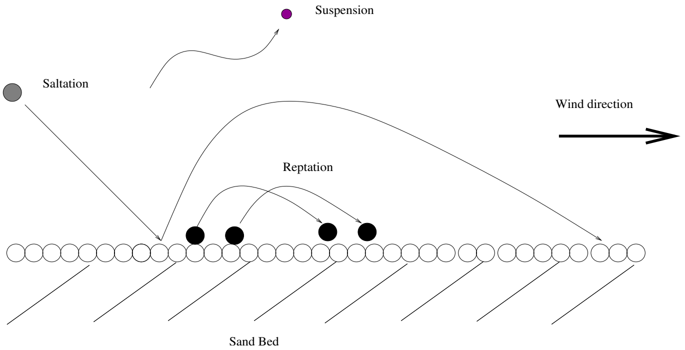
Periodic Trajectories in Aeolian Sand Transport

Alexandre Valance[1], and Jim Jenkins[2]

[1] Institut de Physique de Rennes, Université de Rennes 1, France
[2] Cornell University, Ithaca, USA

Particle Laden flows in Nature, December 15-19, 2013

Saltation transport (Bagnold, 1941)

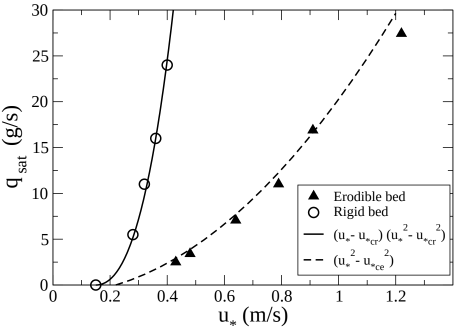


- Particle diameter and density :
  $0.1\ mm < d < 0.6\ mm$ and $\sigma = \rho_p/\rho_{air} = 2200$
- Particle Reynolds and Shields number :
  $1 < Re_p(= d\sqrt{gd}/\nu) < 10$ and
  $0.01 < S(= u^{*2}/\sigma gd) < 0.2$

Crucial role of the bed : Erodible vs Rigid bed

- Wind-tunnel experiments (Ho et al, PRL 11)

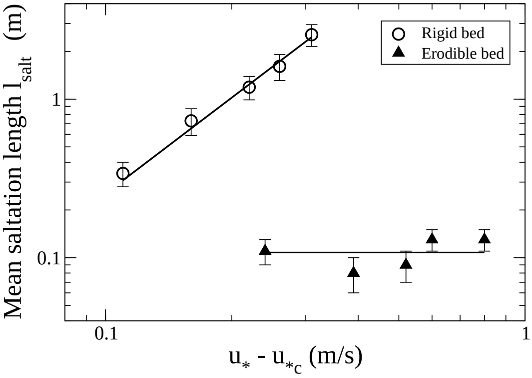
Maximum capacity of transport : Erodible bed vs Rigid bed



- Erodible bed : $Q_{sat} \propto (u^{*2} - u_c^{*2}) \propto (S^* - S_c)$
- Rigid bed :
  $Q_{sat} \propto (u^* - u_c^*)(u^{*2} - u_c^{*2}) \propto (\sqrt{S^*} - \sqrt{S_c})(S^* - S_c)$

Crucial role of the bed : Erodible vs Rigid bed

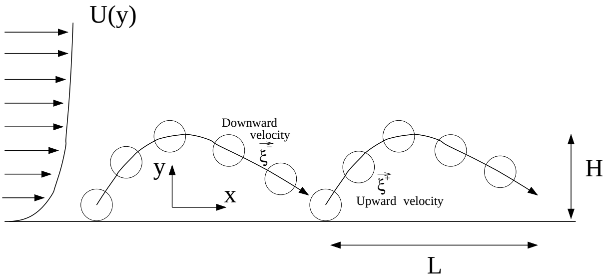
- Mean saltation length : Erodible vs Rigid bed



- Erodible bed : $l_{salt} \approx cst$
- Rigid bed : $l_{salt} \propto (u^{*2} - u_c^{*2}) \propto (S^* - S_c)$

Motivation

- Can Saltation transport be simply described in terms of periodic trajectories ?



