Stratigraphic facies from the physics perspective of emergent phases of self organization

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“Three Sisters” -- aboriginal womans’ place for doing business, near BHPB Yandi iron ore mine

Abstract

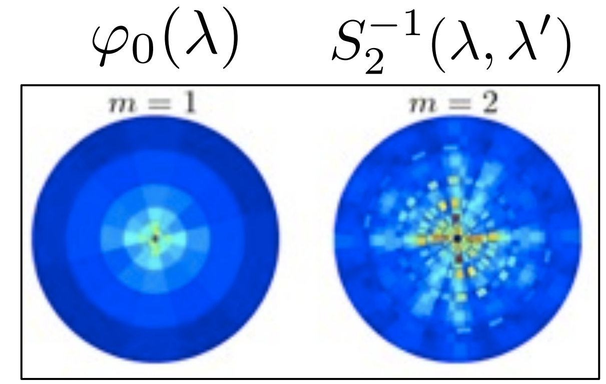
A framework for the analysis of stratigraphic facies as emergent phases of self organization will be presented. An example will be given of turbidite deposition that is governed by a system of partial differential equations. It will be shown how the boundary conditions and coefficients of the PDEs parameterize a phase space that is divided into distinct phases, or what is more commonly called facies. A method of renormalization of the texture of geologic outcrops and well logs will be presented that gives the scale dependance of the PDE coefficients and boundary conditions. This specification of the running coupling coefficients or S-matrix of the physics gives the form of the PDE as well as the coefficients and boundary conditions. Practically this gives a unique fingerprint (or technically a metric) of the geologic facies.

Roadmap

- the big picture -- emergent behavior of self organization
- what is a physical phase and phase diagram (example of water)
- example of sediment wave formation with multiple flows
- Mallat Scattering Transformation (MST) as a metric of self organization
  - ultimate “attribute” of geology for identification and scale extrapolation
  - stratigraphic inversion objective function
- relationship of physics to the Mallat Scattering Transformation
  - why does the MST work so well
  - a new perspective on renormalization of field theory and the S-matrix
- conclusion
  - it’s the physics
  - one-to-one correspondence between geologic facies and phases of physical self organization of system
  - S-matrix (MST) is the ultimate metric of geology

The big picture -- emergent behavior (facies) of self organizing system

$$\frac{\partial c_i}{\partial t} + (\vec{u} + u_{si}\hat{g}) \bullet \nabla c_i = \frac{1}{S_c R_e} \nabla^2 c_i$$
$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \bullet \nabla)\vec{u} = -\nabla p + \frac{1}{R_e} \nabla^2 \vec{u} + c\hat{g}$$
$$\nabla \bullet \vec{u} = 0$$
$$(d, \theta_0, H, c_0)$$



ODEs or PDEs with BC & IC → emergent behavior of self organized state → metric of self organization

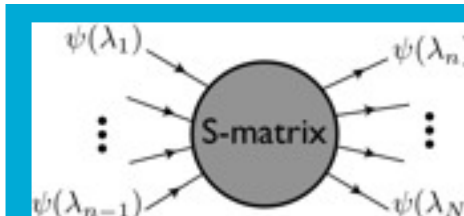
specified by state variables (coefficients of PDEs and BC)

system response

Mallat Scattering Transformation, texture of system response

Lagrangian L

state of system $|\psi\rangle$

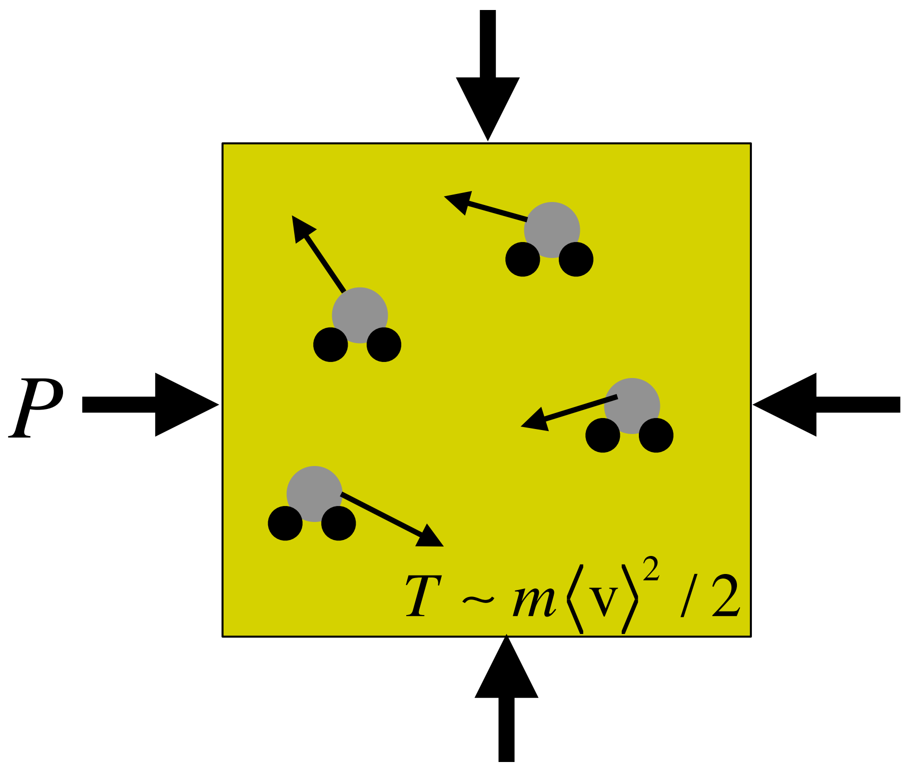


$S_m(|\psi\rangle)$

generalized Green's function, physics as a function of scale running coupling constants (scale)

Lagrangian perspective $\mathcal{F}(L) = S_m(|\psi\rangle)$ canonical perspective

