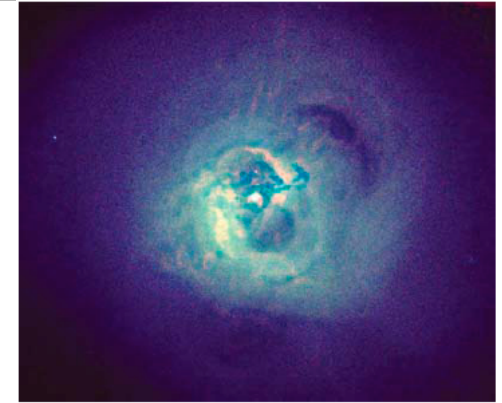
[Schekochihin *et al.*, *ApJ* **629**, 139 (2005)]

Plasma Microinstabilities in the ICM

For typical cluster parameters,

$$\Delta \sim 0.005 \left(\frac{n_e}{0.01 \text{ cm}^{-3}} \right)^{-1} \left(\frac{T_i}{1 \text{ keV}} \right)^{3/2} \left(\frac{\tau_{\text{turb}}}{10 \text{ Myr}} \right)^{-1}$$

$$\frac{2}{\beta} = 0.005 \left(\frac{B}{1 \mu\text{G}} \right)^2 \left(\frac{n_e}{0.01 \text{ cm}^{-3}} \right)^{-1} \left(\frac{T_i}{1 \text{ keV}} \right)^{-1}$$



Magnetic field decreases: $\Delta < 0$

Small, fast and furious...

$$\text{FIREHOSE: } \omega^2 = \frac{k_{\parallel}^2 v_{\text{th}i}^2}{2} \left(\Delta + \frac{2}{\beta_i} \right) \quad \begin{array}{l} \gamma_{\text{peak}}^{\perp} \sim |\Delta|^{1/2} \Omega_i \sim 10^{-3} \text{ s}^{-1} \quad k_{\parallel} \rho_i \sim 1 \\ \gamma_{\text{peak}}^{\parallel} \sim |\Delta| \Omega_i \sim 10^{-4} \text{ s}^{-1} \quad k_{\parallel} \rho_i \sim |\Delta|^{1/2} \end{array}$$

Magnetic field increases: $\Delta > 0$

$$\text{MIRROR: } \gamma = \frac{|k_{\parallel}| v_{\text{th}i}}{\sqrt{\pi}} \left(\Delta - \frac{1}{\beta_i} \right) \quad \begin{array}{l} \gamma_{\text{peak}} \sim \Delta^2 \Omega_i \sim 10^{-6} \text{ s}^{-1} \quad k_{\parallel} \rho_i \sim \Delta \\ k_{\perp} \rho_i \sim \Delta^{1/2} \end{array}$$

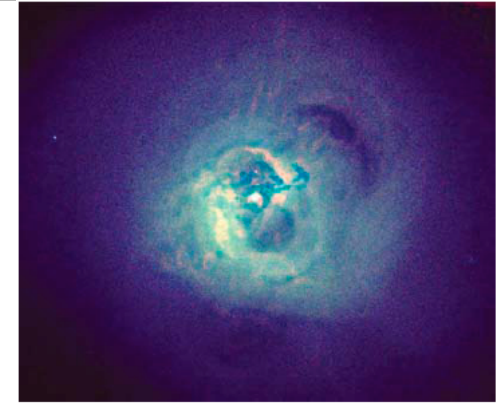
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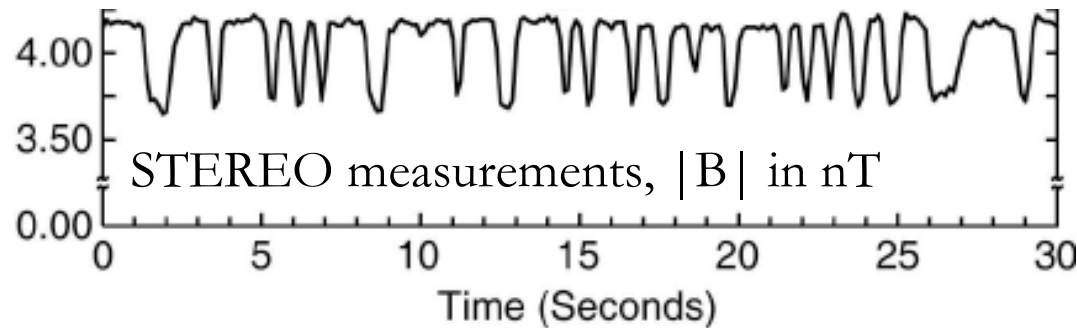
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Magnetic field increases: $\Delta > 0$

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Is ICM in the marginal state with respect to plasma microinstabilities?

Solar Wind: Laboratory for Nanoastrophysics



Mirror modes in the SW
from Russell et al. (2009)

*We will never observe Larmor scales directly in the ICM,
but in near-Earth space, we can measure them in situ with amazing detail and precision
(which is why astronomers should love and cherish space physicists
and pay attention to missions like MMS or SCOPE)*

Magnetic field decreases: $\Delta < 0$

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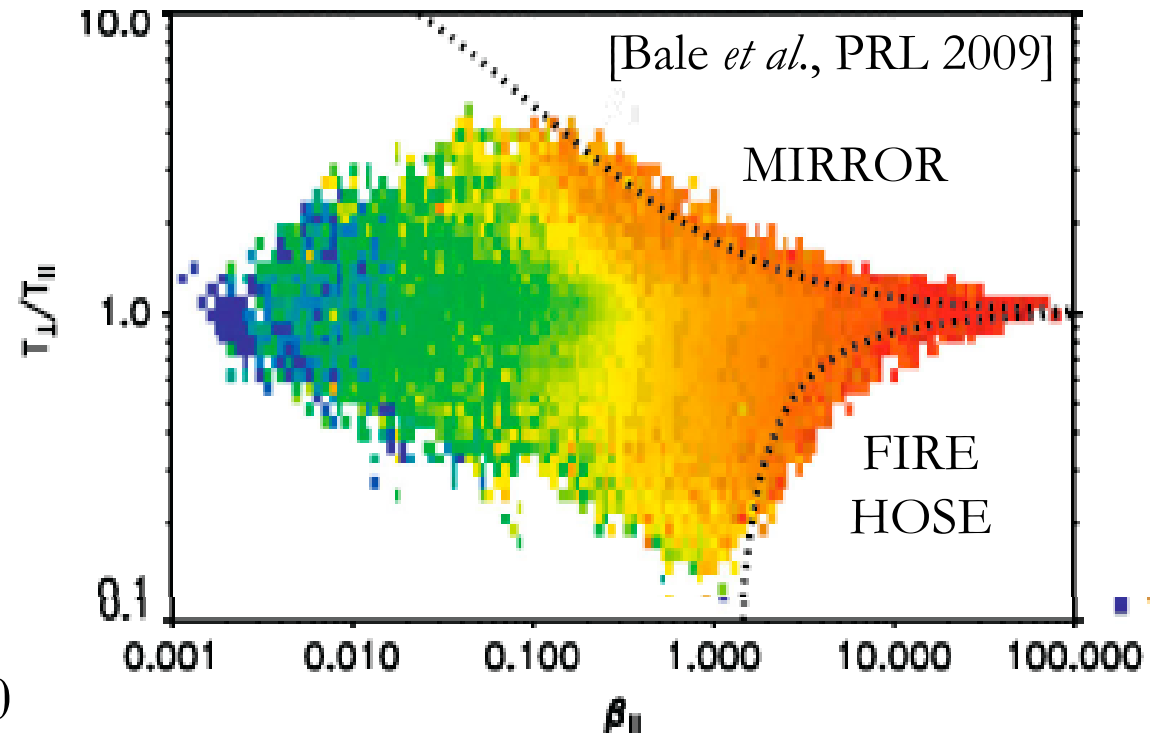
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Solar Wind: Laboratory for Nanoastrophysics



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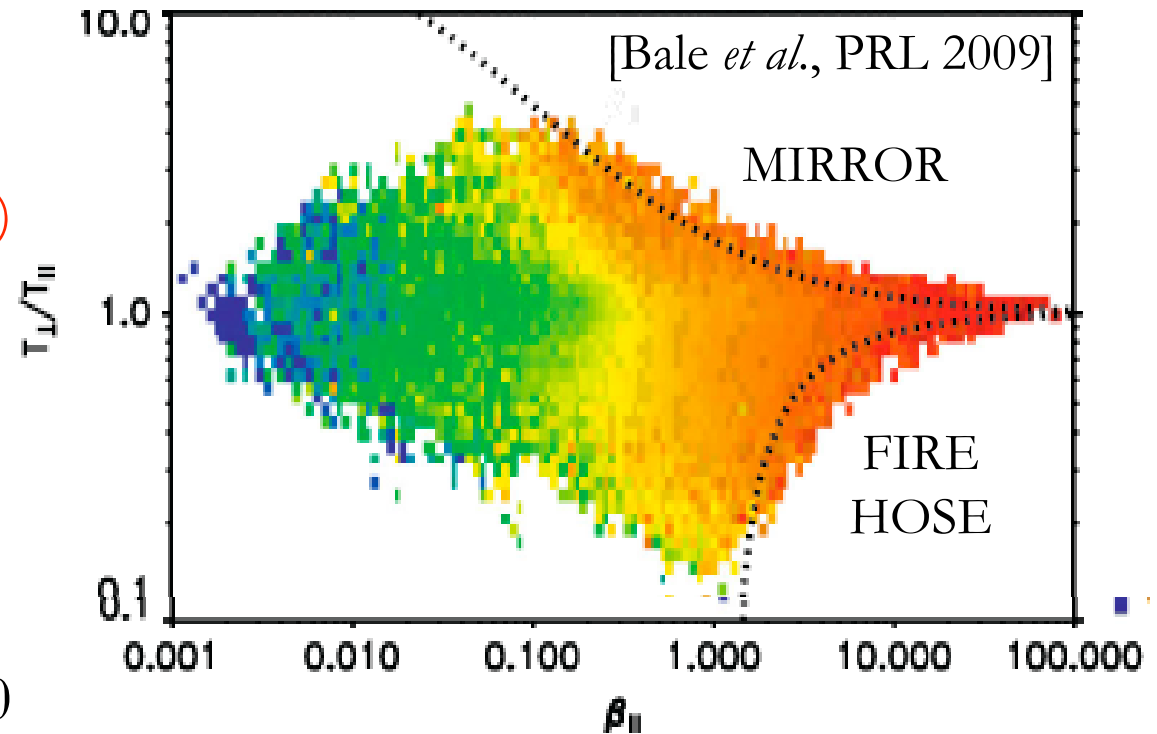
Is ICM in the marginal state with respect to plasma microinstabilities?

[Schekochihin *et al.*, *ApJ* **629**, 139 (2005)]

A Macrophysical Fudge: Marginal ICM

To leapfrog having to do an honest microphysical job, simply assume closure (fudge)

$$\Delta = \frac{2\xi}{\beta_i}, \quad \xi \in [-1, 0.5]$$



Magnetic field decreases: $\Delta < 0$

$$\text{FIREHOSE: } \omega^2 = \frac{k_{\parallel}^2 v_{thi}^2}{2} \left(\Delta + \frac{2}{\beta_i} \right)$$

Magnetic field increases: $\Delta > 0$

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[Kunz *et al.*, MNRAS 410, 2446 (2011)]

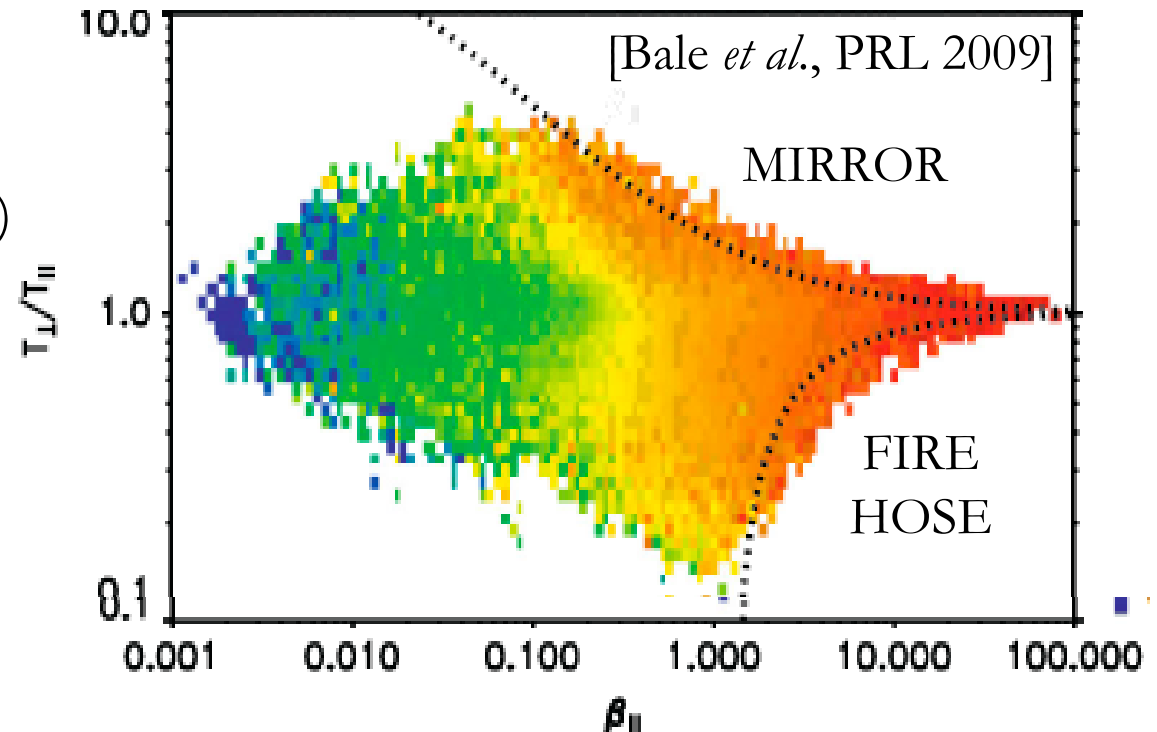
A Microphysical Dilemma

To leapfrog having to do an honest microphysical job, simply assume closure (fudge)

$$\Delta = \frac{2\xi}{\beta_i}, \quad \xi \in [-1, 0.5]$$

How is this achieved?

