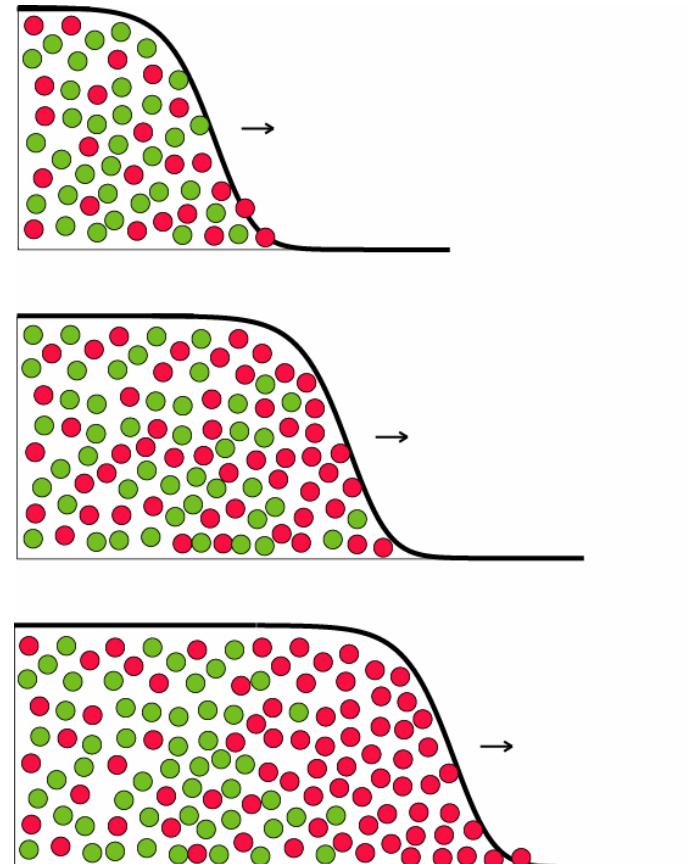


# Evolutionary dynamics in spatially structured (as opposed to well-mixed) asexuals.

Oskar Hallatschek  
MPI-Dynamics and Self-Organization  
Göttingen

<http://www.ds.mpg.de/Forschung/index.php>

P.h.D. and postdoc positions available!



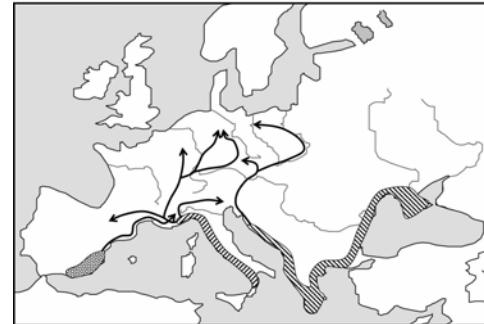
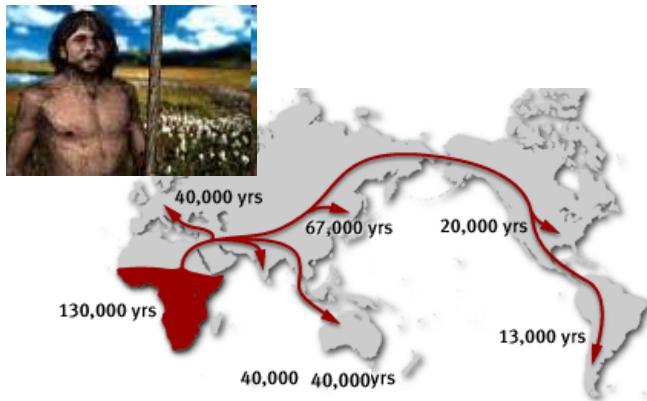
# Outline

- Neutral drift during range expansions
- Speed of adaptation and genetic load in 1d models



Hill & Robertson

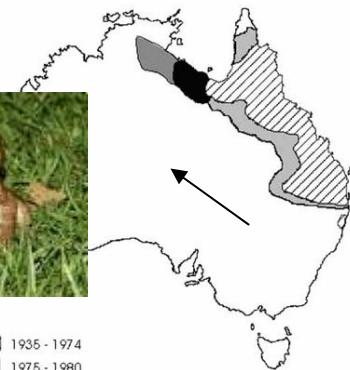
# Rang expansions are ubiquitous



Glacial cycles



■ 1935 - 1974  
■ 1975 - 1980  
■ 1981 - 1986  
■ 1986 - 2001

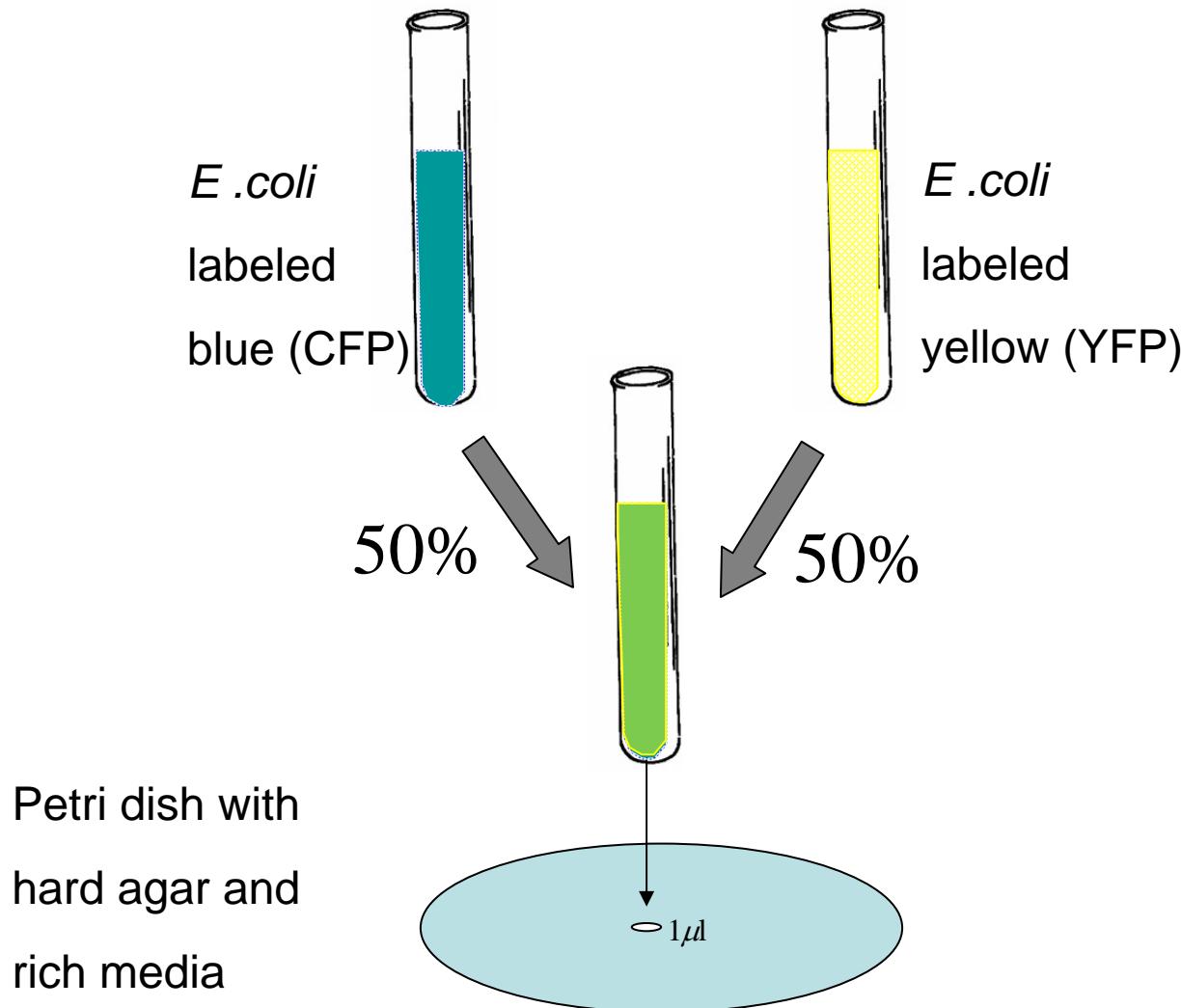


What is the impact of a range expansion on the gene pool of a population ?