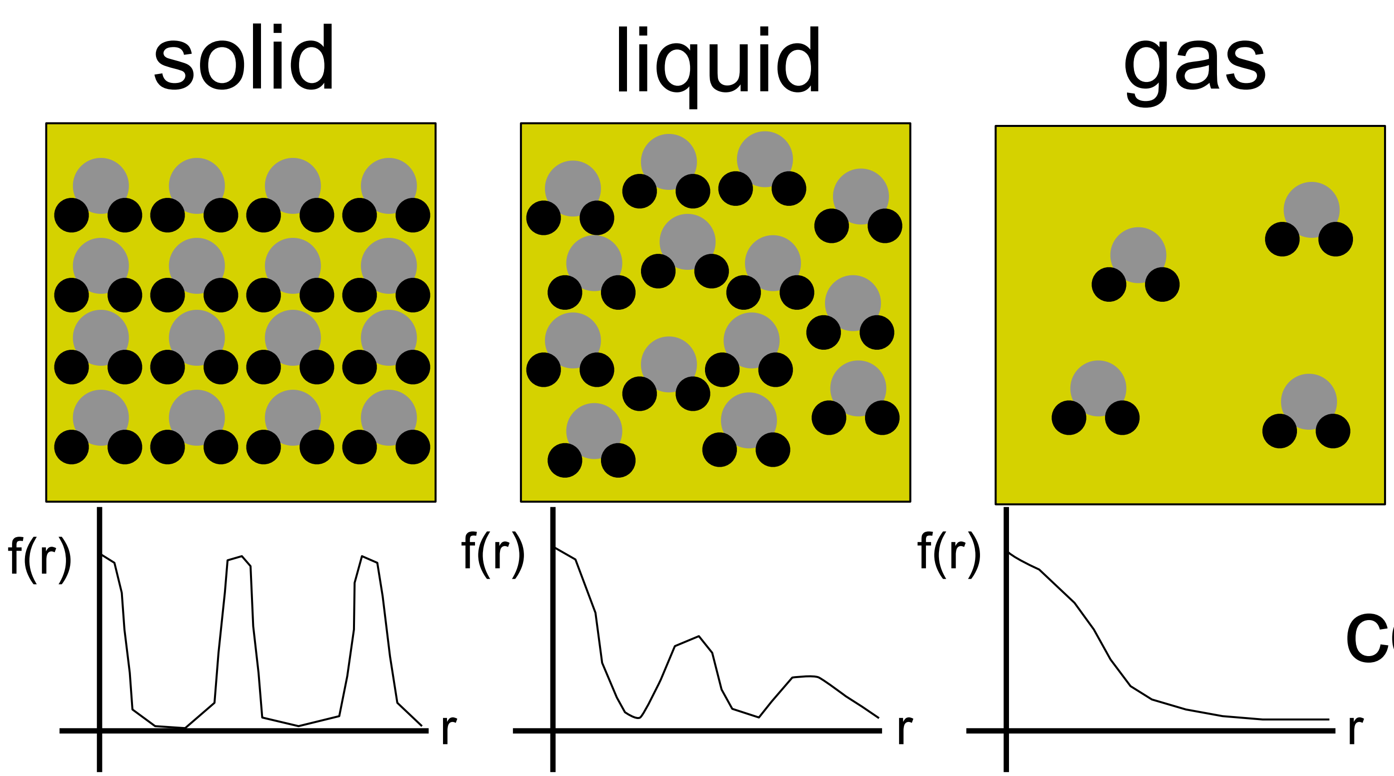
Phases of water



system parameters: Lagrangian L

(a) temperature, T

(b) pressure, P



state of system $|\psi\rangle$

phases:

(a) solid

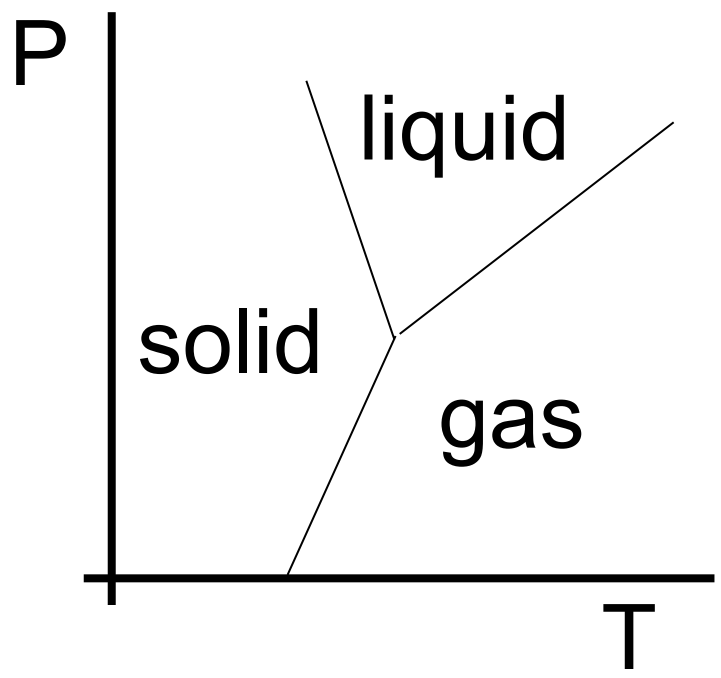
(b) liquid

(c) gas

correlation properties

S-matrix $S_m(|\psi\rangle)$

Phase diagram of water



- temperature
- pressure

→

- flow size
- grain size
- sorting
- slope

- solid
- liquid
- gas

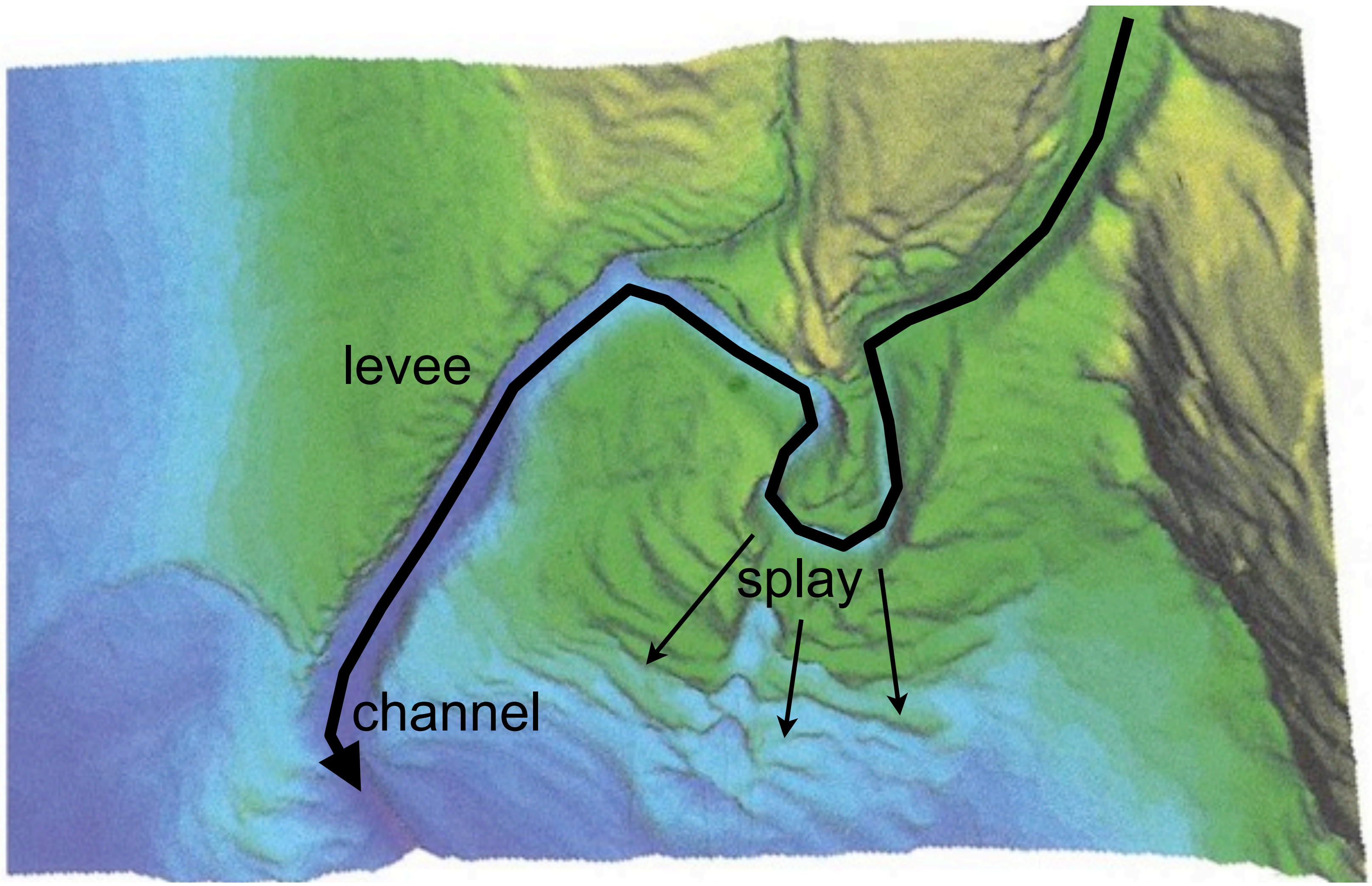
→

- channel
- levee
- fan

phase is determined by the value of the system parameters, system parameter space is divided into regions for each phase