- Enhanced particle scattering isotropises pressure
AND/OR
- Magnetic field structure and evolution modified to offset change



Why This Is An Important Question

To leapfrog having to do an honest microphysical job, simply assume closure (fudge)

$$\Delta = \frac{2\xi}{\beta_i}, \quad \xi \in [-1, 0.5]$$

$$\frac{d\Delta}{dt} \sim \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{u} - \nu_{ii}\Delta$$

How is this achieved?

- Enhanced particle scattering isotropises pressure
AND/OR

Model by limiting Δ
(more collisionality \rightarrow **less** viscosity)
[Sharma et al. 2006;
Schekochihin & Cowley 2006]

- Magnetic field structure and evolution modified to offset change

Model by limiting rate of strain
(in a sense, **more** viscosity)
[Kunz et al. 2011]

Why This Is An Important Question

To leapfrog having to do an honest microphysical job, simply assume closure (fudge)

$$\Delta = \frac{2\xi}{\beta_i}, \quad \xi \in [-1, 0.5]$$

I believe this is going to be hard to justify because microinstabilities are not sufficiently close to the Larmor scale, so can't have much scattering

How is this achieved?

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Model by limiting rate of strain
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Nonlinear Firehose

Principle of nonlinear evolution: *firehose fluctuations cancel on average the change in the mean field to keep anisotropy at marginal level*

$$\Delta \sim \frac{1}{\nu_{ii}} \frac{1}{B} \frac{dB}{dt} \sim \frac{1}{\nu_{ii}} \left(\underbrace{- \left| \frac{d \ln B_0}{dt} \right|}_{\text{macroscale field}} + \underbrace{\frac{1}{2} \frac{d \overline{|\delta \mathbf{B}_\perp|^2}}{dt B_0^2}}_{\text{microscale fluctuations}} \right) \rightarrow -\frac{2}{\beta_i}$$

Schekochihin *et al.*, *PRL* **100**, 081301 (2008)

Rosin *et al.*, arXiv:1002.4017 (2010)

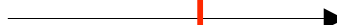
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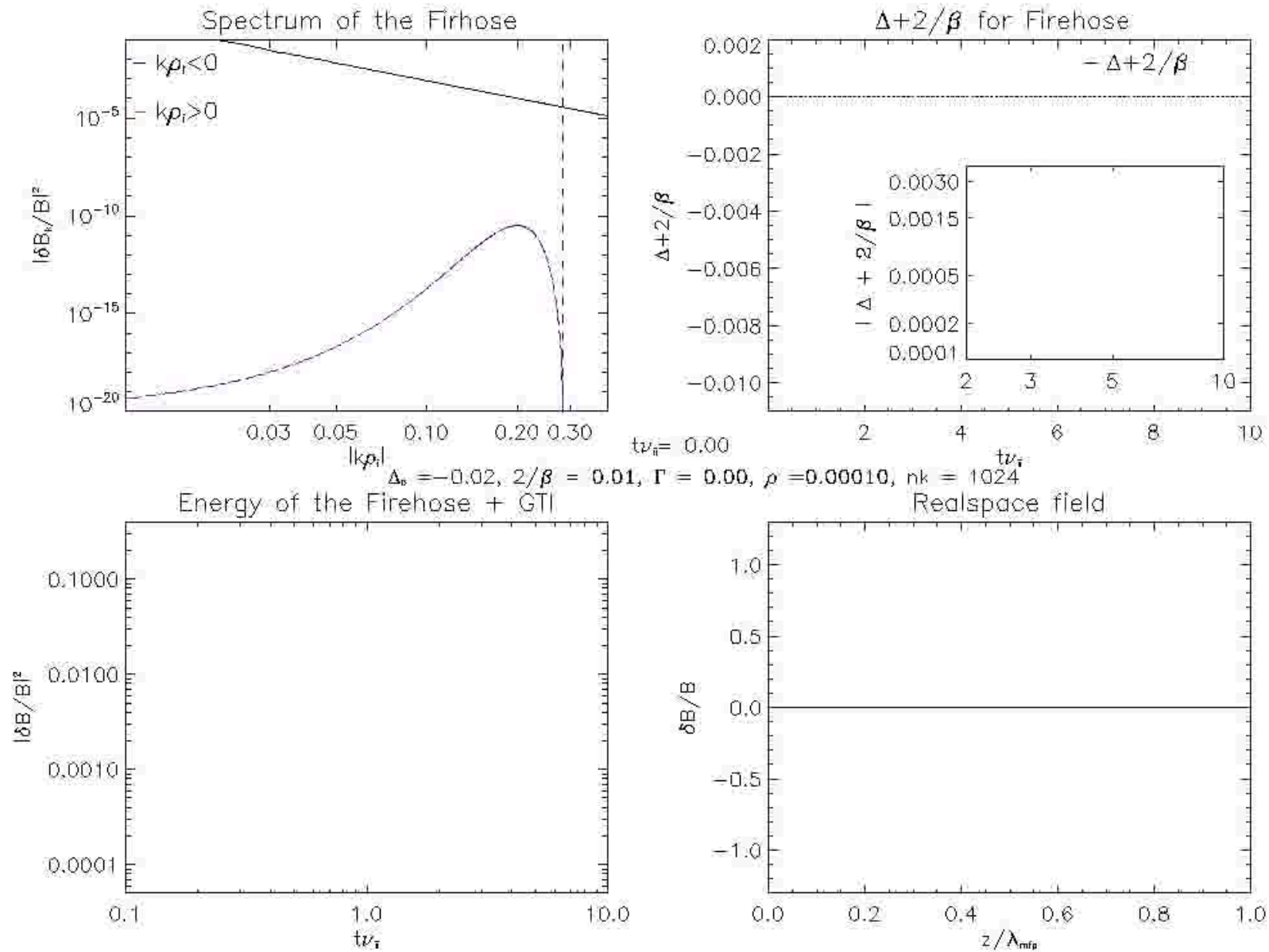
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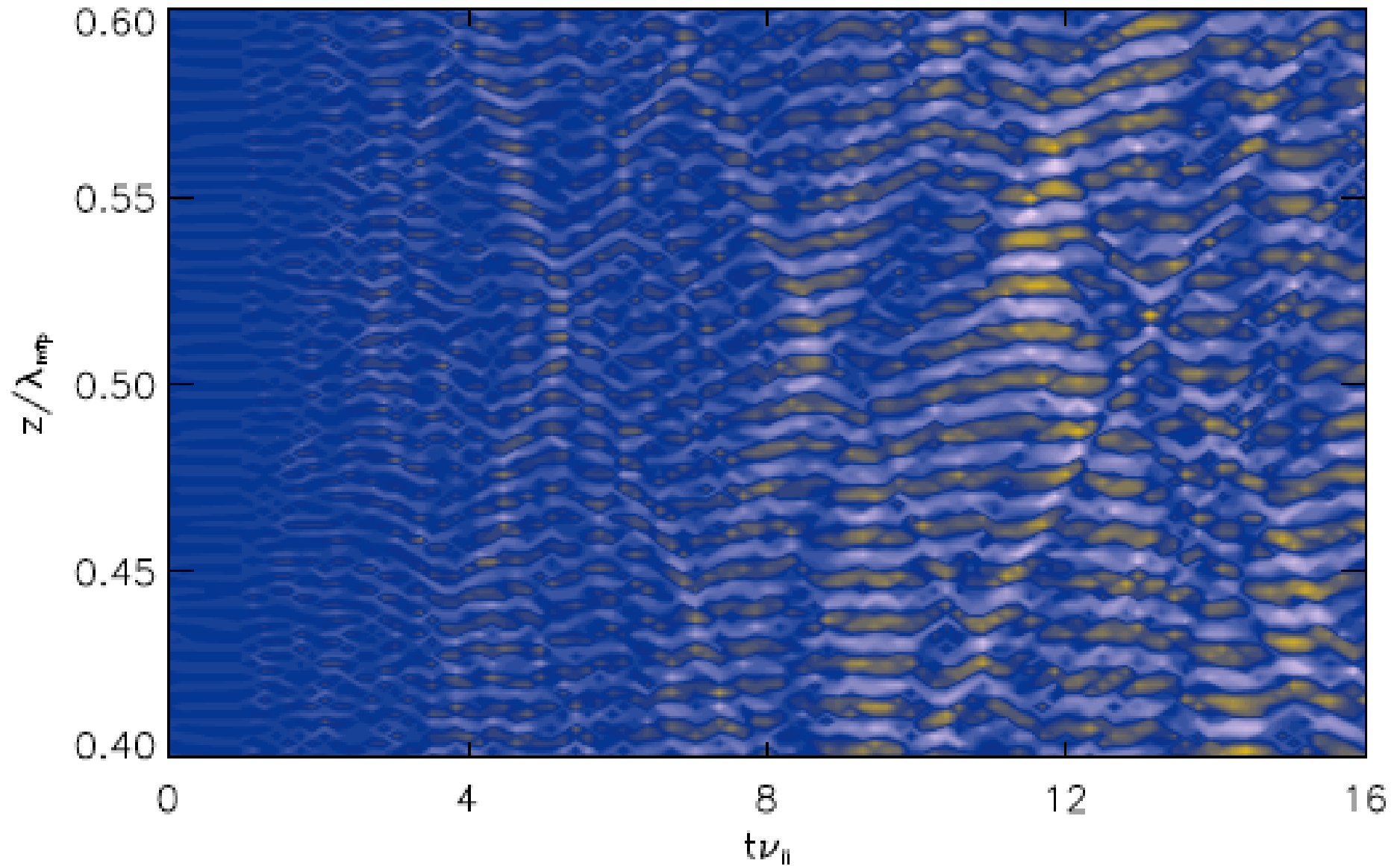


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Nonlinear Firehose



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Monsters, Inc.: Astro & Cosmology with G. Clusters, KITP, 14.03.11



Part II: A Thermally Stable Heating Mechanism for the ICM

Matt Kunz (*Oxford*)
Alex Schekochihin (*Oxford*)
Steve Cowley (*CCFE*)
James Binney (*Oxford*)
Jeremy Sanders (*Cambridge*)

Kunz *et al.*, MNRAS **410**, 2446 (2011)

Heating in Marginal ICM

$$\frac{3}{2} n \frac{dT}{dt} = -nT \nabla \cdot u - \underbrace{\sigma_i : \nabla u}_{\text{heating } Q^+} - \nabla \cdot q_e - \underbrace{n_i n_e \Lambda}_{\text{cooling } Q^-}$$

$\Lambda \propto T^{1/2}$

$$Q^+ = -\sigma_i : \nabla u = p_i \Delta_i \left(bb : \nabla u - \frac{1}{3} \nabla \cdot u \right) = 0.35 p_i \nu_{ii} \Delta_i^2$$

$$-\sigma_i = - \left(bb - \frac{1}{3} \mathbf{I} \right) p_i \Delta_i$$

viscous stress tensor

In the Braginskii limit,

$$\nu_{ii} \Delta_i = 2.9 \left(bb : \nabla u - \frac{1}{3} \nabla \cdot u \right)$$

Heating in Marginal ICM

$$\frac{3}{2}n\frac{dT}{dt} = -nT\nabla\cdot u - \underbrace{\sigma_i:\nabla u}_{\text{heating } Q^+} - \nabla\cdot q_e - \underbrace{n_i n_e \Lambda}_{\text{cooling } Q^-} \quad \Lambda \propto T^{1/2}$$

$$Q^+ = 0.35 p_i \nu_{ii} \Delta_i^2$$

Heating in Marginal ICM

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$\Lambda \propto T^{1/2}$

$$Q^+ = 0.35 p_i \nu_{ii} \Delta_i^2 = 0.35 \frac{\nu_{ii}}{p_i} \left(\frac{\xi B^2}{4\pi} \right)^2$$

*NB: fixing pressure
anisotropy,
not collisionality!*

$$\Delta = \frac{2\xi}{\beta_i}, \quad \xi \in [-1, 0.5]$$

Heating in Marginal ICM

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$$\begin{aligned}
 Q^+ &= 0.35 p_i \nu_{ii} \Delta_i^2 = 0.35 \frac{\nu_{ii}}{p_i} \left(\frac{\xi B^2}{4\pi} \right)^2 \\
 &= 10^{-25} \xi^2 \left(\frac{B}{10 \mu\text{G}} \right)^4 \left(\frac{T}{2 \text{keV}} \right)^{-5/2} \text{ erg s}^{-1} \text{ cm}^{-3}
 \end{aligned}$$

Heating vs. Cooling in Marginal ICM

$$\frac{3}{2} n \frac{dT}{dt} = -nT \nabla \cdot u - \underbrace{\sigma_i : \nabla u}_{\text{heating } Q^+} - \nabla \cdot q_e - \underbrace{n_i n_e \Lambda}_{\text{cooling } Q^-}$$

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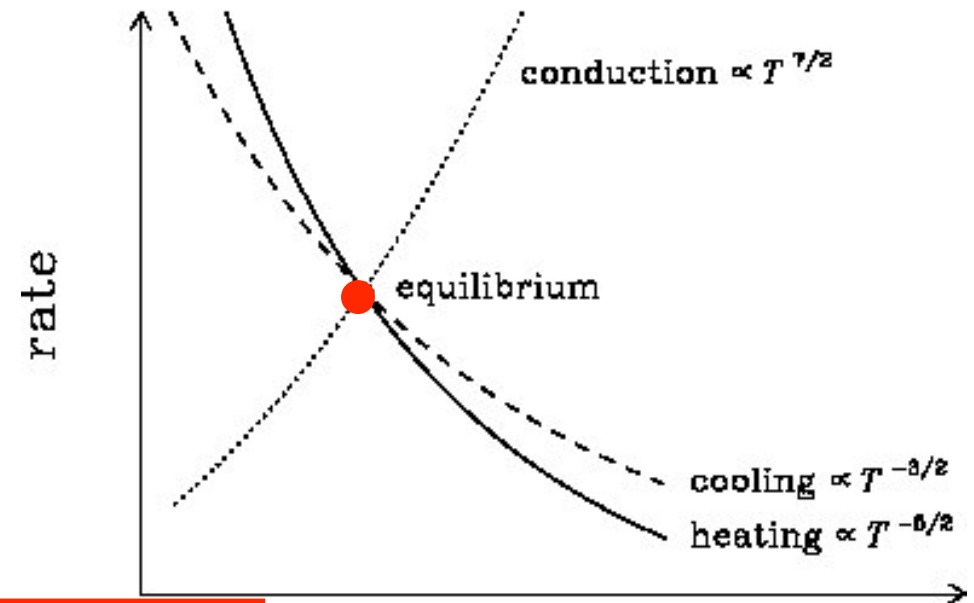
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 \end{aligned}$$

