Asymmetric segregation velocities in granular breaking size segregation waves

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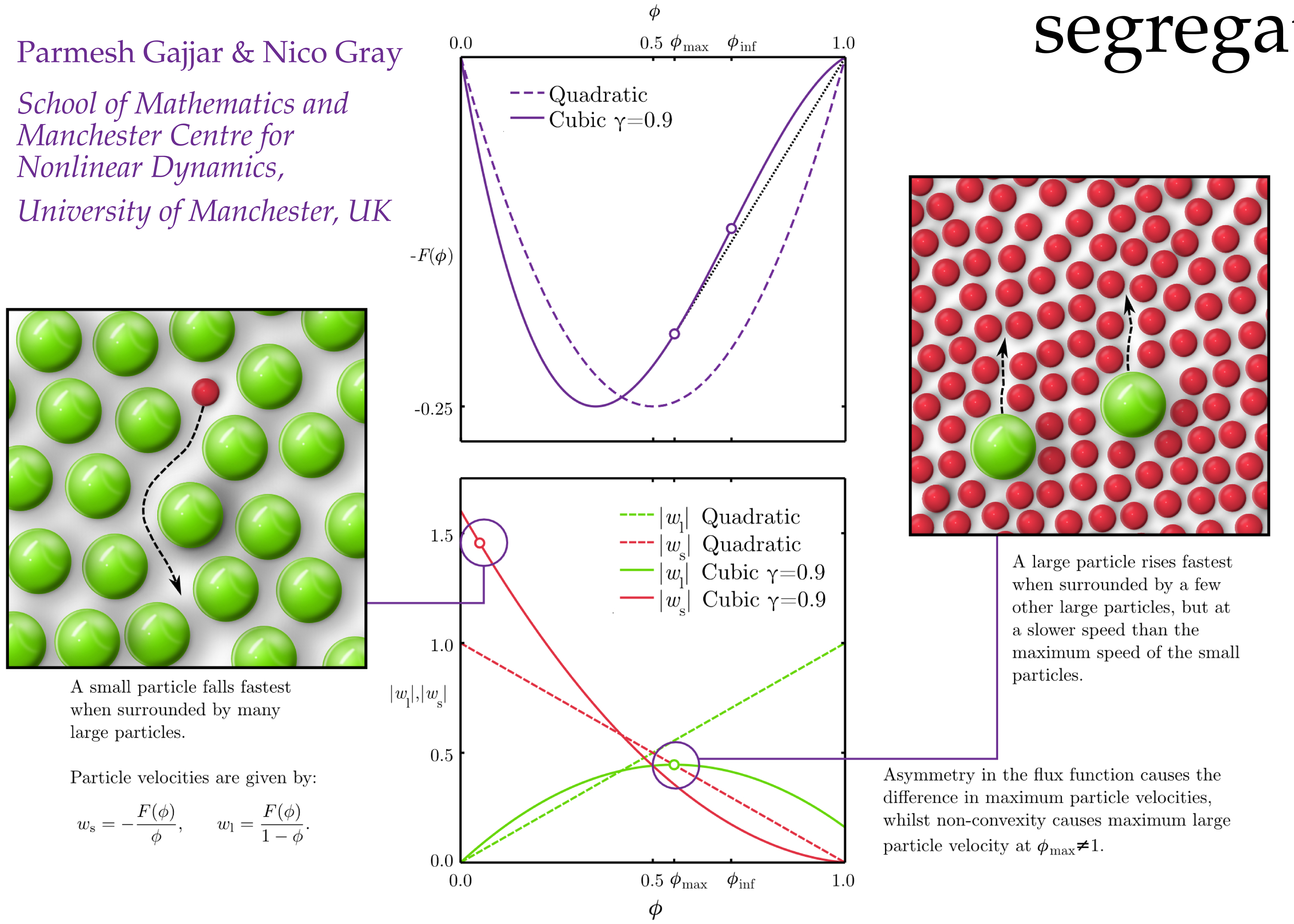

FIGURE 1: Asymmetric flux functions are needed to model asymmetric particle velocities (left, right). The cubic form is a simple asymmetric flux function (top) that gives asymmetric segregation velocities and also a maximum large particle velocity $|w_l|$ away from $\phi = 1$ (bottom). These are not found with the original quadratic model that is shown for comparison.