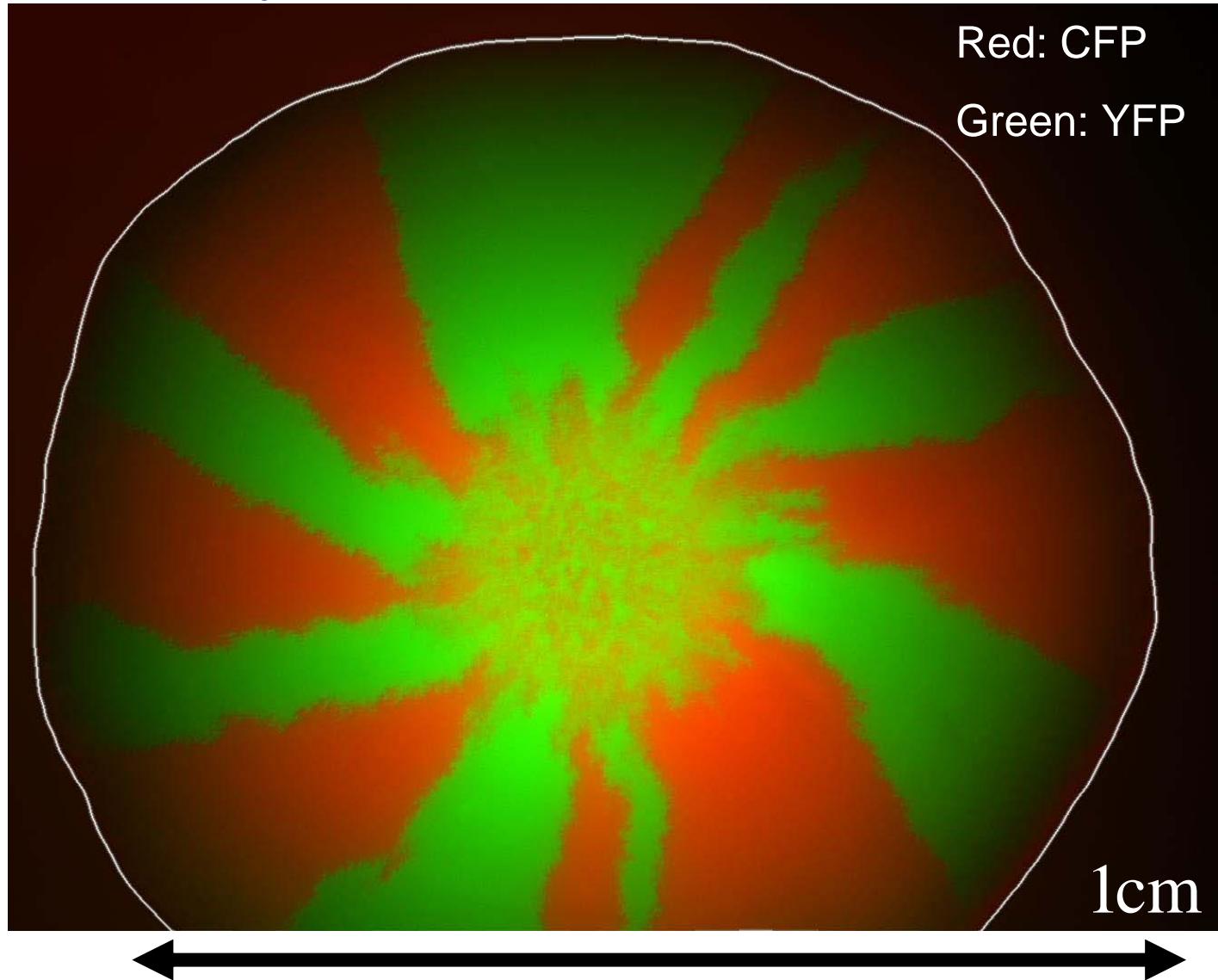
Edmonds, C. A., Lillie, A.S., & Cavalli-Sforza LL. (2004) PNAS.

Ramachandran S, Deshpande O, Roseman CC, Rosenberg NA, Feldman MW & Cavalli-Sforza LL. (2005) PNAS.

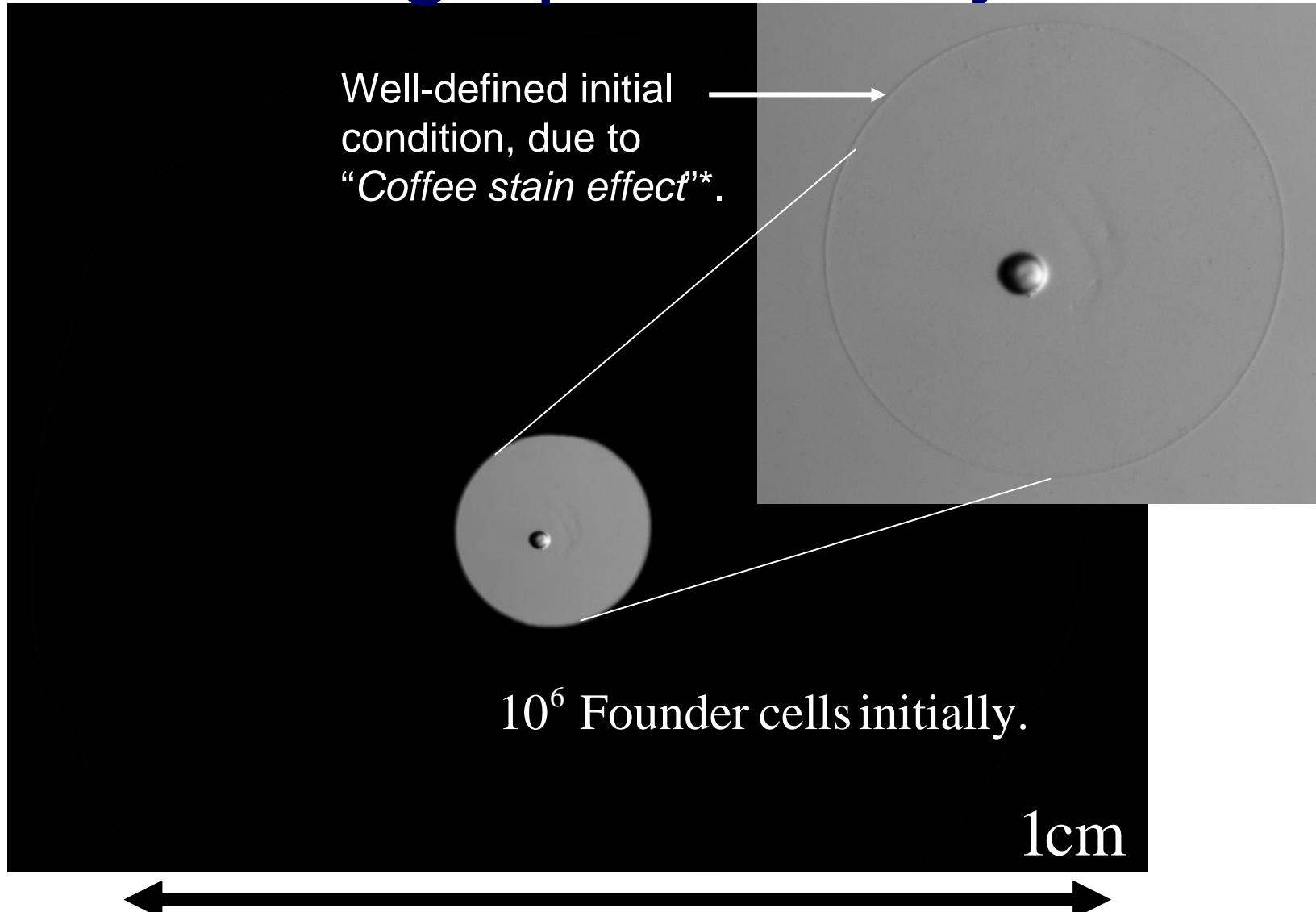
# Tracer dynamics in microbial range expansions