Compare this with Bremsstrahlung cooling:

$$Q^- = 1.4 \times 10^{-25} \left(\frac{n_e}{0.1 \text{ cm}^{-3}} \right)^2 \left(\frac{T}{2 \text{keV}} \right)^{1/2} \text{ erg s}^{-1} \text{ cm}^{-3}$$

Thermal Stability

The balance between heating and cooling is **thermally stable**, while balance between cooling and conduction is not.



$$Q^+ = 0.35 p_i \nu_{ii} \Delta_i^2 = 0.35 \frac{\nu_{ii}}{p_i} \left(\frac{\xi B^2}{4\pi} \right)^2 = Q^-$$

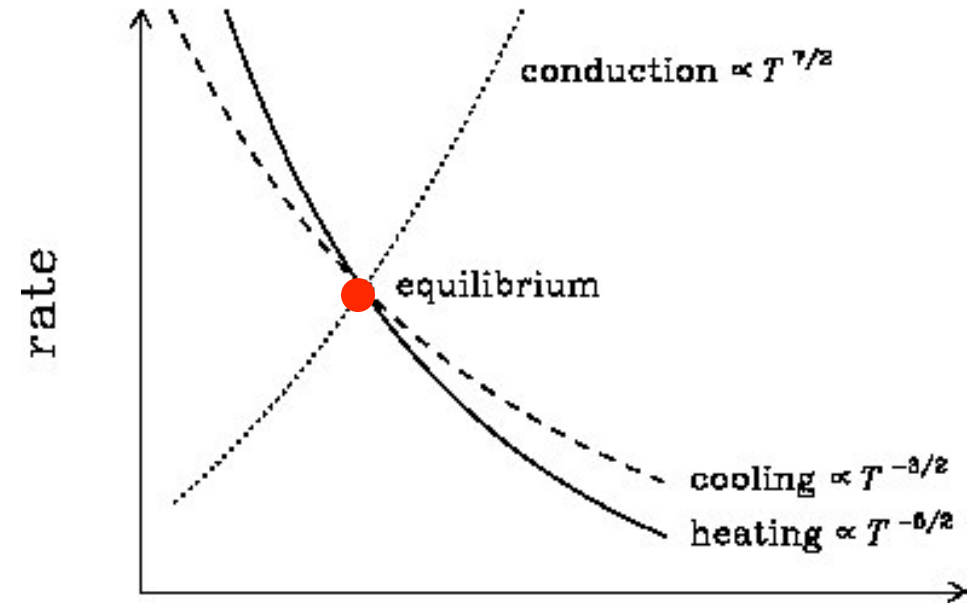
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Corollary: B vs. n and T

The balance between heating and cooling is thermally stable, while balance between cooling and conduction is not.



$$Q^+ = 0.35 p_i \nu_{ii} \Delta_i^2 = 0.35 \frac{\nu_{ii}}{p_i} \left(\frac{\xi B^2}{4\pi} \right)^2 = Q^- \quad T$$

$$B \simeq 11 \xi^{-1/2} \left(\frac{n_e}{0.1 \text{ cm}^{-3}} \right)^{1/2} \left(\frac{T}{2 \text{ keV}} \right)^{3/4} \mu\text{G}$$

NB: Magnetic field is a function both of density and temperature!

Corollary: B vs. n and T

Cluster name	$n_{e,c}$ (10^{-2} cm^{-3})	T_c (keV)	$B_{c,theory}$ ($\xi^{-1/2} \mu\text{G}$)	$B_{c,obs}$ (μG)
Cool-core clusters				
A1835	10	2.85	13.8	–
Hydra A	7.2	3.11	12.4	12 ^a
A478	15.2	1.72	12.1	–
A2199	10	$\simeq 2$	$\simeq 11$	15 ^b
M87	10.8	1.62	9.8	35 ^b
A1795	5.4	2.26	8.6	9.7 ^b
Centaurus	9.5	1.24	7.7	8
A262	3.7	1.54	5.5	–

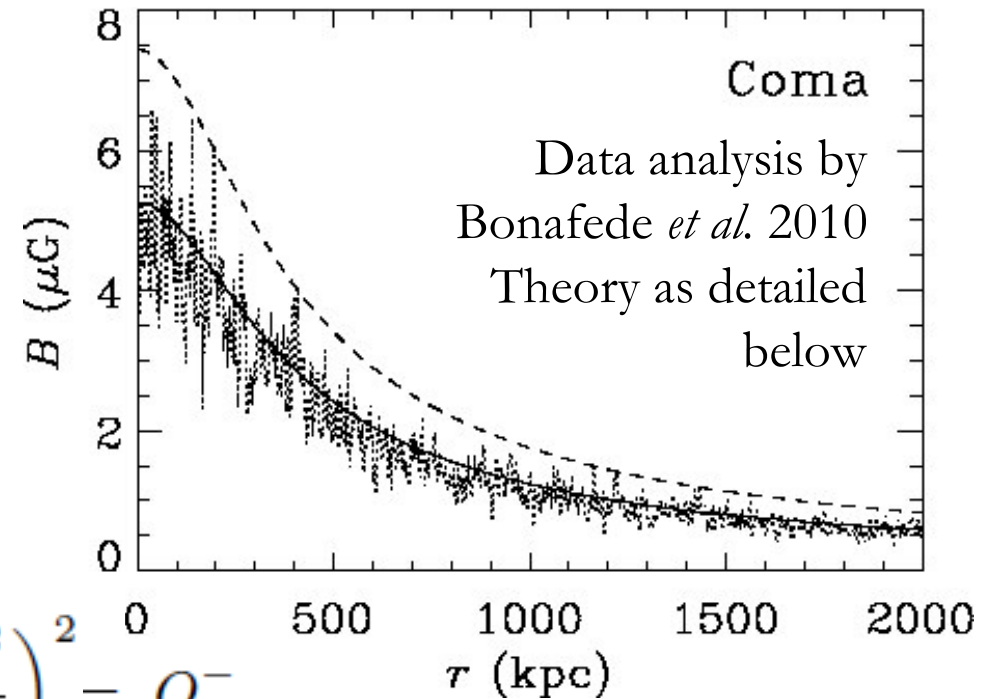
$$Q^+ = 0.35 p_i \nu_{ii} \Delta_i^2 = 0.35 \frac{\nu_{ii}}{p_i} \left(\frac{\xi B^2}{4\pi} \right)^2 = Q^-$$

$$B \simeq 11 \xi^{-1/2} \left(\frac{n_e}{0.1 \text{ cm}^{-3}} \right)^{1/2} \left(\frac{T}{2 \text{ keV}} \right)^{3/4} \mu\text{G}$$

NB: Magnetic field is a function both of density and temperature!

Corollary: B vs. n and T

Caveat: Coma is a dodgy case to look at because cooling times are so long; however, we do not have field profiles for other, colder isothermal clusters (overall field strengths there seem reasonable)



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Cluster name	$n_{e,c}$ (10^{-2} cm^{-3})	T_c (keV)	$B_{c,theory}$ ($\xi^{-1/2} \mu\text{G}$)	$B_{c,obs}$ (μG)
Non-cool-core clusters				
A2142	1.87	8.8	13.0	RM ^c
Ophiucus	0.80	10.3	9.5	RM ^c
A401	0.70	8.3	7.6	RM ^c
A2382	0.50	2.9	3.1	3
A2634	0.28	3.7	2.7	3.5 ^b
A2255	0.2	3.5	2.2	2.5
A400	0.24	2.3	1.8	2.9 ^b

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Corollary: Properties of Turbulence vs. n and T

1) Heating \sim cooling

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2) Dynamo saturates at equipartition

$$\frac{1}{2} m_i n_i U_{\text{rms}}^2 \simeq \frac{B^2}{8\pi}$$

$$U_{\text{rms}} \simeq 70 \xi^{-1/2} \left(\frac{T}{2 \text{ keV}} \right)^{3/4} \text{ km s}^{-1}$$

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3) Turbulent energy absorption adjusts to heating rate

$$m_i n_i \frac{U_{\text{rms}}^2}{\tau_{\text{turb}}} \simeq Q^+ \implies \tau_{\text{turb}} \simeq 2 \xi^{-1} \left(\frac{n_e}{0.1 \text{ cm}^{-3}} \right)^{-1} \left(\frac{T}{2 \text{ keV}} \right) \text{ Myr}$$

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$$L \equiv U_{\text{rms}} \tau_{\text{turb}} \simeq 0.2 \xi^{-3/2} \left(\frac{n_e}{0.1 \text{ cm}^{-3}} \right)^{-1} \left(\frac{T}{2 \text{ keV}} \right)^{7/4} \text{ kpc}$$

$$\kappa_{\text{turb}} \sim U_{\text{rms}}^2 \tau_{\text{turb}} \simeq 3 \times 10^{27} \xi^{-2} \left(\frac{n_e}{0.1 \text{ cm}^{-3}} \right)^{-1} \left(\frac{T}{2 \text{ keV}} \right)^{5/2} \text{ cm}^2 \text{ s}^{-1}$$

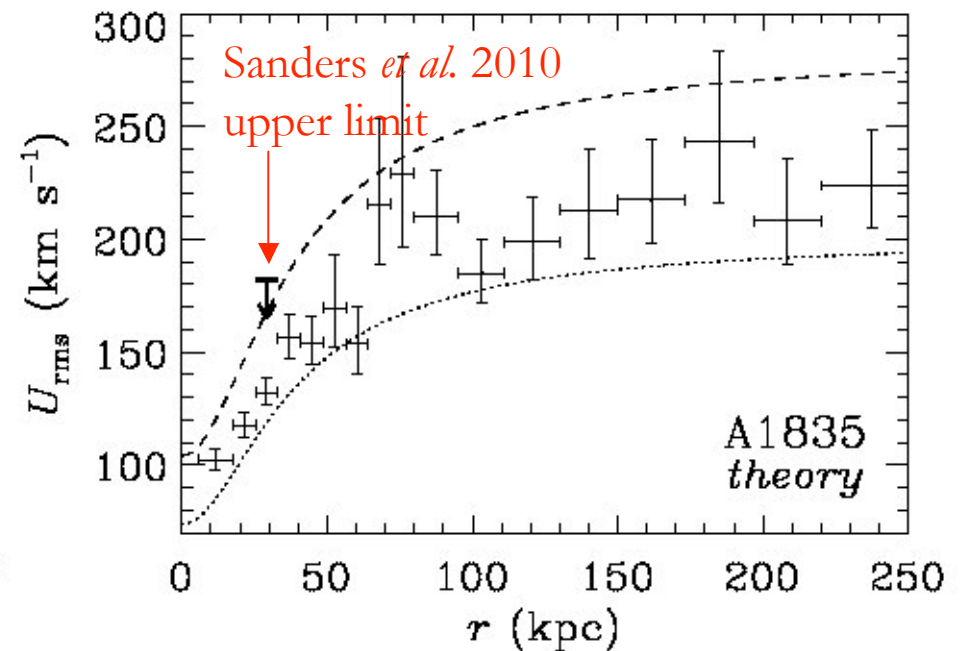
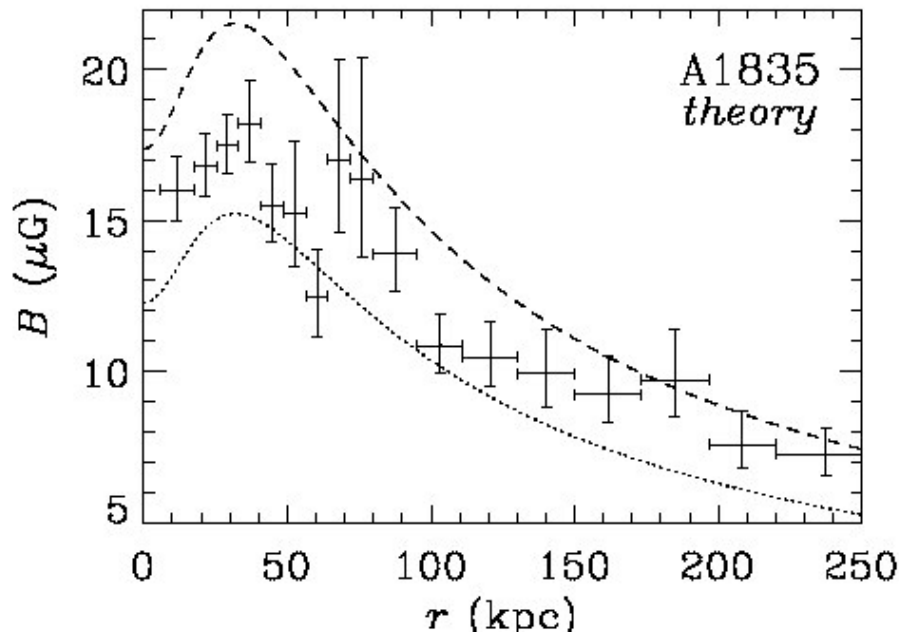
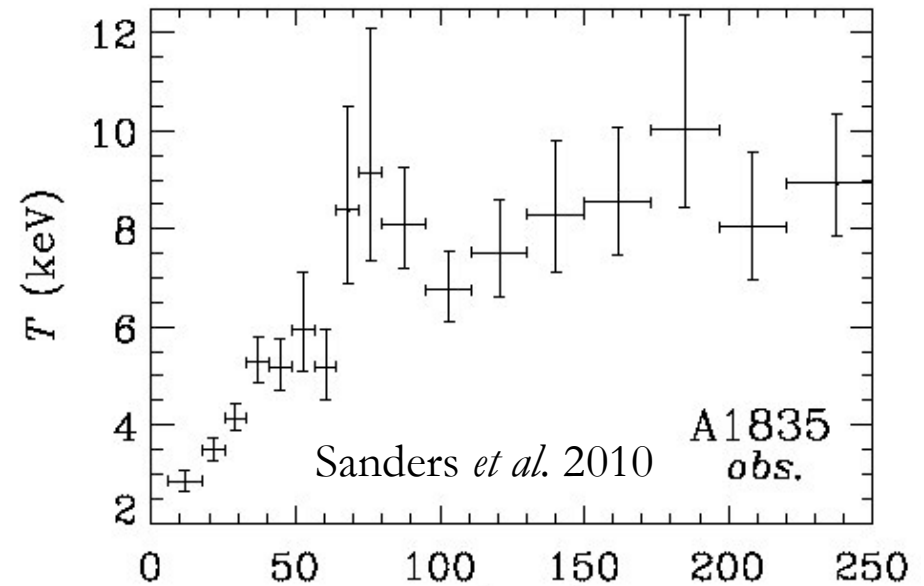
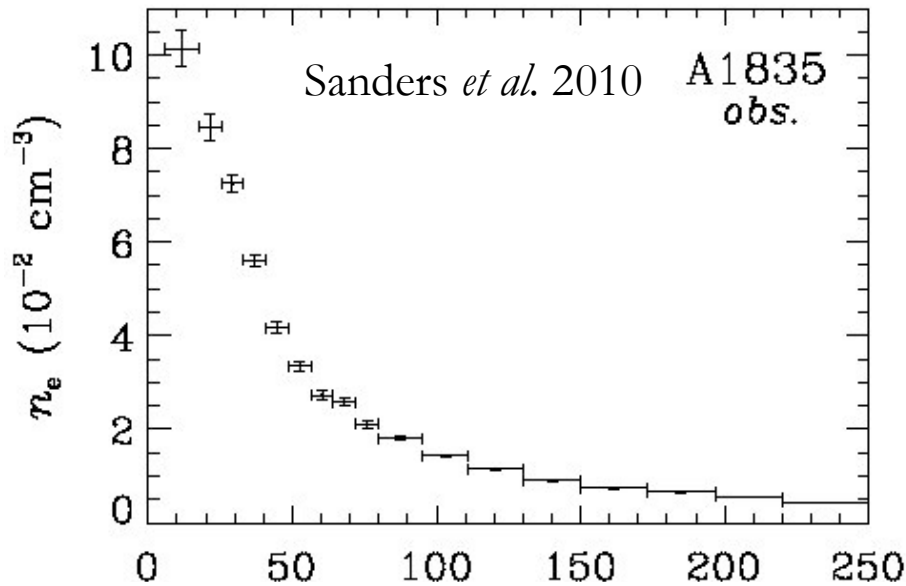
Corollary: Properties of Turbulence vs. n and T