- We developed a simple Eulerian/Lagragian approach
- We investigated firt the transport over a rigid bed

Particle Motion

Ascending Motion

$$a_x^+ = \xi_y^+ \frac{d\xi_x^+}{dy} = D^+(U - \xi_x^+)\,, \tag{1}$$

$$a_y^+ = \xi_y^+ \frac{d\xi_y^+}{dy} = -D^+\xi_y^+ - 1\,, \tag{2}$$

Descending Motion

$$a_x^- = \xi_y^- \frac{d\xi_x^-}{dy} = D^-(U - \xi_x^-)\,, \tag{3}$$

$$a_y^- = \xi_y^- \frac{d\xi_y^-}{dy} = -D^-\xi_y^- - 1\,, \tag{4}$$

Drag coefficient : $D = (0.3\sqrt{(U-\xi_x)^2 + \xi_y^2} + 18/R)/\sigma$

Particle Reynolds number : $R = \rho^f d(gd)^{1/2}/\mu^f$.

Density Ratio : $\sigma = \rho^s/\rho^f$

Particle Motion

Downstream position x^+ and x^- are functions of y

$$\xi_y^+ \frac{dx^+}{dy} = \xi_x^+ , \tag{5}$$

$$\xi_y^- \frac{dx^-}{dy} = \xi_x^- . \tag{6}$$

Fluid Motion

Steady and fully developped turbulent boundary layer

- Clear flow : $S_{air}(y) = constant = S^*$

$$\frac{dU}{dy} = \frac{[S^* \sigma]^{1/2}\,\sigma}{\kappa y}$$

(Classical Mixing length turbulence model)

- Particle laden flow : $S_{air}(y) + s_{particle}(y) = constant = S^*$

$$\frac{dU}{dy} = \frac{[(S^* - s)\sigma]^{1/2}\,\sigma}{\kappa y} \tag{7}$$

Particle Shear Stress

We consider a saltation transport state consisting in an ensemble of particles with periodic trajectories

We define c^+ and c^- respectively as the concentration of ascending and descending particles

- Particle average velocity :

$$u(y) = (c^+\xi_x^+ + c^-\xi_x^-)/(c^+ + c^-)$$

- Particle shear stress :

$$\begin{aligned} s(y) = -\left(c^+\xi_y^+\xi_x^+ + c^-\xi_y^-\xi_x^-\right) &= -c^+\xi_y^+\left(\xi_x^+ - \xi_x^-\right) \\ &= -\phi_0^+\left(\xi_x^+ - \xi_x^-\right) \end{aligned}$$

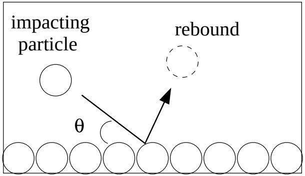
Two-point boundary value problem

- Seven first order differential equations for :
  ξ_x^+,ξ_x^-, ξ_y^+, ξ_y^-, x^+, x^- and U
  to be resolved from $y = 0$ to $y = H$.
- Three additional unknown parameters :
  - the heigth H and the length L of the trajectory
  - the upward vertical mass flux at the bed ϕ_0^+

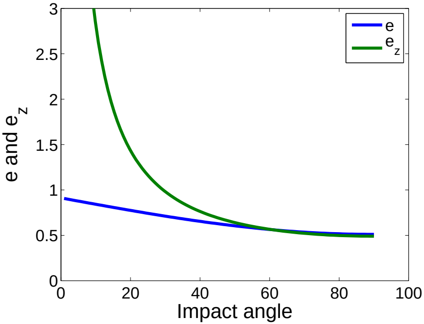
We need 10 **boundary conditions but continuity relations provide only 7 conditions :**

- $\xi_x^+(H) = \xi_x^-(H)$, $\xi_y^+(H) = 0$ and $\xi_y^-(H) = 0$
- $x^+(0) = 0$, $x^-(0) = L$ and $x^+(H) = x^-(H)$
- $U(0) = 0$

Rebound on a rigid and bumpy surface



Rigid and bumpy bed



$$\begin{aligned}\xi^+(0) &= e(\theta)\,\xi^-(0)\\ &= (A - B\sin\theta)\xi^-(0)\\ \xi_y^+(0) &= -e_y(\theta)\,\xi_y^-(0)\\ &= -(A_y/\sin\theta - B_y)\xi_y^-(0)\end{aligned}$$

$A = 0.9$, $B = 0.4$, $A_y = 0.5$ and $B_y = 0$ are constants drawn from D.E.M simulation ($e_n = 0.8$)