# Tracer dynamics in *E. coli* colonies

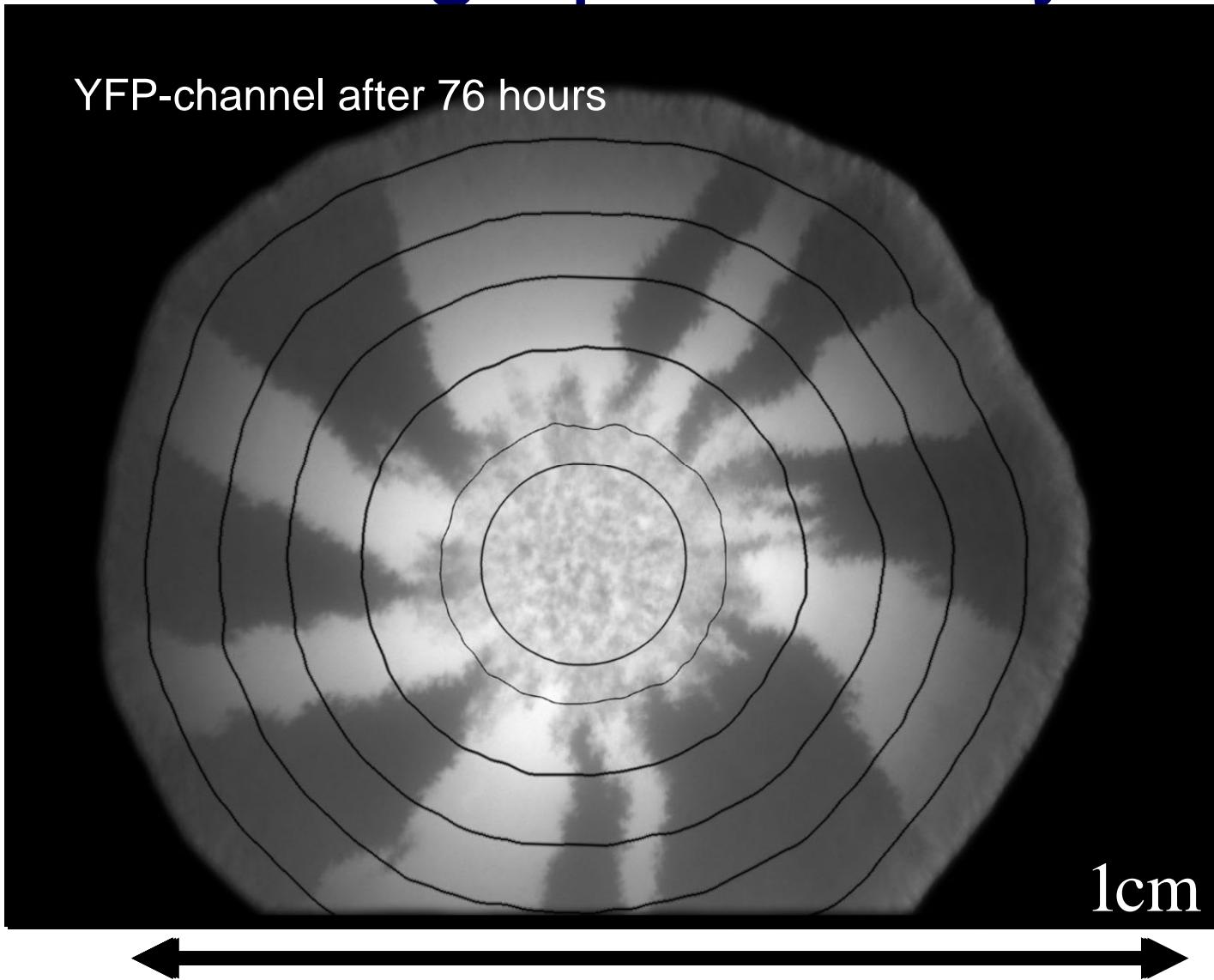


# Demographic history

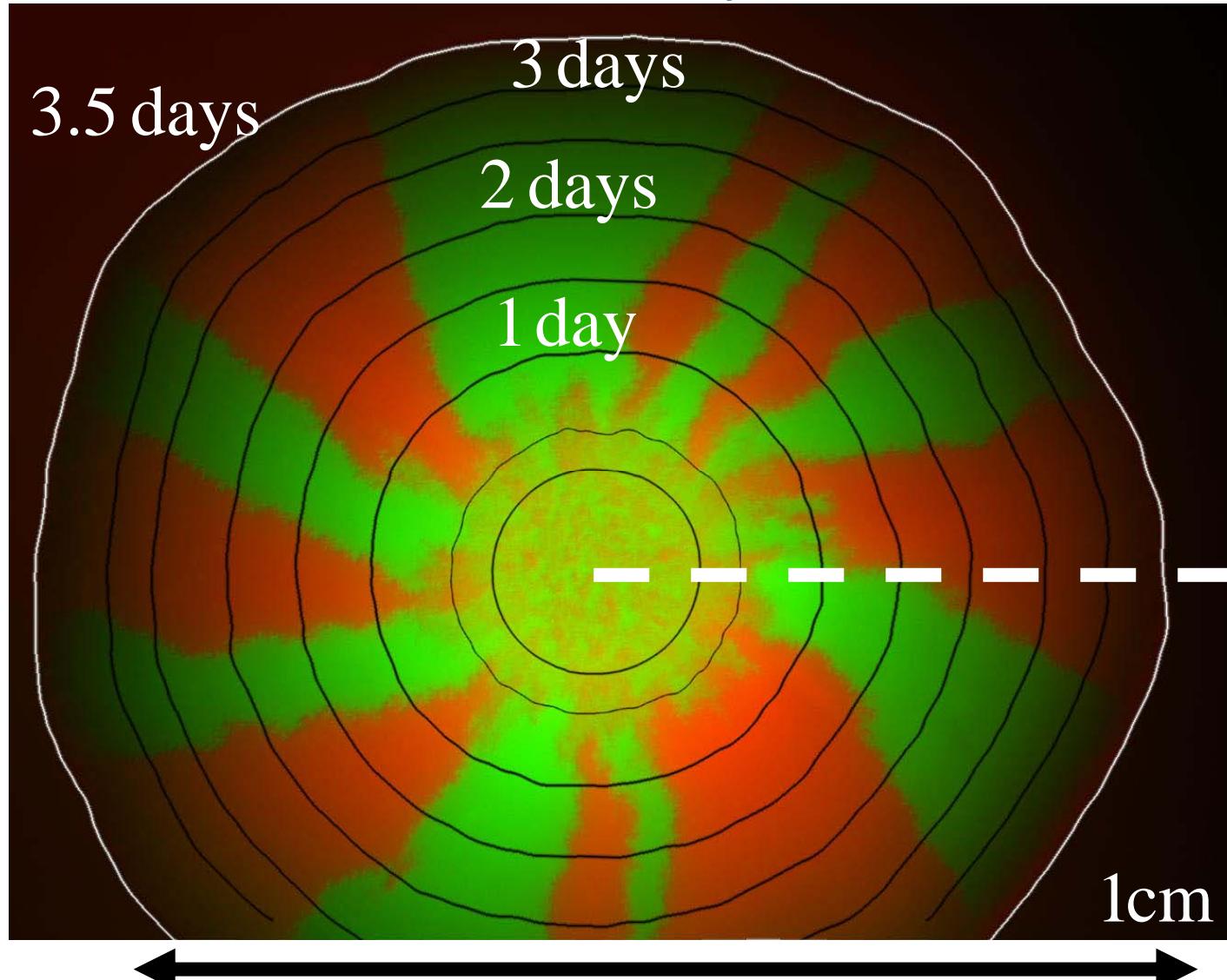


# Demographic history

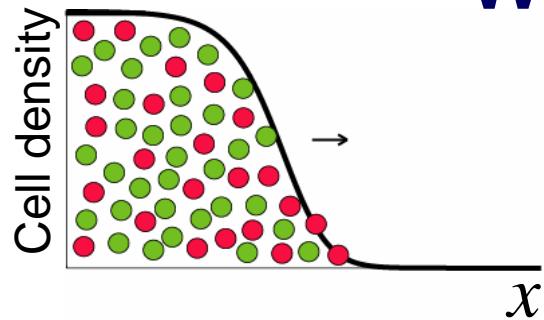
YFP-channel after 76 hours



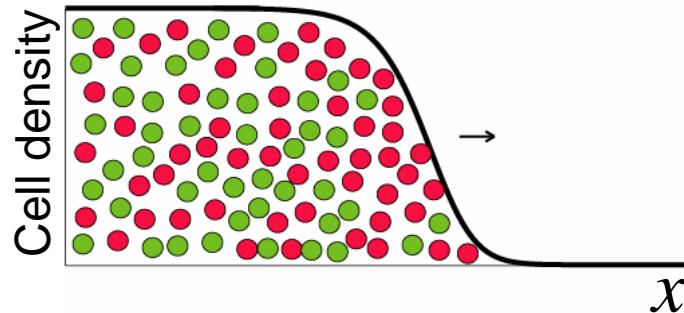
# Marker diversity in *E. coli* colonies



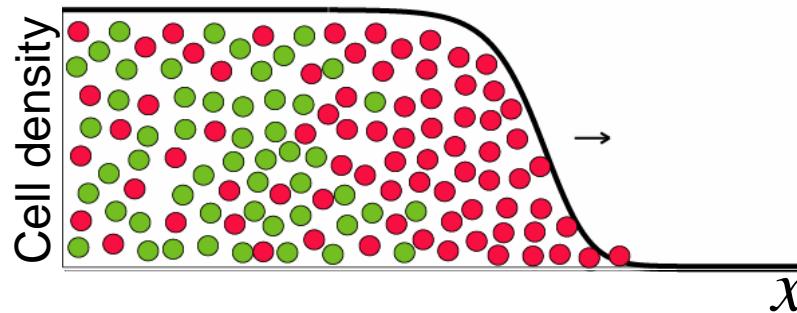
# Density waves with internal states



$$\partial_t c(x, t) = D \Delta c + s \left( 1 - \frac{c}{K} \right) c + \text{noise!}$$

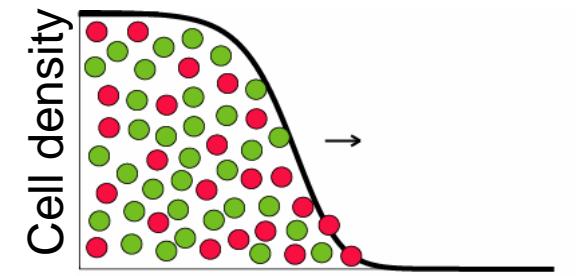


$$\text{velocity} = 2\sqrt{Ds} \times \left\{ 1 - O(\ln^{-2} K) \right\}$$

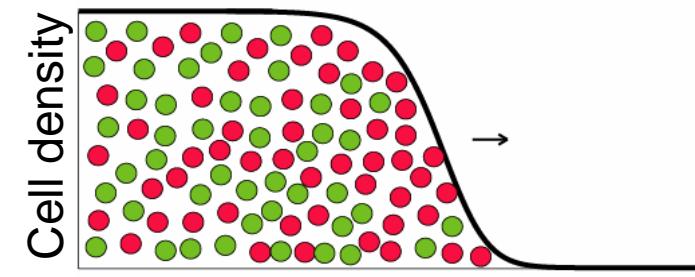


R. A. Fisher (1937)  
E. Brunet, B. Derrida (1997)

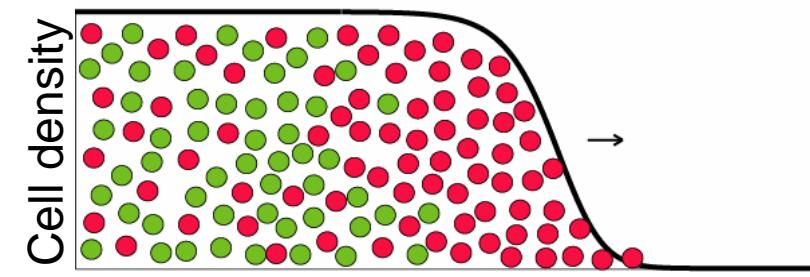
# Number fluctuations cause demixing in 1d