5 parameters: B , U_{rms} , L , n_e , T

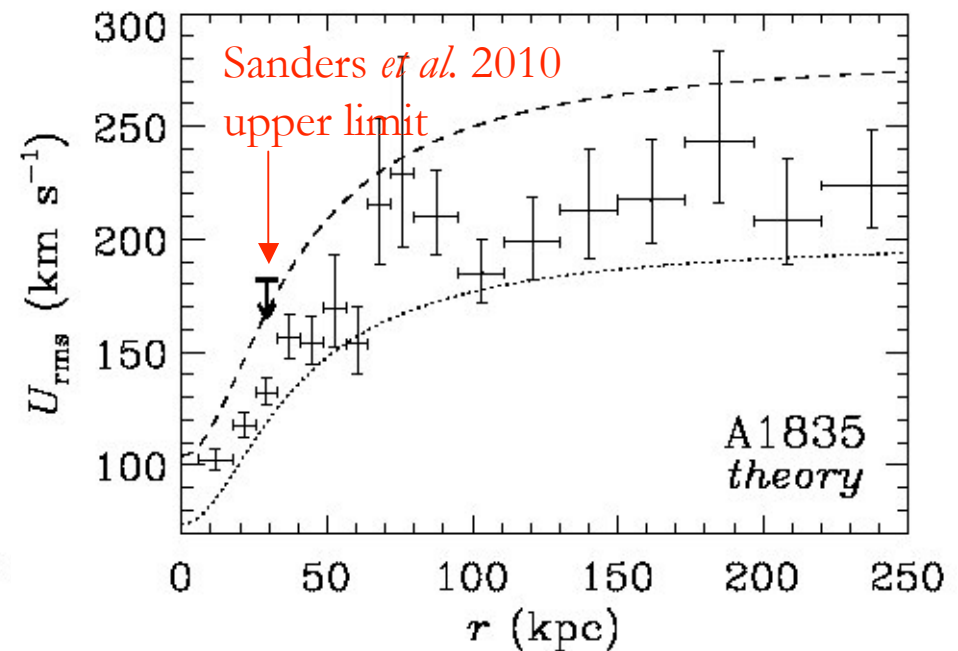
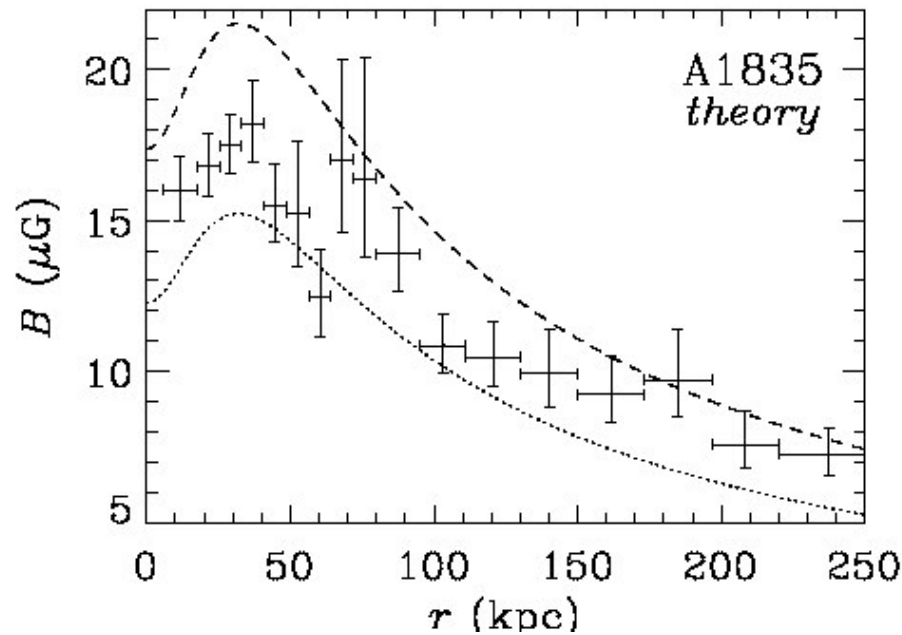
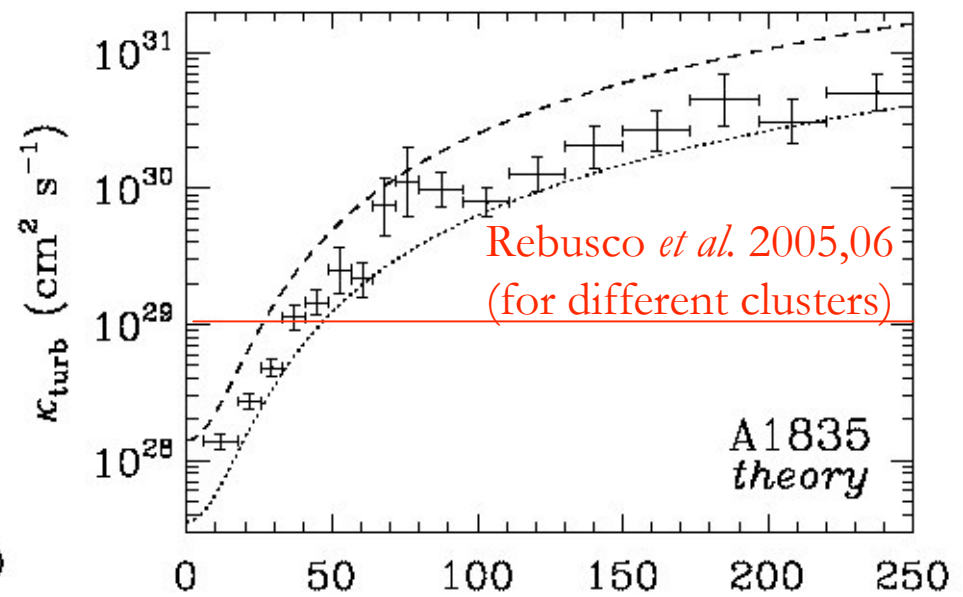
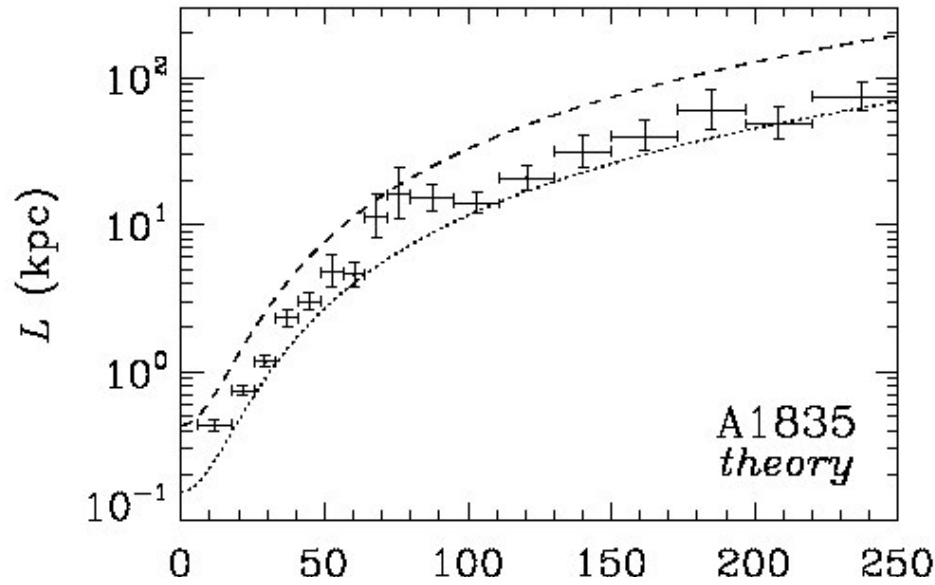
If observations provide 2 of these, we can predict the other 3;
usually n_e and T provided, so we'll predict B , U_{rms} , L

N.B. But no specific causal relationship is implied!

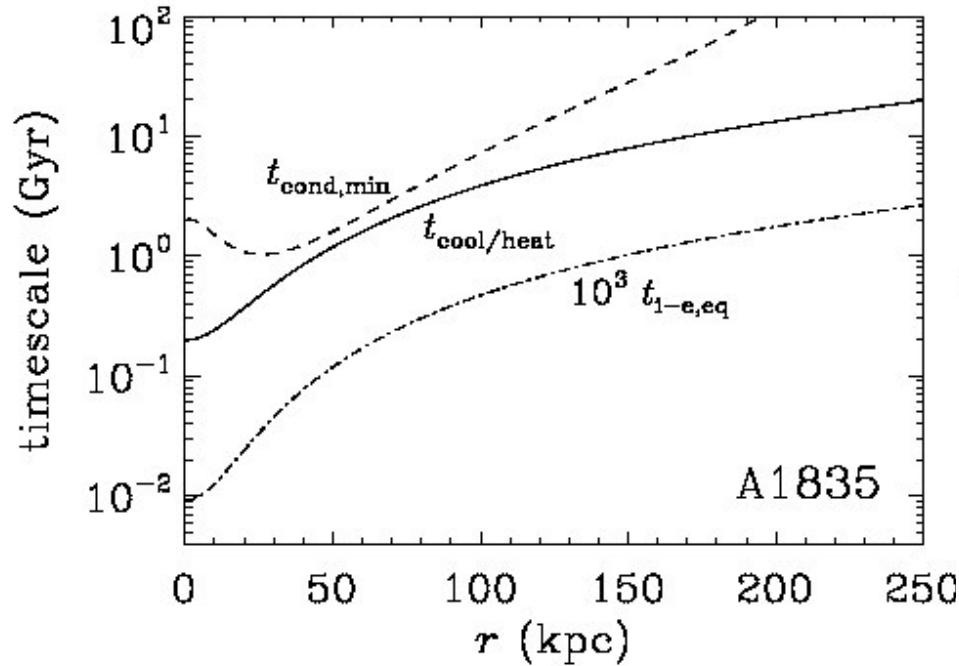
Once Upon a Cluster... (A1835)



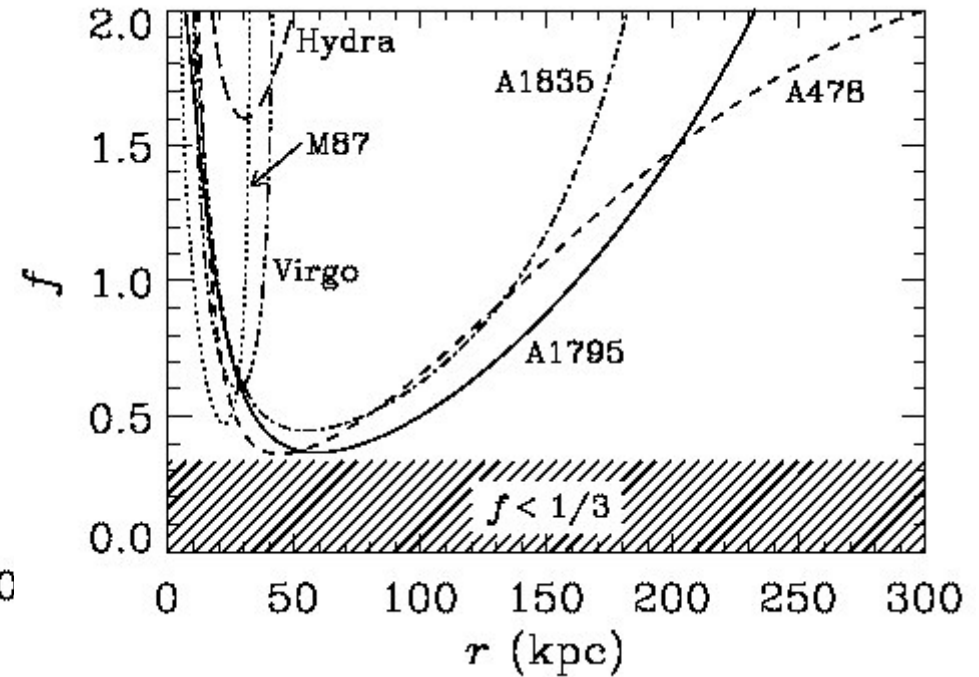
Once Upon a Cluster... (A1835)