Additional boundary conditions

- Two addition conditions :
  $\xi^+(0) = e(\theta)\,\xi^-(0)$

  $\xi_y^+(0) = -e_y(\theta)\,\xi_y^-(0)$

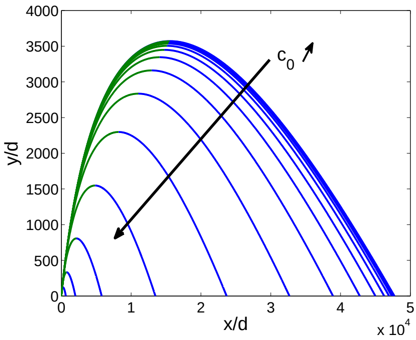
- We have now $(7+2)$ boundary conditions for 7 first order differential equations + 3 unknown parameters

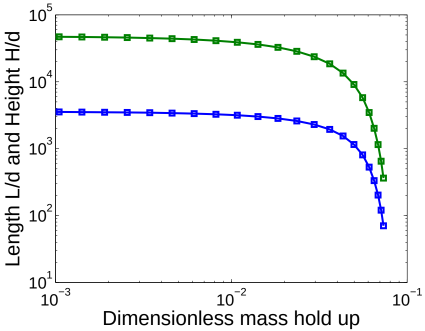
- There is therefore one free parameter (the vertical mass flux ϕ_0^+) which should be prescribed

Particle trajectory

Particle parameter : $d = 0.23\,mm$, $\sigma = 2200$ and $Re_p = 0.73$
Fixed Shields number : $S^* = 0.06$

- Influence of the mass holdup $m = \int_0^H c(y)dy$



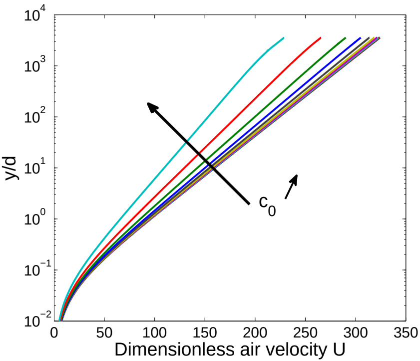


Smaller trajectories with increasing mass holdup

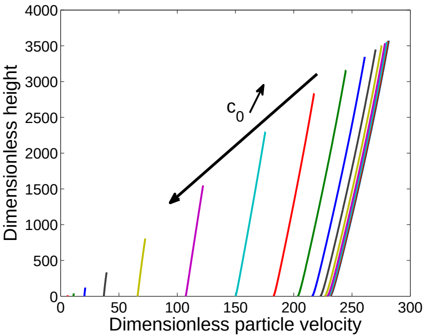
Air and particle velocity

- Influence of the mass holdup at a fixed Shields number : $S = 0.06$

Air flow profile

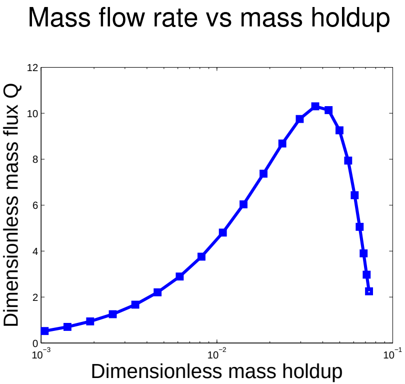


Particle velocity profile



Mass flow rate

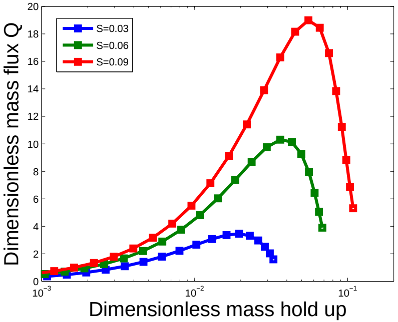
- Influence of the mass holdup at a fixed Shields number : $S = 0.06$



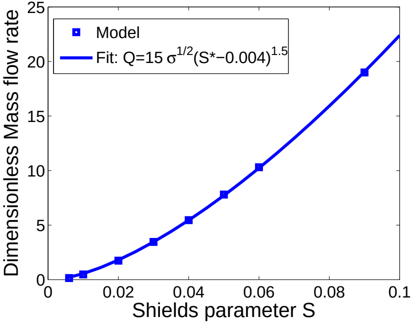
Mass flow rate

- Influence of the Shields number

Mass flow rate vs mass holdup for different Shields numbers



Maximum capacity of transport vs Shields



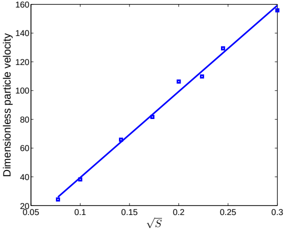
Features of the flow at maximum capacity