Mixed initial condition



Number fluctuations

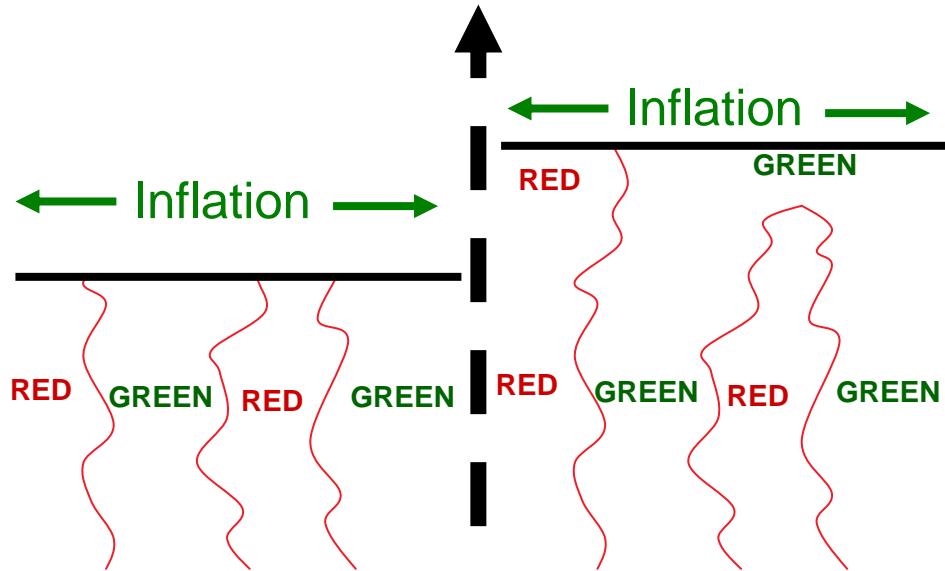


Demixed frontier

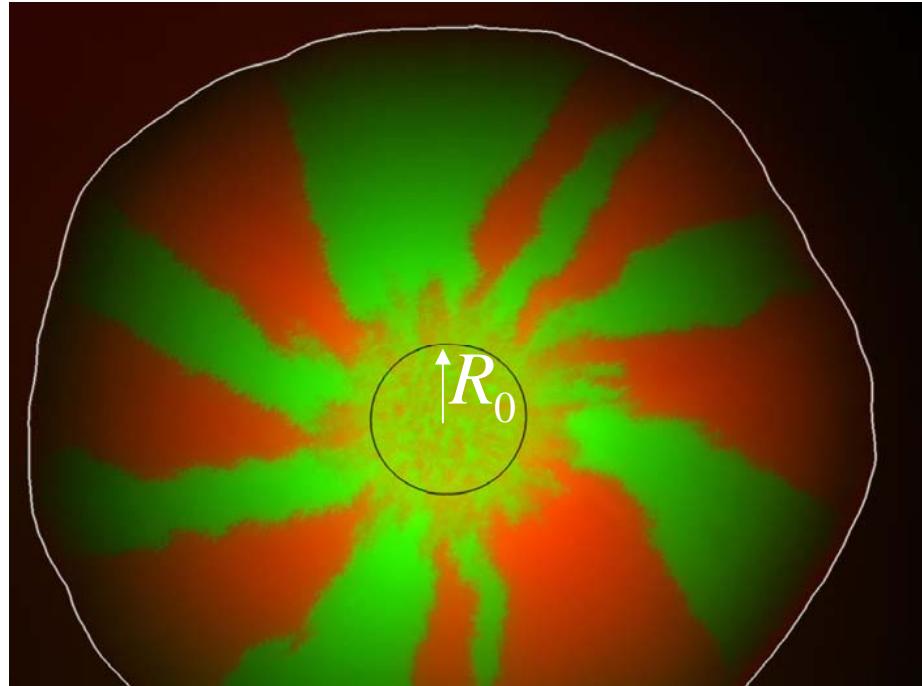
= **absorbing** state

$$T_{\text{fixation}} = O(\ln^\alpha K)$$

# Annihilating random walker model with “inflation”



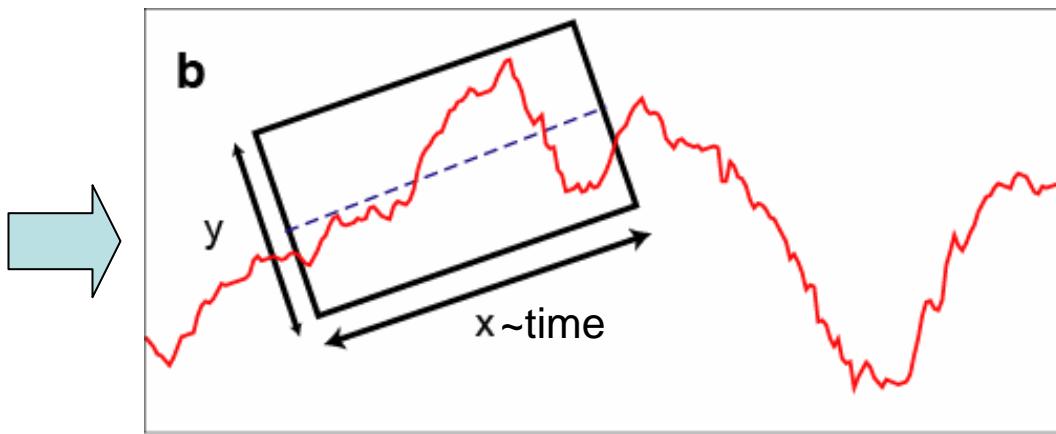
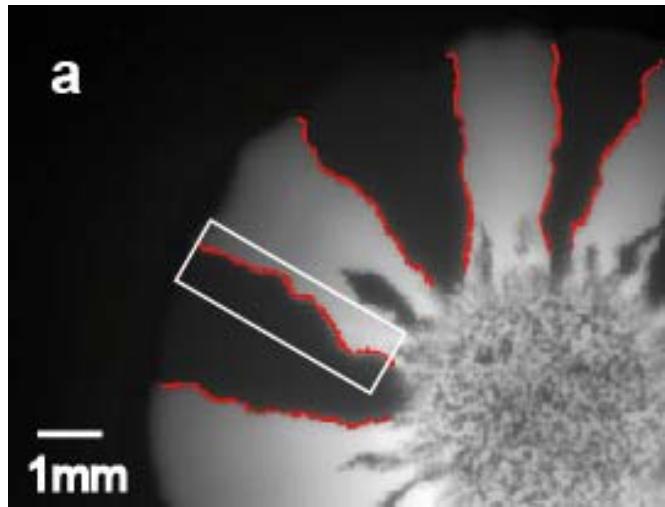
$$t_c \propto R_0$$



$$\langle \# \text{Sectors} \rangle = \sqrt{2\pi R_0 / D_W}$$

... assuming regular diffusion.

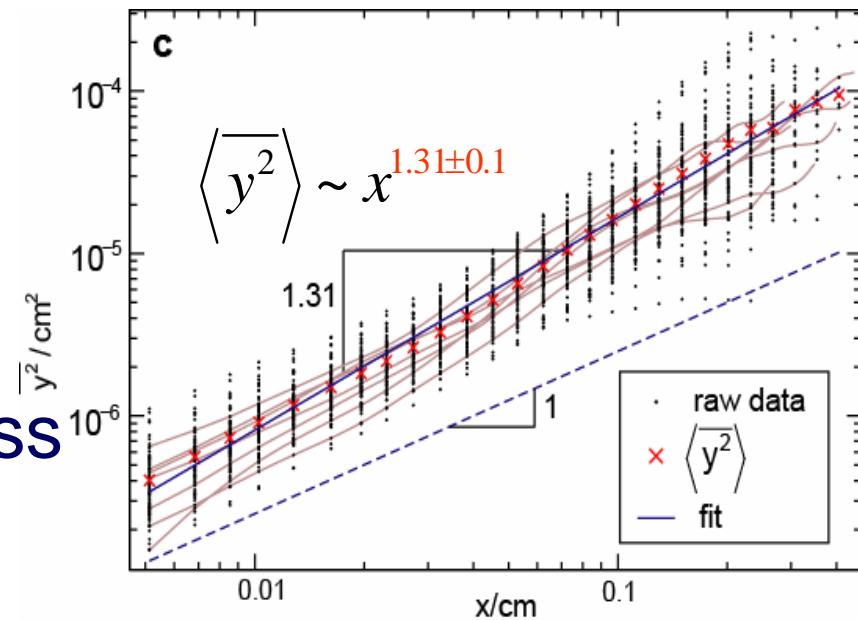
# Meandering of domain boundaries



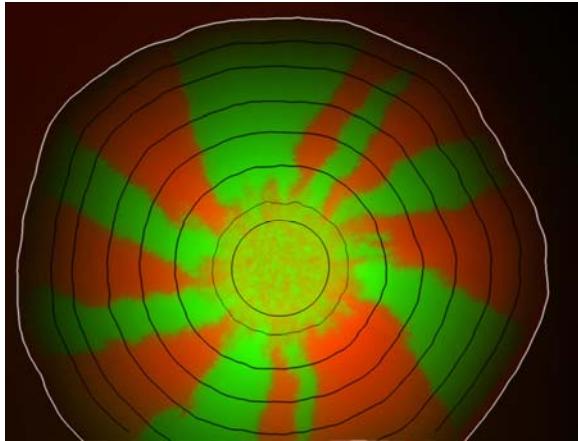
Wandering is **superdiffusive!**

Why?