P.S. Thermal Conduction Seems Hopeless

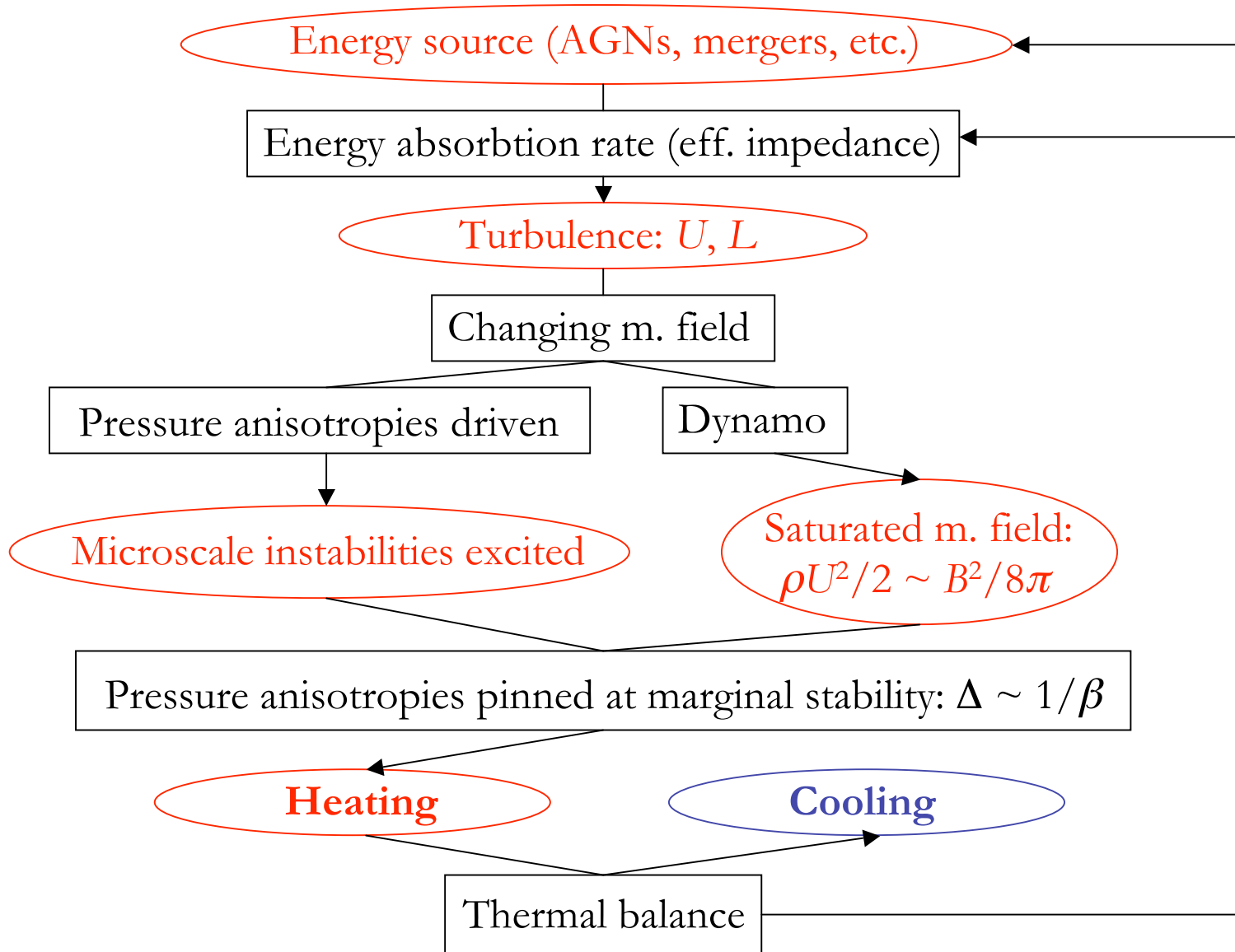


This is a comparison of three relevant timescales:
conduction,
cooling/heating,
i-e temperature equilibration



This plot shows how much enhancement/suppression of Spitzer conductivity is needed to balance cooling in various clusters

Summary





Monsters, Inc.: Astro & Cosmology with G. Clusters, KITP, 14.03.11

Oxford
Physics™

Part III: ICM Dynamo

Schekochihin *et al.*, *ApJ* **629**, 139 (2005)

Schekochihin & Cowley, *Phys. Plasmas* **13**, 056501 (2006)

Important for:

- General understanding of *magnetogenesis* (nice word!)
- Making sense of the size and structure of observed magnetic fields
 - Now that we know magnetic field (via β_j) is likely to set the dissipation rate in the ICM, we also need it to calculate macro-scale dynamics

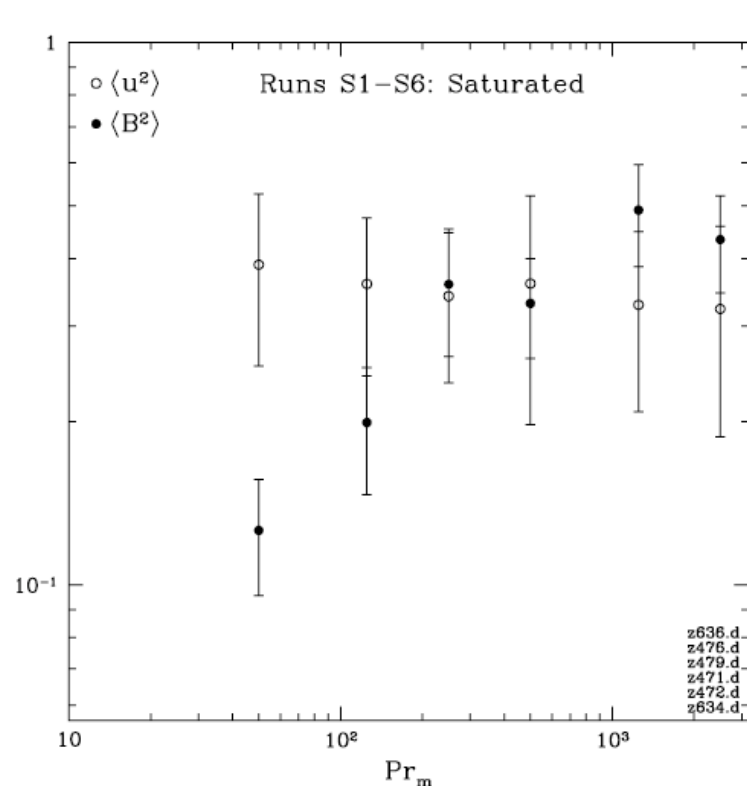
But

this is a complicated and very embarrassing subject...

Fluctuation Dynamo in the ICM

Nobody knows how fluctuation dynamo works in a weakly collisional plasma — and numerics can't answer this because we can't do a kinetic simulation of dynamo (HUGE computing resources required for that).

However, on general grounds, **it must work somehow**: indeed, anywhere we look (ISM, ICM, old clusters, young clusters, cool-core clusters, unrelaxed clusters, etc.), we find $\sim 1\text{-}10 \mu\text{G}$ fields, or, more importantly,



$$\frac{B^2}{8\pi} \sim \frac{\rho u^2}{2}$$

In MHD numerical simulations, there can be a factor < 1 , which, however, seems to increase with magnetic Prandtl number

[Schekochihin *et al.*, *ApJ* **612**, 276 (2004)]

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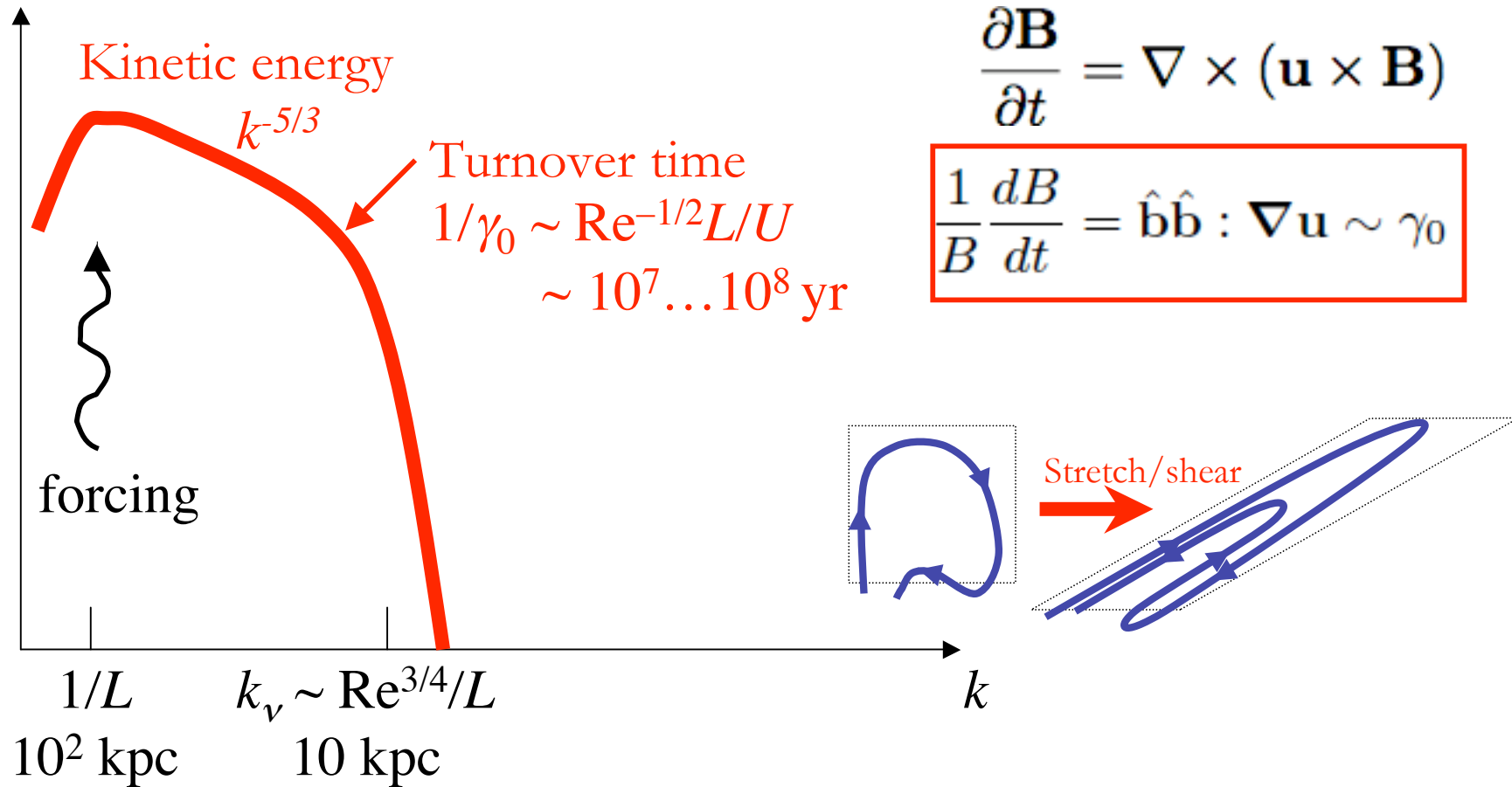
It is easy to argue hand-wavingly that this will happen FAST:

$$\frac{1}{B} \frac{dB}{dt} \sim \nu_{ii} \Delta \sim \frac{\nu_{ii}}{\beta_i} \propto B^2$$

So, **explosive growth**? (If true, no need to count e -folding times!)

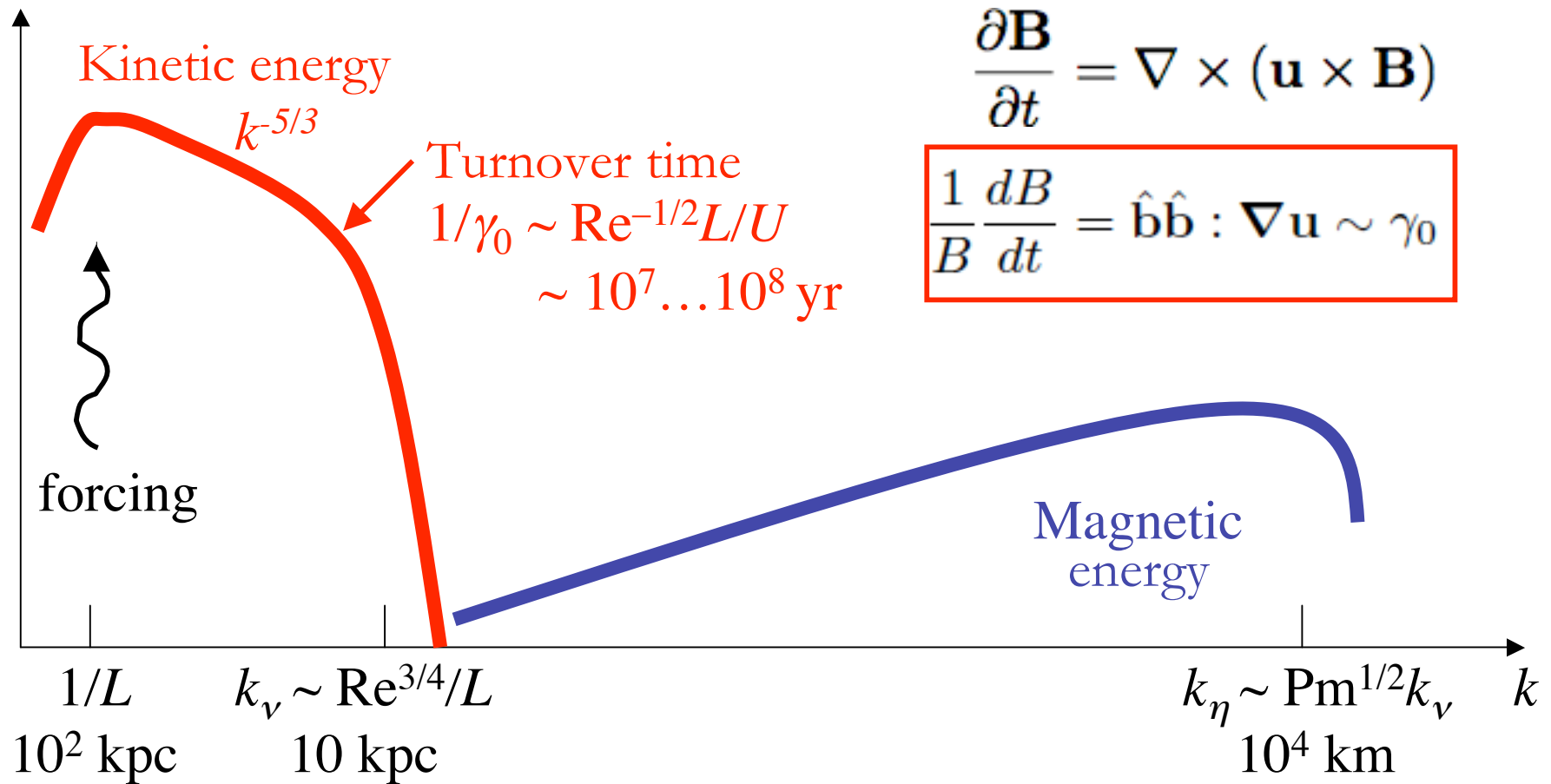
We still have no idea what sets the field's scale...