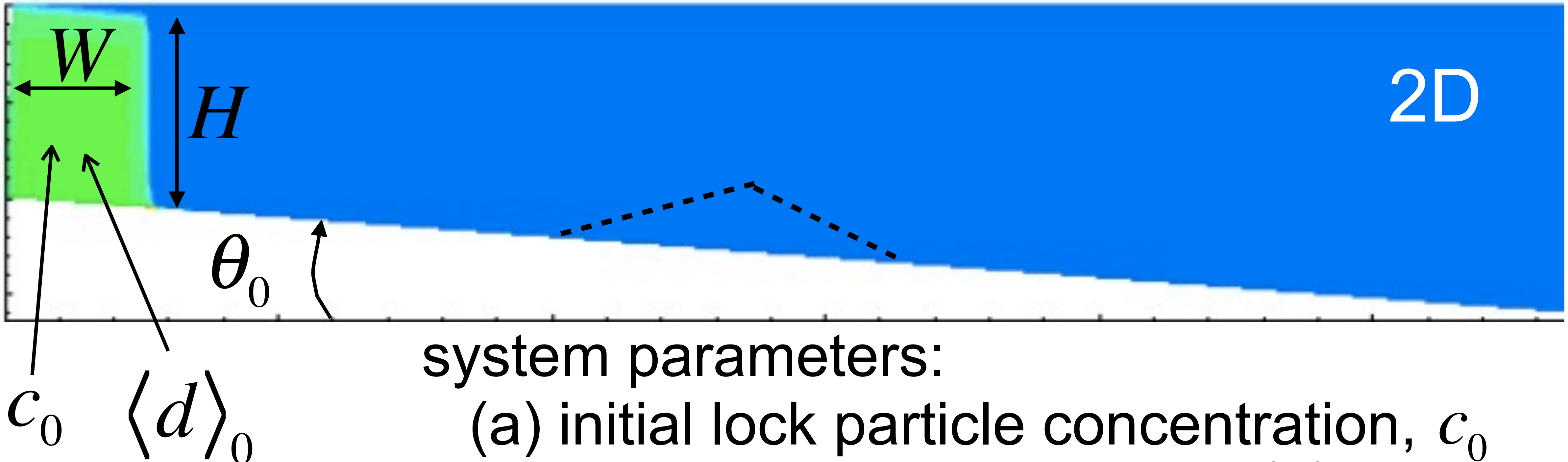
A real example of a sediment wave

Monterey Channel, offshore California USA



breached channel levee (splay)

What are the phases of turbidite deposition in a channel?



system parameters:
(a) initial lock particle concentration, c_0
(b) average particle diameter, $\langle d \rangle_0$
(c) slope angle, θ_0
(d) current size, $HW \sim H$
(e) initial aspect ratio, H / W (insensitive)

Strauss and Glinsky, JGR 2012

Simulation of the fluid and suspended grains

mass continuity equations for each grain size

$$\frac{\partial c_i}{\partial t} + \left(\vec{u} + u_{si}\hat{g}\right) \bullet \nabla c_i = \frac{1}{S_c R_e} \nabla^2 c_i$$

settling velocity particle diffusion

momentum continuity equation, ma=F

$$\frac{\partial \vec{u}}{\partial t} + \left(\vec{u} \bullet \nabla\right)\vec{u} = -\nabla p + \frac{1}{R_e}\nabla^2 \vec{u} + c\hat{g}$$

pressure force viscous drag force gravity force

incompressibility, EOS

$$\nabla \bullet \vec{u} = 0$$

Blanchette et al., 2005, 2006

$$R_* \equiv \frac{\rho_g - \rho_f}{\rho_f}$$

x and y	scaled by	L_0	= 250 m
c	scaled by	c_0	= 0.8 %
u	scaled by	$u_b \equiv \sqrt{g R_* L_0 c_0}$	= 5.4 m/s
t	scaled by	L_0 / u_b	= 46 s
d_i	scaled by	$d_0 \equiv \sqrt[3]{\nu^2 / R_* g}$	= 41 µm

$R_e \equiv u_b L_0 / \nu = \left(L_0 / d_e\right)^{3/2}$ = 10^9

$d_e \equiv \sqrt{\nu^2 / R_* c_0 g}$ = 200 µm

$R_{pi} \equiv \left(d_i / d_0\right)^{3/2}$ = 42

$u_{si} = f(R_{pi})$

scale of particle dissipation

scale of fluid dissipation

Simplified equations

eliminate pressure, set S_c=1 (particles transported as fluid) and write in terms of stream function and vorticity

$$\frac{\partial c_i}{\partial t} + \left(\vec{u} + u_{si}\hat{g}\right) \bullet \nabla c_i = \frac{1}{R_e}\nabla^2 c_i$$

$$\frac{\partial \omega}{\partial t} + \left(\vec{u} \bullet \nabla\right)\omega = \frac{1}{R_e}\nabla^2 \omega + \left(\hat{g} \times \nabla c\right)_z$$

where

$$\omega = -\nabla^2 \psi = F(\psi) \qquad \omega \equiv \left(\nabla \times \vec{u}\right)_z$$

$$\vec{u} \equiv \left(\hat{x}\frac{\partial}{\partial y} - \hat{y}\frac{\partial}{\partial x}\right)\psi = G(\psi)$$

only $\{c_i\}$ and ψ to solve for

$$d_i \qquad \theta_0 \qquad HW = HL_0 \sim H$$

parameters are: $(\{\cancel{u}_{si}\}, \cancel{\hat{g}}, \cancel{R}_e; \cancel{\psi}_0) \qquad \to (d, \theta_0, H)$

Resuspension brings initial concentration back into problem

Garcia and Parker resuspension model

$$J_i = u_{si}(\hat{g}_y c - \varepsilon_{si})$$

settling resuspension

$$\varepsilon_{si} = \frac{a}{c_0}\frac{z_i^5}{1+\frac{a}{0.3}z_i^5}$$

explicit dependance on c_0

$$z_i \equiv \alpha_1 \frac{u_*}{u_{si}} R_{pi}^{\alpha_2} = f(u_*, R_{pi})$$

$$u_* = \sqrt{\frac{1}{R_e}\frac{\partial u_x}{\partial y}}$$ limit to $$u_* = \sqrt{\frac{\omega_b}{f_{\text{shr}} R_e}}$$