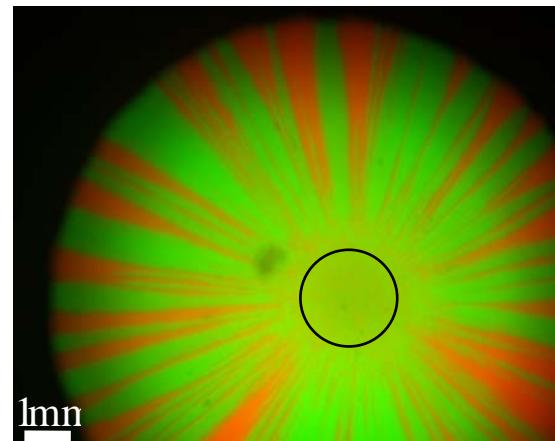
Hypothesis: surface roughness guides wandering  $\Rightarrow \langle \overline{y^2} \rangle \sim x^{4/3}$



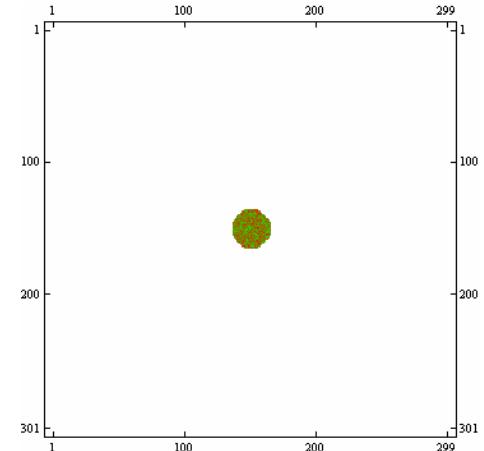
# Demixing & coarsening are generic



*E. coli*



*S. Cerevisiae*

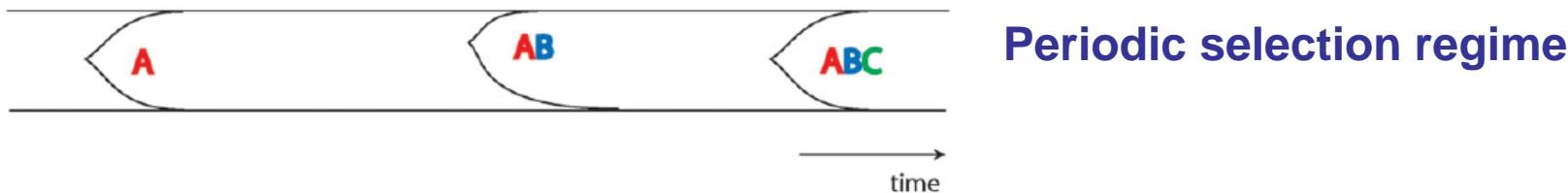


*Simulations*

## Summary:

- Genetic sectoring patterns are characteristic for simple range expansions.
- Can be explained by effects of chance alone.

# Speed of adaptation in asexuals – Rep.: The wellmixed case

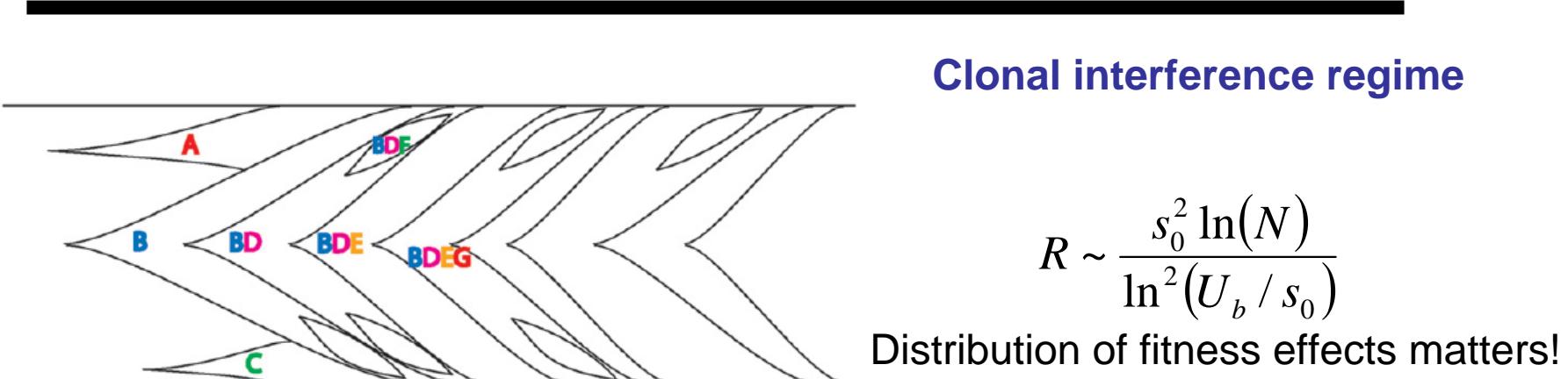


Beneficial mutations occur at rate  $NU_b$   
become established with probability  $2s_0$

$$R \equiv \langle \partial_t W \rangle = (NU_b)(2s_0)s_0$$

Fixation time  $\log(Ns_0)/s_0 \ll (2s_0NU_b)^{-1}$  waiting time

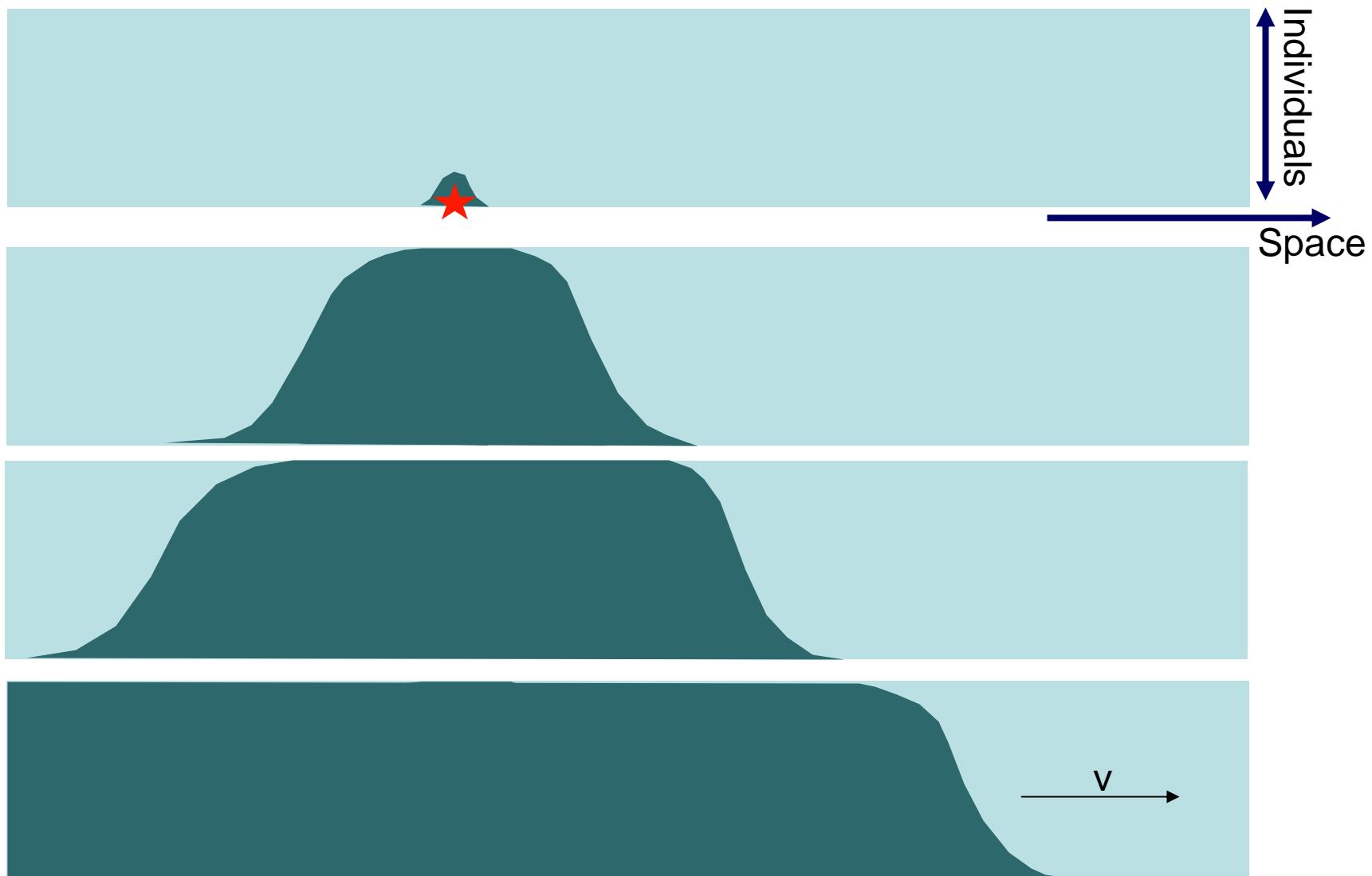
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$$R \sim \frac{s_0^2 \ln(N)}{\ln^2(U_b/s_0)}$$

Distribution of fitness effects matters!