SIZE segregation is a common observation with granular materials, with small particles found at the bottom and large particles at the top. In a gravity driven shear flow, the small particles preferentially percolate through gaps and collect at the bottom, in a process known as kinetic sieving, whilst the resulting mass flux levers large particles upwards to the top, which is known as squeeze expulsion. A continuum model describing size segregation was proposed by Gray and Thornton (2005):

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x}(\phi u) + \frac{\partial}{\partial y}(\phi v) + \frac{\partial}{\partial z}(\phi w) - \frac{\partial}{\partial z}(S_r F(\phi)) = 0, \quad (1)$$

where ϕ is the small particle concentration, (u, v, w) is the bulk velocity, S_r is a non-dimensional segregation number and $F(\phi)$ is the segregation flux. Existing models use the quadratic flux $F(\phi) = \phi(1-\phi)$. However, this simple symmetric flux is unable to capture some of the physics.

Experimentally, it has been observed that the maximum segregation speeds of a small particle falling and large particle rising are different. A small particle falls fastest when surrounded by only large particles, and at a faster rate than the maximum speed of a large particle rising through a region of many small particles. In addition it has been observed that the large particle moves fastest when surrounded by a few other large particles.

Non-convex asymmetric flux functions are needed to model these observations, such as the simple cubic form

$$F(\phi) = A_\gamma \phi(1-\phi)(1-\gamma\phi), \qquad 0.5 \leqslant \gamma \leqslant 1.0, \quad (2)$$

where A_γ is a normalisation constant to give the same maximum amplitude as the quadratic flux.

The asymmetry about $\phi = 0.5$ leads to the asymmetry in the maximum segregation speeds, whilst the non convexity (presence of an inflexion point in the segregation flux function) leads to the maximum large particle velocity to occur when there are other large particles in proximity, i.e. $\phi_{max} \neq 1$ (figure 1).

In a gravity driven shear flow, the bulk downstream velocity increases with the height of the flow z. This can lead to small particles being sheared on top of large particles. Size segregation means that this configuration of small on top of large is dynamically unstable. The small particles would percolate downwards to a region of lower velocity, whilst the large particles would be levered upwards to a region of greater velocity. A breaking size segregation wave (Thornton and Gray 2008) forms in which large and small particles are recirculated. The wave propagates downstream with the flow. An exact solution for the structure of the steady breaking size segregation wave may be derived using the method of characteristics in a frame moving with the speed of the wave.

The asymmetric particle velocities cause an additional tail to the original 'lens-like' structure of the quadratic flux (figure 2). Most large particles recirculate quickly in the lens, but several large particles circulate more slowly at the rear. In addition a sharp concentration shock is formed at the upper left portion of the lens, where small particles fall rapidly through the region of large particles. The rapid falling of the small particles means that the lens contains a higher concentration of large particles. The upper point of the lens x_b is also shifted to the right as a result of the slower motion of the large particles.

Further work is currently being undertaken to validate the theoretical model against experimental data.