Fluctuation Dynamo in MHD



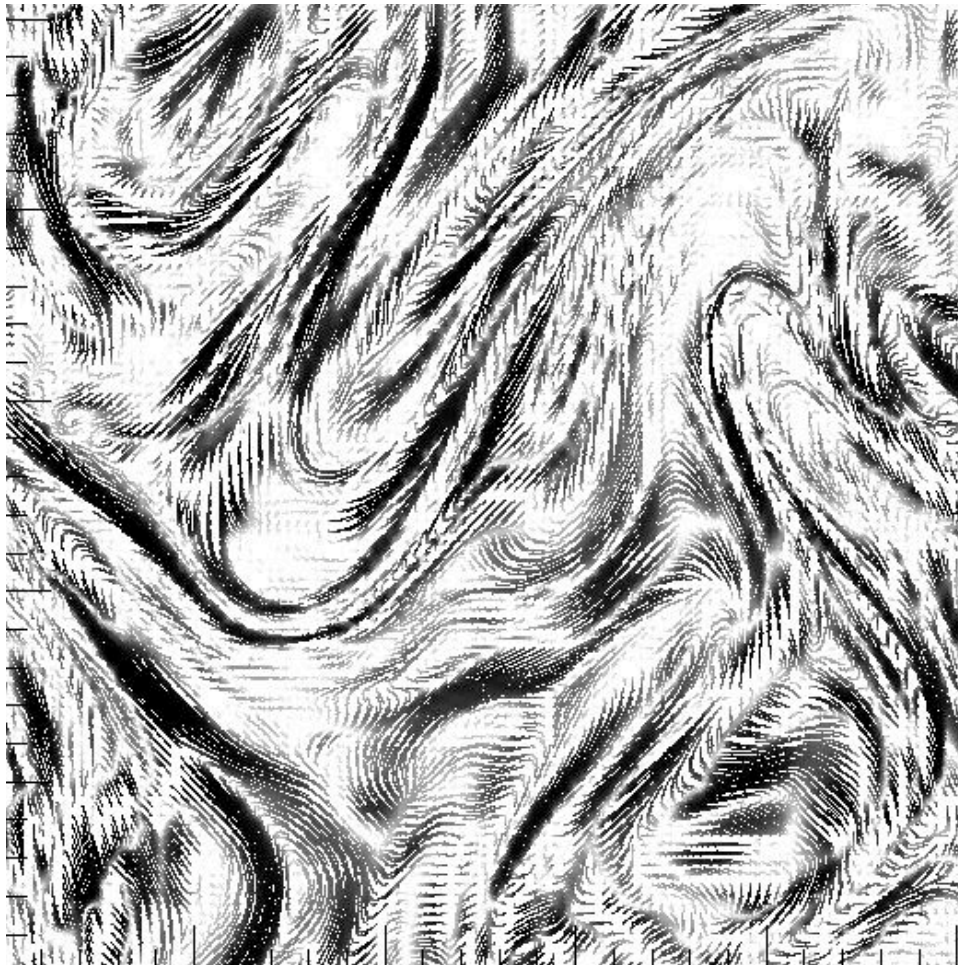
[chatty historical review for bed-time reading:
 Schekochihin & Cowley, astro-ph/0507686]

Fluctuation Dynamo in MHD



The field grows at the resistive scale and, as far as we know, saturates with energy at the smallest scales available to it. All simulations will likely have magnetic field at the Nyquist scale.

Fluctuation Dynamo in MHD



$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\frac{1}{B} \frac{dB}{dt} = \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla \mathbf{u} \sim \gamma_0$$

Magnetic
energy

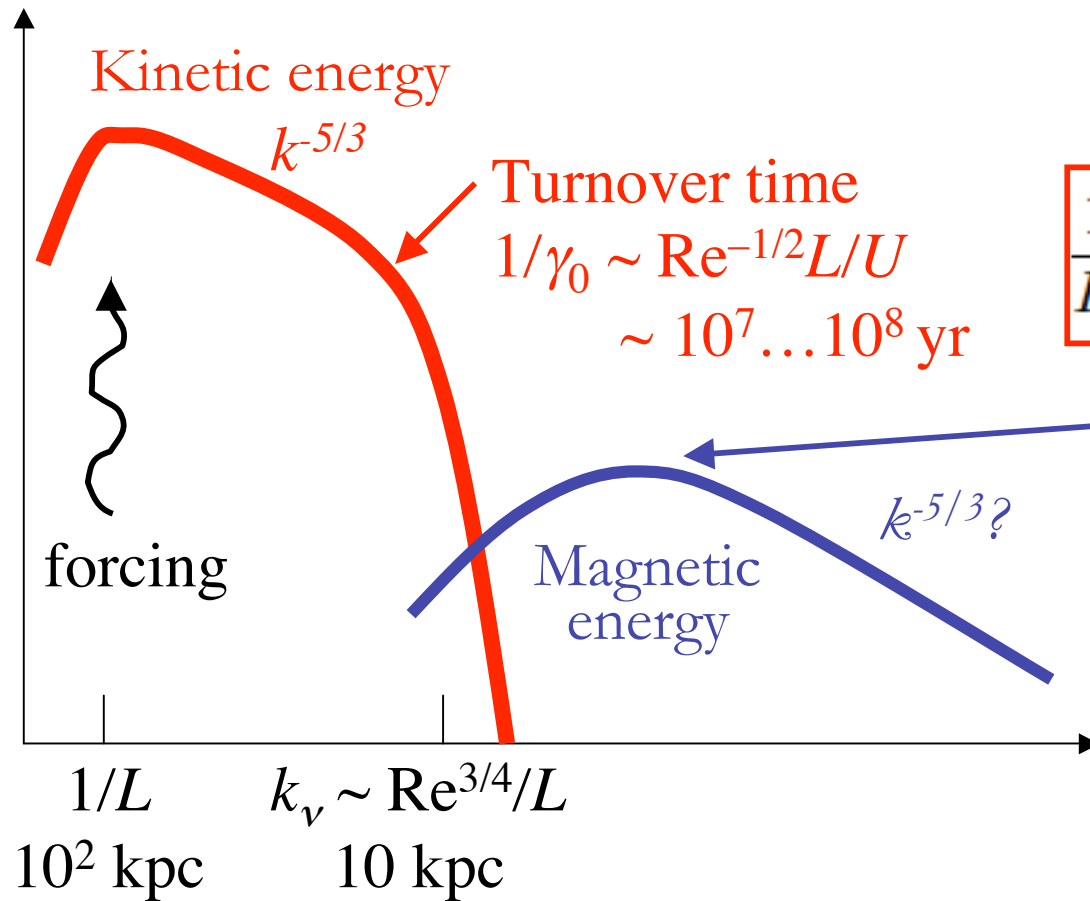
$$k_\eta \sim \text{Pm}^{1/2} k_\nu$$

10^4 km

k

The field grows at the resistive scale and, as far as we know, saturates with energy at the smallest scales available to it (“folds”). All simulations will likely have magnetic field at the Nyquist scale.

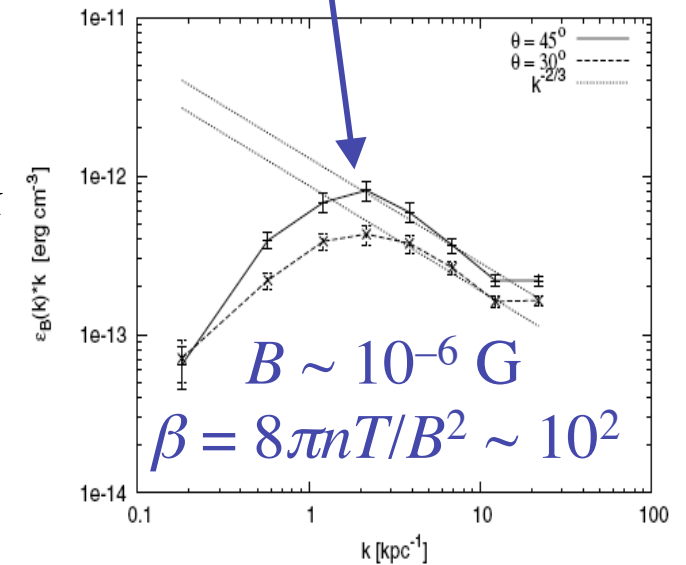
Fluctuation Dynamo in the ICM



$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\frac{1}{B} \frac{dB}{dt} = \hat{\mathbf{b}} \hat{\mathbf{b}} : \nabla \mathbf{u} \sim \gamma_0$$

What determines this scale is unclear



In contrast, observationally, while folded fields are seen, the reversal scale is not that small...

[Vogt & Enßlin 2005, *A&A* 434, 67]

Magnetic Fields in ICM: Summary

- If magnetic field is saturated, it **MUST** be at a dynamically important level (induction equation is linear in B , so if B has a steady value, velocity must know about it)
- Numerical simulations are in agreement with this. It is quite easy to get a factor of up to ~ 10 between magnetic and kinetic energies by tuning Re and Pm , both of which are model parameters and have very little to do with real ICM. This is a finite-resolution effect.
- In MHD simulations, the field tends to sit at the smallest resolved scales. What this implies for ICM cannot be right! (This is where plasma physics must come in. We do not yet have the answer.)
- I am willing to bet that **all fields we observe are saturated**, i.e., either there is always enough seed field or the dynamo is very fast (or both).
- In view of the above, I believe **the key theoretical and observational question is spatial structure of the saturated field**, not the growth time or the overall field strength.



Monsters, Inc.: Astro & Cosmology with G. Clusters, KITP, 14.03.11



Part IV: Ion Heat Flux Regulation
(more dirt under the rug)

Schekochihin *et al.*, MNRAS **405**, 291 (2010)
Rosin *et al.*, MNRAS, in press; arXiv:1002.4017

More Microphysics...

If one does microphysical theory (linear and nonlinear) carefully,
there is always a chance of finding new things....

MRI, MVI, MTI, HBI, HBO, RCO...

(Balbus, Quataert et al.: everything is always unstable)

So, for the aficionados of three-letter instabilities, I give you

GTI

(The GyroThermal Instability)

Gyrothermal Instability: Equations

- Keep the gyroviscous terms in the “Braginskii” stress (this is valid even without collisions and is necessary to get the fastest growing mode for the firehose)
- Keep pressure anisotropies and **parallel ion heat fluxes**

$$mn \frac{du}{dt} = -\nabla \left(p_{\perp} + \frac{B^2}{8\pi} \right) + \nabla \cdot \left[bb \left(p_{\perp} - p_{\parallel} + \frac{B^2}{4\pi} \right) - \mathbf{G} \right]$$

$$\mathbf{G} = \frac{1}{4\Omega} [\mathbf{b} \times \mathbf{S} \cdot (\mathbf{I} + 3bb) - (\mathbf{I} + 3bb) \cdot \mathbf{S} \times \mathbf{b}] + \frac{1}{\Omega} [\mathbf{b} (\boldsymbol{\sigma} \times \mathbf{b}) + (\boldsymbol{\sigma} \times \mathbf{b}) \mathbf{b}]$$

$$\mathbf{S} = (p_{\perp} \nabla u + \nabla q_{\perp}) + (p_{\perp} \nabla u + \nabla q_{\perp})^T$$

$$\boldsymbol{\sigma} = (p_{\perp} - p_{\parallel}) \left(\frac{d\mathbf{b}}{dt} + \mathbf{b} \cdot \nabla u \right) + (3q_{\perp} - q_{\parallel}) \mathbf{b} \cdot \nabla \mathbf{b}$$

- Consider just $k_{\perp} = 0$.
(Alfvénically polarised parallel-propagating modes – they decouple and can be calculated without knowing pressures or heat fluxes)

Gyrothermal Instability: Linear Theory

Instability criterion:

$$\Lambda \equiv \Gamma_T^2 - \frac{(1 - \delta)^2}{2} \left(\Delta + \frac{2}{\beta} \right) > 0$$

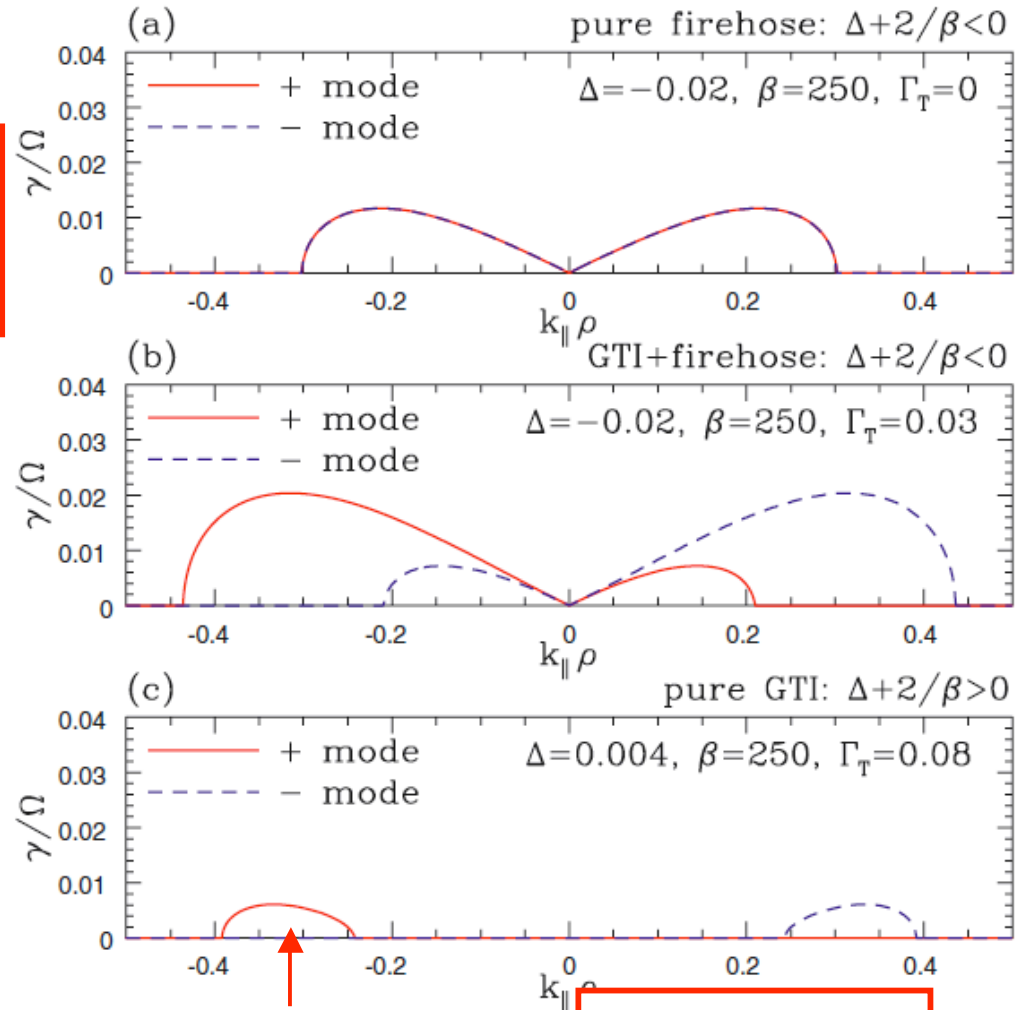
$$\Delta = \frac{p_{\perp i} - p_{\parallel i} + p_{\perp e} - p_{\parallel e}}{p_{\parallel i}}$$

$$\delta = \frac{p_{\perp i} - p_{\parallel i} - (p_{\perp e} - p_{\parallel e})}{p_{\parallel i}} - \frac{2}{\beta}$$

$$\Gamma_T = \frac{2q_{\perp i} - q_{\parallel i}}{p_{\parallel i} v_{th}}$$

In the collisional limit,

$$q_{\perp} = \frac{1}{3} q_{\parallel} = -\frac{1}{2} n \frac{v_{th}^2}{v} \mathbf{b} \cdot \nabla T$$