# Selective sweeps in 1 dimension



# Selective sweeps in 1 dimension

$$\partial_t p(x,t) = D\Delta p + sp(1-p) + \frac{\eta}{K} \sqrt{p(1-p)}$$

Noisy Fisher-Kolmogorov equation

- Establishment probability:  $2s$

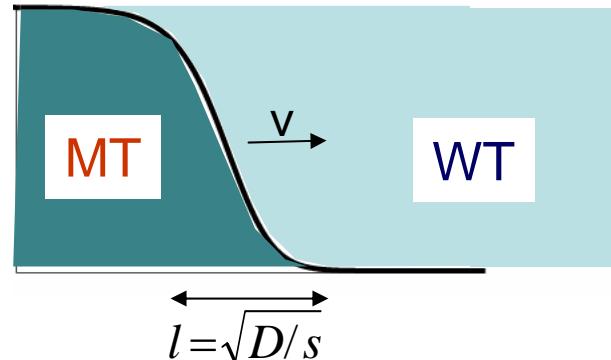


- Wave speed depends on

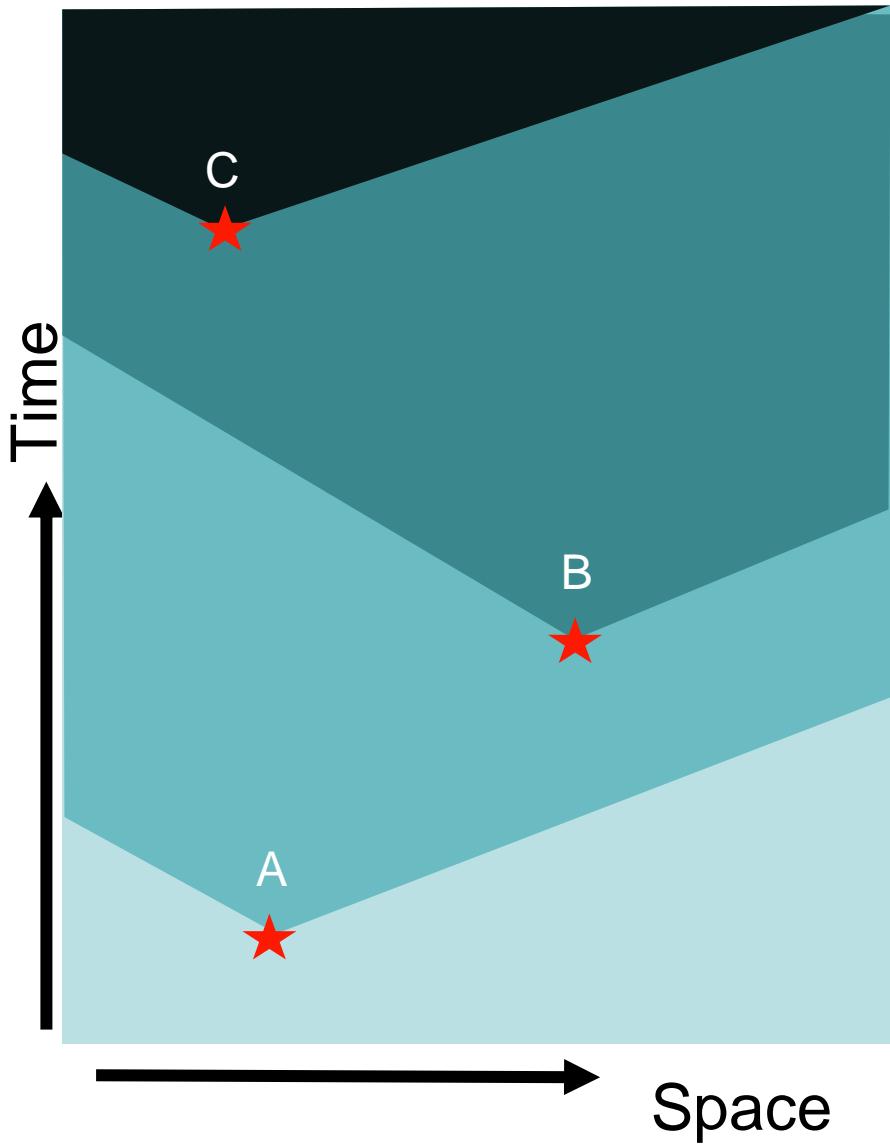
$$"Ns" \equiv Kls = K\sqrt{D/ss} = K\sqrt{Ds}$$

$$\left\{ \begin{array}{l} "Ns" \gg 1, \quad v = 2\sqrt{Ds} \\ \text{R. A. Fisher (1937)} \\ "Ns" \ll 1, \quad v = 2KDs \end{array} \right.$$

Doering et al. (2003)



# Speed of evolution – $R \equiv \langle \partial_t W \rangle = ?$



Suppose: all new mutations confer the same selective advantage  $s_0$

$$\rightarrow R_{1D} = N U_b (2s_0) s_0 = R_{0D}^! \quad (\text{periodic selection})$$

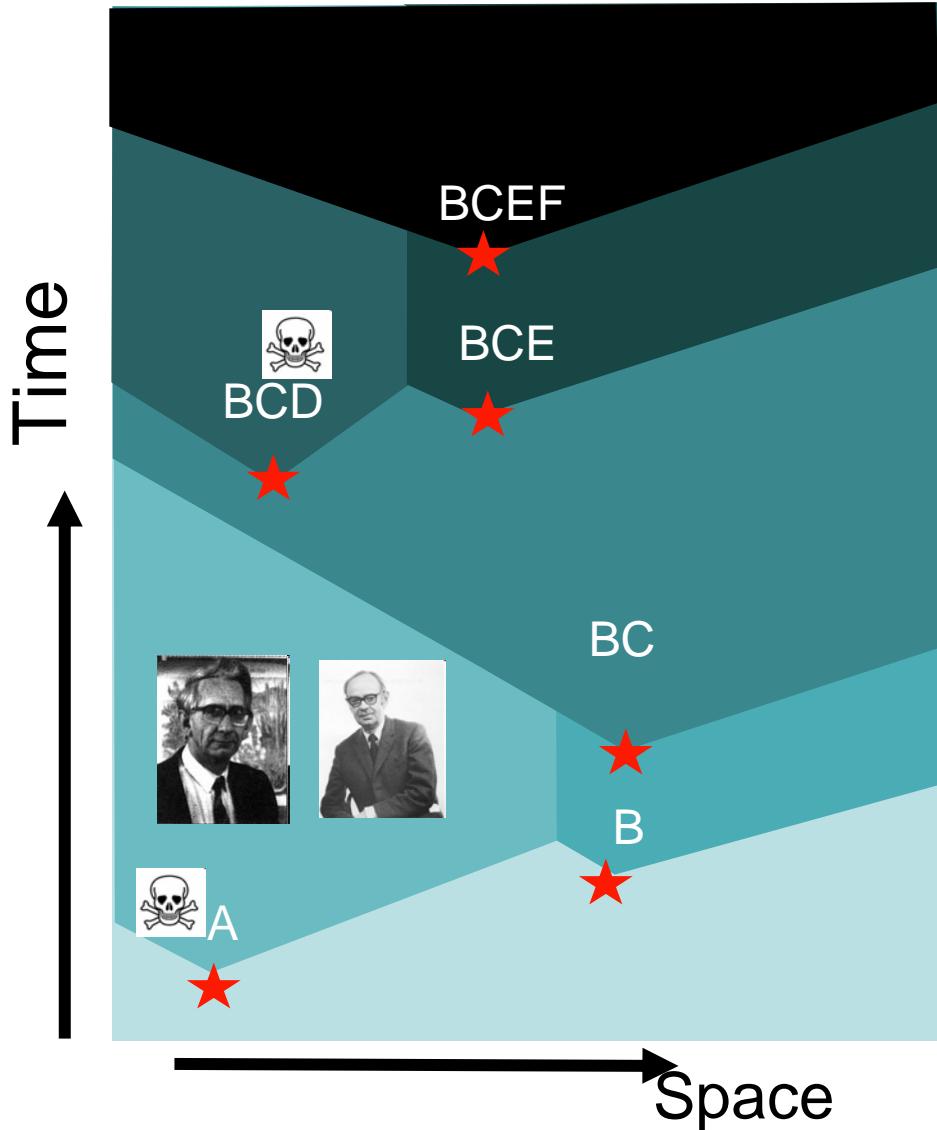
Dimensional analysis:

$$r \equiv R / s_0 = [T^{-1}] \text{ can only depend on } \mu_b \equiv U_b K(2s_0) \quad [(LT)^{-1}] \\ v \quad [L/T] \\ L_{\text{tot}} \quad [L]$$

Periodic selection for  $\mu_b L_{\text{tot}} \ll v/L_{\text{tot}}$

$$r_{1D} = \mu_b L_{\text{tot}}$$

# Speed of evolution – clonal interference



In general, a fraction  $F$  of beneficial mutations is lost by Hill & Robertson.

$$r = L_{tot} \mu_b F \left( \frac{L_{tot}}{\sqrt{v/\mu}} \right)$$

Expectation:

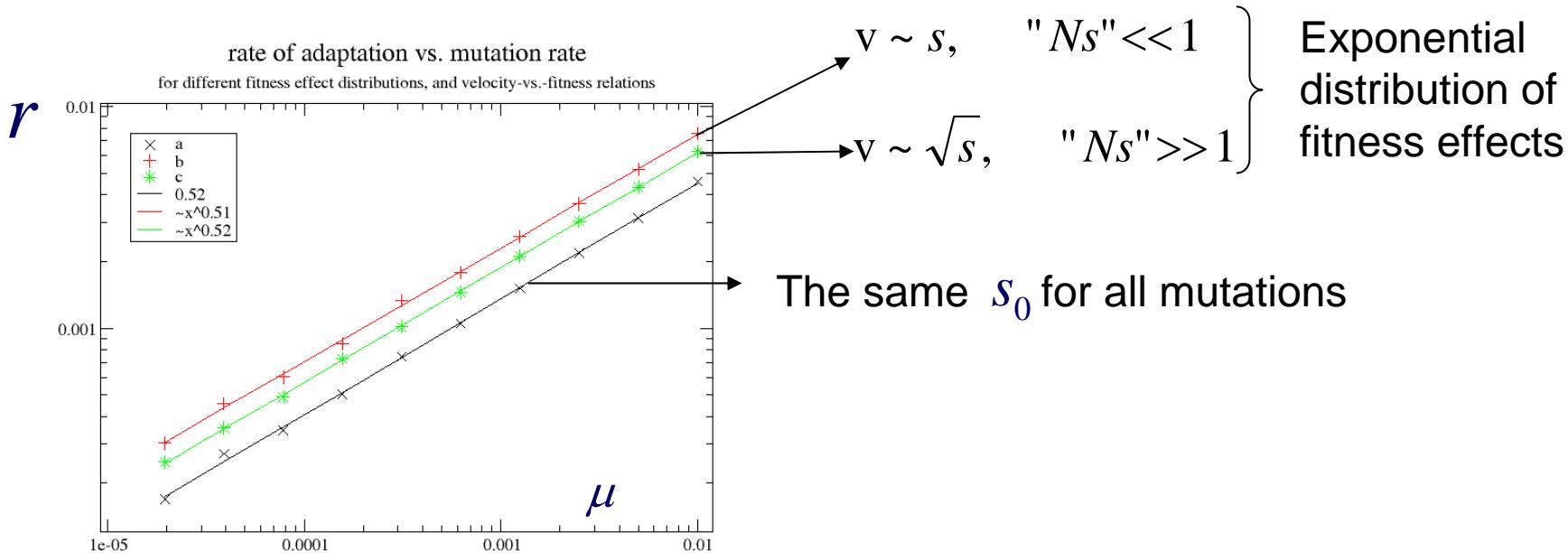
$$F(x) \rightarrow \begin{cases} 1, & \text{for } x \rightarrow 0 \\ \text{const.}/x, & \text{for } x \rightarrow \infty \end{cases}$$

i.e., in the clonal interference regime, we have

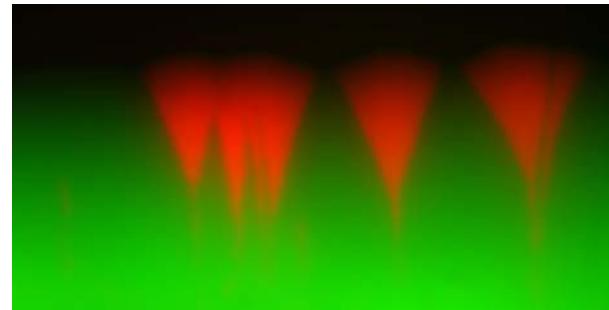
$$R_{1D} \sim \sqrt{U_b K v s_0^{3/2}}$$

Compare with  $R_{0D} \sim \frac{s_0^2 \ln(N)}{\ln^2(U_b / s_0)}$

# Scaling seems robust



- The scaling  $R_{1D} \propto \sqrt{U_b K}$  for large systems seems independent of the distribution of fitness-effects and on relation between velocity and fitness advantage!
- generalization to D>1 straight forward.
- could possibly be tested in microbial experiments.



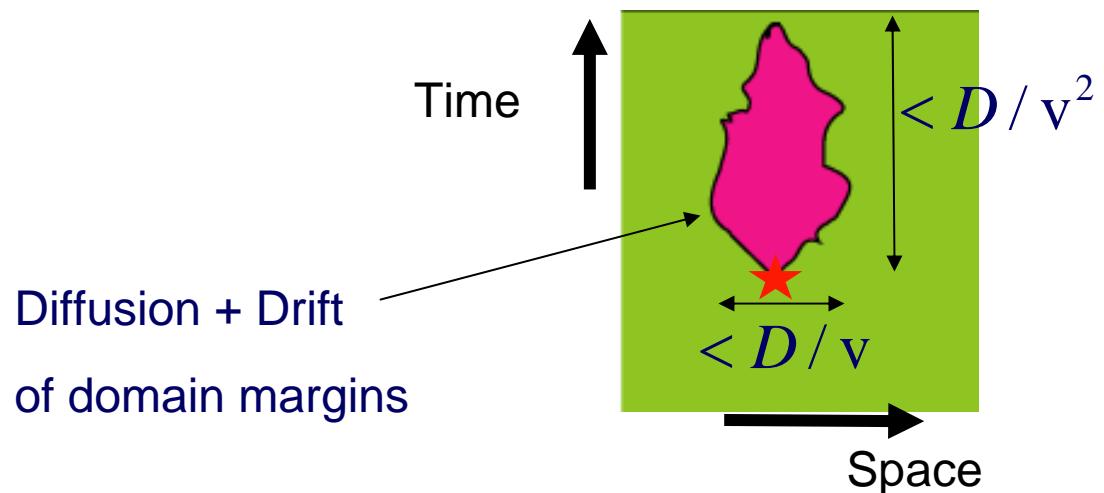
# Genetic load

- $U_d$  mutation rate of wildtype into less fit mutant type.
- $s$  selective disadvantage.
- Question: What is the fraction of mutants in equilibrium?

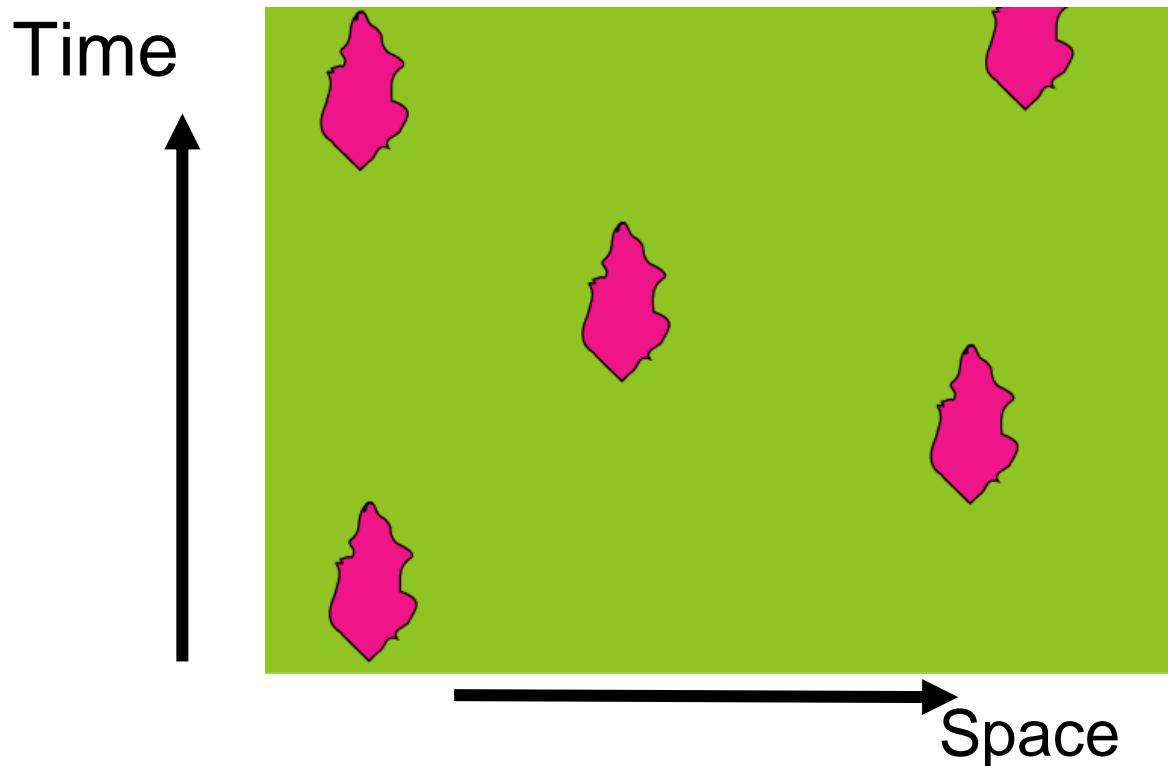
" $Ns$ ">>1, Deterministic limit → Genetic load is as in the well-mixed case

i.e., fraction of mutants  $q = \frac{U_d}{s}$

" $Ns$ "≡  $K\sqrt{Ds} \ll 1$  Weak selection limit → “bubbles”