$$Q = \int c(y)u(y)dy \approx c_0\ u_0\ H$$

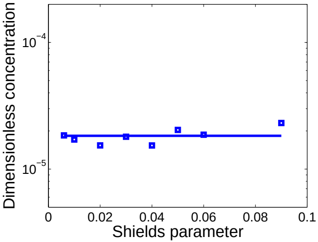
- $c_0 \approx \mathit{Invariant}$
- $u_0 \propto \sqrt{S} - \sqrt{S_c}$
- $H \propto (S - S_c)$

$$\Rightarrow Q \propto (\sqrt{S} - \sqrt{S_c})\ (S - S_c)$$

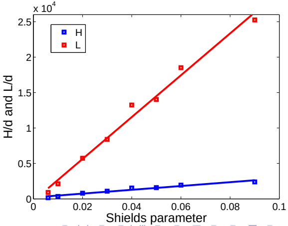
Particle velocity u_0 vs $\sqrt{S}$



Particle concentration c_0 vs S



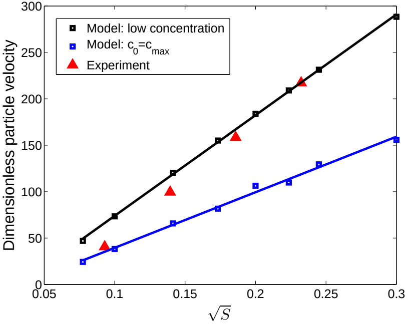
Height and lenght vs S



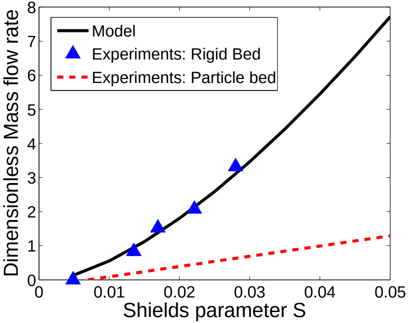
Comparison with Experiments

- Wind-tunnel experiments on rigid and bumby bed (Ho, Ph Thesis 2012, Rennes) :
  Particle parameters : $d = 0.230\ mm$ and $\sigma = 2200$

Particle velocity vs Shields



Maximum capacity of transport vs Shields

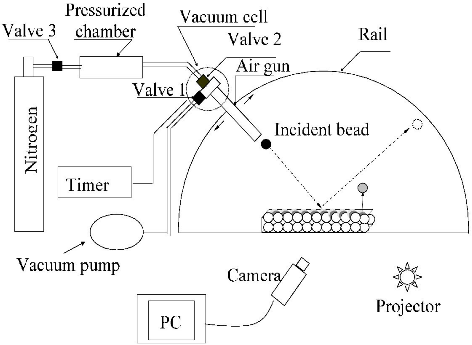


Extension of the Model

- Can we apply this simple model for saltation transport over an erodible bed ?
- How to account for the splash process ?

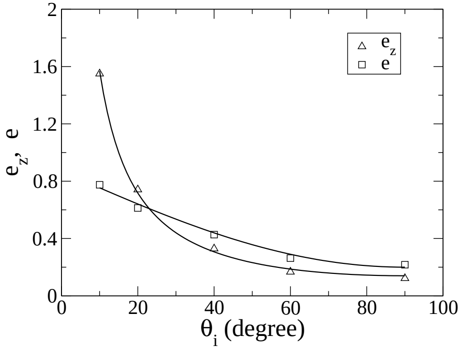
Splash Process

- Model Collision Experiment (Beladjine et al, PRE 2006)



Splash Process : Rebound

Experimental outcomes *(Beladjine et al, PRE 2007)*



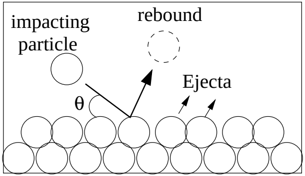
Rebound law over an erodible bed :

$$\begin{aligned} \xi^+(0) &= e(\theta)\,\xi^-(0) \\ &= (A - B\sin\theta)\xi^-(0) \\ \xi_y^+(0) &= -e_y(\theta)\,\xi_y^-(0) \\ &= -(A_y/\sin\theta - B_y)\xi_y^-(0) \end{aligned}$$