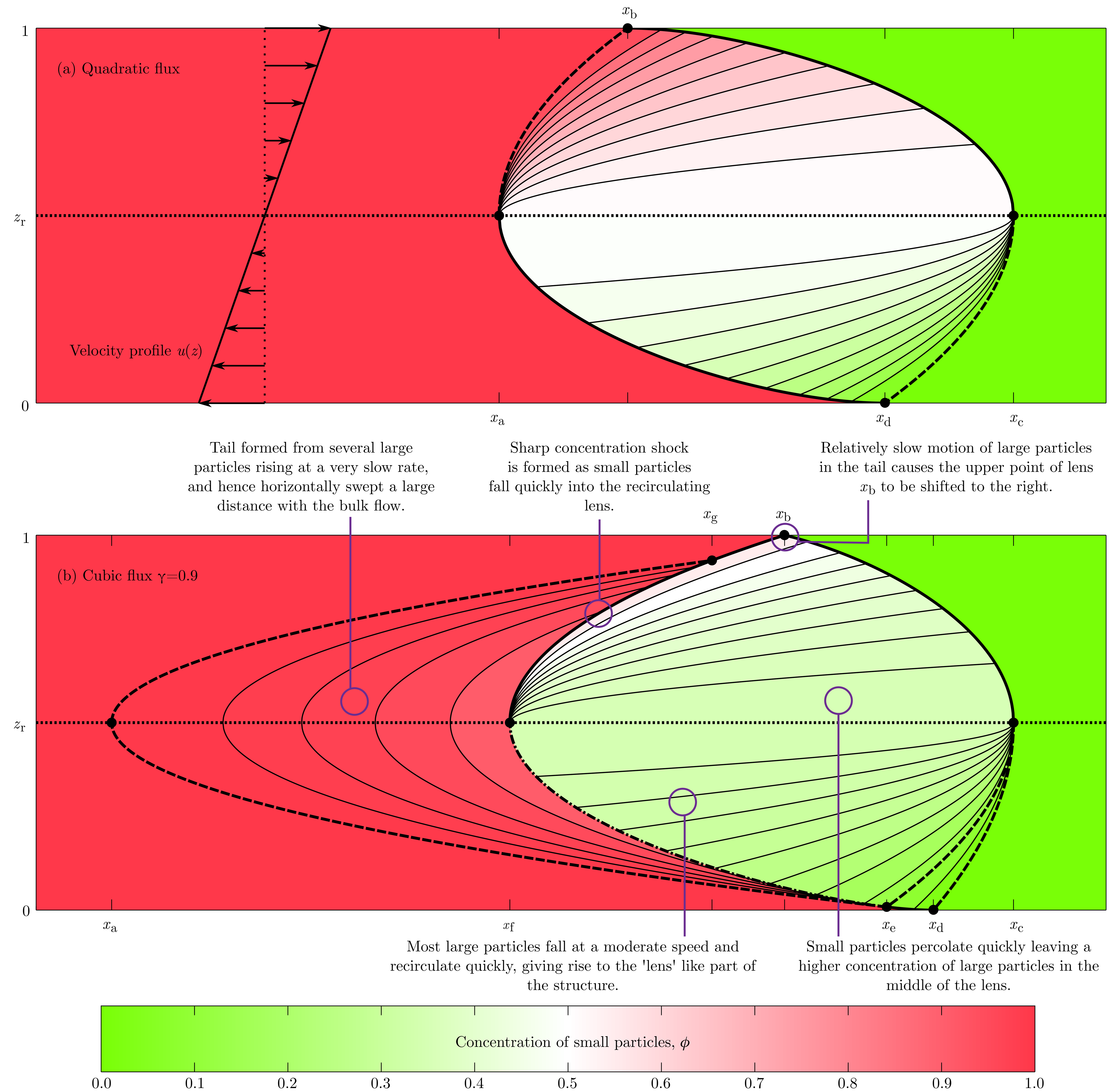

FIGURE 2: An exact solution for the structure of the steady breaking wave may be derived in a frame moving with the speed of the wave, with the transformed velocity profile $(u(z), 0, 0)$ sketched in (a). The segregation equation (1) reduces to a scalar hyperbolic equation, which may be solved by the method of characteristics. Shocks are shown with bold solid lines, one-sided discontinuities with bold dash-dot lines and the edges of rarefaction fans are shown with bold dashed lines. Characteristics are shown with thin solid lines. The concentration field for the quadratic flux is shown in (a), whilst the concentration field for the cubic flux (2) ($\gamma = 0.9$) is shown in (b). New features of the asymmetry model are annotated.

- Breaking size segregation waves in granular avalanches.
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- Breaking size-segregation waves and particle recirculation in granular avalanches.
  Thornton A.R. & Gray J.M.N.T. (2008) J. Fluid Mech. **596**, 261–284.
- A theory for particle size segregation in shallow granular free-surface flows.
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