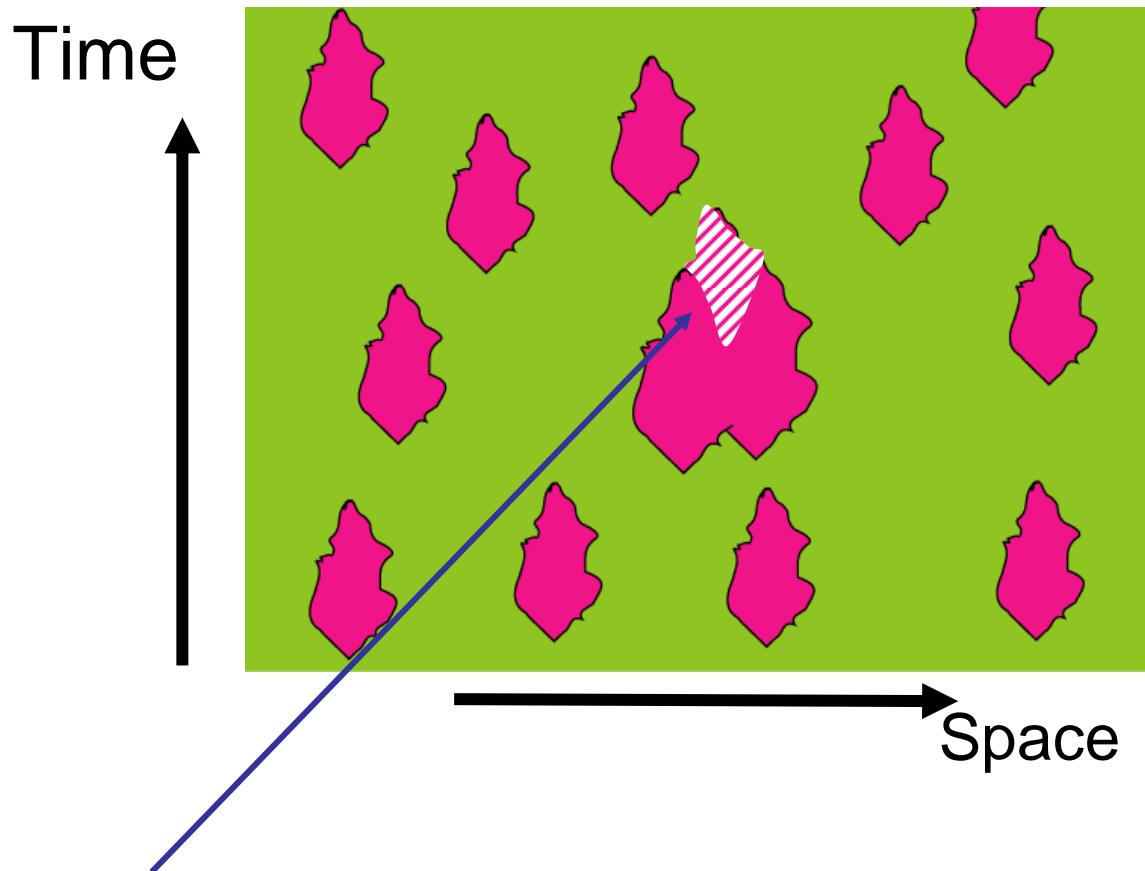


# Genetic load independent bubbles



$$\text{fraction of mutants} \equiv q = \frac{U_d D}{2v^2}, \quad \text{indep. of } K$$

# Genetic load, interacting bubbles



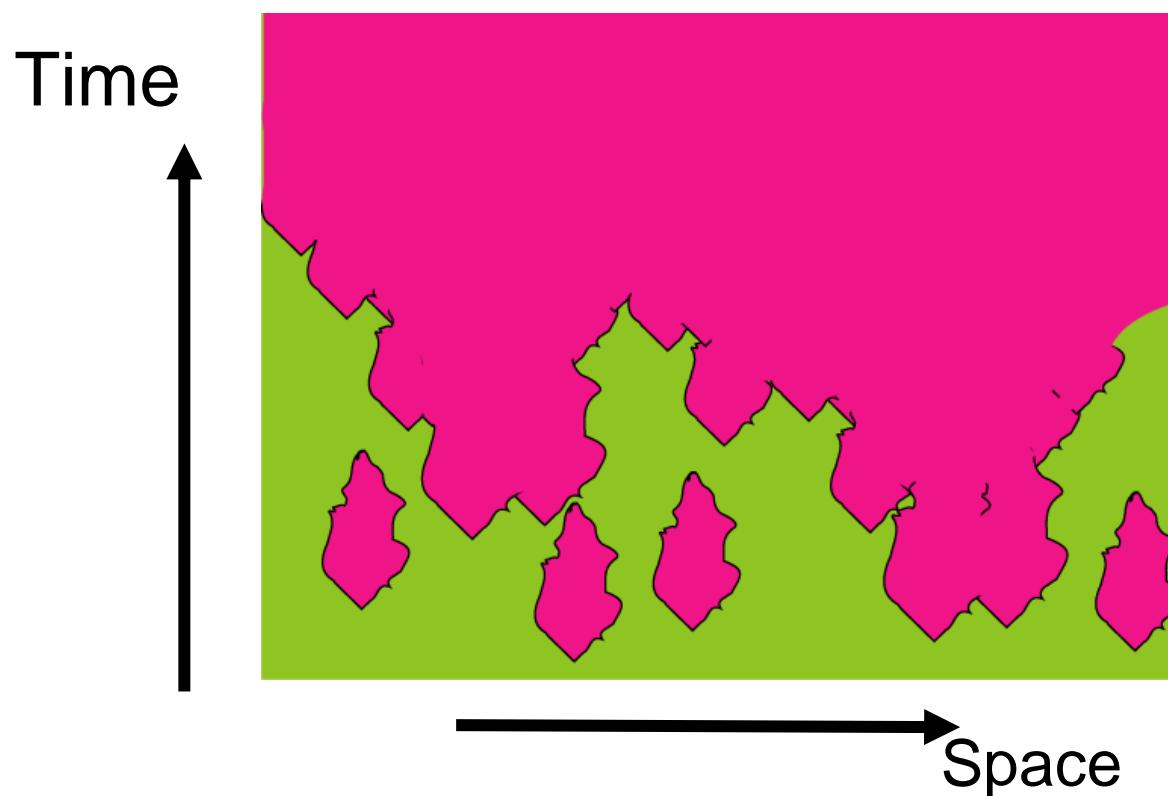
2 x Expected Area  
of a single bubble

Expected Area  
of a doublet

$$q = f\left(\frac{U_d D}{2v^2}\right)$$

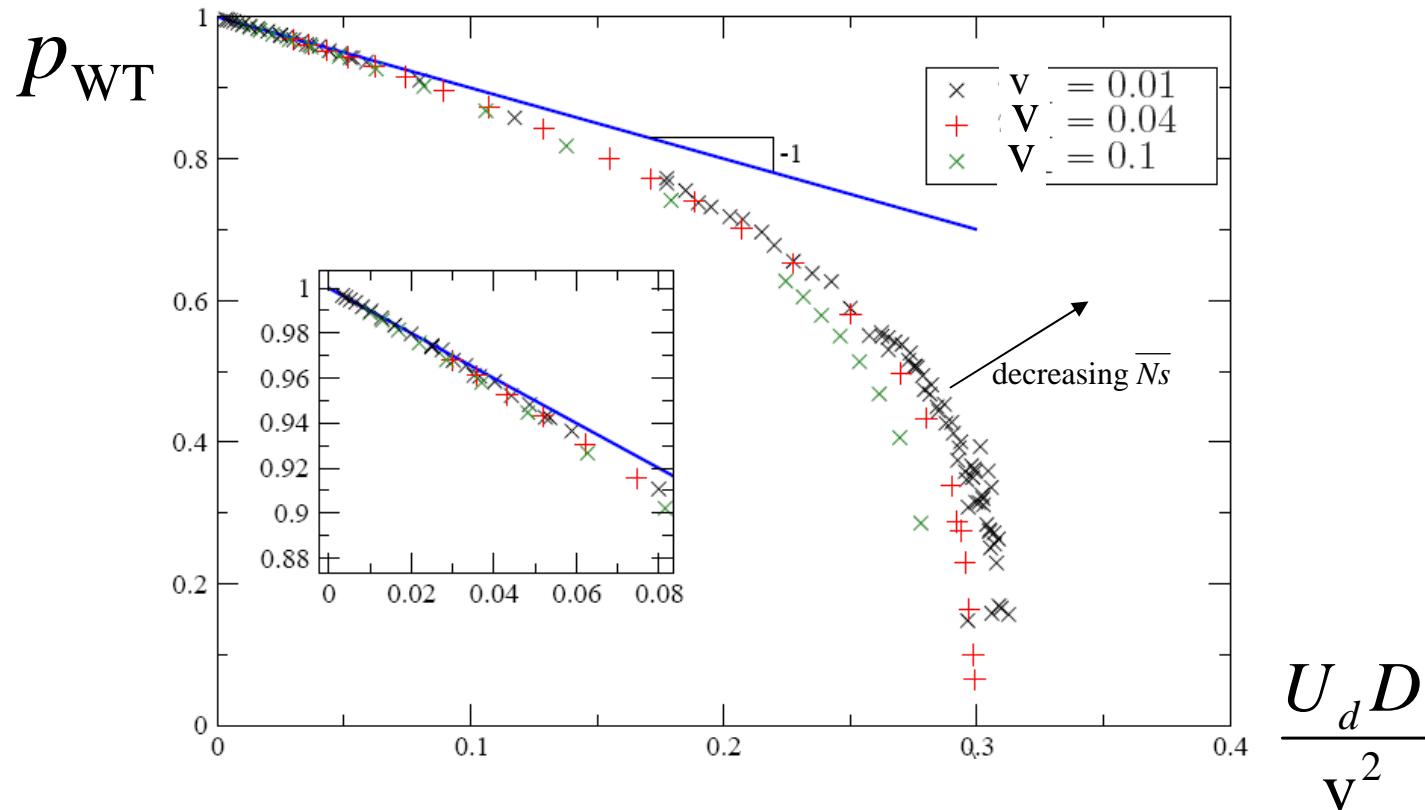


# Genetic load



Loss of wildtype for  $U_d > U_{crit} \sim \frac{v^2}{D} \sim D(Ks)^2$ , "Ns" < 1

# Genetic load is a function of one compound parameter



Universality class of  
“directed percolation”

# Summary

Compared to well-mixed populations, Hill-Robertson effects in spatial systems

- have qualitatively different impact
- are more common
- could possibly be tested with microbes

# Thanks to ...

- David R. Nelson, Sharad Ramanathan and Pascal Hersen
- Tom Shimizu, Howard Berg, Erin O'Shea, J. Krug
- Andrew Murray, Nilay Karahan
- DFG & DAAD for financial support

For more information:

- O. H., D. R. Nelson (2008) *Theoretical Population Biology* 73, 158-170  
O. H., P. Hersen, S. Ramanathan, David R. Nelson (2007) *Proceedings of the National Academy of Sciences USA* 104, 19926-19930  
O.H., D. R. Nelson (2008) ArXiv:0810.0053 [q-bio.PE]  
L. Excoffier, N. Ray, *Trends in Ecology & Evolution*, 23(7):347–351, 7 2008.