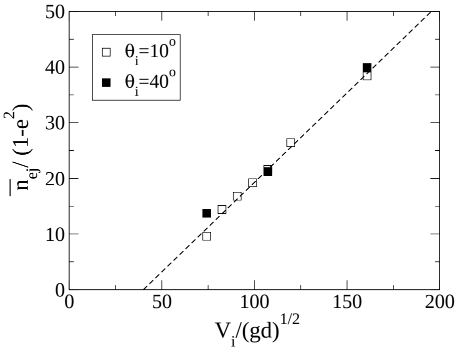
$A = 0.87$, $B = 0.72$, $A_y = 0.3$ and $B_y = 0.15$ are constants drawn from experiments

Splash process : Ejected particles

- Experimental Outcomes *(Beladjine et al, PRE 2007)*



$$N_{tot} = N_{rebound} + N_{ej}$$



$$N_{tot}(\xi) = \begin{cases} 1 + N_0(1-e^2)\,(\xi/\xi_c - 1) & \text{if } \xi > \xi_c \\ 1 & \text{if } 1 \le \xi \le \xi_c \\ 0 & \text{if } \xi \le 1 \end{cases}$$

Consequences

- Vertical mass flux at the bed :

$$\phi = \phi^+ + \phi^- = \int (1 - N_{tot})\xi_y^- f(\xi_x^-, \xi_y^-) d\xi_x^- d\xi_y^-$$

- Steady condition : $\phi = 0$
  - If $f = \delta(\xi_x - u_0, \xi_y)$, $\phi = 0 \Rightarrow 1 < u_0 < \xi_c$
  - If $f = f_{Gaussian}(\xi_x - u_0) \times f_{Gaussian}(\xi_y)$, $\phi = 0 \Rightarrow u_0 \approx \xi_c/2$ (Creyssels et al, 2007)

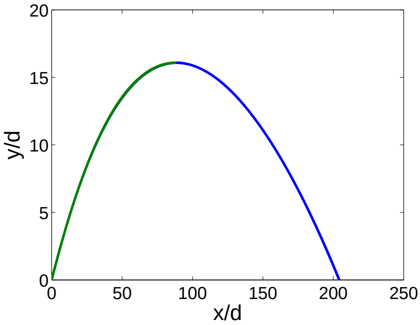
Additional boundary condition

$\Rightarrow$ **Two-point boundary value problem without any free parameter**

Periodic trajectories

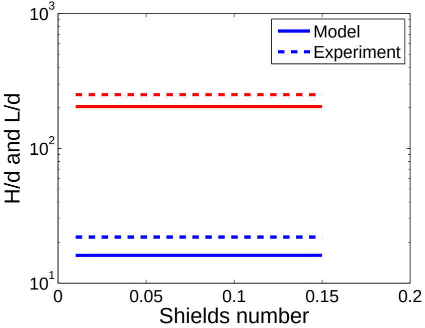
Particle parameters : $\sigma = 2200$ and $d = 0.23\ mm$

- Saltation trajectory



Trajectory invariant with Shields

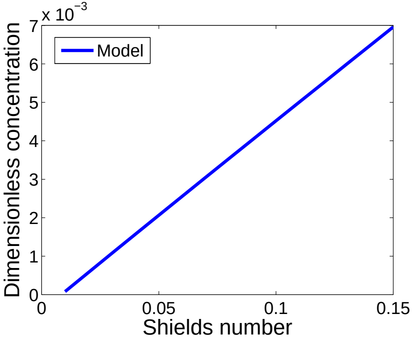
- Saltation Height and Length



H and *L* invariant with Shields

Particle concentration and mass flux

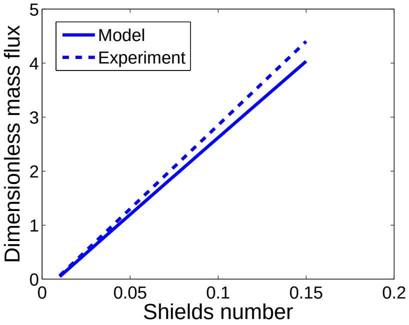
- Concentration at the bed



$$c_0 \propto (S^* - S_c)$$

- Mass flux Q

(Experiments : Creyssels et al, JFM, 2009)



$$Q \propto (S^* - S_c)$$

Conclusion and Perspective

- Conclusion
  - Simple predictive model for saltation transport over erodible and rigid bed
- Perspective
  - Model applicable to bed load transport in water
  - Possible extension of the model to unsteady and inhomogenous situations