Preferred scale
in marginal state:

$$k_{\parallel} \rho_i \sim \frac{\lambda_{mfp}}{l_T}$$

Gyrothermal Instability: Linear Theory

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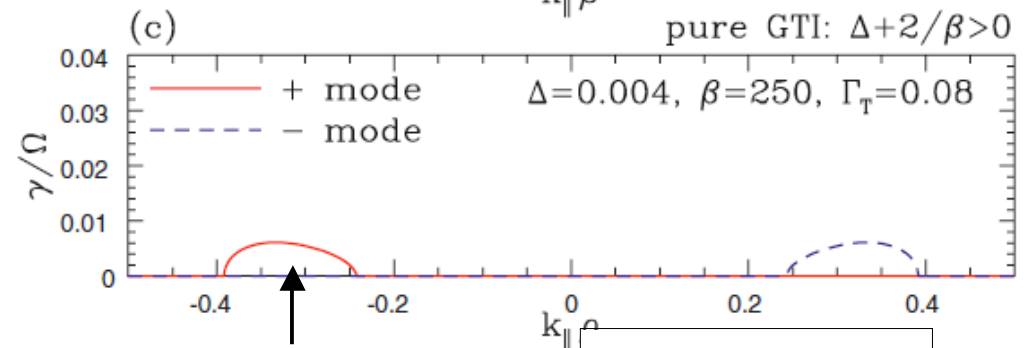
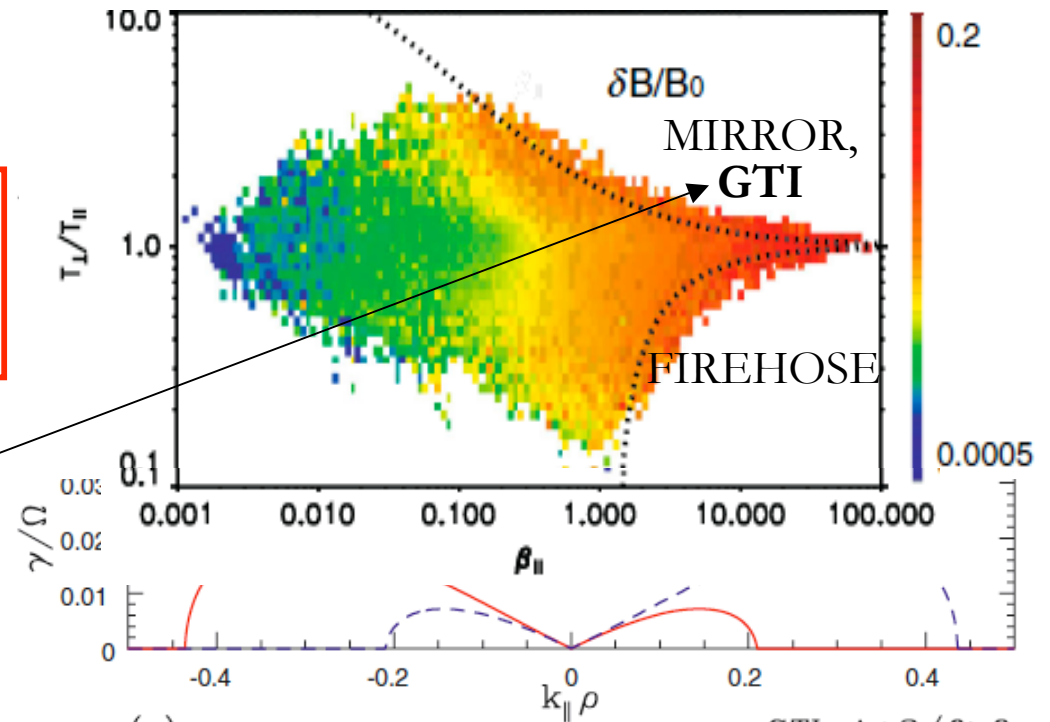
$$\Lambda \equiv \Gamma_T^2 - \frac{(1-\delta)^2}{2} \left(\Delta + \frac{2}{\beta} \right) > 0$$

So, Alfvénically polarised perturbations can be unstable at $\Delta > 0$!

$$\Gamma_T = \frac{2q_{\perp i} - q_{\parallel i}}{p_{\parallel i} v_{th}}$$

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Gyrothermal Instability: Nonlinear Theory

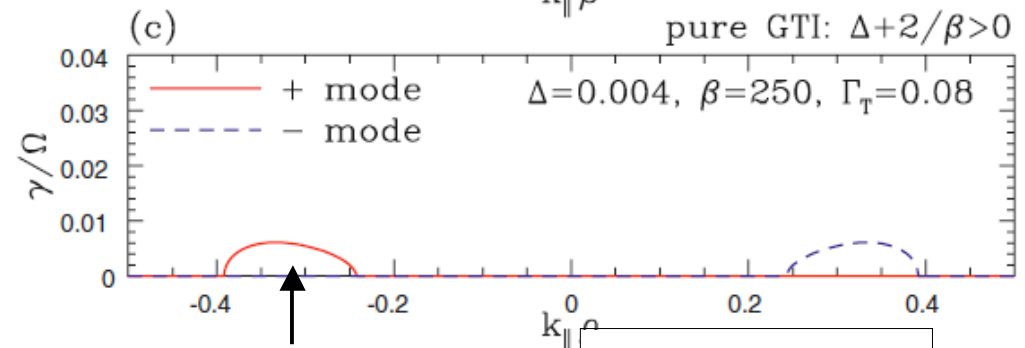
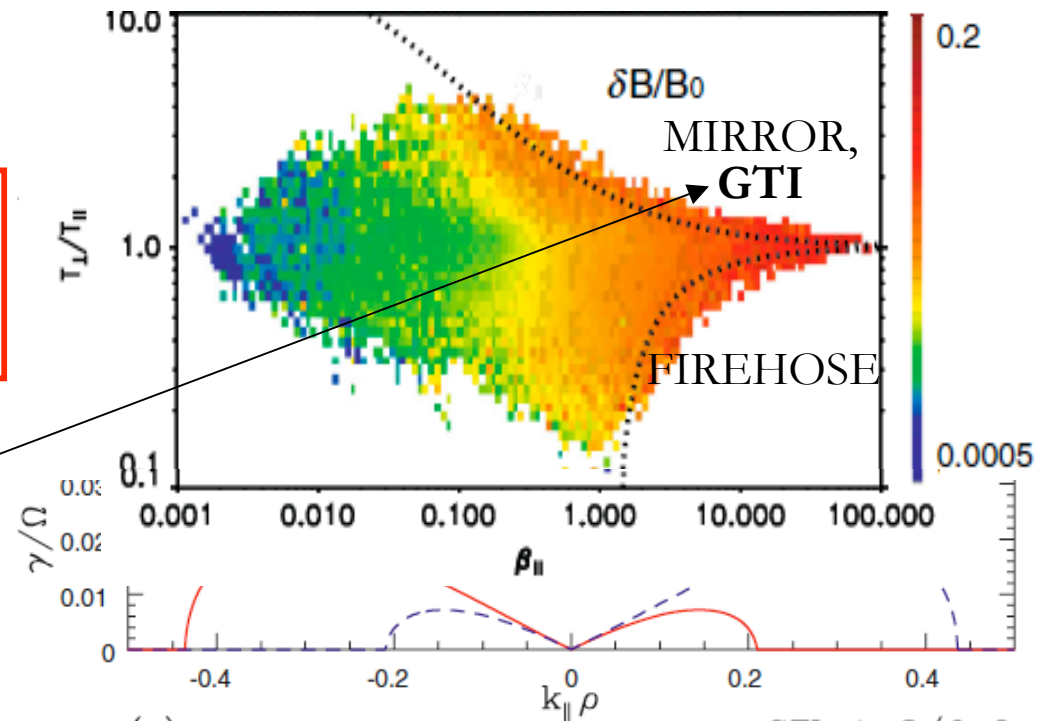
Instability criterion:

$$\Delta \equiv \Gamma_T^2 - \frac{(1-\delta)^2}{2} \left(\Delta + \frac{2}{\beta} \right) > 0$$

So, Alfvénically polarised perturbations can be unstable at $\Delta > 0$!

GTI saturates by the same mechanism as the firehose: magnetic fluctuations adjusting (increasing) Δ

[It might actually destabilise mirror — no idea what then]

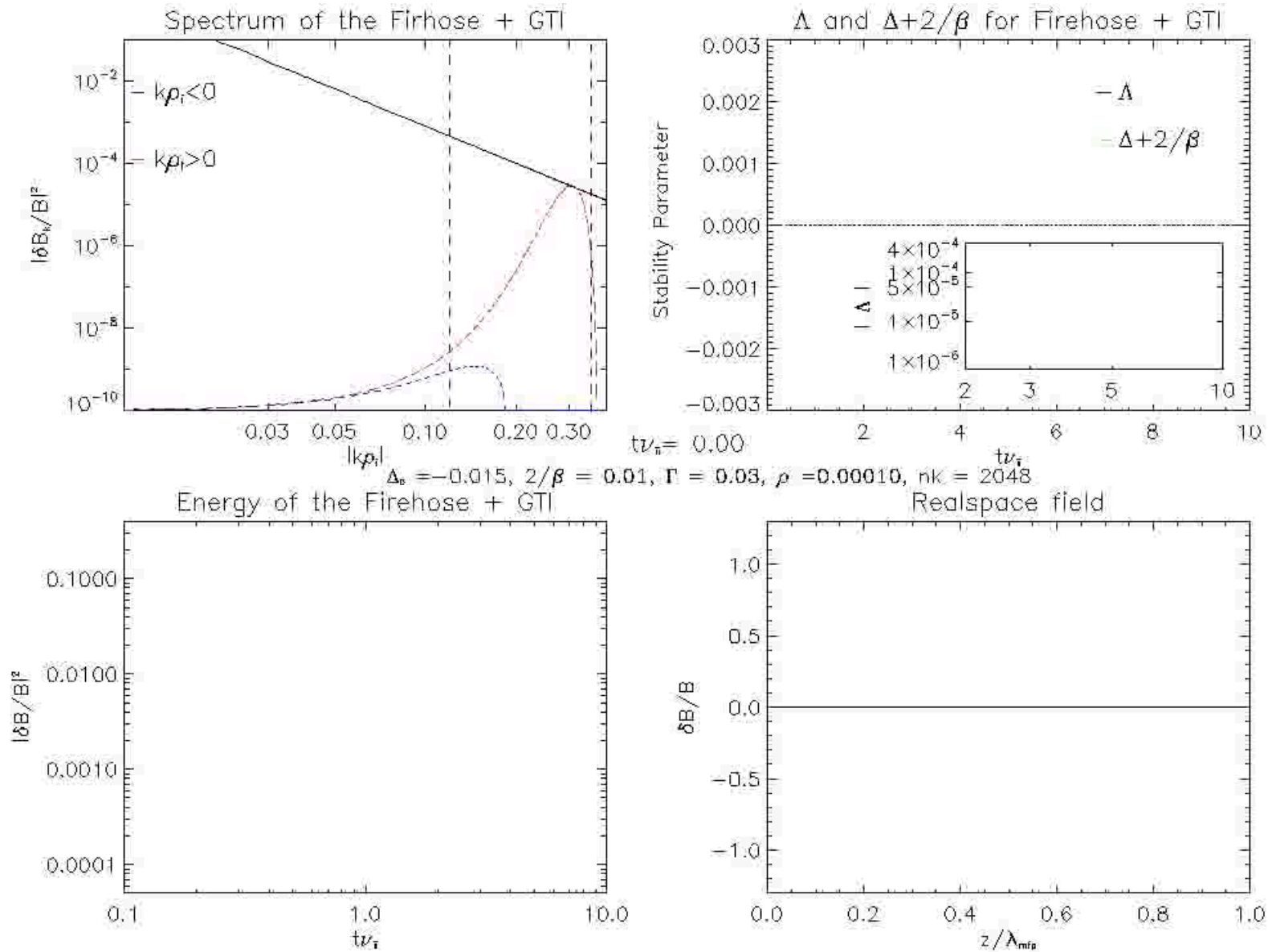


Preferred scale in marginal state:

$$k_{\parallel} \rho_i \sim \frac{\lambda_{\text{mfp}}}{l_T}$$

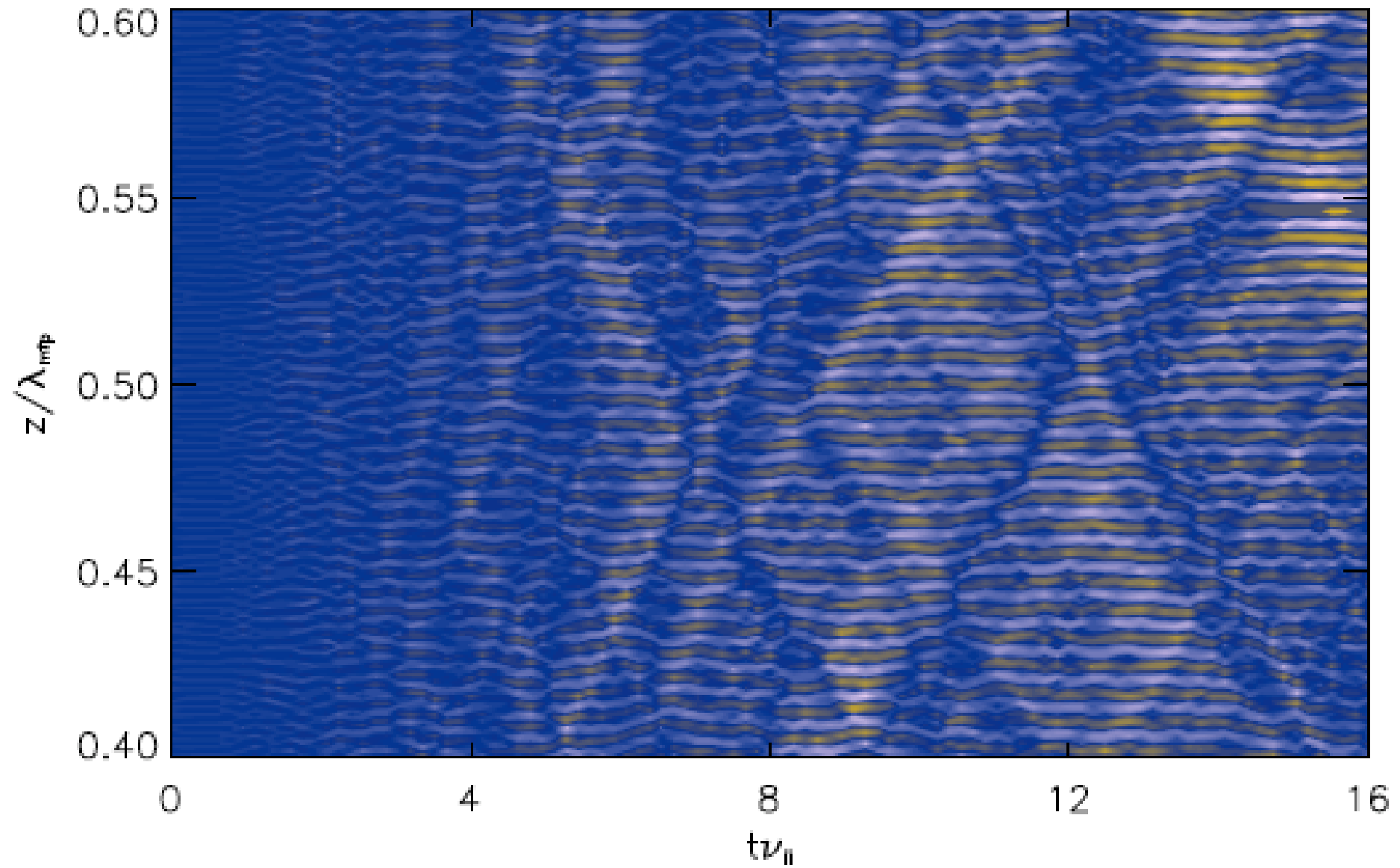
[Rosin *et al.*, arXiv:1002.4017 (2010)]

Nonlinear GTI



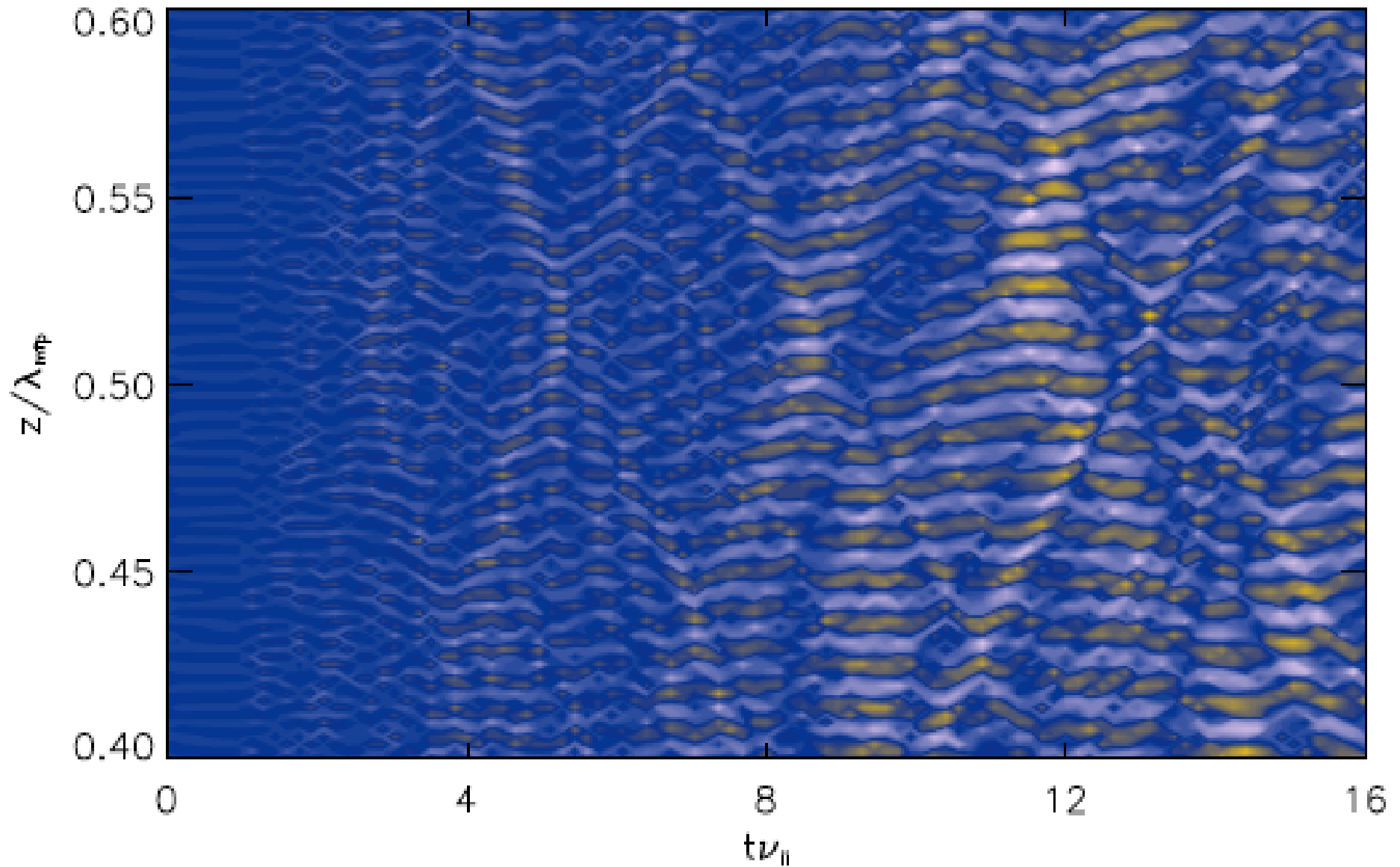
[Rosin *et al.*, arXiv:1002.4017 (2010)]

Nonlinear GTI



[Rosin *et al.*, arXiv:1002.4017 (2010)]

[Cf. Nonlinear Firehose]



[Rosin *et al.*, arXiv:1002.4017 (2010)]

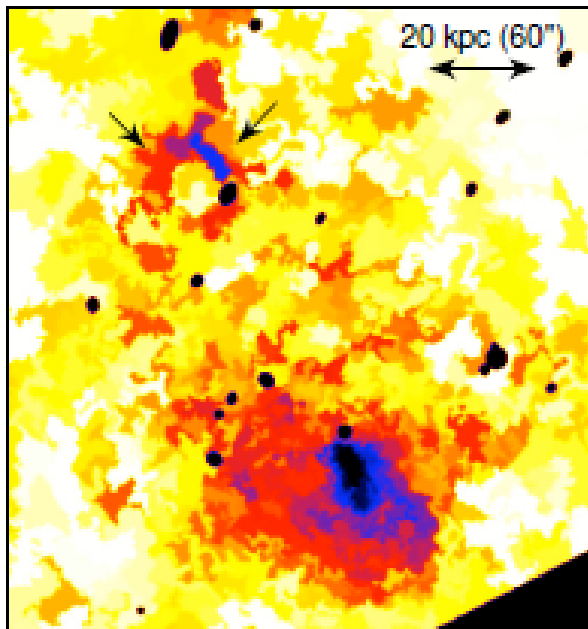
GTI in ICM?

Theoretical condition for GTI marginal stability $\Gamma_T^2 \lesssim 2/\beta$
translates into this: for the temperature scale $l_T^{-1} = b \cdot \nabla \ln T$

$$l_T \gtrsim 0.3 \left(\frac{n_e}{0.01 \text{ cm}^{-3}} \right)^{-1/2} \left(\frac{T_i}{1 \text{ keV}} \right)^{5/2} \left(\frac{B}{1 \mu\text{G}} \right)^{-1} \text{ kpc.}$$

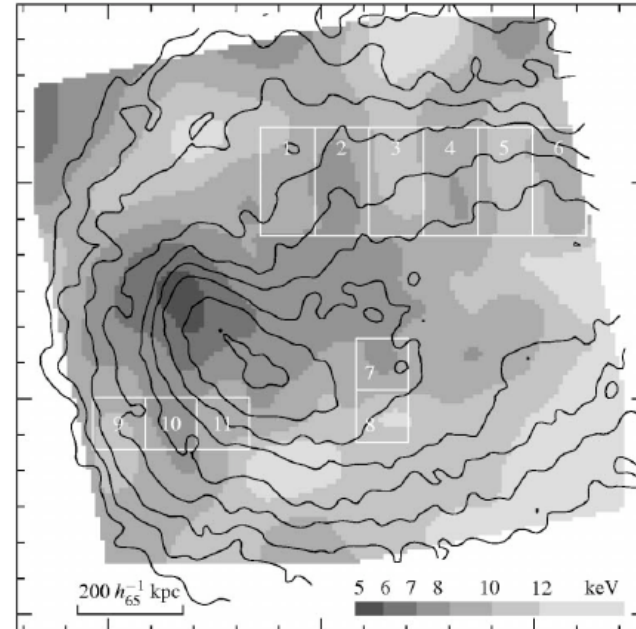
[Schekochihin *et al.*, *MNRAS* **405**, 291 (2010)]

CORES: $\sim 1\text{-}10$ kpc



A262, Sanders *et al.* (2010)

BULK: ~ 100 kpc

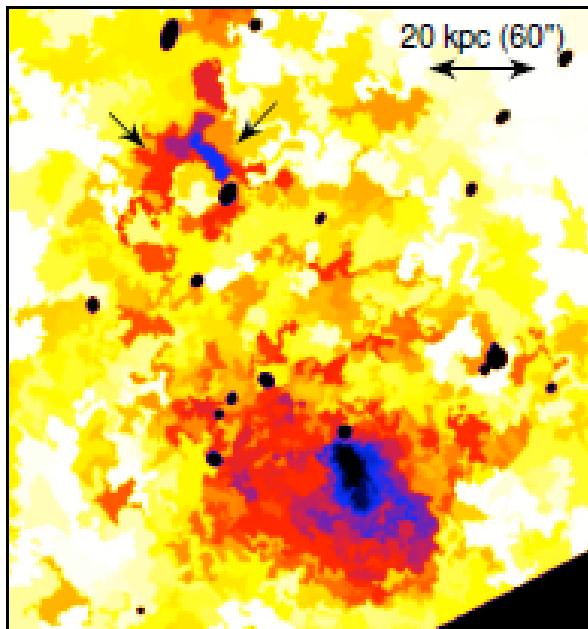


A754, Markevitch *et al.* (2003)

GTI in ICM?

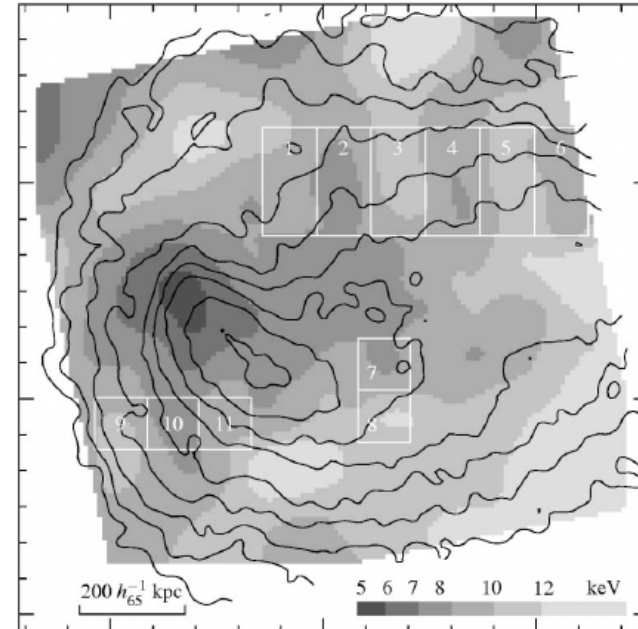
So, it may well turn out that microinstabilities fix not only the pressure anisotropy (i.e., viscosity) but also heat fluxes (i.e., thermal conductivity)

CORES: $\sim 1-10$ kpc




A262, Sanders et al. (2010)

BULK: ~ 100 kpc



A754, Markevitch et al. (2003)

Conclusions

- **Microscale instabilities determine transport, heating, etc.** 
Ab initio theory still incomplete (and painful, but interesting)
- Assuming **pressure anisotropies are pinned at marginal values** is supported by SW data and gives reasonable results for ICM
- Special cases that we have worked out suggest this happens via field modification, not enhanced particle scattering (but who knows)
- Given enough turbulence, **ICM is thermally stable**
- Can predict radial profiles of B , U_{rms} , L , κ_{turb} in ICM (give us your favourite cluster's n and T , we'll give you everything else)
- **Magnetic field depends both on n and T : $B \propto n_e^{1/2} T^{3/4}$**
- **ICM dynamo may be explosively fast** (so cluster age doesn't matter)
- Found a new instability, driven by ion heat flux (GTI)
→ **heat fluxes set by microphysics as well?**

Further reading:

- Schekochihin *et al.*, MNRAS, 405, 291 (2010); Rosin *et al.*, arXiv:1002.4017
- Kunz *et al.*, MNRAS, 410, 2446 (2011)