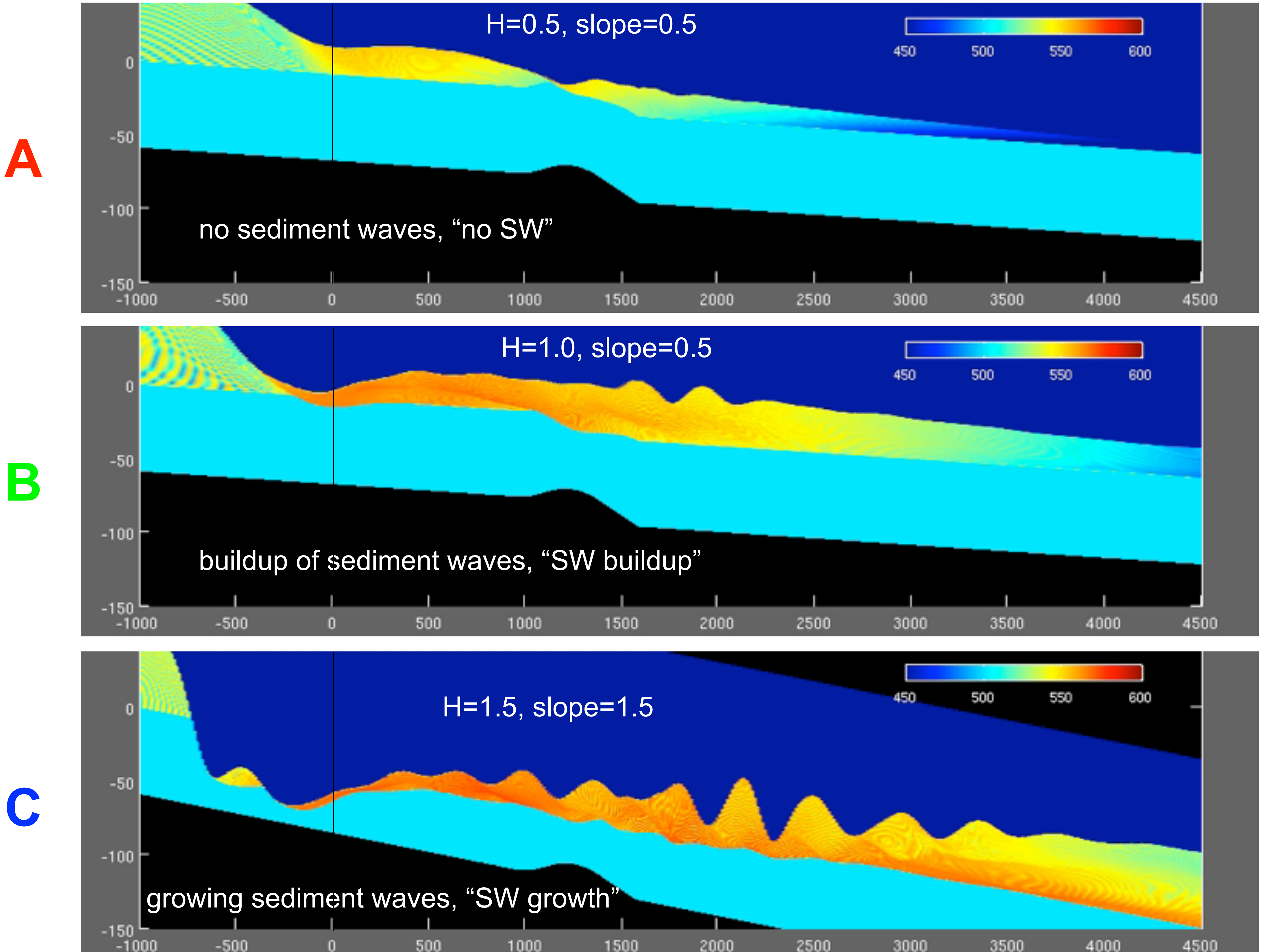
turbulent closure

since R_e simulated is 10^3 instead of real value of 10^9

parameters are: (d, θ_0, H, c_0)

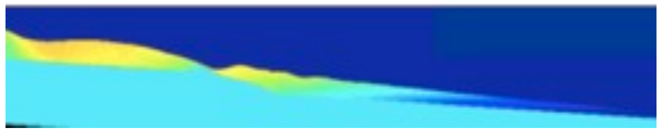
Garcia and Parker, JGR 1993

Three phases of multiple flow turbidite deposition



Characteristics of multiple flow turbidite deposition

no SW



slope never unstable to SW growth

(a) no development of SW
(b) no periodic structures in flow
(c) monotonically decreasing mass
(d) no significant erosion
(e) suppressed front velocity
(f) no evidence of individual flows in bedding
(g) one massive bed fining downslope, coarsing from bottom to top

SW buildup

slope sometimes unstable to SW growth

(a) rapid local SW development to steady state profile
(b) periodic flow structure
(c) relatively constant mass with maximum
(d) no appreciable erosion
(e) reference front velocity
(f) little evidence of individual flows in bedding
(g) one massive bed fining downslope, oscillatory bottom to top structure

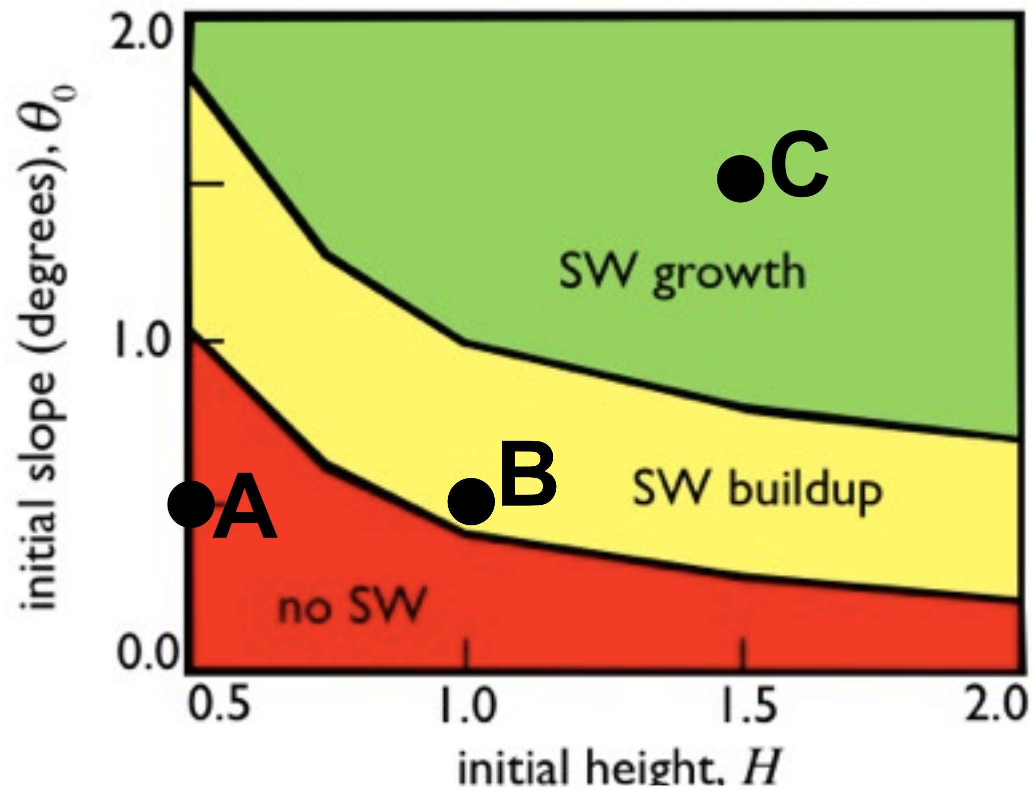
SW growth

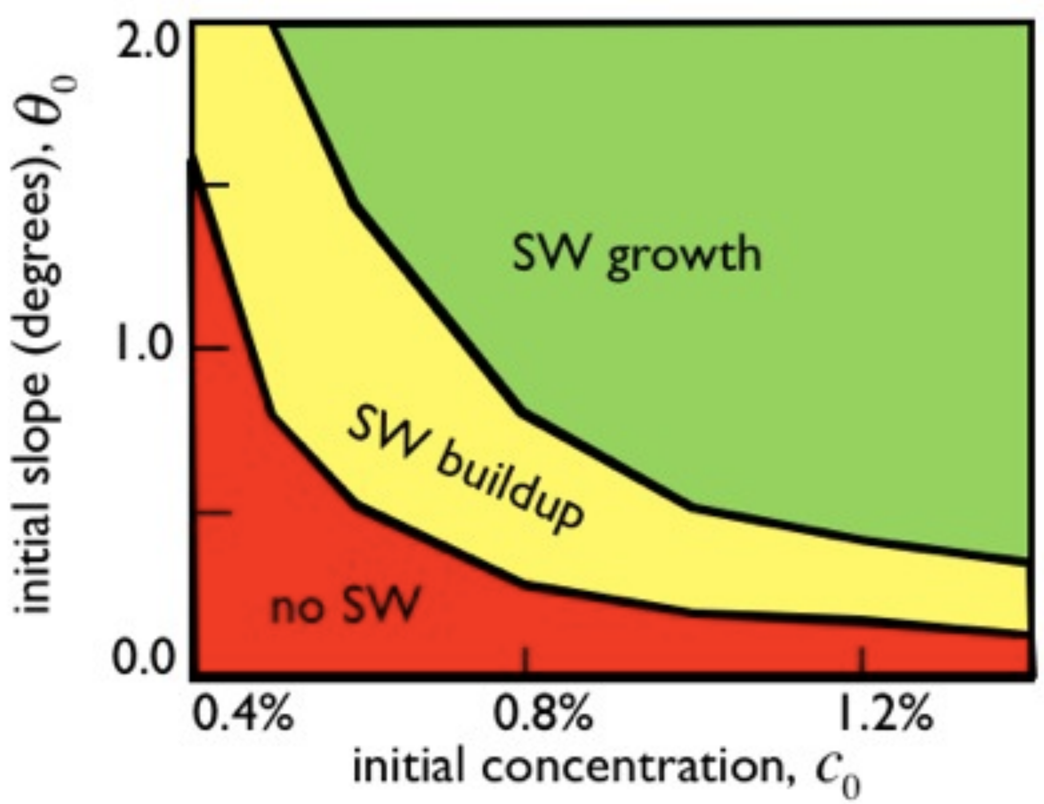
slope always unstable to SW growth

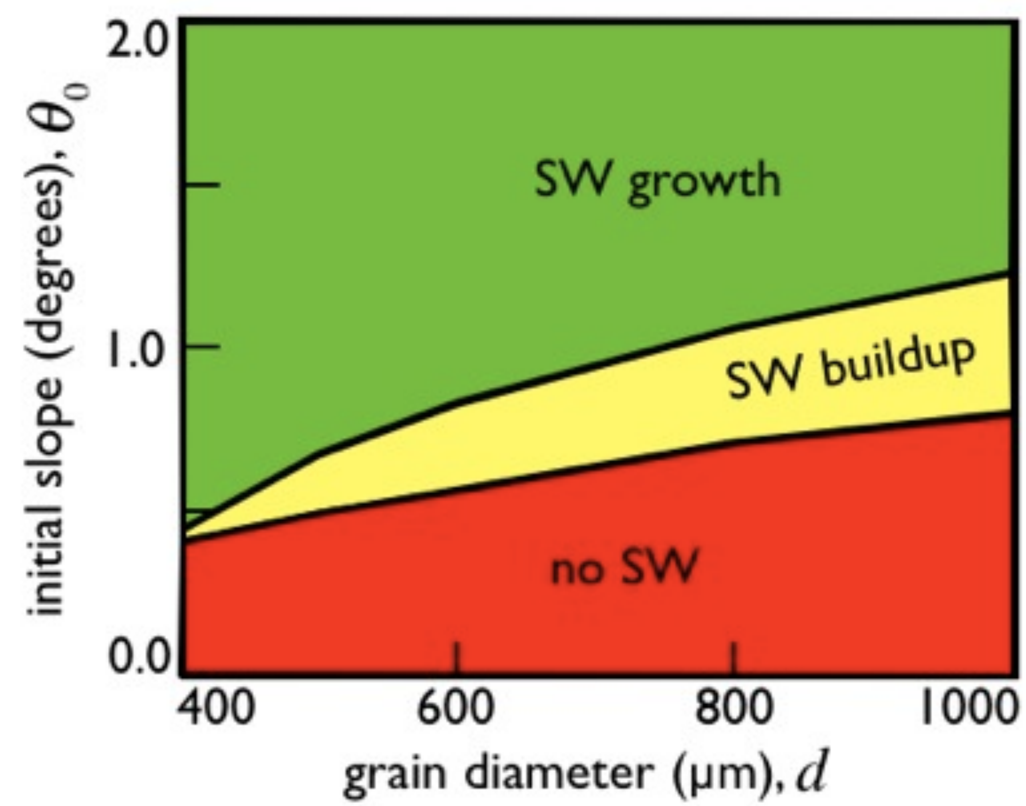
(a) initially exponential growth of global SW
(b) periodic flow & erosion structure
(c) monotonically increasing mass
(d) significant erosion, exponentially growing updip within flow
(e) enhanced front velocity
(f) evidence of individual flows in bedding
(g) complex bed structure

Phase diagram of multiple flow turbidite deposition

in
(θ_0, H, c_0, d)
space







What is wrong with Fourier?

- Invariant of coordinates
- NOT stable to small changes in the dynamics
  - at small scale, small changes in signal lead to large changes in transform
  - origin of divergences in field theory, leading to renormalization, and scale dependant coupling constants

Mallat, arXiv:1101.2286

Bruna and Mallat, arXiv:1112.1120

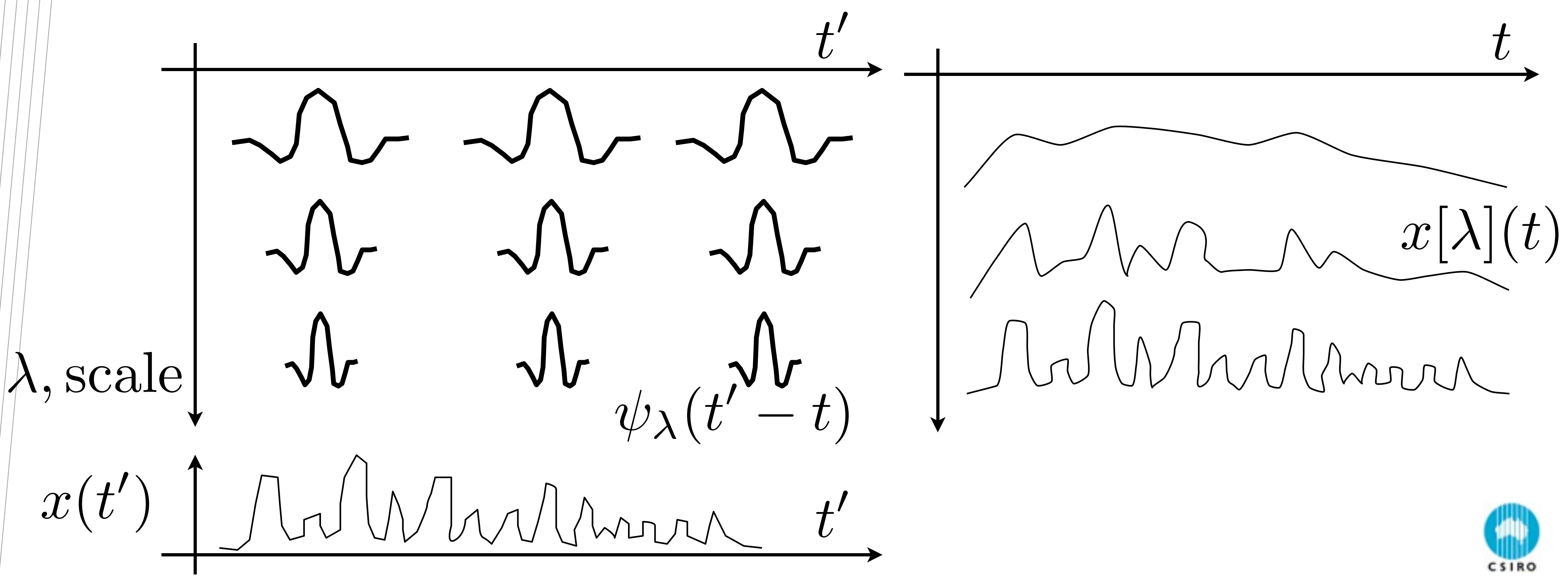
Bruna and Mallat, arXiv:1311.0407

Bruna et al., arXiv:1311.4104

What is wrong with the wavelet transform?

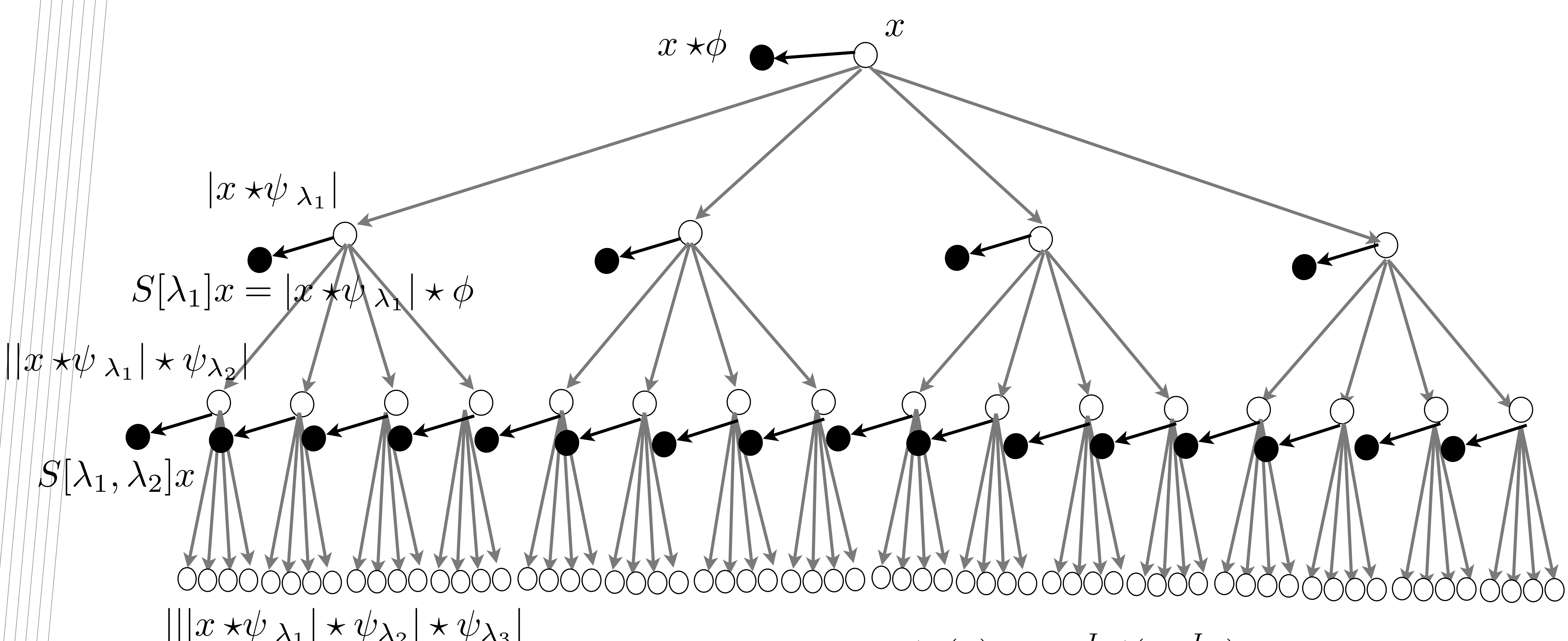
- stable to small changes in the dynamics
- NOT invariant of coordinates

$$x[\lambda](t) = x \star \psi_\lambda = \int dt' \psi_\lambda(t'-t)\, x(t')$$



Mallat Scattering Transformation (MST)

- Iteration on $Ux = \{x \star \phi \,,\, |x \star \psi_{\lambda}|\}_{\lambda}$, contracting.



x is realization of distribution X

$\mathrm{MST} = S[p]X \qquad p \equiv (\lambda_1, \ldots, \lambda_N)$

$\phi_J(x) \equiv 2^{-J}\ \phi(2^{-J}x)$

is windowing function for finite discrete transform

$\psi_{\lambda}(x) \equiv 2^{j}\ \psi_0(2^{j}x) \qquad 0 > j > -J$

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How do we analyze texture, phases, or facies?

X

$|\psi\rangle$

$S[p]X$

$S_m(|\psi\rangle)$

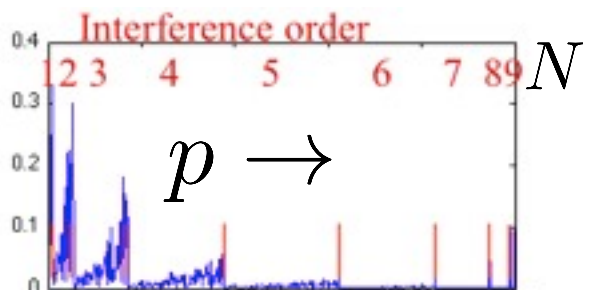
$\mathcal{F}^{-1}(S_m(|\psi\rangle))$

phase #1

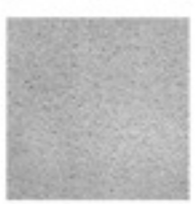
S-matrix
"actions"

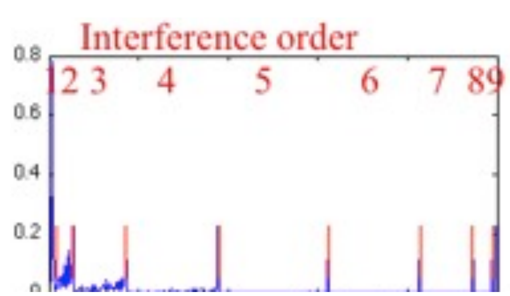
+

random phases
"angles"



phase #2





"geostatistical
simulation"

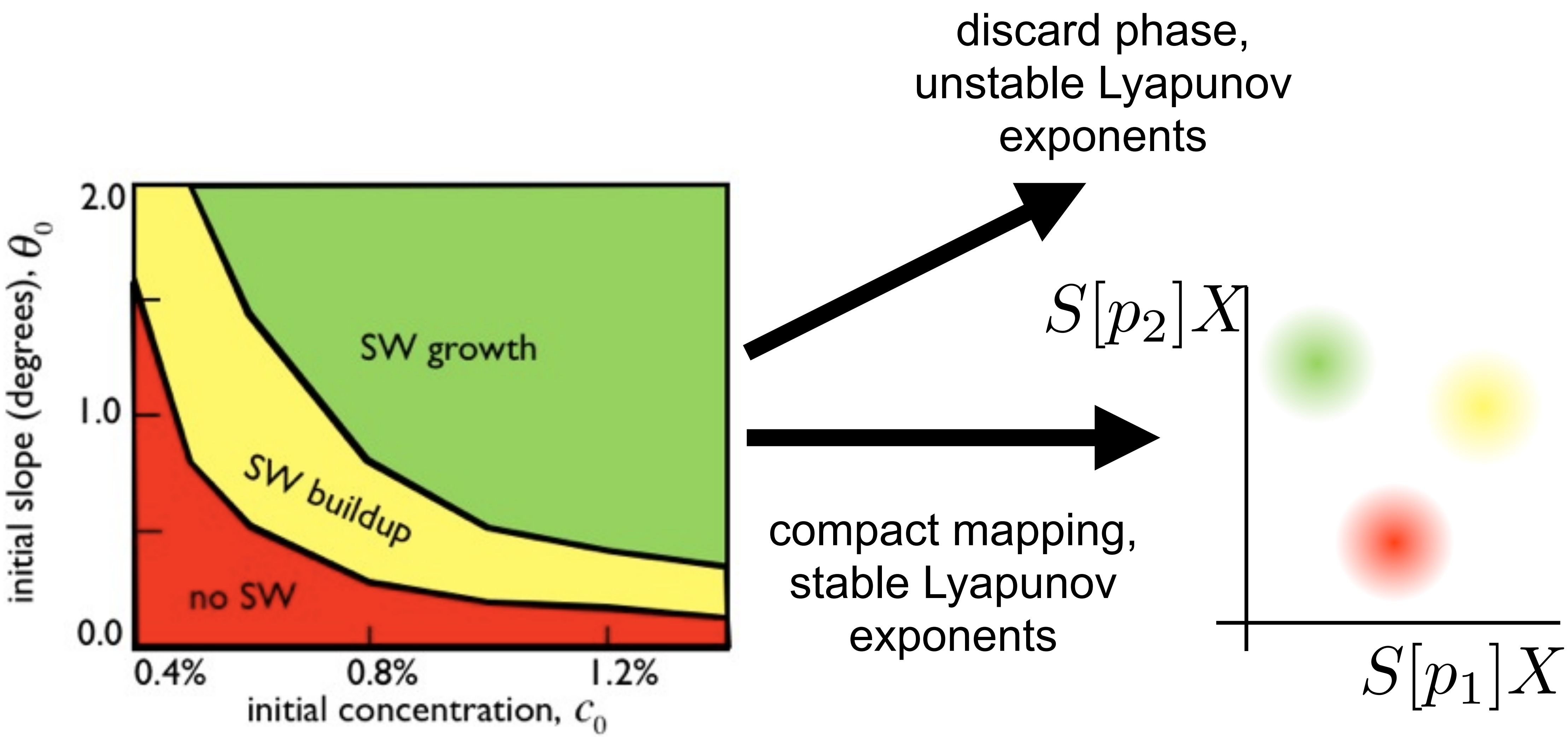
Reconstruction examples

- Natural Sounds
  - Hammer
  - Helicopter
  - Insect
  - Train
  - Water
  - Wind
  - Applause

Large class of stochastic processes described and identified with only second order scattering

Process	first order $T(\lambda, \emptyset) = \varphi_0(\lambda)$	second order $T(\lambda_1, \lambda_2) = S_2^{-1}(\lambda, \lambda')$
White Gaussian	$\lambda^{1/2}$	$(\lambda_1^{-1}\lambda_2)^{1/2}$
Dirac measure	$\|\psi\|_1$	$\|\psi\|_1$
Fractional Brownian Noise $B_H(t)$	λ^H	$(\lambda_1^{-1}\lambda_2)^{1/2}$
Poisson pp density α	$\|\psi\|_1$ if $\lambda < \alpha$ $\lambda^{1/2}$ if $\lambda \geq \alpha$	$\|\psi\|_1$ if $\lambda_1 + \lambda_2 < \alpha$ $(\lambda_1^{-1}\lambda_2)^{1/2}$ if $\lambda_1 + \lambda_2 \geq \alpha$
Mandelbrot cascade	$\lambda^{-\gamma_1}$	C
Log-Normal $Y = \exp(\sigma B_H(t))$	λ^{kH}	$C(\sigma)(\lambda_1^{-1}\lambda_2)^{B(\sigma)(1-H)}$
NASDAQ:AAPL	$\lambda^{-2/3}$	$(\lambda_1^{-1}\lambda_2)^{-0.15}$

S-matrix clustering of phases and classification

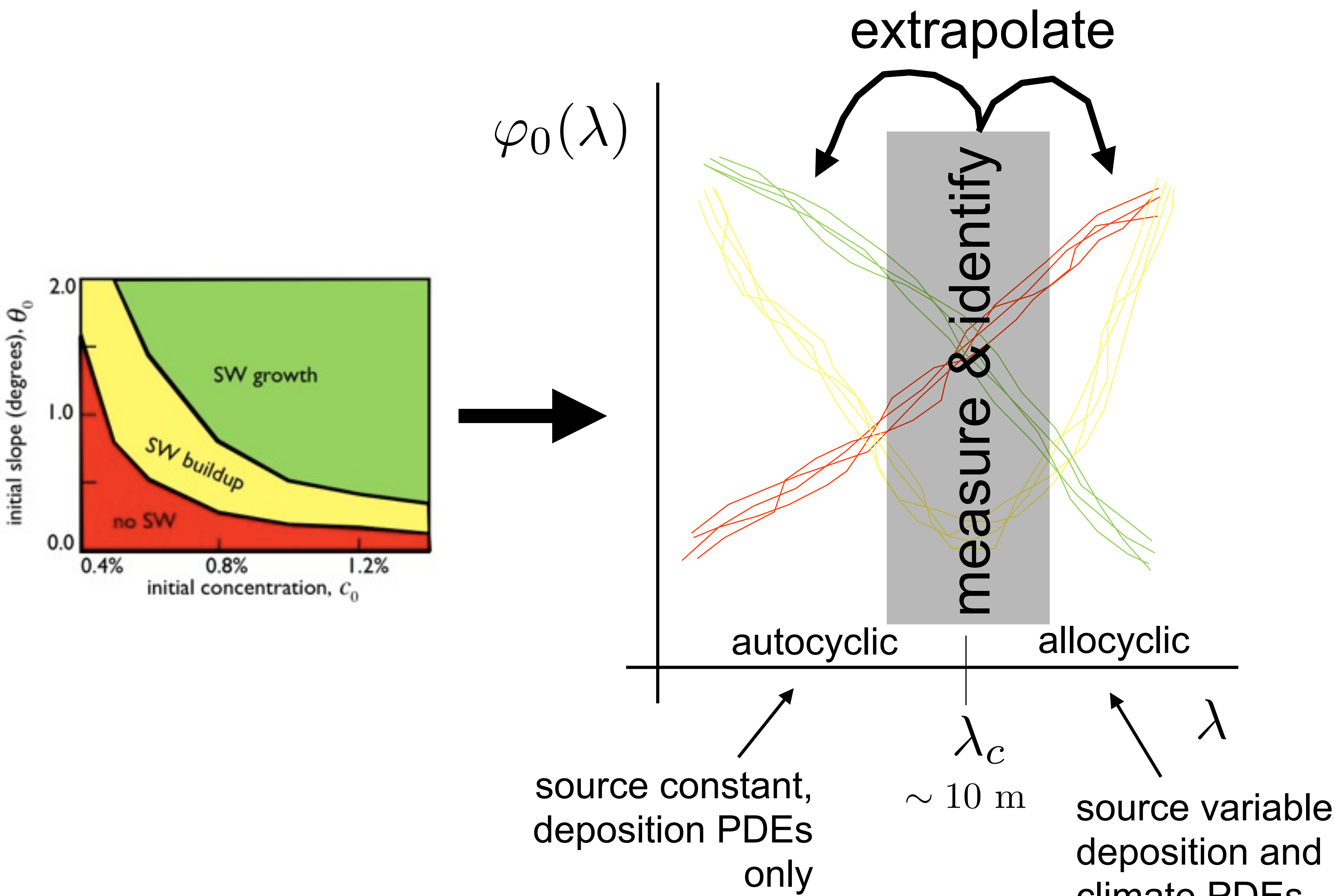


Lagrangian L

S-matrix $S_m(|\psi\rangle)$

classification of phase or texture from potentially limited scale range & scale extrapolation (to small and large)

Phase identification from limited scale S-matrix and S-matrix scale extrapolation



S-matrix "metric" as objective in stratigraphic inversion

$$\|S_{\text{observed}}(|\psi\rangle) - S_{\text{modeled}}(L)\|$$

or explicit inverse, that is simulation

$$\mathcal{F}^{-1}(S_m(|\psi\rangle))$$

Relationship of MST to QFT

mathematics MST	physics QFT	
t	x	coordinate
$x(t)$	$f(x)$	field
$X(x)$	$F(f) = \vert\psi\rangle$	state or distribution of fields
$E(S[p]X(x))$	$E(T_\lambda(\hat{\psi}(\lambda_1)\ldots\hat{\psi}(\lambda_N))\,F(f))$	MST or S-matrix

where

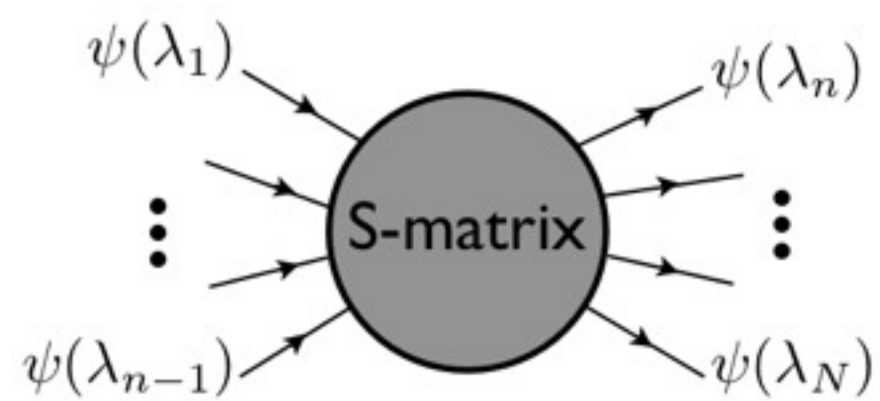
$$\hat{x} \to \hat{\lambda}$$

$$\hat{f}(x) \to \hat{\psi}(\lambda)$$

$$p \equiv (\lambda_1, \ldots, \lambda_N) \qquad \langle \psi, \lambda | f, x \rangle = \psi_\lambda(x) \star f(x)$$

$$S[p]\,x(t) = \lim_{J\to\infty} ||||x \star \psi_{\lambda_1}| \star \psi_{\lambda_2}| \ldots | \star \psi_{\lambda_N}| \star \phi_J$$

not momentum basis where $\langle x|k\rangle = e^{ikx}$



Glinsky, arXiv:1106.4369

Key connection of Lagrangian to the canonical perspective

from the Lagrangian perspective define generating function:

$$Z[J] = N \int [d\psi(\lambda)]\, e^{(i/\hbar)S_0[\psi(\lambda)] + (i/\hbar)\int dx J(\lambda)\psi(\lambda)}$$

the connection to the canonical formulation is:

$$S_m(|\psi\rangle) = E(T_\lambda(\hat{\psi}(\lambda_1)\ldots\hat{\psi}(\lambda_N))\, F(f)) = \frac{1}{Z[J]}\frac{\delta}{\delta J(\lambda_1)}\cdots\frac{\delta}{\delta J(\lambda_N)} Z[J]\Big|_{J=0} = \mathcal{F}(L)$$

Calculation of the effective action to second order

define the effective action through a Legendre transformation

$$S[\varphi(\lambda)] = -\ln Z[J] + \int d\lambda\ J(\lambda)\ \varphi(\lambda)$$

expanding in S and φ it can be shown that:

$$E(\hat{\psi}(\lambda)F(f)) = \frac{1}{Z[J]}\frac{\delta Z[J]}{\delta J(\lambda)}\bigg|_{J=0} = \varphi_0(\lambda)$$

= classical action averaged over fluctuations as a function of renormalization scale

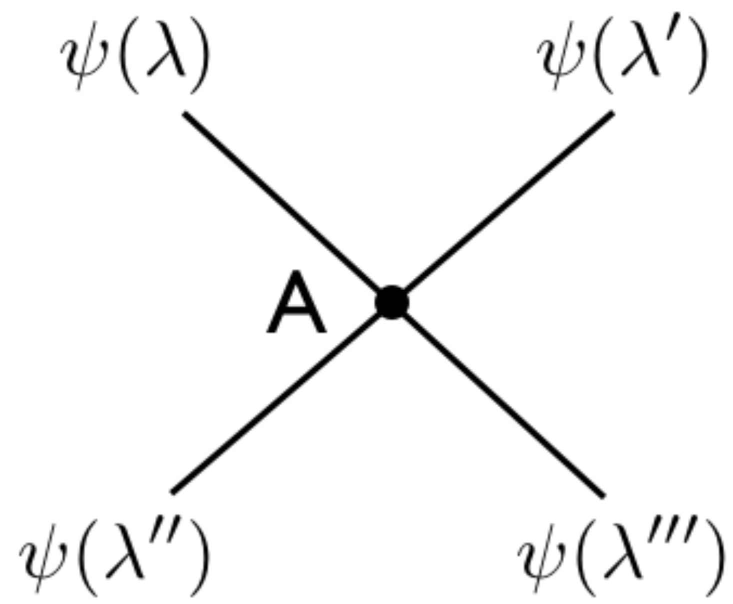
$$E(\hat{\psi}(\lambda)\hat{\psi}(\lambda')F(f)) = \frac{1}{Z[J]}\frac{\delta^2 Z[J]}{\delta J(\lambda)\delta J(\lambda')}\bigg|_{J=0}$$
$$= -S_2^{-1}(\lambda,\lambda')$$

= transfer matrix (scale dependent renormalization mass) as a function of initial and final renormalization scale

effective physics as a function of scale
physics averaged at that scale
running coupling constants
renormalization

notes: (1) 1/J is equivalent to $i\epsilon$ needed for convergence of Gaussian integrals, (2) modulus comes from evaluation of Gaussian integral via stationary phase

Example of phi^4 field theory



the action is:

$$S_0[f(x)] = \int dx\, \frac{1}{2}\left(\frac{df}{dx}\right)^2 - \frac{m^2}{2} f^2 - \frac{\beta}{4!} f^4$$

the transformed action is:

$$S_0[\psi(\lambda)] = \int d\lambda \frac{\lambda^2 - m^2}{2} \psi^2(\lambda) - \frac{\beta}{4!} \int d\lambda\, d\lambda'\, d\lambda''\, d\lambda''' \\ A(\lambda, \lambda', \lambda'', \lambda''')\, \psi(\lambda)\psi(\lambda')\psi(\lambda'')\psi(\lambda''')$$

some interesting moments

$$\frac{\delta^2 S_0[\psi(\lambda)]}{\delta\psi(\lambda)\delta\psi(\lambda')} = (\lambda^2 - m^2)\, \delta(\lambda - \lambda') - \frac{\beta}{2!} \int d\lambda''\, d\lambda'''$$

$$\frac{\delta^4 S_0[\psi(\lambda)]}{\delta\psi(\lambda)\delta\psi(\lambda')\delta\psi(\lambda'')\delta\psi(\lambda''')} = -\beta\, A(\lambda, \lambda', \lambda'', \lambda''')$$

$$\frac{\delta^5 S_0[\psi(\lambda)]}{\delta\psi(\lambda)\delta\psi(\lambda')\delta\psi(\lambda'')\delta\psi(\lambda''')\delta\psi(\lambda'''')} = 0$$

(1) because the interaction is of 4th order, S-matrix coefficients of greater than 4th order will be zero, (2) because there are only three running coupling constants in this theory

$$m(\lambda), \beta(\lambda), \text{and}\, f_0(\lambda)$$

the dimension of the S-matrix will be limited to $3 \times \dim(\mathbb{R})$

Why is S-matrix so simple and compact?

- simple form of physical actions
  - low order
  - limited number of terms and coupling constants and fields

Conclusion

It’s the physics

Geologic facies are physical phases

S-matrix (MST) is ultimate geologic metric

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